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Exercises on Radio-Waves Propagation through Ionospheric Irregularities

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Exercises on Radio-Waves Propagation through Ionospheric Irregularities

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Outline

1. Hybrid Scintillation Propagation Model (**HSPM**) of the transionospheric fluctuating propagation channel
2. Extension of the HSPM model to enable description of the effects due to meso-scale local ionospheric inhomogeneities (bubbles, patches, etc.)
3. Dual frequency extension of the HSPM
4. Demonstration of some simulation programs

1. Hybrid model

Propagation from a satellite to the Earth's surface is calculated in two steps:

• Spatial spectra of the field phase and amplitude are calculated by the ***complex phase method*** for the points of observations below the ionosphere. The spectra are employed to generate a random screen below the ionosphere (physically based - not an equivalent phase screen!)

• The propagation problem for a ***random screen*** is rigorously solved to obtain the field on the Earth's surface.

This hybrid technique provides a full wave-type solution to the propagation problem. It allows:

- The case of strong scintillation of the field amplitude to also be considered;
- Achievement of the higher efficiency of numerical modelling of realistic cases of propagation compared with the case of straightforward purely numerical multiple phase screen calculations.

The complex amplitude of the field passed through the ionosphere is represented as follows

$$E(\mathbf{r},\omega,t)=E_0(\mathbf{r},\omega)R(\mathbf{r},\omega,t)$$

undisturbed field   random factor

Random factor $R(\ ,\omega,t)$ is treated in terms of the complex phase

$$R = e^{\psi}, \qquad \underbrace{\psi}_{\text{Complex phase}} = \underbrace{\chi}_{\text{log-amplitude}} + i\underbrace{S}_{\text{phase}}$$

To formulate a random screen under the ionosphere, the two-dimensional spatial spectra for phase and log-amplitude are produced, which are then employed to generate two-dimensional realisations of χ and S.

Spatial spectra of phase, log-amplitude and their cross-correlation

$$B_S(\kappa_n,\kappa_\tau)=\frac{\pi k^2}{2}\int_0^{s_0}\frac{ds}{J(s)\varepsilon_0(s)}B_\varepsilon(s,0,\eta_n,\eta_\tau)\cos^2\left\{\frac{1}{2k}\left[\eta_n^2 D_n(s)+\eta_\tau^2 D_\tau(s)+2\eta_n\eta_\tau D_{n\tau}(s)\right]\right\}$$

$$B_\chi(\kappa_n,\kappa_\tau)=\frac{\pi k^2}{2}\int_0^{s_0}\frac{ds}{J(s)\varepsilon_0(s)}B_\varepsilon(s,0,\eta_n,\eta_\tau)\sin^2\left\{\frac{1}{2k}\left[\eta_n^2 D_n(s)+\eta_\tau^2 D_\tau(s)+2\eta_n\eta_\tau D_{n\tau}(s)\right]\right\}$$

$$B_{S\chi}(\kappa_n,\kappa_\tau)=\frac{\pi k^2}{4}\int_0^{s_0}\frac{ds}{J(s)\varepsilon_0(s)}B_\varepsilon(s,0,\eta_n,\eta_\tau)\sin\left\{\frac{1}{k}\left[\eta_n^2 D_n(s)+\eta_\tau^2 D_\tau(s)+2\eta_n\eta_\tau D_{n\tau}(s)\right]\right\}$$

J(s) – Jacobian of transformation from transversal spectral variables (κ_n,κ_τ) at the point of observation to the transversal spectral variables (η_n,η_τ) in the ***local ray-centred co-ordinate system*** along generally curved ray

In the numerical simulation, the ***anisotropic*** inverse power law ***spatial spectrum*** of fluctuations of the ionospheric electron density is employed

$$B_\varepsilon(s,\boldsymbol{\kappa}) = C_N^2\left[1-\varepsilon_0(s)\right]^2\sigma_N^2(s)\left(1+\frac{\kappa_x^2+\beta\kappa_y^2+\alpha\kappa_z^2}{\kappa_0^2}\right)^{-p/2}$$

C_N^2 – normalisation coefficient

$\sigma_N^2(s)$ – variance of the fractional electron density fluctuations

$\varepsilon_0(s)$ – distribution of the dielectric permittivity of the background ionosphere along the reference ray – Ionosphere model

$\kappa_0 = \dfrac{2\pi}{L}$, L – outer scale p – spectral index

Generation of the random realization of the complex phase

$B_S = \left\langle SS^T \right\rangle$ - correlation matrix of phase

G - matrix of random normally distributed numbers with unity dispersion:

$\left\langle G \right\rangle = 0, \ \left\langle GG^T \right\rangle = I.$

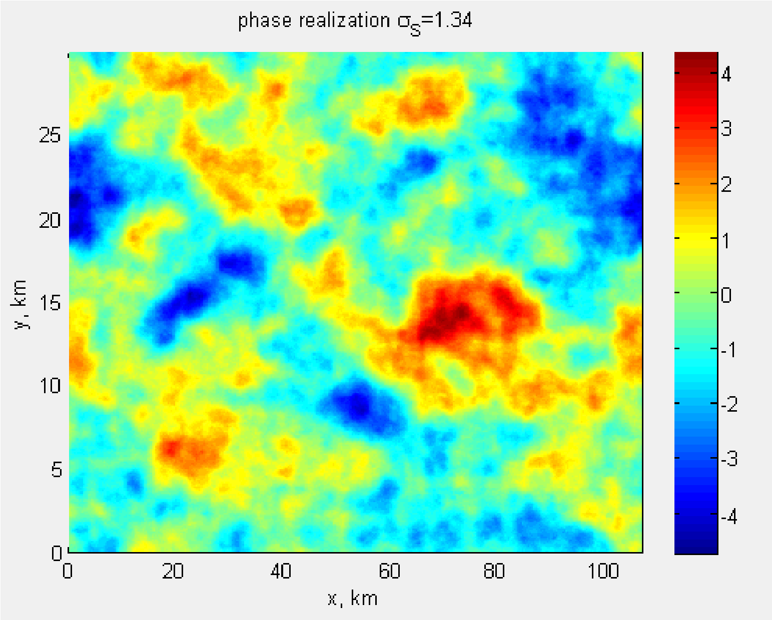
$B_S = A_S A_S^T \rightarrow A_S$ - lower triangular matrix (Cholesky factorization)

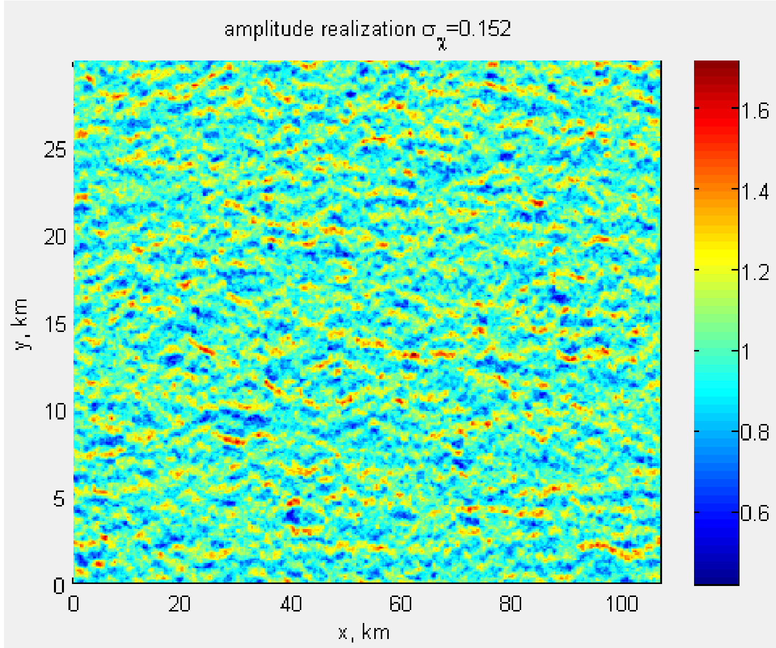
$\left\langle SS^T \right\rangle = \left\langle A_S G G^T A_S^T \right\rangle = A_S A_S^T = B_S$

