The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization

IAEA International Atomic Energy Agency

2326-8

School on Strongly Coupled Physics Beyond the Standard Model

16 - 24 January 2012

Composite Higgs Models - Lecture note 2

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The Higgs as a composite PNGB

The Higgs model ($a=b=c=1$) assumes the theory stays perturbative up to UV scales and the Higgs boson is <u>elementary</u> (up to those energies).

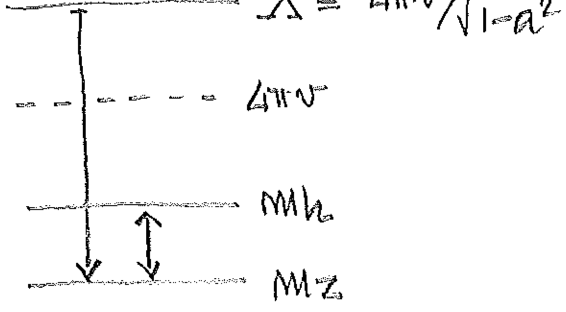
However, it might well happen that the extra (light) scalar is a composite, bound state of some new dynamics. After all we have <u>not</u> discovered <u>any fundamental</u> scalar field so far!

In such scenario, $a \neq 1$ would help relaxing the tension with the EW tests performed at LEP.
As a bonus, the exchange of the scalar would delay the loss of perturbativity up to a scale $\Lambda \approx 4\pi v/\sqrt{1-a^2}$.

Notice that in the EW fit

(*) the UV contribution to $\hat{S}, \hat{T}$ scales like $\sim \frac{1}{\Lambda^2} \sim (1-a^2)$

(*) the IR contribution scales like $(1-a^2)\log(1-a^2)$



By setting $\Lambda = \frac{1.2\,\text{TeV}}{\sqrt{|1-a^2|}}$

one obtains ($m_H = 120$ GeV)

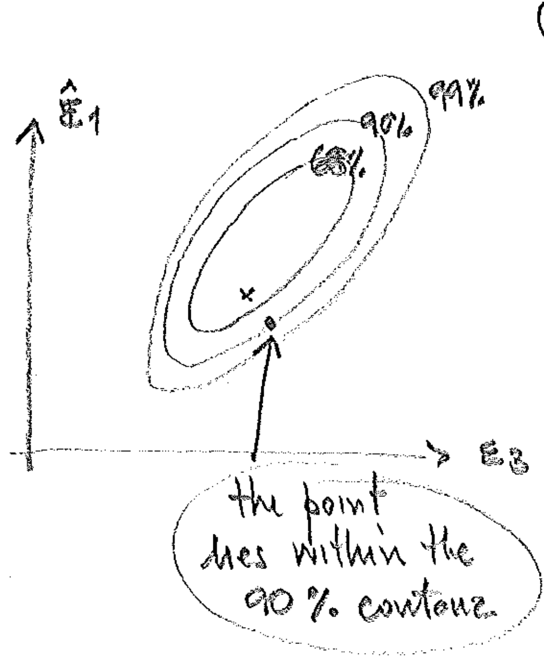
$\boxed{0.77 \leq a^2 \leq 1.55}$ @ 99% CL

For example :

$1-a^2 = 0.1$ (10% tuning)

$m_h = 120$ GeV $\quad \Lambda = 9.8$ TeV

$\left(\frac{m_h}{\Lambda}\right)^2 = 1.5 \times 10^{-4}$



An important example of composite scalars is given by the pions in QCD :

(*) they are $(q\bar{q})$ bound states of more fundamental constituents : the quarks

(*) $\pi\pi$ scattering gets strong at the QCD scale

$$A(\pi\pi \to \pi\pi) \sim E^2/f_\pi^2 \quad \to \quad \Lambda_{QCD} \sim 4\pi f_\pi \sim 1\,\text{GeV}$$

at which new resonances appear ($m_\rho \sim 770$ MeV)

If the Higgs is a composite scalar of some new dynamics there are however two important unanswered issues :

(1) why $m_h \ll \Lambda$?

