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School on Strongly Coupled Physics Beyond the Standard Model

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Composite Higgs Models - Lecture note 1

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Why we need a Higgs boson

The physics discovered so far can be economically described by the following EW chiral Lagrangian (let us neglect the lepton sector for simplicity and consider only the quarks)

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{EWSB}$$

$$\mathcal{L}_0 = -\frac{1}{4g'^2} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4g^2} W^a_{\mu\nu} W^{\mu\nu,a} - \frac{1}{4g_s^2} G^\alpha_{\mu\nu} G^\alpha_{\mu\nu}$$
$$+ \sum_{j=1}^{3} \left(\bar{q}_L^{(j)} i \not{D} q_L^{(j)} + \bar{u}_R^{(j)} i \not{D} u_R^{(j)} + \bar{d}_R^{(j)} i \not{D} d_R^{(j)} \right)$$

$$\mathcal{L}_{EWSB} = \frac{v^2}{4} \mathrm{Tr}\left[(D_\mu \Sigma)^\dagger (D^\mu \Sigma) \right] - \frac{v}{\sqrt{2}} \sum_{i,j} \left(\bar{u}_L^{(i)}, \bar{d}_L^{(i)} \right) \begin{pmatrix} \lambda^u_{ij} \, u_R^{(j)} \\ \lambda^d_{ij} \, d_R^{(j)} \end{pmatrix}$$
$$+ \text{h.c.}$$

where $\Sigma \equiv \exp\left(i \sigma^a \chi^a(x)/v \right) \qquad a = 1,2,3$

$$D_\mu \Sigma \equiv \partial_\mu \Sigma + i W^a_\mu \sigma^a/2 \, \Sigma - i \Sigma \, \sigma^3/2 \, B_\mu$$

The Lagrangian is invariant under local $SU(2)_L \times U(1)_Y$ transformations, under which

$$\Sigma \to U_L \Sigma U_Y^\dagger \qquad U_Y(x) \equiv \exp\left(i \alpha_Y(x) \sigma^3/2 \right)$$
$$U_L(x) \equiv \exp\left(i \alpha_L^a(x) \sigma^a/2 \right)$$

The fields χ^a are the three Goldstone bosons of the spontaneous breaking $SU(2)_L \times U(1)_Y \to U(1)_{em}$.

This can be seen by performing a global $SU(2)_L \times U(1)_Y$ transformation, under which the fields χ^a transform non-linearly and non-homogeneously

$$\frac{\chi^a(x)}{v} \longrightarrow \frac{\chi^a(x)}{v} + \frac{\alpha_L^a(x)}{2} - \frac{\alpha_Y(x)}{2}\delta^{a3} + O(\chi^2)$$

For this reason the $SU(2)_L \times U(1)_Y$ symmetry is said to be non-linearly realized.

In the unitary gauge $\Sigma = \mathbb{1}$ the χ^a fields are eaten and form the longitudinal polarizations of W and Z. Will come back to this later.

[1] As other chiral Lagrangians, the EW one describes a theory which becomes strongly coupled if extrapolated up to energies of the order of a cut-off scale $\Lambda \sim 4\pi v$

This can be most easily seen by working in a ξ gauge and introducing the following GF term:

$$\mathcal{L}_{GF} = -\frac{1}{2\xi}\left(\partial_\mu W^{3}_\mu + \xi\, g\frac{v}{2}\chi^3\right)^2 - \frac{1}{2\xi}\left(\partial_\mu B^\mu + \xi\, g'\frac{v}{2}\chi^3\right)^2$$

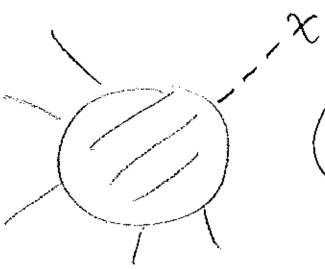
$$-\frac{1}{2\xi}\left|\partial_\mu W^+_\mu + \xi\, g\frac{v}{2}\chi^+\right|^2$$

$$\chi^\pm = \frac{\chi^1 \pm i\chi^2}{\sqrt{2}}$$

In this gauge the χ field propagate and can be considered as external states of Green functions.

The Equivalence Theorem states that

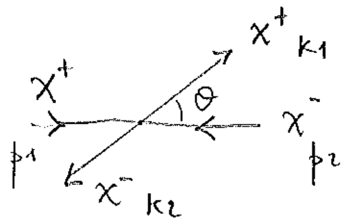
$$= \quad \left(1 + O\left(\frac{m_W^2}{E^2} \right) \right)$$

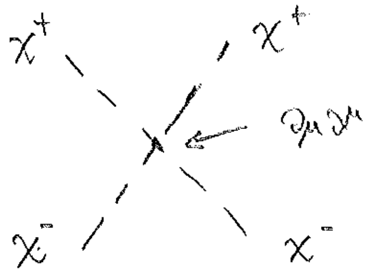


So that Green functions with external longitudinally-polarized W's and Z's can be computed at high energy by the corresponding amplitudes with external χ fields.

