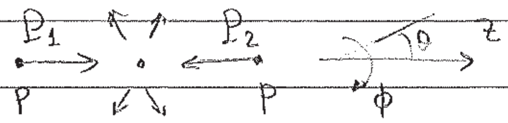
Introduction to Collider Physics

I. Introduction

Collider Experiment: e.g. LHC



"Lab frame" $\vec{P}_1 + \vec{P}_2 = 0$ $\quad s = (p_1+p_2)^2 = 4E_1^2$

$\sqrt{s} = 2E_1$ = "c.-of-M. energy" (e.g. 2 TeV @ the Tevatron
7 TeV → 14 TeV @ LHC)

After collision: detect $e^\pm, \mu^\pm, \gamma, q/\bar{q}, g, \tau^\pm$ (?)

↗ if within "acceptance"

$q/\bar{q}, g$ → hadrons → jets { b-tag: $c\tau \approx 400$-$500\,\mu m$; c-tag: $c\tau \approx 100$-$300\,\mu m$; g vs q: maybe

+ stable, el. charged or colored BSM particles

↑ $c\tau \gtrsim 10\,m \Rightarrow \tau \gtrsim 3\times 10^{-8}$ sec

cf "natural" lifetime $\tau_{nat} \sim \frac{1}{M} \lesssim \frac{1}{100\text{ GeV}} \sim 10^{-27}$ sec for BSM.

do not detect ν + stable, el. neutral, uncolored BSM particles

↑ ex: WIMP dark matter candidates!

measure $E_i, \vec{p}_i$ $\quad i = 1 .. N_{vis}$ – visible external particles

↑ in some cases, need to use $E^2 - \vec{p}^2 = m^2$.

"Event record": $\mathcal{E} = \{ \{PID_i, p_i^\mu\}_1, \{PID_i, p_i^\mu\}_2, \dots \}$

En.-mom. conservation: $P_1^\mu + P_2^\mu = \sum_{i=1}^{N} p_i^\mu$

Proton remnant escapes unobserved ⇒ useless for $\mu = 3, 0$.

But useful for transverse momentum: $\sum_{i=1}^{N} \vec{p}_i^{\,T} = 0$

If this sum is non-zero for visible particles, define

$$\text{"Missing } \vec{p}_T\text{"} \equiv \vec{\not{p}}_T = - \sum_{i=1}^{N_{vis}} \vec{p}_i^{\,T}$$

$$\text{"Missing transverse energy"} \equiv \not{E}_T = |\vec{\not{p}}_T|.$$

This is the total mom. of invisible particles, and is the only information available about them.

The goal is to connect $\mathcal{E}$ to "fundamental theory": $\mathcal{L}$

$\mathcal{L} \to \mathcal{E}$ is algorithmic (at least if pert. theory applies):

$\mathcal{L} \to$ Feynman rules $\to$ S matrix $\to \frac{d\sigma}{d\Pi}$, $d\Pi = \prod_{i=1}^{N} \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i}$ "phase space"

$\to \frac{dN}{d\Pi} = L_{int} \cdot \frac{d\sigma}{d\Pi}$ $\to$ $\mathcal{E}_{th}$: a realization of this distribution (typically generated numerically with a "Monte-Carlo generator")

↑ "integrated luminosity" ∝ # of collisions reported by acc. operators

Compare $\mathcal{E}_{th}$ with measured $\mathcal{E}$ ⇒ theory OK or ruled out!
(roughly, χ^2 test)

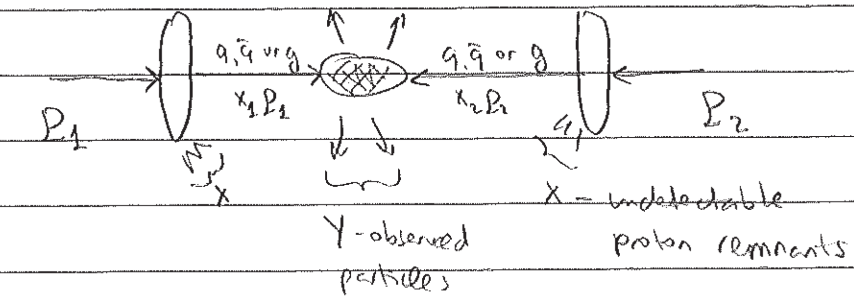
This is all pretty straightforward, but not completely trivial:

- p is composite, q/g → hadrons - some non-pert. physics enters
- Pert. theory has IR divergences

If deviation from the SM is discovered, we will also need to construct $\mathcal{E} \to \mathcal{L}$ ("the inverse problem"). There is no algorithmic solution. Of course, one can just try many $\mathcal{L}$'s until get a good fit, but need some intuition to know what to try! As usual, intuition comes from collecting and internalizing a set of examples. We will try to do it in these lectures.

II. Hello World: Z production at the LHC (or Tevatron)

1) Parton model



"parton distribution functions - pdf's"

$$\sigma(p(P_1)+p(P_2) \to Y + X) = \int_0^1 dx_1 \int_0^1 dx_2 \sum_{f_1, f_2} f_{f_1}(x_1, Q^2)\, f_{f_2}(x_2, Q^2)$$

$$\times\, \sigma(f_1(x_1 P_1) + f_2(x_2 P_2) \to Y)$$

inclusive - sum over all possible X

roughly, $P_Y^2 = M^2_{inv}(Y)$

$f_1, f_2 = u, \bar{u}, d, \bar{d}, \ldots, g$ (some refinements are needed - later!)

"Parton frame" $x_1 \vec{P}_1 + x_2 \vec{P}_2 = 0$

Parton c-of-m energy: $\hat{E}_{cm} \equiv \sqrt{\hat{s}} = \sqrt{(x_1 P_1 + x_2 P_2)^2} = \sqrt{2 x_1 x_2 P_1 \cdot P_2} = \sqrt{x_1 x_2 s}$
$\leq \sqrt{s}$!

Boost lab → parton: $\hat{E} = \sqrt{x_1 x_2 s} = \gamma(1-\beta) x_1 \sqrt{s} = \gamma(1+\beta) x_2 \sqrt{s}$

$$\Rightarrow \beta = \frac{x_1 - x_2}{x_1 + x_2}, \quad \gamma = \frac{x_1 + x_2}{2\sqrt{x_1 x_2}}$$

s fixed, $\hat{s}, \beta, \gamma$ vary event-by-event, according to p.d.f.'s and cross sections.

Facts about pdf's: - not calculable: reflect non-pert. physics of confinement
- obtained by fits to exp. data (mainly DIS)
⇒ error bars, dep. on f_i and x may be large.
- available at
durpdg.dur.ac.uk/HEPDATA/PDF

[slides, comments on relative importance of g, valence/sea quarks]

2) Z prod.: total cross-section

Parton-level, tree-level: $\sigma(gg \to Z) = \sigma(qg \to Z) = 0$

$$\sigma(q\bar{q} \to Z) = \frac{4}{3}\pi^2 \frac{\Gamma(Z \to q\bar{q})}{M_Z} \delta(s - M_Z^2)$$ (see P&S, p.151)

$$\Rightarrow \sigma_{LO}(pp \to Z + X) = \frac{4}{3}\pi^2 \frac{\Gamma_Z}{M_Z} \int_0^1 dx_1 \int_0^1 dx_2 \sum_q{}' \, 2 f_q(x_1) f_{\bar{q}}(x_2)$$

$$\times Br(Z \to q\bar{q}) \times \delta(x_1 x_2 S - M_Z^2).$$

x_2 integral: $\int_0^1 dx_2 \, f_{\bar{q}}(x_2)\, \delta(x_1 x_2 S - M_Z^2) = f_{\bar{q}}\left(\frac{M_Z^2}{S x_1}\right) \cdot \frac{1}{x_1 S} \cdot \theta(x_1 S - M_Z^2).$

$$\Rightarrow \sigma = \frac{4\pi^2}{3} \frac{\Gamma_Z}{M_Z} \cdot \frac{1}{S} \sum_q{}' \int_{M_Z^2/S}^{1} \frac{dx_1}{x_1} \cdot 2 f_q(x_1) f_{\bar{q}}\left(\frac{M_Z^2}{S x_1}\right) \cdot Br(Z \to q\bar{q}).$$