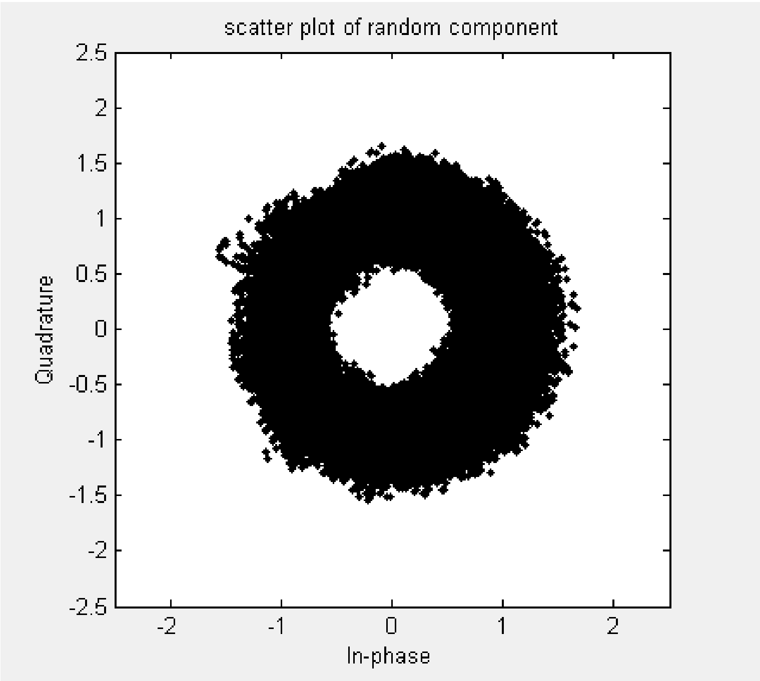
$S = A_S G$ - generating the phase time series

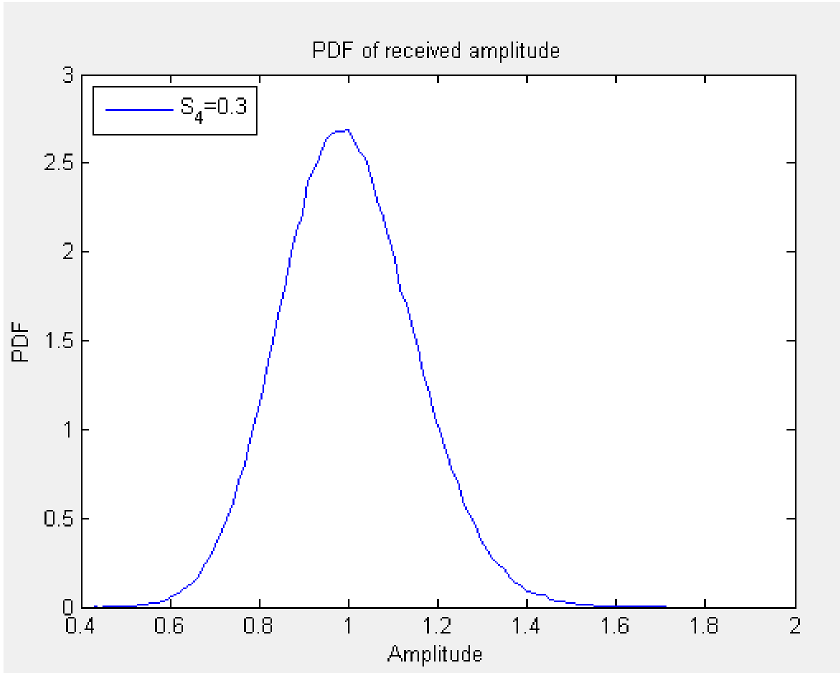
The random spectrum $\tilde{E}(0, \boldsymbol{\kappa})$ of the field on the screen is then transferred to the level of the Earth's surface employing the following relationship of the theory of a random screen

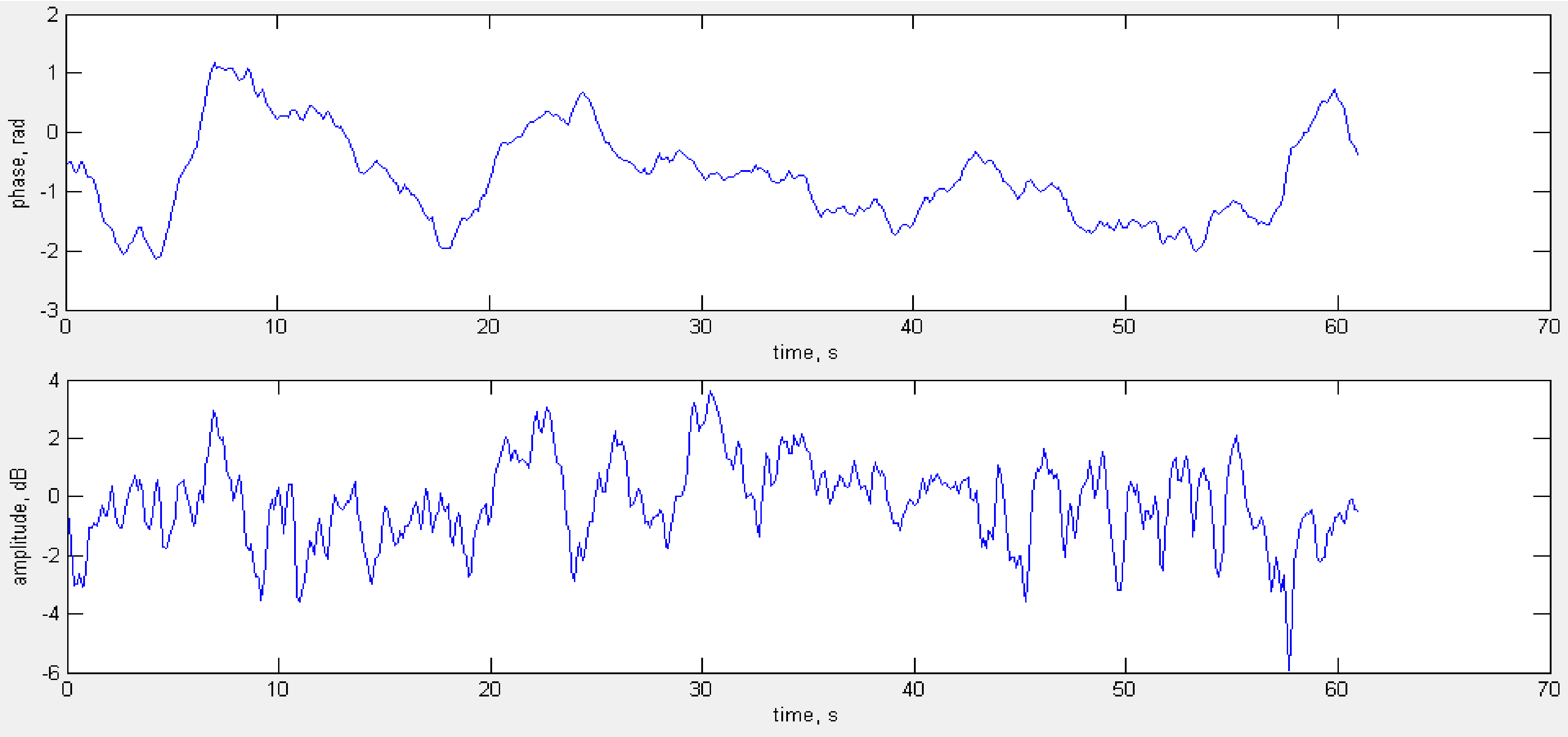
$$\tilde{E}(z, \boldsymbol{\kappa}) = e^{ikz} \tilde{E}(0, \boldsymbol{\kappa}) \exp\left(- \frac{i\boldsymbol{\kappa}^2 z}{2k} \right)$$

