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School on Strongly Coupled Physics Beyond the Standard Model

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Technicolor Theories - Lecture 1

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Trieste Lectures on Technicolor

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Lecture 1:
Technicolor

Introduction

TC (classical defn): models where EW not broken by Higgs boson, instead by non-perturbative effects in a strongly-coupled theory.

LHC 12/13/2011: $m_h = 125$ GeV?

So why study TC?

- needs confirmation
- strong dynamics may be part of picture (e.g. composite or partly composite Higgs)
- adds to field theory toolkit

Plan of Lectures

Lecture 1: TC as theory of EWSB; barrier of pheno; intro to problems

Lecture 2: conformal/walking TC; approaches to flavor

Lecture 3: SUSY + TC

Standard Model Wallet Card

$$SU(3)_c \times SU(2)_W \times U(1)_Y$$

L Weyl fermions

$$\left.\begin{aligned} q &\sim (3,2)_{\frac{1}{6}} \\ u^c &\sim (\bar{3},1)_{-\frac{2}{3}} \\ d^c &\sim (\bar{3},1)_{\frac{1}{3}} \\ l &\sim (1,2)_{-\frac{1}{2}} \\ e^c &\sim (1,1)_1 \end{aligned}\right\} \times 3$$

scalar

$$H \sim (1,2)_{\frac{1}{2}}$$

$$\begin{aligned} D_\mu q = \partial_\mu q &- i g_s G_{\mu A} \tfrac{1}{2}\lambda_A q \\ &- i g W_{\mu a} \tfrac{1}{2}\tau_a q \\ &- i g' B_\mu \left(\tfrac{1}{6}\right) q \end{aligned}$$

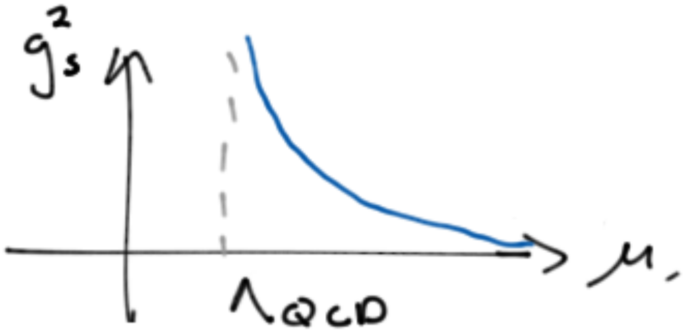
$$\begin{aligned} D_\mu u^c = \partial_\mu u^c &- i g_s G_{\mu A}\left(-\tfrac{1}{2}\lambda_A^T\right) u^c \\ &- i g' B_\mu \left(-\tfrac{2}{3}\right) u^c \end{aligned}$$

$\vdots$

SM w/o Higgs

QCD breaks EW!

$$\mu \frac{d}{d\mu} g_s^2 = -\frac{1}{8\pi^2}\left(11 - \frac{2}{3} N_f\right)$$



$$\Lambda_{QCD} \sim \underbrace{1\,\text{GeV}}_{\sim m_\rho}$$

One generation ($N_f = 2$)

$E \lesssim \Lambda_{QCD}$: non-perturbative

$$\frac{g_s^2 N_c}{16\pi^2} \sim 1$$

Confinement, χSB: mysterious, non-perturbative

Confinement: physical spectrum $=$ hadrons ($\pi, \rho, \dots$)

Chiral Symmetry Breaking (XSB)

QCD sector has approx global symmetry

$$SU(2)_L \times SU(2)_R \times U(1)_B$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix} \sim (2,1)_{\frac{1}{3}}$$

$$q^c = \begin{pmatrix} u^c \\ d^c \end{pmatrix} \sim (1,\bar{2})_{-\frac{1}{3}}$$

$$\langle q^a q^c_b \rangle \sim \Lambda^3_{QCD}\, \delta^a{}_b$$

$\Rightarrow$ spontaneous breaking

$$SU(2)_L \times SU(2)_R \longrightarrow SU(2)_{L+R}$$

$(U(1)_B$ unbroken$)$

Breaks EW symmetry!