In particular

$$A(\chi^+\chi^- \to \chi^+\chi^-) = \frac{1}{v^2}(s+t)\left(1 + O\left(\frac{m_W^2}{E^2}\right)\right)$$





$$(p_1+p_2)^2 \equiv s$$
$$(p_1-k_1)^2 \equiv t = -(s-4m_W^2)\,\frac{1-\cos\theta}{2}$$
$$(p_1-k_2)^2 \equiv u = -(s-4m_W^2)\,\frac{1+\cos\theta}{2}$$

[Ex: derive this formula starting from

$$\mathcal{L} = \frac{v^2}{4}\,\mathrm{Tr}\left[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right] = \frac{1}{6v^2}\left[(\chi^a \partial_\mu \chi^a)^2 - \chi^a\chi^a(\partial_\mu\chi^b\partial^\mu\chi^b)\right] + \frac{1}{2}(\partial_\mu \chi^a)^2 + O(\chi^6)$$
]

Hence, at energies $E \sim 4\pi v$ one has

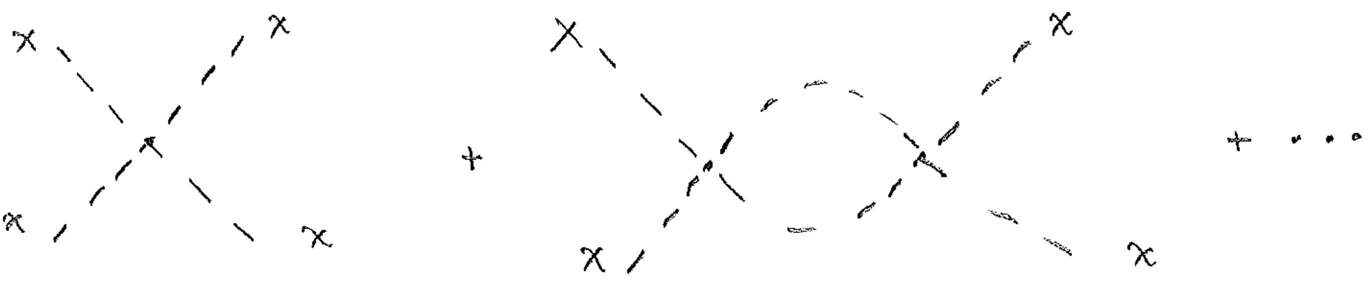
$$A \sim \left(\frac{E}{v}\right)^2 \sim 16\pi^2$$

which implies a strong scattering.

To see this: you should think of E/v as a coupling strenght which depends on the energy

$$g(E) = E/v \qquad \text{so that} \quad A \sim g^2(E)$$

For $g(E) \sim 4\pi$ perturbativity is lost:



$$g^2(E) \qquad\qquad \frac{1}{16\pi^2}\, g^4(E) \times \log(\Lambda^2/E^2)$$

Hence $\left(g^2(E)/16\pi^2\right)$ is the expansion parameter.

Incidentally, unitarity in the $\chi\chi$ scattering is also lost at around the same scale $\Lambda = 4\pi v$; to see this we have to consider the partial wave amplitudes

$$a_\ell = \frac{1}{32\pi} \int_{-1}^{+1} d\cos\theta \;\; A(s,\theta)\; P_\ell(\cos\theta)$$

where $P_\ell(x)$ are Legendre polinomials

$$P_0(x) = 1$$
$$P_1(x) = x$$
$$P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$$
$$\vdots$$

Taking into account a factor π coming from the Log when extracting the imaginary part, one estimates for example

$$\mathrm{Re}(a_0) \sim \frac{1}{32\pi} \left(\frac{E}{v}\right)^2$$

$$\mathrm{Im}(a_0) \sim \frac{1}{32\pi} \cdot \pi \left(\frac{E}{v}\right)^2 \left(\frac{E^2}{16\pi^2 v^2}\right)$$

unitarity requires

$$\frac{\mathrm{Im}(a_0)}{\mathrm{Re}(a_0)} \lesssim \pi \quad \longrightarrow \quad E \lesssim 4\pi v$$

Notice however that the loss of unitarity is only within perturbation theory. The main point here is that the perturbative expansion breaks down and the theory becomes strongly coupled.