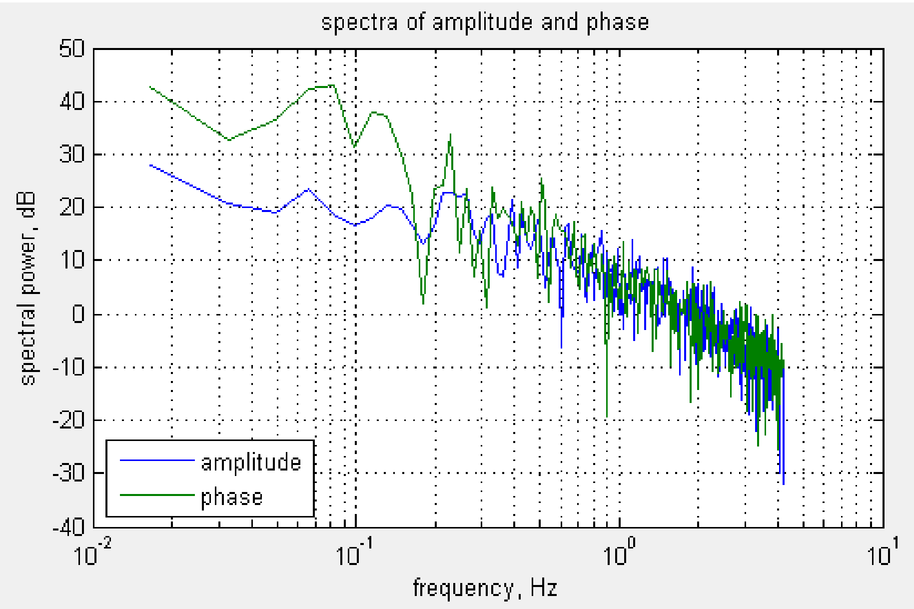

phase realization σS=1.34
y, km
x, km


amplitude realization σχ=0.152
y, km
x, km

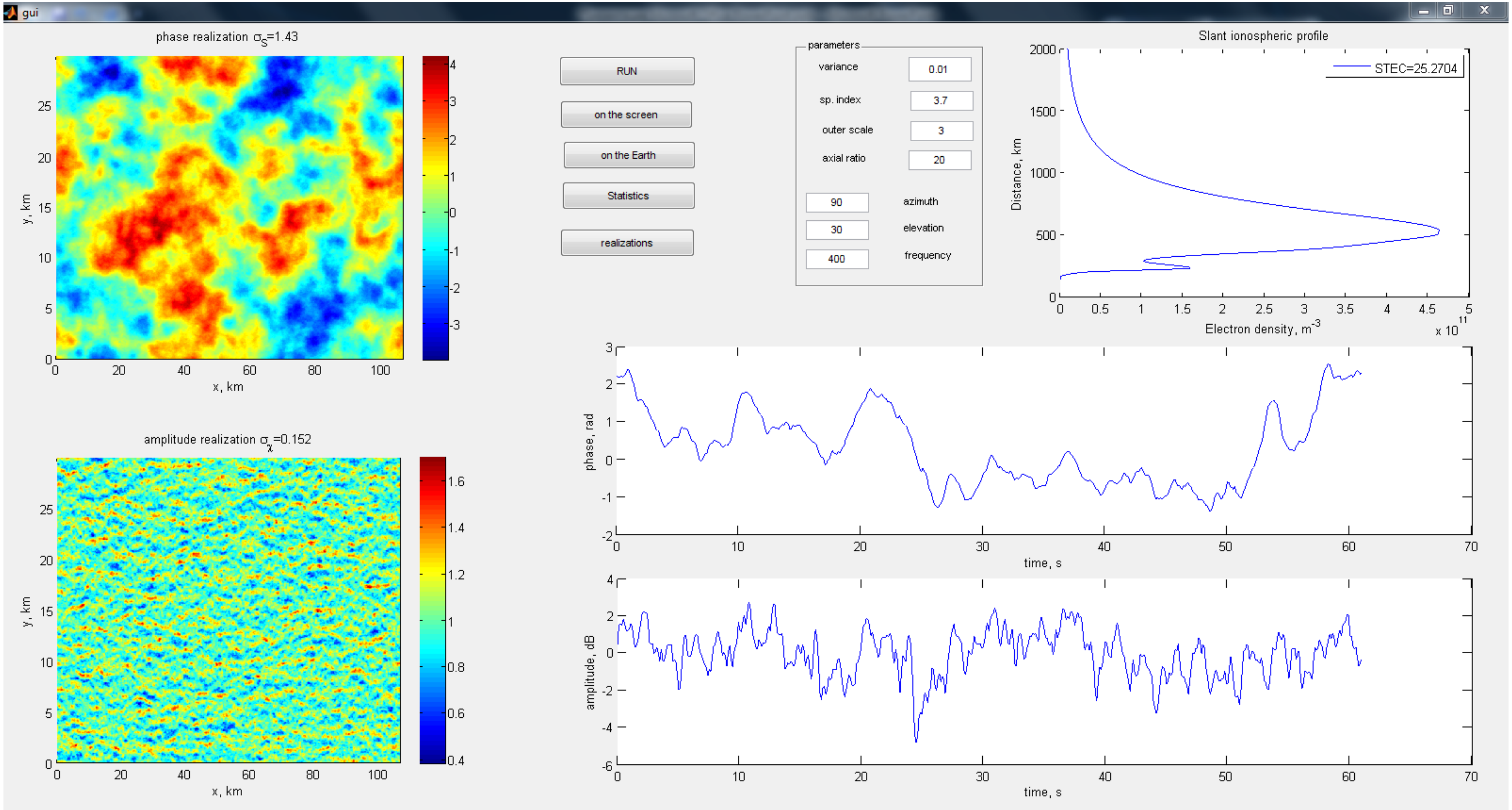

scatter plot of random component
Quadrature
In-phase


PDF of received amplitude
S4=0.3
PDF
Amplitude


phase, rad
time, s
amplitude, dB
time, s


spectra of amplitude and phase
spectral power, dB
amplitude
phase
frequency, Hz

A screenshot of the GUI to the HSPM program



2. Extension of the hybrid model to the case of non-stationary structures

The meso-scale three-dimensional evolving structures (e.g. ionospheric bubbles), which are embedded into the background ionosphere, are treated as a superposition of ***deterministic*** and the ***stochastic*** inhomogeneities of the turbulent nature. These stochastic electron density fluctuations are also statistically ***non-homogeneous*** and ***non-stationary***.

As a result, the statistical moments of the random field (correlation and cross-correlation functions of the field's phase and amplitude) depend not only on the co-ordinate and time separations, but on the space-time co-ordinates of the both points (general non-homogeneous case).

Main steps to model the non-stationary scintillation effects:

1. Calculating the *background* slant electron density distributions along the ray paths connecting the receiver and transmitter for all given moments of time during the event to be simulated;
2. Superimposing the model of a meso-scale structure on the background distributions - *deterministic*;
3. Specifying the model distributions of the variances of the fractional electron density fluctuations - *stochastic*;
4. Calculating the two-point correlation and cross *correlation functions* of the phase and log-amplitude of the field for the height just below the ionospheric layer;
5. Generating the statistically *inhomogeneous* random field producing the *random screen*;
6. Calculating the *time series* of the field at the receiver position;
7. Estimating the statistical characteristics of the received field.

As the *background model* of the ionospheric electron density the NeQuick FORTRAN code is employed

It is capable of producing the slant electron density profiles along the ray path specified by its initial and final points for the given moments of time.

$$Ne_0(height, latitude, longitude, time)$$

The model of a meso-scale bubble structure is superimposed on the background electron density distribution to produce a deterministic part of the refractive index distribution.

