



**The Abdus Salam  
International Centre for Theoretical Physics**



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**Preparatory School to the Winter College on Optics and the Winter College on  
Optics: Advances in Nano-Optics and Plasmonics**

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**BLOCH SURFACE WAVES ON PHOTONIC CRYSTALS  
APPLICATIONS TO GAS SENSING AND BIOPHOTONICS**

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# **BLOCH SURFACE WAVES ON PHOTONIC CRYSTALS**

## **APPLICATIONS TO GAS SENSING AND BIOPHOTONICS**



**SAPIENZA Università di Roma**

***Department of Basic and Applied Sciences for Engineering  
Molecular Photonics Laboratory***

Francesco Michelotti

***International Centre for Theoretical Physics, Trieste, February 2010***

# Collaboration and Credits



SAPIENZA Università di Roma  
Dipartimento di Scienze di Base Applicate per l'Ingegneria  
F.Michelotti, L. Dominici, A.Sinibaldi, G.Figliozzi



POLITECNICO di Torino  
 $\chi$ Lab and Dipartimento di Scienza dei Materiali ed Ingegneria Chimica  
E. Descrovi, M. Ballarini, G. Digregorio, F. Frascella, P. Rivolo, B. Sciacca, F. Geobaldo, F. Giorgis, M. Quaglio, M. Cocuzza and F. Pirri



IMT- Ecole Polytechnique Fédérale de Lausanne (EPFL) - Neuchatel  
T. Sfez, L. Yu, and H.-P. Herzig  
NAM- Ecole Polytechnique Fédérale de Lausanne (EPFL)  
D. Brunazzo and O. J. F. Martin

IOF - Applied Optics and Fine Mechanics – Jena  
N.Danz

IWS - Materials and Beam Technology – Dresden  
F.Sonntag



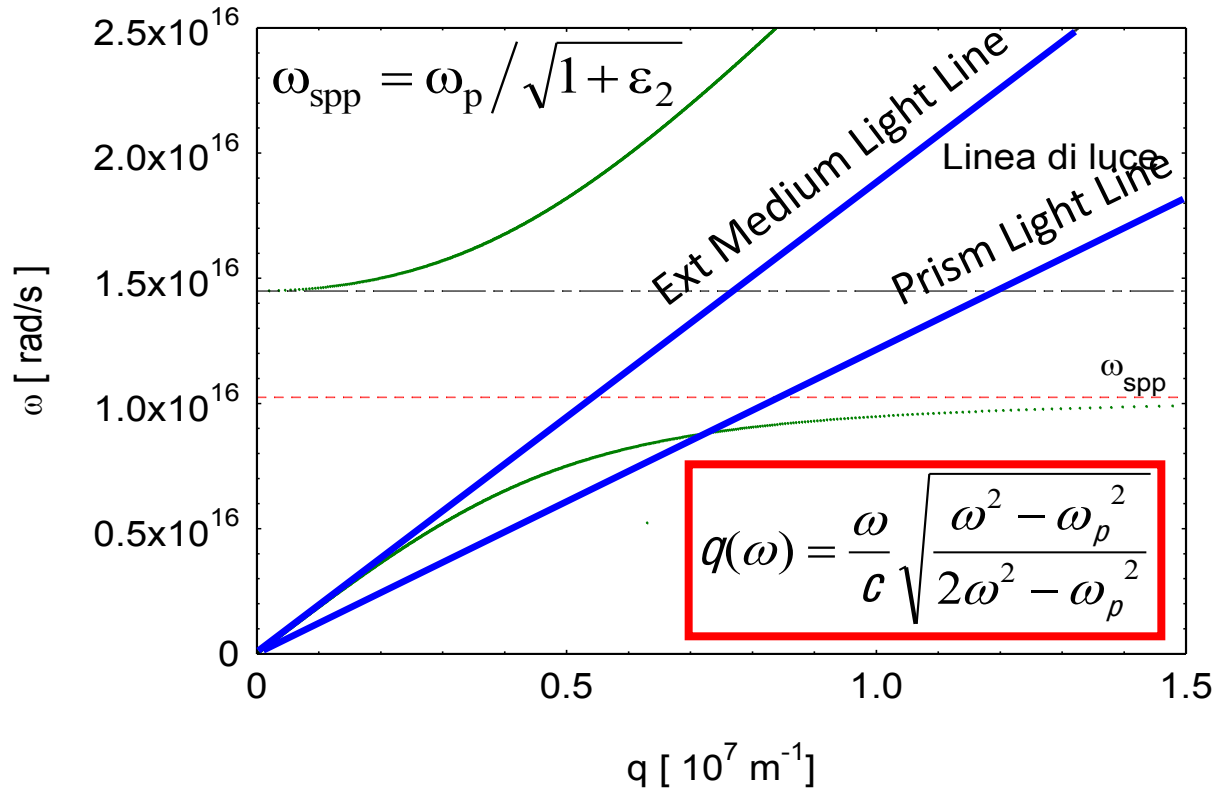
# Plasmonics



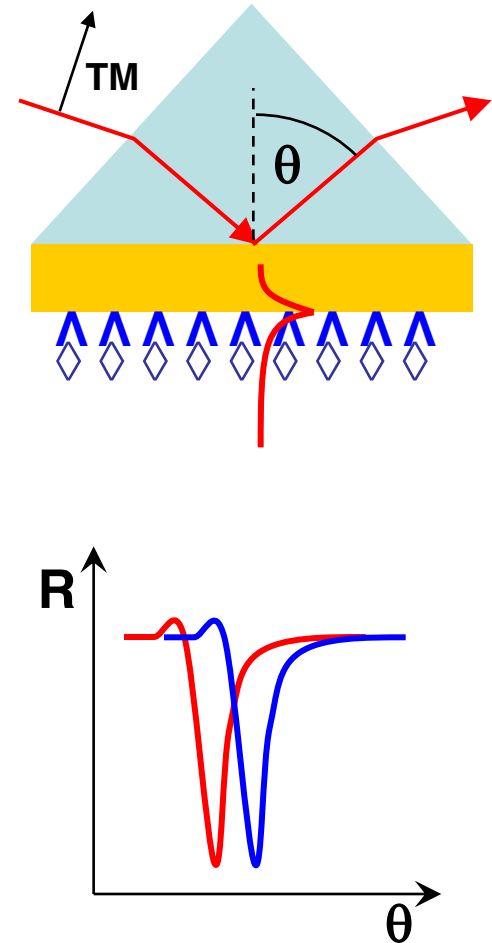
- DURING THESE TWO WEEKS MANY PLASMONICS EXPERIMENTS AND APPLICATIONS WERE OR WILL BE DESCRIBED
- PLASMONICS RECENTLY BECAME A VERY HOT RESEARCH FIELD. SO POPULAR THAT ....
- APPLICATIONS BASED ON PLASMONS SHOW SOME LIMITATIONS WHICH CAN BE OVERCOME ADOPTING ALTERNATIVE APPROACHES

ONE OF THE POSSIBLE APPROACHES ARE **BLOCH SURFACE WAVES ON PHOTONIC CRYSTALS**

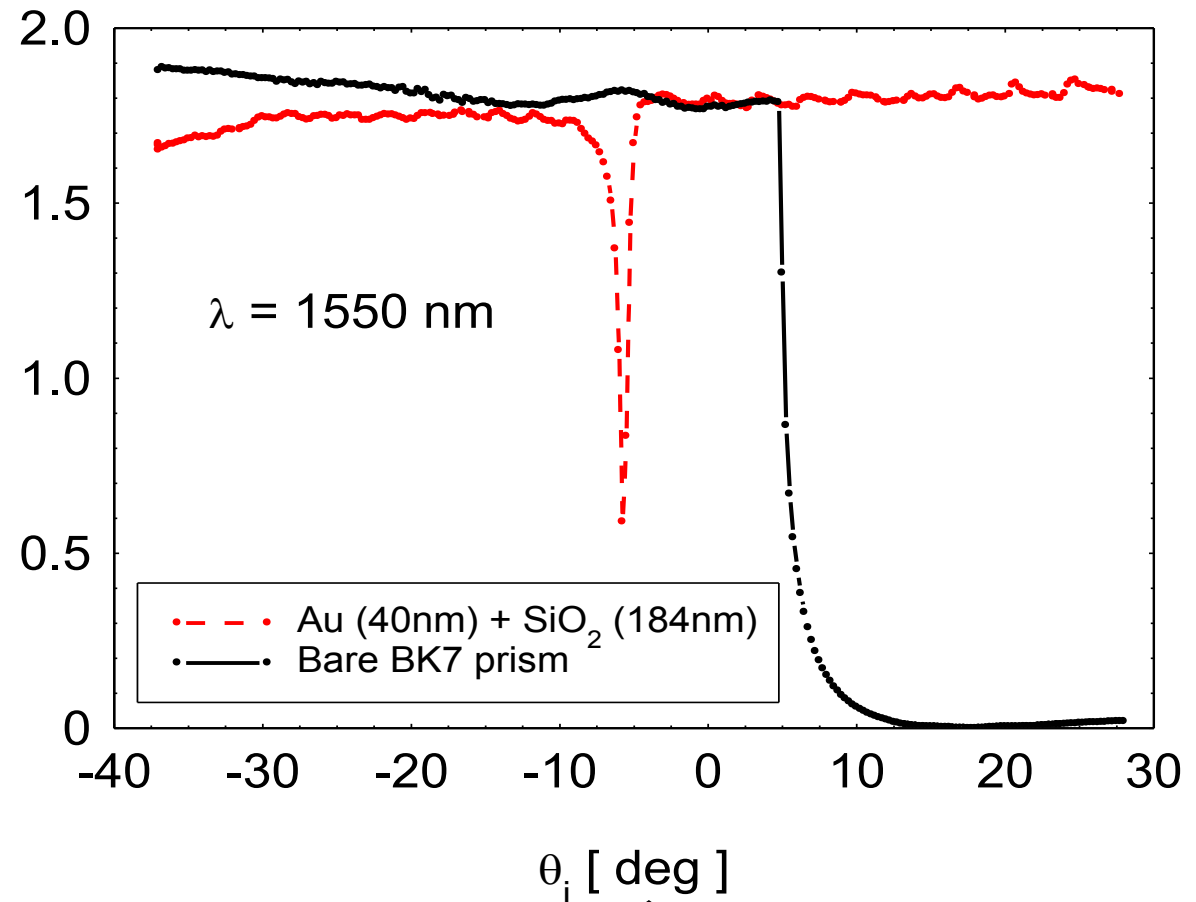
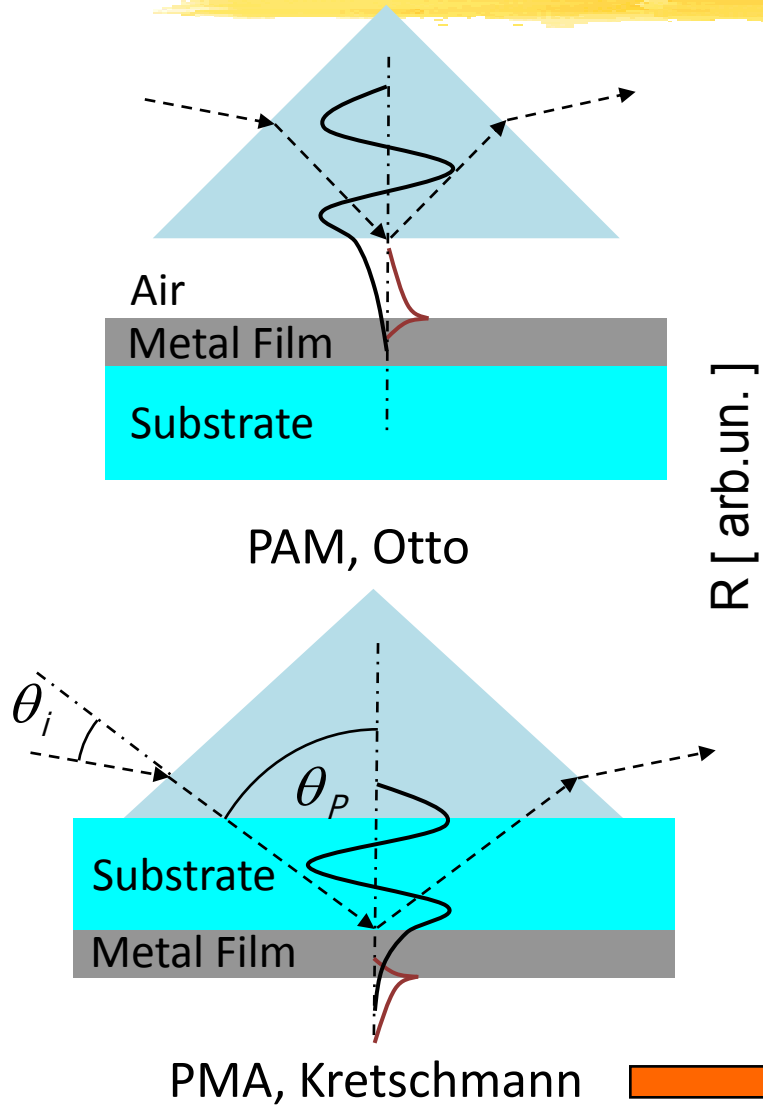
# Examples – SPR Biosensing



SPP dispersion at a metal(ideal)/dielectric interface



# Examples – SPR Biosensing



# Examples – SPR Biosensing



Unmatched productivity



**Analytical sensors**  
By Sensata Technologies



**Small. Fast. Low Cost. Accurate.**

# Examples – SPR Biosensing

- Absorption losses in metal layers give rise to broad resonances and limit the sensitivity of SPR devices
- The limit of resolution is  $\Delta n = 2 \cdot 10^{-7}$  RIU (Biacore)
- The resolution does not permit to detect small molecules (<250 dalton)
- SPR devices never really accessed the Point-of-Care level
- The sensitivity can be improved by making use of long range surface plasmon polaritons but problems due to the symmetry of dielectric layers arise.