EW by QCD

$$T_a q = \frac{1}{2}\tau_a q \qquad T_a q^c = 0$$

$$Y q = \frac{1}{6} q \qquad Y q^c = \left(-\frac{1}{6} 1_2 - \frac{1}{2}\tau_3\right) q^c$$

$\Rightarrow Q = T_3 + Y$ unbroken

$\Rightarrow W, Z$ massive, γ = massless

$SU(2)_{L+R}$ = custodial symmetry

$$\Rightarrow \frac{m_W^2}{m_Z^2} = \cos^2\theta_W \quad (T \text{ small})$$

Chiral Lagrangian

Effective theory for $E \ll \Lambda_{QCD}$

Only light hadrons = $\pi^0, \pi^\pm$

$SU(2)_L \times SU(2)_R$ nonlinearly realized

$$\Sigma(x) = e^{i\Pi(x)/f} \in SU(2)$$

$$\Pi(x) = \Pi_a(x) T_a = \frac{1}{2}\begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$\Sigma \mapsto L \Sigma R \qquad L \in SU(2)_L,\ R \in SU(2)_R$$

$$D_\mu \Sigma = \partial_\mu \Sigma - i g W_\mu \frac{\tau_a}{2} \Sigma - i g' B_\mu \left(-\frac{1}{2}\right) \Sigma$$

$$\mathcal{L}_{eff} = \frac{f^2}{2} \mathrm{tr}\,(D^\mu \Sigma^\dagger D_\mu \Sigma) + \mathcal{O}(\partial^4)$$

Infinitely many higher deriv terms suppressed by powers of $E^2/\Lambda_{QCD}^2 \Rightarrow$ predictive low energy expansion.

W, Z Masses

Unitary gauge: $\Pi(x) \equiv 0$

$$\mathcal{L}_{eff} = \frac{f^2}{2} \operatorname{tr} \left| i g W_{\mu a} \frac{\tau_a}{2} - i g' B_\mu \frac{1}{2} \right|^2$$

$$+ \cdots$$

$$\Rightarrow m_W^2 = g^2 \frac{f^2}{2} \cdot \frac{1}{4} \cdot 2 = \frac{g^2 f^2}{4}$$

$$m_Z^2 = \frac{(g^2 + g'^2) f^2}{4}$$

QCD: $f = f_\pi = 90$ GeV

SM: want $f = v = 246$ GeV

Note: explicit W, Z mass terms...

Naive Dimensional Analysis

Note $f_\pi \simeq 90\ \text{GeV} \sim \frac{1}{10} m_\rho$

QCD has no small param $\Rightarrow$

should both be $\sim \Lambda_{QCD}$

More precise definition of "scale of strong dynamics" (count powers of 4π)

QCD: match perturbative QCD to hadronic theory

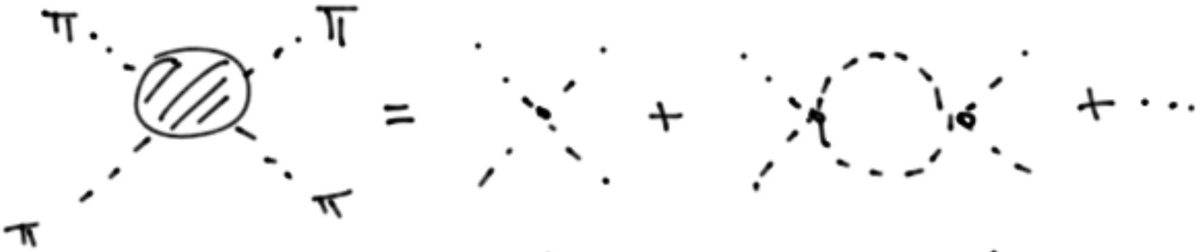
Pert QCD gets strong at scale Λ_{QCD}

$$\frac{g^2(\Lambda_{QCD})}{16\pi^2} \sim 1$$

(assume $N_c \sim 1$)

Expect $\Lambda_{QCD} \sim \underbrace{m_\rho}_{\text{lowest-lying hadron}}$

Chiral Lagrangian gets strong in UV



$$\sim \frac{E^2}{f^2} + \frac{1}{16\pi^2}\frac{E^4}{f^4}\ln E + \cdots$$

Breaks down at

$$E \sim \Lambda_{\chi PT} \sim 4\pi f$$

NDA assumption: $\Lambda_{\chi PT} \sim \Lambda_{QCD}$

Motivated by fact that QCD is a theory defined by a single scale with no small parameter

Assume all couplings in low-E effective theory get strong at Λ

Technicolor (Weinberg; Susskind)

Copy QCD!