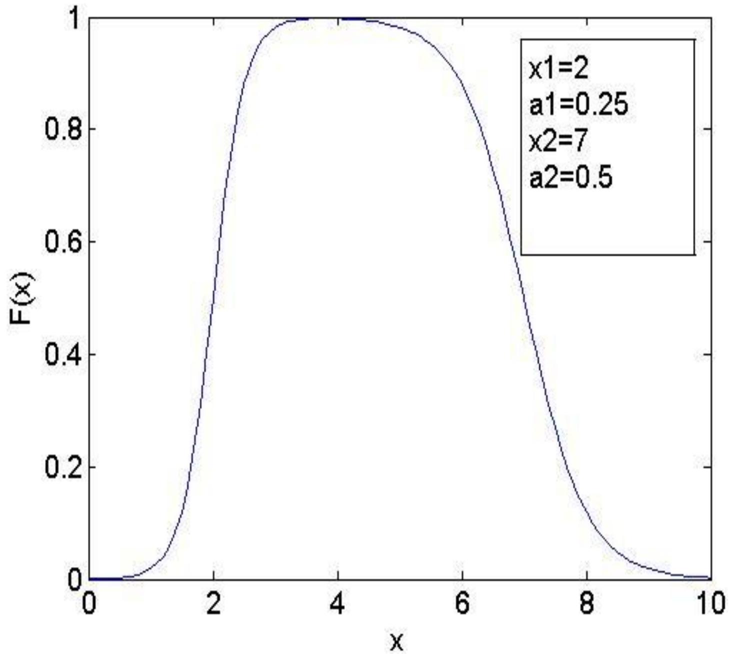
$$Ne(h, lat, lon, t) = Ne_0(h, lat, lon, t) \cdot Model(h, lat, lon, t)$$

In the framework of this approach any suitable (analytical, numerical, empirical) model of the bubble structure can be incorporated to assess the propagation effects.

To explain and predict the radio propagation effects the ‘effective’ model of the non-homogeneous depletion structure is employed:

1. Two-dimensional (the scale size along the magnetic field lines is much greater than across);
2. Not-evolving in time (frozen drift assumption);

$$F(x) = \frac{1}{1+\exp\left[-\frac{x-x_1}{a_1}\right]} - \frac{1}{1+\exp\left[-\frac{x-x_2}{a_2}\right]}$$



Dependence on longitude and time (longitudinal drift)

$$x_1 = lon_1(h,t),\; x_2 = lon_2(h,t)$$

$$F_1(h,lon,t) = F(lon)$$

Dependence on height and time (vertical drift)

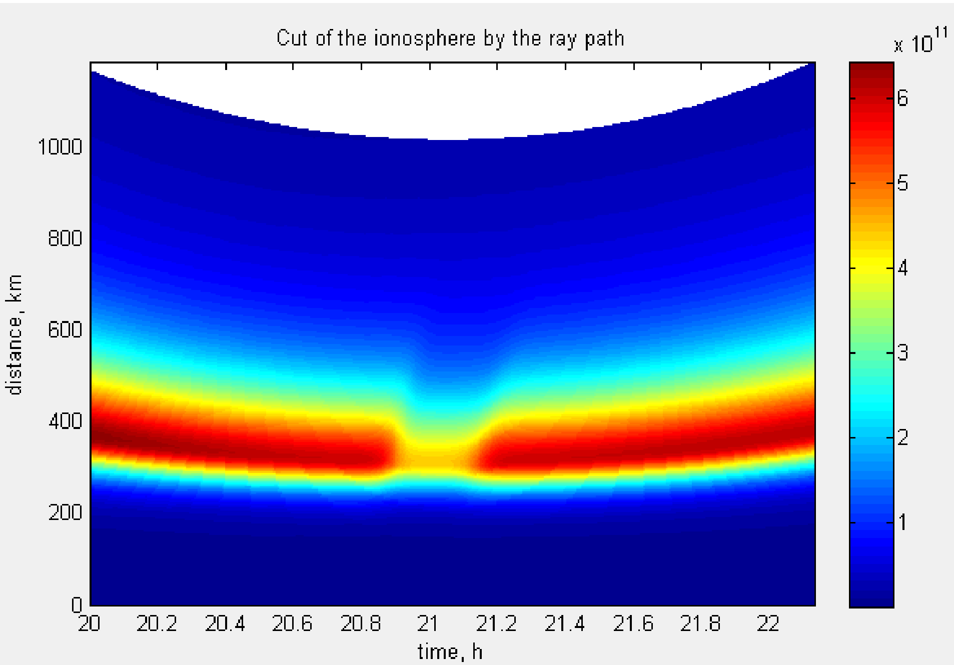
$$x_1 = h_1(t),\; x_2 = h_2(t)$$

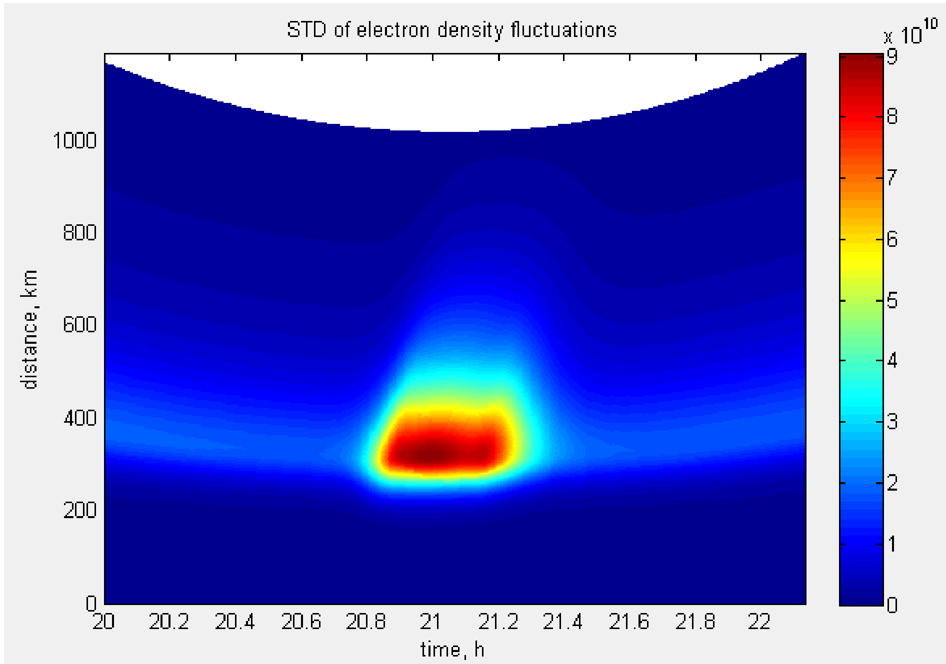
$$F_2(h,lon,t) = F(h)$$

$$Ne(h,lat,lon,t) = Ne_0(h,lat,lon,t)\cdot\left[1 - A\cdot F_1(h,lon,t)\cdot F_2(h,lon,t)\right]$$

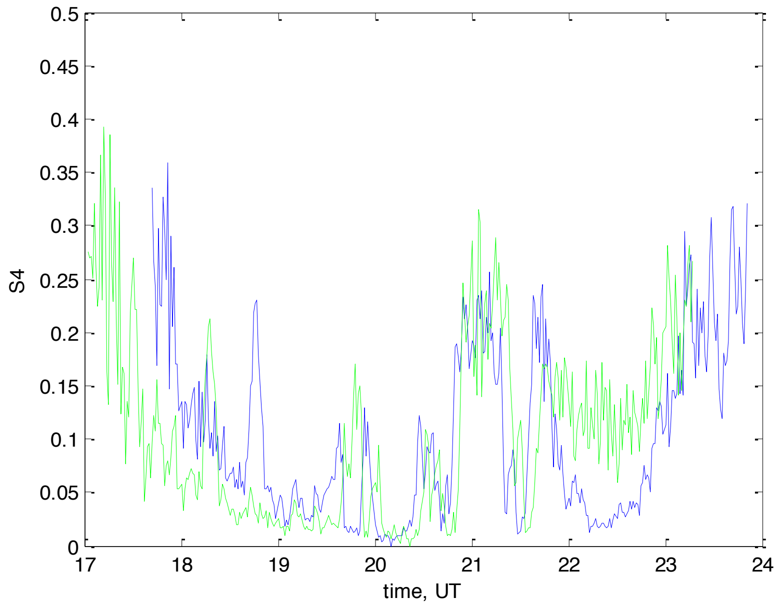
As a result of non-linear evolution of the bubble the strong plasma turbulence is generated inside it, so that the variance of the fractional electron density fluctuations inside the bubble and around it is greater than in the ambient ionosphere.
We model this non-uniform distribution of the variance empirically employing the same basic function as for the modelling of the depletion.