↑ if $p\bar{p}$ collision, $2 f_q f_{\bar{q}} \to f_q f_q + f_{\bar{q}} f_{\bar{q}}$

since $f_{\bar{q}}^{\bar{p}} = f_q^{p}$ and vice versa!

Note: $x_{min} = \frac{M_Z^2}{S} \approx 2.5 \times 10^{-3}$ @ Tevatron, 1.6×10^{-4} @ LHC-7

or 4.0×10^{-5} @ LHC-14

in general, $x_{min} \simeq \frac{E_{min}^2}{S}$ for a process requiring $\sqrt{\hat{s}} \geq E_{min}$.

"typical" x: $f(x) \cdot f\left(\frac{x_{min}}{x}\right)$ maximal if $x \sim \sqrt{x_{min}}$

(rough rule-of-thumb; depends on σ and other details!)

For example, $u\bar{u}$ contribution:

$$x u(x) = \begin{cases} -1.2 - 0.9 \log_{10} x, & 10^{-2} < x < 10^{-3} \\ 0.7, & 10^{-2} < x < 0.1 \\ -0.2 - 0.9 \log_{10} x, & 0.1 < x < 0.6 \\ 0, & x > 0.6 \end{cases}$$

Tevatron: $\int_{x_{min}}^{1} \frac{dx_1}{x_1} u(x_1) u\left(\frac{x_{min}}{x_1}\right) \simeq 600$, $\int_{x_{min}}^{1} \frac{dx_1}{x_1} \bar{u}(x_1) \bar{u}\left(\frac{x_{min}}{x_1}\right) \simeq 5$ - valence-quarks dominate!

Plug in numbers $\Rightarrow$ $\sigma(p\bar{p}\to Z) \approx 3$ nb, $u\bar{u}$ alone, LO

Actual $\sigma_{NLO} = 8.2$ nb [slide] not bad!

LHC 14: $\sigma(p\bar{p}\to Z) \approx 60$ nb ($u\bar{u}$ alone, LO) $\sigma_{NLO} = 60$ nb (luck!)
sea quarks dominate: $x \approx \sqrt{4\times10^{-5}} \approx 6\times10^{-3}$ - in the "sea" range...

[data-slide] HW: Do it for LHC-7

Note: cross section grows with $\sqrt{s}$, due to growth of pdf's at low x.

Replace Z with heavy Z' (same couplings):

$$\sigma(pp\to Z'X) = \frac{8}{3}\pi^2 \frac{\Gamma_{Z'}}{M_{Z'}} \cdot \frac{1}{s} \cdot \Bigg[\underbrace{\int_{M_{Z'}^2/s}^{1} \frac{dx}{x} f_u(x) f_{\bar{u}}\left(\frac{M_{Z'}^2}{sx}\right)}_{F_u(M_{Z'}^2/s)} + \ldots \Bigg]$$

Very roughly, $F_u \approx 0.09\left(\frac{\sqrt{s}}{M_{Z'}}\right)^3$ ($M_{Z'} \sim 0.1 - 1$ TeV)

Generally $\Gamma_{Z'} \propto M_{Z'} \Rightarrow \sigma \propto M_{Z'}^{-3}$.