# Examples – Fluorescence Imaging

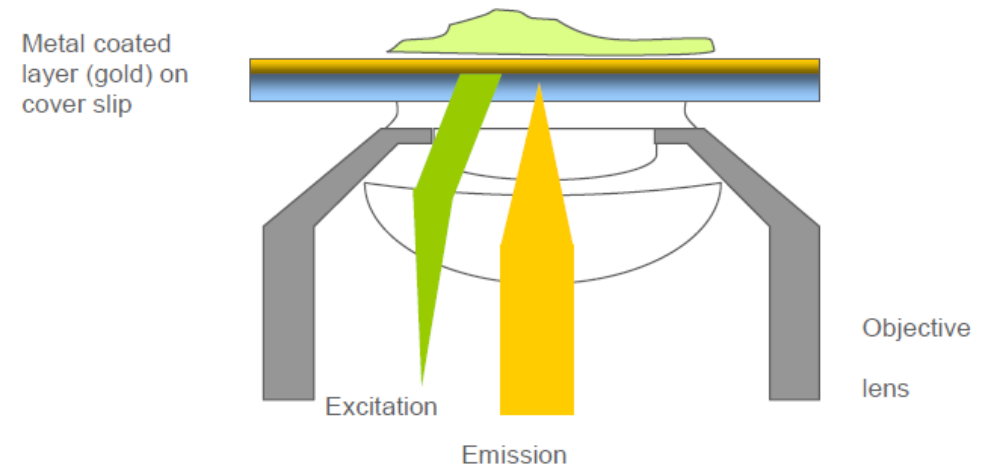
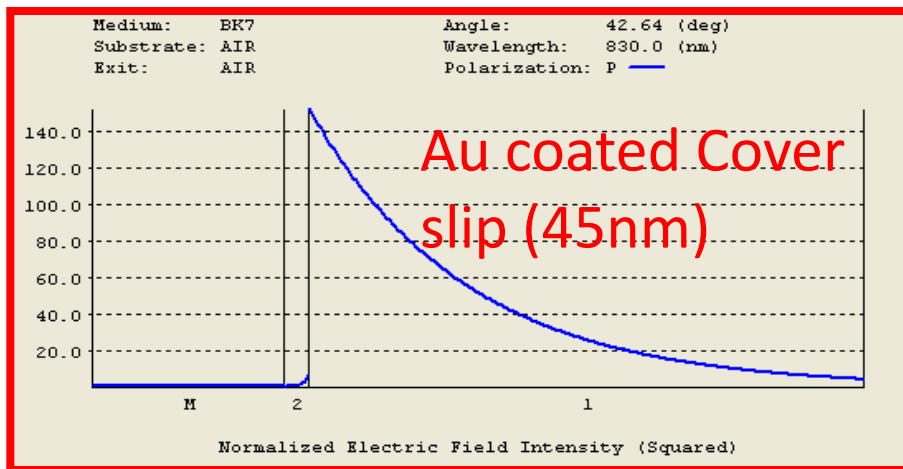
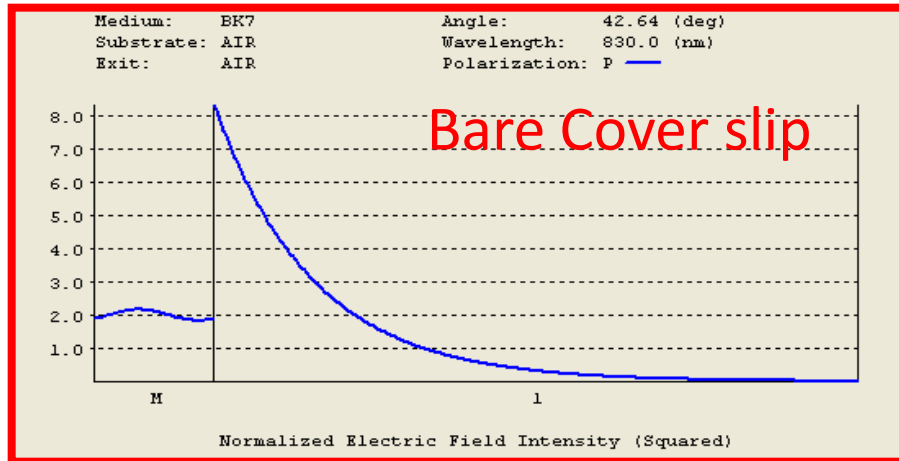
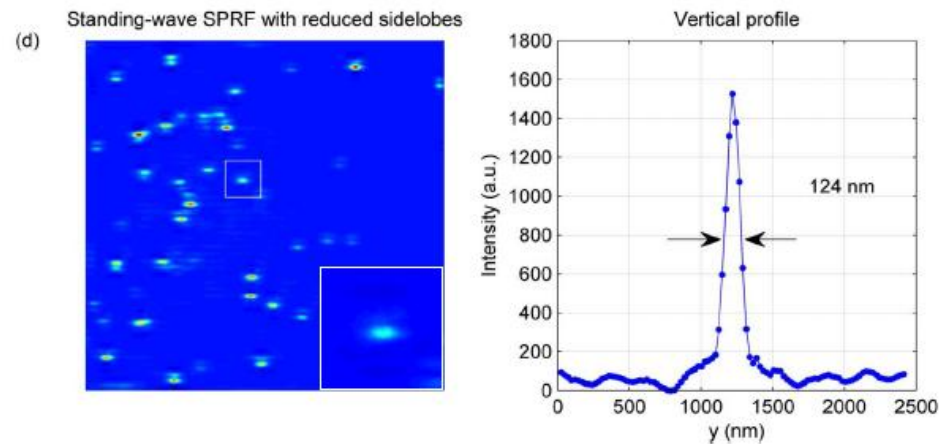
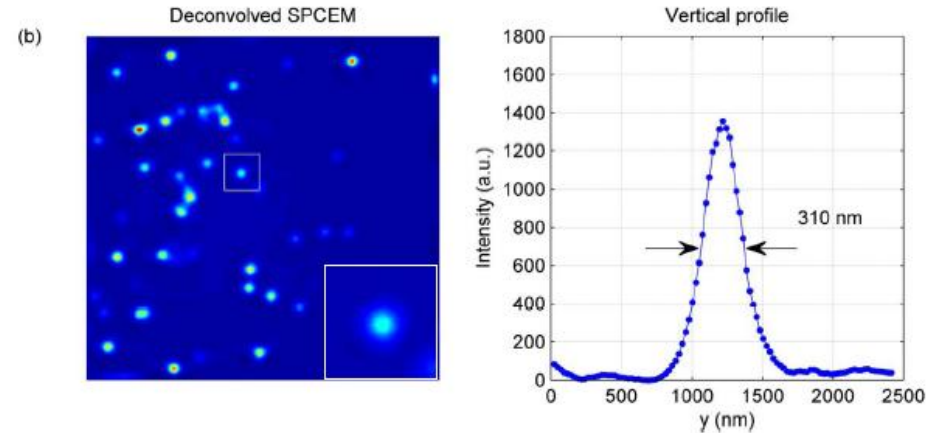
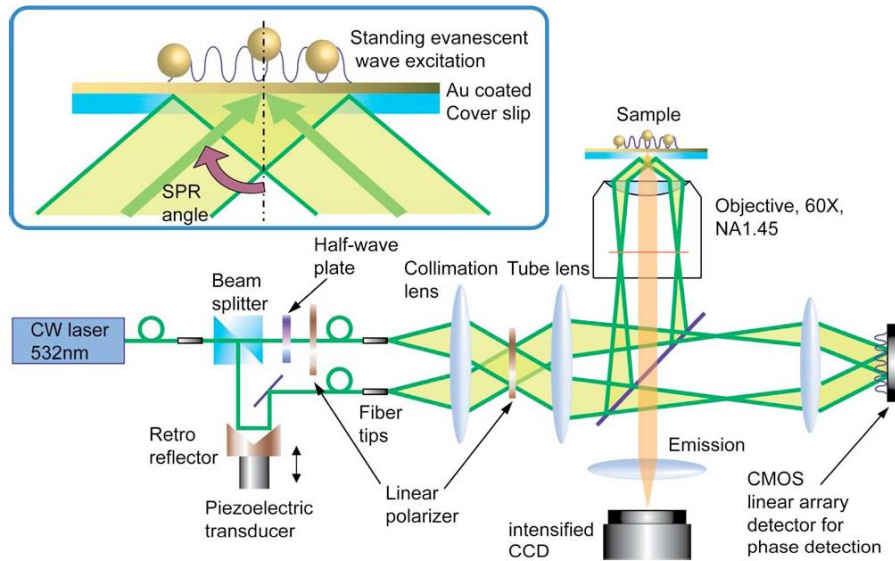


Figure 2-2. An objective-launched set-up for SPCE imaging.

**Surface Plasmon Coupled Emission (SPCE) and Surface Plasmon Field-enhanced Fluorescence (SPFS)**

# Examples – SW-SPCF Fluorescence Imaging



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## Wide-field extended-resolution fluorescence microscopy with standing surface-plasmon-resonance waves

Euiheon Chung,<sup>1,2</sup> Yang-Hyo Kim,<sup>1</sup> Wai Teng Tang,<sup>3</sup> Colin J. R. Sheppard,<sup>4</sup> and Peter T. C. So<sup>1,5,\*</sup>

# Examples – DLSPPW and LRSPP Waveguiding

**Dielectric  
thickness  
500nm**

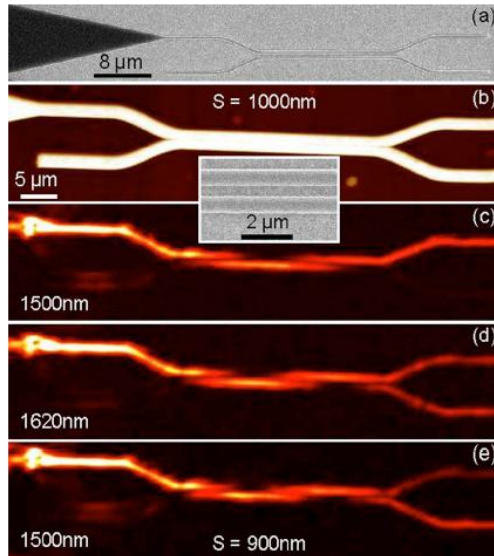


Fig. 1. (Color online) (a) Scanning electron microscope image of the fabricated DC showing the funnel structure facilitating the DLSPPW excitation. (b) Topographical and (c)–(e) near-field optical [ $\lambda$ =(c) 1500, (d) 1620, and (e) 1500 nm] SNOM images of 45  $\mu\text{m}$  long DCs with the separations (b)–(d)  $S=1000$  nm along with an inset showing SEM image of the coupling region and (e)  $S=900$  nm.