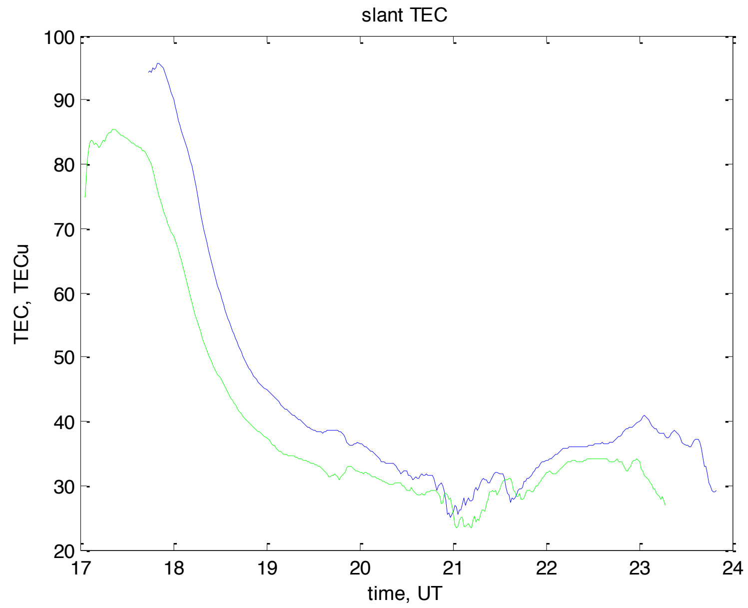
$$\sigma(h,lat,lon,t)=\sigma_0\cdot\left[1+B\cdot F_3(h,lat,t)\cdot F_4(h,lat,t)\right]$$

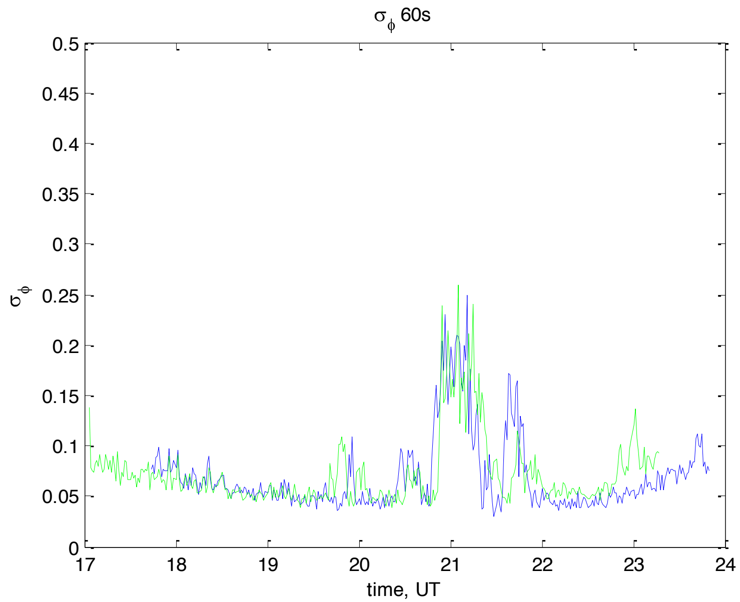


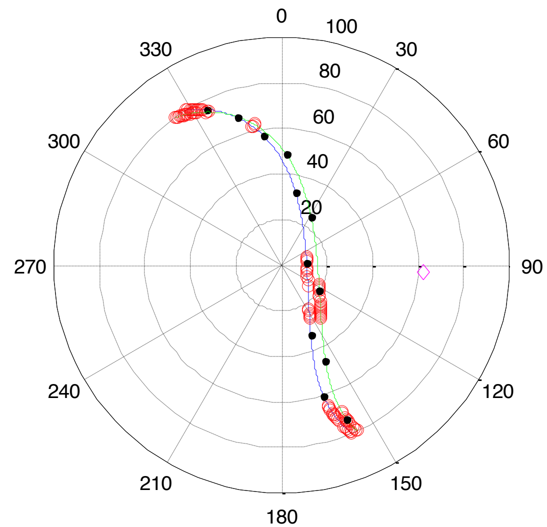


Experimental data: Douala, Cameroon, 07 June 2004 (courtesy of Dr B. Arbesser-Rastburg)
blue – PRN26, green – PRN29