$\boxed{\text{NOTICE}}$! Having a cutoff scale, the Lagrangian $\mathcal{L}$ must be thought of as an *effective* one which includes a whole series of operators organized in a chiral expansion in powers of (∂_μ/Λ) [chiral expansion]

[2] Information from LEP precision tests

(i) [there's an approximate custodial invariance, that is: the associated global coset is $SO(4)/SO(3) \sim \frac{SU(2)\times SU(2)}{SU(2)}$ instead of $SU(2)_L \times U(1)_Y / U(1)_{em}$.]

In the unitary gauge $\Sigma = \mathbb{1}$

$$\frac{v^2}{4}\,\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger (D^\mu\Sigma)\right] = \frac{1}{2}\,\frac{v^2}{4}\left[\left(g W_\mu^3 - g' B_\mu\right)^2 + \left((W_\mu^1)^2 + (W_\mu^2)^2\right) g^2\right]$$

after canonically normalizing the gauge fields

$$= m_W^2\, W_\mu^+ W_\mu^- + \frac{1}{2}\, m_Z^2\, Z_\mu^2$$

where
$$\begin{cases} m_W^2 = \dfrac{g^2 v^2}{4} \\[2mm] m_Z^2 = \dfrac{g^2+g'^2}{4}\, v^2 = \dfrac{g^2 v^2}{4\cos^2\theta_W} \end{cases} \qquad \left(\begin{aligned} e &= g\sin\theta_W \\ &= g'\cos\theta_W \end{aligned}\right)$$

Hence
$$\boxed{\rho \equiv \frac{m_W^2}{\cos^2\theta_W\, m_Z^2} = 1}$$

Had we introduced the additional operator:

$$\Delta \mathcal{L} = a_T \frac{v^2}{8} \left(\mathrm{Tr}[\Sigma^\dagger (D_\mu \Sigma)\, \sigma^3] \right)^2$$

which is also $SU(2)_L \times U(1)_Y$ invariant, and contributes only to m_Z, we would have found

$$\Delta \mathcal{L} \overset{\text{unitary gauge}}{=} \Delta m_Z^2 \; Z_\mu^2 \qquad \Delta m_Z^2 = \frac{1}{2} a_T \frac{v^2}{4} (g^2 + g'^2)$$

$$\boxed{\rho = \frac{1}{1 + a_T}}$$

LEP precision tests at the Z pole constrains

$$\Delta \rho \lesssim \text{a few ‰}$$

which indicates that $\Delta \mathcal{L}$ is very much suppressed.

Now we notice that the Lagrangian $\mathcal{L}$, though <u>not</u> $\Delta \mathcal{L}$, is invariant under a <u>global</u>

$$SU(2)_L \times SU(2)_R \,/\, SU(2)_C \qquad U_L = \exp\left(i\, \alpha_L^a \frac{\sigma^a}{2} \right)$$

$$\Sigma \to U_L \Sigma U_R^\dagger \qquad U_R = \exp\left(i\, \alpha_R^a \frac{\sigma^a}{2} \right)$$

in the limit $g' = 0$, $\lambda^u = \lambda^d$. The residual $SU(2)_C$ is called a <u>custodial</u> <u>symmetry</u>

Under $SU(2)_c$ the $\chi^a(x)$ transform like a triplet, which implies that they must get the same mass:

$$m_W = M_Z$$

Turning on g' leads to $\rho \neq 1$ (at tree-level).

This shows that any additional physics which is to UV complete $\mathcal{L}$ <u>must</u> approximately preserve the custodial symmetry.

(ii) LEP sets a strong constraint also on another operator, that of the S parameter:

$$\Delta \mathcal{L}_{EWSB} = a_T \frac{v^2}{8} \left(\mathrm{Tr}[\Sigma^{+} (D_\mu \Sigma) \sigma^3] \right)^2$$

$$+ a_S \, \mathrm{Tr}\left[\Sigma^{+} W^a_{\mu\nu} \frac{\sigma^a}{2} \Sigma \, B_{\mu\nu} \frac{\sigma^3}{2} \right]$$