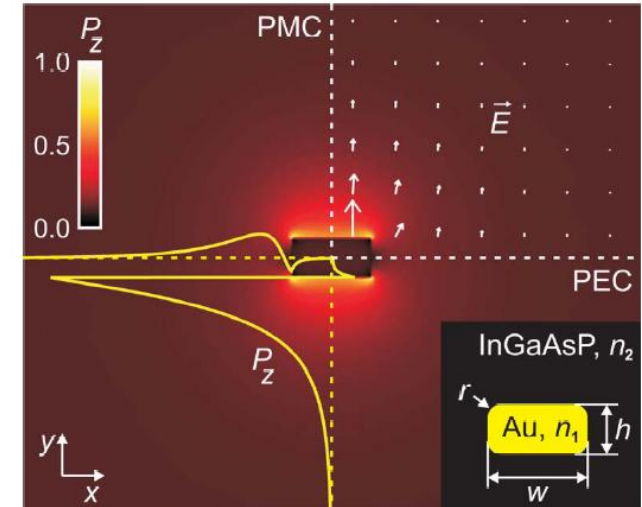


Fig. 1. (Color online) Field map of power flow  $P_z$  of the long-range SPP mode supported by a 200 nm  $\times$  100 nm waveguide at  $\lambda = 1550$  nm. Solid curves correspond to the vertical and horizontal cross sections of the power flow  $P_z$ . Arrows show the direction of the electric field. Inset: studied Au( $n_1 = 0.55-11.5i$ )/InGaAsP( $n_2 = 3.3737$ ) metallic wire waveguide with width  $w$  and height  $h$ , having corners rounded with  $r = 5$  nm.

## Wavelength-selective directional coupling with dielectric-loaded plasmonic waveguides

Zhuo Chen,<sup>1</sup> Tobias Holmgaard,<sup>1</sup> Sergey I. Bozhevolnyi,<sup>1,2,\*</sup> Alexey V. Krasavin,<sup>3</sup> Anatoly V. Zayats,<sup>3</sup> Laurent Markey,<sup>4</sup> and Alain Dereux<sup>4</sup>

## Numerical analysis of long-range surface plasmon polariton modes in nanoscale plasmonic waveguides

Alexey V. Krasavin\* and Anatoly V. Zayats

# Can we do similar things in a different way?

We can exploit the propagation of surface electromagnetic waves (SEW) at the truncation interface of finite one dimensional photonic crystals (1DPC)

We shall refer to such waves with the name

**Bloch Surface Waves (BSW)**

To demonstrate such possibility in the following we will describe the:

Mon	10 <sup>??</sup> – 11 <sup>00</sup>	General properties of BSW on 1DPC (Theory)
Tue	10 <sup>00</sup> - 11 <sup>00</sup>	Experimental techniques for the detection of SPP and BSW (Experimental)
Tue	11 <sup>30</sup> – 12 <sup>30</sup>	Applications of BSW to gas sensing (Experimental)
Wed	14 <sup>30</sup> - 15 <sup>30</sup>	Applications of BSW to biophotonics (Experimental)



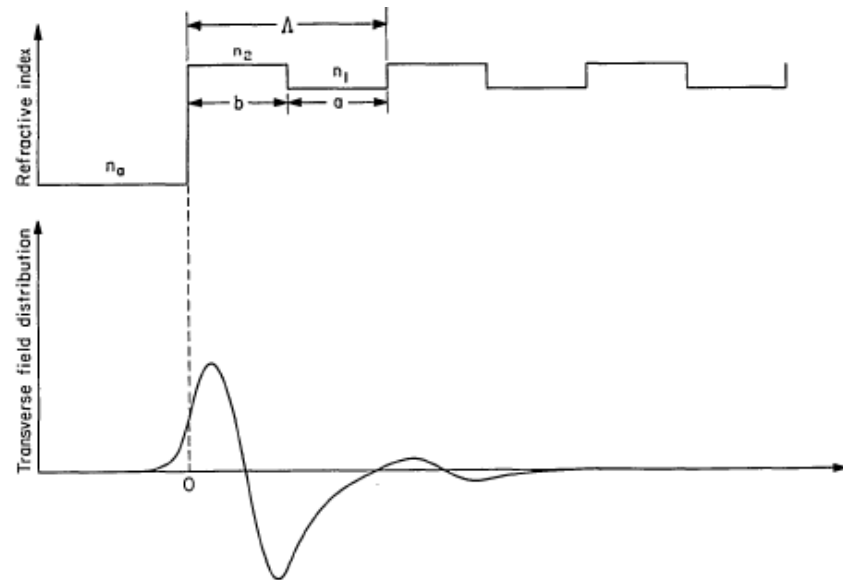
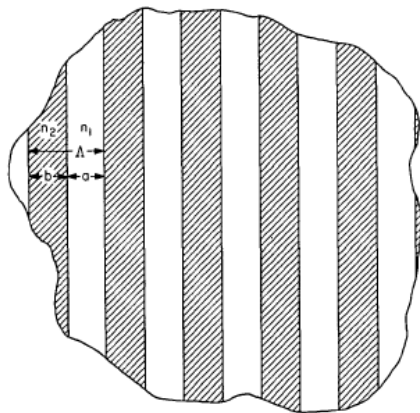
**Lecture 1**  
**General properties of BSW on 1DPC**  
**(Theory)**

# BSW at the truncation interface of 1DPC

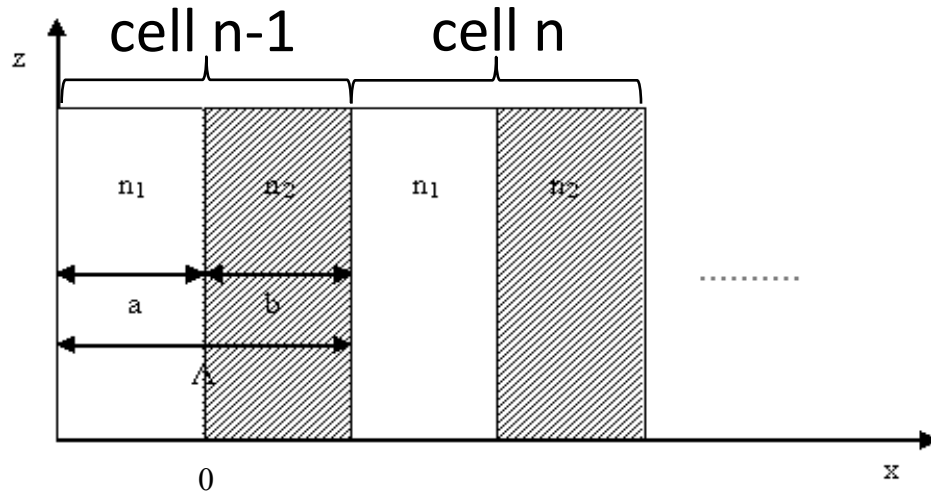
## Electromagnetic propagation in periodic stratified media. I. General theory\*

Pochi Yeh, Amnon Yariv, and Chi-Shain Hong  
California Institute of Technology, Pasadena, California 91125  
(Received 8 November 1976)

The propagation of electromagnetic radiation in periodically stratified media is considered. Media of finite, semi-infinite, and infinite extent are treated. A diagonalization of the unit cell translation operator is used to obtain exact solutions for the Bloch waves, the dispersion relations, and the band structure of the medium. Some new phenomena with applications to integrated optics and laser technology are presented.



# Propagation of light in infinite 1DPC



$$n(x) = n_2 \quad \text{for} \quad 0 < x < b$$

$$n(x) = n_1 \quad \text{for} \quad b < x < \Lambda$$

with  $n(x + \Lambda) = n(x)$

$E(x, z) = E(x)e^{j\beta z}$  In each layer the field can be expressed as the superposition of **forward** and **backward** propagating waves

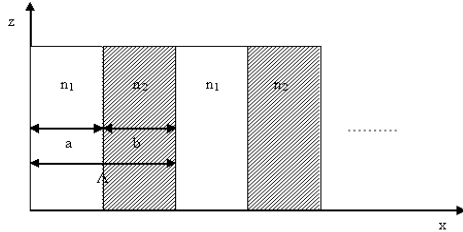
$$E(x, z) = (a_n^{(\alpha)} e^{ik_{\alpha x}(x-n\Lambda)} + b_n^{(\alpha)} e^{-ik_{\alpha x}(x-n\Lambda)}) e^{j\beta z}$$

With  $k_{\alpha x} = \left\{ \left[ (\omega/c)n_{\alpha} \right]^2 - \beta^2 \right\}^{\frac{1}{2}}, \quad \alpha=1,2$

**In a vectorial representation**

→  $\begin{pmatrix} a_n^{(\alpha)} \\ b_n^{(\alpha)} \end{pmatrix}$

# Propagation of light in infinite 1DPC



For the sake of notation simplicity

$$\begin{pmatrix} a_n^{(1)} \\ b_n^{(1)} \end{pmatrix} = \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad \begin{pmatrix} a_n^{(2)} \\ b_n^{(2)} \end{pmatrix} = \begin{pmatrix} c_n \\ d_n \end{pmatrix}$$

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} = T$$

The unit cell  $\Lambda$  translation operator is unimodular **AD-CB=1**

## TE case

$$A = e^{-ik_{1x}a} \left[ \cos k_{2x}b - \frac{1}{2}i \left( \frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x}b \right]$$

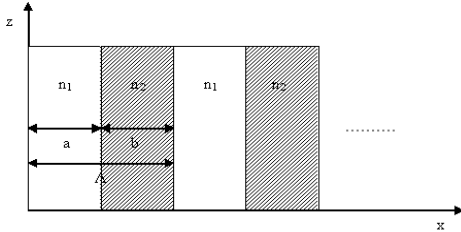
$$B = e^{ik_{1x}a} \left[ -\frac{1}{2}i \left( \frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x}b \right]$$

$$C = e^{-ik_{1x}a} \left[ \frac{1}{2}i \left( \frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x}b \right]$$

$$D = e^{ik_{1x}a} \left[ \cos k_{2x}b + \frac{1}{2}i \left( \frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x}b \right]$$



# Propagation of light in infinite 1DPC



For the sake of notation simplicity

$$\begin{pmatrix} a_n^{(1)} \\ b_n^{(1)} \end{pmatrix} = \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad \begin{pmatrix} a_n^{(2)} \\ b_n^{(2)} \end{pmatrix} = \begin{pmatrix} c_n \\ d_n \end{pmatrix}$$

$$\begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix} = T$$

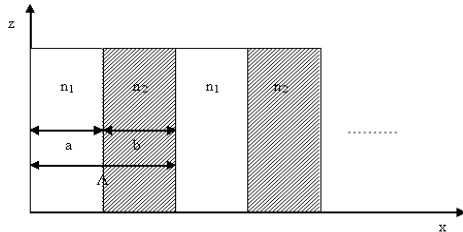
The unit cell  $\Lambda$  translation operator is unimodular **AD-CB=1**

**TM case**

$$A = e^{-ik_{1x}a} \left[ \cos k_{2x}b - \frac{1}{2} i \left( \frac{n_2^2 k_{1x}}{n_1^2 k_{2x}} + \frac{n_1^2 k_{2x}}{n_2^2 k_{1x}} \right) \sin k_{2x}b \right] \quad B = e^{ik_{1x}a} \left[ -\frac{1}{2} i \left( \frac{n_2^2 k_{1x}}{n_1^2 k_{2x}} - \frac{n_1^2 k_{2x}}{n_2^2 k_{1x}} \right) \sin k_{2x}b \right]$$

$$C = e^{-ik_{1x}a} \left[ \frac{1}{2} i \left( \frac{n_2^2 k_{1x}}{n_1^2 k_{2x}} - \frac{n_1^2 k_{2x}}{n_2^2 k_{1x}} \right) \sin k_{2x}b \right] \quad D = e^{ik_{1x}a} \left[ \cos k_{2x}b + \frac{1}{2} i \left( \frac{n_2^2 k_{1x}}{n_1^2 k_{2x}} + \frac{n_1^2 k_{2x}}{n_2^2 k_{1x}} \right) \sin k_{2x}b \right]$$

# Propagation of light in infinite 1DPC



Only one vector is independent

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-n} \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} D & -B \\ -C & A \end{pmatrix}^n \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$$

