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School on Strongly Coupled Physics Beyond the Standard Model

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Technicolor Theories - Lecture 2

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Lecture 2: Walking/Conformal Technicolor

Dimensional Analysis and EW

SM: $\dim(H) = 1$

$\Rightarrow \dim(H^\dagger H) = 2$

$$\Delta\mathcal{L} = m_H^2 H^\dagger H \qquad m_H^2 \sim \Lambda_{UV}^2$$

$\Rightarrow \dim(H^\dagger q t^c) = 4$

$$\Delta\mathcal{L} = y_t H^\dagger q t^c \qquad y_t \sim 1$$

TC: $H \rightarrow Q U^c \qquad \dim(H) = 3$

$\Rightarrow \dim(H^\dagger H) = 6$

no UV sensitivity

$\dim(H^\dagger q t^c) = 6$

$\Rightarrow$ must address flavor at low scales

Are there theories in between?

Want: "dangerous dimensions"

$\dim(H) = d$

$\Rightarrow \dim(H^\dagger q t^c) = 3 + d$ = small

$\dim(H^\dagger H) = \Delta > 4$

$\Delta \neq 2d$ in interacting theories

$$\mathcal{L}_t \sim \frac{1}{\Lambda_t^{d-1}} (Q U^c)^\dagger q t^c$$

Λ_t = scale where top int strong

$$m_t \sim \Lambda_{TC} \left(\frac{\Lambda_{TC}}{\Lambda_t} \right)^{d-1}$$

$$\Lambda_t \sim \Lambda_{TC} \left(\frac{\Lambda_{TC}}{m_t} \right)^{1/(d-1)}$$

dimension in exponent

$$\sim \begin{cases} 10 \text{ TeV} & d = 3 \\ 40 \text{ TeV} & d = 2 \\ 600 \text{ TeV} & d = 1.5 \end{cases}$$

Scale invariant QFT

$$S = \int d^4x \, \frac{1}{2} (\partial\varphi)^2$$

$$\varphi(x) \longmapsto \underbrace{e^{\omega}}_{\text{scale fields}} \varphi(\underbrace{e^{\omega}}_{\text{scale coords}} x)$$

$\int d^4x \, \varphi^2(x)$ not invariant

$\int d^4x \, \varphi^4(x)$ invariant

Broken by quantum effects

$\lambda \rightarrow \lambda(\mu)$

μ = dimensionful

$\Rightarrow$ breaks scale invariance

dim analysis:

$$0 = \left[\mu \frac{\partial}{\partial\mu} + \sum_i x_i^\mu \frac{\partial}{\partial x_i^\mu} + n \right] \langle \varphi(x_1) \cdots \varphi(x_n) \rangle$$

renorm group:

$$0 = \mu \frac{d}{d\mu} \langle \cdots \rangle$$

$$= \left[\mu \frac{\partial}{\partial\mu} + \beta(\lambda) \frac{\partial}{\partial\lambda} - \frac{n}{2} \gamma(\lambda) \right] \langle \cdots \rangle$$

$$\beta = \mu \frac{d\lambda}{d\mu}$$

$$\gamma = \mu \frac{d}{d\mu} \ln Z \qquad \mathcal{L} = \frac{1}{2} Z (\partial\varphi)^2 + \cdots$$

$$\varphi \mapsto \varphi \quad Z \to Z^{-1/2} \Rightarrow \varphi^n \sim Z^{-n/2}$$

CZ eqn:

$$0 = \left[\sum_i x_i^\mu \frac{\partial}{\partial x_i^\mu} - \beta \frac{\partial}{\partial \lambda} + n\left(1 + \frac{\gamma}{2}\right) \right] \langle \cdots \rangle$$

fixed pt:

$\lambda \longrightarrow \lambda_*$

$\Rightarrow \beta(\lambda_*) = 0$

$\gamma(\lambda_*) = \gamma_* = \text{const}$

$$\Rightarrow 0 = \left[x_i^\mu \frac{\partial}{\partial x_i^\mu} + n \underbrace{\left(1 + \frac{\gamma_*}{2}\right)}_{\text{anomalous dim}} \right] \langle \cdots \rangle$$

Banks - Zaks Fixed Point

$SU(N_c)$ gauge theory, N_f flavors

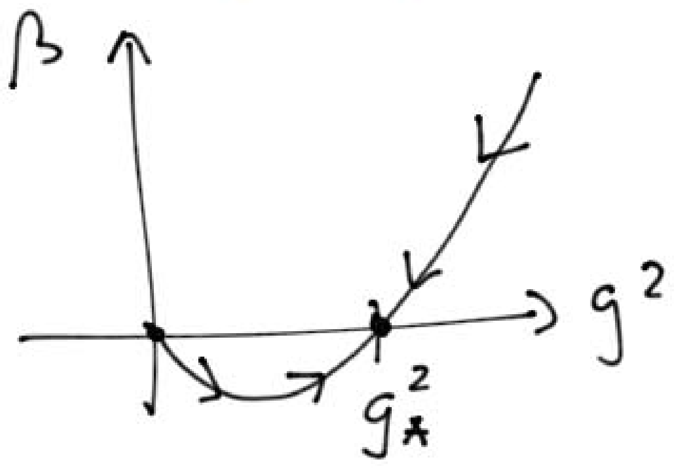
$$N_c \gg 1 \Rightarrow x = \frac{N_f}{N_c} = \text{constant}$$

$$\mu \frac{d}{d\mu}\left(N_c g^2\right) = \frac{1}{8\pi^2}\underbrace{\left(\frac{11}{3} - \frac{2}{3}x\right)}_{x_c = \frac{11}{2}}\left(N_c g^2\right)^2$$

$$+\left(\frac{1}{8\pi^2}\right)^2(\sim 1)\left(N_c g^2\right)^3$$

$$\frac{N_c g^2}{8\pi^2} \sim \quad \frac{11}{3} - \frac{2}{3}x = \varepsilon$$

loop expansion param



Conformal Symmetry

scale: $\delta\mathcal{O} = \omega\left[x^\mu\partial_\mu + \Delta\right]\mathcal{O}$

$\Longleftrightarrow \delta g_{\mu\nu} = -2\omega g_{\mu\nu}$

(+ action on fields)

ω = scale param
= param of scale trans

conformal: most general trans that leaves metric invariant up to scale: has more trans params.

Primary ops transform like

scale $\Rightarrow$ conformal?

- No 4D counterexample known
- Can prove in some cases

e.g. fixed pt in pert theory
see also talks during conference

Conformal symmetry
$\Rightarrow \Delta$ = real, diagonal
($\partial_\mu \mathcal{O} \neq$ primary ...)

Relevant and Irrelevant Ops

Perturb CFT:

$$\mathcal{L} = \mathcal{L}_{CFT} + c \cdot \mathcal{O}$$

$$\langle \Phi(x_1) \cdots \Phi(x_n) \rangle$$

$$= \underbrace{\langle \cdots \rangle_{CFT}} + \underbrace{\int d^4x \langle \mathcal{O}(x) \cdots \rangle} + \cdots$$

which dominates in IR?

$\Delta < 4$: dominates in IR
"relevant"

$\Delta > 4$: "irrelevant"

$\Delta = 4$: marginal

QCD Conformal Window

$x = \frac{N_f}{N_c}$

conformal for $\underbrace{x_c}_{=?} < x < \frac{11}{2}$

Theory w/ $x \simeq x_c$ is strongly coupled: fixed pt exists as long as pert theory is valid.