(2) why $1-a^2 \ll 1$?

The example of QCD is very instructive in this regard. Indeed, it is a fact that

$$m_\pi \ll m_\rho \sim \Lambda_{QCD} \approx 4\pi f_\pi$$
$$(135\,\text{MeV}) \qquad (770\,\text{MeV})$$

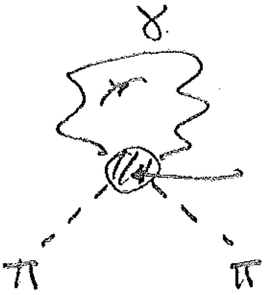
The reason for such parametric gap is that the pion is a Nambu - Goldstone boson of the global chiral invariance of QCD :

$$SU(2)_L \times SU(2)_R \longrightarrow \underbrace{SU(2)_V}_{\text{3 real NG bosons : } \pi^{i=1,2,3}}$$

$$SU(2)_L : \quad q_L \to e^{i\alpha_L} q_L$$
$$SU(2)_R : \quad q_R \to e^{i\alpha_R} q_R$$

Its mass comes from the <u>explicit</u> breaking of the chiral symmetry.

For example, in the chiral limit $m_q = 0$, the dominant contribution to m_π is the electromagnetic one :



Form Factor dumps large virtual momenta $q^2 \gg \Lambda_{QCD}^2 \sim m_\rho^2$

$$\Delta m_\pi^2 \sim \frac{e^2}{16\pi^2} m_\rho^2 \simeq (20\,\text{MeV})^2$$

while experimentally

$$\Delta m_\pi^2 \simeq (35\,\text{MeV})^2$$

The explicit breaking comes from the <u>partial</u> gauging of the (unbroken) subgroup of global symmetry

$$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_V$$

$$\downarrow$$

$$U(1)_{em}$$

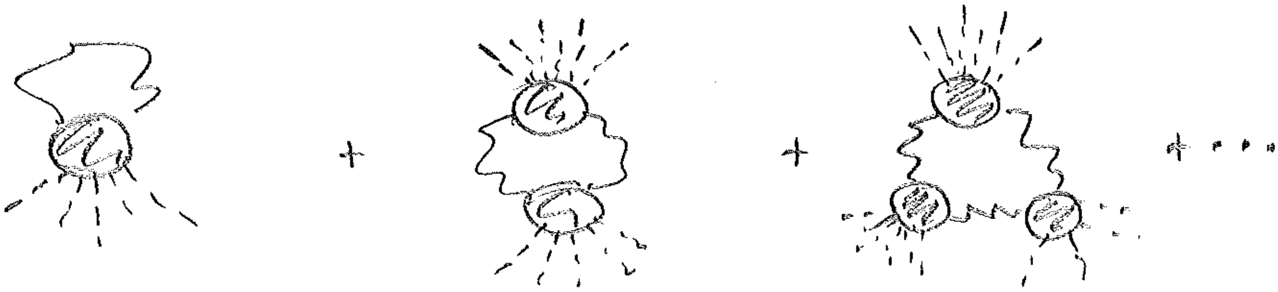
$$Q = T_{3L} + Y$$

$$Y = T_{3R} + B/2$$

Hence, the pion is a <u>pseudo-NG boson</u> after the EM interactions are turned on.