Note that:  $\begin{pmatrix} c_n \\ d_n \end{pmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}^{-n} \begin{pmatrix} c_0 \\ d_0 \end{pmatrix}$  and that:  $\begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \neq \begin{pmatrix} A & B \\ C & D \end{pmatrix}$

The unit cell translation operator is such that:

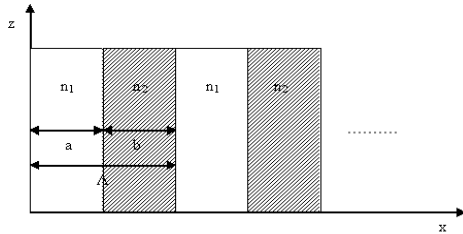
$$Tx = x + \Lambda \quad T \cdot E(x) = E(T^{-1}x) = E(x - \Lambda).$$

**Floquet theorem** for a wave propagating in a periodic medium:

$$\begin{cases} E_K(x, z) = E_K(x) e^{iKx} e^{i\beta z} \\ E_K(x + \Lambda) = E_K(x) \end{cases}$$

$K :=$  Bloch wavenumber

# Propagation of light in infinite 1DPC



$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{iK\Lambda} \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix}$$

Periodicity



$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{-iK\Lambda} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

Eigenvalues

$$e^{-iK\Lambda} = \frac{1}{2}(A + D) \pm \left\{ \left[ \frac{1}{2}(A + D) \right]^2 - 1 \right\}^{1/2}$$

Eigenvectors

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} B \\ e^{-iK\Lambda} - A \end{pmatrix}$$

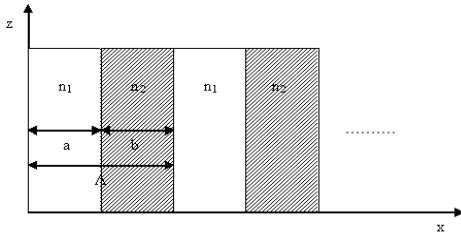
Unimodularity of T implies that:

$$K(\beta, \omega) = \frac{1}{\Lambda} \cos^{-1} \left[ \frac{1}{2}(A + D) \right] \quad \text{Dispersion relation}$$

The field in the  $n_1$  layer of the  $n$ -th cell is:

$$E_K(x) e^{iKx} = \left[ \left( a_0 e^{ik_{1x}(x-n\Lambda)} + b_0 e^{-ik_{1x}(x-n\Lambda)} \right) e^{-iK(x-n\Lambda)} \right] e^{iKx}$$

# Propagation of light in infinite 1DPC



$$\left| \frac{1}{2}(A + D) \right| < 1 \Rightarrow K \in R$$

Propagating waves (permitted)

$$\left| \frac{1}{2}(A + D) \right| > 1 \Rightarrow K = \frac{m\pi}{\Lambda} + iK_i \in C$$

Evanescent waves (forbidden in  $\infty$  1DPC)

$$\left| \frac{1}{2}(A + D) \right| = 1$$

Band edges

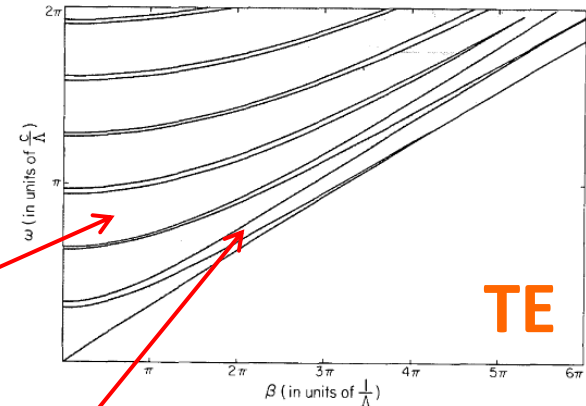


FIG. 3. TE waves (E perpendicular to the direction of periodicity) band structure in the  $\omega$ - $\beta$  plane. The dark zones are the allowed bands.

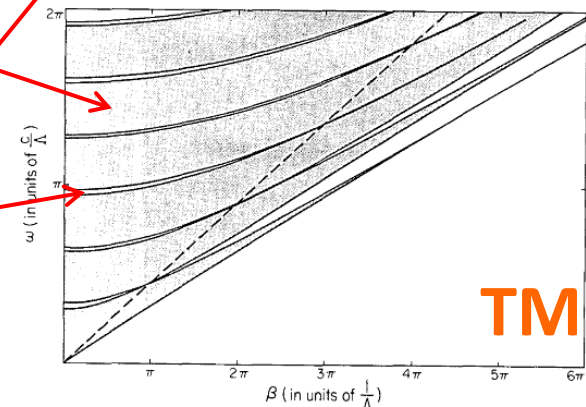
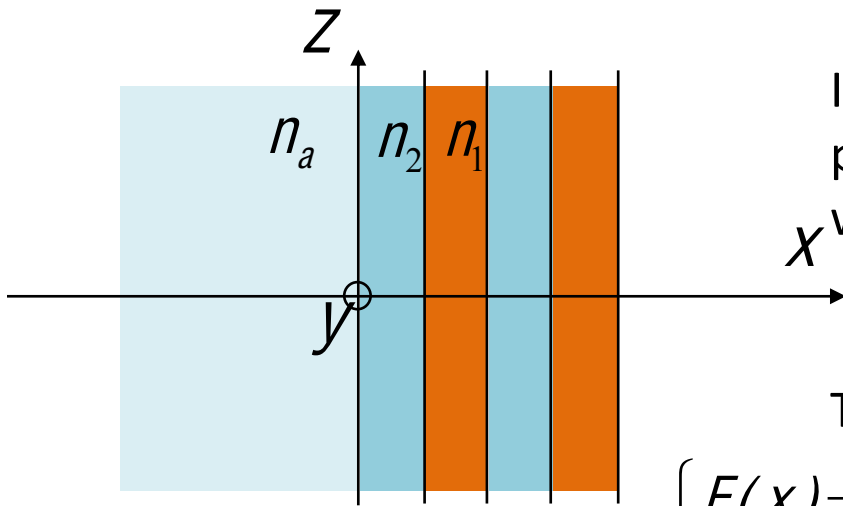


FIG. 4. TM waves (H perpendicular to the direction of periodicity) band structure in the  $\omega$ - $\beta$  plane. The dashed line is  $\beta = (\omega/c)n_2 \sin \phi_B$ . The dark zones are the allowed bands.

# BSW at the truncation interface of 1DPC



If the 1DPC is finite the evanescent solutions are permitted at the interface and decay in the 1DPC with an envelope (Bloch Surface Wave - BSW):

$$e^{-K_i x}$$

The BSW field has the transverse structure:

$$\begin{cases} E(x) = \alpha e^{q_a x} & x \leq 0 \\ E(x) = E_K(x) e^{iKx} & x \geq 0 \end{cases} \quad \text{with} \quad q_a = \left\{ \beta^2 - [(\omega/c)n_a]^2 \right\}^{1/2}$$

$$K \in \mathbb{C}$$



BSW live in the forbidden bands

Continuity of  
E and  $\partial E / \partial x$



BSW dispersion relation

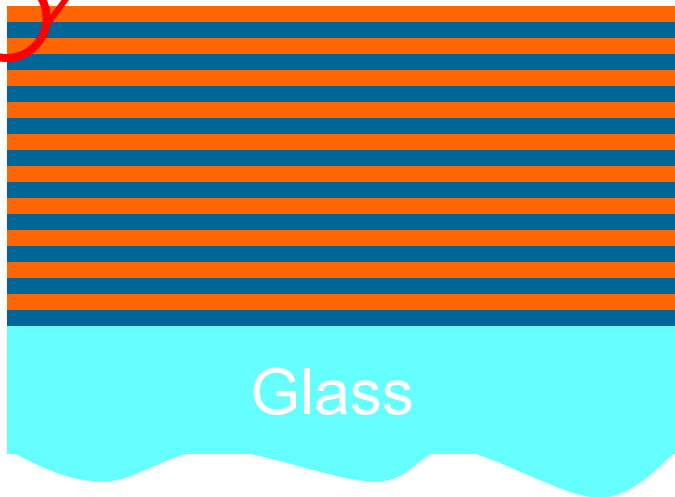
$$q_a = q \left( e^{-iK\Lambda} - A - B \right) / \left( e^{-iK\Lambda} - A + B \right)$$

# EXAMPLE: a-Si<sub>1-x</sub>N<sub>x</sub>:H 1DPC

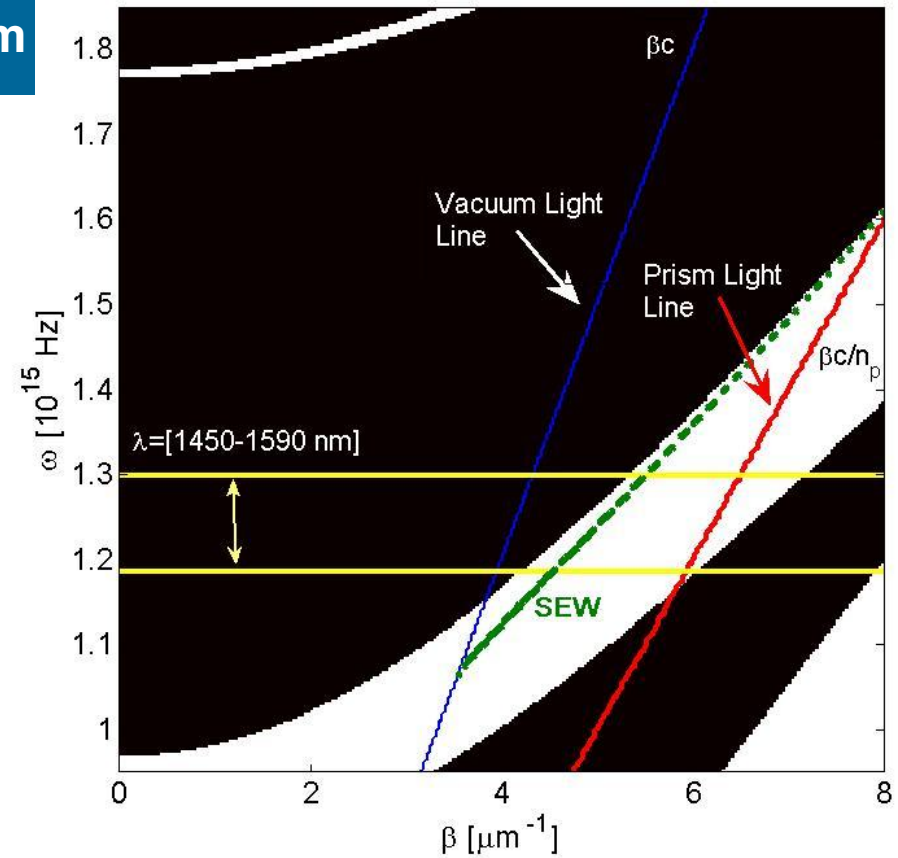
**L** t=294nm n=1.75 @ λ=1530nm

**H** t=240nm n=2.23 @ λ=1530nm

10 periods



## Band Diagram

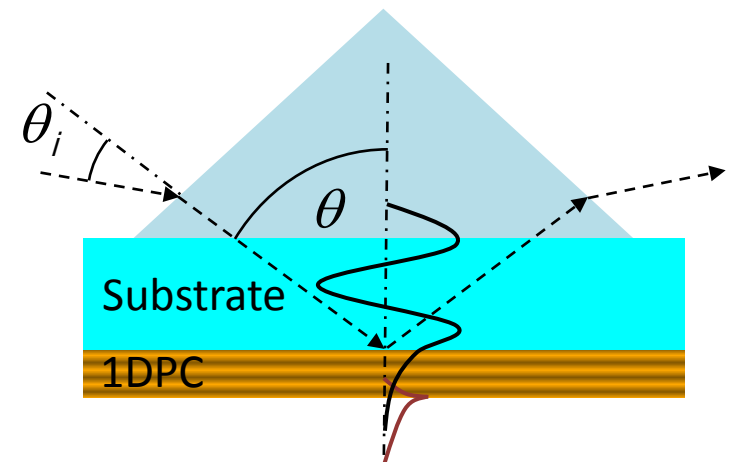
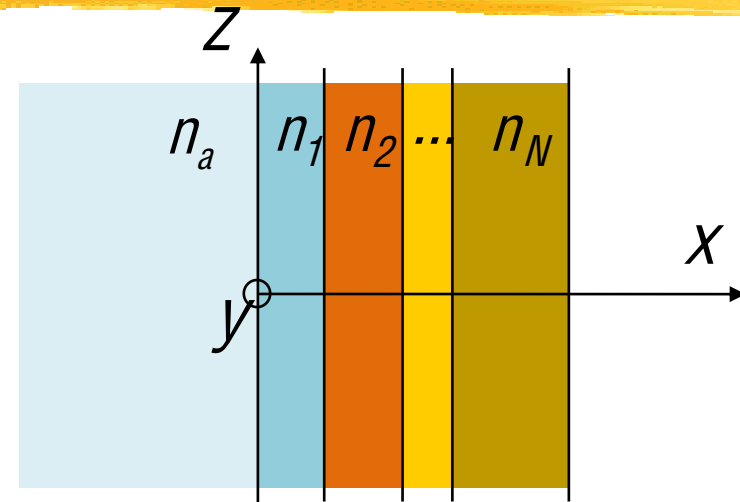


# Finite and non periodic 1DPC

In the case the 1DPC is finite and/or non periodic the analytical approach from Yariv cannot be applied. The solution must be sought numerically.