Add to SM

$$SU(N)_{TC} \times SU(2)_W \times U(1)_Y$$

$$Q \sim (N, 2)_y$$

$$U^c \sim (\bar{N}, 1)_{-y-\frac{1}{2}}$$

$$D^c \sim (\bar{N}, 1)_{-y+\frac{1}{2}}$$

g_{TC} strong at $\Lambda_{TC} \sim 1$ TeV

$$Q^c = \begin{pmatrix} U^c \\ D^c \end{pmatrix} \Rightarrow SU(2)_L \times SU(2)_R$$

approx symmetry

$$\langle Q^a Q^c_b \rangle = \Lambda^3_{TC} \delta^a{}_b$$

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)$$

$\Rightarrow$ electroweak breaking...

Why is this a good idea?

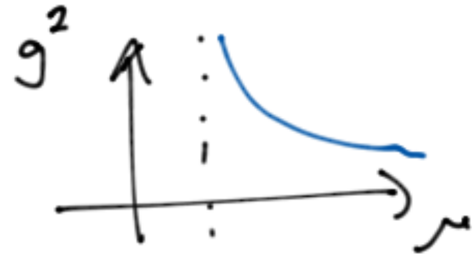
Naturalness

Why is $M_W \ll M_P$ (or M_{GUT})?

$M_W \sim 100\text{ GeV} \qquad M_P \sim 10^{19}\text{ GeV}$

Technicolor:

Asymptotic freedom



$$\Rightarrow \Lambda_{TC} = \mu_0\, e^{-8\pi^2 / b\, g_{TC}^2(\mu_0)}$$

$$b = \frac{11}{3} N - \frac{2}{3} \underbrace{N_f}_{=2}$$

$$g_{TC}(M_{GUT}) \sim 1 \Rightarrow \underbrace{\Lambda_{TC} \ll M_{GUT}}_{\text{generic!}}$$

Unnaturalness *

SM w/ elementary Higgs

$$V = m_H^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$m_H^2 < 0$
$\lambda > 0$

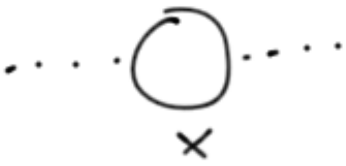
$$m_H^2 < 0 \Rightarrow v^2 = -\frac{m_H^2}{\lambda}$$

Perturbative control of WW scatt

$\Rightarrow |m_H| \lesssim \text{TeV} \ll M_P$

m_H^2 = effective mass param at weak scale

$$= m_{H0}^2 + \underbrace{\frac{y_X^2}{16\pi^2} M_X^2}_{} + \cdots$$



$$\Rightarrow \text{tuning} \sim \frac{m_H^2}{m_X^2}\,\frac{16\pi^2}{y_X^2} \sim 10^{-26}$$

for $y_X m_X \sim M_{GUT}$

MSSM *

Good example of <u>un</u>naturalness!

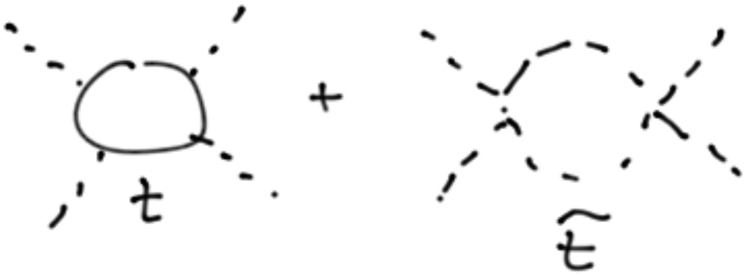
Tree level:

$$m_h^2 = \text{physical Higgs mass}$$

$$\sim \lambda v^2 \sim g^2 v^2 \sim m_Z^2$$

$\Rightarrow$ need loop correction to λ

$\Delta\lambda \sim$



$$\sim \frac{N_c y_t^4}{16\pi^2} \underbrace{\ln \frac{m_{\tilde{t}}}{m_t}}$$

$\Rightarrow$ need large $m_{\tilde{t}}$

$$\text{But } \Delta m_H^2 \sim \frac{N_c y_t^2}{16\pi^2} m_{\tilde{t}}^2$$

$\Rightarrow \sim 1\%$ tuning

WW scattering

Chiral Lagrangian breaks down for $E \gtrsim \Lambda_{QCD}$.