$$\frac{g_*^2 N_c}{16\pi^2} \sim 1 \quad \text{for } x \simeq x_c$$

Conformal TC

$SU(N) \times SU(2)_W \times U(1)_Y$

$$\left.\begin{aligned} Q &\sim (N, 2)_y \\ U^c &\sim (\bar{N}, 1)_{-y-\frac{1}{2}} \\ D^c &\sim (\bar{N}, 1)_{-y+\frac{1}{2}} \end{aligned}\right\} \text{min TC}$$

$$\left.\begin{aligned} \Psi &\sim (N, 1)_0 \\ \Psi^c &\sim (\bar{N}, 1)_0 \end{aligned}\right\} \times K$$

can make other choices

$$\Delta\mathcal{L} = m\Psi\Psi^c$$

explicit ~~conformal~~

$$d = \dim(QU^c) = \cdots = \dim(\Psi\Psi^c) < 3$$

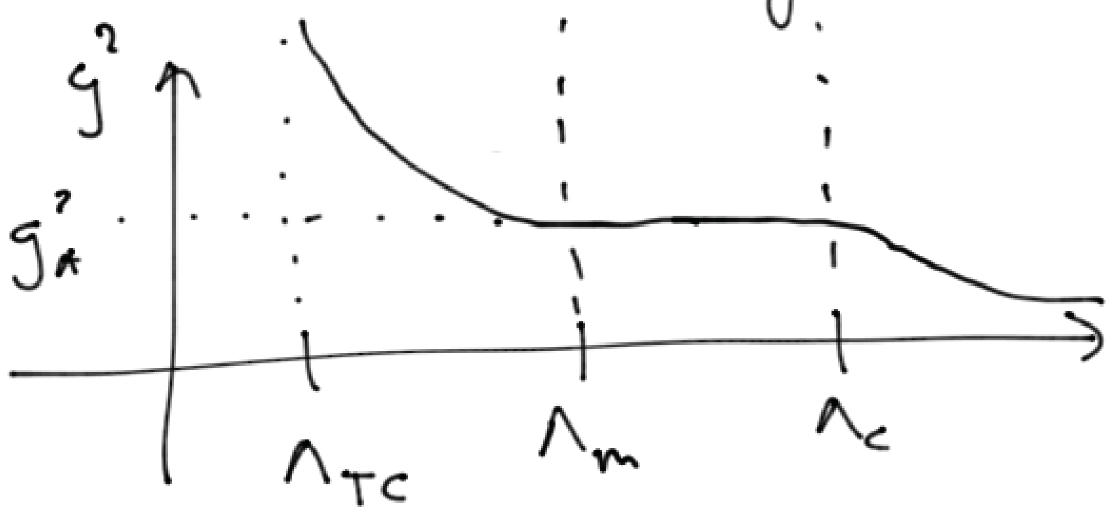
- soft breaking of conformal
- gets strong in IR

What happens when it gets strong?

$\frac{g_*^2 N_c}{8\pi^2} \ll 1 \Rightarrow$ integrate out massive fermion

... at scale $\Lambda_m \sim m^{1/(4-d)}$

Effective theory below Λ_m is asymptotically free



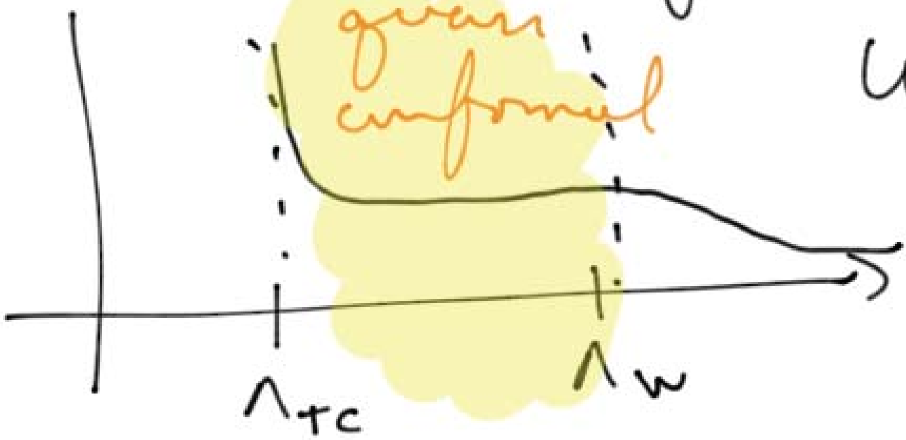
$$\Lambda_{TC} = \Lambda_m e^{-8\pi^2/b g_*^2}$$

