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International Centre for Theoretical Physics**



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**School on Synchrotron and FEL Based Methods and their Multi-Disciplinary
Applications**

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Inelastic x-ray scattering: principles and applications

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Inelastic x-ray scattering: principles and applications



Filippo Bencivenga



OUTLINE

Introduction

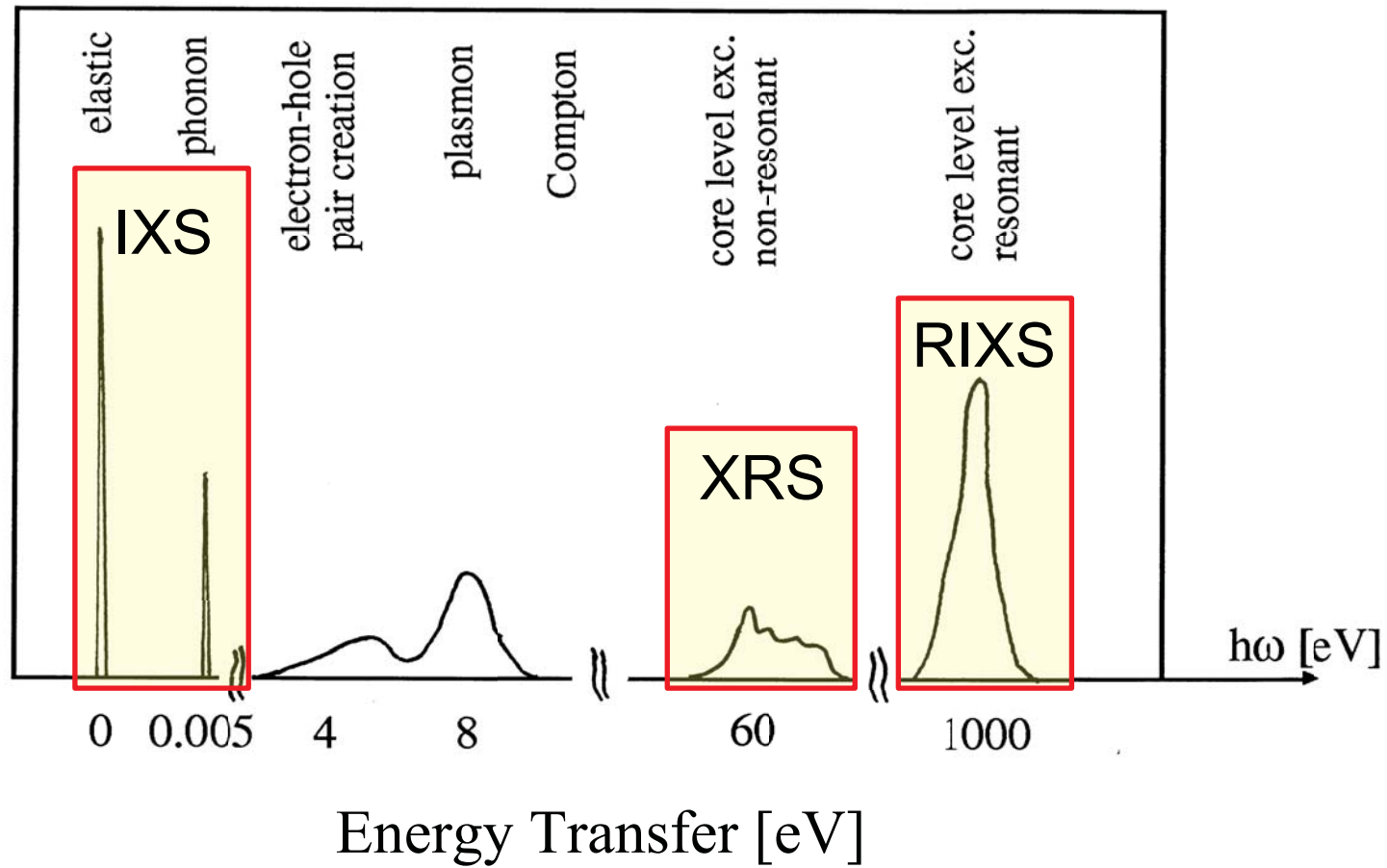
High resolution inelastic x-ray scattering (IXS)

- Collective atomic dynamics
- Neutrons vs. X-rays
- Basic theory and instrumentation
- Experimental highlights

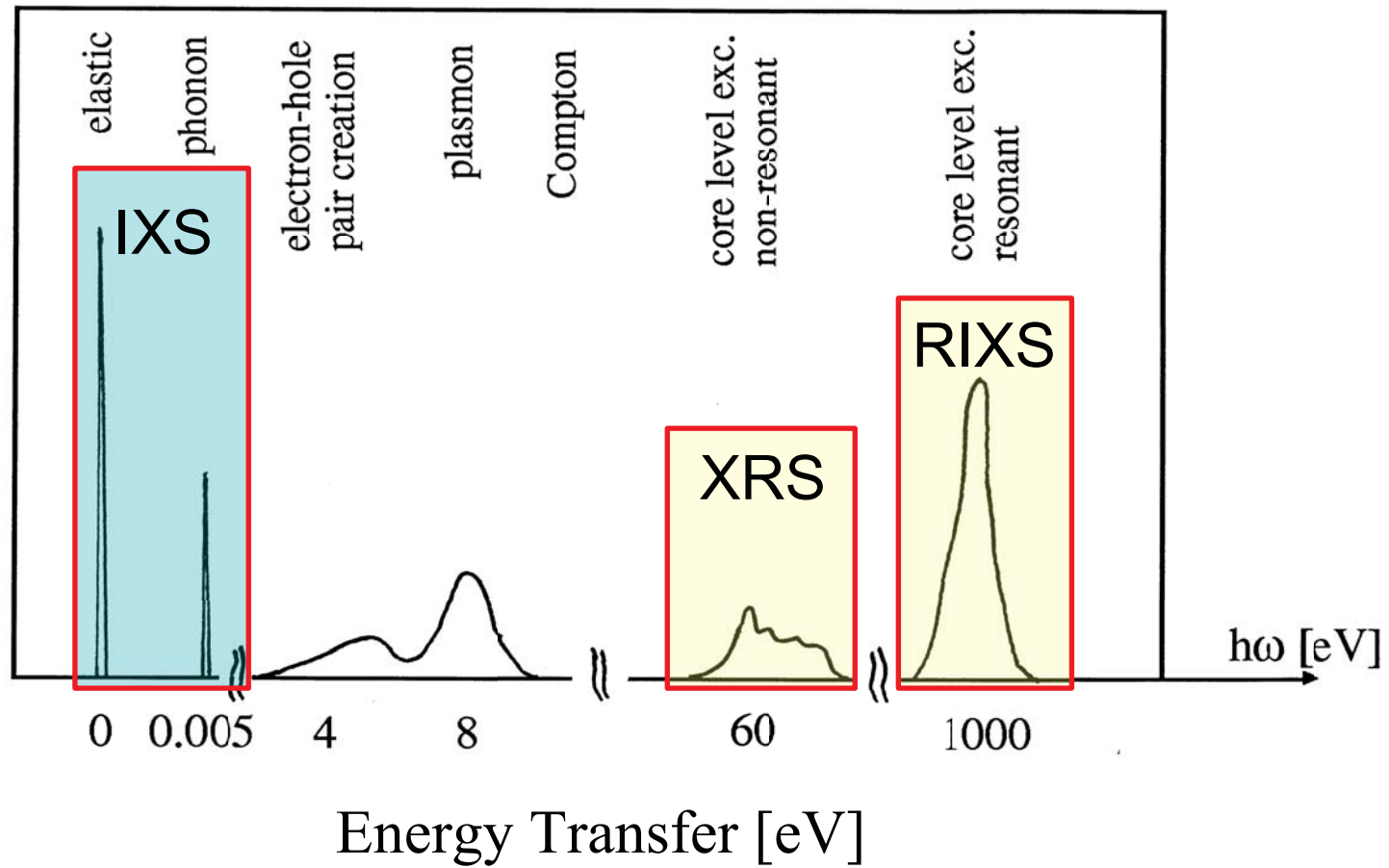
Inelastic x-ray “Raman” scattering (XRS)

- Experimental/theoretical aspects
- Scattering vs. absorption spectroscopy
- Experimental highlights

Introduction: inelastic X-ray spectrum

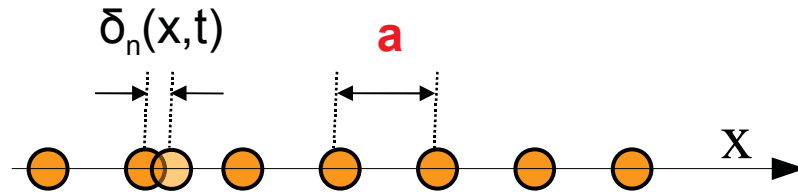


high-resolution Inelastic X-ray Scattering (IXS)



IXS: collective atomic dynamics

The simpler case



$$U = -\beta x^2$$

Information:

- Interatomic Structure (**a**)
- Interaction Potential (**β**)

Phonons



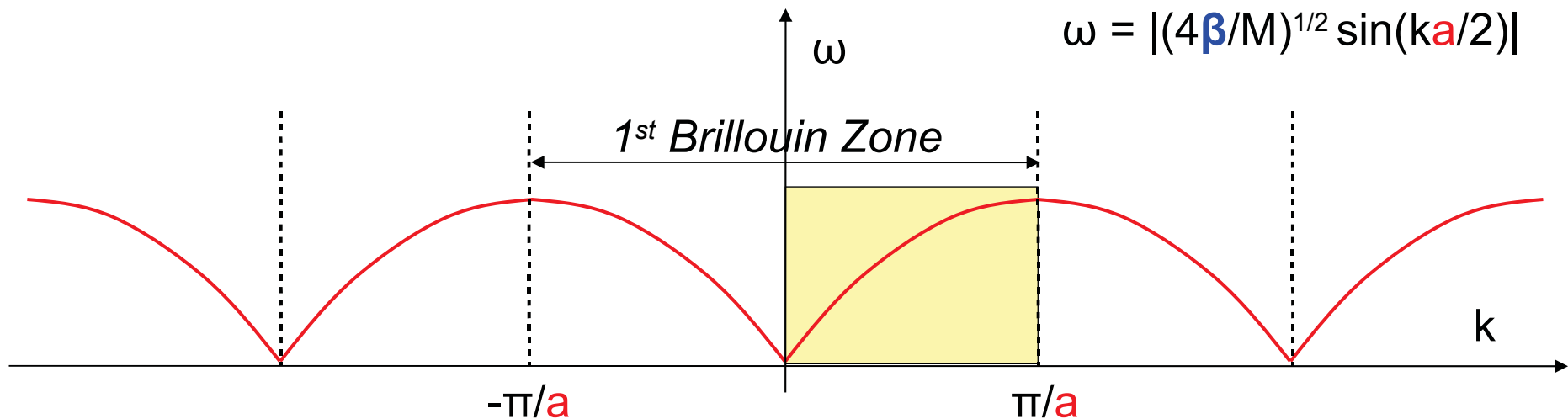
Eigenstates of vibrational field

$$\delta_n(x,t) = \delta_{n,0} \exp[i(kx - \omega t)]$$

$\omega(k)$

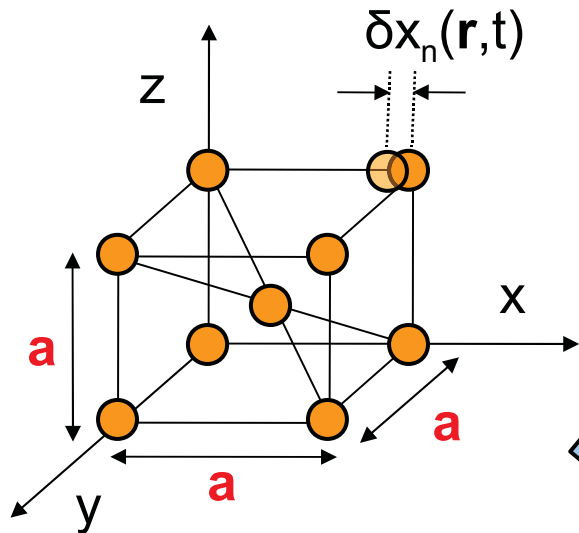


Dispersion relation



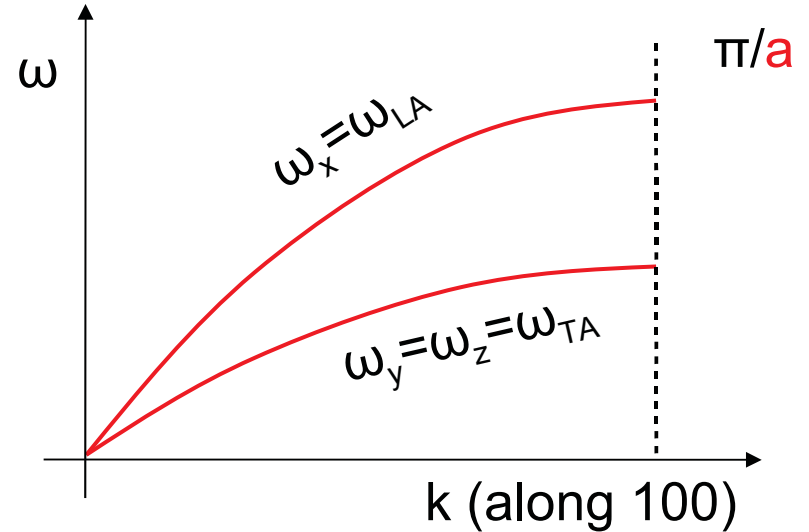
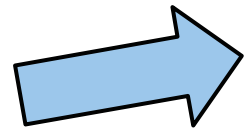
IXS: collective atomic dynamics

One step forward: 3D lattice

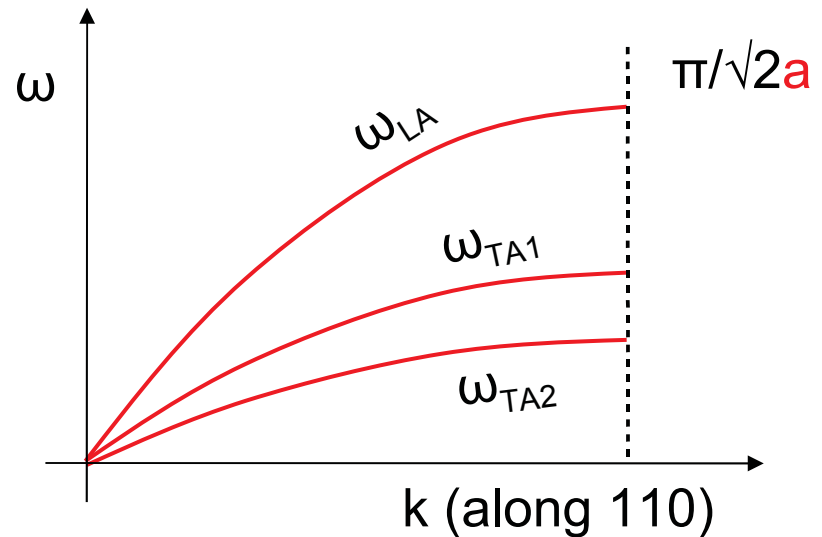


$$U = -\beta |\mathbf{r}|^2 \quad \mathbf{r} = (x, y, z)$$

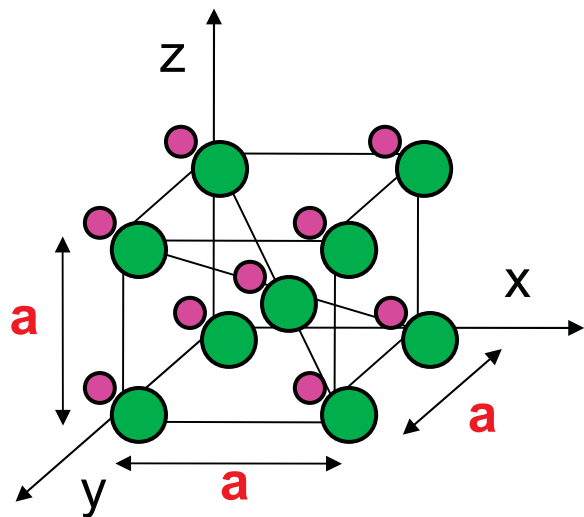
- Information:**
- Interatomic Structure (\mathbf{a})
 - Interaction Potential (β)
 - Anisotropy (elasticity: \mathbf{c}_{ij})



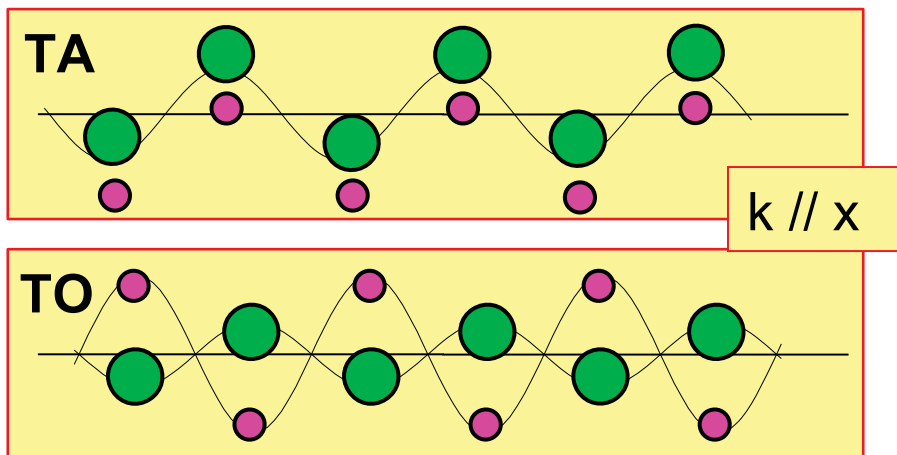
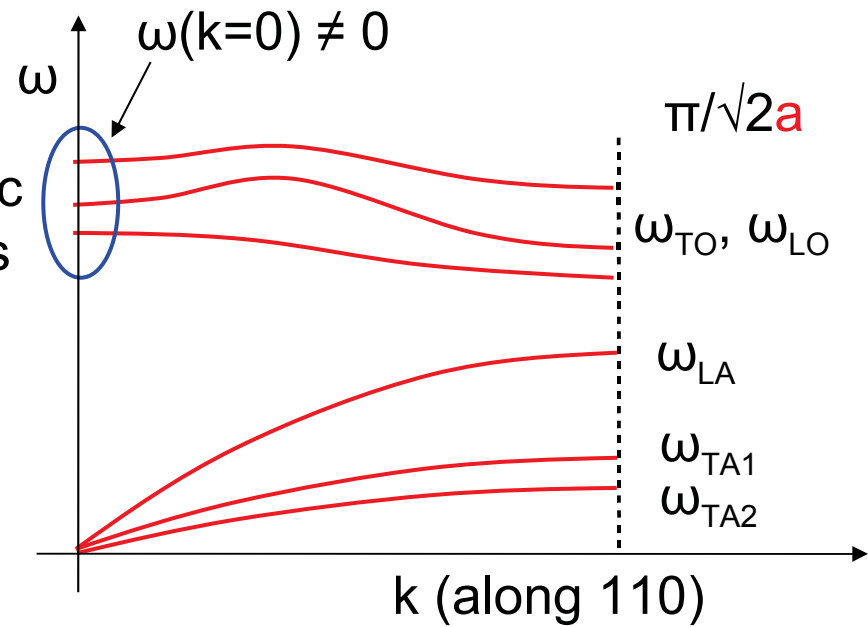
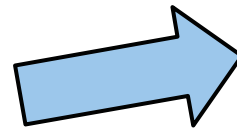
$$\omega = |(\mathbf{c}_{ij}/\rho)^{1/2} \sin(\mathbf{k}\mathbf{a}^*)|$$



IXS: collective atomic dynamics



3N-3 optic branches

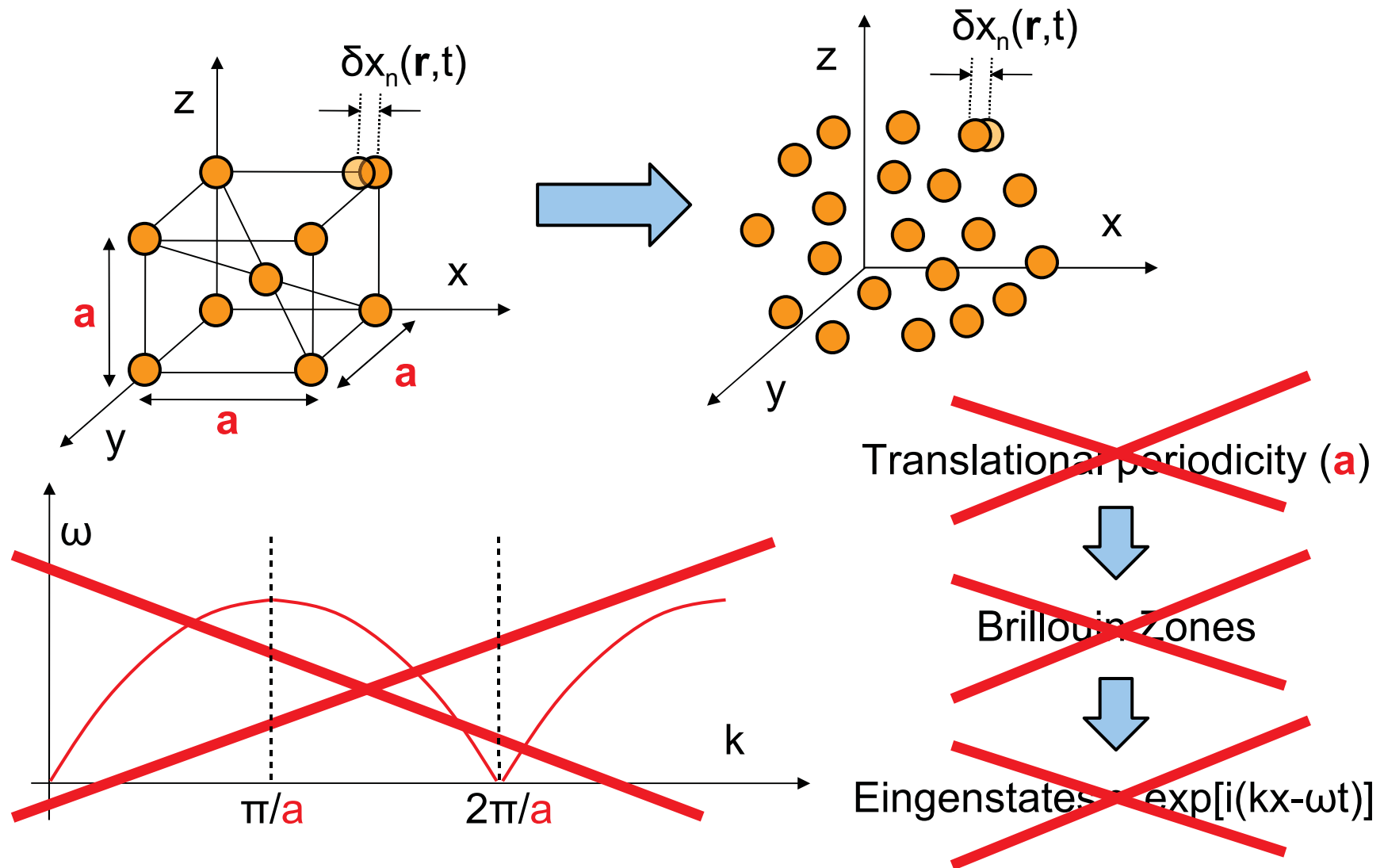


Information:

- Interatomic Structure (a)
- Interaction Potential (β)
- Anisotropy (elasticity: c_{ij})
- Intramolecular vibrations

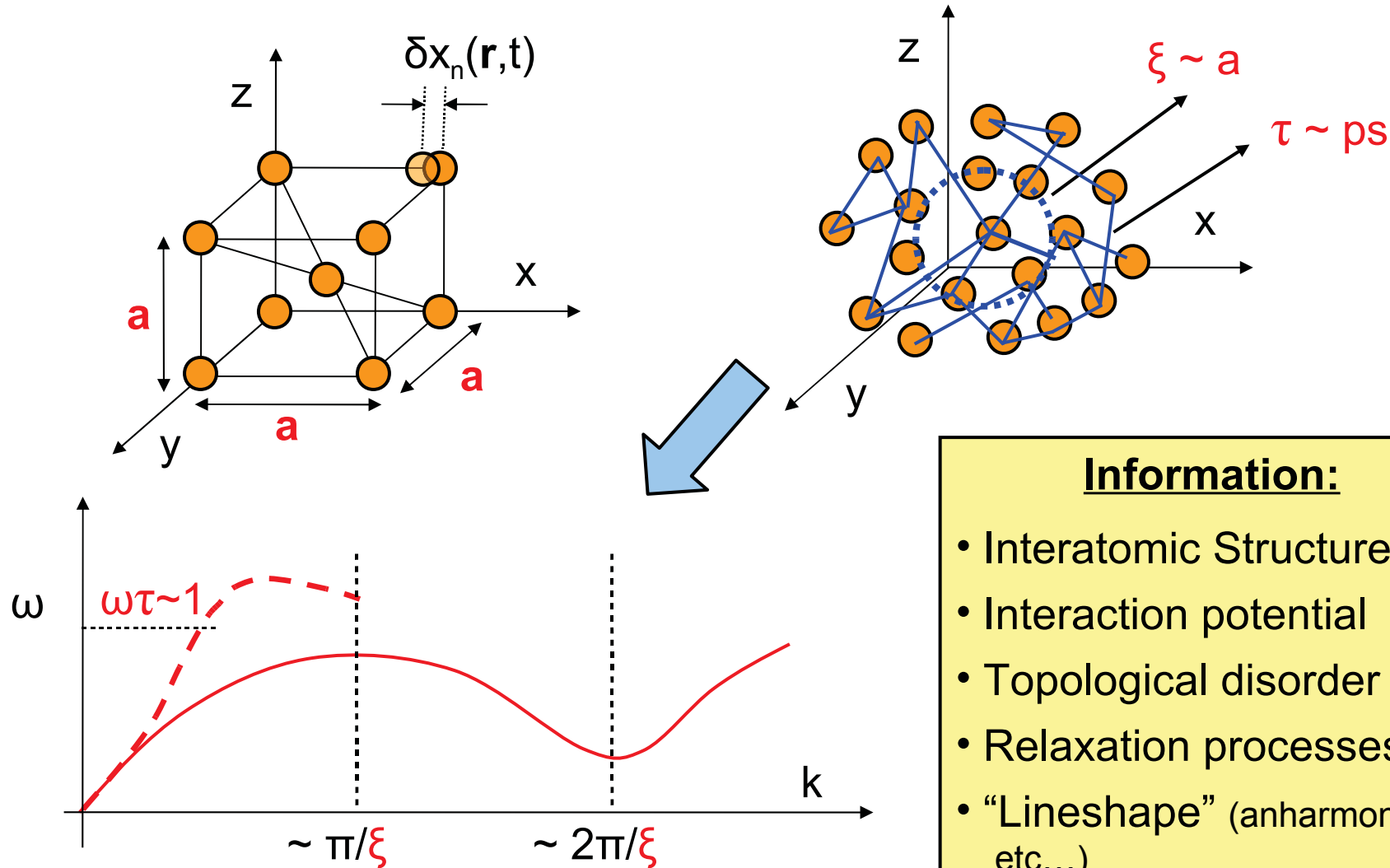
IXS: collective atomic dynamics

The most complex case: disordered systems



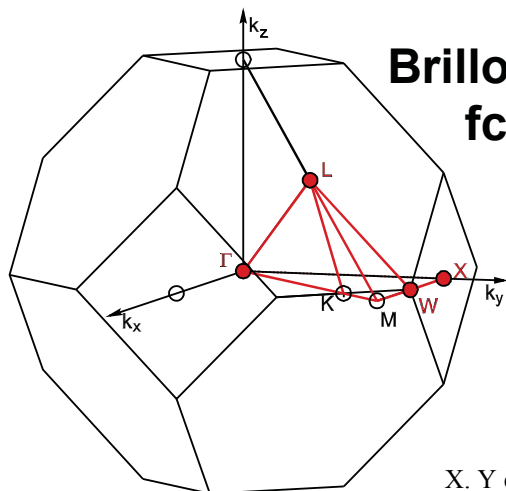
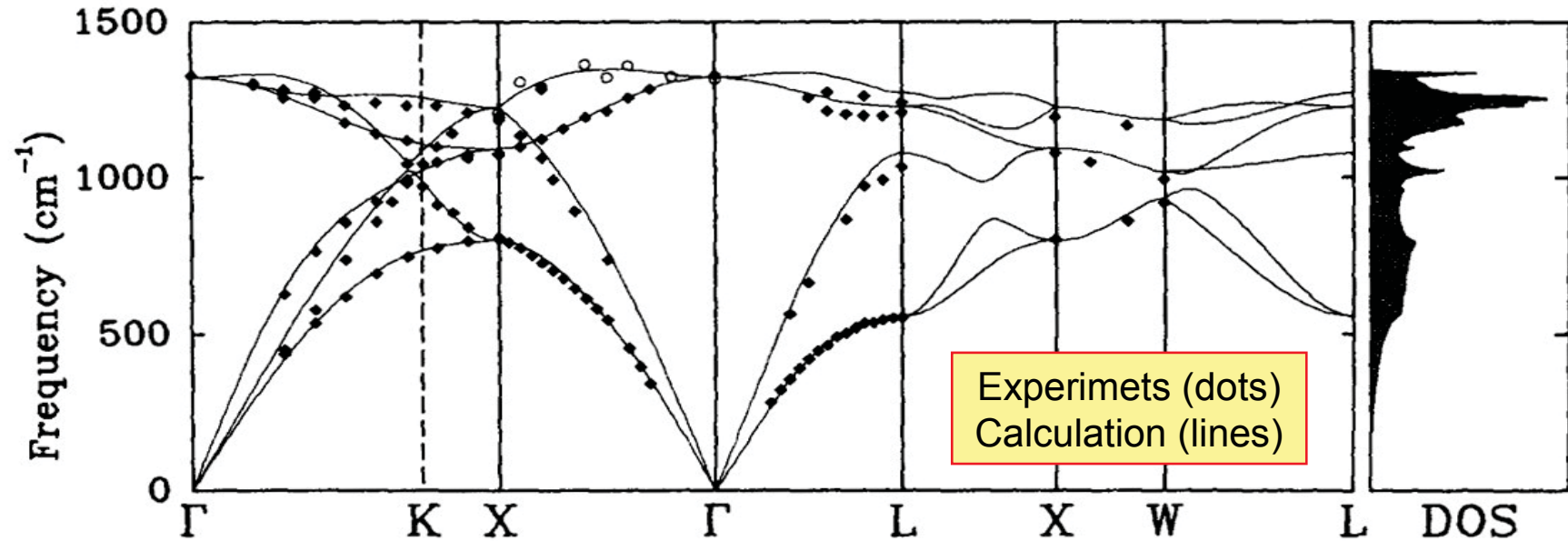
IXS: collective atomic dynamics

The most complex case: disordered systems



An example...

