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Galaxy Formation. Advanced Models

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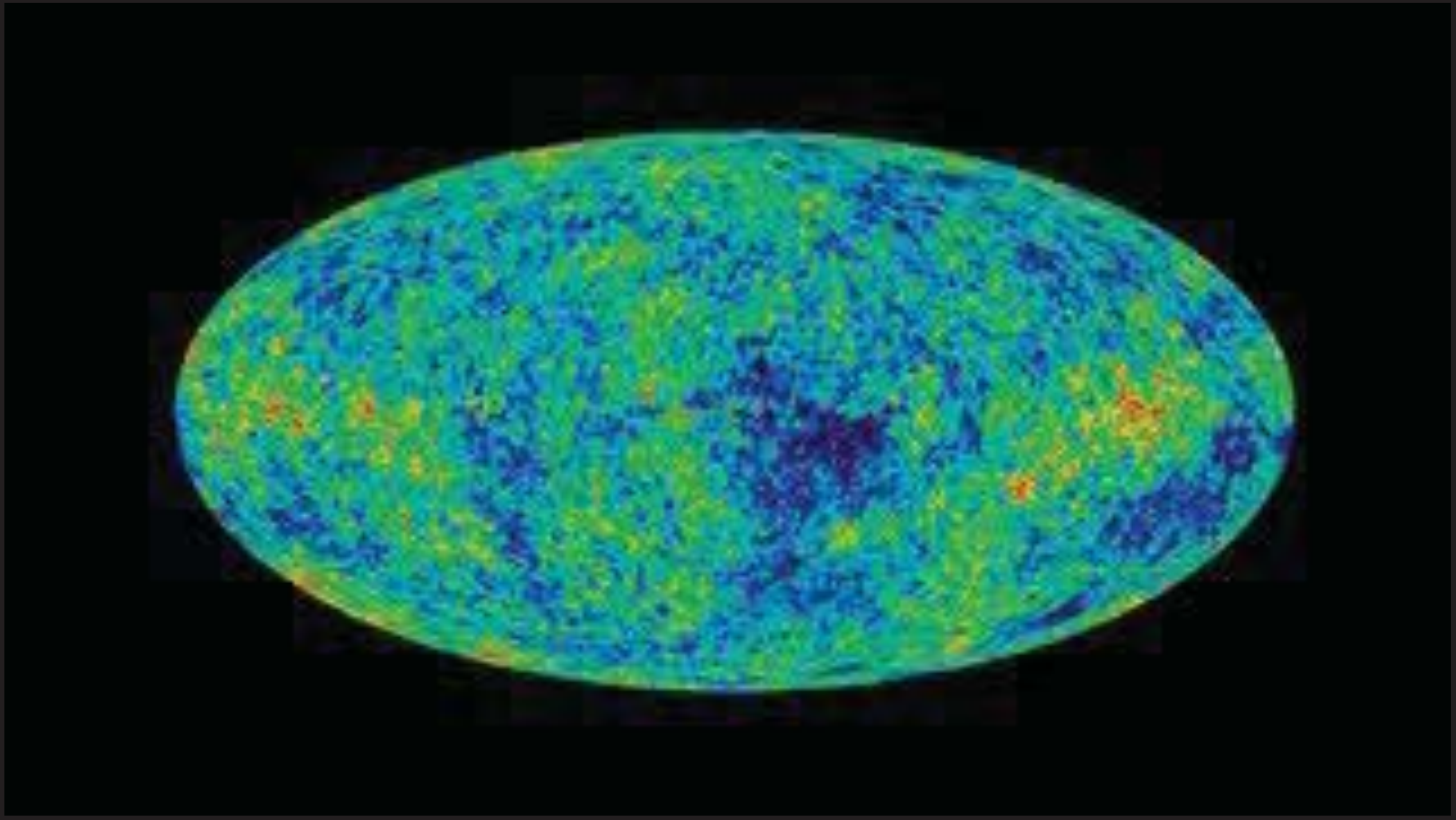
Italy

Galaxy formation

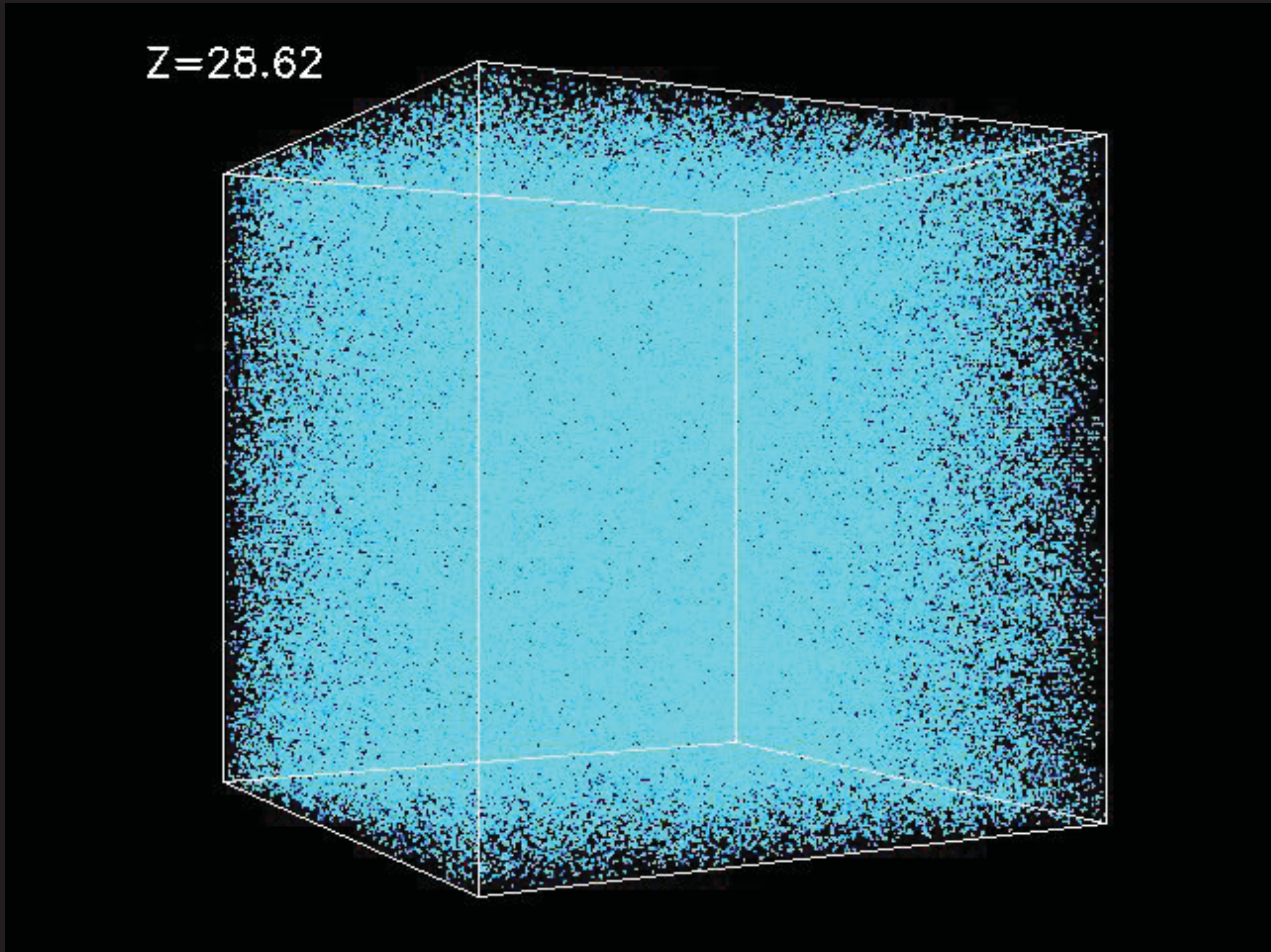
1. Hierarchical clustering
2. Galaxies

Ravi K Sheth (ICTP/Upenn)