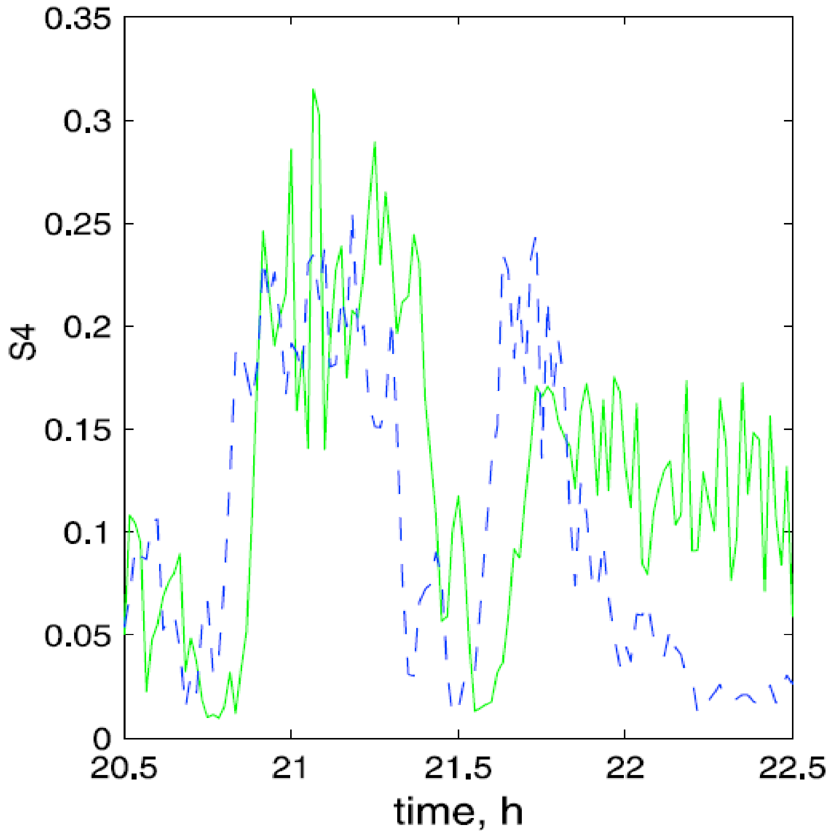




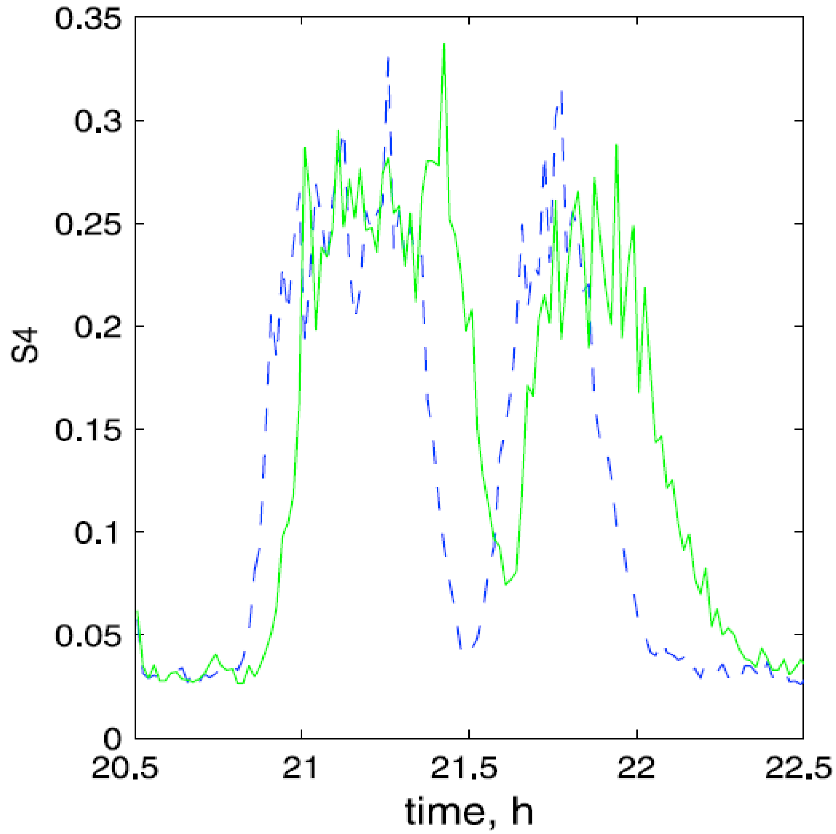




Modeling of the two bubbles passage (inverse problem)