3) Z decays and observables

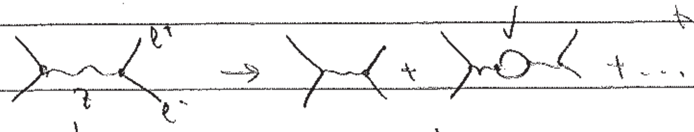
Z decays "promptly" ($c\tau \ll 1\,\mu m$) $\Rightarrow$ see decay products

Decays: $Z \to q\bar{q} \to$ jets $\leftarrow$ invisible: $\sigma(Z) \lll \sigma(\text{jets})$ (plot)
$Z \to e^+e^-, \mu^+\mu^-$ $\leftarrow$ easily visible! (event display)

"Invariant mass" $M_{inv} = \sqrt{(p_{e^+} + p_{e^-})^2}$ is frame-independent
$\Rightarrow$ compute in parton frame, measure in lab frame.

LO: $M_{inv} = M_Z$, $\frac{d\sigma}{dM_{inv}} \propto \delta(M_{inv} - M_Z)$

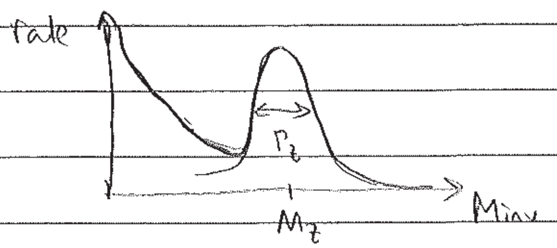
NLO: Breit-Wigner peak



$\frac{1}{p^2 - M_Z^2} \to \frac{1}{p^2 - M_Z^2 + i\Gamma_Z M_Z}$ if $\Gamma_Z \ll M_Z$

"narrow-width approximation"

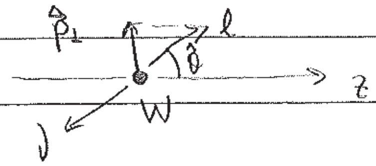
$$\rightarrow \frac{d\sigma}{dM_{inv}} \propto \frac{1}{(M_{inv} - M_Z)^2 + M_Z^2\Gamma_Z^2}$$



– "inv. mass peak" or "bump" – signature of unstable new particle.
[plot]

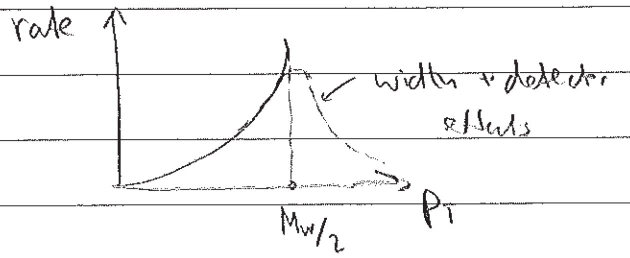
What if $Z \to W$? $W \to \ell\nu$ – p_ν unknown!

- Lepton p_T: partonic c-of-m frame = W rest frame



$$p_T = |\vec{p}_\perp| = |\hat{\vec{p}}_\perp| = \hat{E}\sin\hat{\theta} = \frac{1}{2} M_W \sin\hat{\theta} \quad \Rightarrow \quad p_T^{max} = \frac{1}{2} M_W.$$

$$\frac{d\sigma}{dp_T} = \frac{d\sigma}{d\cos\hat{\theta}} \frac{d\cos\hat{\theta}}{dp_T} = \frac{p_T}{\sqrt{\left(\frac{M_W}{2}\right)^2 - p_T^2}} \frac{d\sigma}{d\cos\hat{\theta}}$$



"Jacobean peak" [slide]

• "Transverse mass" $\quad \vec{p}_\perp = -\vec{p}_{\perp,\ell} \equiv \hat{\vec{p}}_{\perp,1}$

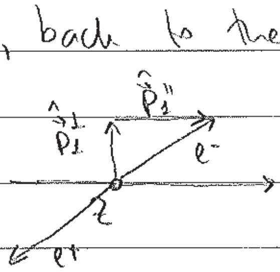
$$M_T^2 \equiv \left(|\vec{p}_\perp^{\,\nu}| + |\vec{p}_\perp^{\,\ell}|\right)^2 - \left(\vec{p}_\perp^{\,\nu} + \vec{p}_\perp^{\,\ell}\right)^2$$

<u>HW</u>: Show that $\quad 0 \le M_T^2 < M_W^2$

M_T distribution also has a Jacobean peak [slide]

$\Rightarrow$ Tevatron $\quad \frac{\Delta M_W}{M_W} \approx \frac{31\,MeV}{80.4\,GeV} \sim 4 \times 10^{-4}$ best in the world!