We can apply the Transfer Matrix Method (TMM) to calculate the reflectance of an arbitrary structure.

Suppose we wish to calculate the reflectance of a finite 1DPC in the Kretschmann prism coupling configuration.

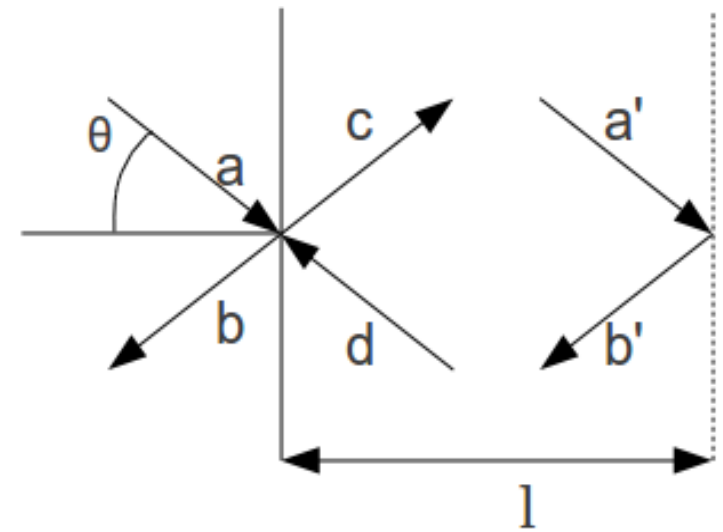


PMA, Kretschmann

# Finite and non periodic 1DPC

Step 1 – The single layer transfer matrix is:

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{t_{12}} \underbrace{\begin{pmatrix} 1 & r_{12} \\ r_{12} & 1 \end{pmatrix}}_{\text{Interface matrix}} \underbrace{\begin{pmatrix} e^{jkl} & 0 \\ 0 & e^{-jkl} \end{pmatrix}}_{\text{Propagation matrix}} \begin{pmatrix} a' \\ b' \end{pmatrix}$$



$$\begin{cases} c = t_{12}a + r_{21}d \\ b = r_{12}a + t_{21}d \end{cases} \quad \text{TE case} \quad \begin{aligned} r_{ij} &= \frac{n_i \sin(\vartheta_i) - n_j \sin(\vartheta_j)}{n_i \sin(\vartheta_i) + n_j \sin(\vartheta_j)} & i, j = 1, 2 \quad i \neq j \\ t_{ij} &= 1 + r_{ij} & \vartheta_j = \arcsin\left(\frac{n_i}{n_j} \sin(\vartheta_i)\right) \end{aligned}$$



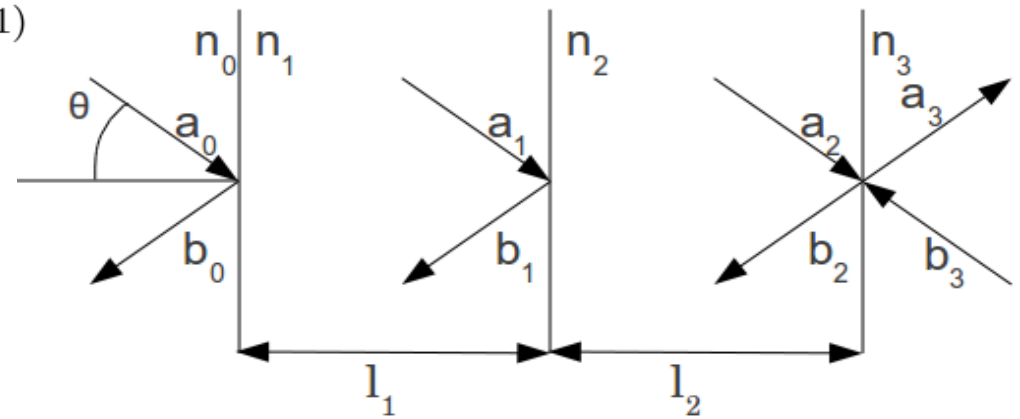
# Finite and non periodic 1DPC

Step 2 – The multilayer transfer matrix is:

$$\bar{M} = \left( \prod_{k=1}^N M_{(k-1)k} T_k \right) M_{N(N+1)}$$

$$M_{ij} = \frac{1}{t_{ij}} \begin{pmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{pmatrix}$$

$$T_j = \begin{pmatrix} e^{jk_j l_j} & 0 \\ 0 & e^{-jk_j l_j} \end{pmatrix}$$



$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \bar{M} \begin{pmatrix} a_{N+1} \\ b_{N+1} \end{pmatrix}$$

# Finite and non periodic 1DPC

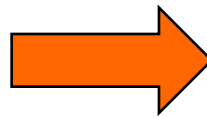
**Step 3 – The reflectance and transmittance are calculated imposing that  $b_{N+1}=0$ :**

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} a_{N+1} \\ 0 \end{pmatrix}$$



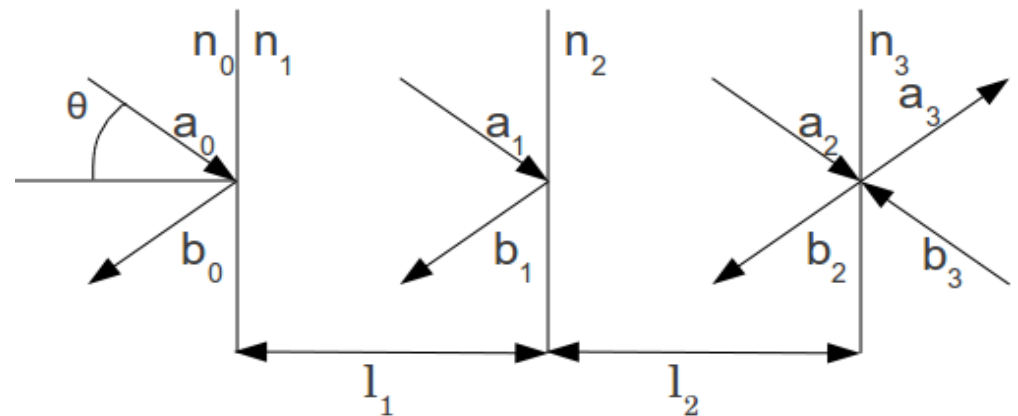
$$r = \frac{b_0}{a_0} = \frac{m_{21}}{m_{11}}$$

$$t = \frac{a_{N+1}}{a_0} = \frac{1}{m_{11}}$$



$$R = |r|^2$$

$$T = |t|^2$$



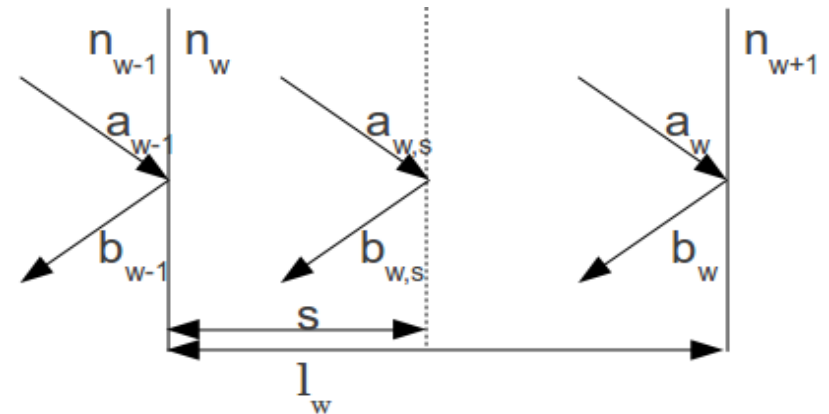
# Finite and non periodic 1DPC

**Step 4 – The field distribution at any position inside the structure is calculated imposing that  $a_0=1$  and  $b_0=r$ :**

$$\begin{pmatrix} a_{w,s} \\ b_{w,s} \end{pmatrix} = \hat{T}_w(s) \left( \prod_{k=1}^{w-1} \hat{M}_{(w-k)(w-k+1)} \hat{T}_{w-k}(l_{w-k}) \right) \hat{M}_{01} \begin{pmatrix} 1 \\ r \end{pmatrix}$$

$$\hat{T}_j(s) = \begin{pmatrix} e^{-jk_j s} & 0 \\ 0 & e^{jk_j s} \end{pmatrix}$$

$$\hat{M}_{ij} = M_{ij}^{-1} = \frac{1}{1 - r_{ij}} \begin{pmatrix} 1 & -r_{ij} \\ -r_{ij} & 1 \end{pmatrix}$$

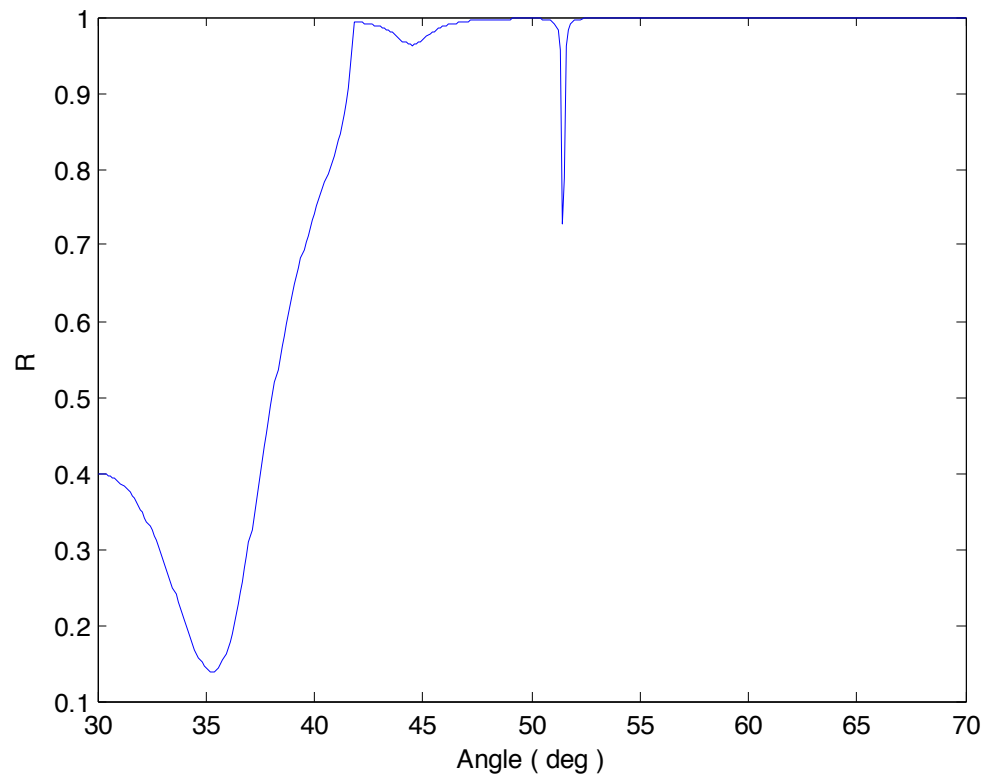
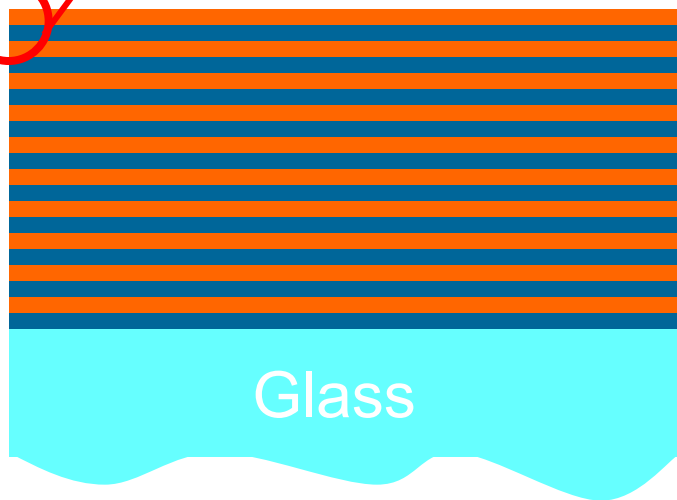