Early Universe



+ Structure Formation

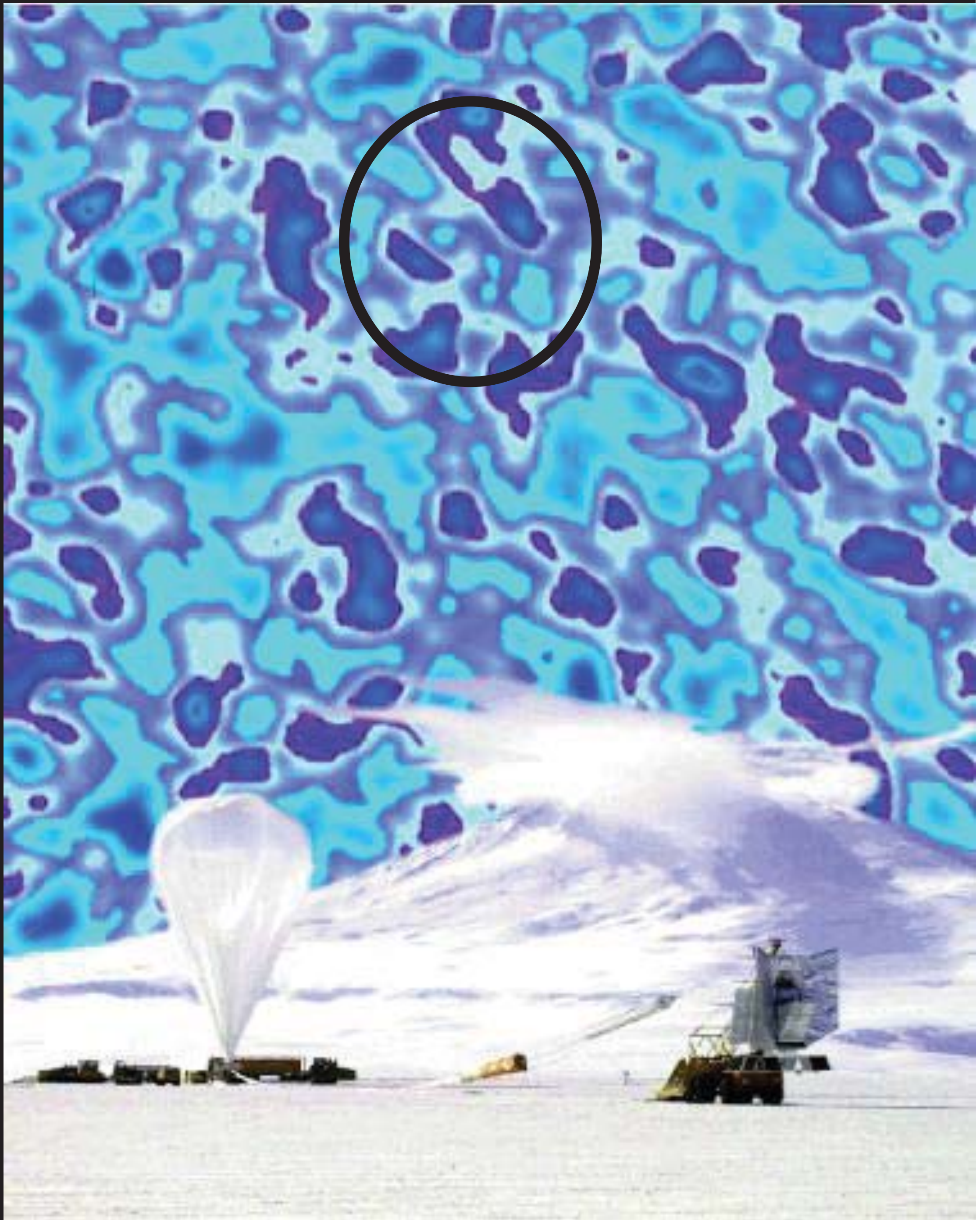


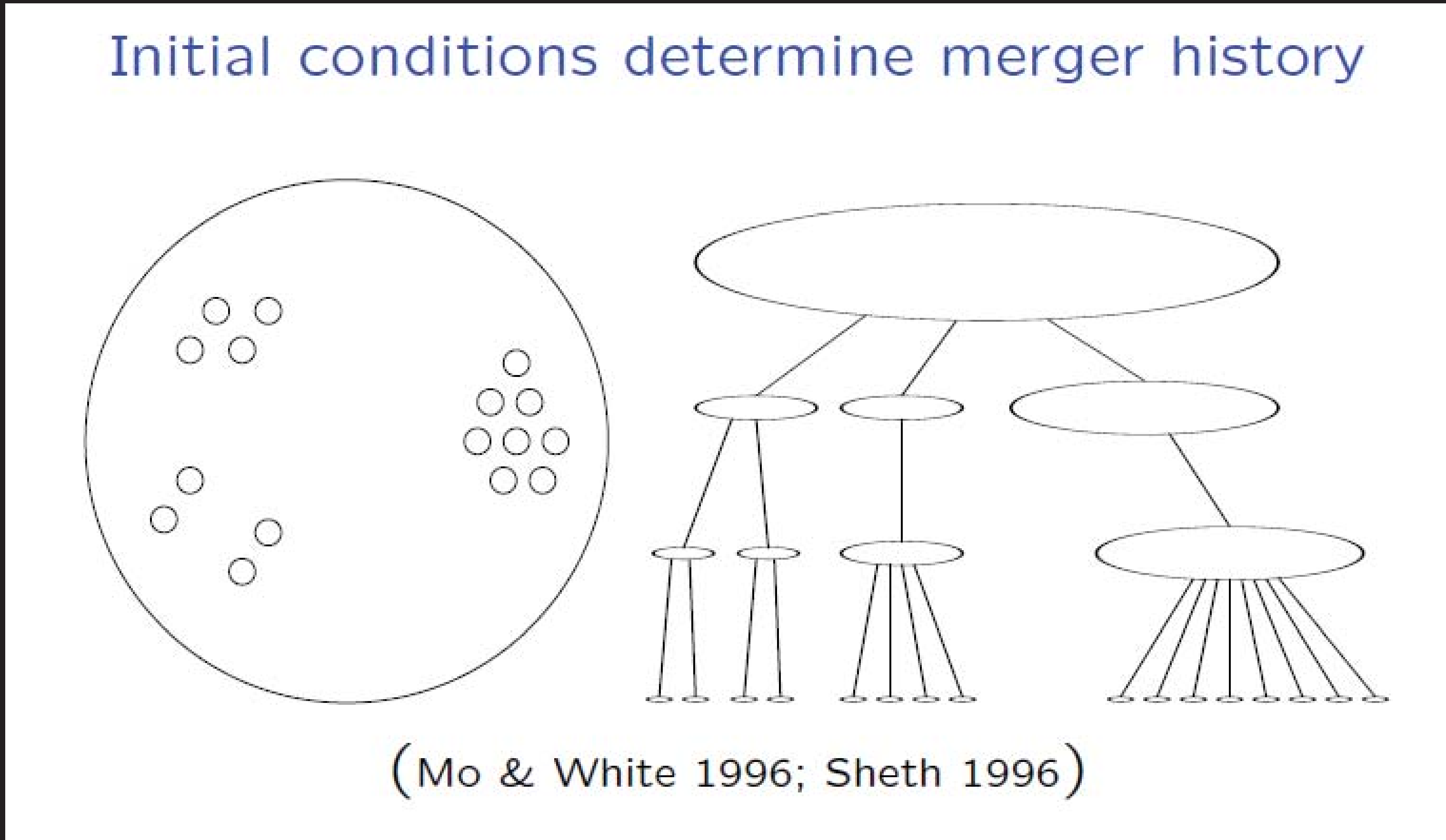
In Cold Dark Matter models, mass doesn’t move very far

The Cosmic Background Radiation

Cold: 2.725 K
Smooth: 10^{-5}
Simple physics

Gaussian fluctuations
= seeds of subsequent structure formation
= simple(r) math
Logic which follows is general





Birkhoff's theorem important

Use initial conditions (CMB)

+

model of nonlinear

gravitational clustering

to make inferences about

late-time, nonlinear structures

N-body simulations of

gravitational clustering

in an expanding universe

Assume a spherical cow …

(Gunn & Gott 1972)

Spherical evolution model

$$d^2R/dt^2 = -GM/R^2 + \Lambda\, R$$
$$= -\rho\,(4\pi G/3H^2)\, H^2 R + \Lambda\, R$$
$$= -\tfrac{1}{2}\,\Omega(t) H(t)^2 R + \Lambda\, R$$

- Note: currently fashionable to modify gravity. Should we care that only $1/R^2$ or R give stable circular orbits?

Spherical evolution model

- Initially, $E_i = -GM/R_i + (H_iR_i)^2/2$
- Shells remain concentric as object evolves; if denser than background, object pulls itself together as background expands around it
- At 'turnaround': $E = -GM/r_{\max} = E_i$
- So $-GM/r_{\max} = -GM/R_i + (H_iR_i)^2/2$
- Hence $(R_i/r) = 1 - H_i^2R_i^3/2GM$

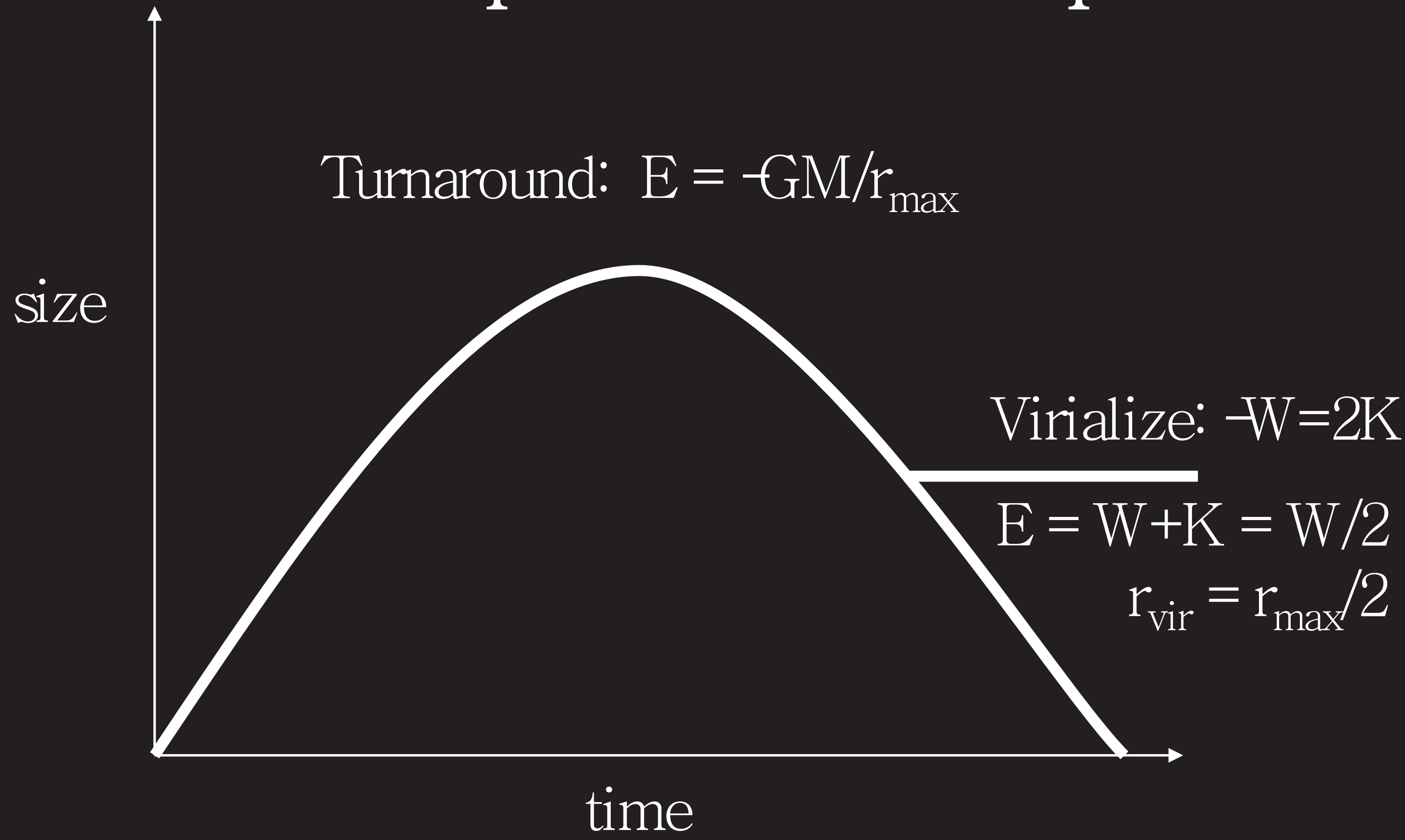
$$= 1 - (3H_i^2/8\pi G)\,(4\pi R_i^3/3)/M$$

$$= 1 - 1/(1+\Delta_i) = \Delta_i/(1+\Delta_i) \approx \Delta_i$$

Virialization

- Final object virializes: $-W = 2K$
- $E_{vir} = W+K = W/2 = -GM/2r_{vir} = -GM/r_{max}$
  - so $r_{vir} = r_{max}/2$:
- Ratio of initial to final size = (density)$^{1/3}$
  - final density determined by initial overdensity
- To form an object at present time, must have had a critical overdensity initially
- Critical density ↔ Critical scale-length
- To form objects at high redshift, must have been even more overdense initially

Spherical collapse



Modify gravity → modify collapse

Exact Parametric Solution

(R_i/R) vs. θ and (t/t_i) vs. θ

very well approximated by…

$$(R_{\rm initial}/R)^3$$
$$= \text{Mass}/(\rho_{\rm com}\text{Volume})$$
$$= 1 + \delta(t) \approx (1 - D_{\rm Lin}(t)\delta_{\rm init}/\delta_{\rm sc})^{-\delta sc}$$

Dependence on cosmology from $\delta_{sc}(\Omega,\Lambda)$, but this is rather weak

$$1 + \delta \approx (1 - \delta_{Linear}/\delta_{sc})^{-\delta_{sc}}$$

- As $\delta_{Linear} \rightarrow \delta_{sc}$, $\delta \rightarrow$ infinity
  - This is virialization limit
- As $\delta_{Linear} \rightarrow 0$, $\delta \approx \delta_{Linear}$
- If $\delta_{Linear} = 0$ then $\delta = 0$
  - This does not happen in modified gravity models where $D(t) \rightarrow D(k,t)$!
  - Related to loss of Birkhoff's theorem when r^{-2} lost?

Virial Motions

- $(R_i/r_{vir}) \sim f(\Delta_i)$: ratio of initial and final sizes depends on initial overdensity
- Mass $M \sim R_i^3$ (since initial overdensity « 1)
- So final virial density $\sim M/r_{vir}^3 \sim (R_i/r_{vir})^3$ ~function of critical density: Hence, all virialized objects have the same density, $\Delta_{vir}\, \rho_{crit}(z)$, whatever their mass
- $V^2 \sim GM/r_{vir} \sim (Hr_{vir})^2 \Delta_{vir} \sim (HGM/V^2)^2\, \Delta_{vir} \sim (HM)^{2/3}$: massive objects have larger internal velocities or temperatures; H decreases with time, so, for a given mass, virial motions (or temperature) higher at high z

Open questions

- Virial density scales with background or critical density?
  - In ΛCDM, critical seems more reasonable
  - Can address by running simulations beyond present epoch!
- Tri-axial collapse from initially spherical or tri-axial patches?
  - How best to incorporate tidal effects?
  - What is equivalent of virial size?
  - Predicting final axial ratios is tough problem

Spherical collapse with DM + DE + νs!