Diamond: fcc symmetry + 2 C atoms each lattice site @ $\pm(1/4, 1/4, 1/4)$



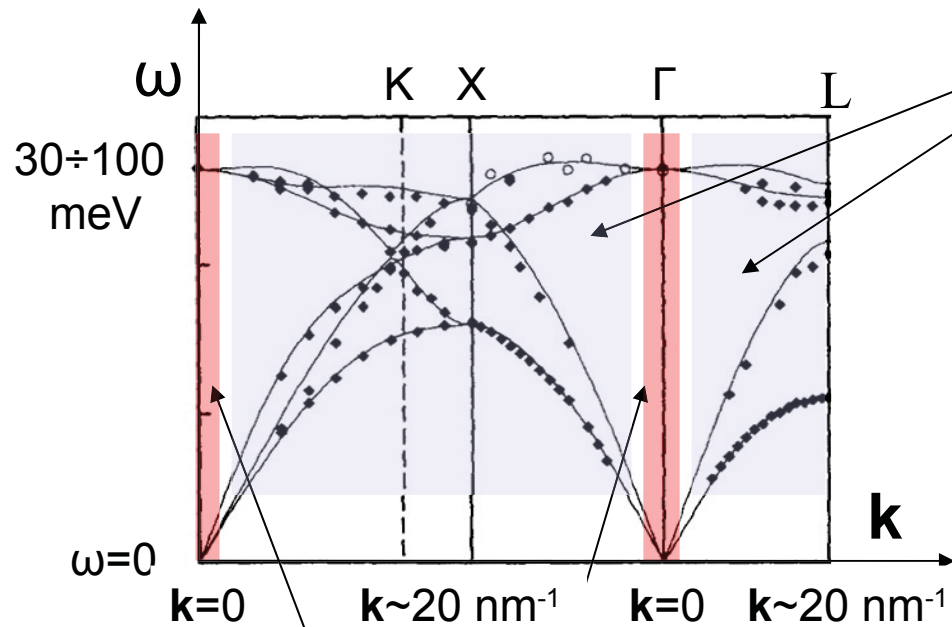
Brillouin zones for fcc symmetry

X. Y et al., PRB 48 (1993)

Information:

- **Structure** and **Elasticity** (sound velocities)
- **Interaction potential** and **Anharmonicity**
- **Dynamical instabilities** (phonon softening)
- **Electron-phonon coupling**
- **Thermodynamics** (c_V , λ , Θ_D , S_D , etc ...)

How can we measure Atomic Dynamics?



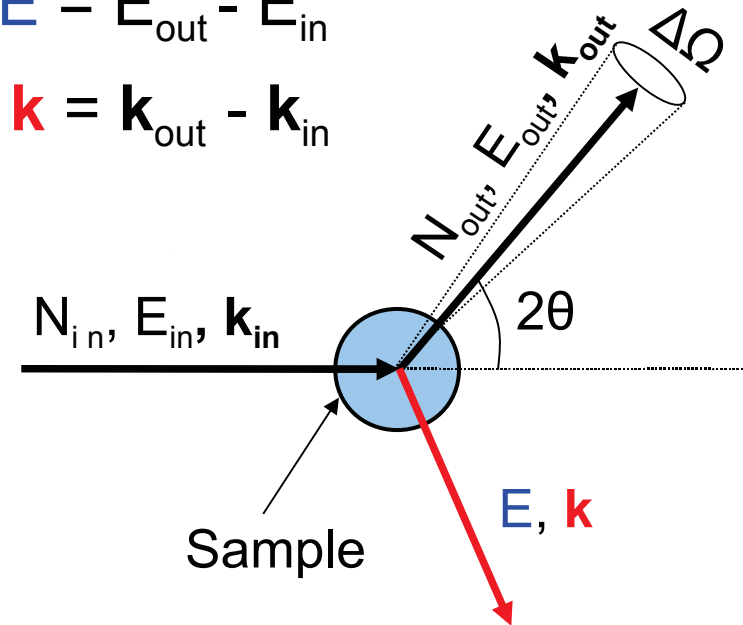
- Probe wavelength ($2\pi/|k|$) < 0.1 nm
- Probe energy (E) $> 30\div 100$ meV

- Inelastic Light Scattering (Brillouin & Raman)
- Ultrasonics
- Transient Grating
- Etc ...

Inelastic scattering:

$$E = E_{\text{out}} - E_{\text{in}}$$

$$k = k_{\text{out}} - k_{\text{in}}$$



Cross section: $\frac{\partial^2 \sigma}{\partial \Omega \partial E} \sim N_{\text{out}}/N_{\text{in}}$

Neutrons

vs.

X-rays

$$\lambda_{in} = 1\text{\AA} \Rightarrow E_{in} = 82 \text{ meV}$$

$$\lambda_{in} = 1\text{\AA} \Rightarrow E_{in} = 12.4 \text{ keV}$$

$$E > 4 \text{ meV} \rightarrow \Delta E/E_{in} = 0.05$$

$$E > 4 \text{ meV} \rightarrow \Delta E/E_{in} = 3 \cdot 10^{-7}$$

Moderate energy resolution

Very high energy resolution

100 INS instruments

+

Spin sensitive

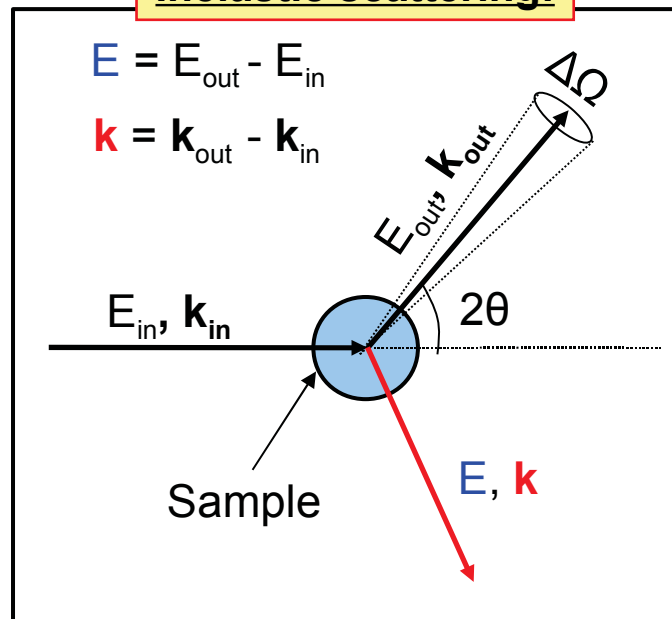
Better contrast

“Older” technique

Inelastic scattering:

$$E = E_{out} - E_{in}$$

$$\mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$



4 IXS instruments

Why X-rays?

Neutrons

vs.

X-rays

$$\lambda_{in} = 1 \text{ \AA} \Rightarrow E_{in} = 82 \text{ meV}$$

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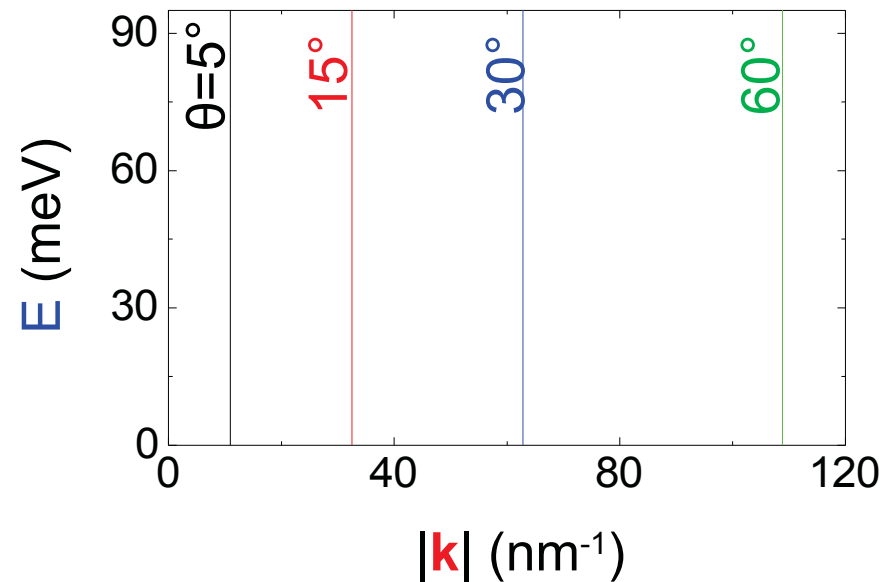
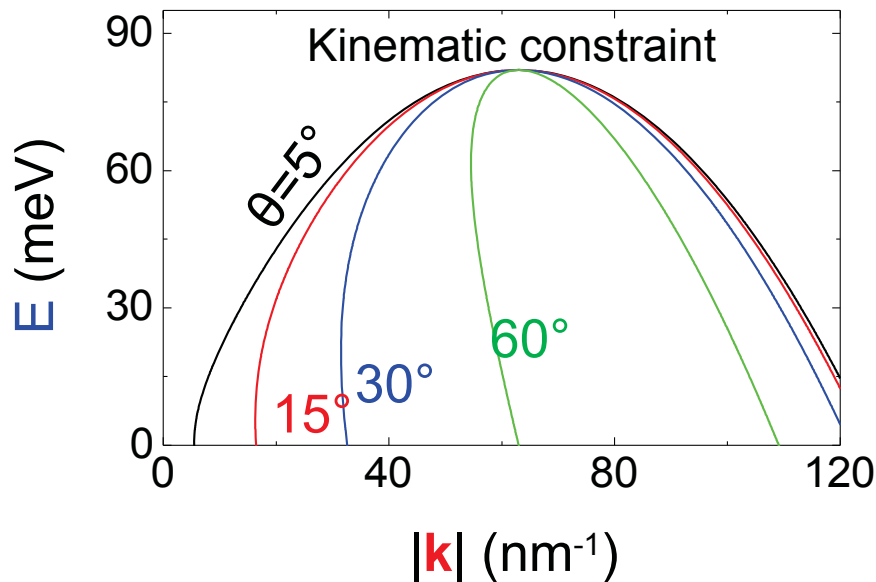
$$E_{out} \neq E_{in}$$

$$E = E_{out} - E_{in} \quad \& \quad \mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$

$$E_{out} \approx E_{in}$$

$$\frac{|\mathbf{k}|^2}{2|\mathbf{k}_{in}|^2} = 1 - E/E_{in} + \cos(2\theta)(1 - 2E/E_{in})^{1/2}$$

$$|\mathbf{k}| = 2|\mathbf{k}_{in}|\sin(\theta)$$



Neutrons

vs.

X-rays

$$\lambda_{in} = 1 \text{ \AA} \Rightarrow E_{in} = 82 \text{ meV}$$

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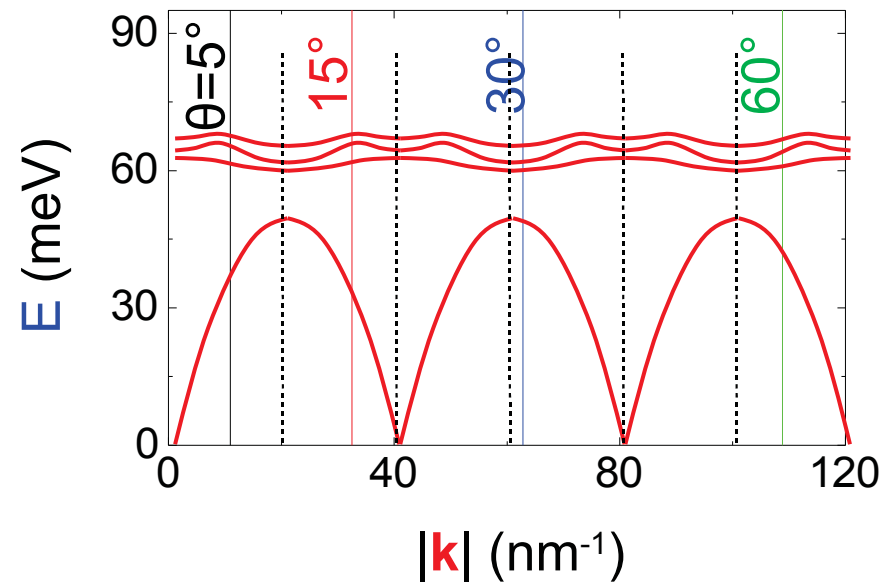
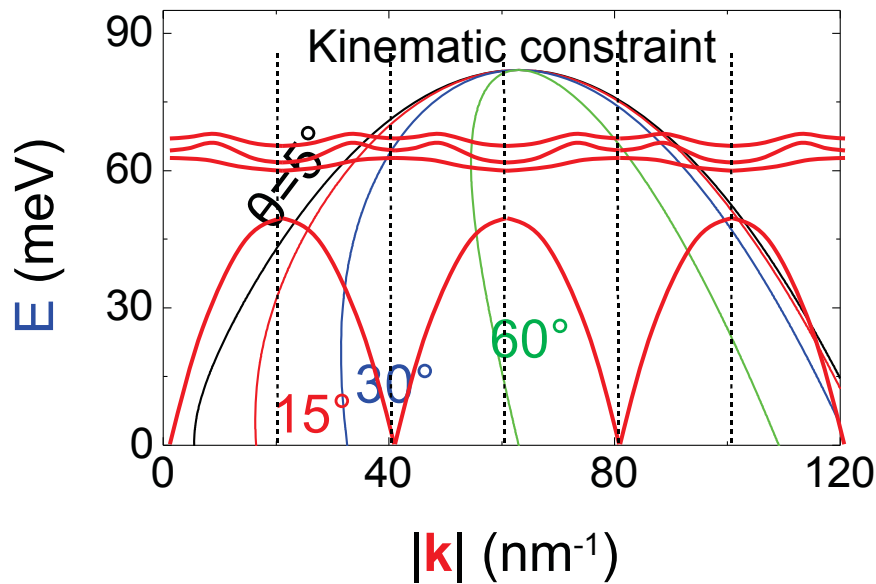
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Neutrons

vs.

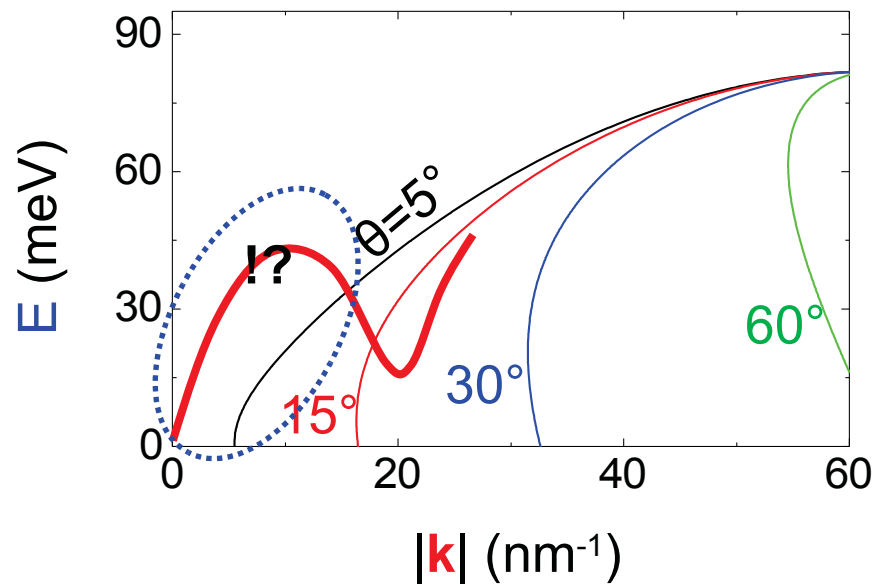
X-rays

Inelastic excitations in disordered systems

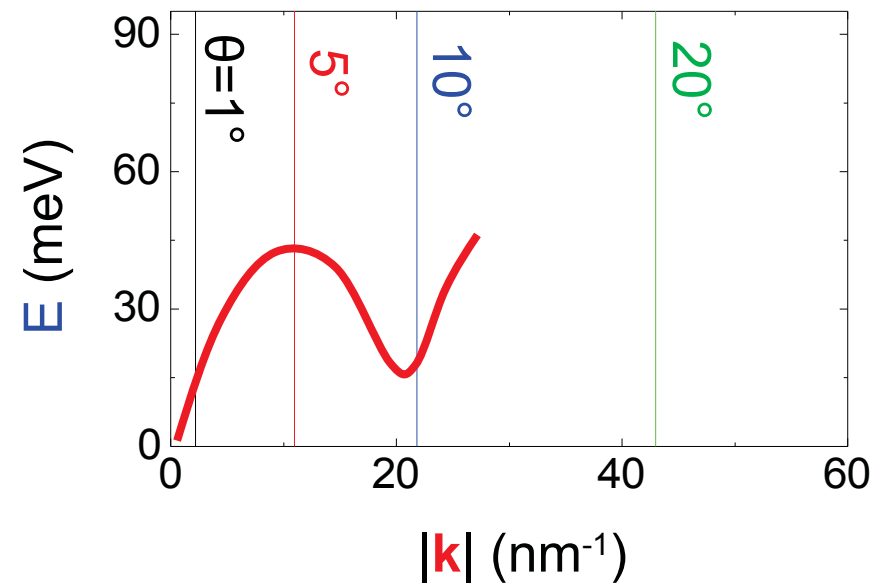
Neutrons

X-rays

$\lambda_{in} = 1 \text{ \AA}$ $E_{in} = 82 \text{ meV}$



$\lambda_{in} = 1 \text{ \AA}$ $E_{in} = 12.4 \text{ keV}$



Neutrons

vs.

X-rays

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Moderate energy resolution

Very high energy resolution

100 INS instruments

3 IXS instruments

+

Spin sensitive

Better contrast

“Older” technique

No kinematical constraints
(Disordered systems)

Small beams
(small samples: high pressure, exotic materials, etc...)

Why X-rays?

No incoherent cross section

Basic theoretical aspects

$$H_{\text{int}} = (e/m_e c) \sum_j [(e/2c) \mathbf{A}_j \cdot \mathbf{A}_j + \mathbf{A}_j \cdot \mathbf{p}_j + \text{magnetic}]$$

\mathbf{A} is the vector potential of electromagnetic field

\mathbf{p} is the momentum operator of the electrons

j is the summation over all electrons of the system

1st order perturbation theory

$\mathbf{A} \cdot \mathbf{A}$ term \rightarrow one photon (non-resonant) scattering

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\varepsilon}_{\text{in}} \cdot \boldsymbol{\varepsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}}) \sum_l P_l |\langle l | \exp\{i \mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_{\text{out}} + E_{\text{in}})$$

Basic theoretical aspects

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\epsilon}_{\text{in}} \cdot \boldsymbol{\epsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}}) \sum_i P_i |\langle I | \exp\{i\mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_F + E_I)$$

The key assumption:

Adiabatic approximation \rightarrow $|I\rangle = |I_n\rangle |I_e\rangle$ and $|F\rangle = |F_n\rangle |F_e\rangle$

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = \underbrace{r_0^2 (\boldsymbol{\epsilon}_{\text{in}} \cdot \boldsymbol{\epsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}})}_{\text{Thomson scattering cross section}} \underbrace{F(|\mathbf{k}|)^2}_{\text{Molecular form factor } (|I_e\rangle, |F_e\rangle)} \underbrace{S(\mathbf{k}, E)}_{\text{Dynamic structure factor}}$$

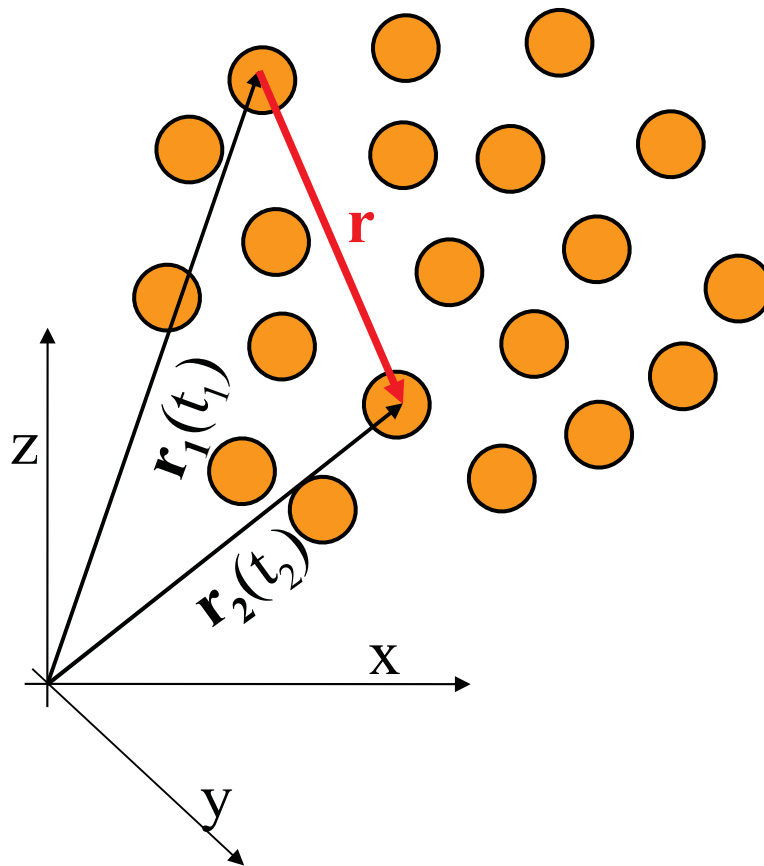
Thomson scattering
cross section

Molecular form factor ($|I_e\rangle, |F_e\rangle$)

Dynamic structure factor

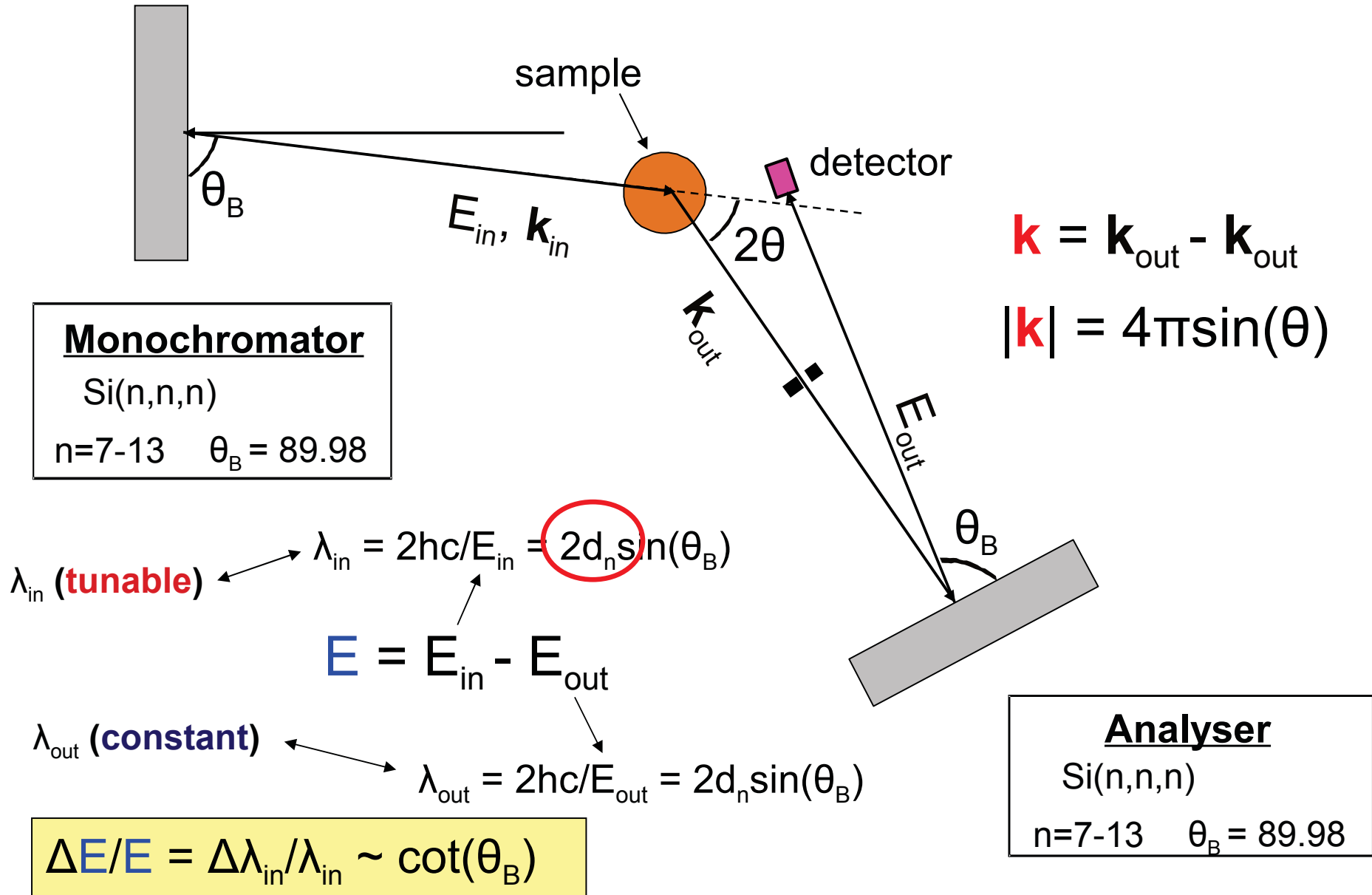
The dynamic structure factor

$S(\mathbf{k}, E)$ is the **SPACE** and **TIME** Fourier transform of $G(\mathbf{r}, t)$

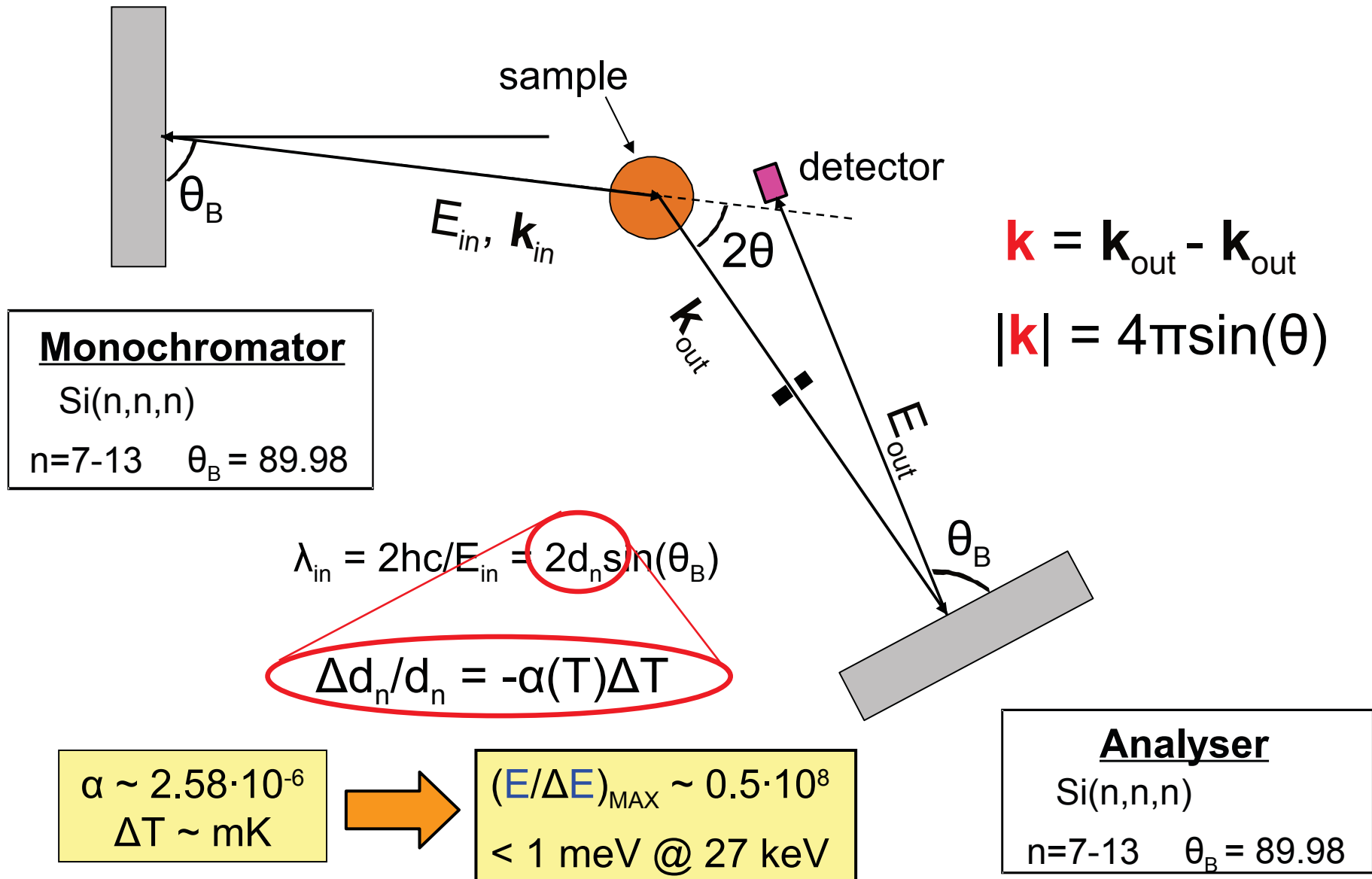


$G(\mathbf{r}, t)$ is the probability to find two distinct particles at positions $\mathbf{r}_1(t_1)$ and $\mathbf{r}_2(t_2)$, separated by the distance $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ and the time interval $t = t_2 - t_1$.