How do we see this in unitary gauge ($\Leftrightarrow$ explicit $m_{W,Z}$)?

W, Z masses do not decouple at high energy

massless spin 1 : 2 pol

massive spin 1 : 3 pol

Study behavior of "extra" polarization at high E

(Lee, Quigg, Thacker)

In cm frame

$$p_W^\mu = \left(\sqrt{p^2+m_W^2}\,,\ 0,\ 0,\ p\right)$$

$p \gg m_W \Leftrightarrow$ high energy scattering

Physical polarizations

$$\varepsilon_T^\mu = (0, *, *, 0)$$

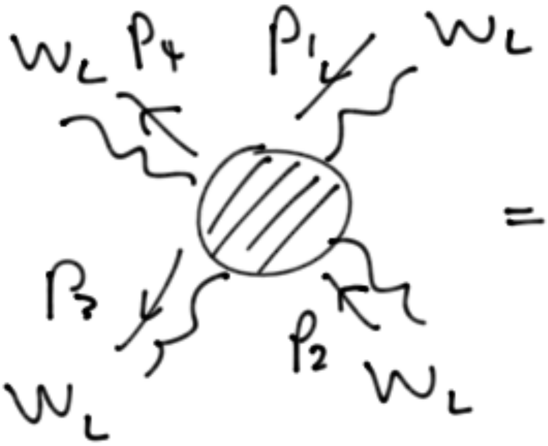
$$\varepsilon_L^\mu = \frac{1}{m_W}\left(p, 0, 0, \sqrt{p^2+m_W^2}\right)$$

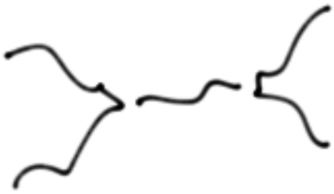
$$\varepsilon_L^2 = \frac{1}{m_W^2}\left[\cancel{p^2} - (\cancel{p^2} + m_W^2)\right] = -1 \quad \boxed{\checkmark}$$

$$p \gg m_W \Rightarrow p^\mu \simeq (p, 0, 0, p)$$

$$\varepsilon_L^\mu \simeq \frac{1}{m_W}(p, 0, 0, p)$$

$$\simeq \frac{p^\mu}{m_W}$$



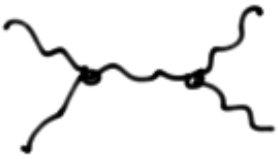
=  + 

+ crossed

 $= g^2\Big[\varepsilon_L(p_1)\cdot\varepsilon_L(p_2)\ \varepsilon_L(p_3)\cdot\varepsilon_L(p_4)$
$+ 2 \text{ more terms}\Big]$

$$\simeq g^2\left[\frac{p_1\cdot p_2 \ \ p_3\cdot p_4}{m_W^4} + \cdots\right]$$

$$\sim \cancel{g^2\frac{E^4}{m_W^4}}$$ leading terms cancel

 $\sim g^2\left[\cancel{\frac{E^4}{m_W^4}} + \frac{E^2}{m_W^2} + \cdots\right]$

 $\sim \dfrac{g^2E^2}{m_W^2} \sim \underbrace{\dfrac{E^2}{f^2}}_{\text{expansion param of chiral Lagrangian!}}$

Expansion valid for

$$\underbrace{m_W}_{\sim gf} \ll E \ll \Lambda_{QCD}$$

requires $g \ll 1$

Unitarity $\Rightarrow$  $\lesssim O(E^0)$

Effective theory "violates unitarity?"

NO! Unitarity guaranteed by Hermitian Hamiltonian!