Okay, back to the Z. Again, parton frame $\to$ Z at rest.



"Pseudo-Rapidity" $\hat{\eta}$:

$$\hat{p}_1^{\parallel} = \hat{p}_1^{\perp} \sinh\hat{\eta}_1$$

$$\hat{E}_1 = \sqrt{(\hat{p}_1^{\parallel})^2 + (\hat{p}_1^{\perp})^2} = \hat{p}_1^{\perp} \cdot \cosh\hat{\eta}_1$$

$\hat{\eta}_2 = -\hat{\eta}_1 \qquad \vec{p} \to (p^{\parallel}, p^{\perp}, \phi) \to (\eta, p^{\perp}, \phi)$ – canonical description

Parton → lab frame: $p^\perp$, ϕ invariant

HW: Show that $\eta_1 = \hat{\eta}_1 + \eta_Z$,

where $\eta_Z = \frac{1}{2}\log\left(\frac{S}{M_Z^2} x_1^2\right)$ – independent of $\hat{\eta}_1$, same for all particles – additive Lor. tr. for η!

$= \tanh^{-1}\beta$

Likewise $\eta_2 = \hat{\eta}_2 + \eta_Z = -\hat{\eta}_1 + \eta_Z$

$\Rightarrow \eta_Z = \frac{1}{2}(\eta_1 + \eta_2)$ So, measure $\eta_1, \eta_2 \Rightarrow Z$ velocity in lab frame ⇒ p.d.f. information.

[slide: NNLO calculation vs. data]

Lab-frame η observable: $p_1^{\parallel} = p_1^\perp \sinh\eta_1$

$$\Rightarrow \tan\theta_1 = \frac{1}{\sinh\eta_1}.$$

$\eta_{max} = \frac{1}{2}\log\left(\frac{S}{M_Z^2}\right)$ = 3.0 Tevatron ($\theta_{min} = 5.7°$)
5.0 LHC ($\theta_{min} = 0.8°$)

Detectors designed roughly to cover out to this η.

"Heavier" final state (i.e. larger $M_{inv}^2 = (\sum_{i=1}^{n} p_i)^2$)

⇒ smaller η_{max} – more "central"!

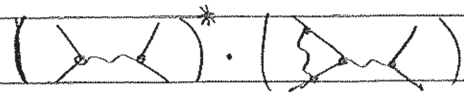
Of course, individual particles may have larger η, may not be detected.

IV. Radiative Corrections and Infrared Divergences

Naively, subleading orders in pert. theory are only important if you need presision. In reality though, certain aspects of beyond-LO physics must be understood even to develop qualitative intuition about collider processes. The reason is IR divergences.

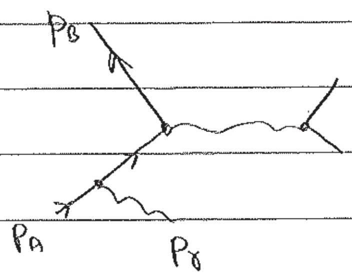
Let's start with a QED example: $e^+e^- \to \mu^+\mu^-$

LO: $\overline{|\mathcal{M}|^2} \propto \alpha^2$

NLO:  $+ \text{h.c.} \propto \alpha^3$ ("loop corr.")