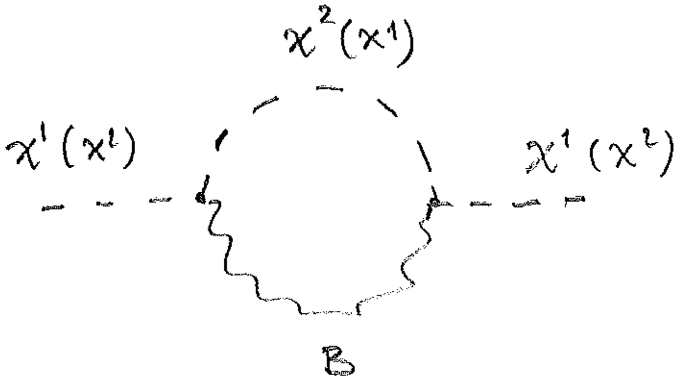
$$+ \dots$$

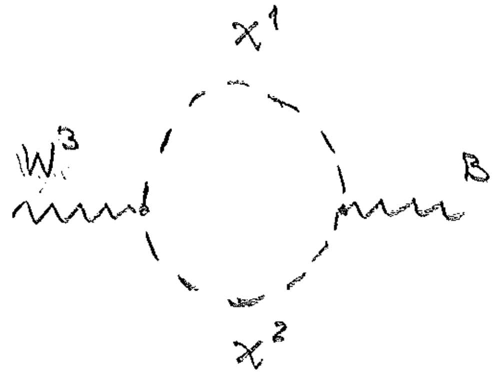
The constraint is on the value of a_T, a_S at the weak scale:

$$\begin{cases} \Delta \hat{T} \equiv \Delta \varepsilon_1 = a_T(m_Z) \\ \Delta \hat{S} \equiv \Delta \varepsilon_3 = g^2 \; a_S(m_Z) \end{cases}$$

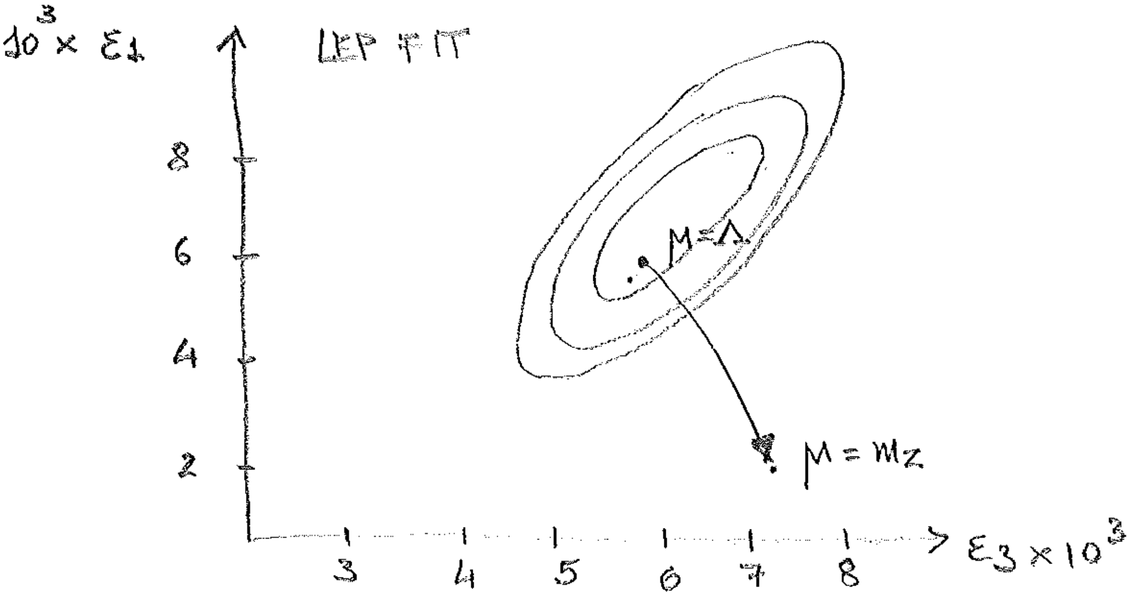
<u>Now</u>: assuming $a_T(\Lambda), a_S(\Lambda) \simeq 0$ does <u>not</u> give a good fit, since their value runs logarithmically at lower scales due to the 1-loop exchange

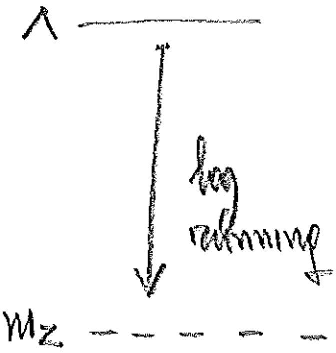
of the Goldstone bosons:





$$\boxed{\begin{gathered}\Delta\varepsilon_{1,3}(m_Z) = \Delta\varepsilon_{1,3} + c_{1,3}\log\frac{\Lambda^2}{m_Z^2} \\ c_1 = -\frac{3}{16\pi^2}\frac{\alpha(m_Z)}{\cos^2\theta_W} \qquad c_3 = +\frac{1}{12\pi}\frac{\alpha(m_Z)}{4\sin^2\theta_W}\end{gathered}}$$