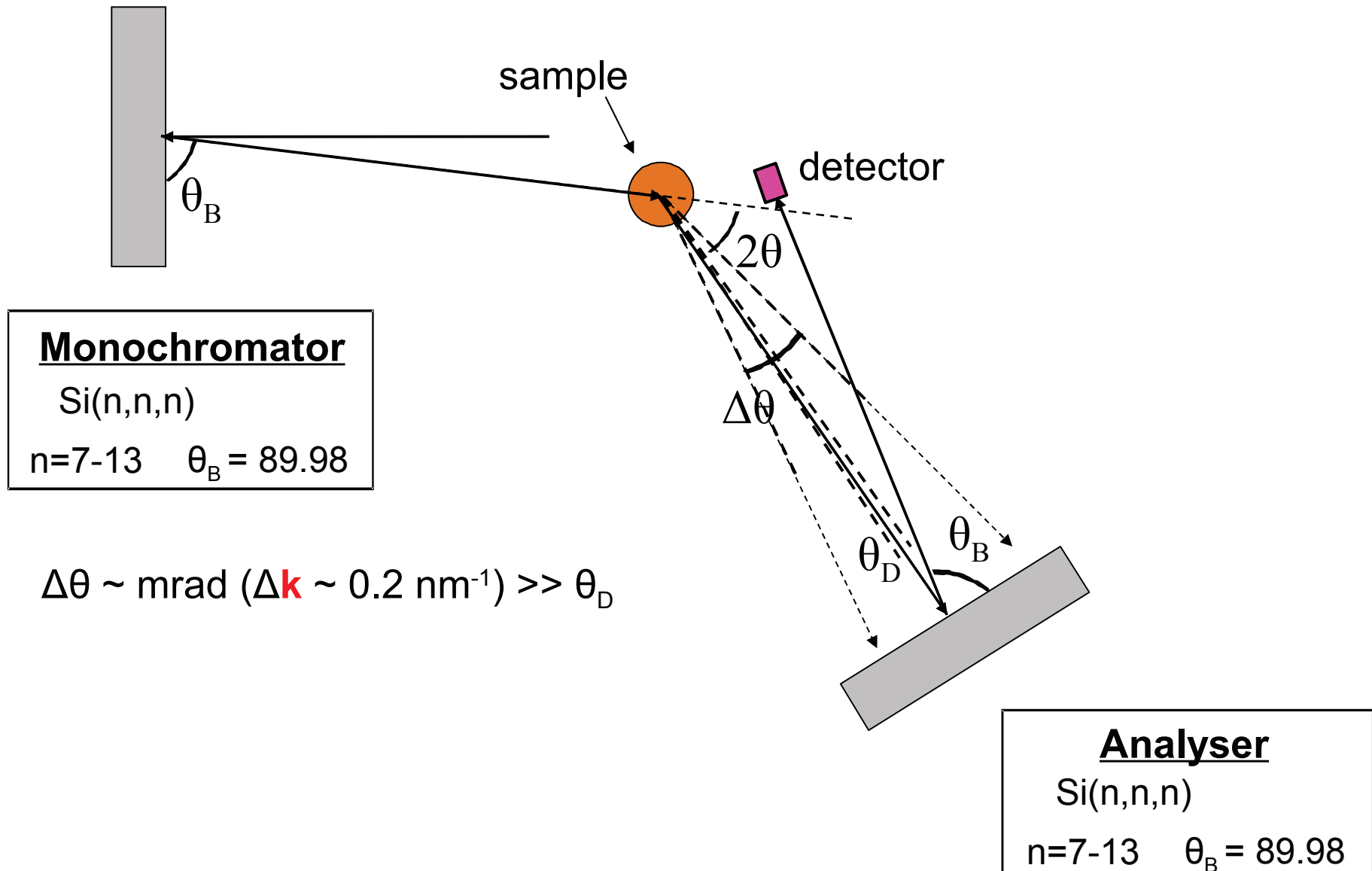
Basic IXS instrumentation



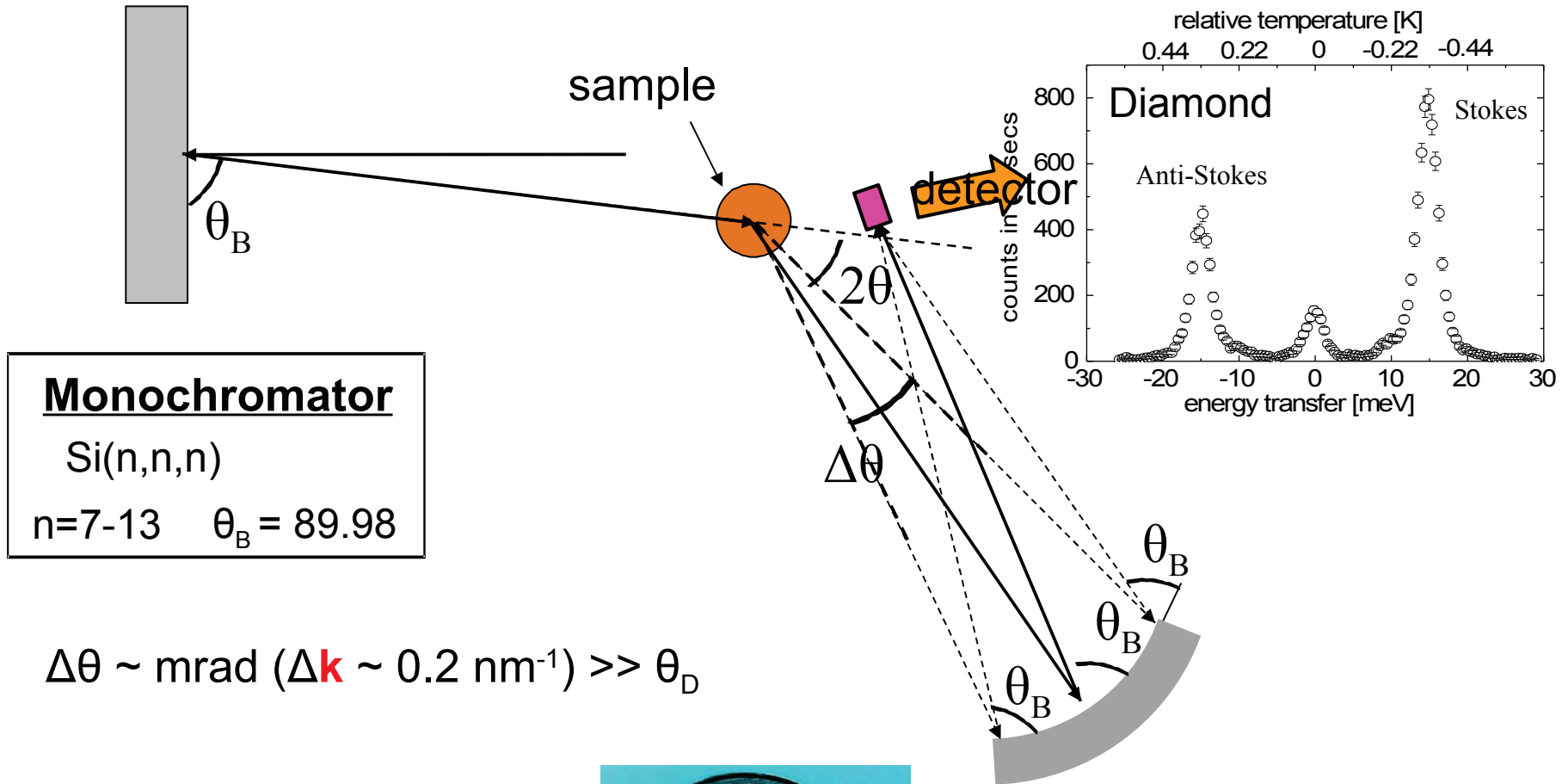
Basic IXS instrumentation



Basic IXS instrumentation



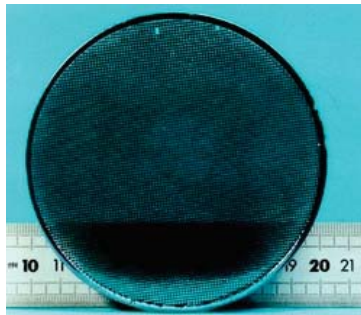
Basic IXS instrumentation



Monochromator
 Si(n,n,n)
 n=7-13 $\theta_B = 89.98$

$\Delta\theta \sim \text{mrad}$ ($\Delta k \sim 0.2 \text{ nm}^{-1}$) $\gg \theta_D$

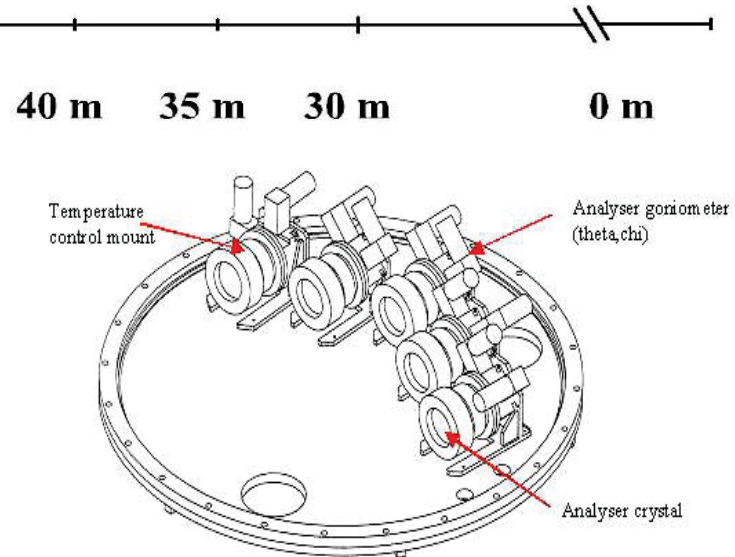
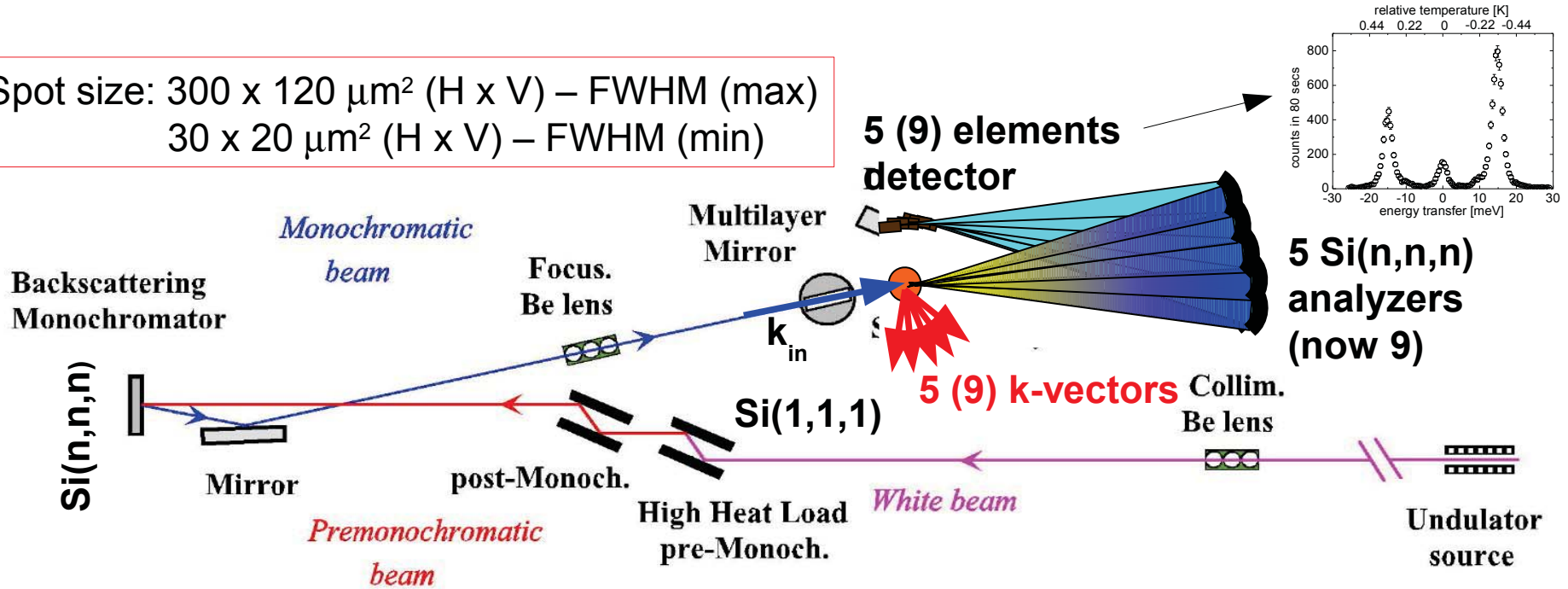
≈ 12000 flat Si “perfect” single crystals ($0.6 \times 0.6 \text{ mm}^2$) that approximate a spherical surface



Analyser
 Si(n,n,n)
 n=7-13 $\theta_B = 89.98$

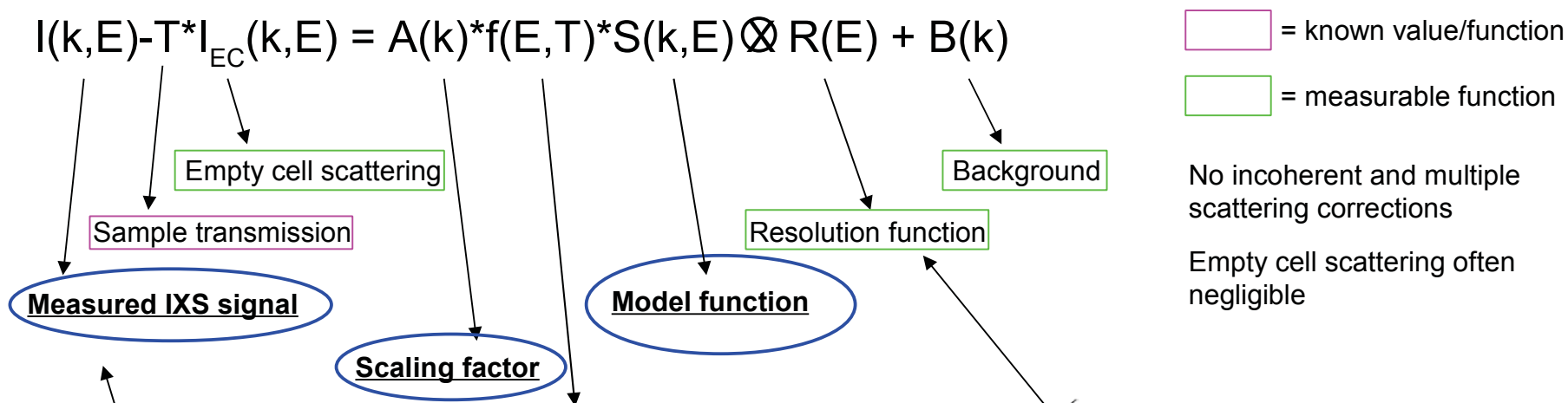
ID-28 at the ESRF (Grenoble, France)

Spot size: $300 \times 120 \mu\text{m}^2$ (H x V) – FWHM (max)
 $30 \times 20 \mu\text{m}^2$ (H x V) – FWHM (min)



Data Analysis

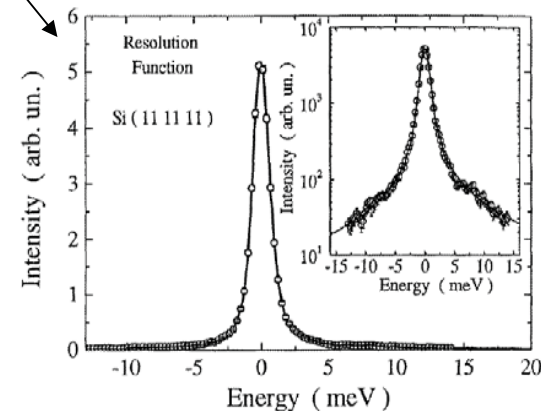
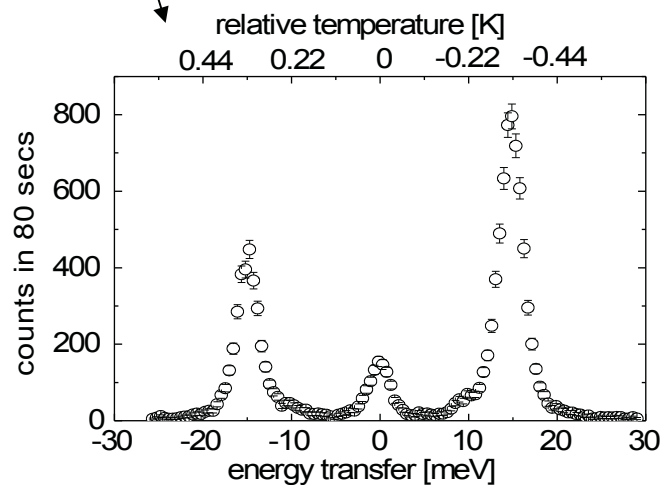
$$I(k,E) - T \cdot I_{EC}(k,E) = A(k) \cdot f(E,T) \cdot S(k,E) \otimes R(E) + B(k)$$



No incoherent and multiple scattering corrections

Empty cell scattering often negligible

Detailed balance (statistical population of vibrational modes at a given T – accounting for the Stokes to Anti-Stokes intensity ratio)



The E-dependence (linashape) of the measured IXS signal can be directly linked to the model S(k,E)

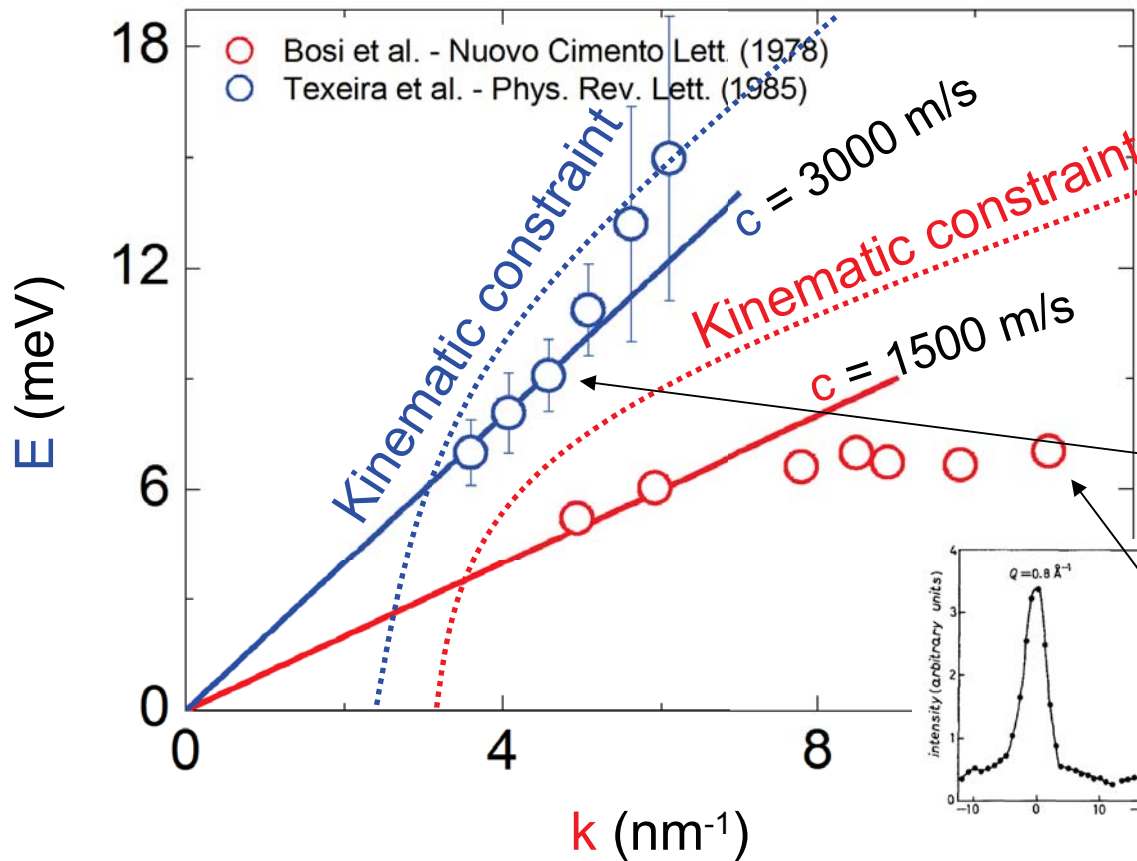
Experimental highlights (1)

Collective dynamics in water (IXS to overcome kinematic constraints)

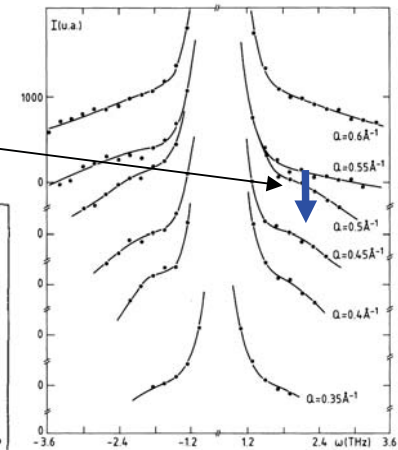
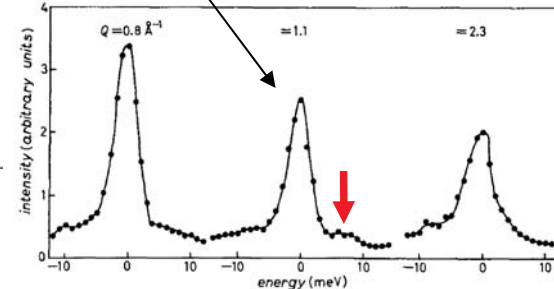
Inelastic Neutron Scattering (D_2O):
2 experiments, 2 results: why?

A possible interpretation:

- High frequency mode → **D**
- Low frequency mode → **O**



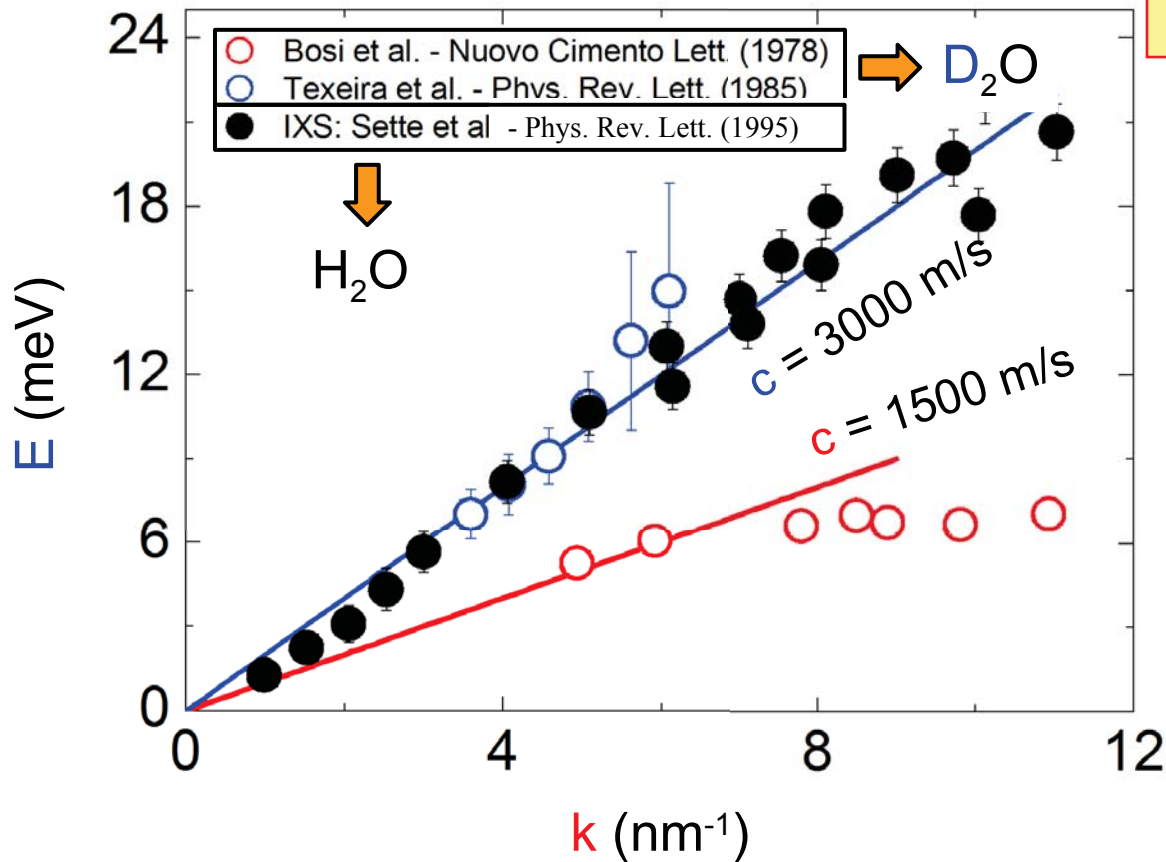
$$\Omega_{HF}/\Omega_{LF} \sim (m_O/m_D)^{1/2} \sim 2$$



Experimental highlights (1)

Collective dynamics in water

Inelastic Neutron Scattering vs.
Inelastic X-ray Scattering (H_2O vs. D_2O)



A possible interpretation:

- High frequency mode → **D**
- Low frequency mode → **O**



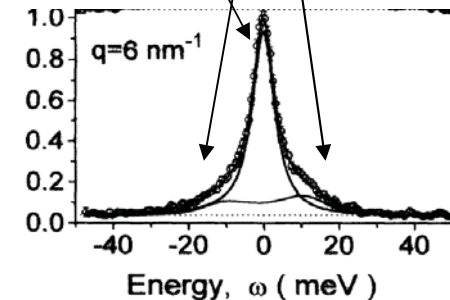
$$\Omega_{HF}/\Omega_{LF} \sim (m_O/m_D)^{1/2} \sim 2$$

Data analysis

$$S(k,E) = \frac{A_{qe}}{E^2 + \Gamma^2} + \frac{A_{inel}}{(E^2 - E_p^2)^2 + (E \cdot \Gamma)^2}$$

Lorentian peak
(quasi elastic
scattering)