Notice that the gauging of $U(1)_{em}$ does not lead to any Higgs mechanism, since the gauged $U(1)_{em}$ is <u>aligned</u> along the linearly-realized $SU(2)_V$

This is true at tree level by construction, but remains true also at the loop level after the pion potential is computed a la Coleman-Weinberg



$$V(\pi) \approx (+)\frac{\alpha_{em}}{4\pi} \sin^2(\pi/f_\pi)$$

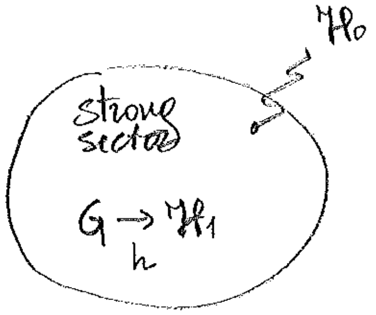
[positive coefficient implies VACUUM ALIGNMENT]

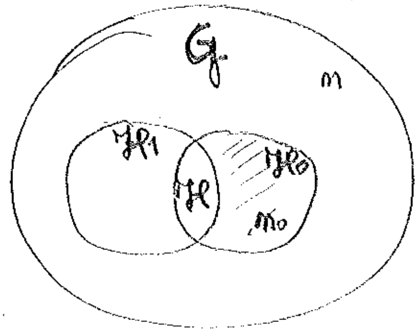
Similarly to the pion, the Higgs boson could be realized as a pseudo-NG boson (Georgi-Kaplan '80). In such scenario:

- the Higgs boson is a composite pNG boson of some global symmetry $\mathcal{G} \to \mathcal{H}$, such that the EW gauging of $\mathcal{G}$ does trigger a Higgs mechanism at tree level.
- At loop level, the explicit breaking of $\mathcal{G}$ leads to a potential for the Higgs and thus to the EWSB.

[the COMPOSITE HIGGS PROGRAM]

In general one can have





Algebra cartoon

$\mathcal{G} \to \mathcal{H}_1$ = global symmetry

$\mathcal{H}_0$ = gauged subgroup

$\mathcal{H} = \mathcal{H}_0 \cap \mathcal{H}_1$ = unbroken gauge group

NG bosons:

$m = \dim(\mathcal{G}) - \dim(\mathcal{H}_1)$

eaten NG Bosons

$m_0 = \dim(\mathcal{H}_0) - \dim(\mathcal{H})$

$(m - m_0)$ are pseudo NG-bosons

Realizing the Higgs as a pNG boson then requires two conditions

(1) the SM group G_{SM} can be embedded into the unbroken global subgroup

$$G_{SM} \subset \mathcal{H}_1$$

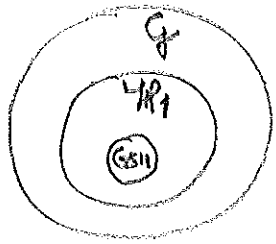
(2) $\mathcal{G}/\mathcal{H}_1$ contains at least one doublet of $SU(2)_L$

By identifying for simplicity the external gauge group with the SM one,

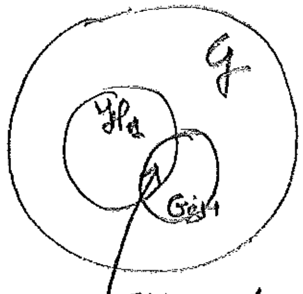
$$\mathcal{H}_0 = G_{SM}$$

this implies that

(*) at the tree level $G_{SM} \subset \mathcal{H}_1$



(*) at the one loop level $G_{SM} \to U(1)_{em}$, which can also be understood as a misalignment of the true vacuum from the gauged subgroup G_{SM}



$U(1)_{em}$ ($\times$ $SU(3)_c$)

eaten NB bosons : $n_0 = \dim(G_{SM}) - \dim(U(1)_{em}) = 3$

pseudo-NG bosons : $n = \dim(\mathcal{G}) - \dim(\mathcal{H}_1)$

Before showing an explicit example of such scenario it is worth recalling a few general results :

(*) definition of quotient space $\mathcal{G}/\mathcal{H}$:

given a subgroup $\mathcal{H}$ of $\mathcal{G}$, the latter can be divided into EQUIVALENCE CLASSES according to the following Equivalence Relation :

$$\forall\ g_1, g_2 \in \mathcal{G} \qquad g_1 \sim g_2$$

$$\text{iff} \quad \exists\ h \in \mathcal{H} \ /$$

$$g_1 = g_2 \cdot h$$

By choosing one representative for each equivalence class, one constructs the quotient space $\mathcal{G}/\mathcal{H}$