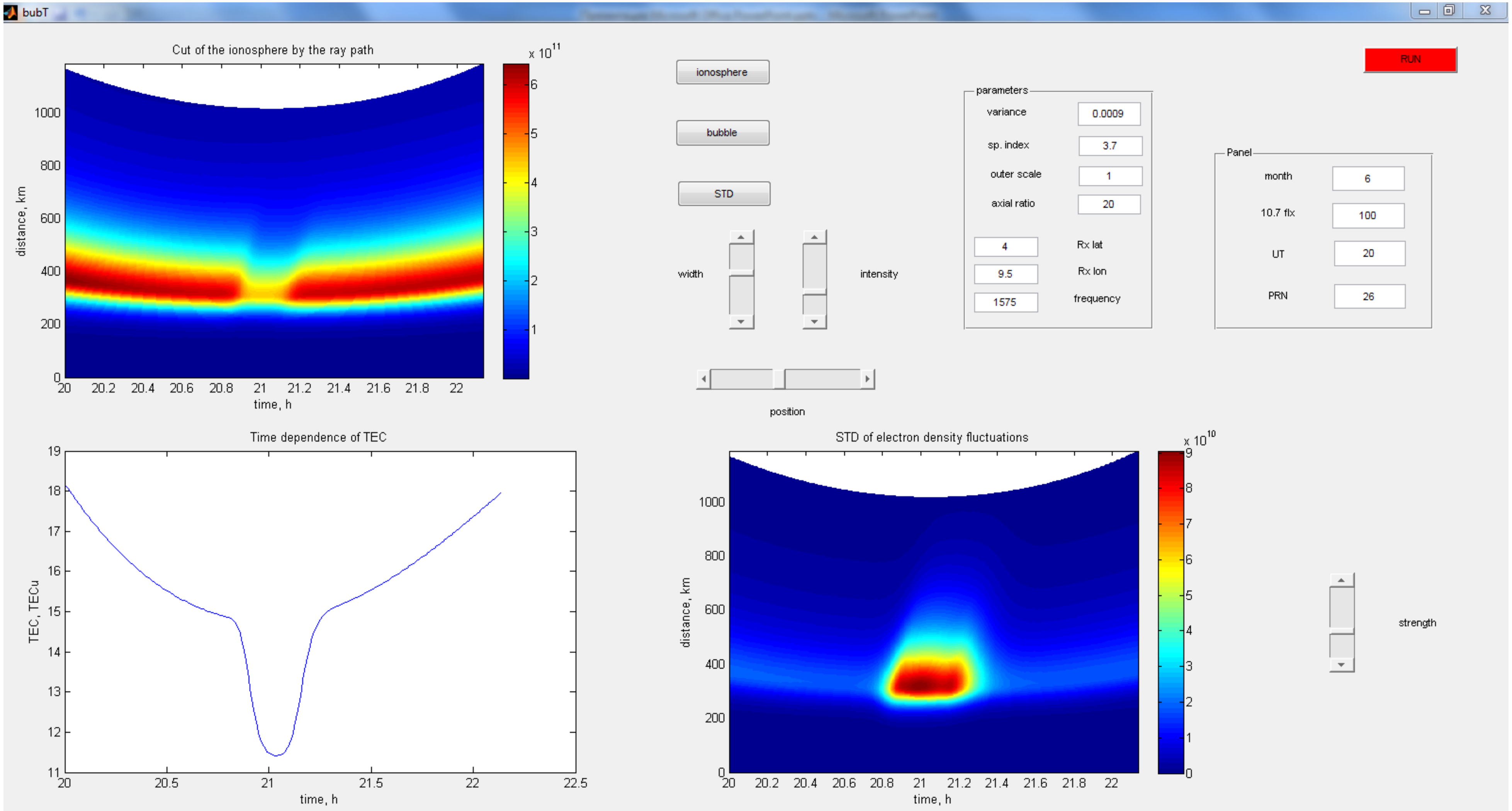
Experimental data



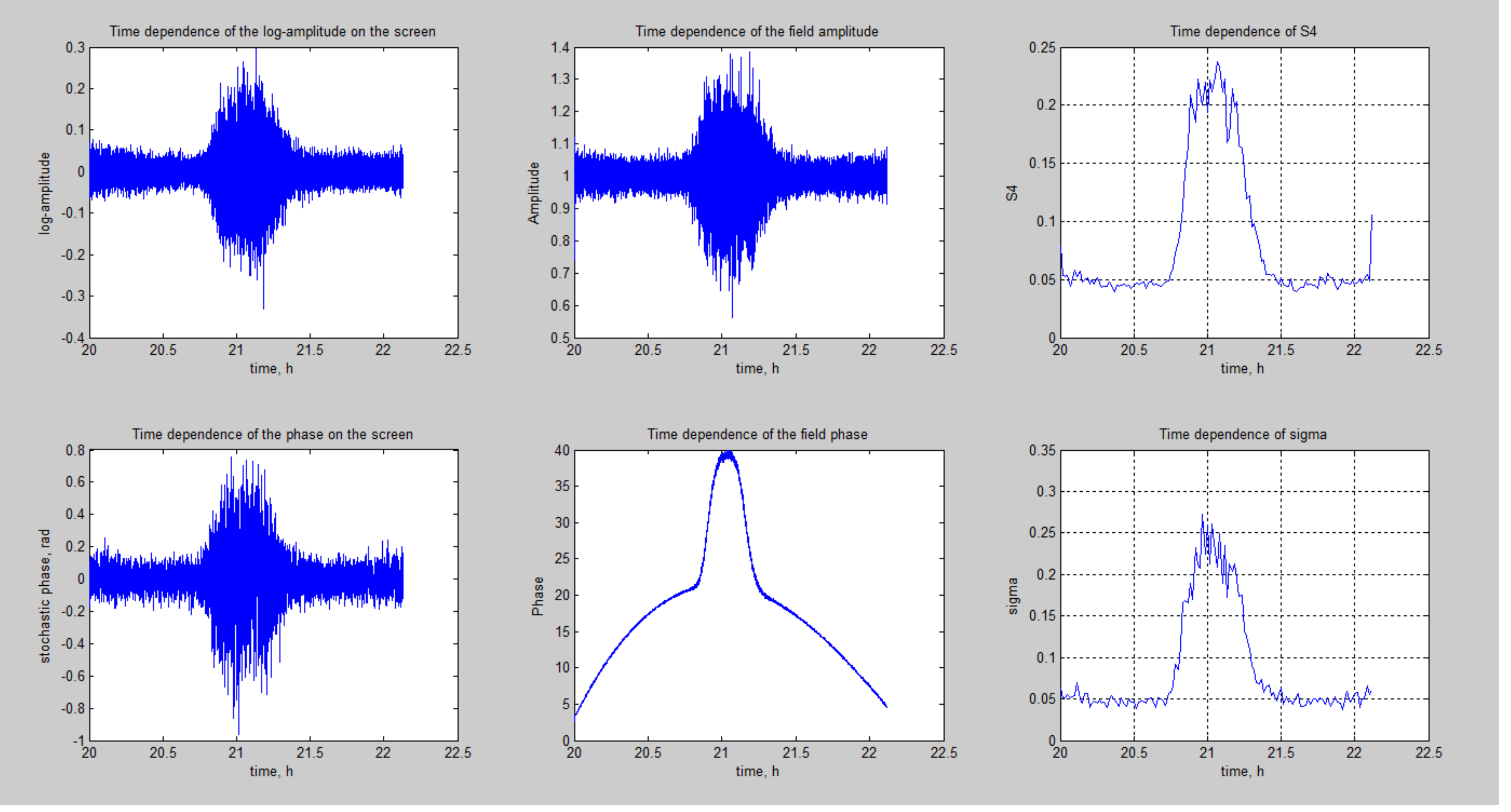
Modeling

Zernov, N. N., V. E. Gherm, and H. J. Strangeways (2009), On the effects of scintillation of low-latitude bubbles on transionospheric paths of propagation, *Radio Sci.*, 44, RS0A14, doi:10.1029/2008RS004074.

A screenshot of the GUI of the program for the simulation of the effects of equatorial plasma bubbles on GPS



Example of the simulation results



3. The dual frequency extension of the HSPM

- To compensate for the background ionosphere in range-finding using satellite navigation systems, a linear combination of signals that travel nearly the same path at the same time but differing frequencies is used such as the L1 and L2 signals broadcast by GPS.

- The state-of-the-art techniques to decode the encrypted GPS L2 signal rely on cross-correlation of the signals received at L1 and L2.

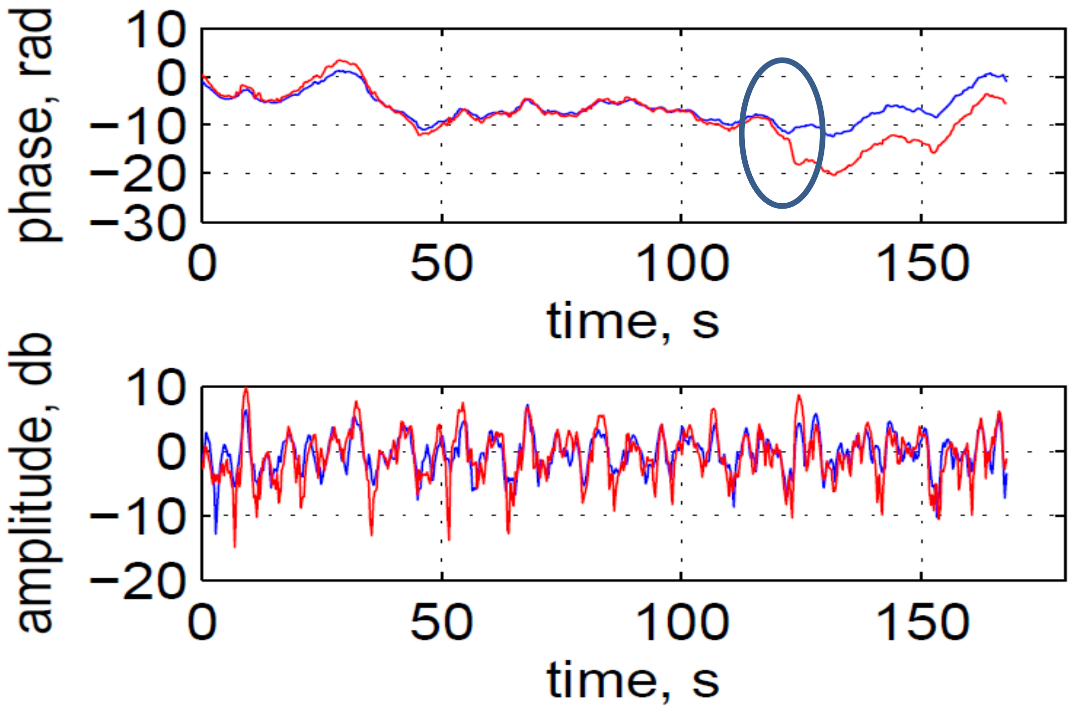
- In the future both GPS and the new Galileo satellite system will broadcast 3 frequencies enabling more advanced 3 frequency correction systems.

- Thus it is important to determine how well these signals are correlated. Lack of correlation will also introduce range error in dual frequency correction using L1 and L2 even in the case of perfect decoding.

• The simulator program has been modified to generate **two realizations of random screens** below the ionosphere, corresponding to the two frequencies.

• The random fields on the screens are generated taking into account the **auto- and cross-correlation functions of phase and log-amplitude at the two frequencies**.

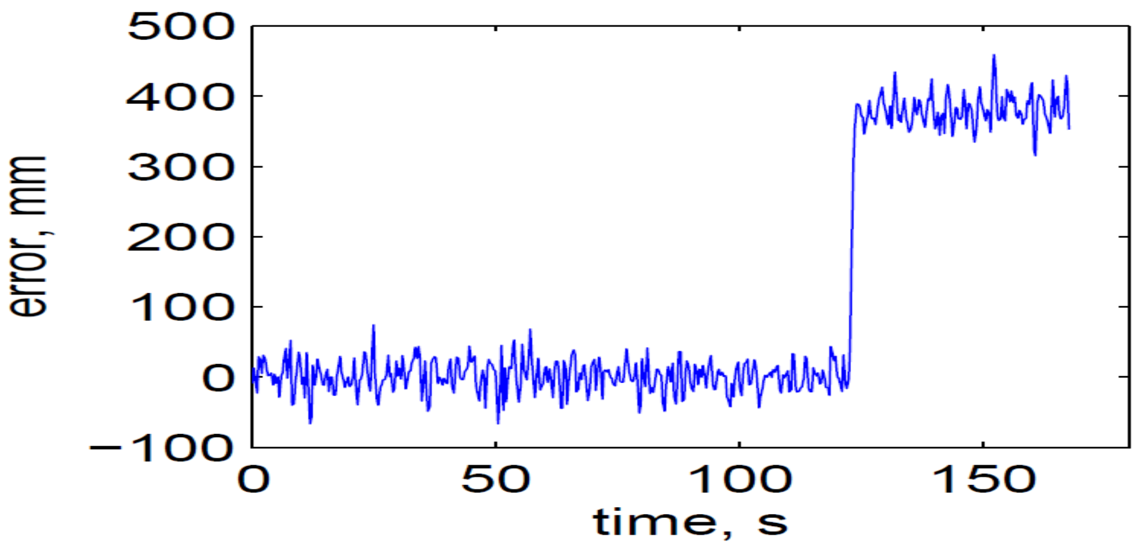
• Then the fields of the both frequencies are simulated on the ground. This thus provides a method of simulating the **stochastic fields at different frequencies, which are properly correlated**.