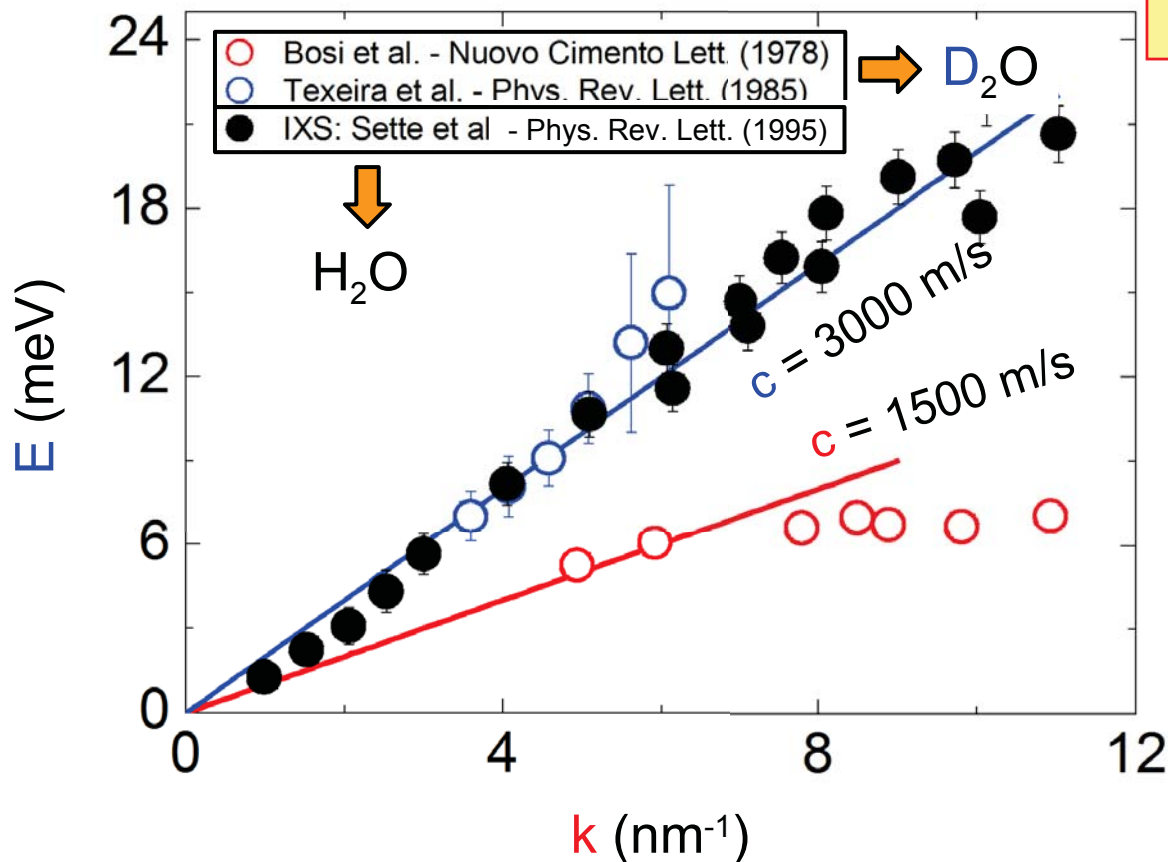
Damped harmonic
oscillator (inelastic
excitations)



Experimental highlights (1)

Collective dynamics in water

Inelastic Neutron Scattering vs. Inelastic X-ray Scattering (H₂O vs. D₂O)



A possible interpretation:

- High frequency mode → **D**
- Low frequency mode → **O**

$$\Omega_{\text{HF}}/\Omega_{\text{LF}} \sim (m_{\text{O}}/m_{\text{D}})^{1/2} \sim 2$$

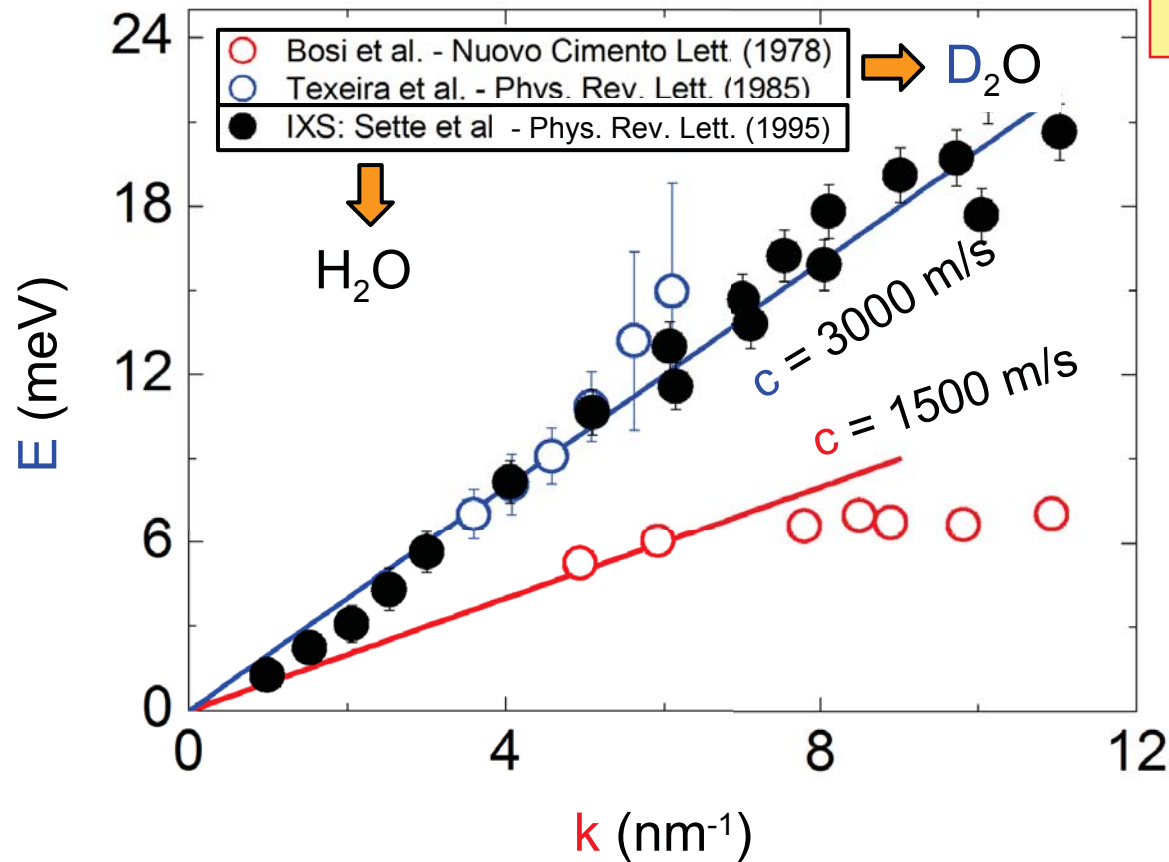
High frequency mode:

Expected for H₂O...
 $\Omega_{\text{IXS}}/\Omega_{\text{INS}} \sim (m_{\text{H}}/m_{\text{D}})^{1/2} \sim 1.4$

Experimental highlights (1)

Collective dynamics in water

Inelastic Neutron Scattering vs. Inelastic X-ray Scattering (H_2O vs. D_2O)



A possible interpretation:

- High frequency mode → **D**
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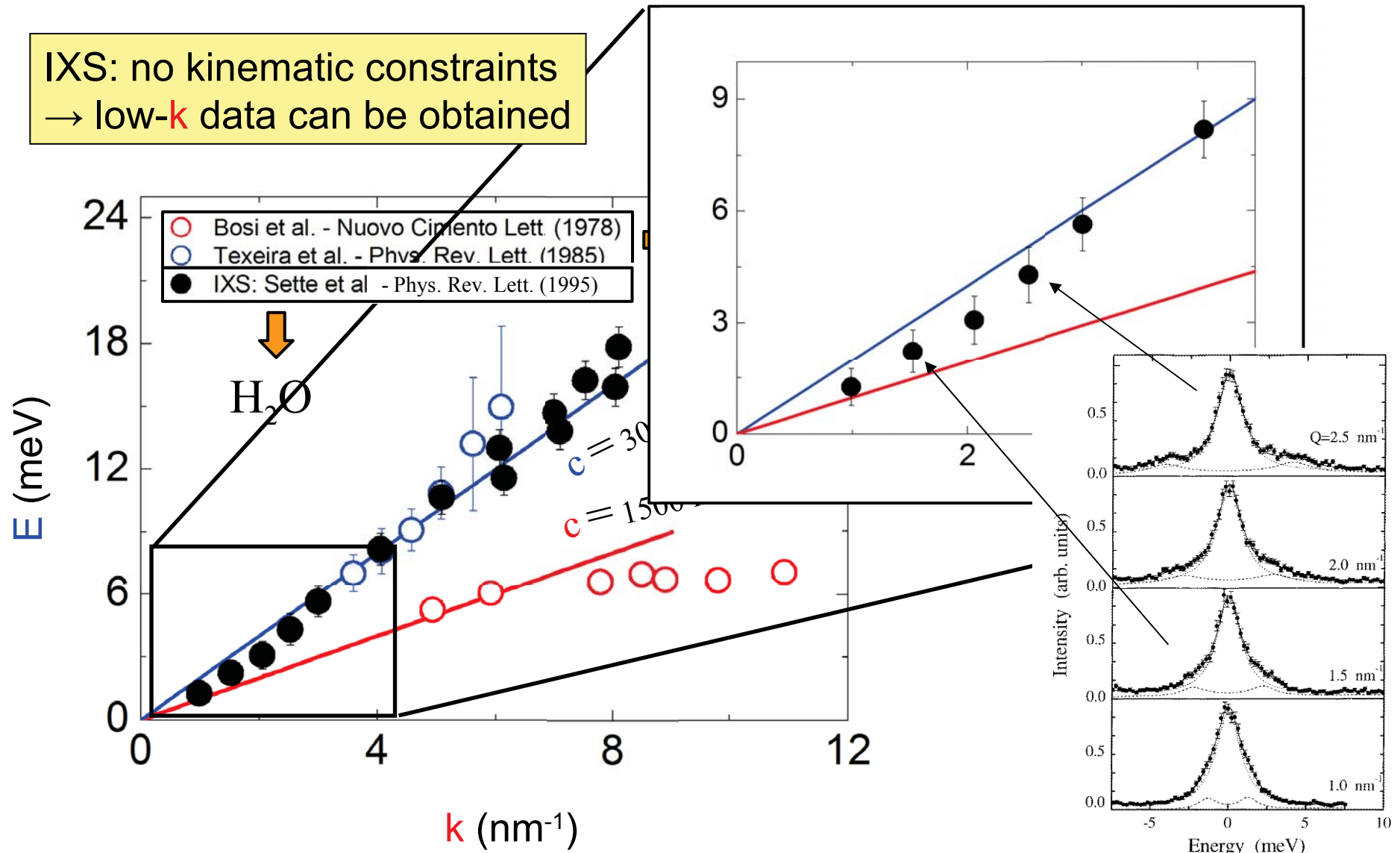
$$\Omega_{\text{HF}}/\Omega_{\text{LF}} \sim (m_{\text{O}}/m_{\text{D}})^{1/2} \sim 2$$

High frequency mode:

$$\text{but... } \Omega_{\text{IXS}} = \Omega_{\text{INS}}$$

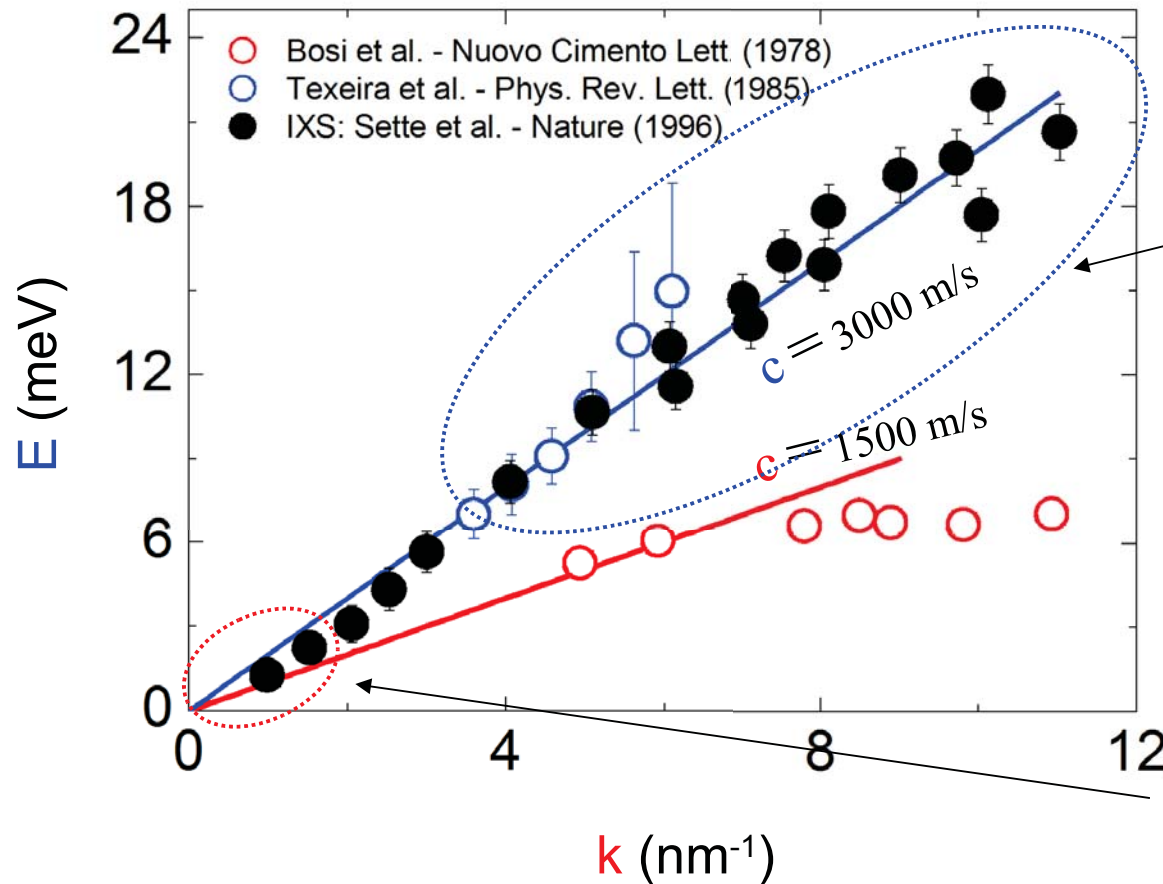
Experimental highlights (1)

Collective dynamics in water

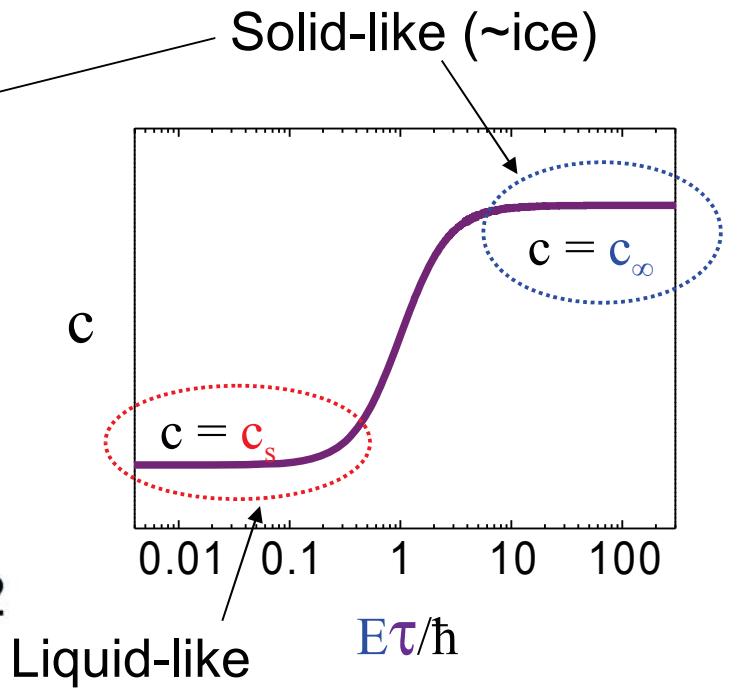


Experimental highlights (1)

Collective dynamics in water (viscoelasticity)



Viscoelasticity:
The sound velocity ($c = E/\hbar k$) is not constant but depends on $E\tau/\hbar$

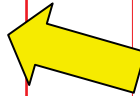


Experimental highlights (1)

Collective dynamics in water (viscoelasticity)

Information we can get from each spectrum:

- 1) High and low frequency sound velocity (c_{inf} and c_0)^{a,b}
- 2) High/low frequency damping (lifetime) of inelastic excitations^{a,b} → Viscosity/anharmonicity/local disorder
- 3) Relaxation time^{a,b} → hints on the physical processes responsible for the viscoelastic transition
- 4) Thermal properties (heat diffusion, specific heat)^b
- 5) Characteristic energy of inelastic excitations (from the peak energy of the function $J_{\perp}(k,E)=E^2S(k,E)$)^{a,b}

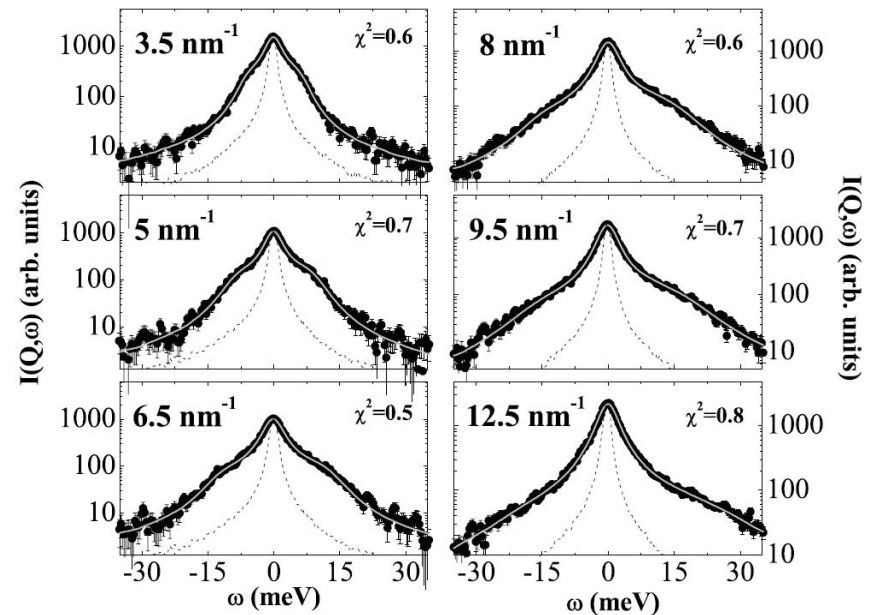
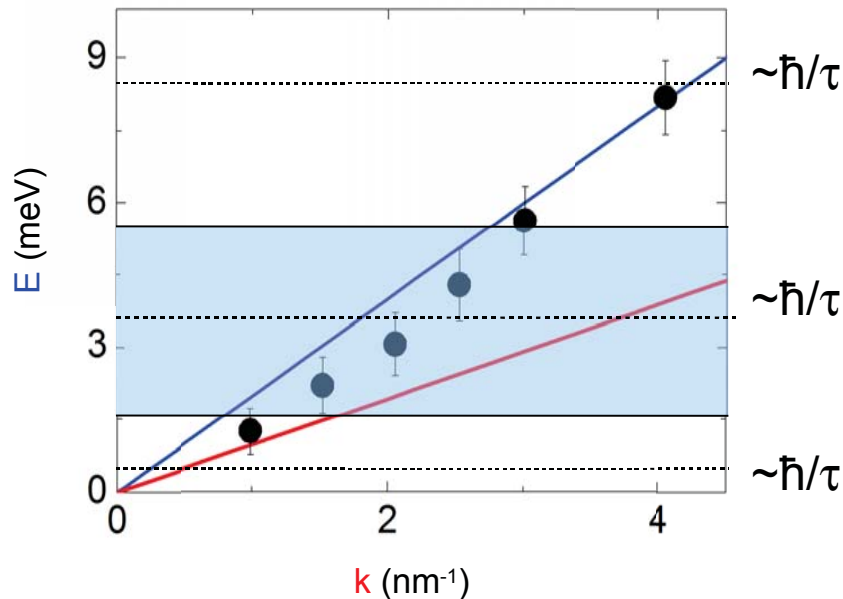


Data analysis

Viscoelastic model function

$$S(k,E) = A_{\text{inel}} \frac{E_0^2(k) * m'(k,E)}{[E^2 - E_0^2(k) - E * m''(k,E)]^2 + [E * m'(k,E)]^2}$$

$m'(k,E)$ and $m''(k,E)$ are the real and imaginary part of the FT of memory function, respectively related with viscous and elastic response of the fluid.



Experimental highlights (1)

Collective dynamics in water (viscoelasticity)

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- 3) Relaxation time^{a,b} → hints on the physical processes responsible for the viscoelastic transition
- 4) Thermal properties (heat diffusion, specific heat)^b
- 5) Characteristic energy of inelastic excitations (from the peak energy of the function $J_L(k,E)=E^2S(k,E)$)^{a,b}

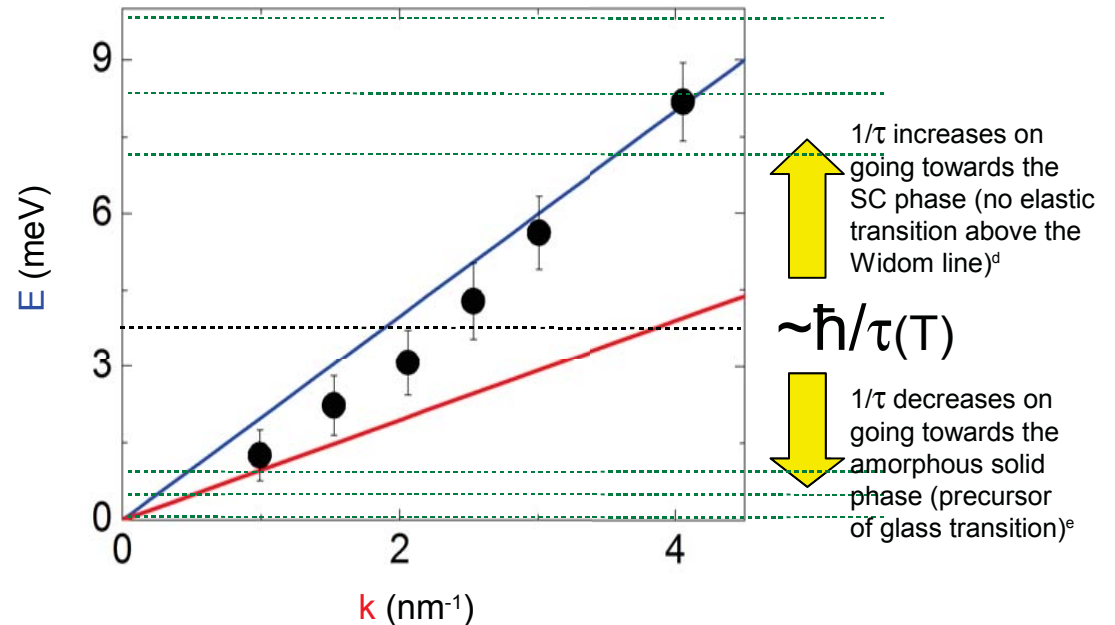
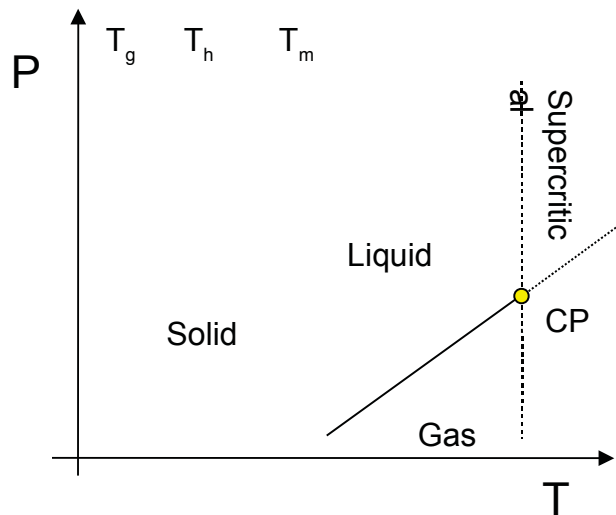
c_0 vs (P,T, ρ) = adiabatic sound velocity (classical hydrodynamics)^{a,b,c}

c_{inf} ~ sound velocity in glassy/crystalline water (elastic medium)^{a,b}

Low frequency damping vs (P,T, ρ) proportional to viscosity (class. hydrod.)^{a,b,c}

High frequency damping as in glassy water (elastic medium)^{a,b}

Arrhenius temperature dependence ($\tau = \text{const} \cdot \exp\{E_a/k_B T\}$) in the liquid phase^{a,b,c}; ~ constant in the supercritical phase^b; “diverging” (vs T) in the supercooled liquid phase (only by IUVS data)^c.

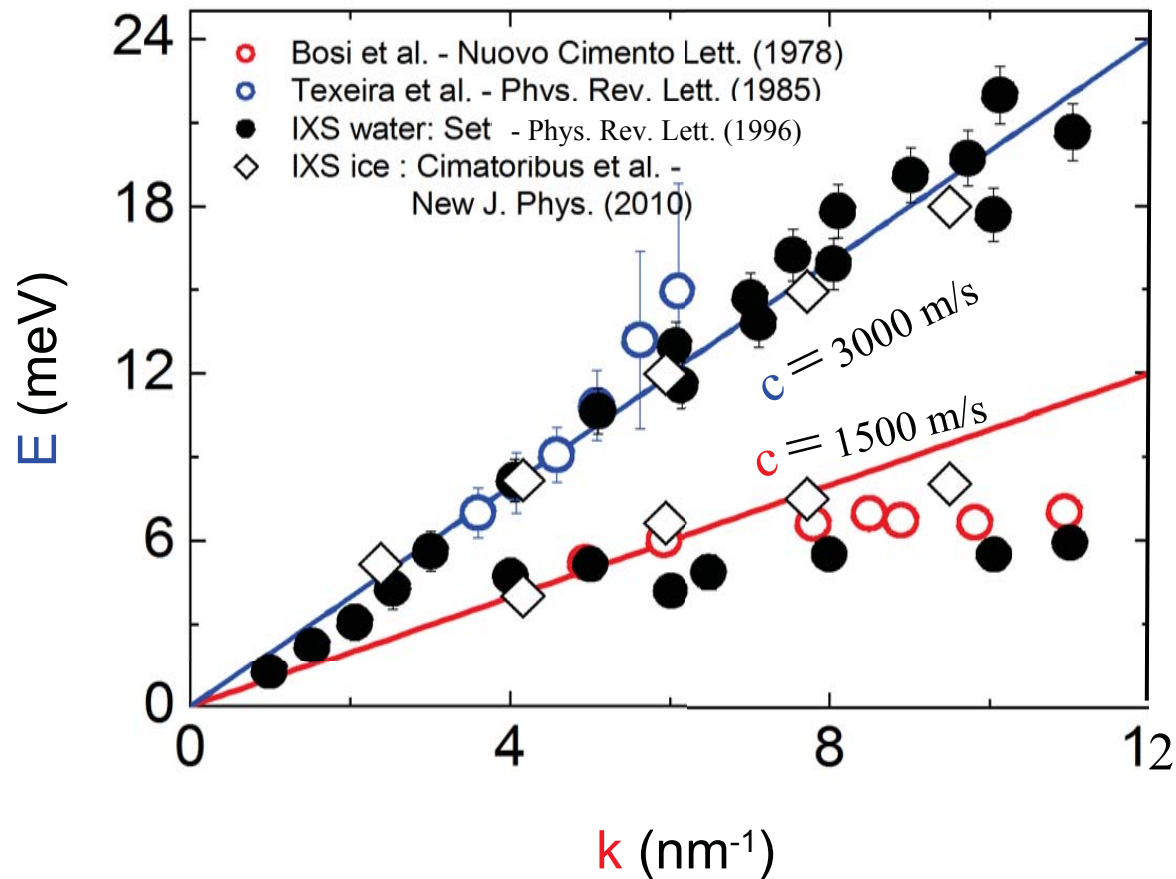


a) G. Monaco et al., PRE 60 (1999) – constant density vs T (liquid)
 b) F. Bencivenga et al., PRE 75 (2007) – vs density and T (liquid and SC)
 c) C. Masciovecchio et al., PRL 92 (2004) – vs T (IUVS data, supercooled)

d) G.G. Simeoni et al., Nat. Phys. 6 (2010).
 e) IXS experiments on various glass formers → F. Sette et al., Nature 280 (1998); T. Scopigno et al., Science 302 (2003)

Experimental highlights (1)

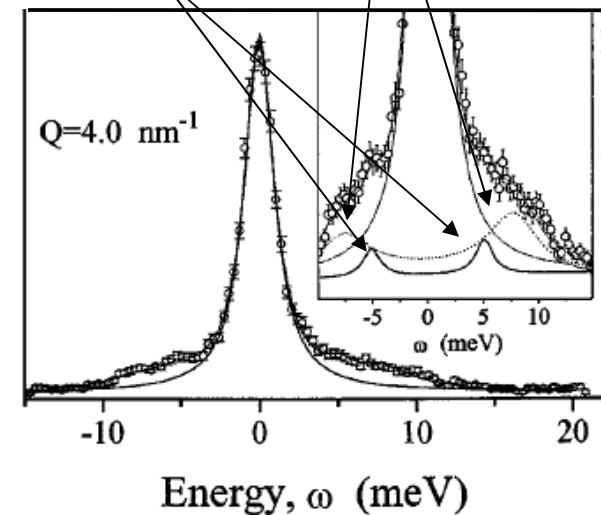
Collective dynamics in water (transverse mode)



Data analysis

Lorentian peak (quasi elastic scattering) plus two damped harmonic oscillator (DHO) functions (two pairs of inelastic peaks) – **1.5 meV resolution**

$$S(k,E) = \frac{A_{qe}}{E^2 + \Gamma^2} + \frac{A_{HF}}{(E^2 - E_{HF}^2)^2 + (E \cdot \Gamma_{HF})^2} + \frac{A_{LF}}{(E^2 - E_{LF}^2)^2 + (E \cdot \Gamma_{LF})^2}$$

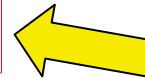


F. Sette et al., PRL 77 (1996)

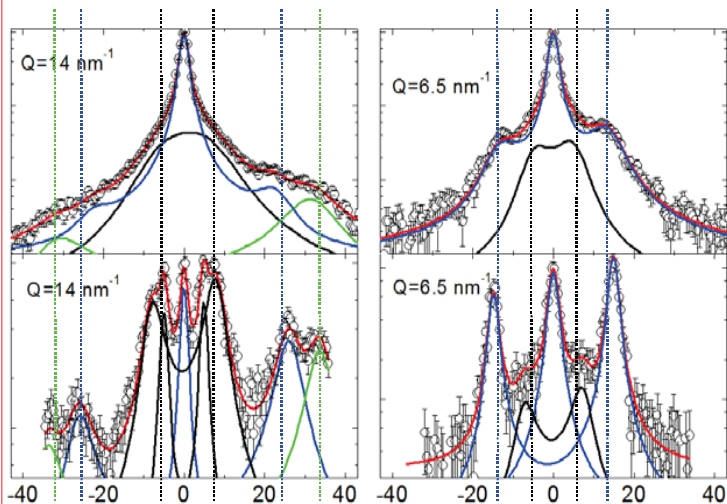
Experimental highlights (1)

Collective dynamics in water (transverse mode)

Improving experiments and data analysis...