Notice: · a naive estimate with cutoff Λ gives (NDA):

$$\Delta\mathcal{L} \supset v^2\Big(O(1)\,\mathrm{Tr}[\Sigma^\dagger D_\mu\Sigma\,\sigma^3]^2 + \frac{O(1)}{\Lambda^2}\,\mathrm{Tr}[\Sigma^\dagger W_{\mu\nu}\Sigma B_{\mu\nu}] + \cdots\Big)$$

that is :

$$\begin{cases} a_T(\Lambda) \sim O(1) \\ a_S(\Lambda) \sim O\left(\frac{1}{16\pi^2}\right) \end{cases} \leftarrow \boxed{\text{requires custodial invariance !}}$$

$\xrightarrow{\text{NDA}}$

$$a_T(\Lambda) \sim \frac{\lambda^2}{16\pi^2}$$

$\lambda =$ spurion to break custodial invariance

Tension even with UV contribution :

$$\Delta\varepsilon_3 \sim \frac{g^2}{16\pi^2} \approx 2.3 \times 10^{-3}$$

while experimentally $\Delta\varepsilon_3 \lesssim 2 \div 3 \times 10^{-3}$

Notice :

$$\boxed{\Delta\varepsilon_3 \sim \frac{g^2 v^2}{16\pi^2 v^2} \sim \frac{m_W^2}{m_\Phi^2}} \leftarrow \text{mass of new states } m_\Phi \approx \Lambda$$

REFERENCES

[1] H. Georgi Nucl Phys B 363 (1991) 301
(on log running of S, T)

[2] Peskin, Takeuchi PRD 46 (1992) 381
Barbieri et al. NPB 703 (2004) 127
(on S, T, W, Y)

[3] Manohar, Georgi NPB 234 (1984) 189
Georgi, Randall NPB 276 (1986) 241
(on NDA)

[3] The most simple addition : 1 new scalar, singlet of the custodial symmetry

Let us show that by a simple addition to the theory the comparison with the LEP data can be sensibly improved :

We assume the existence of one extra (real) scalar, $h(x)$.
We that it to be a singlet of the custodial symmetry

$$h \text{ is } \mathbb{1} \text{ of } SU(2)_c$$

Assuming custodial invariance,
the most general chiral lagrangian at the level of 4 derivatives ($O(p^4)$), which includes all possible cubic interactions with 1 or 2 Higgs and quartic interactions with 2 Higgs, is the following:

$$\boxed{\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial_\mu h)^2 - V(h) \\ & + \frac{v^2}{2} \mathrm{Tr}\left[(D_\mu \Sigma)^2 (D_\mu \Sigma) \right] \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2} + \dots \right) \\ & - \frac{v}{\sqrt{2}} \sum_{i,j} \left(\bar{u}_L^{(i)} \ \bar{d}_L^{(i)} \right) \Sigma \begin{pmatrix} \lambda^u_{ij} & u_R^{(j)} \\ \lambda^d_{ij} & d_R^{(j)} \end{pmatrix} \left(1 + c\frac{h}{v} + c_2\frac{h^2}{v^2} + \dots \right) \\ & + h.c. \\ & + \mathcal{L}^{(4)} \end{aligned}}$$

where

$$\boxed{V(h) \equiv \frac{1}{2} m_h^2 h^2 + d_3 \frac{1}{6}\left(\frac{3 m_h^2}{v}\right) h^3 + d_4 \frac{1}{24}\left(\frac{3 m_h^2}{v^2}\right) h^4 + \dots}$$

and the $O(p^4)$ Lagrangian $\mathcal{L}^{(4)}$ has the form

$$\boxed{\mathcal{L}^{(4)} = \sum_{\{K\}} O_K}$$

where $\{O_K\}$ is the following list of operators :

$$O_1 = \alpha_1 \mathrm{Tr}\left[(D_\mu \Sigma)^\dagger (D^\mu \Sigma)\right] (\partial_\nu h)^2$$

$$O_2 = \mathrm{Tr}\left[(D_\mu \Sigma)^\dagger (D_\nu \Sigma)\right] \partial^\mu \partial^\nu F^{(2)}(h)$$

$$O_{GG} = G_{\mu\nu} G^{\mu\nu} \; F^{(GG)}(h)$$

$$O_{BB} = B_{\mu\nu} B^{\mu\nu} \; F^{(BB)}(h)$$

$$O_W = D_\mu W^a_{\mu\nu} \; \mathrm{Tr}\left[\Sigma^\dagger \sigma^a \, i \overleftrightarrow{D}_\nu \Sigma\right] \; F^{(W)}(h)$$