$$\frac{g_*^2 N_c}{8\pi^2} \sim 1 \Rightarrow \Lambda_{TC} \sim \Lambda_m$$

solves hierarchy problem?
Λ_{TC} put in "by hand"
(via m). <u>soft</u> breaking of
conformal invariance, analogous
to SUSY!

Walking TC (Holdom 1982, ...)

Pure gauge theory



Why?

Plausible that conformal window ends when $\Delta < 4$

$\Rightarrow$ fixed pt unstable

$N_f = (N_f)_c - 1 \quad \Delta = 4 - \varepsilon \quad \varepsilon = \mathcal{O}\left(\frac{1}{N_c}\right)$

$[H^\dagger H]$ mildly relevant

$\Rightarrow \Lambda_{TC} \sim \Lambda_W e^{-1/\varepsilon} \sim \Lambda_W e^{-N_c}$

Requires large N_c

$S \sim N_c \Rightarrow$ bad for precision EW

Dimension of $\mathcal{H}$ iggs

$$d = \dim(\Psi\Psi^c) = ?$$

Lattice: measure dimensions from scaling with m

$$\Delta \mathcal{L} = m\Psi^c\Psi$$

conformal: $\underbrace{\Lambda_{phys}}_{\text{e.g. } \rho \text{ mass}} \sim m^{1/(4-d)}$

confine: Λ_{phys} indep of m

Some simulations see scaling, give crude estimate of d. Field is promising but not mature

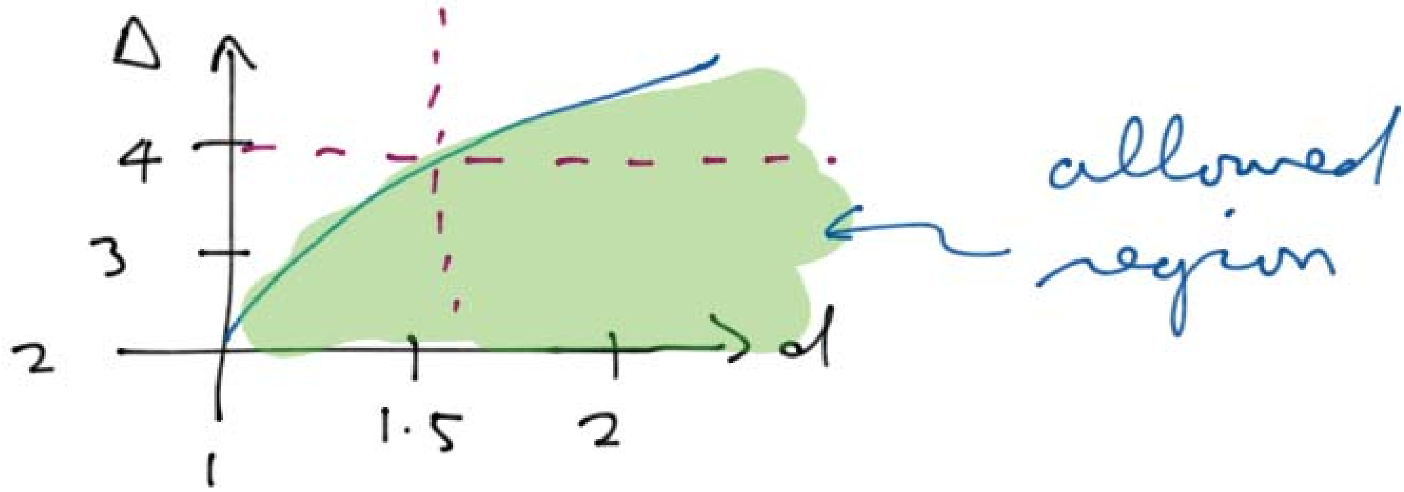
CFT Bounds

Rigorous bounds on d for given $\Delta = \dim(H^\dagger H)$.