# EXAMPLE: a-Si<sub>1-x</sub>N<sub>x</sub>:H 1DPC

**L** t=294nm n=1.75 @ λ=1530nm

**H** t=240nm n=2.23 @ λ=1530nm

10 periods

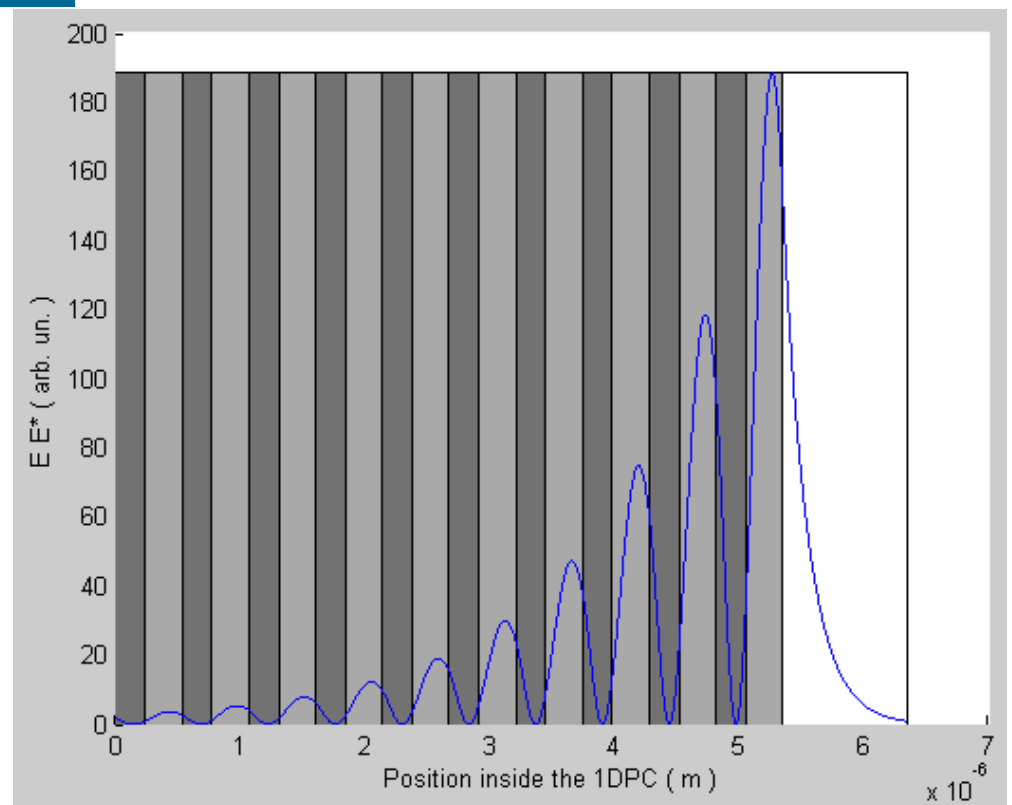
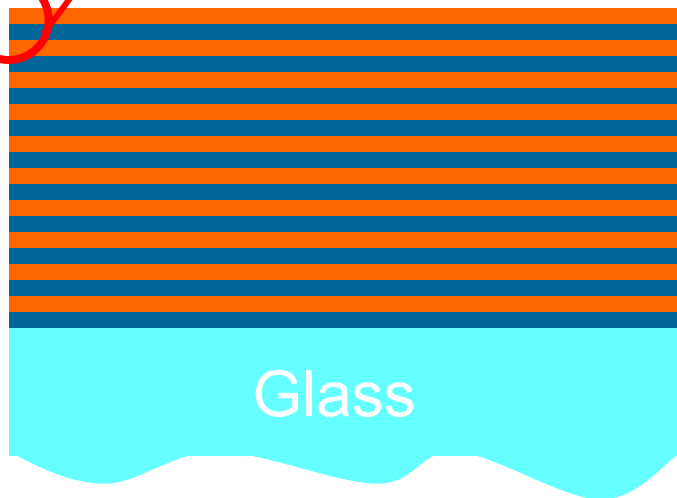


# EXAMPLE: a-Si<sub>1-x</sub>N<sub>x</sub>:H 1DPC

**L** t=294nm n=1.75 @  $\lambda=1530\text{nm}$

**H** t=240nm n=2.23 @  $\lambda=1530\text{nm}$

10 periods

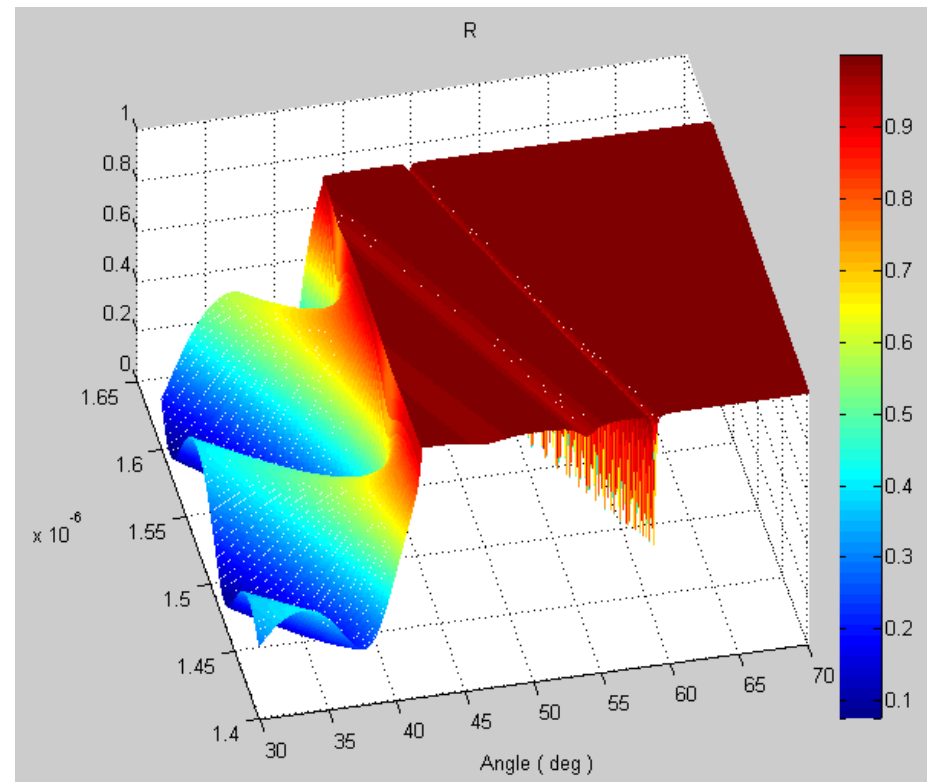
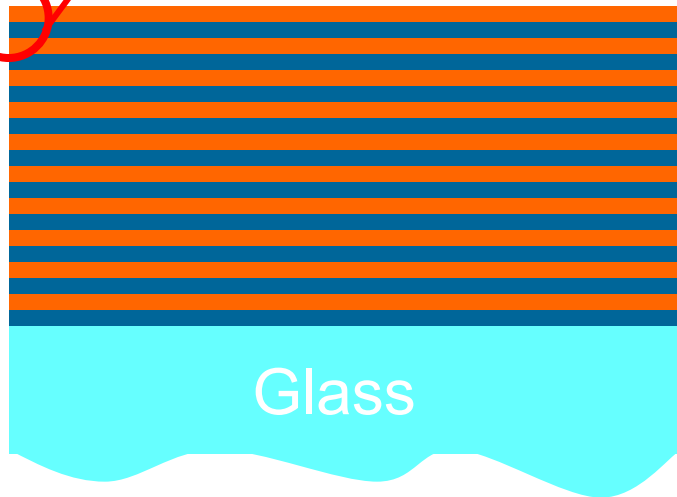


# EXAMPLE: a-Si<sub>1-x</sub>N<sub>x</sub>:H 1DPC

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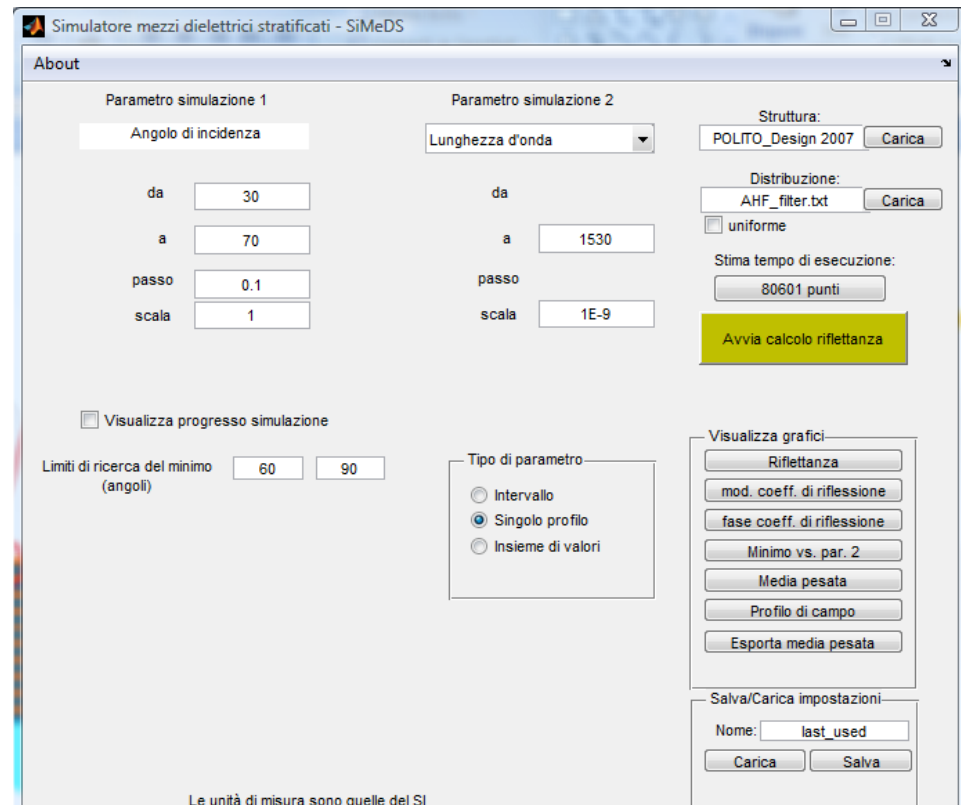
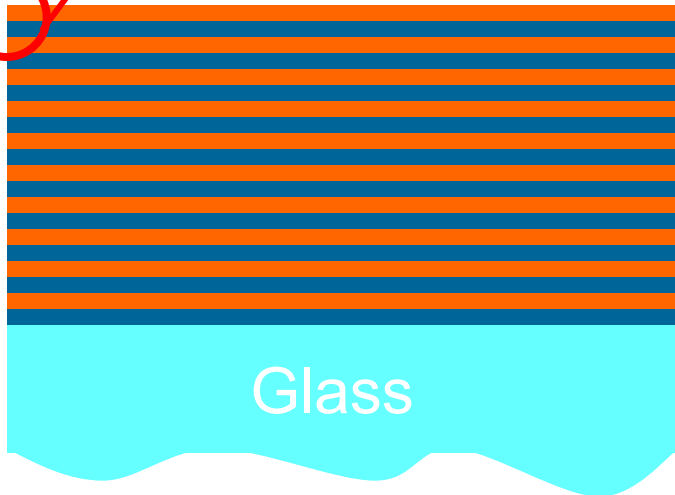