THE EXCURSION SET APPROACH

Halo abundances: Epstein (1983); Bond et al. (1991)
Hale mergers/formation: Lacey & Cole (1993)
Clustering/environment: Mo & White (1996)
Counts-in-cells: Sheth (1998); Lam & Sheth (2008)
Voids: Sheth & van de Weygaert (2004); Paranjape et al. (2011)
Filaments and sheets: Shen et al. (2006)

THE ASTROPHYSICAL JOURNAL, 379:440–460, 1991 October 1

EXCURSION SET MASS FUNCTIONS FOR HIERARCHICAL GAUSSIAN FLUCTUATIONS

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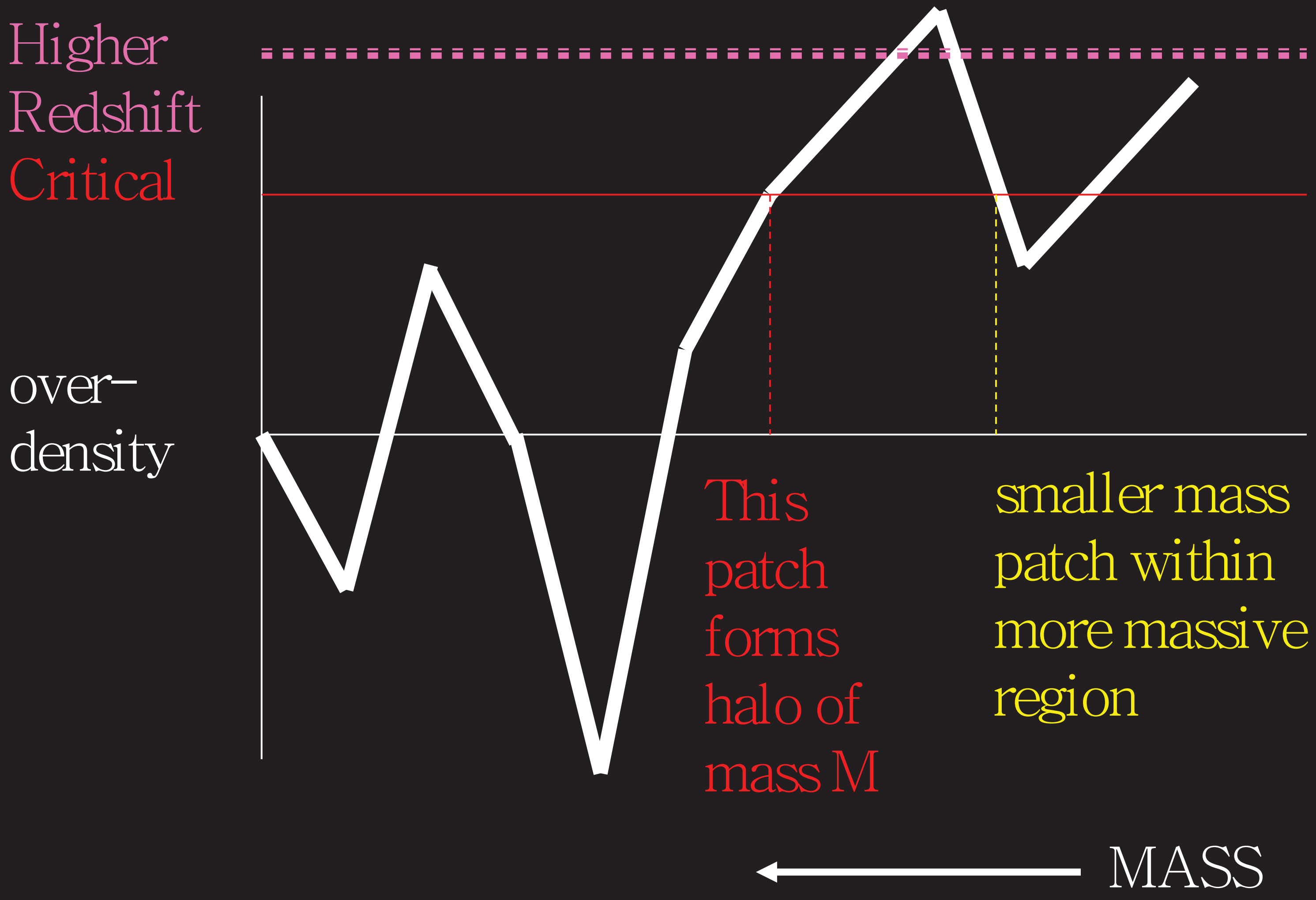
Received 1990 July 23; accepted 1990 December 28

ABSTRACT

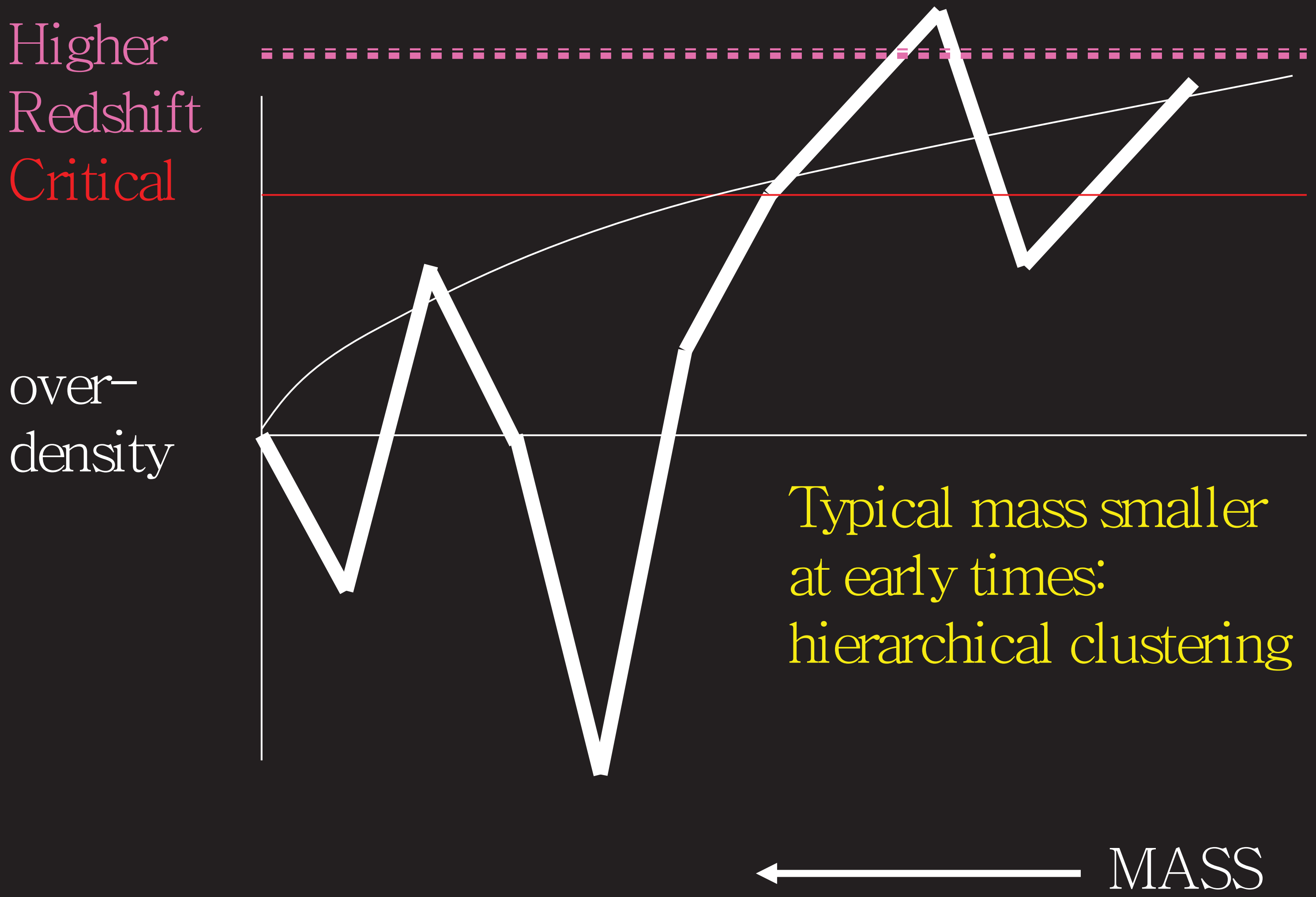
Most schemes for determining the mass function of virialized objects from the statistics of the initial density perturbation field suffer from the "cloud-in-cloud" problem of miscounting the number of low-mass clumps, many of which would have been subsumed into larger objects. We propose a solution based on the theory of the excursion sets of $F(\mathbf{r}, R_f)$, the four-dimensional initial density perturbation field smoothed with a continuous hierarchy of filters of radii R_f. We identify the mass fraction of matter in virialized objects with mass greater than M with the fraction of space in which the initial density contrast lies above a critical overdensity when smoothed on some filter of radius greater than or equal to $R_f(M)$. The differential mass function is then given by the rate of first upcrossings of the critical overdensity level as one decreases R_f at constant position $\mathbf{r}$. The shape of the mass function depends on the choice of filter function. The simplest case is "sharp k-space" filtering, in which the field performs a Brownian random walk as the resolution changes. The first upcrossing rate can be calculated analytically and results in a mass function identical to the formula of Press and Schechter—complete with their normalizing "fudge factor" of 2. For general filters (e.g., Gaussian or "top hat") no analogous analytical result seems possible, though we derive useful analytical upper and lower bounds. For these cases, the mass function can be calculated by generating an ensemble of field trajectories numerically. We compare the results of these calculations with group catalogs found from N-body simulations. Compared to the sharp k-space result, less spatially extended filter functions give fewer large-mass and more small-mass objects. Over the limited mass range probed by the N-body simulations, these differences in the predicted abundances are less than a factor of 2 and span the values found in the simulations. Thus the mass functions for sharp k-space and more general filtering all fit the N-body results reasonably well. None of the filter functions is particularly successful in identifying the particles which form low-mass groups in the N-body simulations, illustrating the limitations of the excursion set approach. We have extended these calculations to compute the evolution of the mass function in regions that are constrained to lie within clusters or underdensities at the present epoch. These predictions agree well with N-body results, although the sharp k-space result is slightly preferred over the Gaussian or top hat results.