<u>Tree level</u> amplitude violates unitarity, but higher order terms are equally important.

What is happening: <u>perturbation theory</u> is breaking down!

Minimal Technicolor

$SU(2)_{TC}: \ 2 \simeq \bar{2}$

$$\Psi^a = \begin{pmatrix} Q \\ U^c \\ D^c \end{pmatrix}$$

$$\Rightarrow SU(2)_L \times SU(2)_R \longrightarrow SU(4)$$

$$\langle \Psi^a \Psi^b \rangle = - \langle \Psi^b \Psi^a \rangle$$

$$= \Lambda^3 \begin{pmatrix} \cos\theta\, \varepsilon & \sin\theta\, 1_2 \\ -\sin\theta\, 1_2 & -\cos\theta\, \varepsilon \end{pmatrix}$$

$\theta = 0$ $\ SU(2)_W \times U(1)_Y$ unbroken

"EW vacuum"

$\theta = \frac{\pi}{2}$: "TC vacuum"

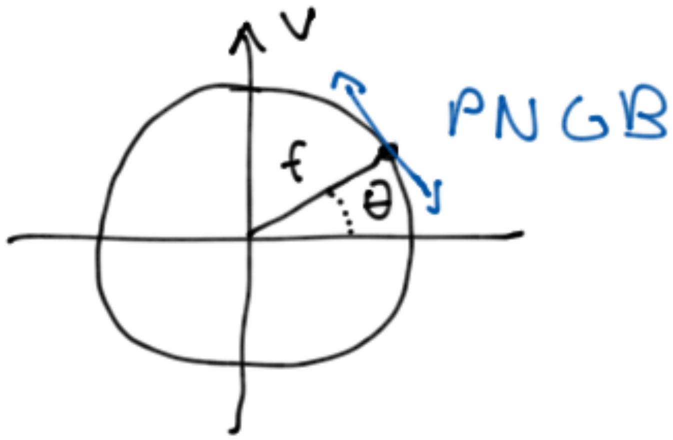
$SU(4) \longrightarrow Sp(4)$
$\quad\;\;\simeq \qquad\qquad\quad \simeq$
$SO(6) \qquad\quad SO(5)$

$\Rightarrow$ 5 NGB's

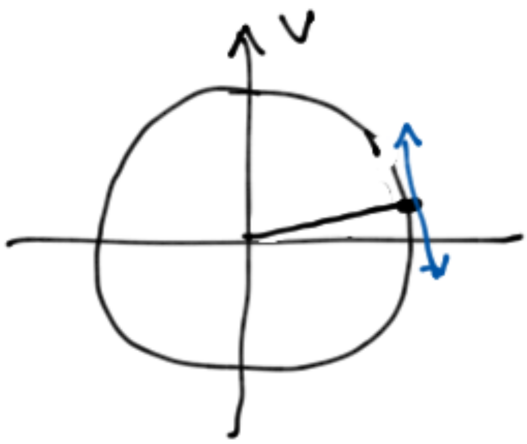
$\pi^\pm, \pi^0$ = eaten

$h^0 = CP+$, $A^0 = CP-$

$v = f \sin\theta \qquad v = 246$ GeV



$\theta \ll 1$:



PNGB $\simeq$ excitation of v

$\Leftrightarrow$ light Higgs boson

Vacuum alignment determined by explicit $SU(4)$ breaking

- EW gauge loops
  favor EW vacuum
- top quark loops
  plausibly favor TC vacuum
- technifermion mass

$$\Delta \mathcal{L} = m\, U D + \tilde{m}\, U^c D^c$$

allowed for $y = 0$

Naturally gives $0 < \theta < \frac{\pi}{2}$;
predicts $m_h \sim \sqrt{3}\, m_t \ldots$

Is this the real minimal composite Higgs model?