# Transfer Matrix Method on MATLAB

L t=294nm n=1.75 @  $\lambda=1530\text{nm}$

H t=240nm n=2.23 @  $\lambda=1530\text{nm}$

10 periods

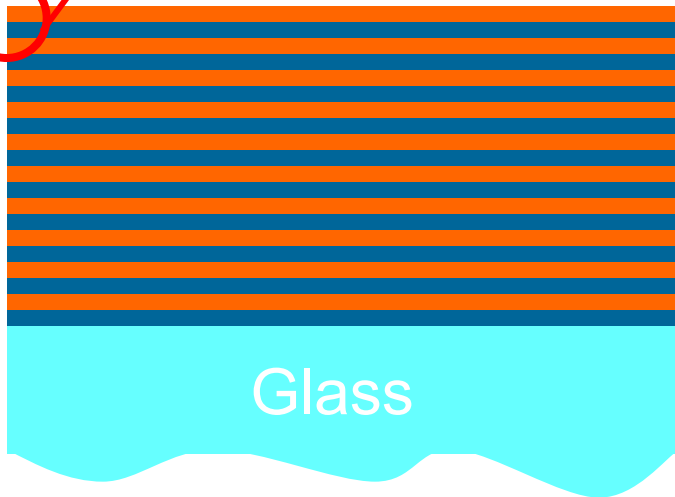


# .... a small preview of Lecture 2

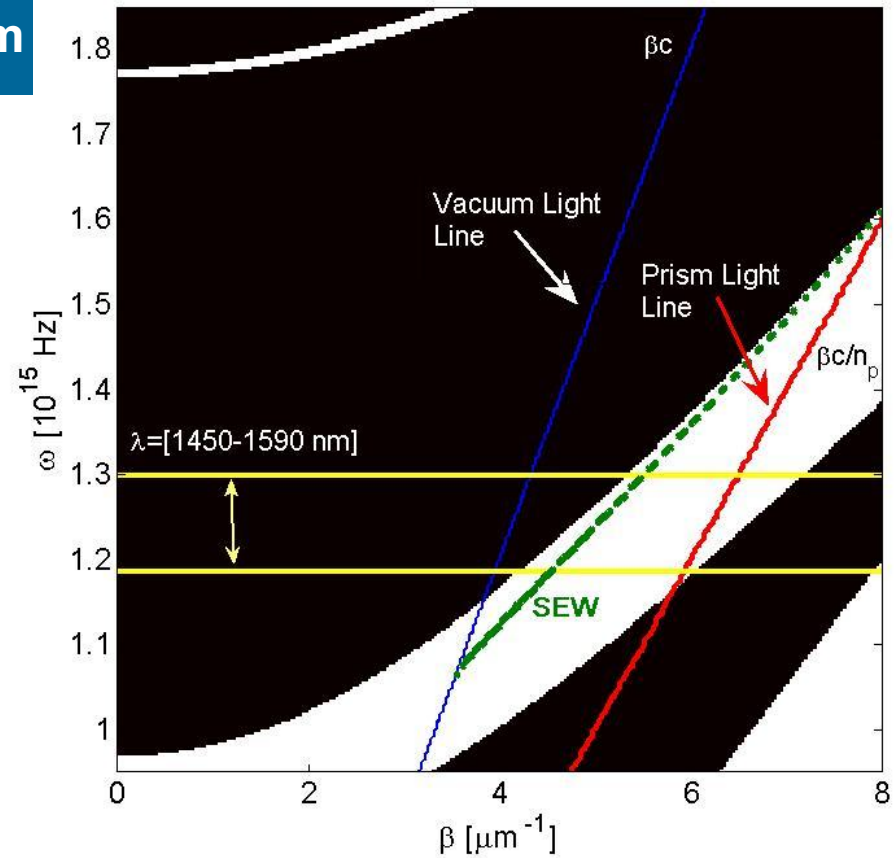
**L**  $t=294\text{nm}$   $n=1.75$  @  $\lambda=1530\text{nm}$

**H**  $t=240\text{nm}$   $n=2.23$  @  $\lambda=1530\text{nm}$

10 periods

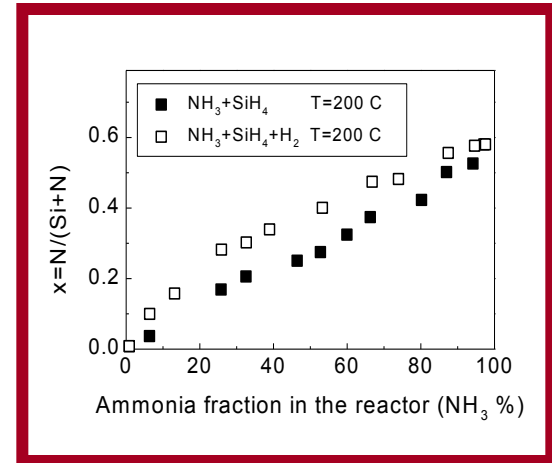
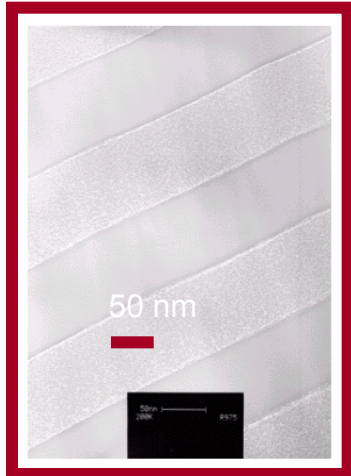


## Band Diagram





# Hydrogenated amorphous silicon nitride ( $\text{Si}_{1-x}\text{N}_x\text{:H}$ ) by PECVD



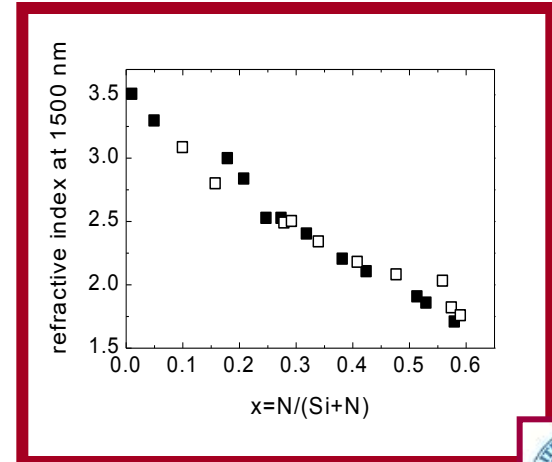
**a-Si<sub>1-x</sub>N<sub>x</sub>:H alloy**

material with tunable optical gap and refractive index varying the N content

**13.56 MHz PECVD**

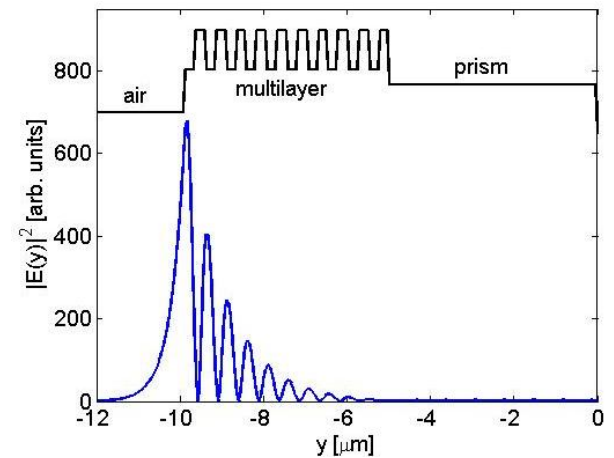
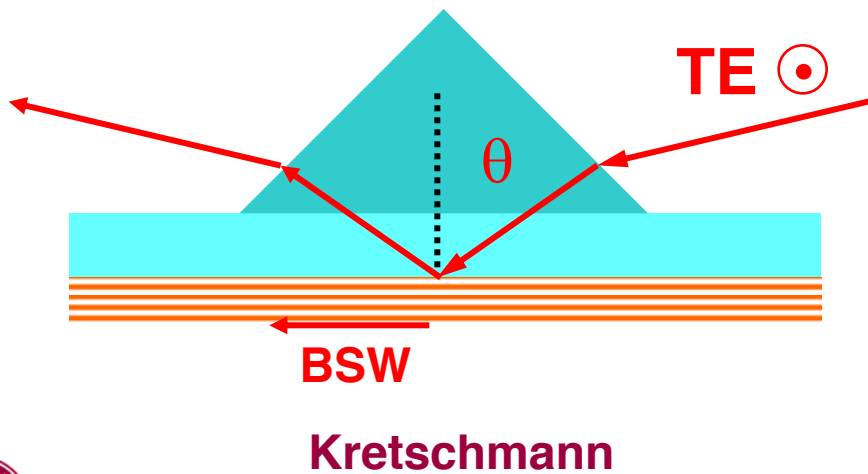
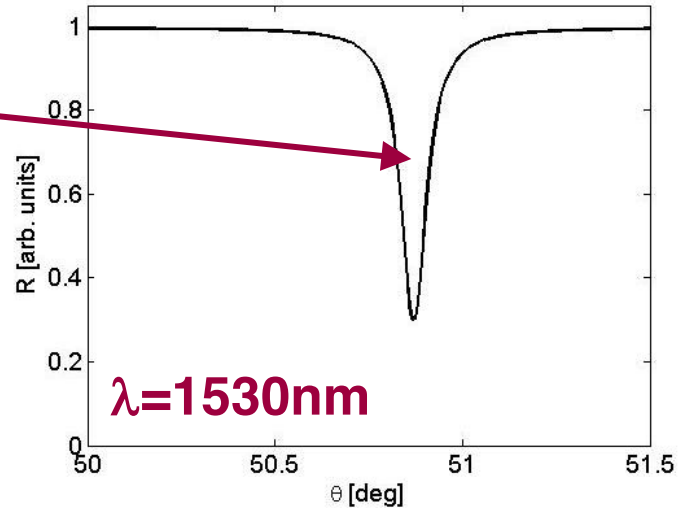
( $\text{SiH}_4 + \text{NH}_3$ )

growth of a-Si<sub>1-x</sub>N<sub>x</sub>:H thin films and multilayers with an excellent control of the thickness and composition