$$O_B = -\partial_\mu B_{\mu\nu} \; \mathrm{Tr}\left[\Sigma^\dagger \, i \overleftrightarrow{D}_\nu \Sigma \, \sigma^3\right] \; F^{(B)}(h)$$

$$\boxed{\begin{aligned} O_{WH} &= i\, W^a_{\mu\nu}\, \mathrm{Tr}\left[(D^\mu\Sigma)^\dagger \sigma^a (D^\nu\Sigma)\right] F^{(WH)}(h) \\ O_{BH} &= -i\, B_{\mu\nu}\, \mathrm{Tr}\left[(D_\mu\Sigma)^\dagger (D^\nu\Sigma)\sigma^3\right] F^{(BH)}(h) \\ O'_{WH} &= \frac{1}{2}\, W^a_{\mu\nu}\, \mathrm{Tr}\left[\Sigma^\dagger \sigma^a\, i \overleftrightarrow{D}_\nu \Sigma\right] \partial^\nu F^{(WH)}(h) \\ O'_{BH} &= -\frac{1}{2}\, B_{\mu\nu}\, \mathrm{Tr}\left[\Sigma^\dagger\, i\overleftrightarrow{D}_\nu \Sigma\, \sigma^3\right] \partial^\nu F^{(BH)}(h) \end{aligned}}$$

plus the operators

$$\boxed{O_{G\tilde{G}},\ O_{B\tilde{B}},\ O_{\tilde{W}H},\ O_{\tilde{B}H},\ O_{\tilde{W}},\ O_{\tilde{B}},\ O'_{\tilde{W}H},\ O'_{\tilde{B}H}}$$

obtained by the corresponding operators above by replacing one field strenght by its dual

ex: $$O_{G\tilde{G}} = \varepsilon^{\mu\nu\rho\sigma}\, G_{\mu\nu}\, G_{\rho\sigma}\, F^{(G\tilde{G})}(h)$$

Here $F^{(i)}(h)$ denotes a polynomial of $h(x)$, of which only the terms up to the quadratic one are of interest here

$$\boxed{F^{(i)} \equiv \alpha_0^{(i)} + \alpha_1^{(i)}\, h(x) + \alpha_2^{(i)}\, h^2(x) + \cdots}$$

The additional operators

$$\begin{cases} O_{WW} = W^a_{\mu\nu} W^a_{\mu\nu} \, \mathcal{F}^{(WW)}(h) \\ O_{WB} = \mathrm{Tr}\left[\Sigma^\dagger W^a_{\mu\nu} \sigma^a \Sigma B_{\mu\nu} \sigma^3 \right] \mathcal{F}^{(WB)}(h) \end{cases}$$

can be rewritten in terms of those in the above list by means of the identities:

$$\begin{cases} O_{WW} - \frac{1}{2} O_{WB} = O_W + 2\left(O_{WH} + O'_{WH} \right) \\ O_{BB} - \frac{1}{2} O_{WB} = O_B + 2\left(O_{BH} + O'_{BH} \right) \end{cases}$$

Similarly, the operators $O_{W\tilde{W}}$, $O_{W\tilde{B}}$ are not linearly independent.

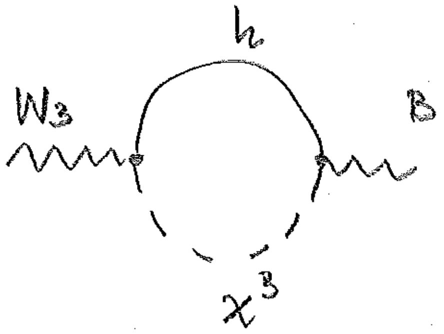
Ex: prove the above identities and thus that

$$\boxed{\hat{S} = 4 g^2 \left(\alpha_0^{(W)} + \alpha_0^{(B)} \right)}$$

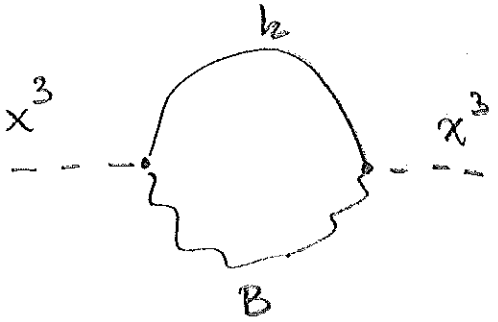
Although some of the operators in $\mathcal{L}^{(4)}$ do affect the phenomenology of h at the LHC, let us concentrate for simplicity on the $O(p^2)$ terms.