Subject headings: cosmology — galaxies: clustering — numerical methods

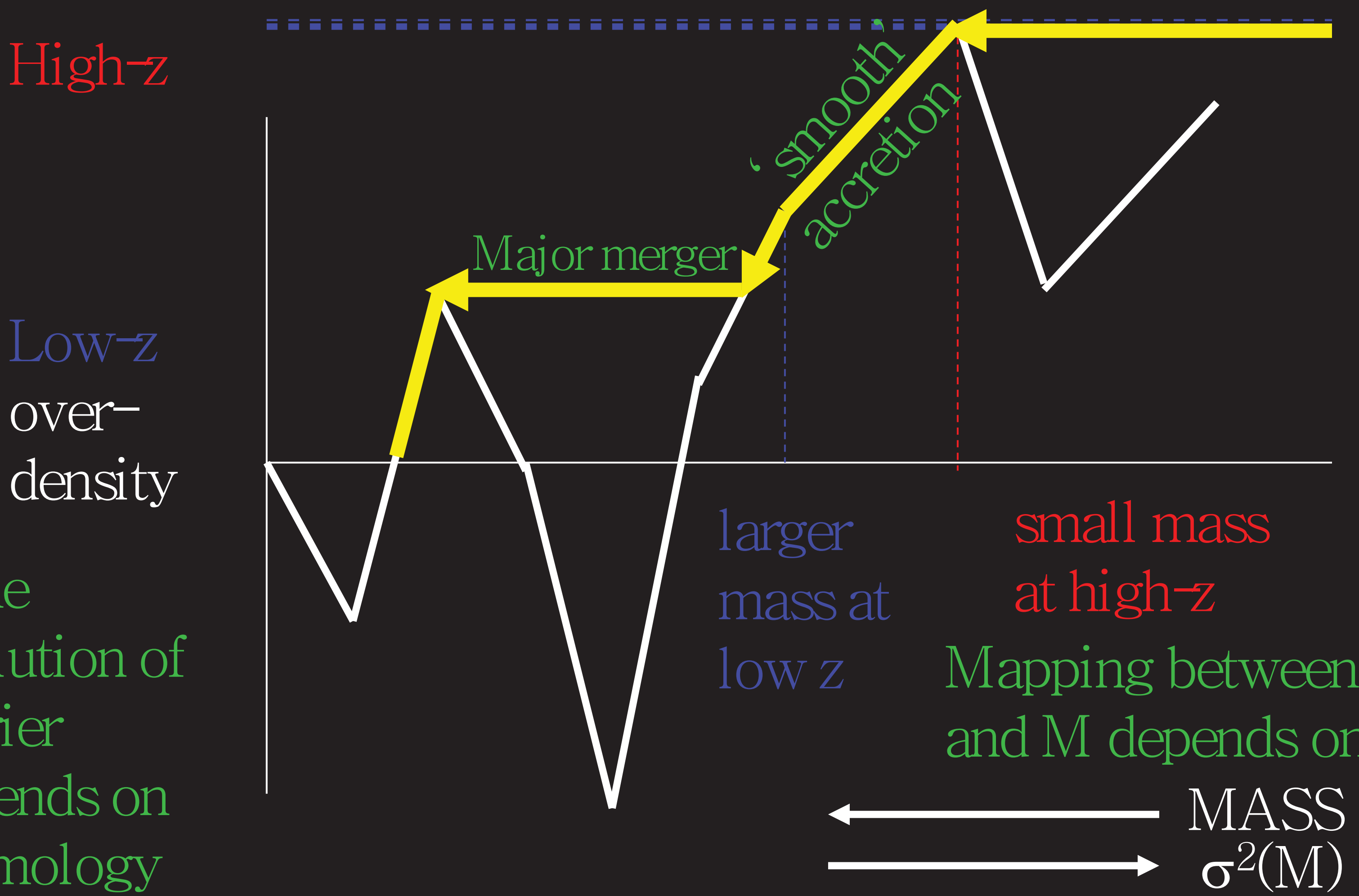
The Random Walk Model



The Random Walk Model



The excursion set approach



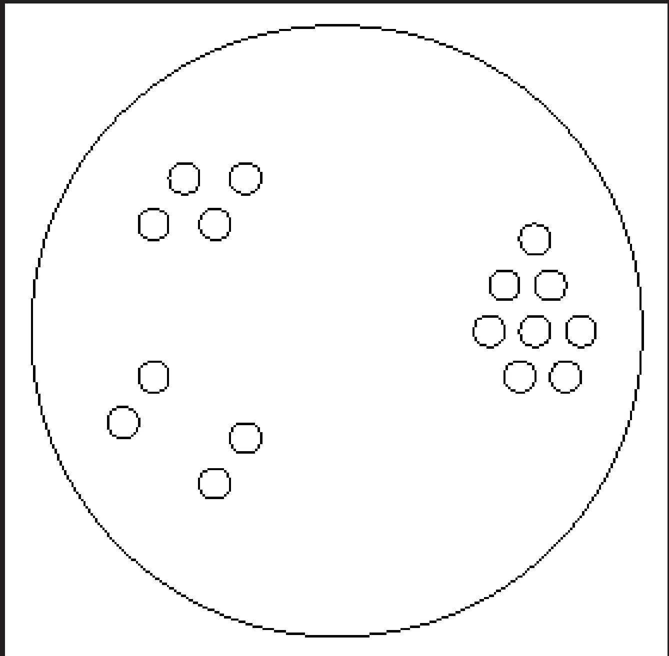
Time evolution of barrier depends on cosmology

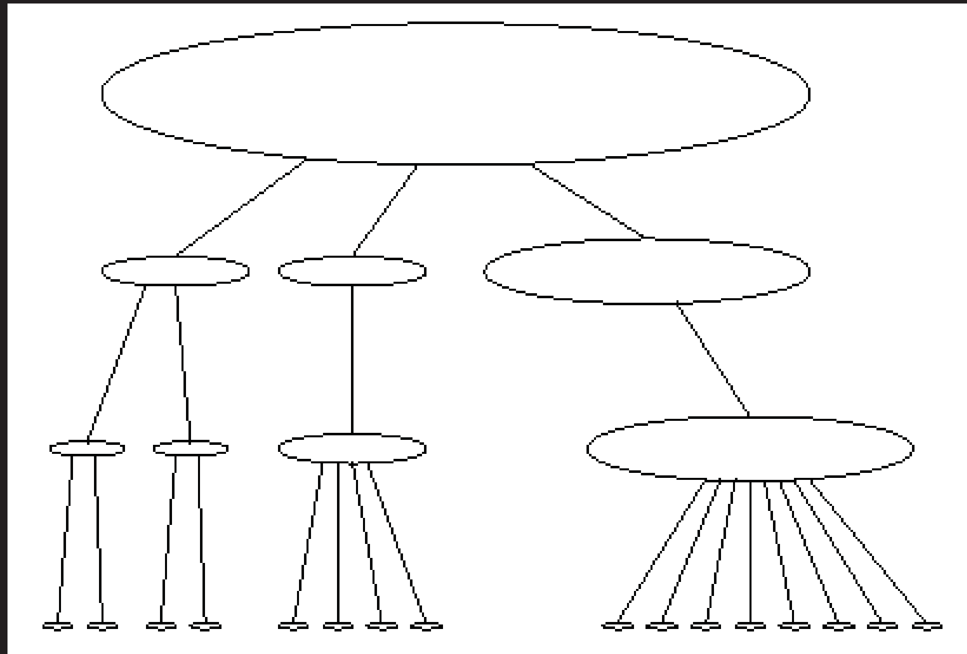
Mapping between σ^2 and M depends on *P(k)*

Simplification because…

- Everything local
- Evolution determined by cosmology (competition between gravity and expansion)
- Statistics determined by initial fluctuation field: since Gaussian, statistics specified by initial power-spectrum *P(k)*
- Fact that only very fat cows are spherical is a detail (*crucial* for precision cosmology); in excursion set approach, mass-dependent barrier height increases with distance along walk

Initial spatial distribution within patch (at z~1000)...



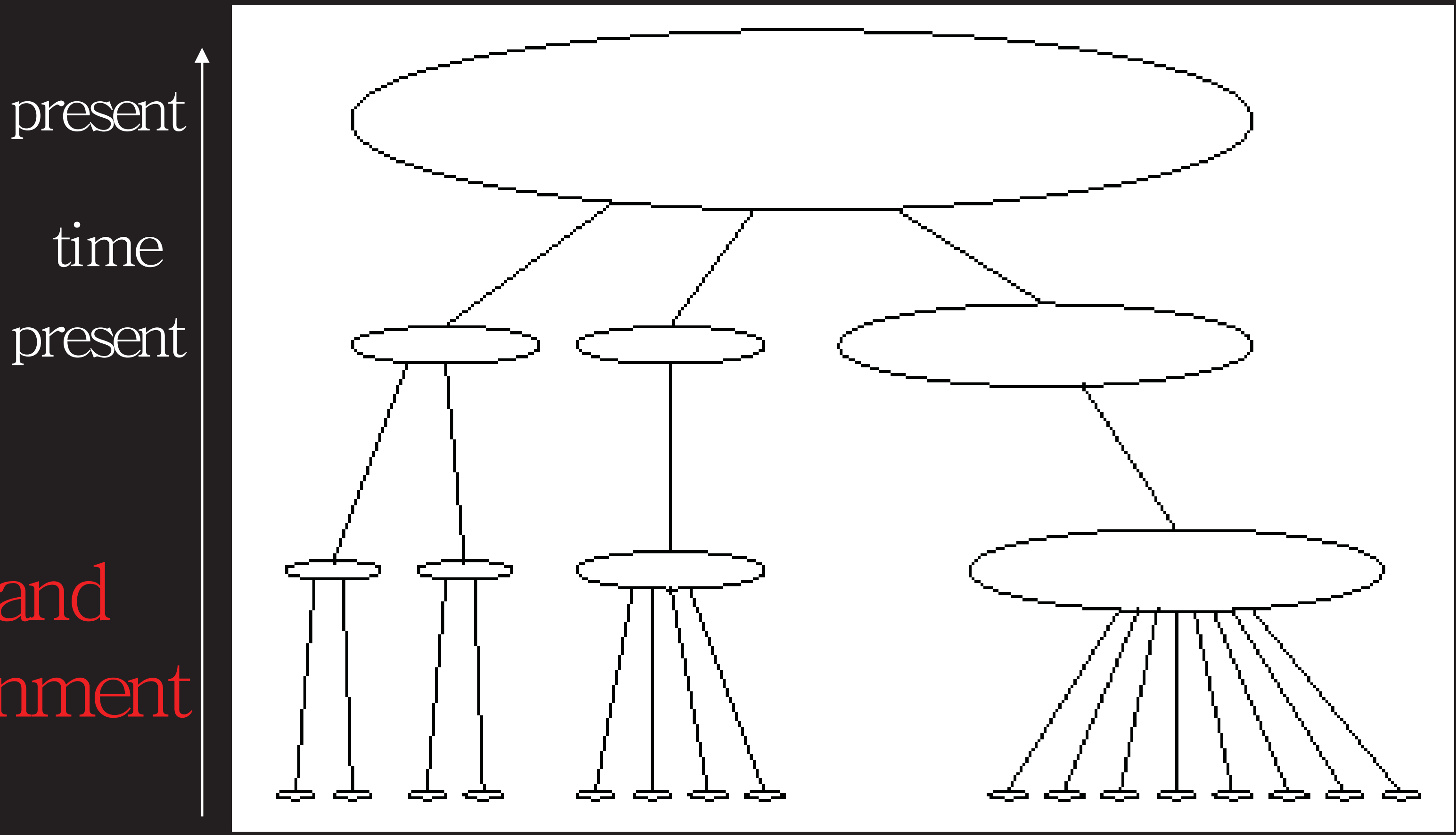


…stochastic (initial conditions Gaussian random field); study 'forest' of merger history 'trees'.