 $\propto \alpha^3$ ("real emission")

Let's focus on real emission, one diagram:



$$= \bar{U}_B \gamma^\mu \frac{\not p_A - \not p_\gamma}{(p_A - p_\gamma)^2} (e\gamma^\nu) U_A \cdot \varepsilon_\nu(p_\gamma) \cdot A_\mu$$

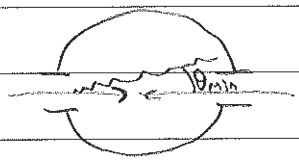
$$p_A = (E, 0, 0, E) \qquad p_\gamma = \left(zE, \vec{p}_\perp, \sqrt{(zE)^2 - \vec{p}_\perp^{\,2}}\right)$$

$$(p_A - p_\gamma)^2 = -2 p_A \cdot p_\gamma = -2zE^2 \left(1 - \sqrt{1 - \frac{\vec{p}_\perp^{\,2}}{(zE)^2}}\right)$$

$\mathcal{M} \to \infty$ when $z \to 0$ "soft singularity"

or $|\vec{p}_\perp| \to 0$ "collinear singularity"

Physically, soft + collinear photons are unobservable:



- Small-θ γ's escape through holes
- Small-E γ's drowned in "noise"

Define Q: $p_\perp > Q$ - observe $2\to3$

$p_\perp < Q$ - observe $2\to2$

~~Small $p_\perp$ ⇒~~ $(p_A-p_\gamma)^2 \to 0 \quad \Rightarrow \quad \not{p}_A - \not{p}_\gamma \approx \sum_s u_s(p_A-p_\gamma)\,\bar{u}_s(p_A-p_\gamma)$

$$(p_A-p_\gamma)^2 \approx -\frac{p_\perp^2}{z}$$

$$\Rightarrow \quad \mathcal{M} \approx -\frac{z}{p_\perp^2}\sum_s{}' \underbrace{\left[e\,\bar{u}_s(p_A-p_\gamma)\,\gamma^\nu u_A\,\varepsilon_\nu(p_\gamma)\right]}_{e\to e\gamma \text{ amp., ind. of the rest - "factorization"}} \cdot \mathcal{M}(p_A-p_\gamma, p_B \to p_1, p_2)$$

$$d\sigma_{2\to3} \approx \frac{2\hat{s}}{2s}\, d\Pi_\gamma \left(\frac{z}{p_\perp^2}\right)^2 \underbrace{\sum_s{}' |\mathcal{M}(e\to e\gamma)|^2}_{\frac{4e^2p_\perp^2}{z(1-z)}\left[\frac{1+(1-z)^2}{z}\right]} \cdot \underbrace{|\mathcal{M}_{2\to2}(\hat{s}=(1-z)s)|^2\, d\Pi_1 d\Pi_2 \frac{1}{2\hat{s}}}_{d\sigma^{LO}_{2\to2}(\hat{s})}$$

- P&S pp. 576-578

$$d\Pi_\gamma = \frac{d^3p_\gamma}{(2\pi)^3}\frac{1}{2E_\gamma} = \frac{p_\perp dp_\perp}{8\pi^2}\frac{dz}{z}$$

$$\Rightarrow \quad \sigma_{2\to3} \approx \int_{z_{min}}^{1}\frac{dz}{z}\int_Q^{\sim E}\frac{p_\perp dp_\perp}{8\pi^2}\cdot(1-z)\cdot\left(\frac{z}{p_\perp^2}\right)^2\cdot\frac{4e^2p_\perp^2}{z(1-z)}\cdot\left[\frac{1+(1-z)^2}{z}\right]\cdot\sigma^{LO}_{2\to2}(\hat{s})$$

$z_{min} = \frac{Q}{E}$

↑ 2 log divergences!

$$\approx \frac{\alpha}{\pi} \log^2 \frac{\sqrt{s}}{Q} \cdot \sigma^{LO}_{2\to 2}(s) + (1 \text{ or } 0 \text{ logs}).$$

↑ "Sudakov double log"