The exchange of the scalar $h(x)$ gives two types of effects:

(i) virtual corrections to the EW parameters $\varepsilon_1, \varepsilon_3$:

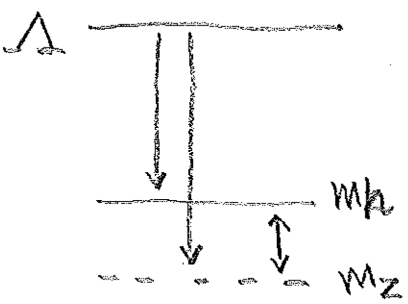


$$\rightarrow \quad \Delta\varepsilon_3 = -c_3\, a^2 \log \frac{\Lambda^2}{m_h^2}$$



$$\rightarrow \quad \Delta\varepsilon_1 = -c_1\, a^2 \log \frac{\Lambda^2}{m_h^2}$$

The net effect in the RG running of $\varepsilon_T(\mu)$, $\varepsilon_S(\mu)$ down to $\mu = m_Z$ is:



$$\Delta\varepsilon_i = c_i \log\left(\frac{m_h^2}{m_Z^2}\right) + (1-a^2)\, c_i \log\left(\frac{\Lambda^2}{m_h^2}\right)$$

$i = 1, 3$

$$\left[\text{so that } a < 1 \rightarrow \quad \begin{matrix} \Delta\varepsilon_3 > 0 \\ \Delta\varepsilon_1 < 0 \end{matrix}\right]$$

Given Λ and m_h, one can thus derive a limit on the value of a^2. It is convenient to do so by first defining an effective value of m_h:

$$\Delta \varepsilon_i \equiv c_i \log\left(\frac{m_h^2|_{\text{eff}}}{M_Z^2}\right)$$

so that

$$\boxed{1-a^2 = \frac{\log\left(\frac{m_h^2|_{\text{eff}}}{m_h^2}\right)}{\log\left(\frac{\Lambda^2}{m_h^2}\right)}} \qquad \boxed{m_h|_{\text{eff}} = m_h\left(\frac{\Lambda}{m_h}\right)^{1-a^2}}$$

Setting $m_t = 173.2$ GeV and performing a fit with two degrees of freedom ($\Delta\chi^2 = 9.21$) one obtains (from LEP)

$$\boxed{23\,\text{GeV} \le m_h|_{\text{eff}} \le 280\,\text{GeV}} \quad @\ 99\%\ \text{CL}$$

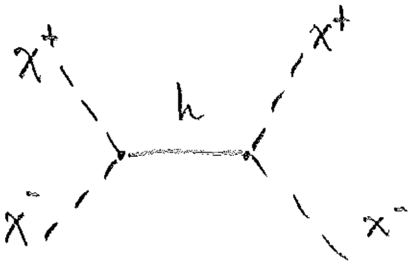
which for $\begin{cases}\Lambda = 1.2\,\text{TeV} \\ m_h = 120\,\text{GeV}\end{cases}$ implies $\boxed{0.63 \le a^2 \le 1.72}$ @ 99% CL

[Extracting the bound on a^2 directly, without introducing m_{eff}, one finds

$$0.7 \le a^2 \le 1.61$$

The difference is due to the fact that the dependence on $\log m_{\text{eff}}$ is not exactly logarithmic]

(ii) tree-level exchange in $\chi\chi$ scattering modifies the scale of strong scattering



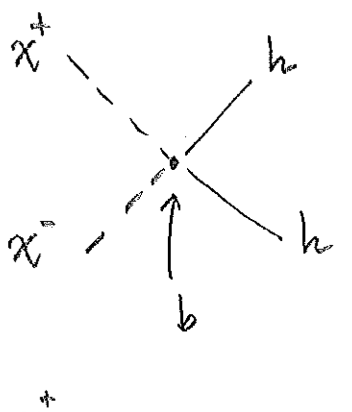
$$\boxed{A(\chi^+\chi^- \to \chi^+\chi^-) \simeq \frac{1}{v^2}\left[s - \frac{a^2 s^2}{s - m_h^2} + (s \leftrightarrow t)\right]}$$

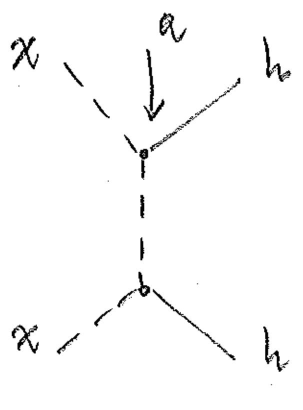
this implies that the loss of perturbativity is delayed up to the scale

$$\Lambda_{CH} \sim \frac{4\pi v}{\sqrt{|1-a^2|}}$$

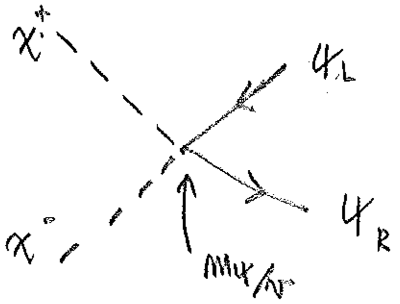
This of course requires the Higgs to be light enough ($m_h \ll 4\pi v$) to be effective in the scattering.