…encodes information about subsequent 'merger history' of object

(Mo & White 1996; Sheth 1996)

Halos and environment



Dense region will collapse soon, so ‘present’ is small look-back time, so halo masses in dense regions relatively large
Present is big look-back time for under-dense region; typical masses smaller in underdense regions

Spherical evolution
very well approximated by
'deterministic' mapping ···

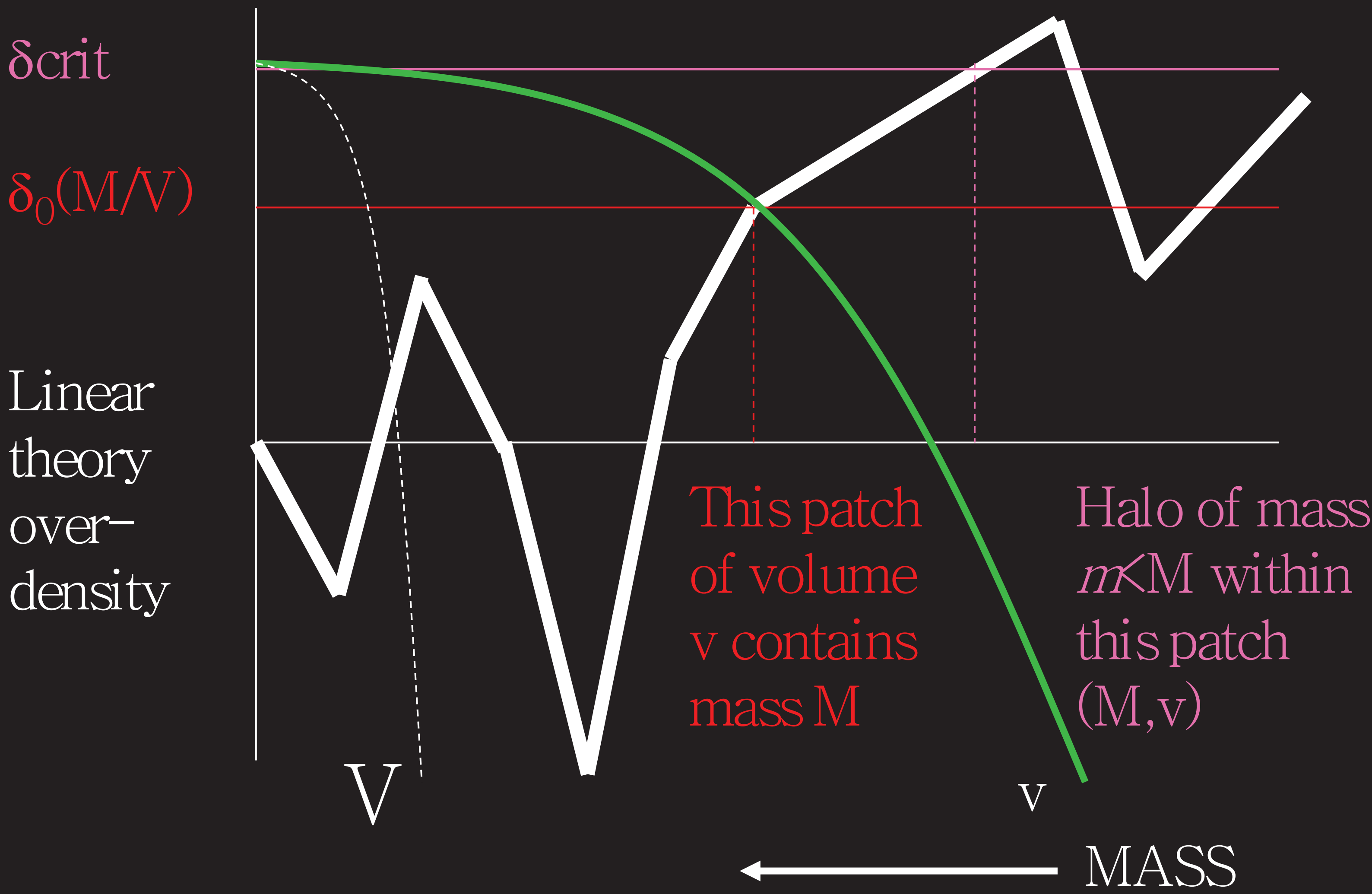
$$(R_{\rm initial}/R)^3 = \ \mathrm{Mass}/(\rho_{\rm com}\mathrm{Volume}) =$$

$$1 + \delta \approx (1 - \delta_0/\delta_{sc})^{-\delta sc}$$

···which can be inverted:

$$(\delta_0/\delta_{sc}) \approx 1 - (1 + \delta)^{-1/\delta sc}$$

The Nonlinear PDF



- Halo mass function is distribution of counts in cells of size $v \to 0$ that are not empty.

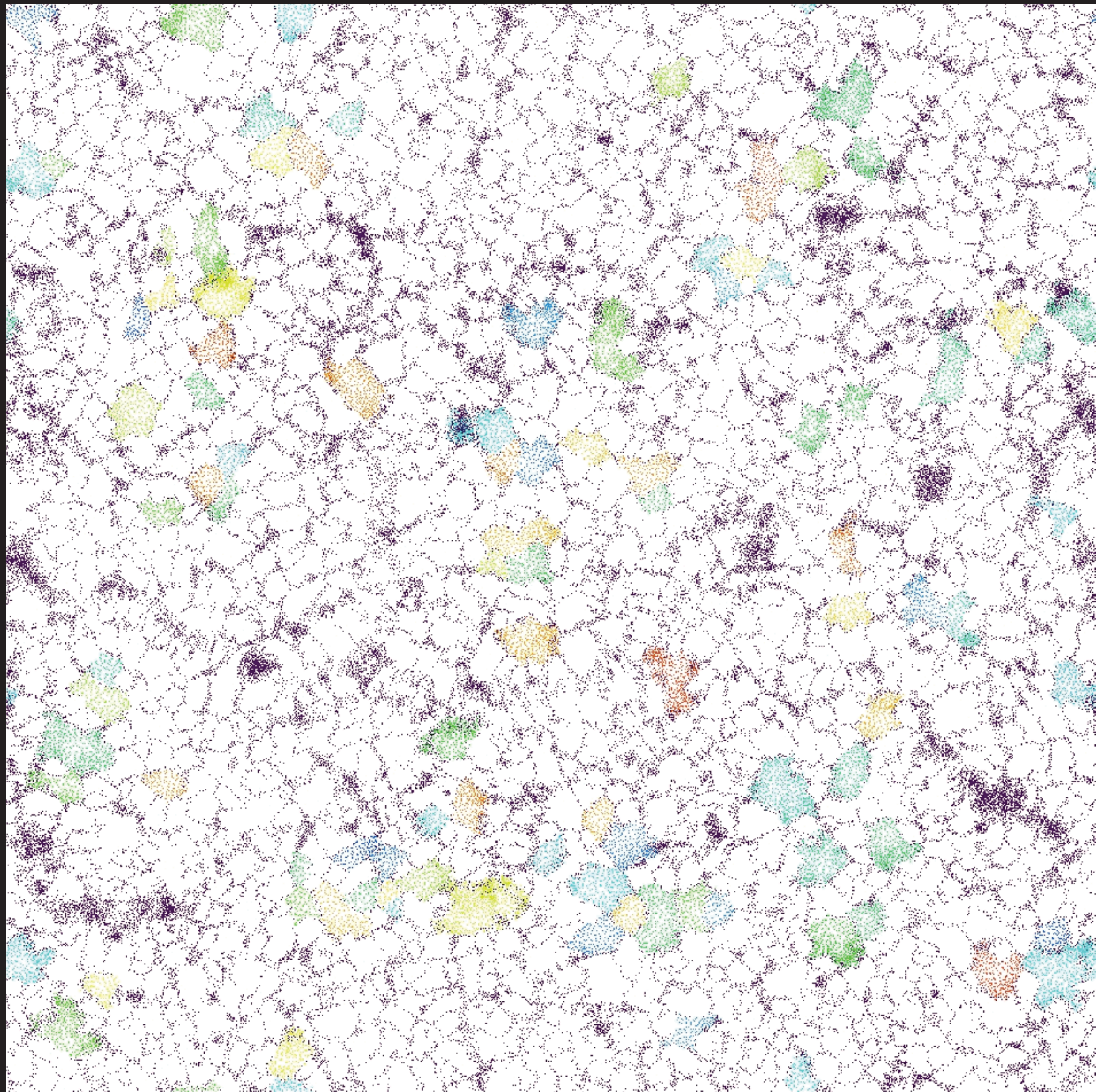
- Fraction f of walks which first cross barrier associated with cell size V at mass scale M,

$$f(M|V)\, dM = (M/V)\, p(M|V)\, dM$$

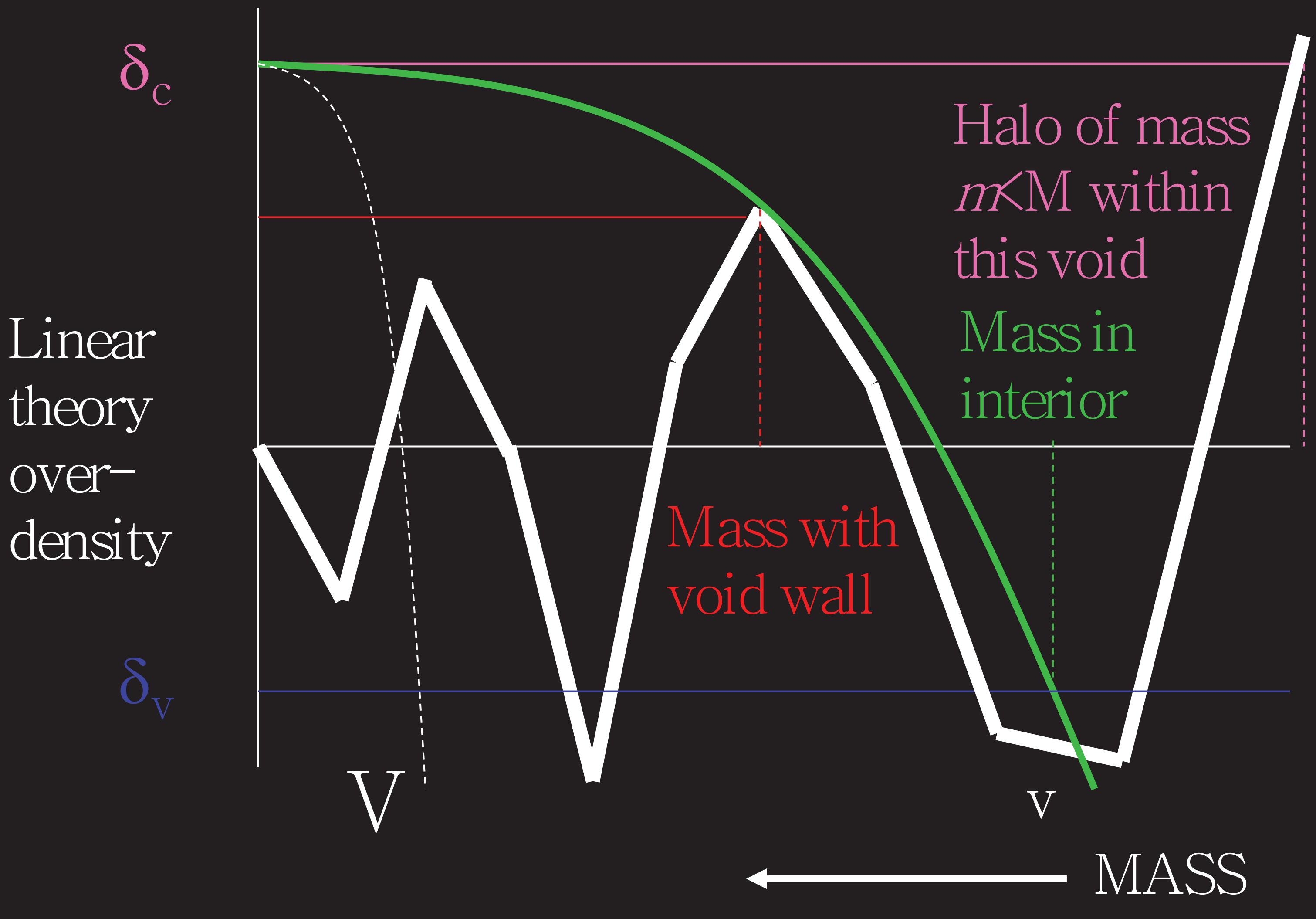
where $p(M|V)\, dM$ is probability randomly placed cell V contains mass M.