[properties of an equivalence relation are :

(1) it's reflexive $\quad g_1 \sim g_1$

(2) it's symmetric $\quad g_1 \sim g_2 \rightarrow g_2 \sim g_1$

(3) it's transitive $\quad g_1 \sim g_2 ,\ g_2 \sim g_3 \rightarrow g_1 \sim g_3$]

Denoting by V^a the generators of $\mathcal{H}$ and by $A^{\hat{a}}$ the remaining ones, the algebra of $\mathcal{G}$ obeys the following commutation relations

$$[V, V] = V$$
$$[V, A] = A$$

$\Big\} \rightarrow$ fixed by the request of $\mathcal{H}$ being a subgroup

$$[A, A] = V + A$$

If there exists an automorphism (grading) of the algebra under which

$$V^a \rightarrow + V^a$$
$$A^{\hat{a}} \rightarrow - A^{\hat{a}}$$

such that

$$[A, A] = V$$

then the quotient space $\mathcal{G}/\mathcal{H}$ is said to be symmetric.

(*) $M_{vacua} \sim \mathcal{G}/\mathcal{H}$

If a field theory possesses a global invariance $\mathcal{G} \to \mathcal{H}$, then the manifold of vacua M_{vacua} is in one-to-one correspondence with the quotient space $\mathcal{G}/\mathcal{H}$.

proof: given one vacuum, described by the field configuration ϕ_0, any other vacuum can be reached by means of a $\mathcal{G}$ transformation

$$\phi = g\,\phi_0 \qquad g \in \mathcal{G}$$

In a neighborhood of the identity, any element $g \in \mathcal{G}$ can be uniquely decomposed as

$$g = e^{i\xi_0 \cdot A}\, e^{i u \cdot V}$$

Since however $e^{iu \cdot V}\phi_0 = \phi_0 \;\; \forall\, u^a$ by definitions, then

$$g = e^{i\xi_0 \cdot A}\, \phi_0$$

that is: the manifold of possible vacua is described by the parameters $\{\xi_0^{\hat{a}}\}$ (they form a set of local coordinates), hence $M_{vacua} \sim \mathcal{G}/\mathcal{H}$.

(*) the NG bosons live on the quotient space $\mathcal{G}/\mathcal{H}$

Since the NG bosons describe massless fluctuations around the vacuum, they too live on $\mathcal{G}/\mathcal{H}$:

$$U(x) = e^{i\xi(x)\cdot A}$$

such that

$$\xi(x) \to \xi_0 \qquad \text{for} \quad |x| \to \infty$$

(*) transformation law of the NG bosons

For any transformation $g \in \mathcal{G}$ connected to the identity, the NG bosons transform as follow:

$$g\, e^{i\xi\cdot A} \equiv e^{i\xi'(\xi,g)\cdot A}\, e^{iu'(\xi,g)\cdot V} \equiv e^{i\xi'(\xi,g)\cdot A}\, h(\xi,g)$$

where $h \in \mathcal{H}$. Hence

$$\xi(x) \to \xi'(\xi(x), g)$$ [non linear, non homogeneous transformation law]

such that

$$\boxed{e^{i\xi'(\xi,g)\cdot A} = g\, e^{i\xi\cdot A}\, h^{-1}(\xi,g)}$$

An important special case is when $\mathcal{G}/\mathcal{H}$ is symmetric. Then there exists a grading R such that

$$g = \exp\left(i\alpha^a V^a + i\alpha^{\hat a} A^{\hat a} \right)$$

$$R:\ g \longrightarrow R(g) \equiv \exp\left(i\alpha^a V^a - i\alpha^{\hat a} A^{\hat a} \right)$$