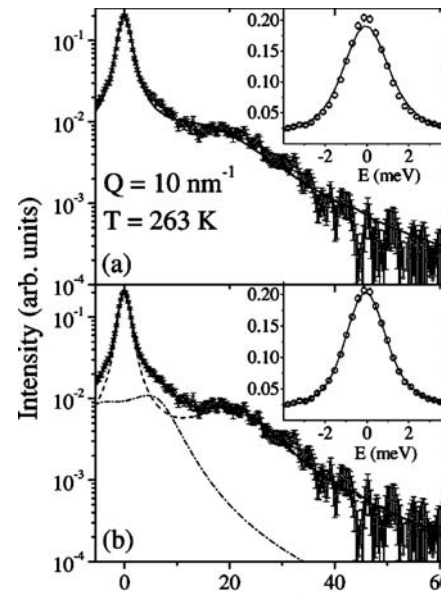


Data analysis



Experiment → High-pressure (3 kbar) and low-T (down to the supercooled liquid phase) for one-to-one comparison with (polycrystalline) ice
 Analysis → Viscoelastic model function (quasi elastic and HF peaks) + DHO function (LF peak and the “new” HF peak)
 Results → Observed in water a “new” inelastic peak (also present in ice) at frequency higher than those of the “old” HF mode and due to a L-T mixing (in agreement with simulations)

A. Cimattoribus et al., New J. Phys. 12 (2010)

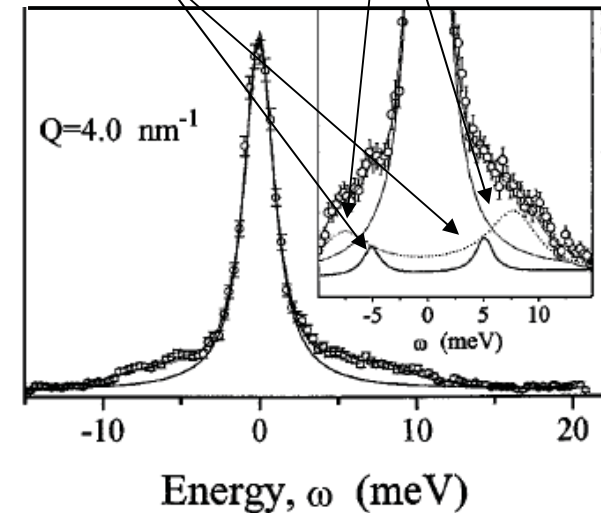


Experiment → High-pressure (3 kbar) vs T
 Analysis → Viscoelastic model (HF and quasi elastic peaks) + DHO function (LF peak)
 Main result → LF peak shows up in the elastic side of the viscoelastic transition

E. Pontecorvo et al., PRE 71 (2005)

Lorentzian peak (quasi elastic scattering) plus two damped harmonic oscillator (DHO) functions (two pairs of inelastic peaks) – **1.5 meV resolution**

$$S(k,E) = \frac{A_{qe}}{E^2 + \Gamma^2} + \frac{A_{HF}}{(E^2 - E_{HF}^2)^2 + (E \cdot \Gamma_{HF})^2} + \frac{A_{LF}}{(E^2 - E_{LF}^2)^2 + (E \cdot \Gamma_{LF})^2}$$



F. Sette et al., PRL 77 (1996)

Experimental highlights (1)

Collective dynamics in water (transverse mode)

Exploiting the complementarity of IXS and INS...

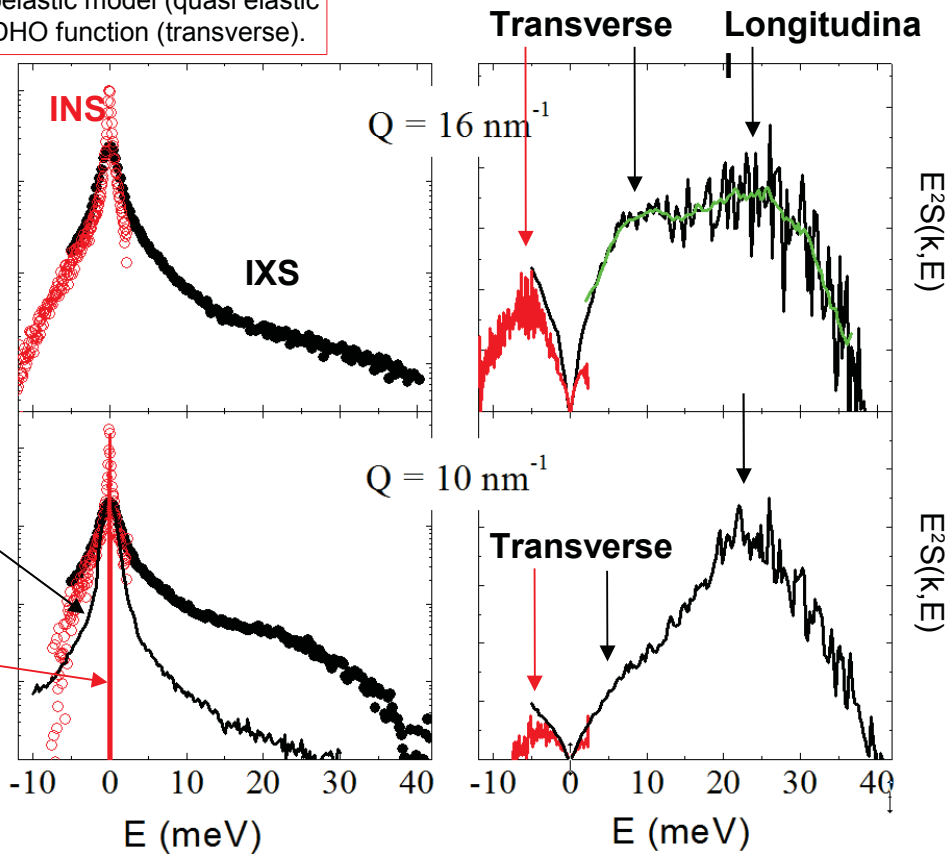
Data analysis

Join (iterative) best fitting of IXS and INS spectra at having the same k ; viscoelastic model (quasi elastic and longitudinal) plus a DHO function (transverse).

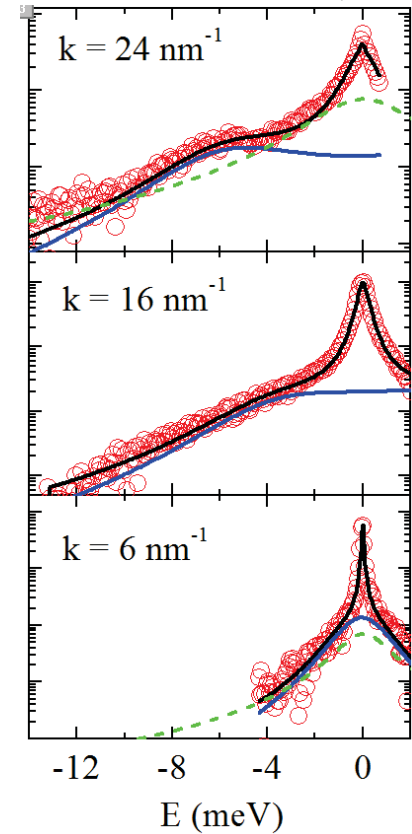
Water @ room-P
and $\sim 10^\circ\text{C}$

I_XS (HERIX @ Argonne
National Lab):
1.6 meV resolution

I_NS (CNCS @ Oak
Ridge National Lab):
0.1 meV resolution using
cold neutrons ($E_n = 3.7$
meV)



INS spectra



— Fit results for the transverse peak
- - - Expected lineshape (Lorentian
with FWHM equal to the shear
viscosity times k^2), according to
classical hydrodynamics

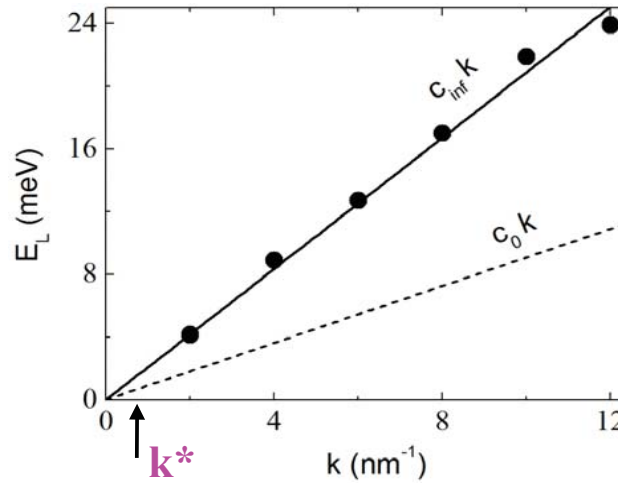
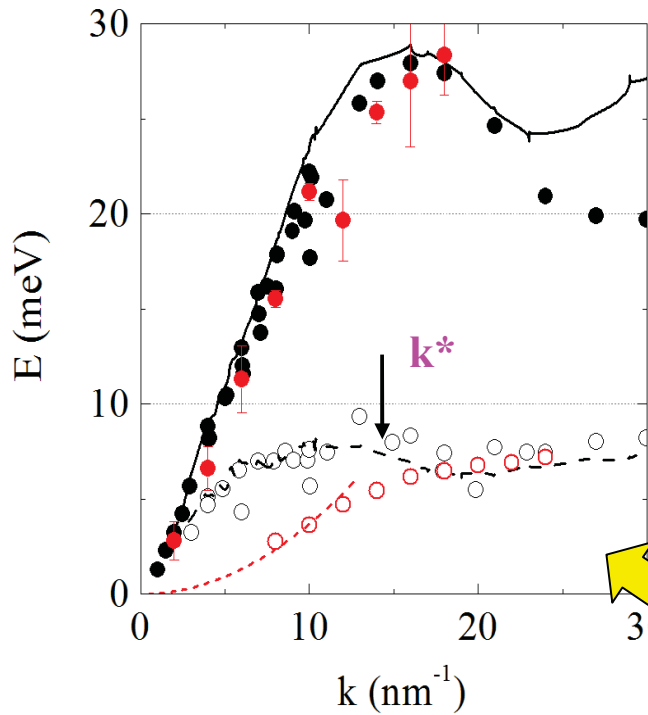
Note

A peak in the $E^2S(k,E)$ lineshape does not necessarily implies a peak (i.e. a propagating inelastic mode) in the corresponding $S(k,E)$!

Experimental highlights (1)

Collective dynamics in water (transverse mode)

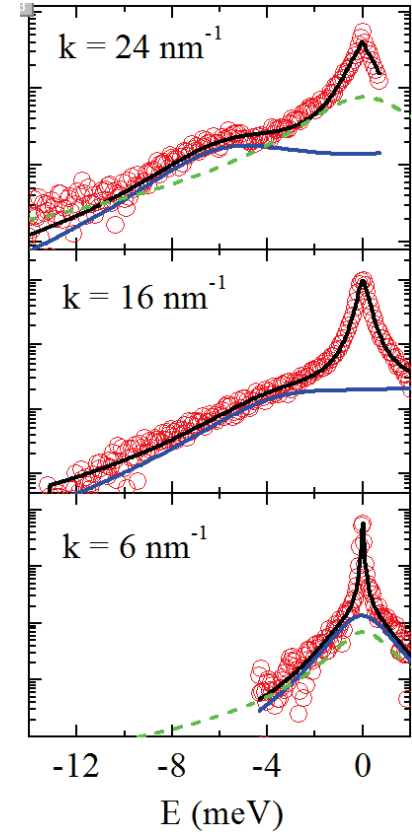
Exploiting the complementarity of IXS and INS...



At $k > 3 \text{ nm}^{-1}$ we are in the “elastic side” of the viscoelastic transition, occurring at some k^* (below 3 nm^{-1} in this case)

We observe a diffusive transverse mode (it is **not propagating**). Propagation occurs for $k > k^{**}$ ($\sim 15 \text{ nm}^{-1}$ in this case)

INS spectra

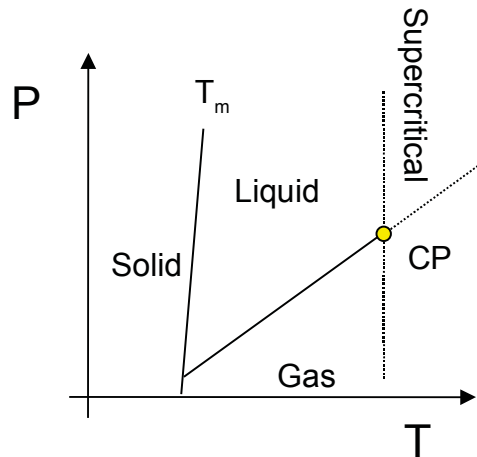
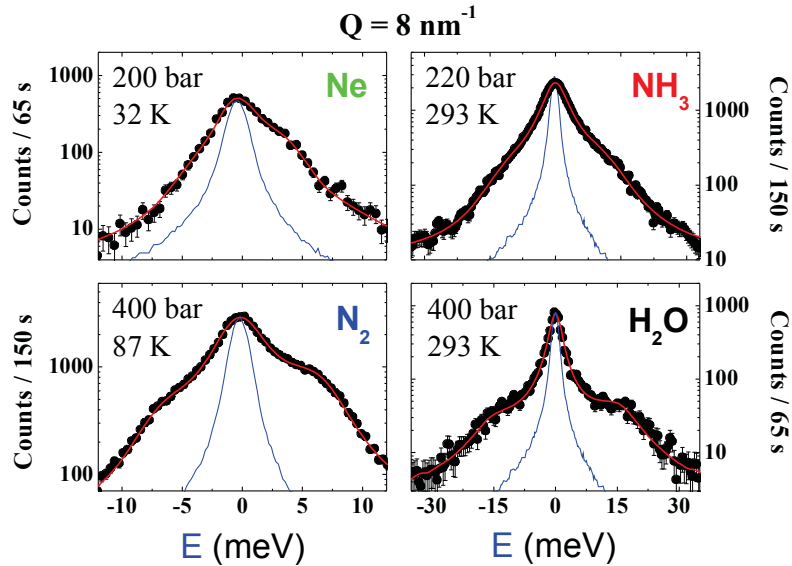


— Fit results for the transverse peak
 - - - - - Expected lineshape (Lorentian with FWHM equal to the shear viscosity times k^2), according to classical hydrodynamics

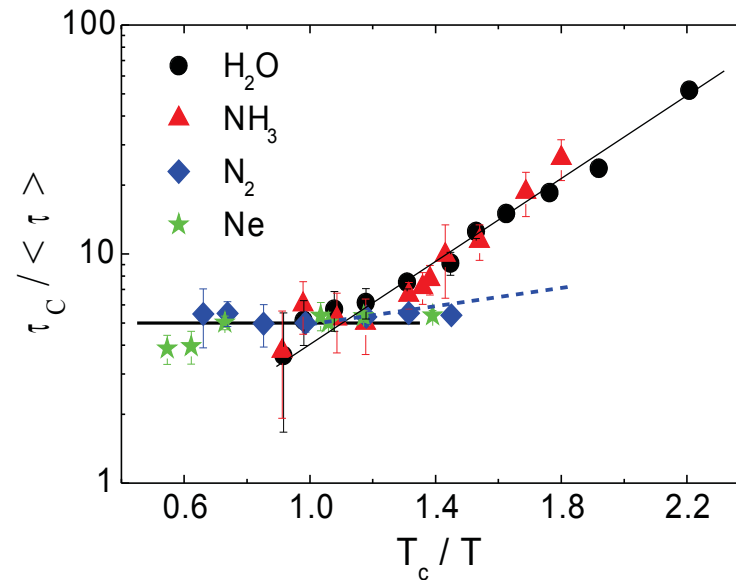
Observability of transverse modes (L-T coupling) and their propagation occurs at different k 's, k^* and k^{**} . The former is related with the viscoelastic crossover, the latter probably not. A strong P-T dependence of k^{**} is expected.

Experimental highlights (2)

Liquids vs supercritical fluids

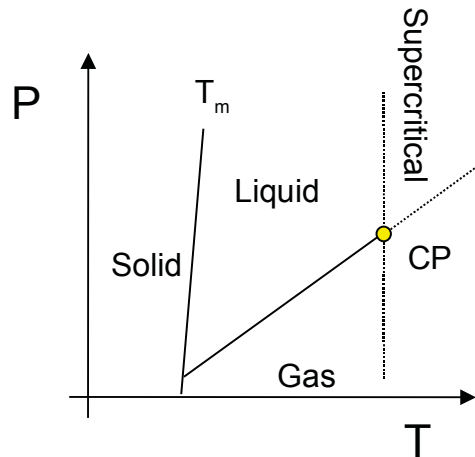
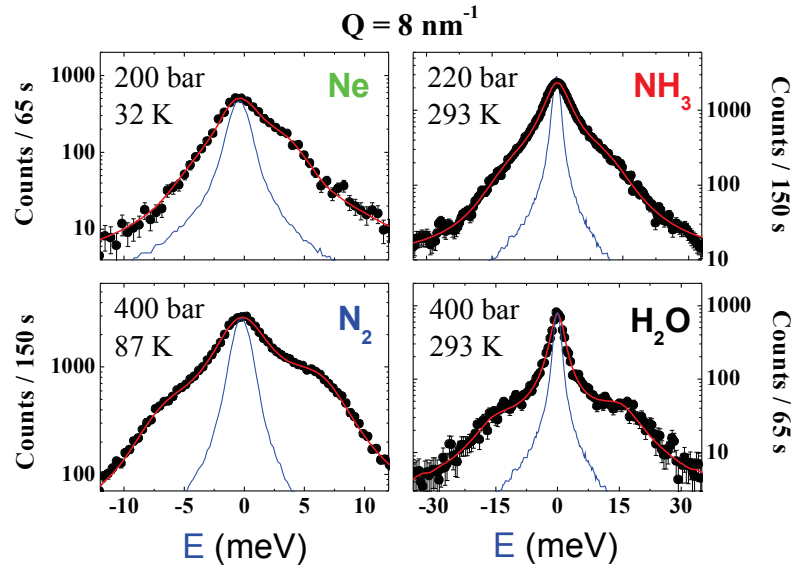


- 1) Positive sound dispersion in all samples; **“negative” sound dispersion** (c_0 from c_s to c_T – adiabatic to isothermal) in some cases.^a
- 2) For $T > T_c$ relaxation time proportional to mean free time between **collisions**; for $T < T_c$ relaxation time proportional to $\exp\{E_a/kBT\}$, where E_a is the energy of intermolecular bonds.^b



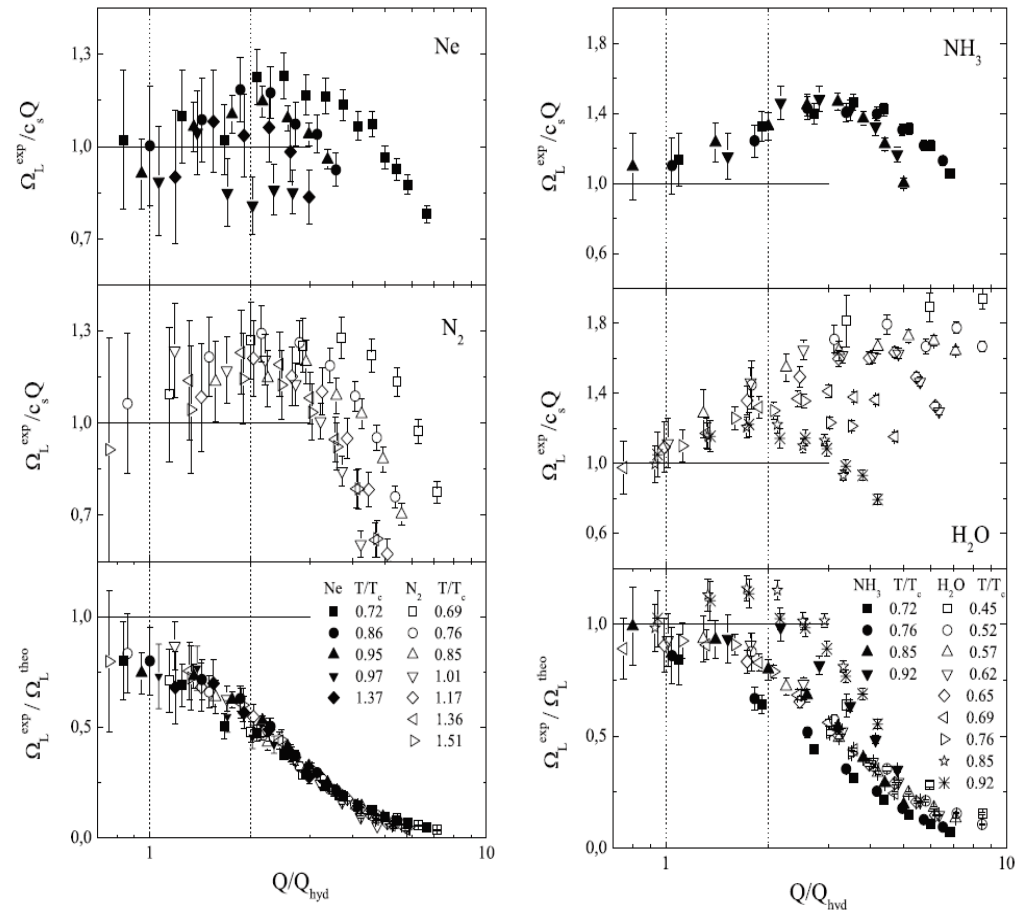
Experimental highlights (2)

Liquids vs supercritical fluids



Test of Generalized Collective Modes (GCM) theory:^a

$$\rightarrow E_{\text{inel}} = c_{\text{hyd}} * k + \beta * k^3; \text{ for } k < k_{\text{hyd}}$$



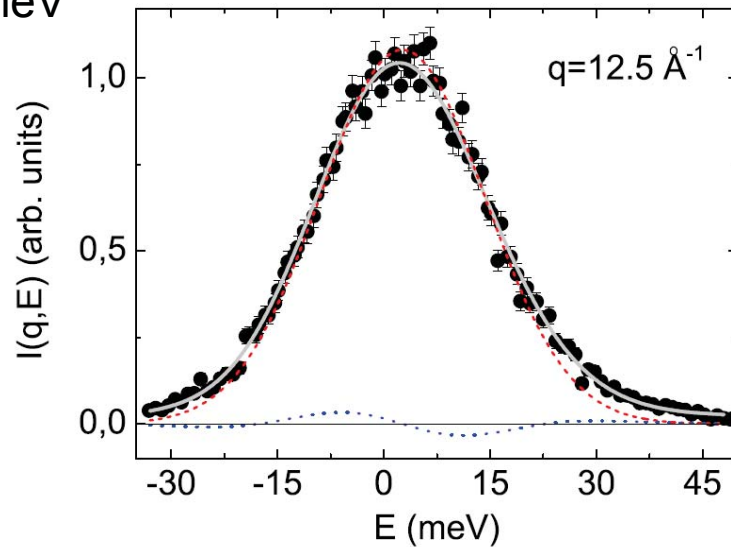
Experimental highlights (3)

Experimental test of Sachs-Teller theory^a

Liquid I₂ (T=388 K);

k = 25 – 150 nm⁻¹ (>> k*~7.5 nm⁻¹);

ΔE = 3 meV

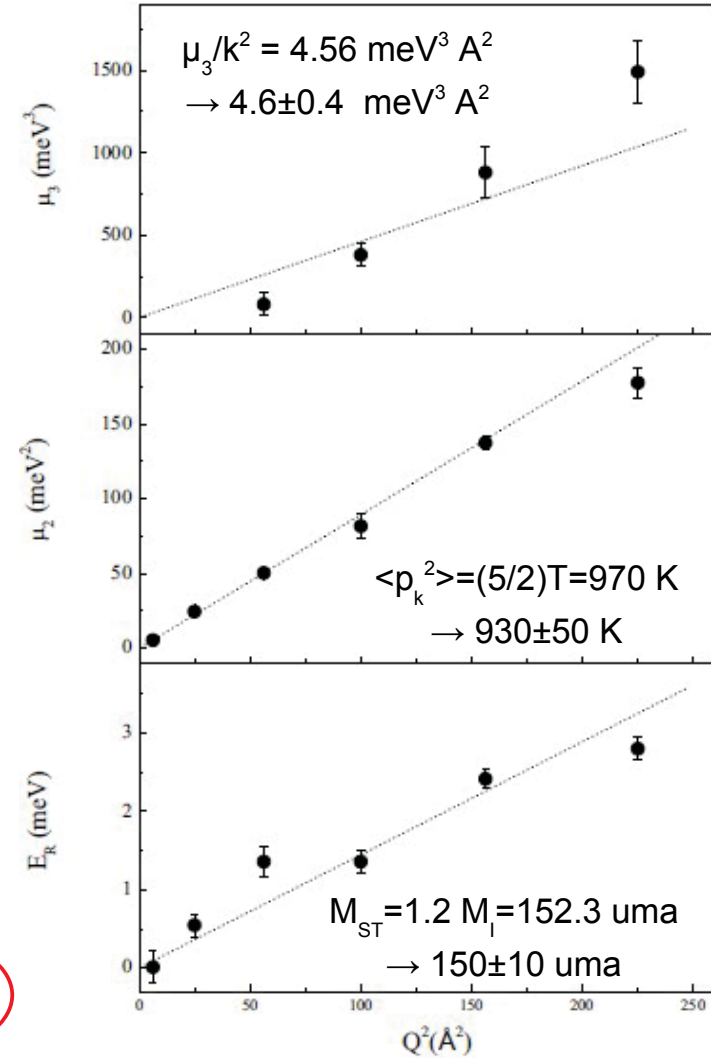


$$S_{IA}(k,E) = (\text{const}/\mu_2) \cdot \exp\left\{-\frac{(E-E_R)^2}{2\mu_2}\right\} \cdot \left[1 - \left(\frac{\mu_3}{2\mu_2}\right) \cdot (E-E_R) \cdot \left(1 - \frac{(E-E_R)^2}{3\mu_2}\right)\right]$$

$$E_R = \frac{\hbar^2 k^2}{2M_{ST}}; \mu_2 = \left(\frac{\hbar^2 k^2}{M_{ST}}\right) \cdot \langle p_k^2 \rangle;$$