- Note: all other crossings irrelevant → stochasticity in mapping between initial and final density

Voids

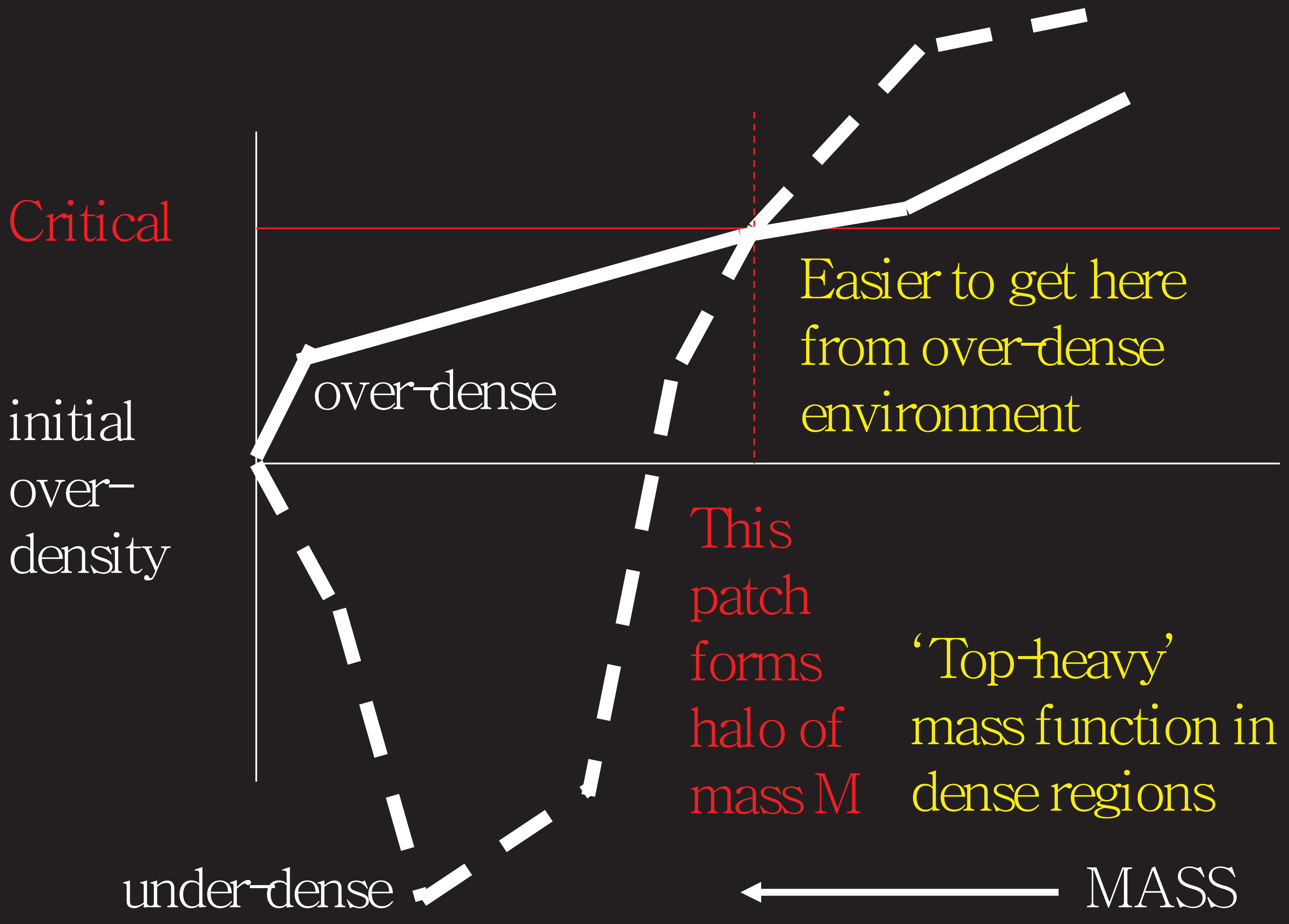


Voids

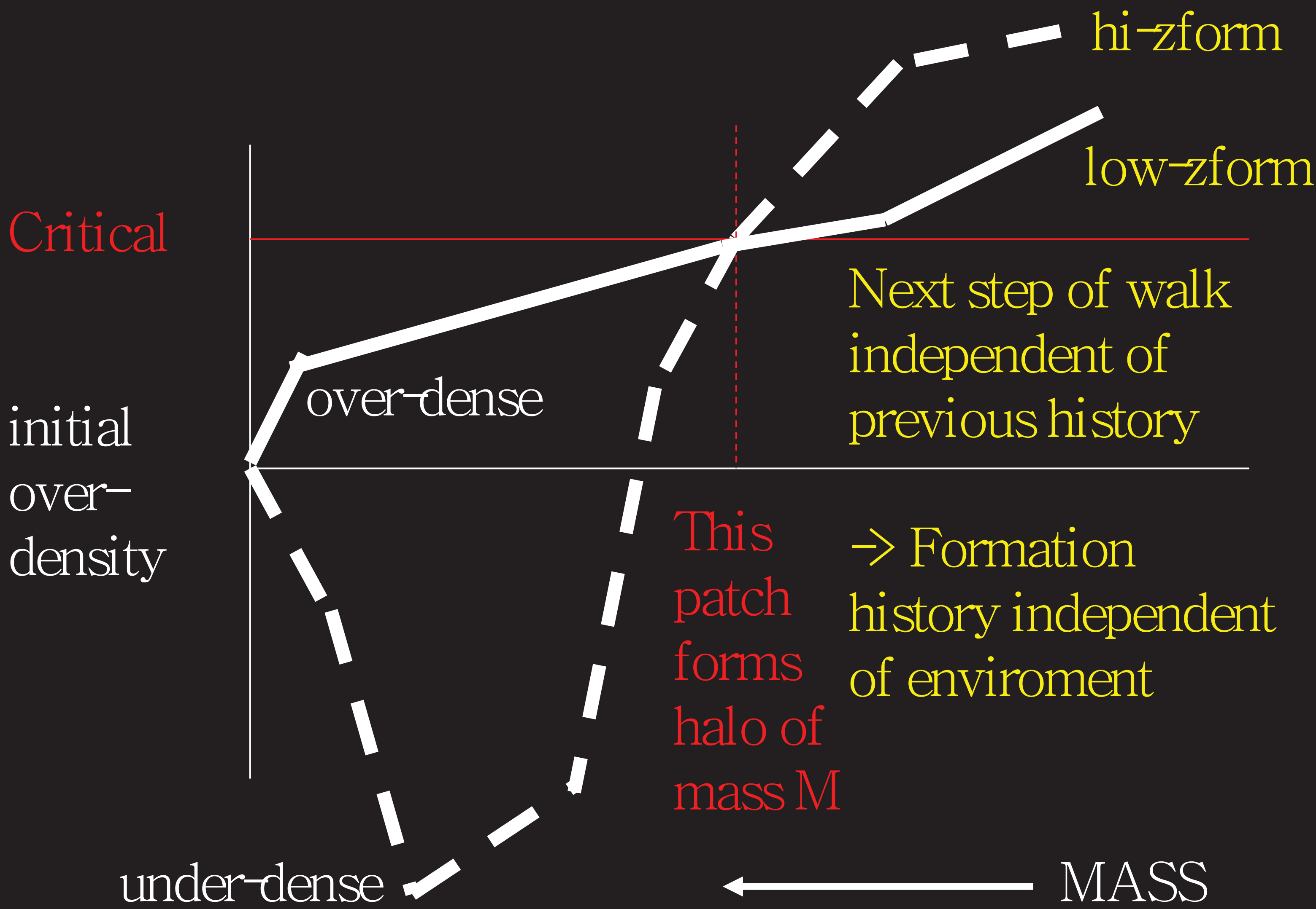


δ_c
Halo of mass
m<M within
this void
Mass in
interior
Linear
theory
over-
density
Mass with
void wall
δ_v
V
v
MASS

Correlations with environment



Correlations with environment



Environmental effects

- In hierarchical models, close connection between evolution and environment (dense region ~ dense universe ~ more evolved)
- Gastrophysics determined by formation history of halo
- Observed correlations with environment test hierarchical galaxy formation models – all environmental effects because massive halos populate densest regions

Galaxy formation

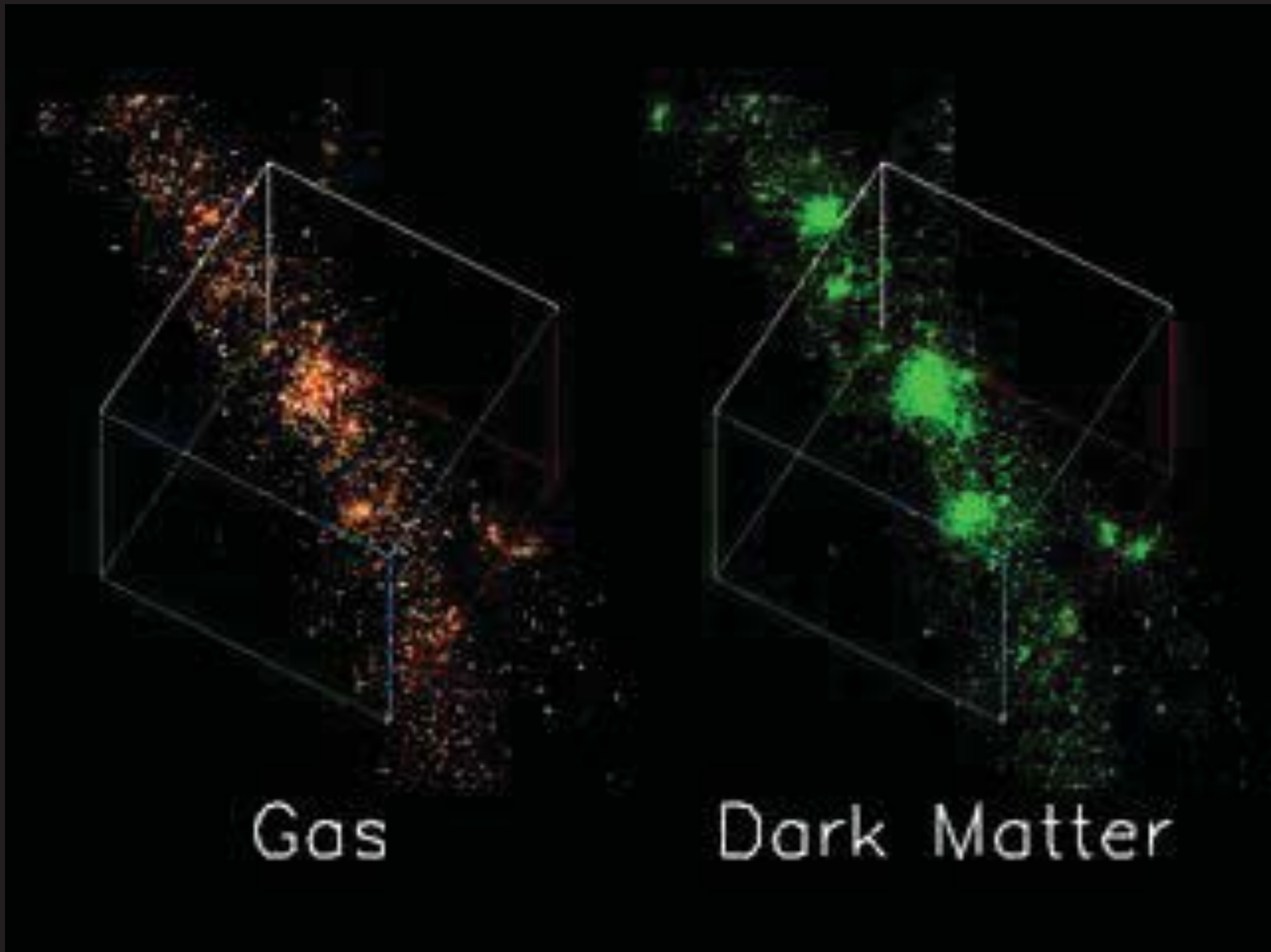
Core condensation in heavy halos: a two-stage theory for galaxy formation and clustering