Fermion Masses

SM : $\Delta\mathcal{L} = y_t H^\dagger q t^c + \cdots$

$$\langle H \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \Rightarrow m_t = \frac{y_t v}{\sqrt{2}}$$

TC : $H \to Q U^c$ & $H^* \to Q D^c$

$\Rightarrow$ need new interactions

$$\Delta\mathcal{L} = \frac{c}{\Lambda_t^2}(Q U^c)^\dagger (q t^c) + \cdots$$

Higher dim op $\Rightarrow$ gets strong in UV

Define Λ_t = scale of strong dynamics

$$\Rightarrow m_t \sim \frac{\Lambda^3}{\Lambda_t^2} \quad (\text{NDA})$$

$m_t(\mu = 2\,\mathrm{TeV}) = 133\ \mathrm{GeV}$
(in a theory w/o Higgs)

$\Rightarrow \Lambda_t \sim 10\ \mathrm{TeV}$

TC requires UV completion at or below this scale

Flavor in TC

$$\Delta \mathcal{L} = \frac{c_{ij}}{M_f^2}(QU^c)^\dagger(q_i u_j^c) + \cdots$$

What about

$$\Delta \mathcal{L} = \frac{d_{ijkl}}{M_f^2}(q_i u_j^c)^\dagger(q_k u_l^c) + \cdots$$

$\Rightarrow$ dangerous FCNC's

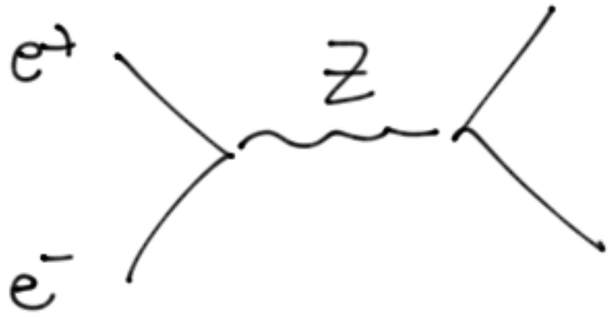
$$\frac{1}{M_f^2}(ds^c)^\dagger(sd^c) \Rightarrow M_f \gtrsim 10^7 \text{ GeV}$$

($K-\bar{K}$ mixing)

$M_f \gg \Lambda_t \Rightarrow$ special flavor structure needed

More on this in lecture 2 ...

Precision EW



, m_W, m_Z, m_t

Sensitive to 1% deviations from SM

$$\Delta\mathcal{L}_{eff} = \frac{\hat{S}}{16\pi^2} g_2 g_R \underbrace{\mathrm{tr}\left(L^{\mu\nu}\Sigma R_{\mu\nu} \Sigma^\dagger\right)}_{SU(2)_L - SU(2)_R \text{ mixing}}$$

$$+ \hat{T}\,\frac{f^2}{2} \underbrace{\mathrm{tr}\left(D^\mu \Sigma^\dagger D_\mu \Sigma \tau_3\right)}_{SU(2)_R \text{ breaking}}$$

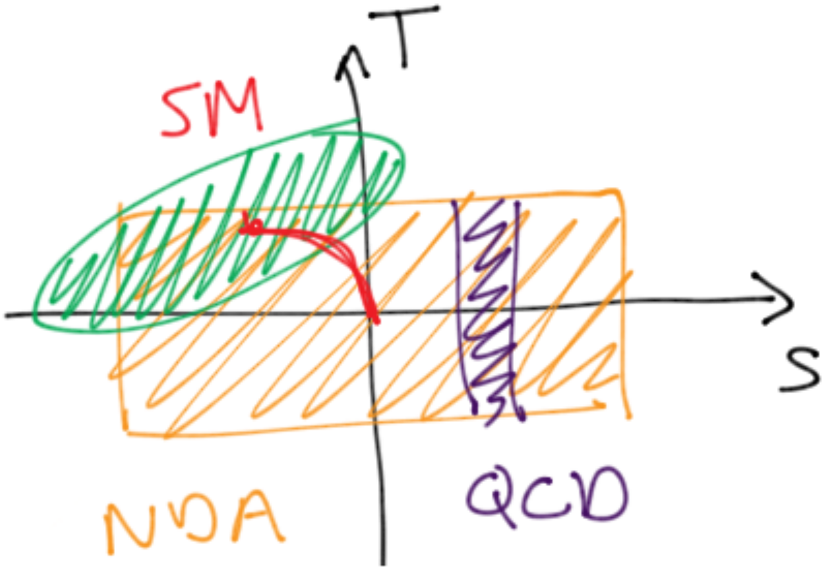
$$S = 16\pi\, \hat{S}(\mu = M_Z)$$

$$T = \frac{4\pi}{e^2}\, \hat{T}(\mu = M_Z)$$

NDA:

$$\hat{S}(\mu \sim \mathrm{TeV}) \sim 1$$

$$\hat{T}(\mu \sim \mathrm{TeV}) \sim \frac{g'^2}{16\pi^2} + \cdots$$



additional custodial symmetry breaking $\Rightarrow$ T adjustable
Need small (or negative) S