Unitarity $\Rightarrow d \geq 1$

$d \to 1 \Rightarrow$ CFT $\to$ free

$\Rightarrow \Delta \to 2$



$\Delta > 4 \Rightarrow d \gtrsim 1.6$

See talk by Rychkov in conference.

Flavor Models

How small does d need to be? Main constraint is top mass:

$$\Lambda_t \sim \begin{cases} 10 \text{ TeV} & d=3 \\ 40 \text{ TeV} & d=2 \\ 600 \text{ TeV} & d=1.5 \end{cases}$$

high enough?

FCNC constraints:

$$\Delta \mathcal{L}_{eff} = \frac{1}{M_f^2}(ds^c)^\dagger(sd^c)$$

$K-\bar{K}$ mixing $\Rightarrow M_f \gtrsim 10^7$ GeV

How to compare w/ Λ_t?

gets strong at

$$\Lambda_f \sim 4\pi M_f \gtrsim 10^8 \text{ GeV} \ldots$$

But: expect some flavor suppression for light fermions, e.g.

$$\frac{1}{\Lambda_f^2} \sim \frac{1}{\Lambda_t^2} \cdot \underbrace{\frac{m_d m_s}{m_t^2}}_{\sim 10^{-8}}$$

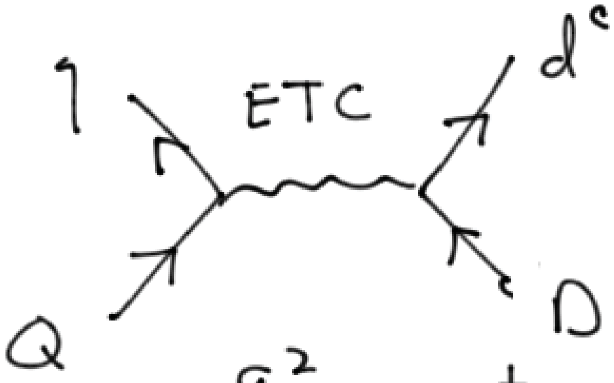
$$\Lambda_f \sim 10^4 \Lambda_t \Rightarrow \Lambda_t \gtrsim 10^4 \text{ GeV}$$

$\Rightarrow$ need additional suppression

To go further, need to look at specific flavor mechanisms

Extended TC

Flavor from gauge int



$$\rightarrow \frac{g_{ETC}^2}{M_{ETC}^2}\left(Q^\dagger \sigma^\mu q\right)\left(D^{c\dagger}\sigma_\mu d^c\right)$$

$$= \frac{g_{ETC}^2}{M_{ETC}^2}\left(Q\,U^c\right)^\dagger\left(q\,u^c\right)$$

$$\Rightarrow \quad m_u = \frac{g_{ETC}^2}{M_{ETC}^2}\langle Q\,U^c\rangle$$

Gauge group unifies SM, TC fermions!

$$\Psi = (Q^1, \ldots, Q^N, q_1^r, q_1^g, q_1^b, q_2^r, q_2^g, q_2^b, q_3^r, q_3^g, q_3^b, l_1, l_2, l_3)$$

$$\Psi^c = (Q_1^c, \ldots, Q_N^c, q_{1r}^c, q_{1g}^c, q_{1b}^c, q_{2r}^c, q_{2g}^c, q_{2b}^c, q_{3r}^c, q_{3g}^c, q_{3b}^c, l_1^c, l_2^c, l_3^c)$$

$$l_i^c = \begin{pmatrix} e_i^c \\ \nu_i^c \end{pmatrix} \quad q_i^c = \begin{pmatrix} u_i^c \\ d_i^c \end{pmatrix} \quad i = 1, 2, 3$$

ETC gauge group = $SU(N+12)$

Need hierarchical breaking of ETC gauge group, w/ no unbroken symmetries that prevent all quarks from mixing

Require:

- hierarchical breaking of ETC gauge groups
- no unbroken symmetries that can prevent all quarks from mixing with each other
- ~~CP~~

⋮

No models even come close...