Simultaneous random realizations of amplitude and phase for two frequencies

blue – L1 (1575.42 MHz)

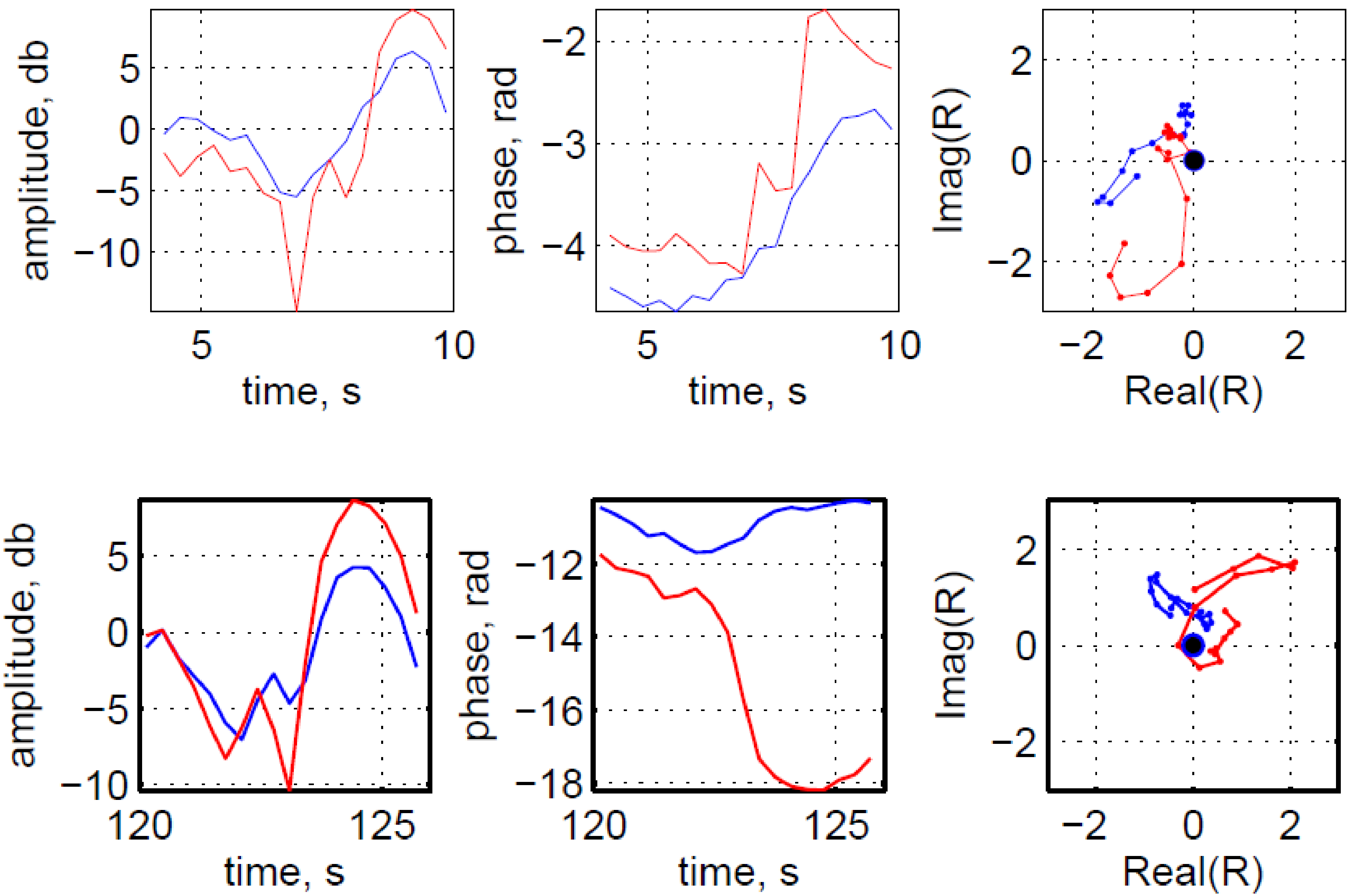
red – L2 (1227.60 MHz)

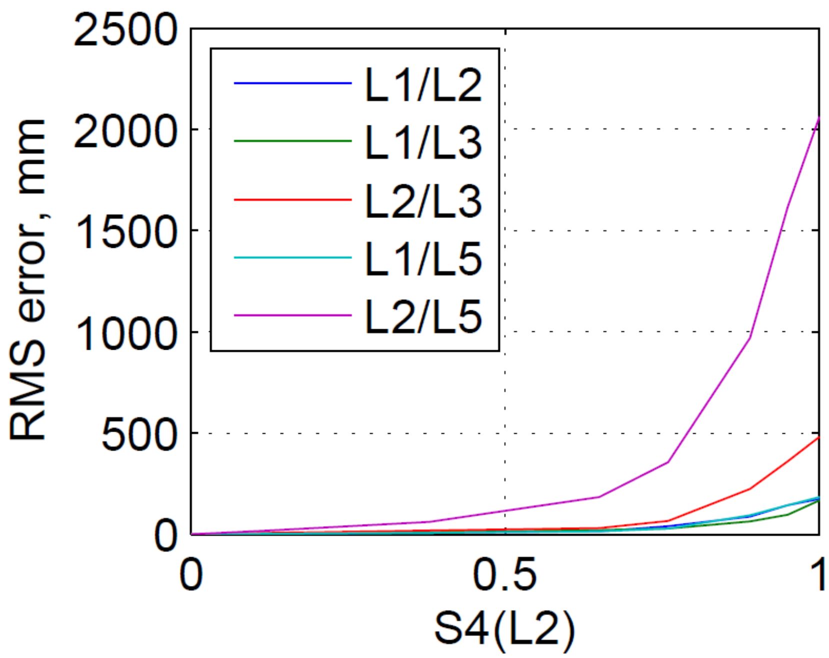


Instantaneous error of dual frequency range finding

$$r = \frac{f_1^2 \left(\varphi_1 / k_1\right) - f_2^2 \left(\varphi_2 / k_2\right)}{f_1^2 - f_2^2}$$

Fragments of realizations of phase, log amplitude and complex amplitude of the field (phasor) without cycle slip (up) and with cycle slip (down)



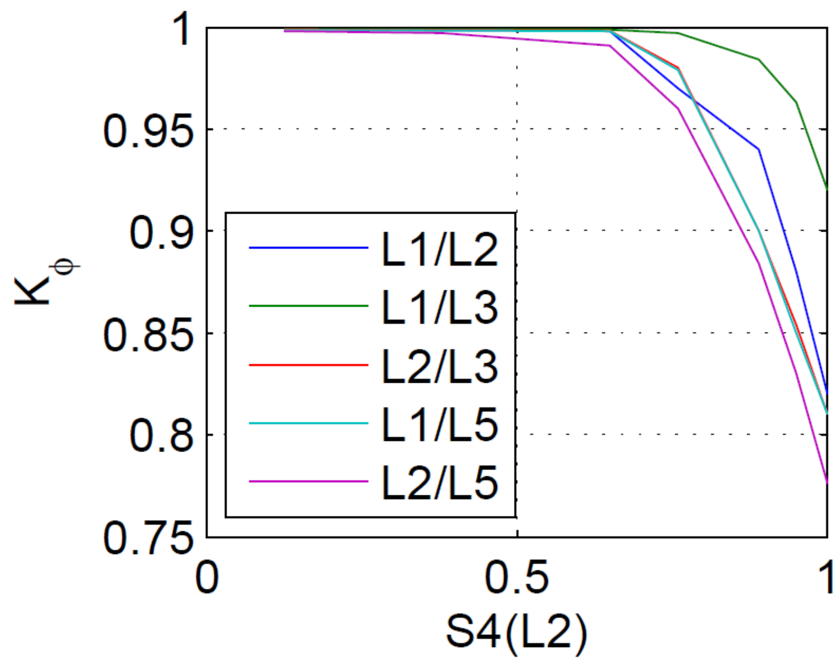


Dependence of RMS error of dual frequency range finding on the scintillation index S4(L2) for different pairs of frequencies

1575.42 (L1), 1227.60 (L2), 1381.05 (L3), 1176.45 (L5)

Dependence of cross-correlation coefficient of phases for different pairs of frequencies on the scintillation index S4(L2)

$$K_\varphi = \frac{\langle \varphi_1 \varphi_2 \rangle}{\left(\langle \varphi_1^2 \rangle \langle \varphi_2^2 \rangle\right)^{1/2}}$$



4. Demonstration of simulating programs