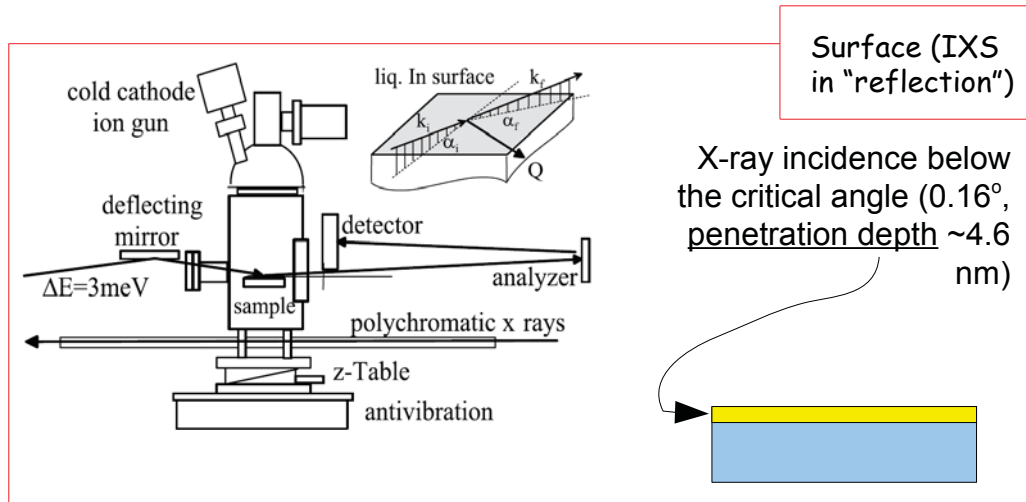
$$\mu_3 = \left(\frac{\hbar^4 k^2}{6M_{ST}^2}\right) \cdot \langle \nabla U(r) \rangle$$

$$M \rightarrow M_{ST}$$

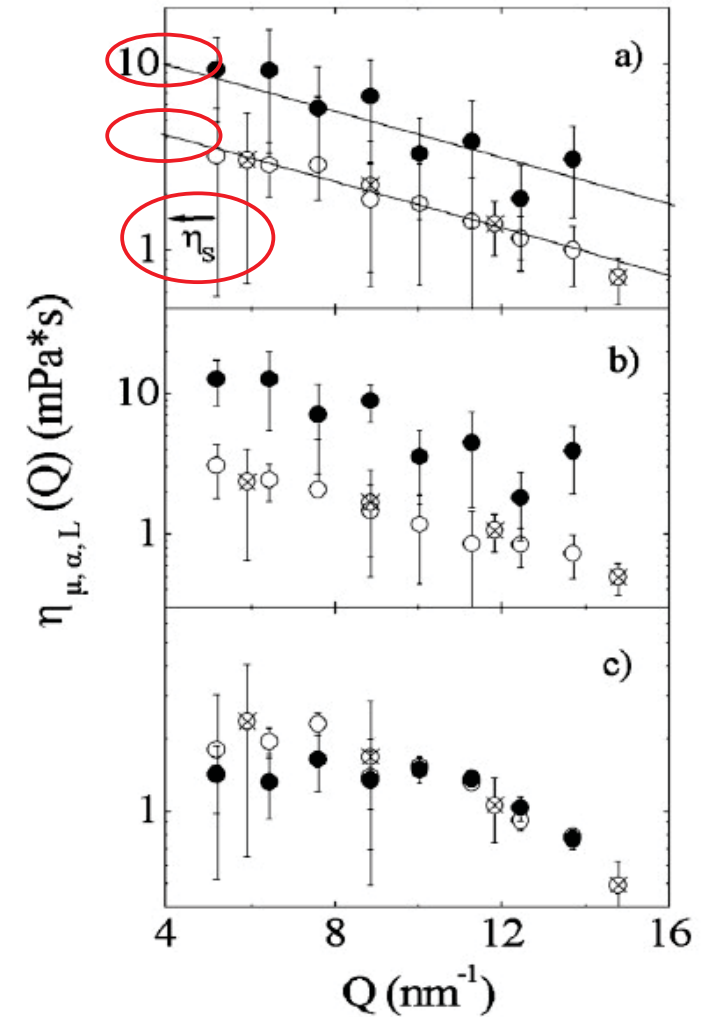
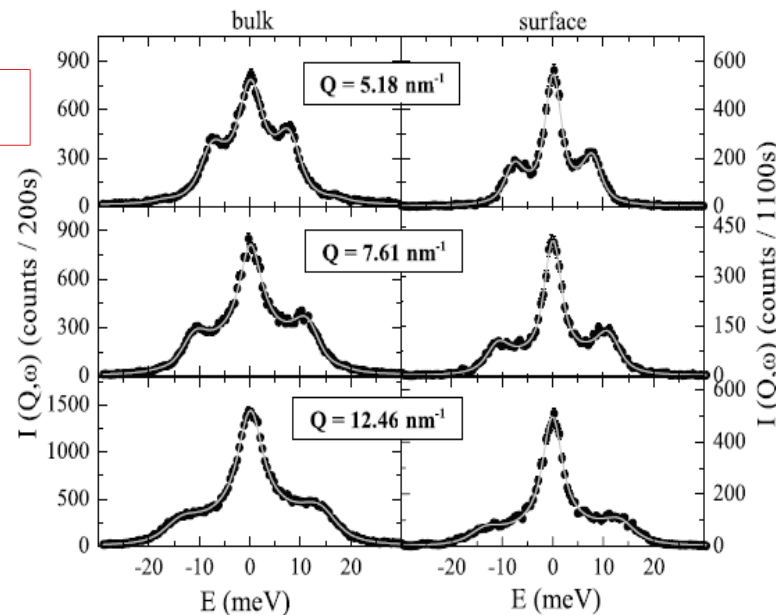
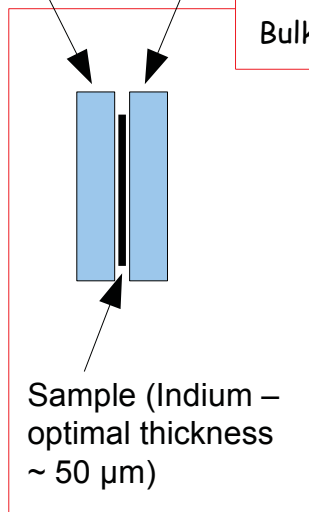


Experimental highlights (4)

(sub)surface sensitivity of IXS



Diamond windows



“Sub-surface viscosity increases by a factor 3 with respect to the bulk

Experimental highlights (5)

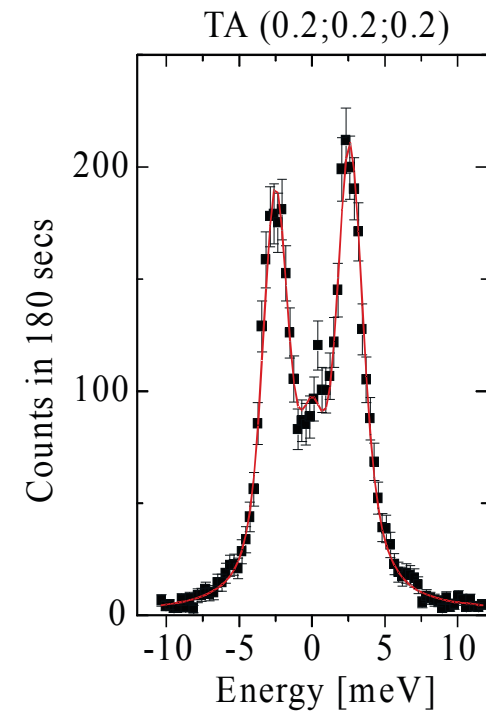
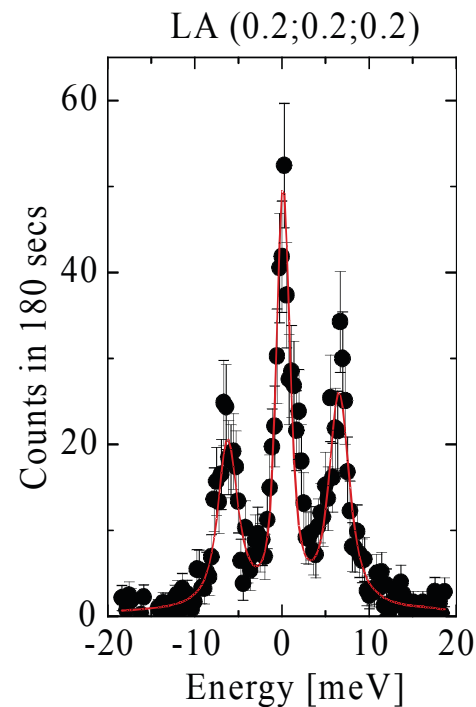
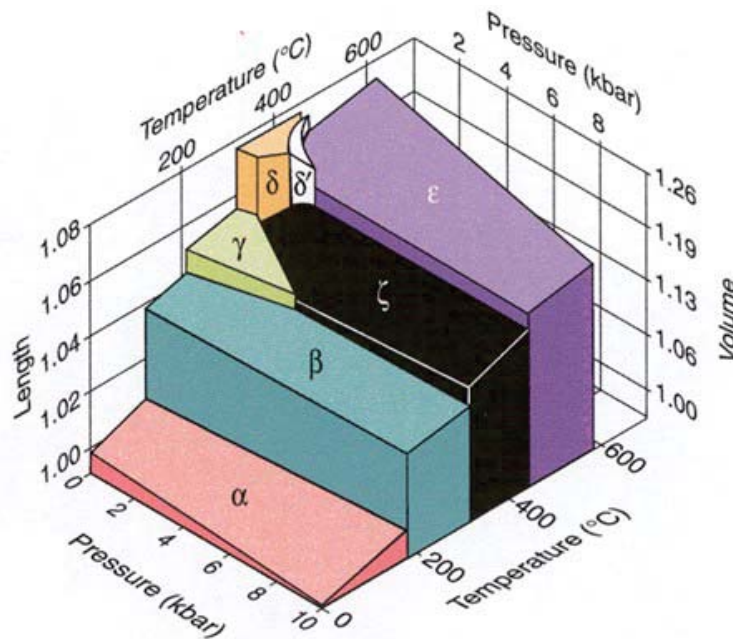
Phonon dispersions in Plutonium: conventional IXS experiment" on a "non-conventional" sample (IXS used for the tight focusing)

Plutonium is one of the most fascinating and exotic element:

- Multitude of unusual properties
- Central role of 5f electrons

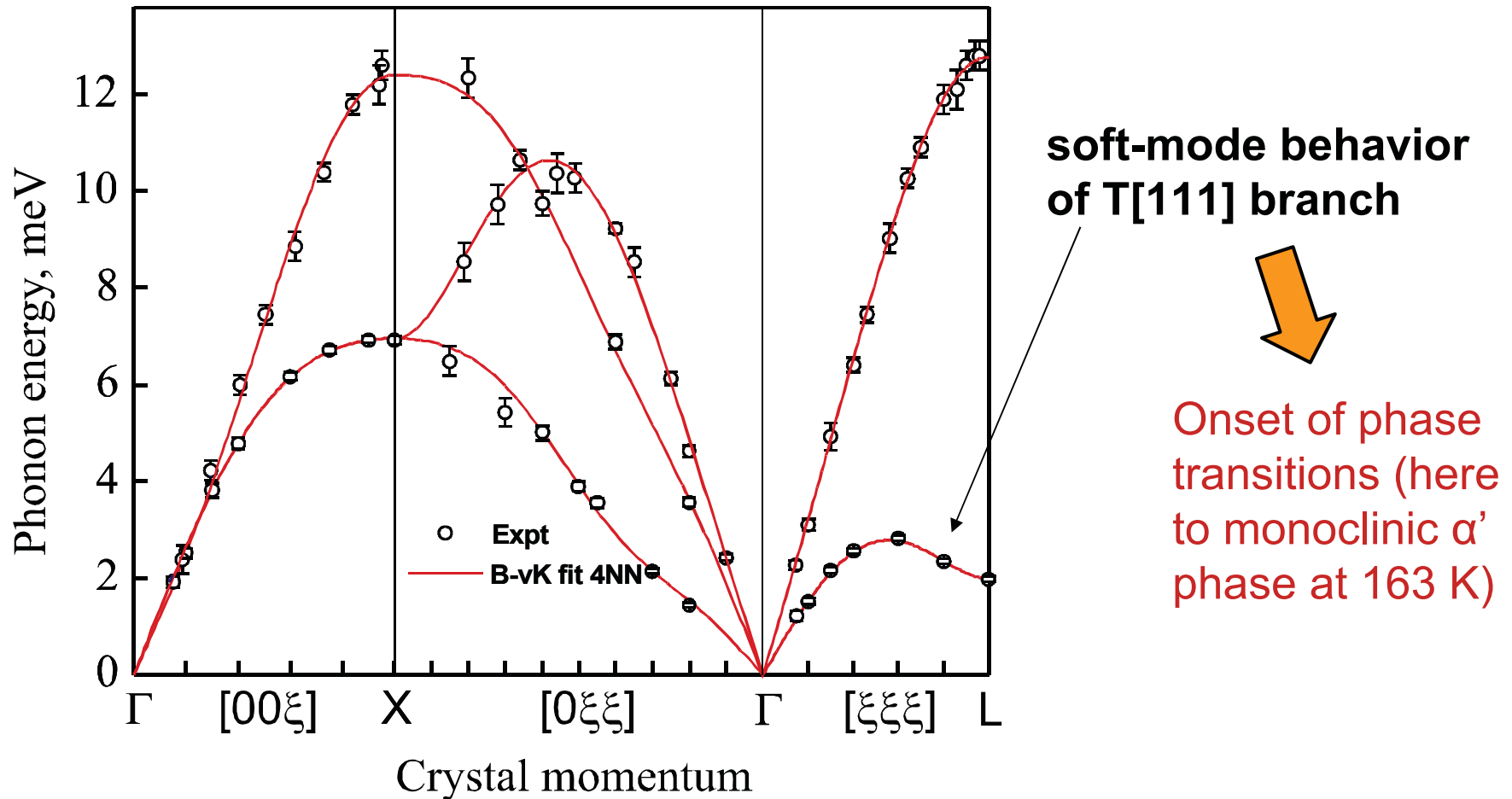
ID28 at ESRF

- Beam size: 20 x 60 μm^2 (HxV)
- Grain size: $\sim 80 \mu\text{m}^2$
- On-line diffraction analysis



Experimental highlights (5)

Phonon dispersions in plutonium



- **Born-von Karman force constant model fit** (fourth nearest neighbors)

Experimental highlights (5)

Phonon dispersions in plutonium

Close to Γ -point: $E = \mathbf{V}q/\hbar$



$$V_L[100] = (C_{11}/\rho)^{1/2}$$

$$V_T[100] = (C_{44}/\rho)^{1/2}$$

$$V_L[110] = ([C_{11} + C_{12} + 2C_{44}]/\rho)^{1/2}$$

$$V_{T1}[110] = ([C_{11} - C_{12}]/2\rho)^{1/2}$$

$$V_{T2}[110] = (C_{44}/\rho)^{1/2}$$

$$V_L[111] = [C_{11} + 2C_{12} + 4C_{44}]/3\rho)^{1/2}$$

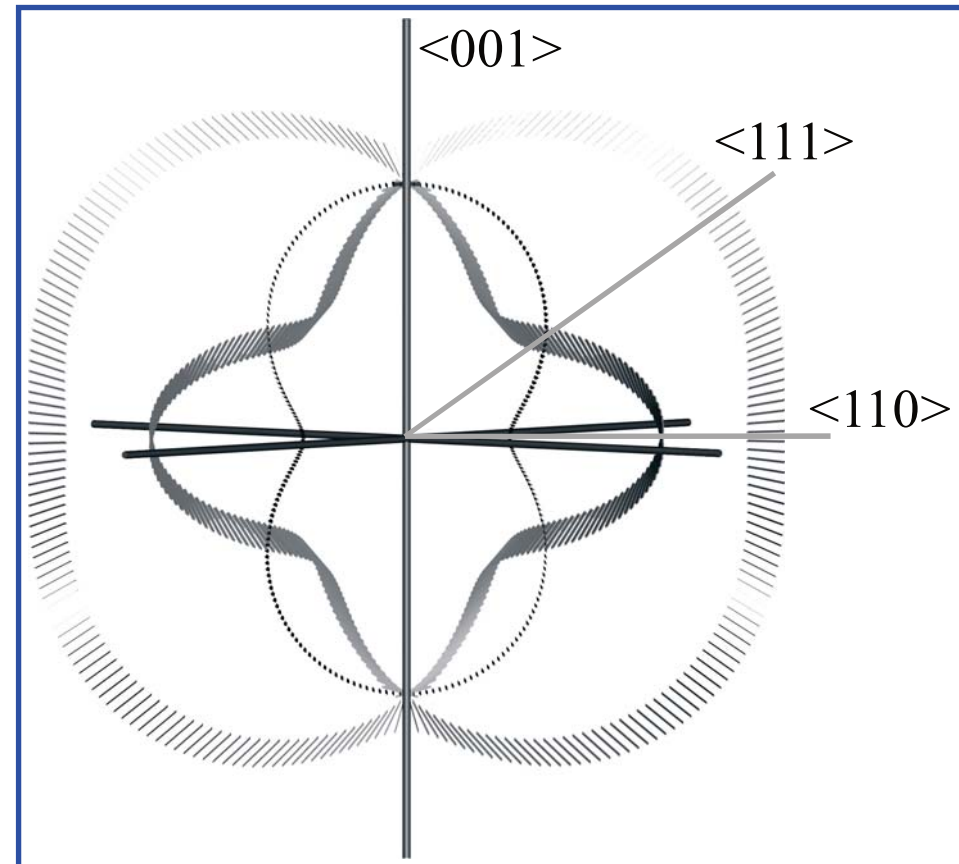
$$V_T[111] = ([C_{11} - C_{12} + C_{44}]/3\rho)^{1/2}$$



$$C_{11} = 35.3 \pm 1.4 \text{ GPa}$$

$$C_{12} = 25.5 \pm 1.5 \text{ GPa}$$

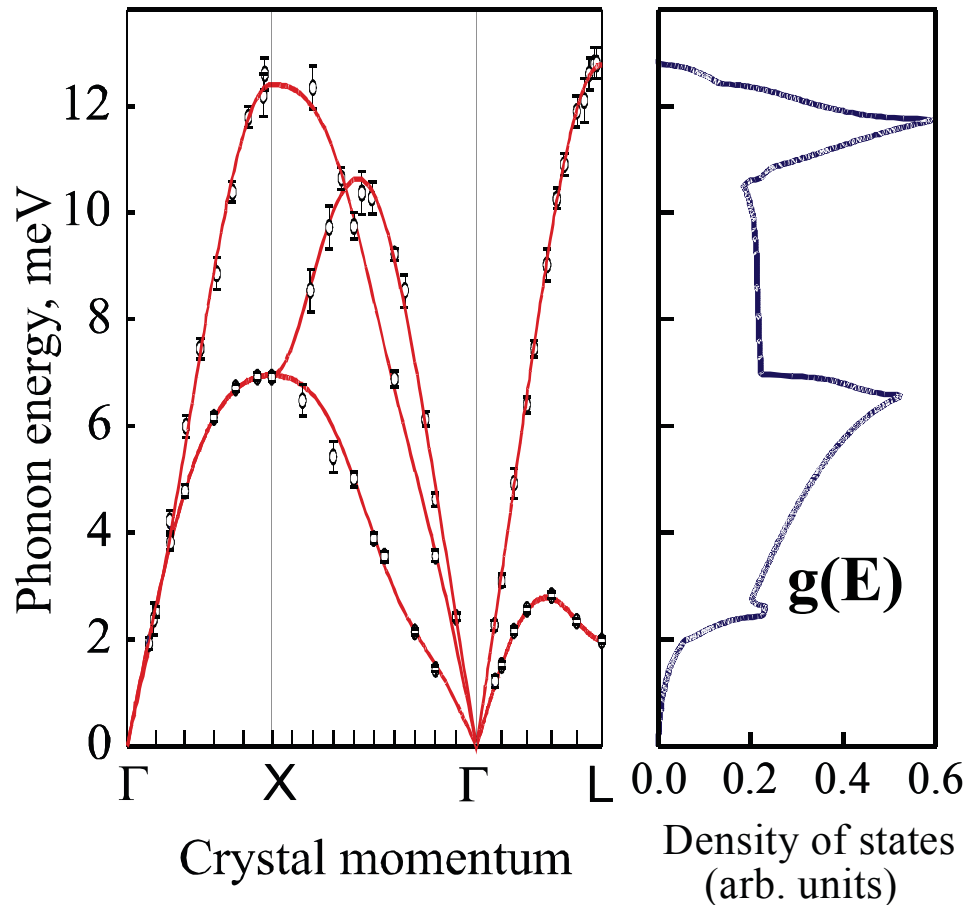
$$C_{44} = 30.5 \pm 1.1 \text{ GPa}$$



**highest elastic anisotropy
of all known fcc metals**

Experimental highlights (5)

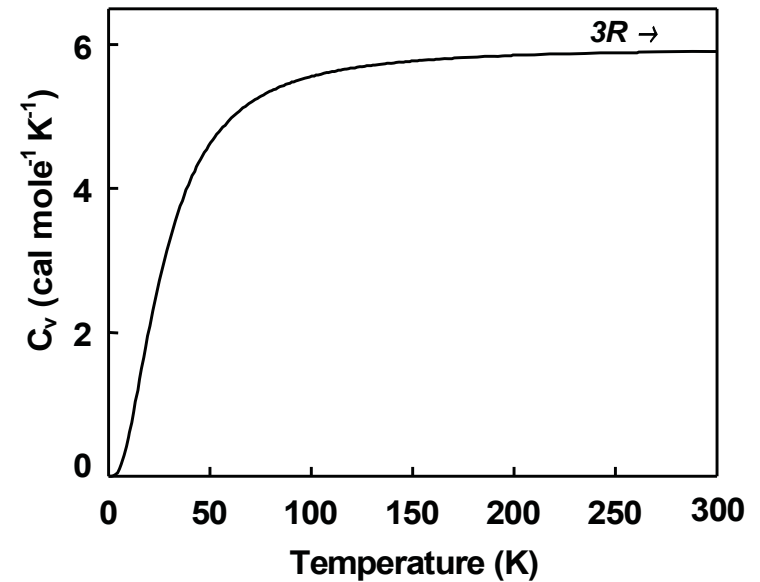
Phonon dispersions in plutonium



- Born-von Karman fit

Specific heat:

$$C_v = 3Nk_B \int_0^{E_{\max}} \left(\frac{E}{k_B T} \right)^2 \frac{\exp(E/k_B T) g(E) dE}{(\exp(E/k_B T) - 1)^2}$$

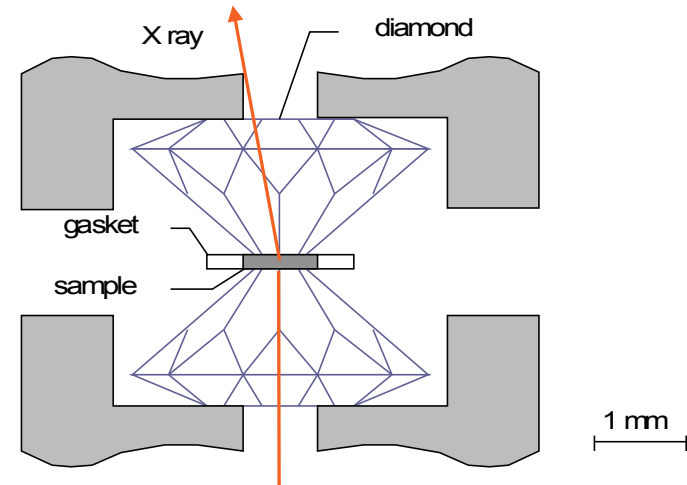
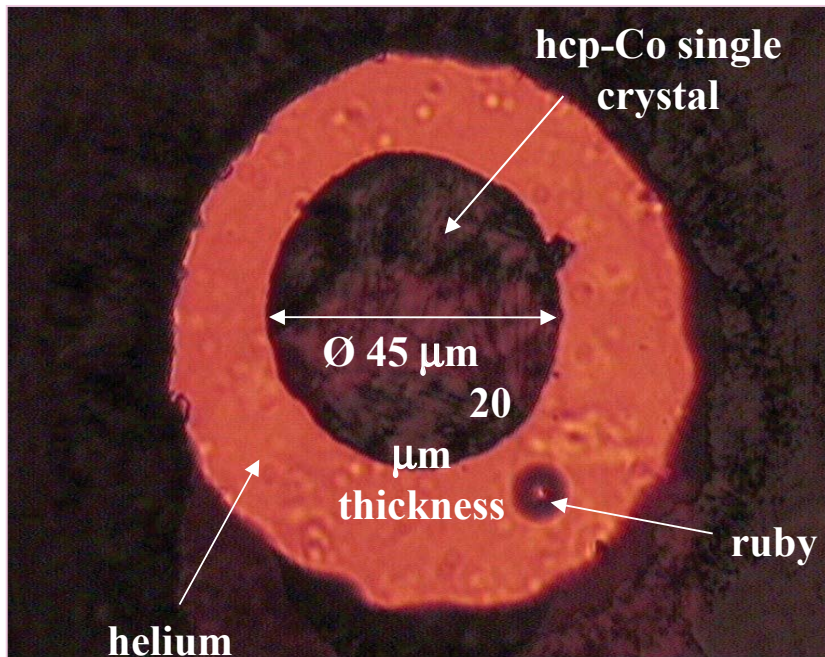


Experimental highlights (6)

Elasticity at high pressure (small focus)

Elasticity of hcp-metals under very high pressure (up to 1 Mbar):

- Geophysical interest (Earth core)
- DAC sample environment + IXS



hcp-structure:

5 independent elastic moduli

$$V_L[001] = (C_{33}/\rho)^{1/2}$$

$$V_L[100] = (C_{11}/\rho)^{1/2}$$

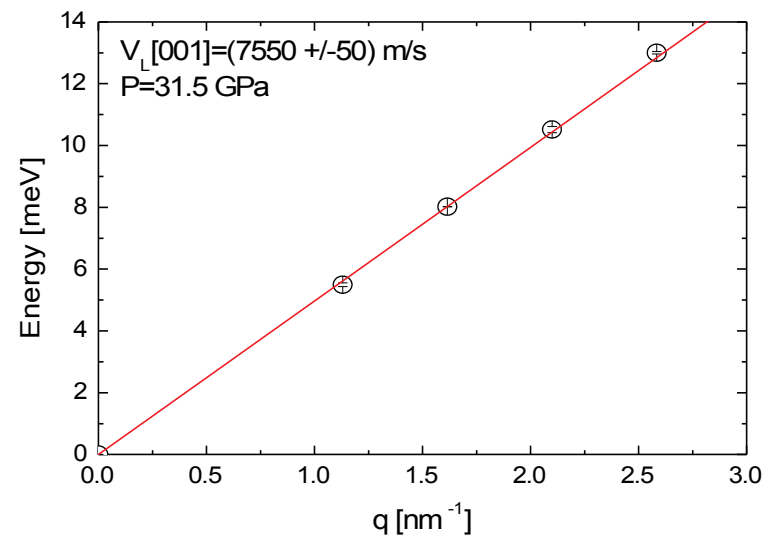
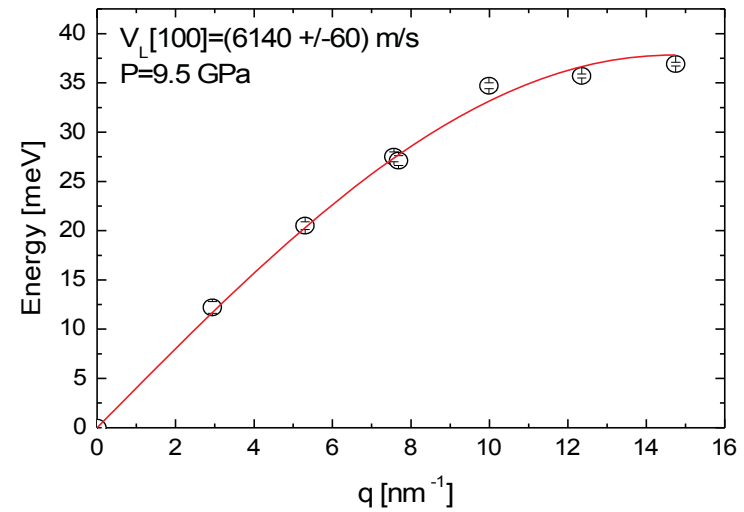
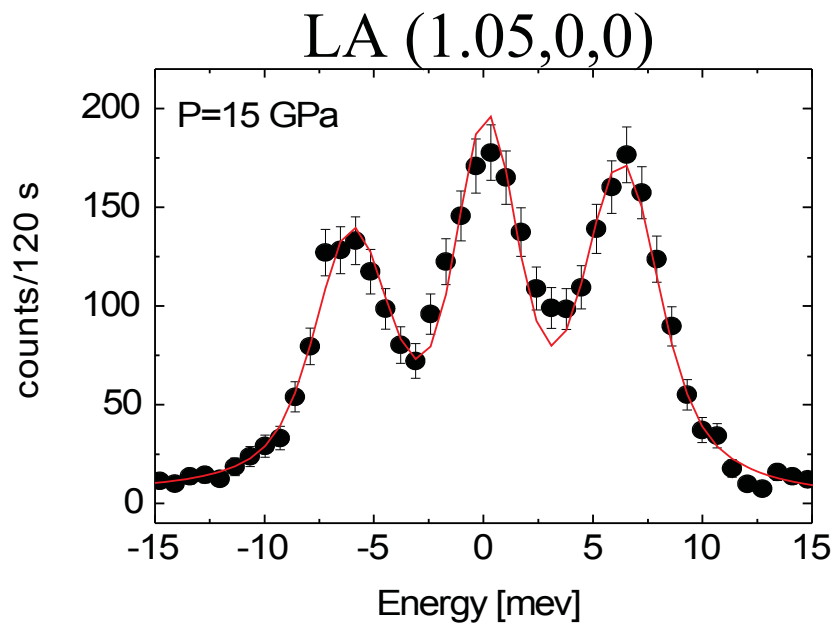
$$V_{T1}[110] = ([C_{11} - C_{12}] / 2\rho)^{1/2}$$