S Parameter in Technicolor

In QCD, $S > 0$ from data (dispersion relation)

Only partial theoretical understanding of this (Weinberg sum rules)

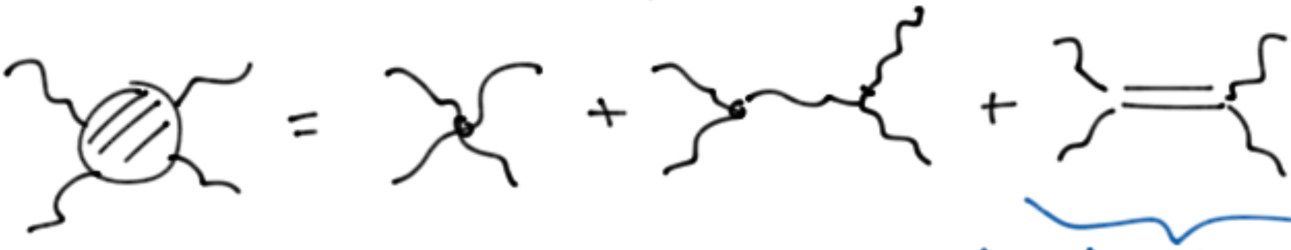
Recent lattice calculations suggest that S/doublet is significantly smaller in theories with larger N_f (where only some flavors are gauged)

Technicolor Phenomenology

Phenomenological approach:
"technicolor" = any strongly coupled theory of EW

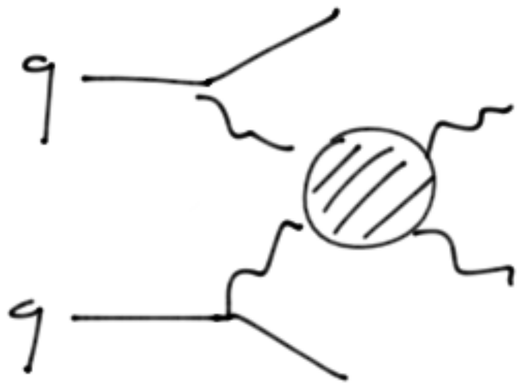
Expect strong WW scattering w/ resonances at ~ 1 TeV

$$\left[m_{\rho_{TC}} \sim m_\rho \times \frac{v}{f_\pi} \simeq 2\,\mathrm{TeV} \right]$$



2-body decay
⇒ broad resonance

At LHC:



need
$\mathcal{O}(100\,\mathrm{fb}^{-1})$
@ 14 TeV...

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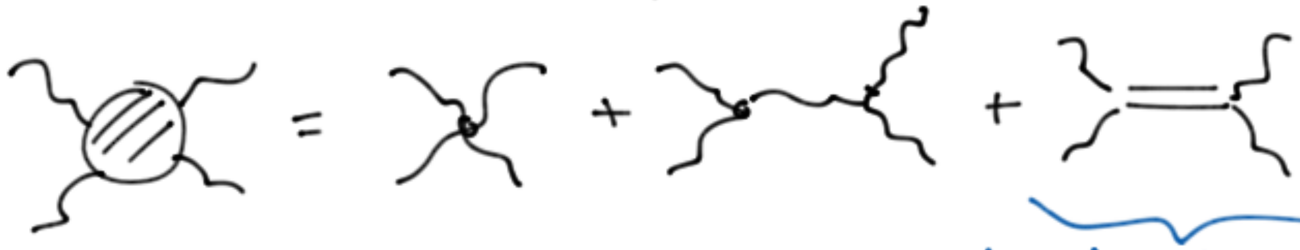