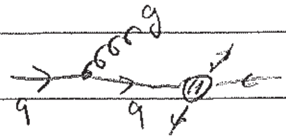
$$d\sigma_{2\to 2} \approx \int_0^{1} \frac{dz}{z} \int_0^{Q} \frac{dp_\perp}{p_\perp} (\ldots)\, \sigma^{LO}_{2\to 2}(\hat{s}) = \int_0^1 dz \frac{\alpha}{2\pi} \left[\frac{1+(1-z)^2}{z} \right] \log \frac{Q}{m_e} \cdot$$

↑ $0 \to m_e$, if you're careful!

$$\times\, \sigma_{2\to 2}(\hat{s} = s(1-z)).$$

This looks like parton model! e beam is really eγ beam, with fraction of γ's depending on Q. Of course, Q is unphysical, so the total rate is independent of it:

$$\frac{d}{dQ}\left(\sigma_{2\to 2} + \sigma_{2\to 3}\right) = 0.$$

Same thing happens in QCD: e.g. 

$p_{T,g} < Q \Rightarrow$ correction to p.d.f.'s (Altarelli-Parisi eq's) $\Rightarrow f(x, Q)$

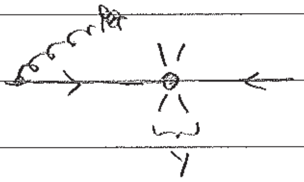
$p_{T,g} > Q \Rightarrow$ extra jet

If final state Y with inv. mass $M_{inv}(Y)$ is produced, then

$$\sigma(Y + \text{jet}, p_T > Q) \approx \frac{\alpha}{\pi} \log^2 \frac{M_{inv}(Y)}{Q} \cdot \sigma_{LO}(Y \text{ with no jet})$$ – lots of low-p_T jets!

inclusive: $\sigma(Y) + \sigma(Y + \text{jet}) = \sigma_{LO}(Y)\left(1 + c\frac{\alpha}{\pi}\right)$ (no logs!)

Soft/Collinear Divergences



$p_{T,g} < Q$ - pdf correction

$p_{T,g} > Q$ - $pp \to Y + \text{jet}$

$$\frac{\sigma(pp \to Y + \text{jet})}{\sigma(pp \to Y)} \sim \frac{\alpha_s}{\pi} \log^2 \frac{M_{inv}^2(Y)}{Q^2}$$

where $M_{inv}(Y) = \left(\sum_{i=1}^{n} p_i\right)^2$

Ex. : $pp \to Z + j$ $\quad \frac{\alpha_s}{\pi} \log^2 \frac{M_Z^2}{Q^2} \sim 1$ for $Q \sim 10$ GeV

$\Rightarrow$ Z prod. is typically accompanied by 1 or more 10 GeV jets!

slide: p_T^Z distribution, jet mult.

Inclusive cross section:

$$\sigma_{inc}(pp \to Y + \geq 0 \text{ jets}) = \int dx_1 dx_2 \, f_{f_1}(x_1, Q^2) \, f_{f_2}(x_2, Q^2)$$

$$\times \Big(\sigma_{LO}(f_1 f_2 \to Y) + \underbrace{\sigma_{NLO}(f_1 f_2 \to Y) + \sigma_{LO}(f_1 f_2 \to Y + j, p_T > Q)}_{\mathcal{O}(\alpha) \cdot \sigma_{LO}} + \dots \Big)$$

$\frac{d\sigma_{inc}}{dQ} = 0$ **but** f's involve "resummation" of leading logs to all orders in α, while partonic σ is fixed-order

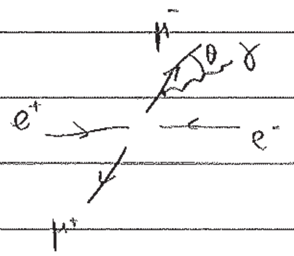
$\Rightarrow$ cancellation is inexact!