$$V_{T2}[110] = (C_{44}/\rho)^{1/2}$$

$$V_{QL}[101] = f(C_{ij}, \rho) \rightarrow C_{13}$$

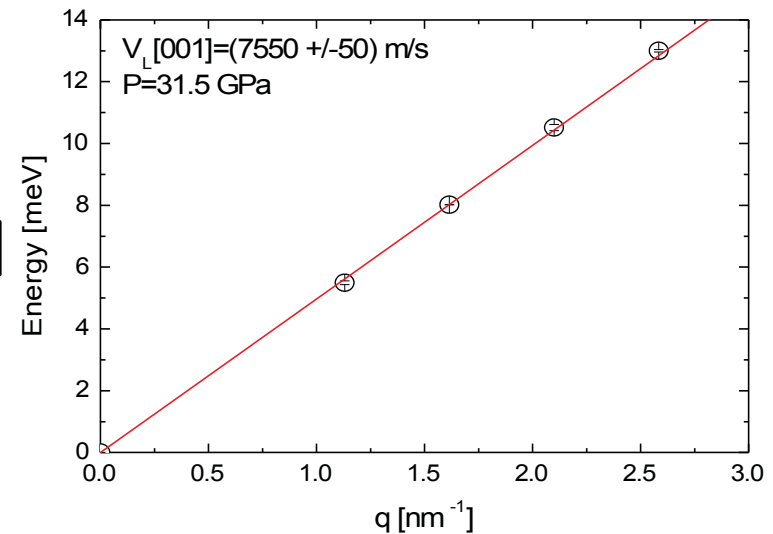
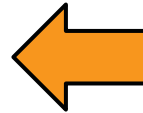
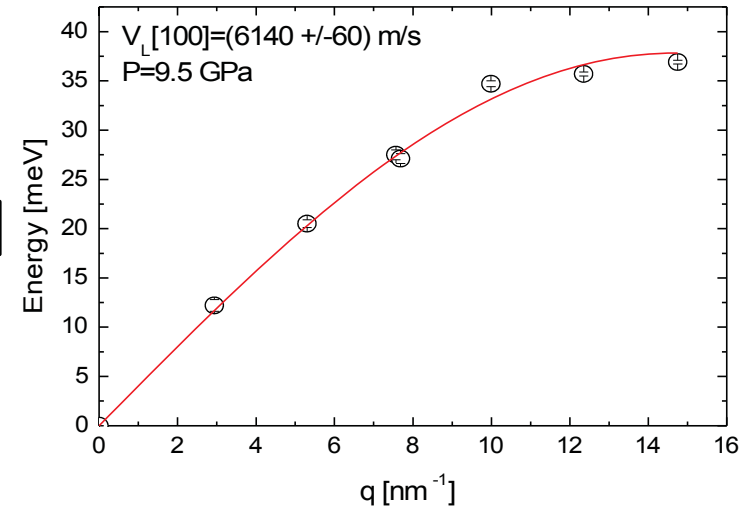
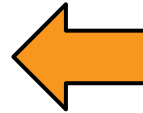
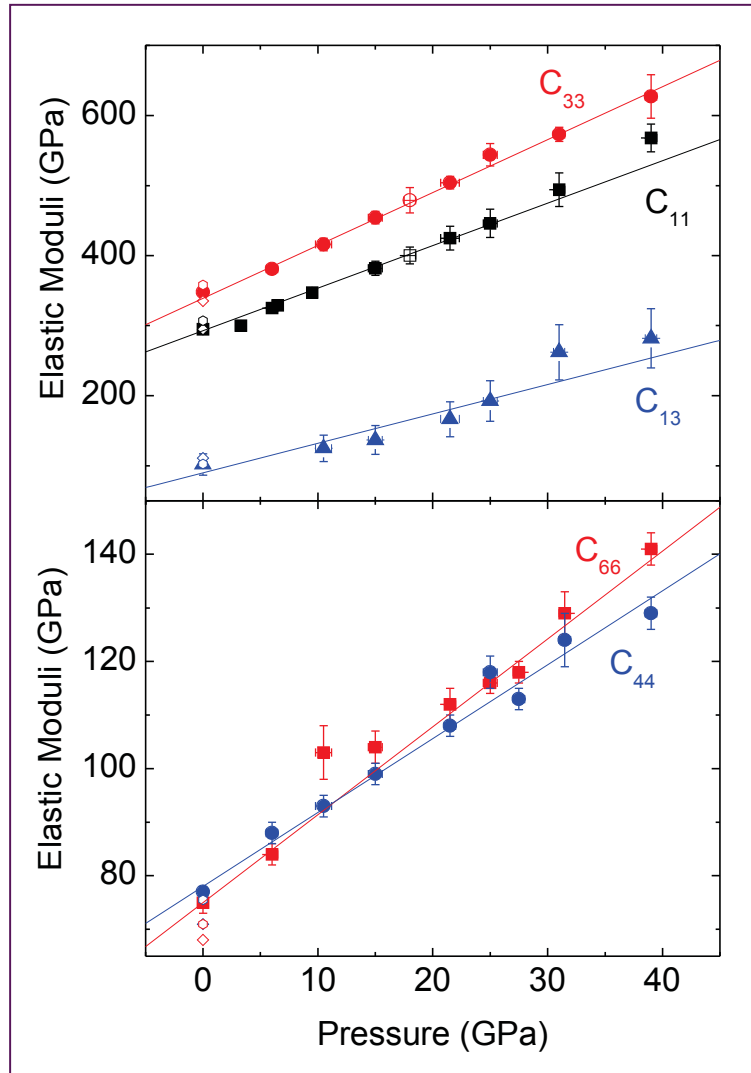
Experimental highlights (6)

Elasticity at high pressure



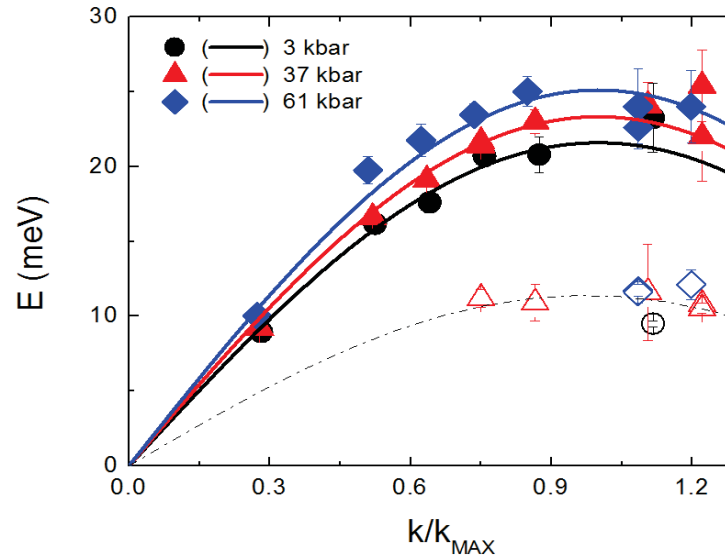
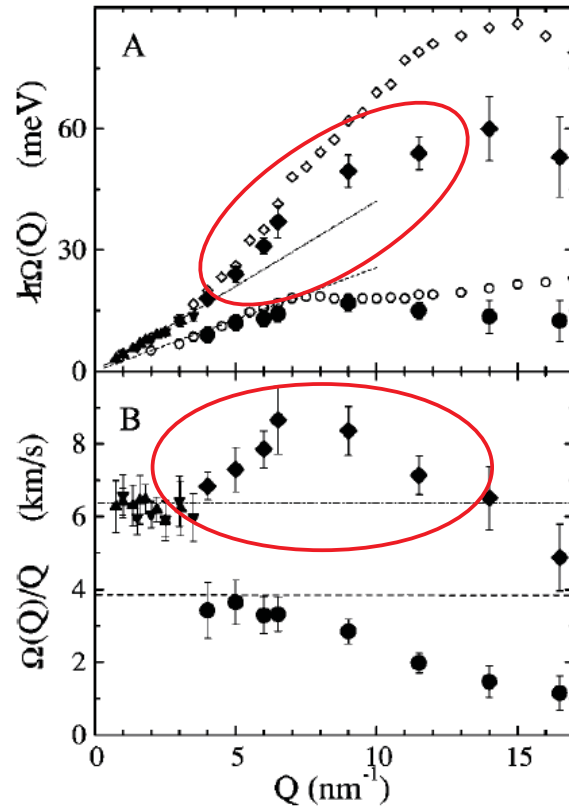
Experimental highlights (6)

Elasticity at high pressure

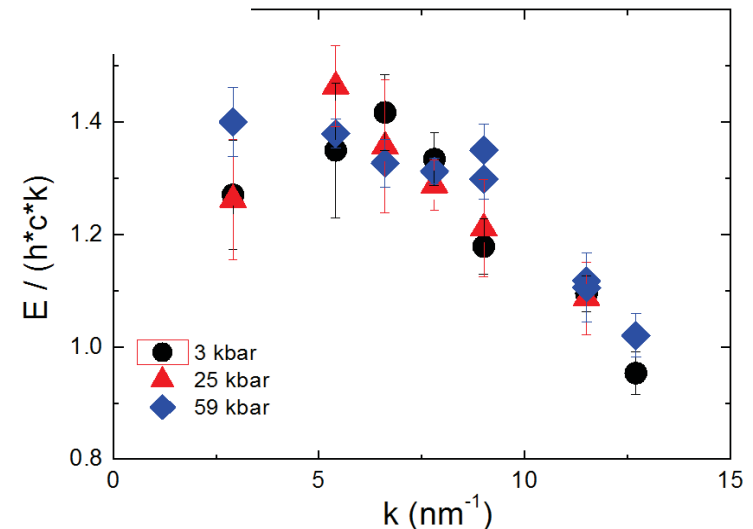


Experimental highlights (7)

Glassy GeO₂ under pressure (kinematic constraints + tight focus)



No “anomalies” observed in the crystalline phase; positive dispersion in the glassy phase (weak P-dependence). Structural transition in glassy GeO₂ at P=0-7 GPa (coordination from 4 to > 10 GPa)

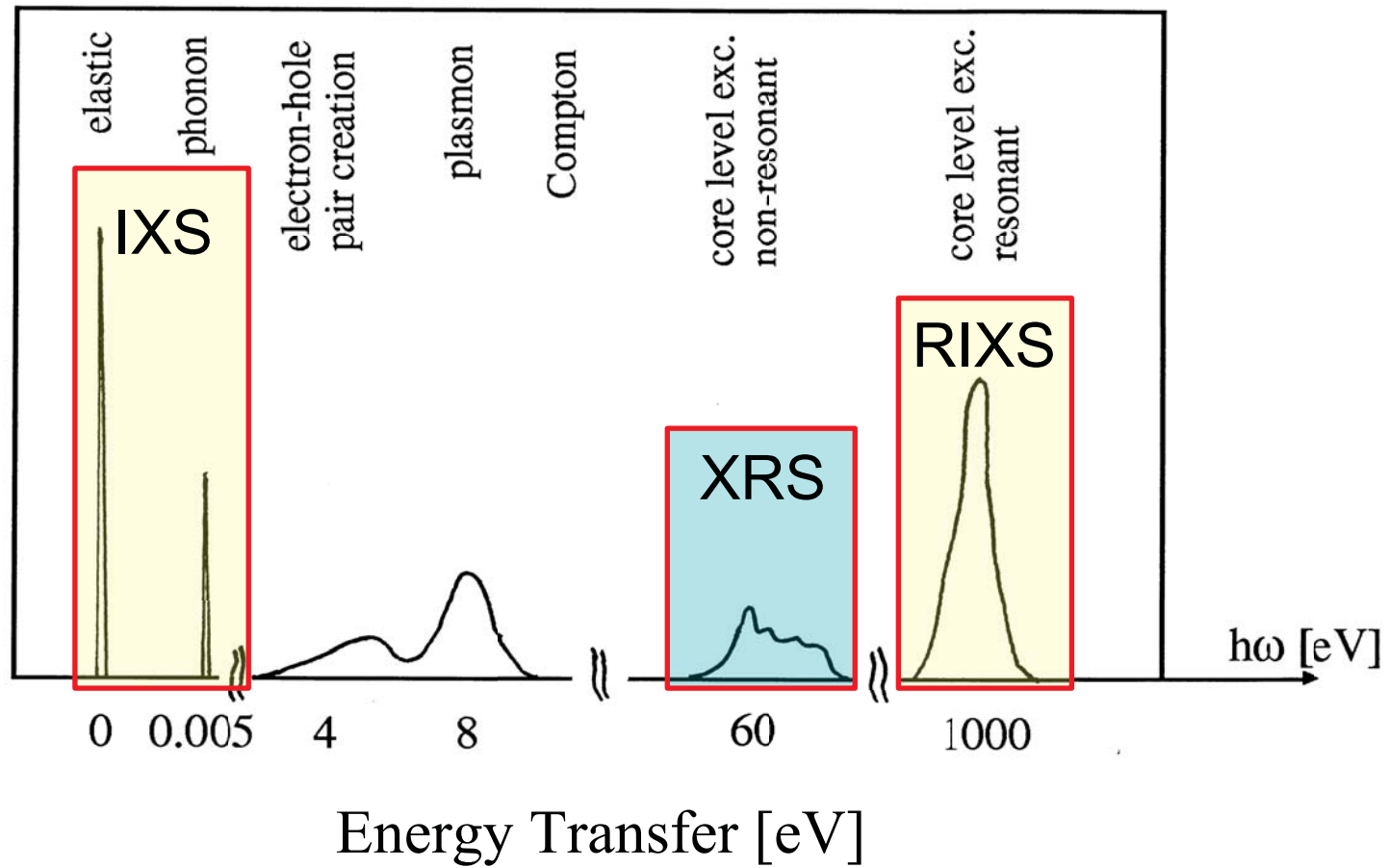


Positive sound dispersion in amorphous **solids** (SiO₂)^a:
 → Also observed in GeO₂ and GeSe₂ (by IXS and INS)
 → Predicted by MD simulations for “harmonic glasses”^b
 → Related to anharmonicity or structural disorder?

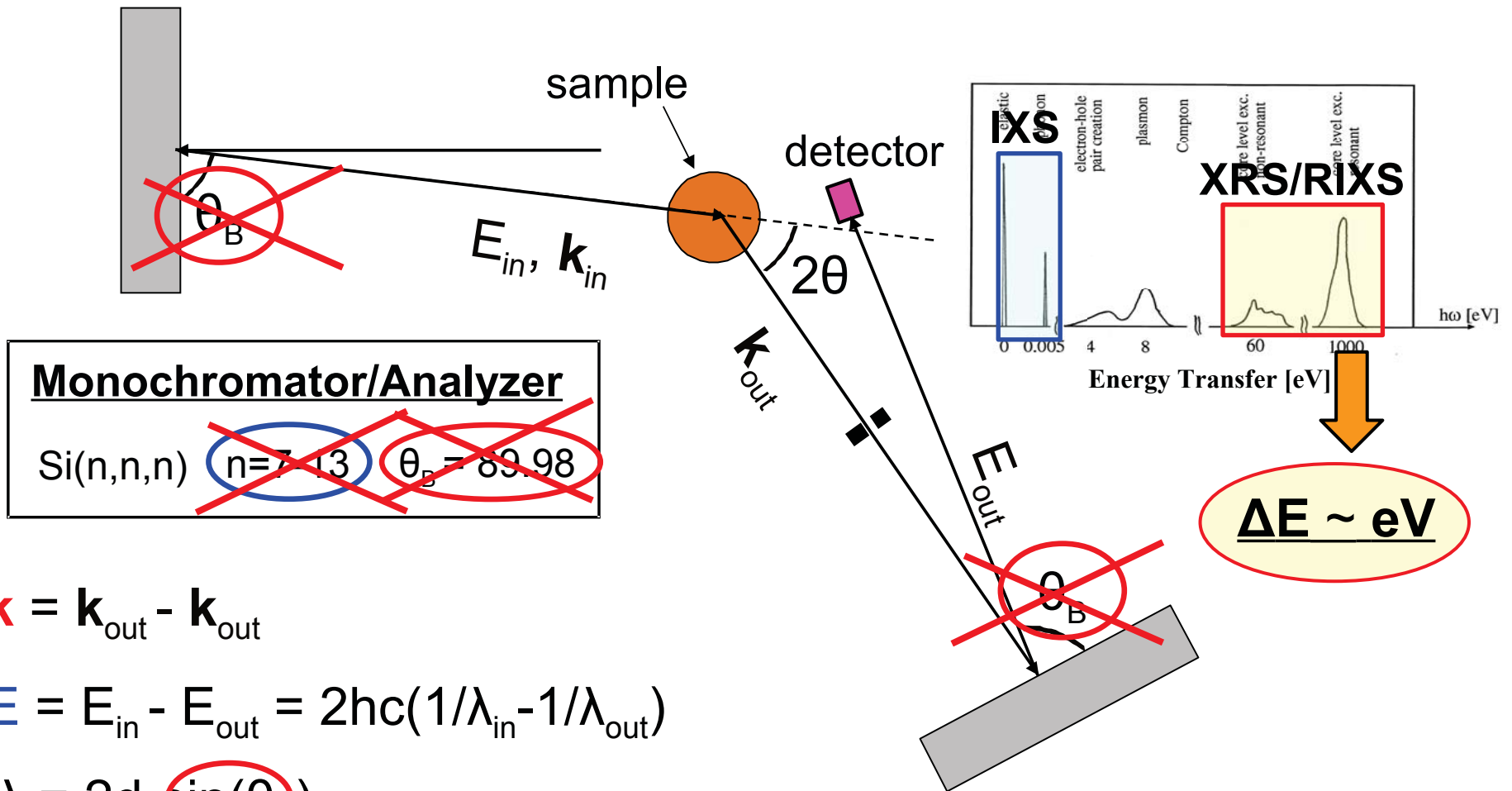
a) B. Ruzicka et al. PRBR 69 (2004); b) G. Ruocco et al., PRL 84 (2000)

F. Bencivenga et al., unpublished

X-ray Raman Scattering (XRS)



Basic instrumentation (XRS/RIXS)



Monochromator/Analyzer
~~Si(n,n,n) $n=7.13$ $\theta_B = 89.98$~~

$$\mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$

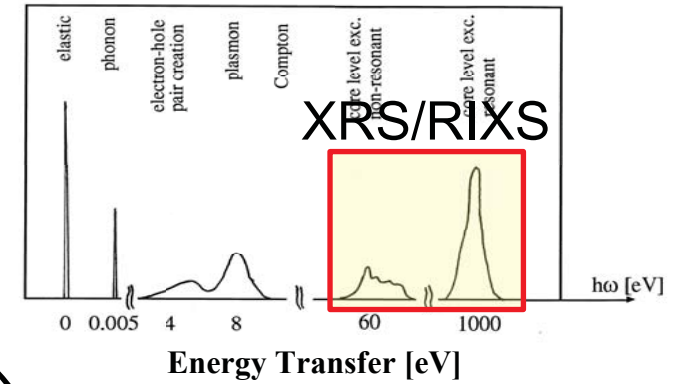
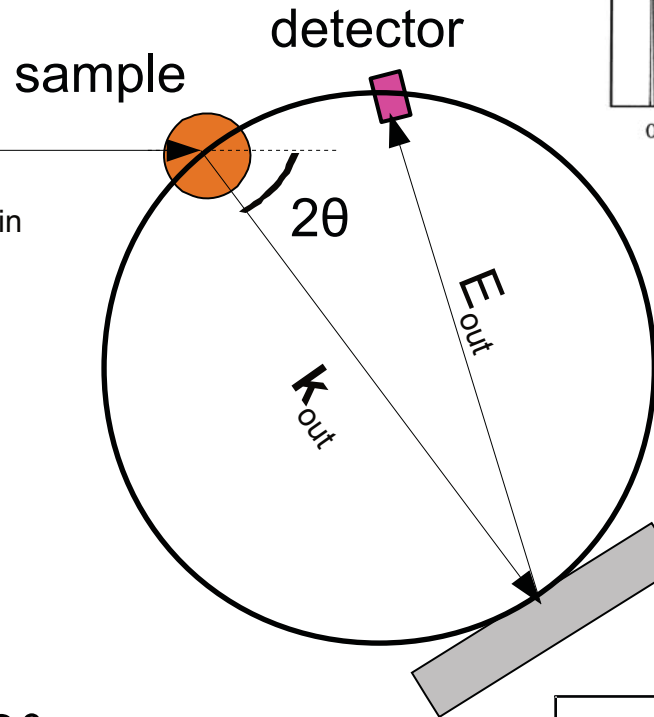
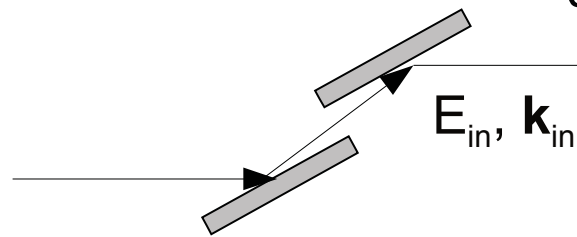
$$E = E_{in} - E_{out} = 2hc(1/\lambda_{in} - 1/\lambda_{out})$$

$$\lambda = 2d_n \sin(\theta_B)$$

~~backscattering~~ + ~~high order reflections~~ = ~~$\Delta E \sim meV$~~

Basic instrumentation (XRS/RIXS)

Monochromator
 Si(1,1,1); (2,2,0); ...
 λ_{in} (tunable)



Rowland circle spectrometer (1 m)

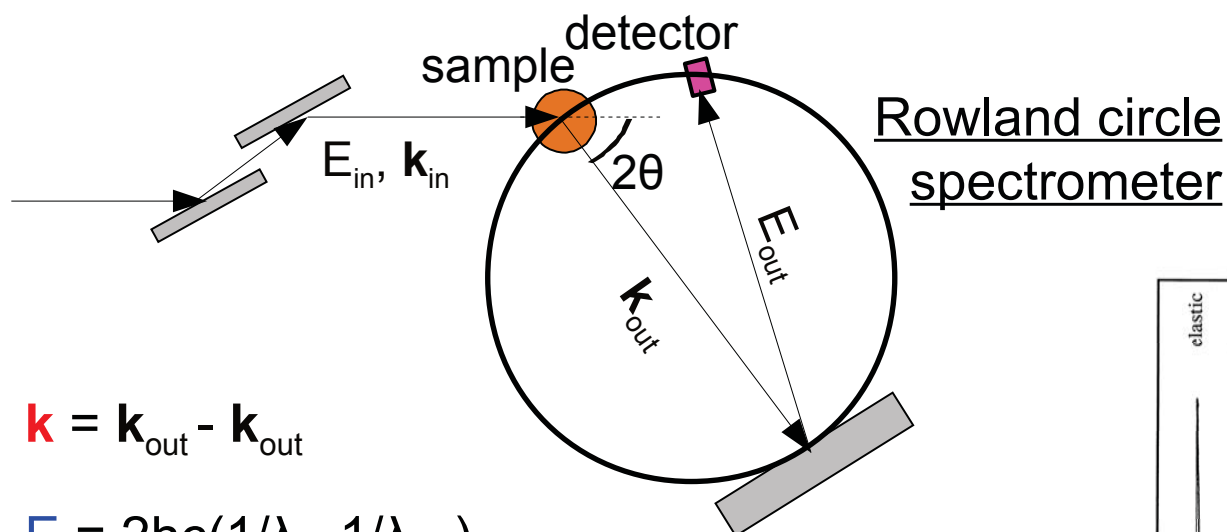
$$\mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$

$$E = 2hc(1/\lambda_{in} - 1/\lambda_{out})$$

$$\lambda = 2d_n \sin(\theta_B); \theta_B < 90^\circ$$

Analyser
 Si(h,k,l) λ_{out} (tunable)

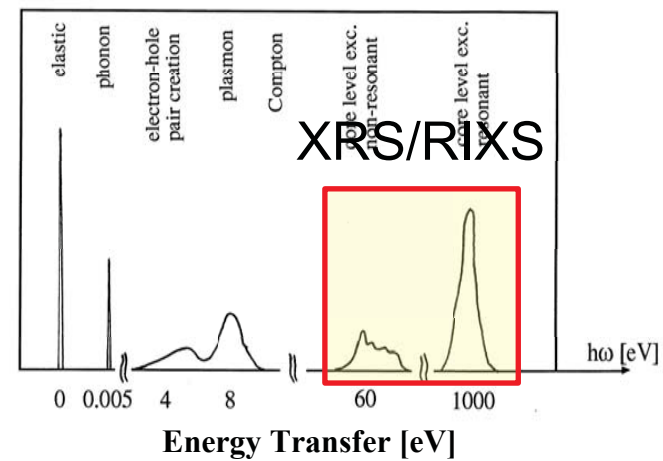
Basic instrumentation (XRS/RIXS)



$$\mathbf{k} = \mathbf{k}_{out} - \mathbf{k}_{in}$$

$$E = 2hc(1/\lambda_{in} - 1/\lambda_{out})$$

$$\lambda = 2d_n \sin(\theta_B); \theta_B < 90^\circ$$



Scanning strategy

1. E_{out} fixed, scanning E_{in} - IXS, XRS, RIXS
2. E_{in} fixed, scanning E_{out} (rotating crystal and follow with the detector) - RIXS
3. Scanning E_{in} and E_{out} keeping E constant - RIXS

Basic theoretical aspects (XRS)

$$H_{\text{int}} = (e/m_e c) \sum_j [(e/2c) \mathbf{A}_j \cdot \mathbf{A}_j + \mathbf{A}_j \cdot \mathbf{p}_j]$$

A: vector potential of electromagnetic field

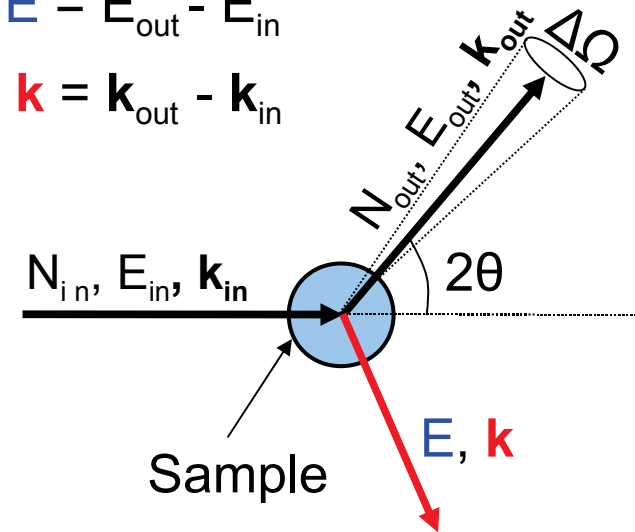
P: momentum operator of the electrons

j: summation over all electrons of the system

Inelastic scattering:

$$E = E_{\text{out}} - E_{\text{in}}$$

$$\mathbf{k} = \mathbf{k}_{\text{out}} - \mathbf{k}_{\text{in}}$$



Cross section: $\frac{\partial^2 \sigma}{\partial \Omega \partial E} \sim N_{\text{out}} / N_{\text{in}}$

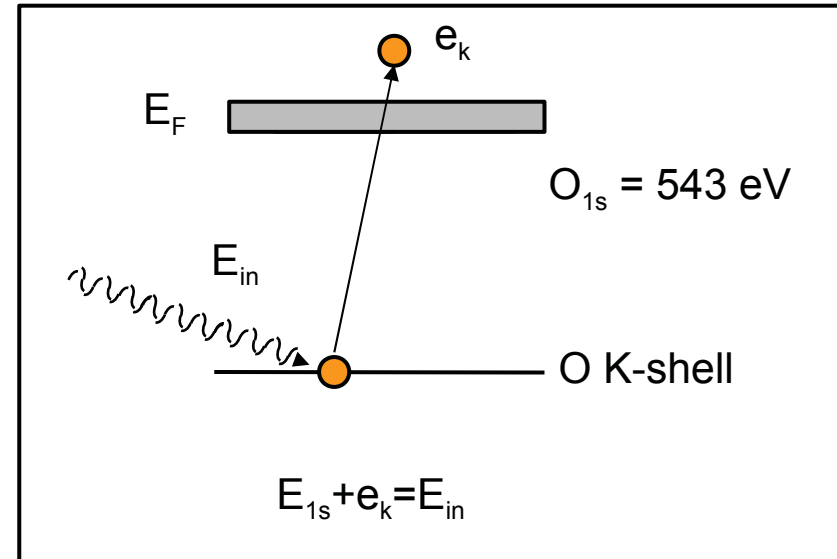
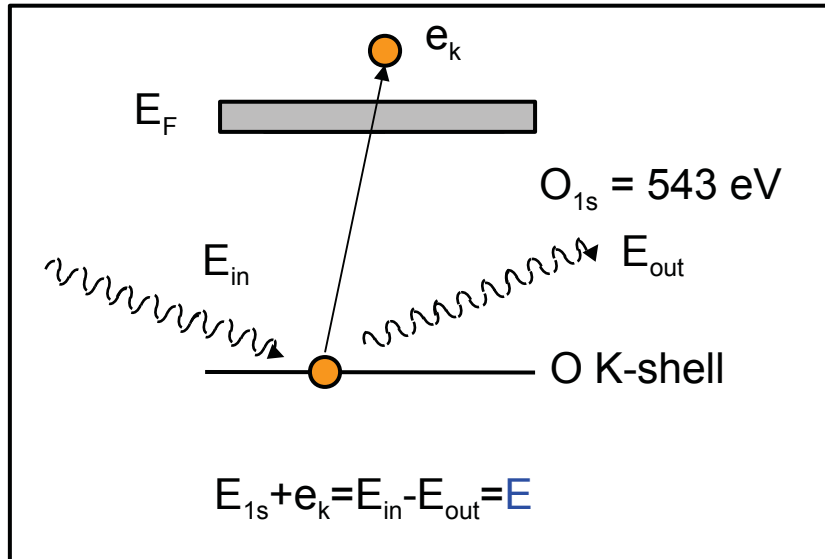
$\mathbf{A} \cdot \mathbf{A}$ → non-resonant scattering (example: IXS)

$\mathbf{A} \cdot \mathbf{p}$ → resonant scattering, absorption followed by emission

Basic theoretical aspects (XRS)

Non resonant IXS cross section:

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\varepsilon}_{\text{in}} \cdot \boldsymbol{\varepsilon}_{\text{out}})^2 \left(\frac{k_{\text{in}}}{k_{\text{out}}} \right) \sum_l P_l |\langle l | \exp\{i\mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_F + E_l)$$



X-ray absorption cross section (dipolar approximation):

$$\frac{\partial \sigma}{\partial E_{\text{in}}} = 4\pi^2 \alpha E_{\text{in}} \sum_l P_l |\langle l | \boldsymbol{\varepsilon}_{\text{in}} \cdot \mathbf{r}_j | F \rangle|^2 \delta(E_{\text{in}} - E_F + E_l)$$

Basic theoretical aspects (XRS)

Non resonant IXS cross section:

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\epsilon}_{\text{in}} \cdot \boldsymbol{\epsilon}_{\text{out}})^2 (k_{\text{in}}/k_{\text{out}}) \sum_l P_l |\langle l | \exp\{i\mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_F + E_l)$$



$$\mathbf{k} \cdot \mathbf{r}_j \ll 1 \rightarrow e^{i\mathbf{k} \cdot \mathbf{r}} \sim 1 + i\mathbf{k} \cdot \mathbf{r}_j$$

$\mathbf{k} \cdot \mathbf{r}_j \ll 1 \rightarrow$ Dipolar regime: identical to photon absorption, where:

i) The momentum transfer (\mathbf{k}) plays the role of the photon polarization vector ($\boldsymbol{\epsilon}_{\text{in}}$)

ii) The energy transfer (E) plays the role of the incident energy (E_{in})

X-ray absorption cross section (dipolar approximation):

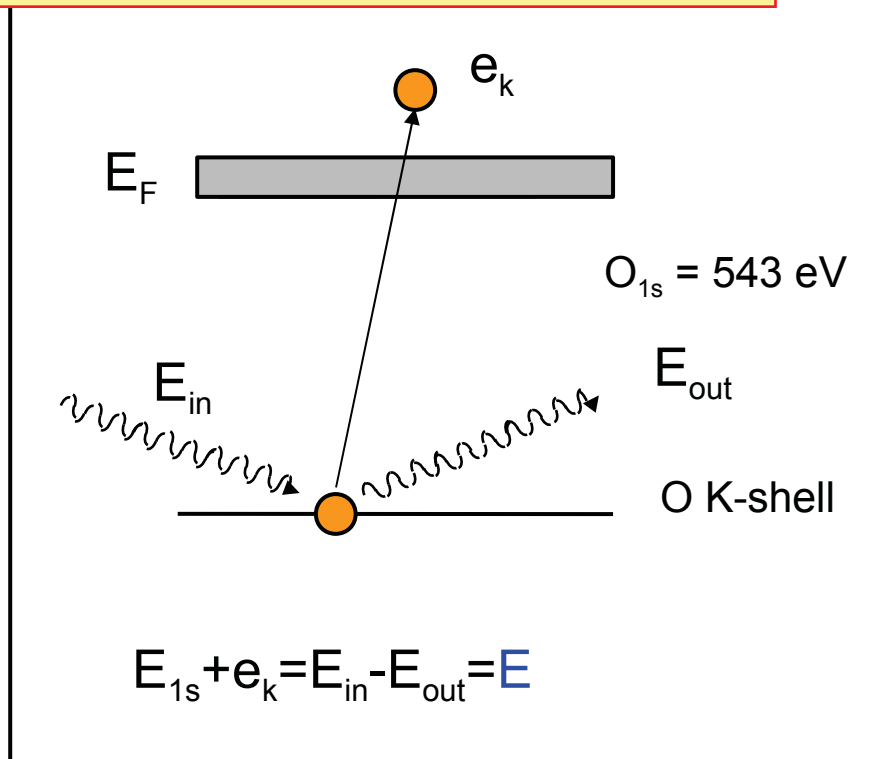
$$\frac{\partial \sigma}{\partial E_{\text{in}}} = 4\pi^2 \alpha E_{\text{in}} \sum_l P_l |\langle l | \boldsymbol{\epsilon}_{\text{in}} \cdot \mathbf{r}_j | F \rangle|^2 \delta(E_{\text{in}} - E_F + E_l)$$

Basic theoretical aspects (XRS)

Non resonant IXS cross section:

$$\frac{\partial^2 \sigma}{\partial \Omega \partial E} = r_0^2 (\boldsymbol{\epsilon}_{\text{in}} \cdot \boldsymbol{\epsilon}_{\text{out}})^2 (k_{\text{in}} / k_{\text{out}}) \sum_l P_l |\langle l | \exp\{i\mathbf{k} \cdot \mathbf{r}_j\} | F \rangle|^2 \delta(E - E_F + E_l)$$

X-ray Raman Scattering (XRS)



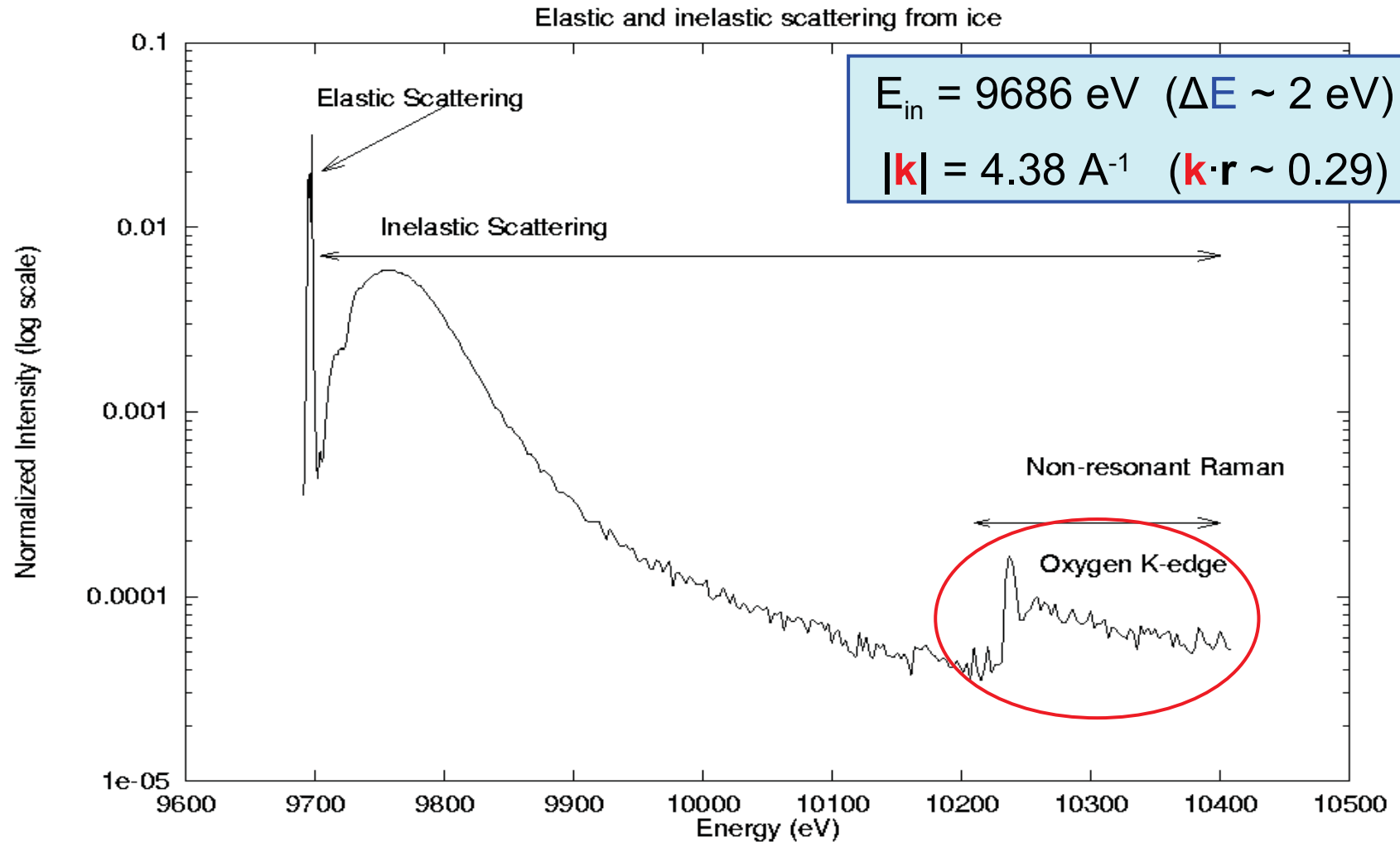
Motivation: element-selective probe for local atomic structure

XRS is alternative to:

- Neutron scattering (with isotopic substitution)
- X-ray (anomalous) scattering
- XANES and EXAFS

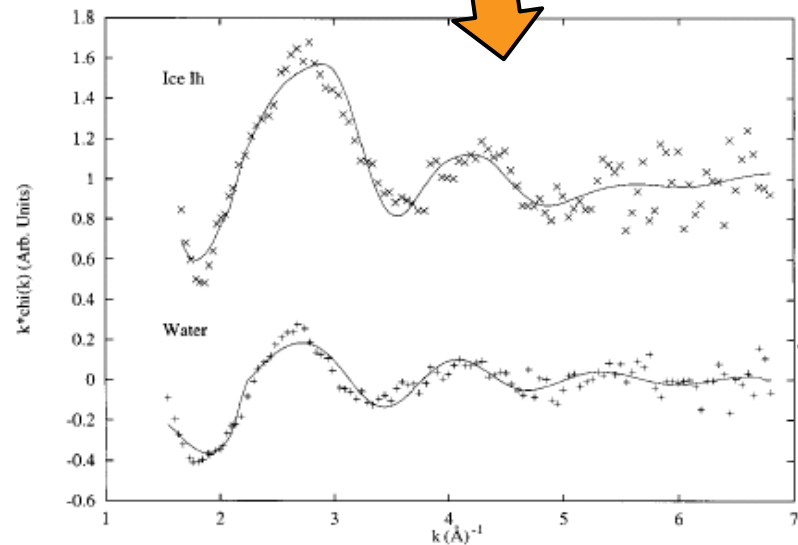
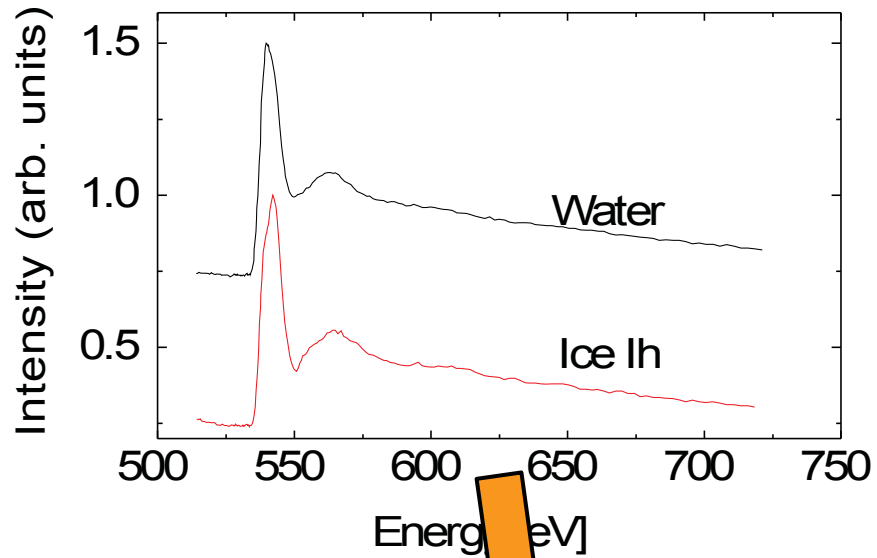
Experimental highlights (XRS)

XRS from O K-edge in water and ice



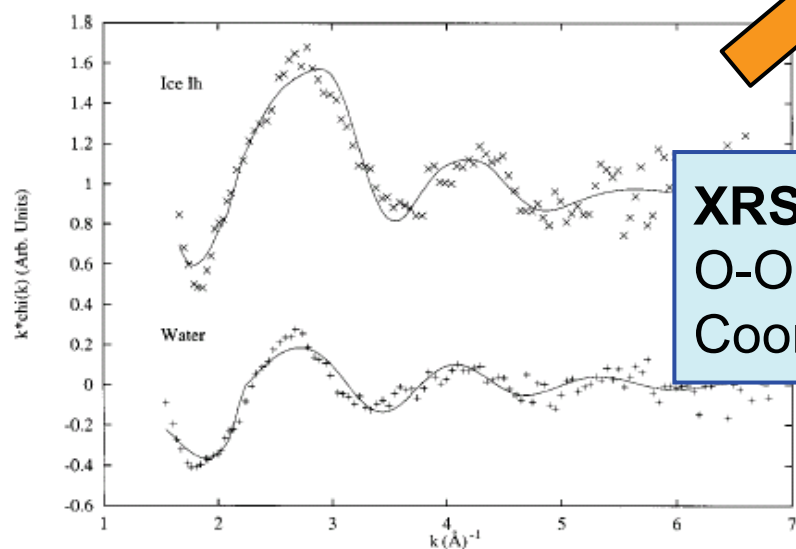
Experimental highlights (XRS)

XRS from O K-edge in water and ice (EXAFS)

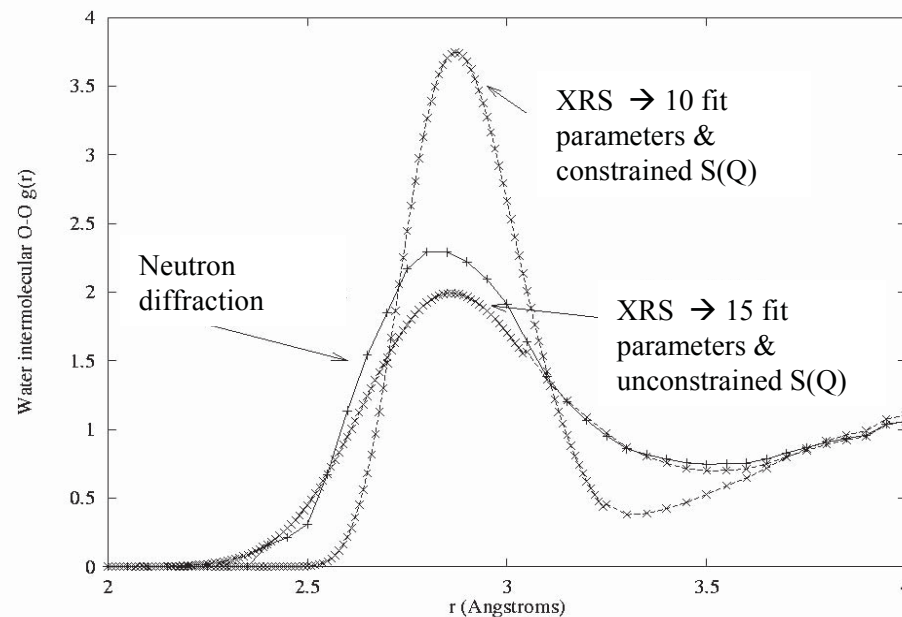


Experimental highlights (XRS)

XRS from O K-edge in water and ice (EXAFS)



XRS:
O-O distance: 2.87 \AA
Coordination: 4 - 7

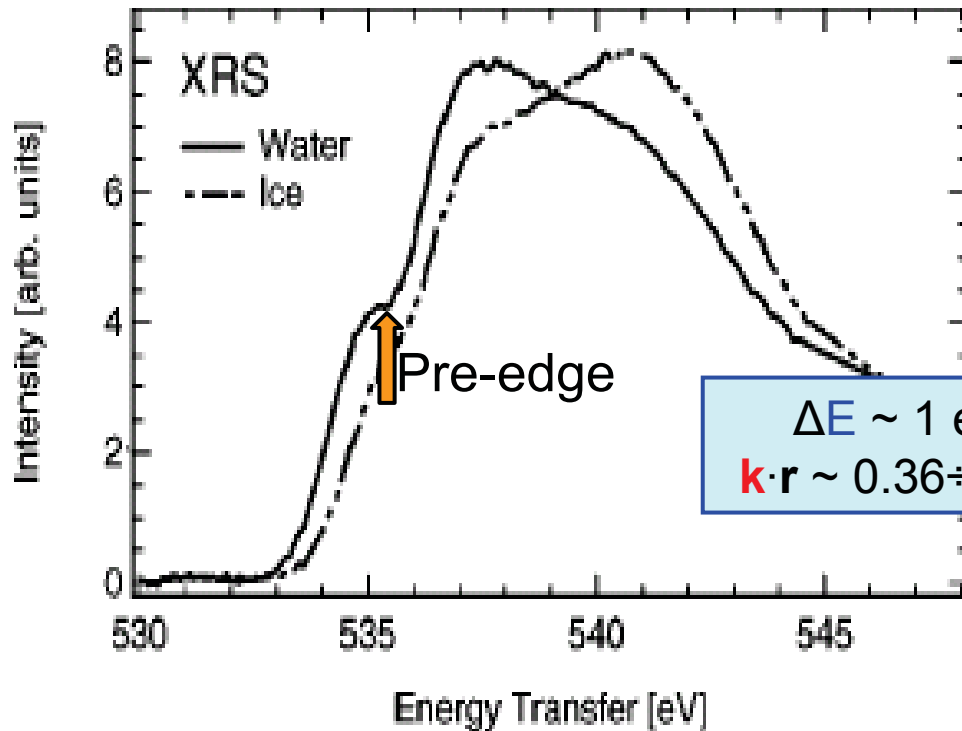


Neutrons:
O-O distance: 2.85 \AA
Coordination: 4.4

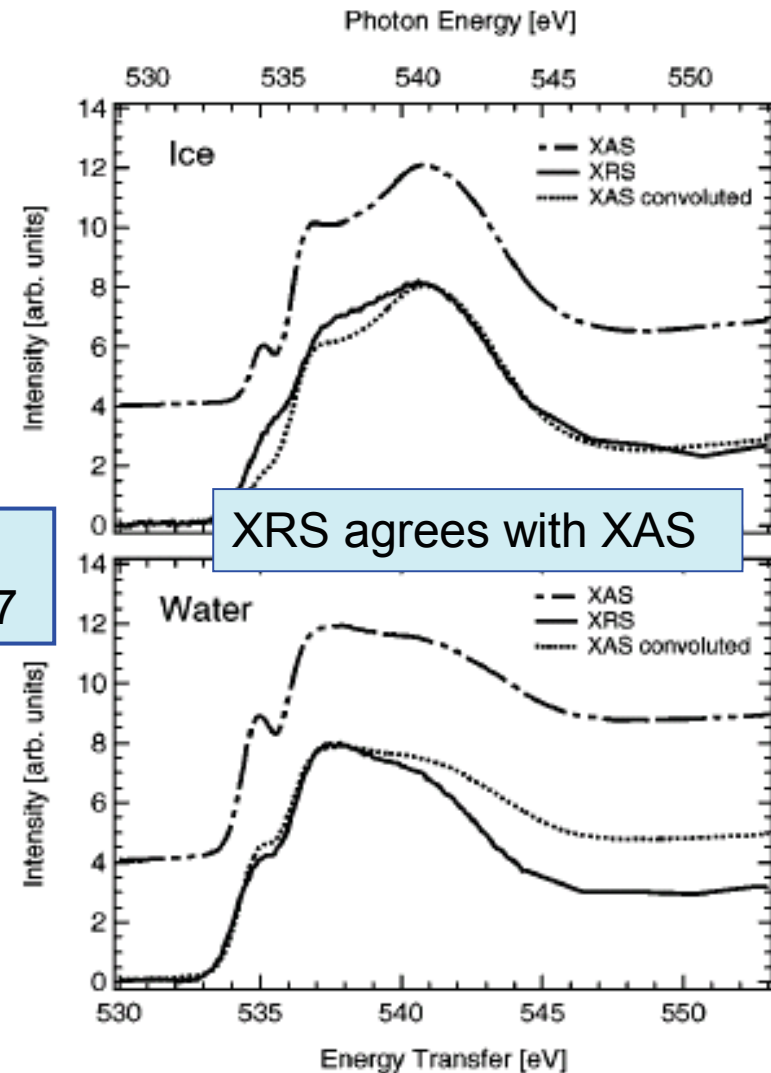
Experimental highlights (XRS)

XRS from O K-edge in water and ice (XANES)

XANES sensitive to the number and “type” of hydrogen bonds (HB)

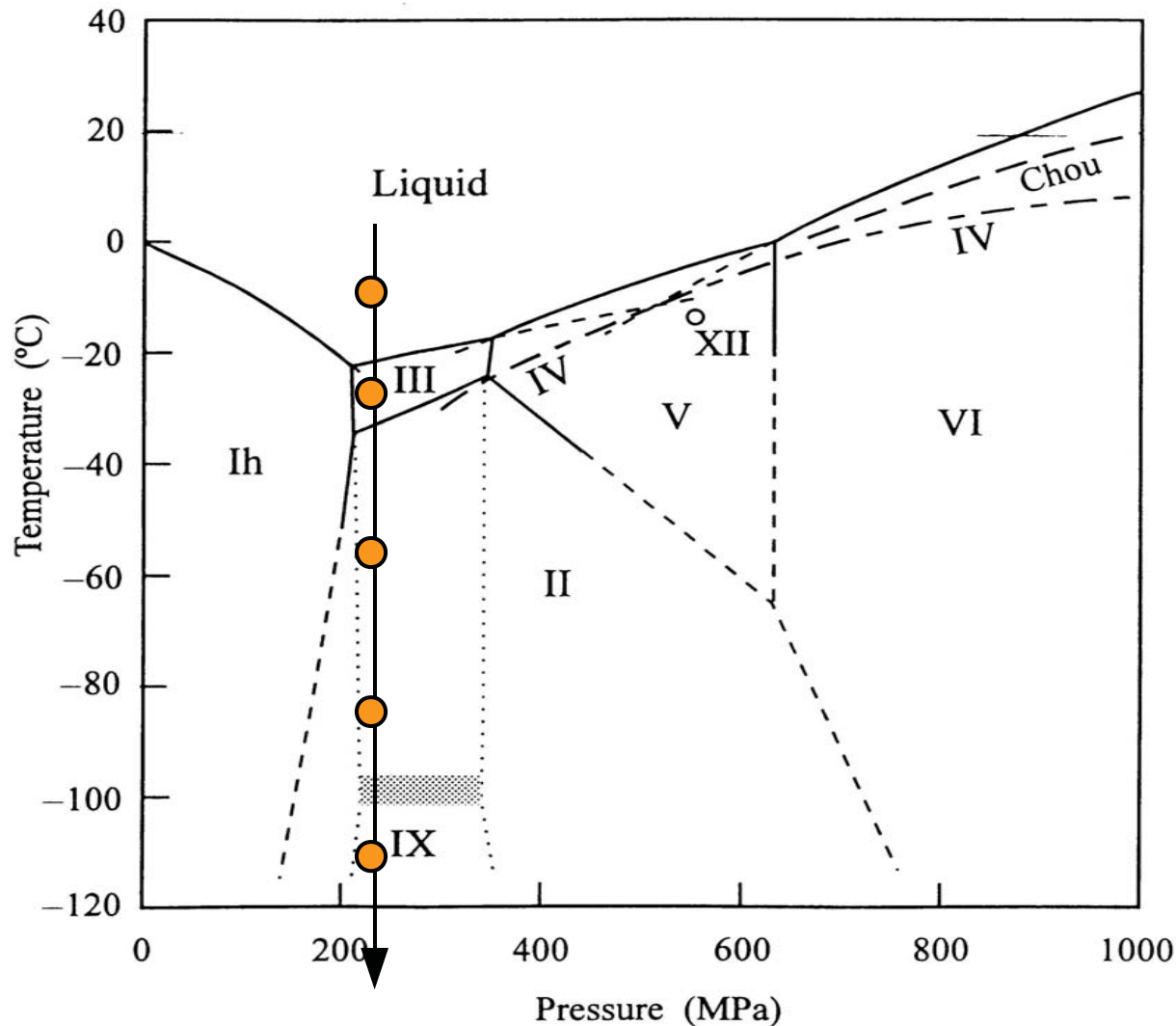


- Pre-edge indicates a large number (~ 70%) of distorted or broken HB (supported by calculation)



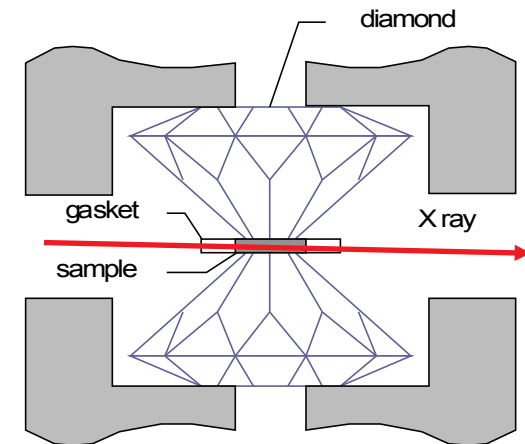
Experimental highlights (XRS)

XRS from O K-edge in ice under high pressure



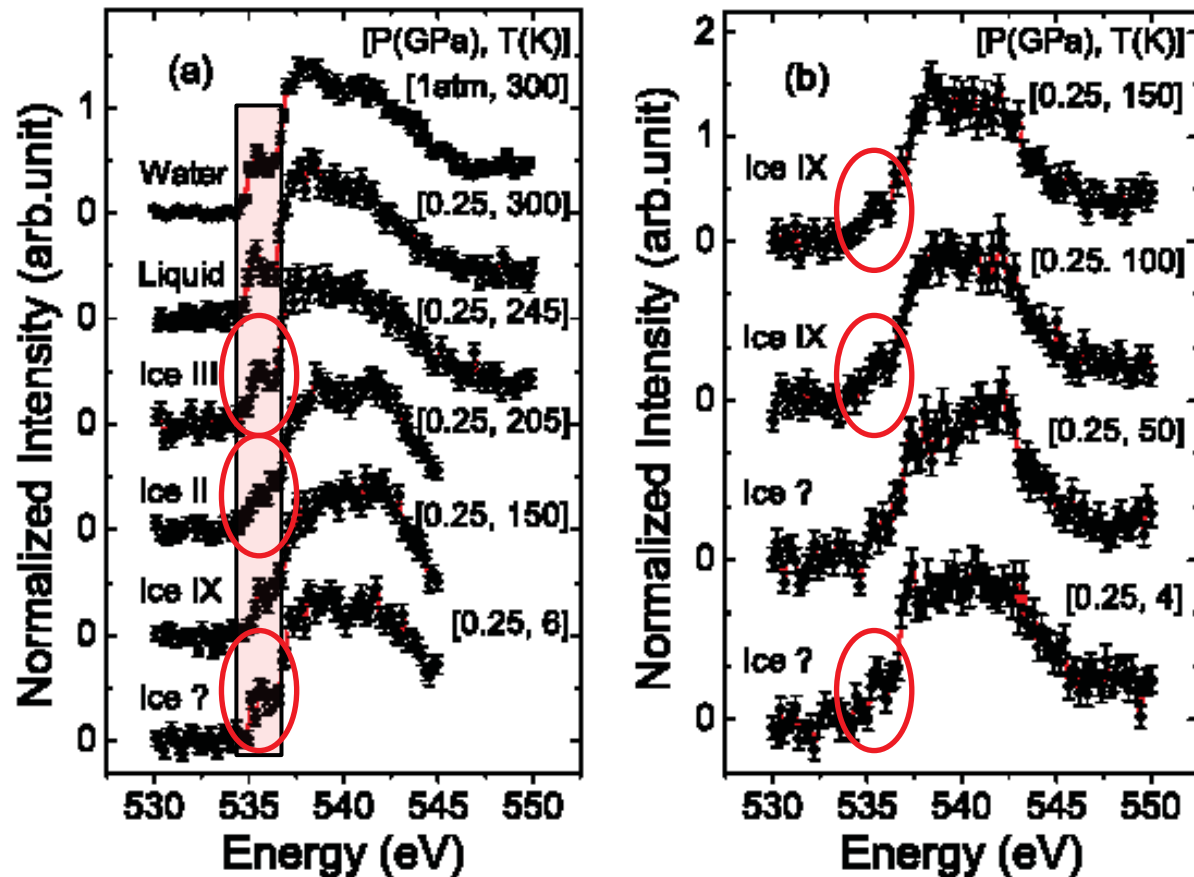
$E_{\text{out}} = 9885 \text{ eV}$
 $(\Delta E \sim 0.2 \div 0.3 \text{ eV})$
 $|\mathbf{k}| \sim 3 \text{ \AA}^{-1}$ ($\mathbf{k} \cdot \mathbf{r} \sim 0.2$)

DAC sample environment:
hard x-ray needed



Experimental highlights (XRS)

XRS from O K-edge in ice under high pressure



- Slight increase of pre-edge with P (larger HB distortion)
- Increasing order of HB from liquid → Ice III (tetrahedral) → Ice II / IX
- New pre-edge increase @ low-T: new Ice phase?

Observation of spectral changes:
Need of much better statistics and theory to extract quantitative information

XRS in summary

Soft x-ray spectroscopy in the hard x-ray regime

Advantages

“simpler” sample environment
(high pressure/temperature,
etc...) + bulk sensitive
→ indicated for studying (bulk)
Oxygen and Carbon

Drawbacks

- “weak probe”
(practically limited to $Z < 14$)
→ limited quality for structural
analysis (EXAFS), reasonable
quality in the XANES region

Exploit information in the near-edge region