S. D. M. White and M. J. Rees *Institute of Astronomy, Madingley Road, Cambridge*

Received 1977 September 26

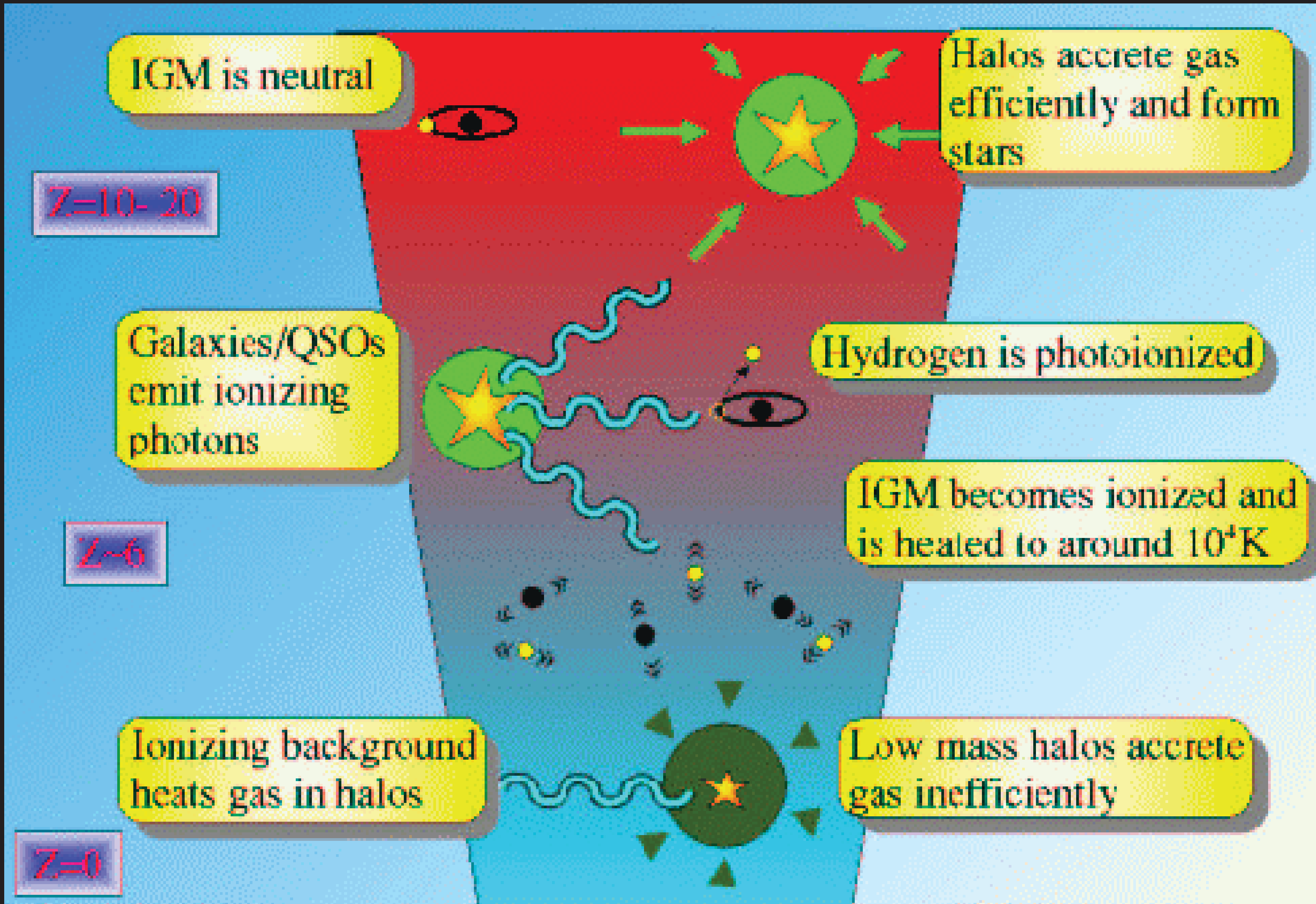
Summary. We suggest that most of the material in the Universe condensed at an early epoch into small 'dark' objects. Irrespective of their nature, these objects must subsequently have undergone hierarchical clustering, whose present scale we infer from the large-scale distribution of galaxies. As each stage of the hierarchy forms and collapses, relaxation effects wipe out its substructure, leading to a self-similar distribution of bound masses of the type discussed by Press & Schechter. The entire luminous content of galaxies, however, results from the cooling and fragmentation of residual gas within the transient potential wells provided by the dark matter. Every galaxy thus forms as a concentrated luminous core embedded in an extensive dark halo. The observed sizes of galaxies and their survival through later stages of the hierarchy seem inexplicable without invoking substantial dissipation; this dissipation allows the galaxies to become sufficiently concentrated to survive the disruption of their halos in groups and clusters of galaxies. We propose a specific model in which $\Omega \sim 0.2$, the dark matter makes up 80 per cent of the total mass, and half the residual gas has been converted into luminous galaxies by the present time. This model is consistent with the inferred proportions of dark matter, luminous matter and gas in rich clusters, with the observed luminosity density of the Universe and with the observed radii of galaxies; further, it predicts the characteristic luminosities of bright galaxies and can give a luminosity function of the observed shape.

- Gas cools in virialized dark matter 'halos'. Physics of halos is nonlinear, but primarily gravitational.
- Complicated gastrophysics (star formation, supernovae enrichment, etc.) mainly determined by local environment (i.e., by parent halo), not by surrounding halos.