Choice of Q: minimize log's in partonic x-section ⇒ $Q^2 = M^2_{inv}(Y)$

Estimate Q dependence: vary up/down by a factor of 2.

Final State Radiation

$e^+e^- \to \mu^+\mu^-\gamma$

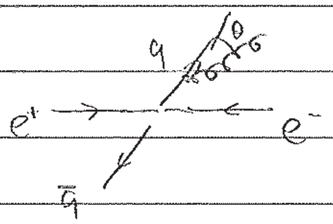


sing.: $E_\gamma \to 0$ (unobs.)

$\theta \to 0$ (cut off by M_μ)

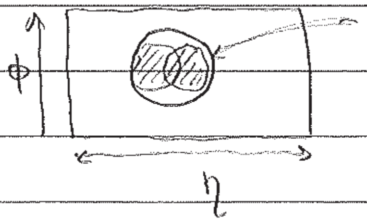
$$\sigma(2\to3, E_\gamma > E_{min}) \approx \sigma_{LO}(2\to2) \cdot \frac{\alpha}{\pi} \log\frac{E_\mu}{E_{min}} \log\frac{E_\mu}{m_\mu}$$ — "Sudakov double-log"

$e^+e^- \to q\bar{q}g$



Same sing., but collinear sing. is cut off in a different way:

Jet



"core" → jet

q and g "merged" for small θ

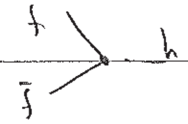
$\Delta R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} < \Delta R_{min}$ – 1 jet

$> \Delta R_{min}$ – 2 jets

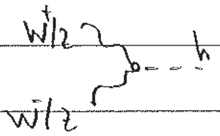
n-jet rates depend <u>strongly</u> on the details of jet-finding algorithm, very hard to predict; inclusive rates are much safer ("IR-safe observable")

Multiple splittings likely → PYTHIA, HERWIG, SHERPA
"showering codes"

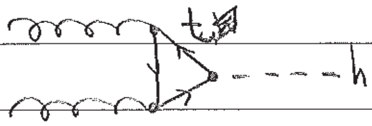
V. Higgs Search Example

1) Couplings: 

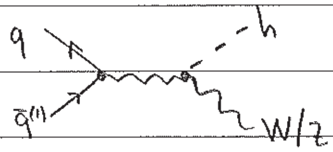
$y_h = \frac{M_f}{v} << 1$ except t



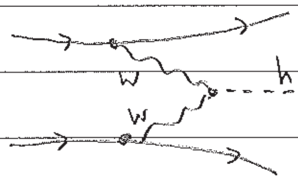
$\sim g v$

2) Processes: 1 h: 

"gluon fusion"

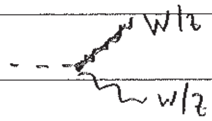
h + W/Z: 

"Bjorken process" or "associated production"

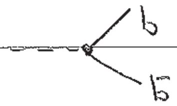
h + qq 

"Vector Boson Fusion"

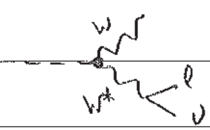
[slides: x-sections]

3) Stable? No! Decays: 

if $M_h \gtrsim 2M_W = 160$ GeV



$\propto y_b^2$ if $M_h < 2M_W$

competes with 3-body 

for 130-160 GeV

Subdominant modes: $h \to c\bar{c}$, $h \to \tau^+\tau^-$

+ 1-loop induced: $h \to gg$, $h \to \gamma\gamma$

[slide: Br's]

(assume $M_h < 130$ GeV)

4). Backgrounds: $h \to b\bar{b}, c\bar{c}, gg$ completely swamped

[x-sections S vs Bg slide]

$h \to \gamma\gamma$ or $h \to \tau^+\tau^-$ - small Br's, but best S/Bg!

[slides: $h \to \gamma\gamma$ search from CMS]

Note: bump hunting: "shoulder-subtraction" technique
$\Rightarrow$ reduced need for MC,
systematics $\to$ statistics!