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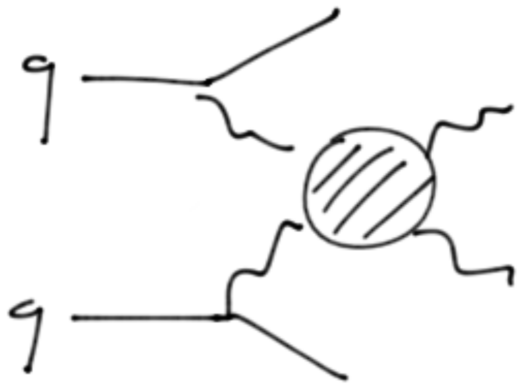


need
$\mathcal{O}(100\ fb^{-1})$
@ 14 TeV...

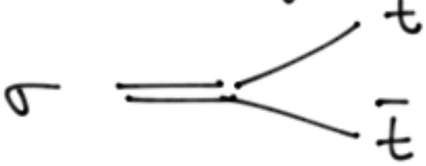
EW must also couple to top:

$$\Delta\mathcal{L} \sim \frac{1}{\Lambda_t^2} \underbrace{(QU^c)^\dagger}_{\text{spin } 0} q t^c$$

Production:

σ = spin 0, neutral

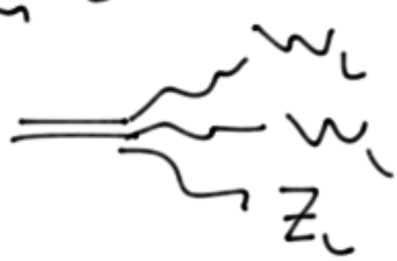
Decay:



$\Gamma/m \sim \frac{1}{10}$

2-body strong decay to $W_L W_L$ may be forbidden by discrete symmetries

$\Rightarrow$ e.g.



may compete

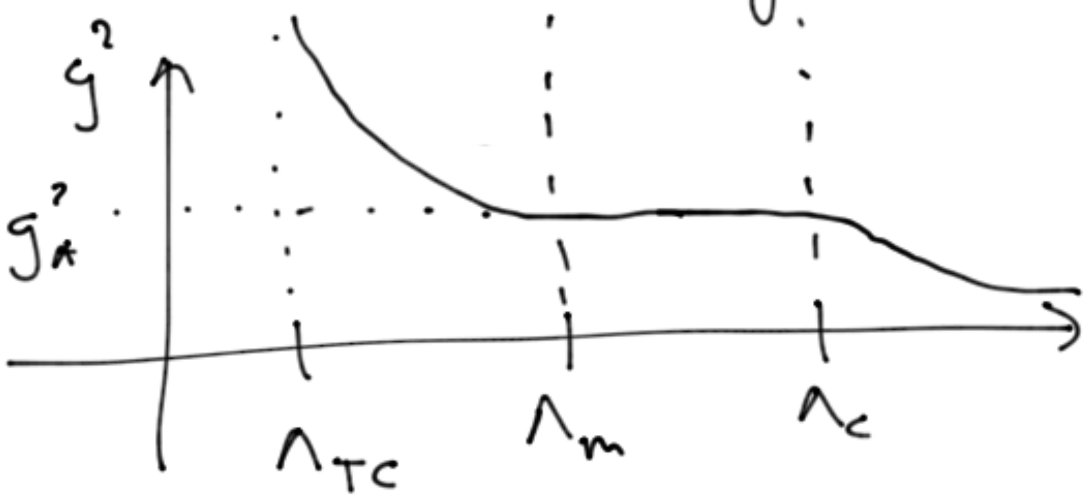
In general: high p_T production of multiple W, Z, t.
(See final states and simplified models at
lhcnewphysics.org)

What happens when it gets strong?

$\frac{g_*^2 N_c}{8\pi^2} \ll 1 \Rightarrow$ integrate out massive fermions

... at scale $\Lambda_m \sim m^{1/(4-d)}$

Effective theory below Λ_m is asymptotically free



$$\Lambda_{TC} = \Lambda_m e^{-8\pi^2/bg_*^2}$$

$$\frac{g_*^2 N_c}{8\pi^2} \sim 1 \Rightarrow \Lambda_{TC} \sim \Lambda_m$$