Gas
Dark Matter

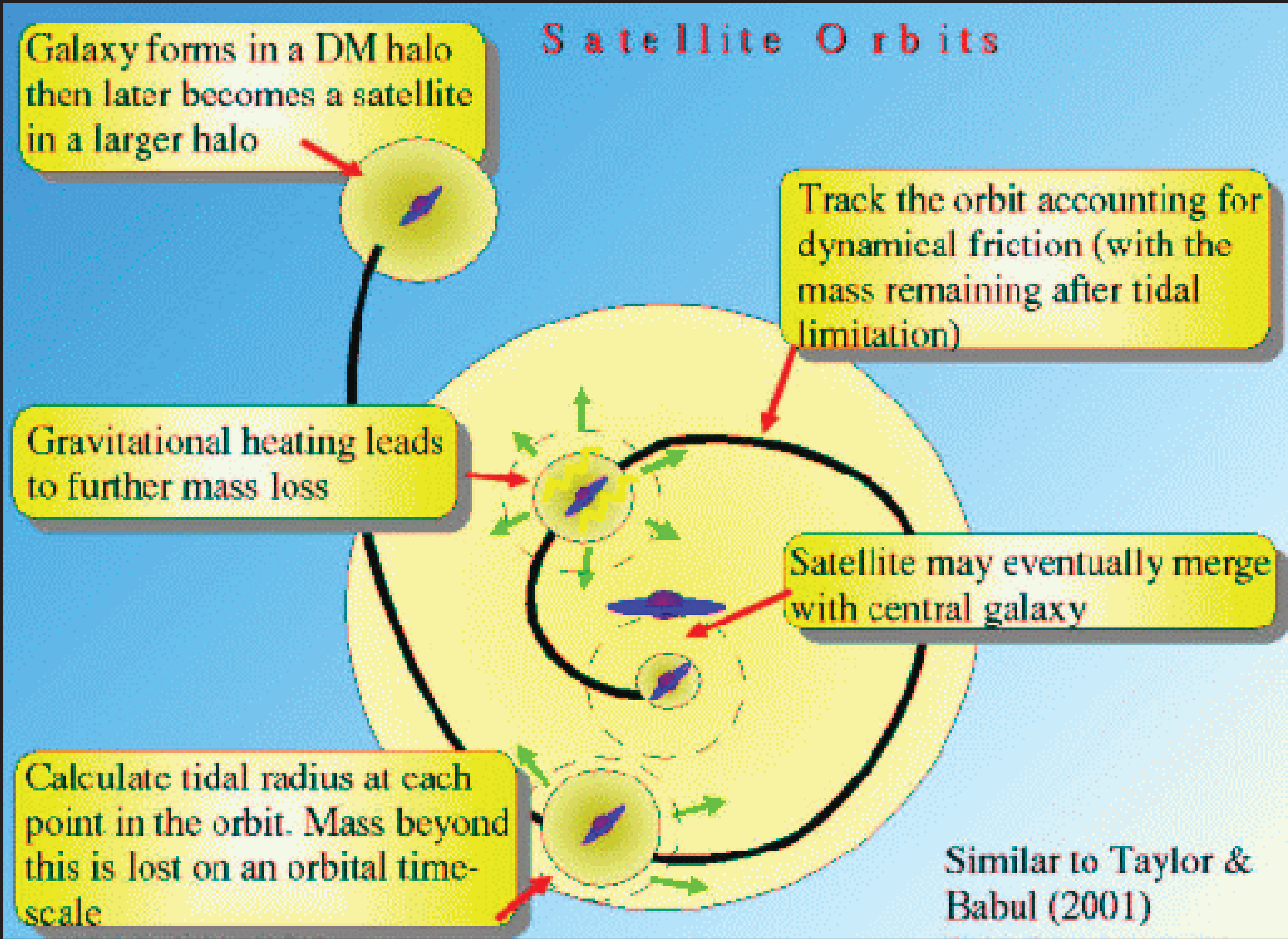
Hierarchical clustering in GR

= the persistence of memory

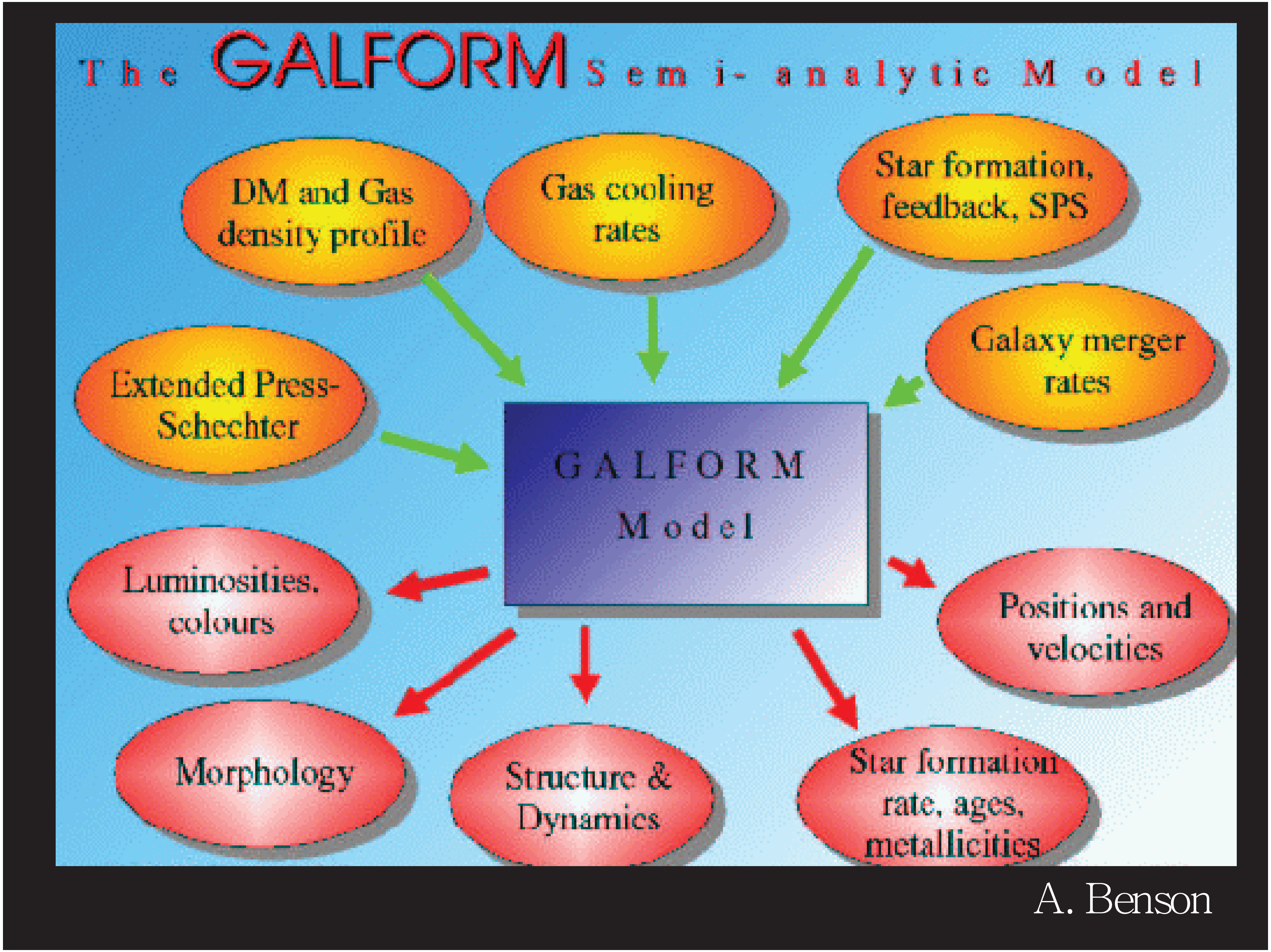


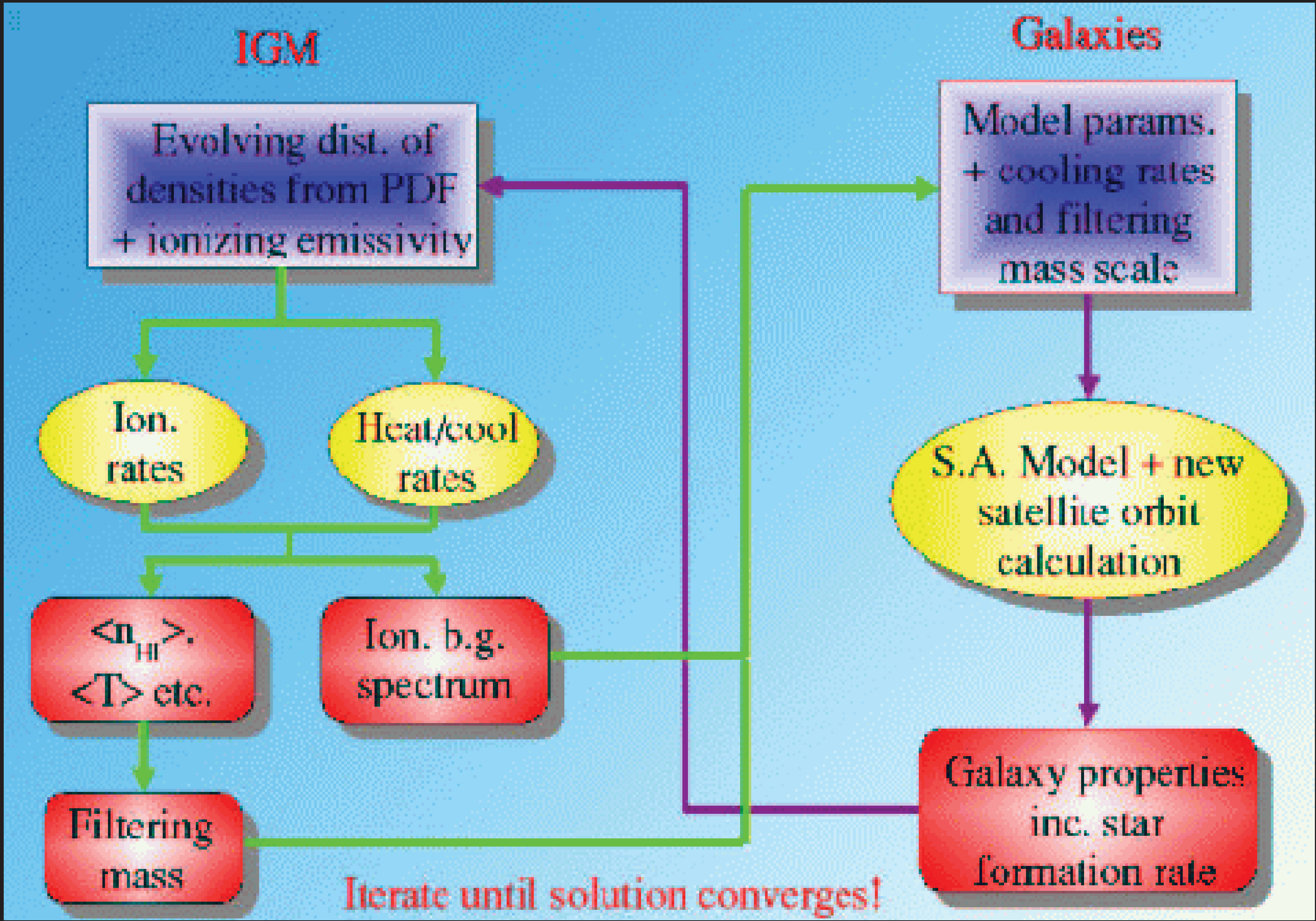
IGM is neutral
Halos accrete gas efficiently and form stars
Z=10- 20
Galaxies/QSOs emit ionizing photons
Hydrogen is photoionized
IGM becomes ionized and is heated to around 10⁴K
Z~6
Ionizing background heats gas in halos
Low mass halos accrete gas inefficiently
Z=0

A. Benson


Satellite Orbits
Galaxy forms in a DM halo then later becomes a satellite in a larger halo
Track the orbit accounting for dynamical friction (with the mass remaining after tidal limitation)
Gravitational heating leads to further mass loss
Satellite may eventually merge with central galaxy
Calculate tidal radius at each point in the orbit. Mass beyond this is lost on an orbital time-scale
Similar to Taylor & Babul (2001)

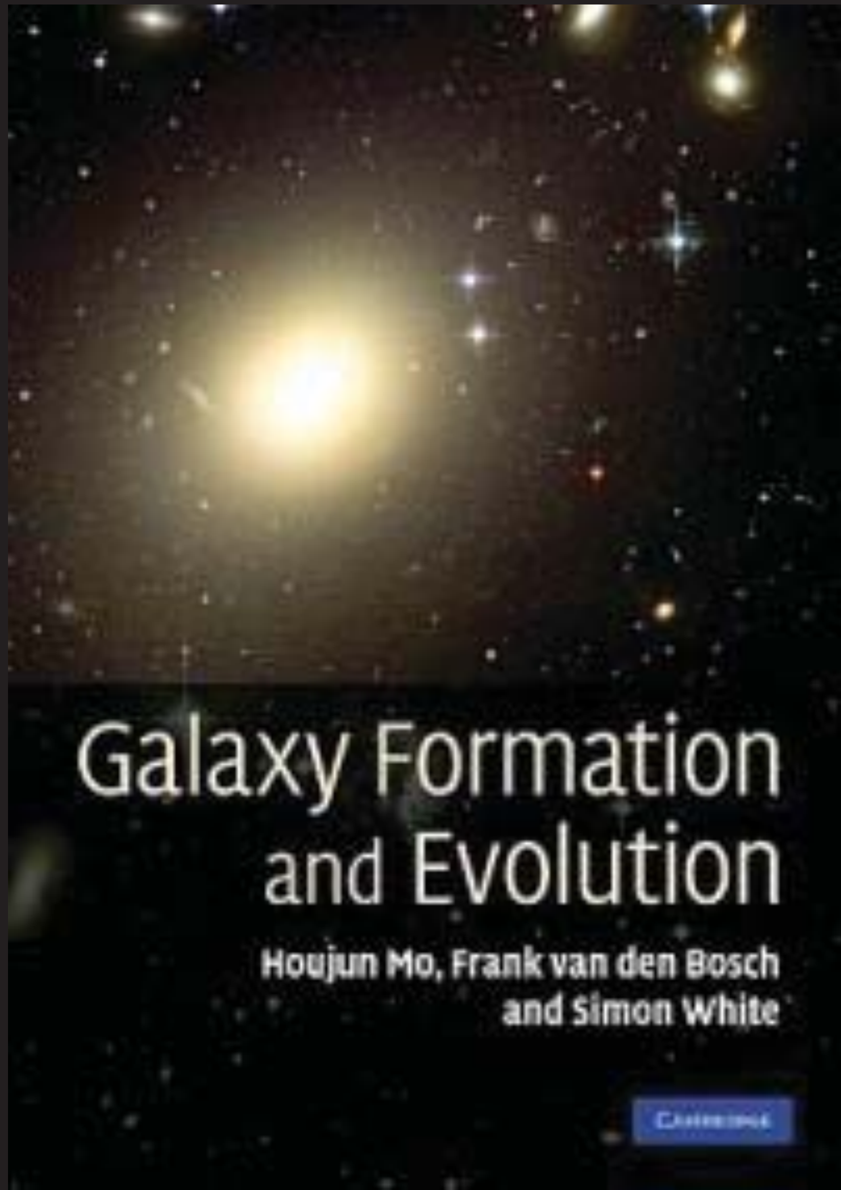
A. Benson


The GALFORM Semi-analytic Model
DM and Gas density profile
Gas cooling rates
Star formation, feedback, SPS
Galaxy merger rates
Extended Press-Schechter
GALFORM Model
Luminosities, colours
Positions and velocities
Morphology
Structure & Dynamics
Star formation rate, ages, metallicities
A. Benson


IGM
Galaxies
Evolving dist. of densities from PDF + ionizing emissivity
Model params. + cooling rates and filtering mass scale
Ion. rates
Heat/cool rates
S.A. Model + new satellite orbit calculation
$\langle n_{HI} \rangle$, $\langle T \rangle$ etc.
Ion. b.g. spectrum
Galaxy properties inc. star formation rate
Filtering mass
Iterate until solution converges!

A. Benson

New public code by A. Benson: Galacticus



Galaxy formation slides can be obtained from Houjon Mo's notes for ICTP school (2010):

http://cdsagenda5.ictp.it/full_display.php?email=0&ida=a09159