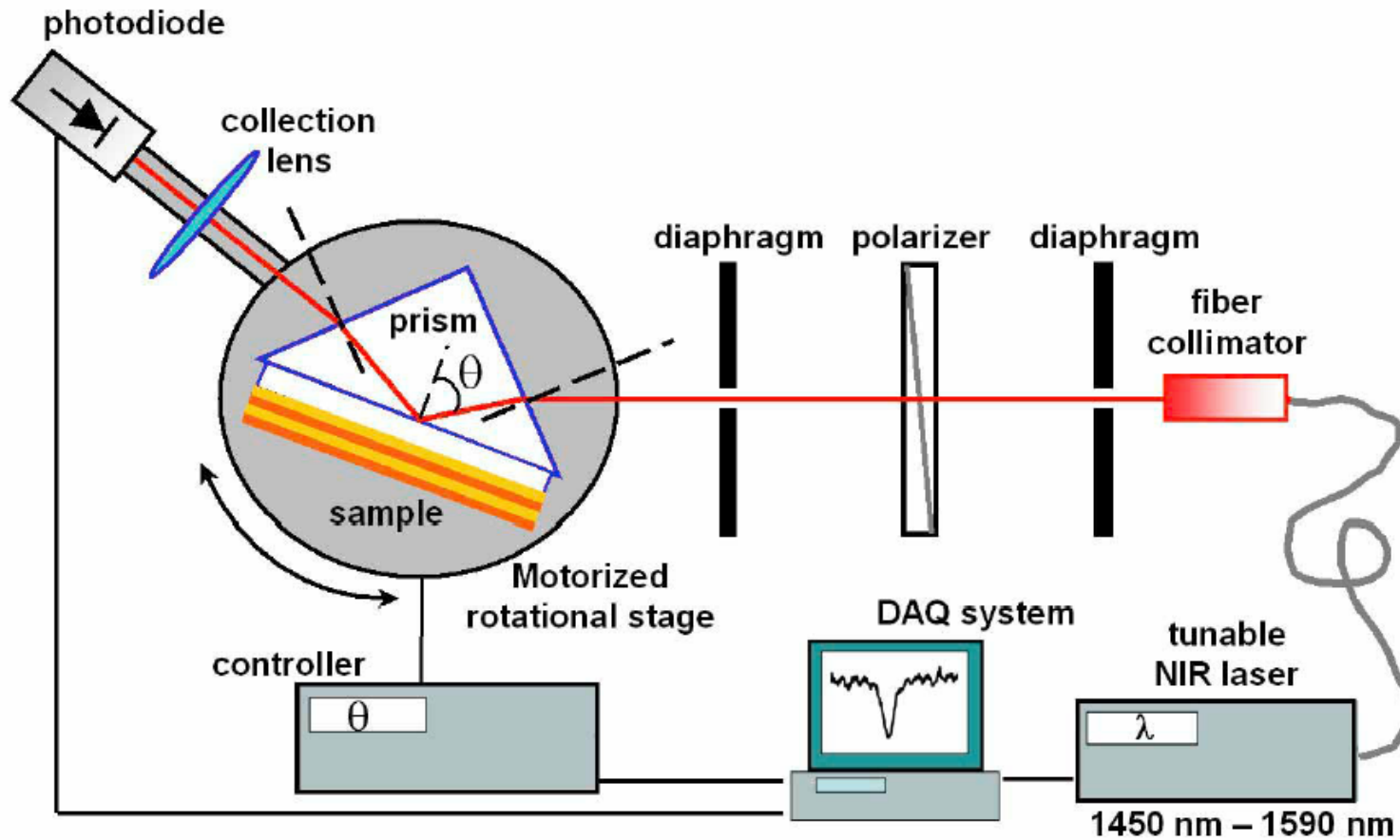


# a-Si<sub>1-x</sub>N<sub>x</sub>:H based 1D Photonic Crystal

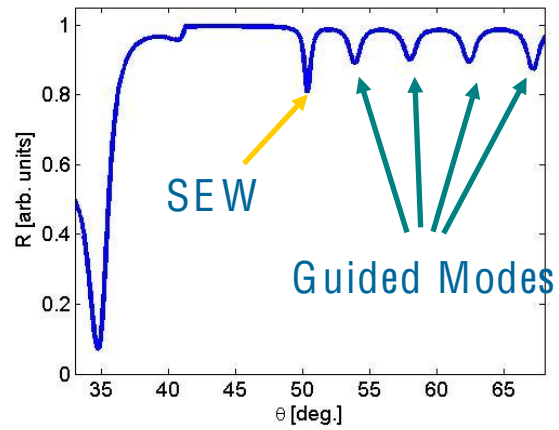
Linewidths are smaller than observed for SPPs



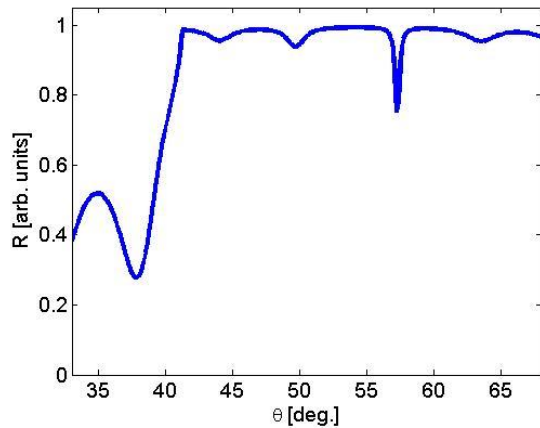
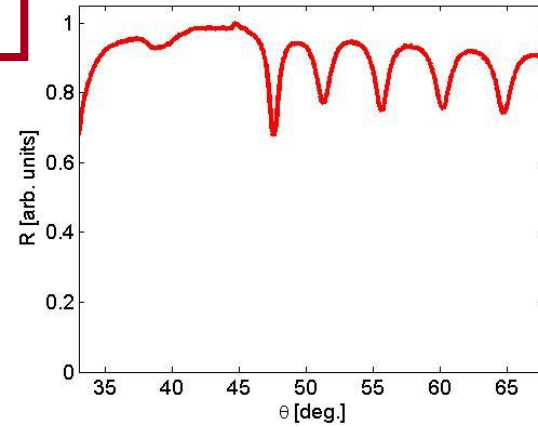
# $a\text{-Si}_{1-x}\text{N}_x\text{:H}$ based 1D Photonic Crystal



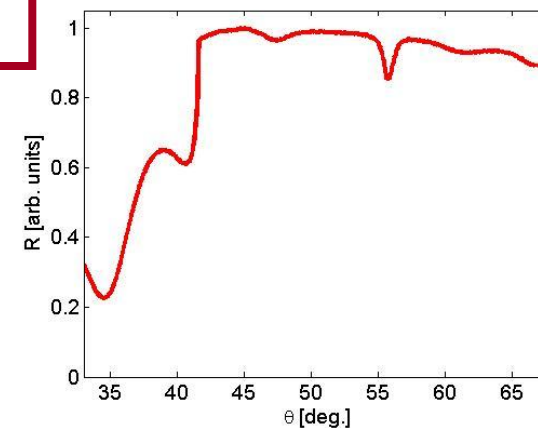
# a-Si<sub>1-x</sub>N<sub>x</sub>:H – Kretschman reflectance



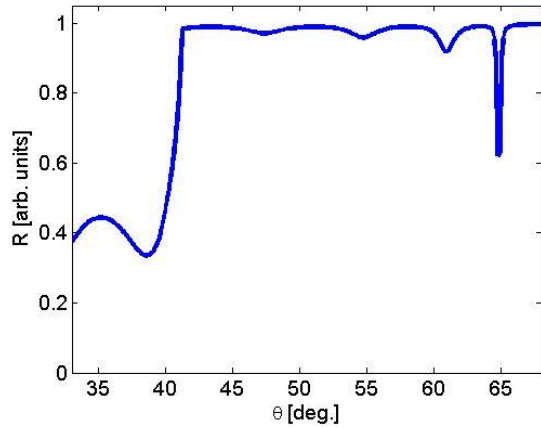
633 nm



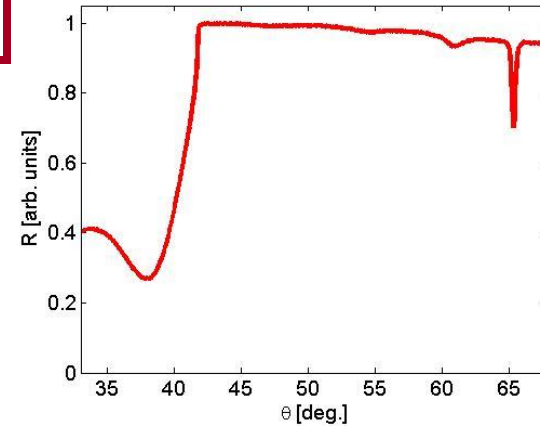
830 nm



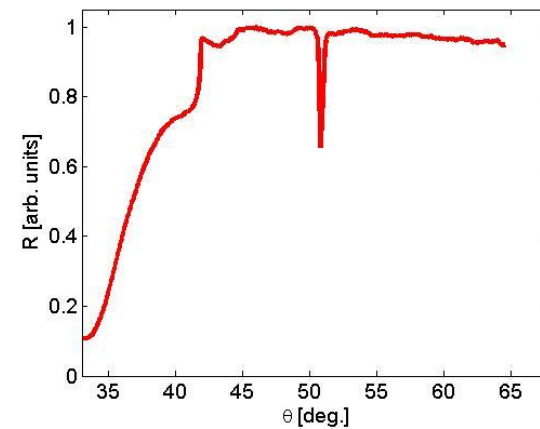
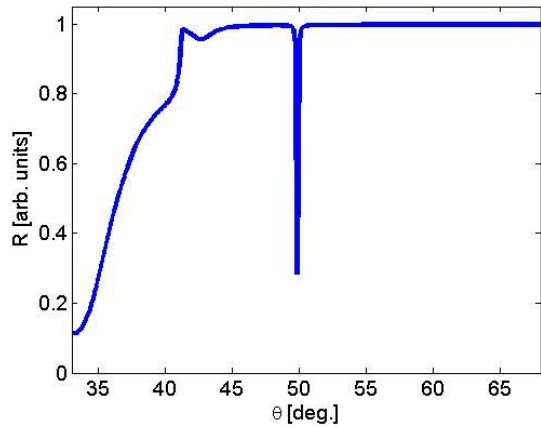
# a-Si<sub>1-x</sub>N<sub>x</sub>:H – Kretschman reflectance



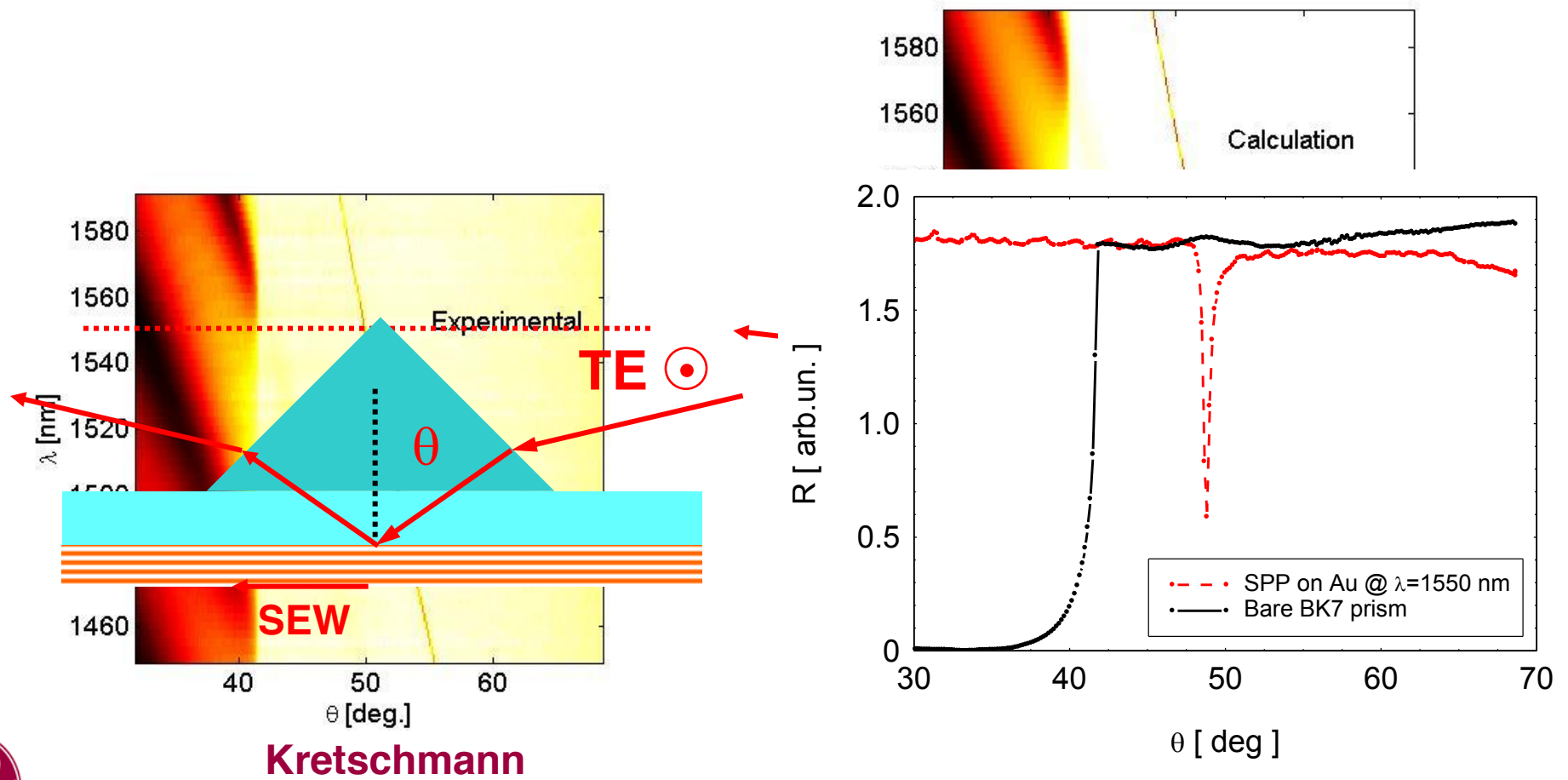
1300 nm



1550 nm



# Kretschman Reflectance Map



Kretschmann