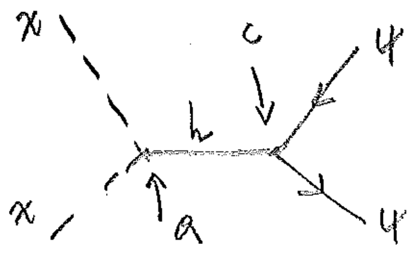
There are also other inelastic channels to monitor:





$$\boxed{A(\chi\chi \to hh) \simeq \frac{s}{v^2}(b - a^2)}$$





$$\boxed{A(\chi\chi \to \psi_L \bar{\psi}_R) \simeq \frac{m_\psi \sqrt{s}}{v^2}(1 - ac)}$$

At this point we notice that by tuning the scalar's couplings to

$$\boxed{\begin{array}{l} a = b = c = 1 \\ \text{all other couplings vanishing} \end{array}}$$

<u>SM point</u>
(Higgs Model)

(*) the theory can be made <u>perturbative</u> up to arbitrary (Planckian) scales

(*) the logarithmic divergence in $a_S(\mu)$, $a_T(\mu)$ cancels out (so that the contribution to the EW parameters is finite)

Both facts are intimately connected to the fact that a theory with $a = b = c = 1$ is renormalizable. This is most easily seen by performing a field redefinition

$$H = \frac{1}{\sqrt{2}}\, e^{i\sigma^a \chi^a(x)/v} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

In terms of the $SU(2)_L$ doublet H the Lagrangian has the form

$$\mathcal{L} = |D_\mu H|^2 + \mu^2 (H^\dagger H) - \lambda (H^\dagger H)^2$$

and it is thus manifestly renormalizable.
I will refer to this special point in the parameter space as to the "<u>Higgs model</u>".

<u>Ex</u>: check it!

*) Notice that, in term of H, UV contributions to $\varepsilon_1, \varepsilon_3$ are parametrized by dimension-6 operators:

$$\varepsilon_1 \leftrightarrow (H^\dagger D_\mu H)^2$$

$$\varepsilon_3 \leftrightarrow (H^\dagger W_{\mu\nu} B^{\mu\nu} H)$$

If the theory is renormalizable (hence the lagrangian at tree level only contains renormalizable operators) by dimensional analysis no divergence other than those corresponding to a renormalization of the Higgs wave functions and gauge kinetic terms will arise.
This implies that $\varepsilon_{1,3}$ will only get finite corrections.

*) Notice also that in the lagrangian of the Higgs model the custodial symmetry is still present, although not manifest.
To see it one can rewrite H in terms of its real components

$$H = \begin{pmatrix} w_1 + i\, w_2 \\ w_3 + i\, w_4 \end{pmatrix} \qquad (H^\dagger H) = \sum_i (w_i)^2$$

and notice that

$V(h) = V(H^\dagger H)$ is invariant under the $SO(4)$ rotating the four w_i's.

In the vacuum

$\langle H^\dagger H \rangle = v^2$ hence $SO(4) \to SO(3)$

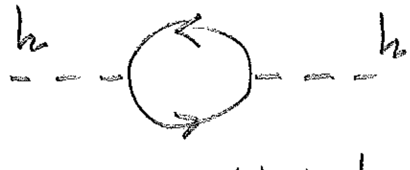
where $SO(3)$ is the custodial symmetry.

Although the Higgs model seems economical and thus attractive we notice that

(1) LEP data only require $a^2 = 1 + O(20\text{-}30\%)$ and put no constraints on the remaining couplings

(2) the perturbative Higgs model is plagued by the hierarchy problem (UV instability under radiative corrections):

$$\delta m_h^2 = \frac{\Lambda^2}{8\pi^2}\left(6y_t^2 - \frac{3}{4}\left(3g^2 + g'^2\right) - 6\lambda_4\right)$$



[the Higgs Model should perhaps be regarded as a parametrization rather than a dynamical explanation of the EWSB]