Partially Composite Fermions
(Kaplan 1991)

$$\Delta \mathcal{L} = y_{iI}\, q_i\, \Psi_I^c + \cdots$$

Ψ_I^c = fermion op in strong theory

"dual" description of what is used in RS models w/ fermions in bulk

$$d = \dim(\Psi_I^c)$$

- flavor coupling marginal for $d = \frac{5}{2}$
- unitarity bound $d \geq \frac{3}{2}$

$\Rightarrow$ plenty of room

Problems:

- no GIM
- tension between T param and $Z \to \bar{b}b$

May be reduced by exotic custodial symmetry

Will not go into details here...

Bosonic TC

(Kagan, Samuel 1990)

allow SUSY $\Rightarrow$ MFV Yukawa

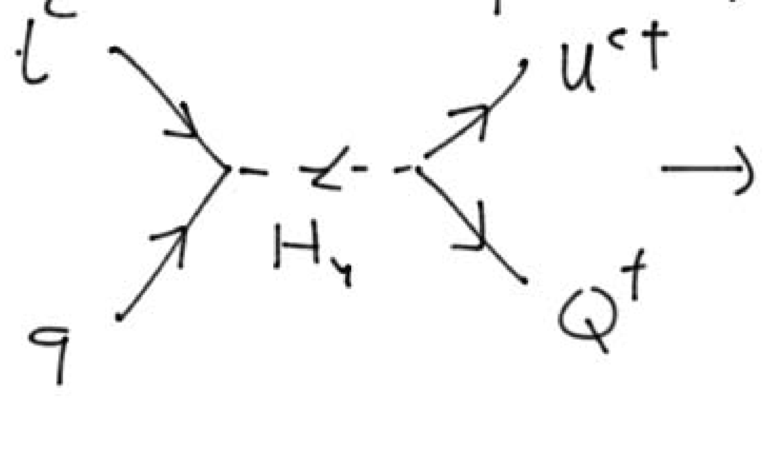
TC needs SUSY, but also vice versa: SUSY tuning

$$M_{SUSY} \gtrsim 10 \text{ TeV} \leftarrow$$ solves SUSY flavor...

$$\underbrace{m^2_{H_u}, m^2_{H_d}}_{\sim M^2_{SUSY}} > 0 \Rightarrow \langle H_{u,d} \rangle = 0$$

chiral superfields ...

$$W = y_t H_u q t^c + y_{TC} H_u Q U^c + \cdots$$



$$\longrightarrow \frac{y_t y_{TC}}{m^2_{H_u}} (Q U^c)^\dagger \times q t^c$$

Note flavor couplings inherit ETC structure from Yukawas!

$$\frac{m_t}{\Lambda_{TC}} \sim \underbrace{\#}_{\sim 1} \left(\frac{\Lambda_{TC}}{M_{SUSY}}\right)^{d-1} \left(\frac{y_t}{4\pi}\right)\left(\frac{y_{TC}}{4\pi}\right)$$

Assumes dim$(QU^c) = d$ from $\underbrace{M_{SUSY}}_{\sim \frac{1}{10}}$ to $\underbrace{\Lambda_{TC} \qquad\qquad\qquad\qquad}_{\sim \frac{1}{10}}$

Only plausible soln

$y_t, y_{TC} \sim 4\pi$ at M_{SUSY}

Is this a coincidence?

Not if it is at a strong fixed pt!

Note: requires top and TC to be part of a strong conformal sector.

This may be the real answer to the question: "Who needs conformal TC"?

Details in lecture 3...