Acting with R on the transformation rule gives

$$e^{-i\xi'\cdot A} = R(g)\, e^{-i\xi\cdot A}\, h^{-1}(\xi, g)$$

or $$e^{i\xi'\cdot A} = h(\xi, g)\, e^{i\xi\cdot A}\, R^{-1}(g)$$

This implies that

$$\boxed{\Sigma(\xi(x)) \equiv \exp(2i\xi\cdot A) = U^2}$$

transforms linearly under $\mathcal{G}$:

$$\boxed{\Sigma \to g\,\Sigma\, R(g)^{-1}}$$

(*) EXAMPLES of SYMMETRIC SPACES

(i) $$\boxed{\frac{SU(N)\times SU(N)}{SU(N)} = SU(N)}$$

This is the case of QCD, with $SU(2)\times SU(2) \to SU(2)$
The NG field is

$$\Sigma = \exp\left(2 i \frac{\pi^i V^i}{f_\pi}\right) = \exp\left(i\, \pi^i \sigma^i / f_\pi \right)$$

The fact that $\Sigma \in SU(2)$ proves that $\mathcal{G}/\mathcal{H} = SU(2)$.

(ii) $$\boxed{\frac{SO(n+1)}{SO(n)} = S^n}$$

proof:

The topology of $\frac{SO(n+1)}{SO(n)}$ is that of the manifold of vacua in a theory with invariance $SO(n+1) \to SO(n)$.
Let $\phi_0 = (0, 0, \dots, 0, 1)$, an $(n+1)$-dimensional unit vector pointing in some direction of $\mathbb{R}^{n+1}$, be one vacuum.
Then any other vacuum of the theory can be obtained by rotating it (acting with an element of $SO(n+1)$).
But the rotation of an $(n+1)$-dimensional unit vector spans the n-dimensional sphere S^n.

For example :

$$\boxed{\frac{SO(4)}{SO(3)} = S^3}$$

same algebra of $SU(2) \times SU(2) \to SU(2)$

Notice :

(*) $SU(2) \sim S^3$ ($SU(2)$ matrices are described by 4 real parameters a_i satisfying $\sum_i a_i^2 = 1$)

hence $SU(2)$ is simply connected, while $SO(3)$ is not

(*) The isomorphism between $SO(4)$ and $SU(2) \times SU(2)$ can be made explicit as follows :

to any vector v^a of $\mathbb{R}^4$ is associated a matrix

$$V \equiv \sigma^a v^a \qquad \sigma^a = (\vec{\sigma}, -i\,\mathbb{1})$$

The isomorphism $SO(4) \sim SU(2) \times SU(2)$ can be proven by looking at the transformation rules of the vector and the matrix :

$$\left[\begin{array}{l} v^a \to S^{ab} v^b \\ S \in SO(4) \\ \|v\| \text{ is invariant} \end{array}\right] \iff \left[\begin{array}{l} V \to L V R^+ \\ L \in SU(2)_L \\ R \in SU(2)_R \\ \det V = -\|v\|^2 \text{ is invariant} \end{array}\right]$$

Then: for each $S \in SO(4)$ there are two $SU(2) \times SU(2)$ transformations that act in the same way

$$S \leftrightarrow \{ (L, R) \; ; \; (-L, -R) \}$$

At the level of group elements the equivalence reads

$$\boxed{SO(4) = \frac{SU(2) \times SU(2)}{\mathbb{Z}_2}}$$

(*) $\quad \text{Adj}[SO(4)] = 6$

$V^i = 3$ of $SO(3)$
$A^i = 3$ of $SO(3)$

Another example is:

$$\boxed{\frac{SO(5)}{SO(4)} = S^4}$$

$\text{Adj}[SO(5)] = 10$

$V^a = 6$ of $SO(4) \leftrightarrow (1,3) + (3,1)$ of $SU(2)_L \times SU(2)_R$
$A^{\hat{a}} = 4$ of $SO(4) \leftrightarrow (2,2)$ of $SU(2)_L \times SU(2)_R$
$\leftrightarrow$ complex 2 of $SU(2)_L$