The Abdus Salam International Centre for Theoretical Physics

2333-15

Workshop on Science Applications of GNSS in Developing Countries (11-27 April), followed by the: Seminar on Development and Use of the Ionospheric NeQuick Model (30 April-1 May)

11 April - 1 May, 2012

Scintillation Modeling

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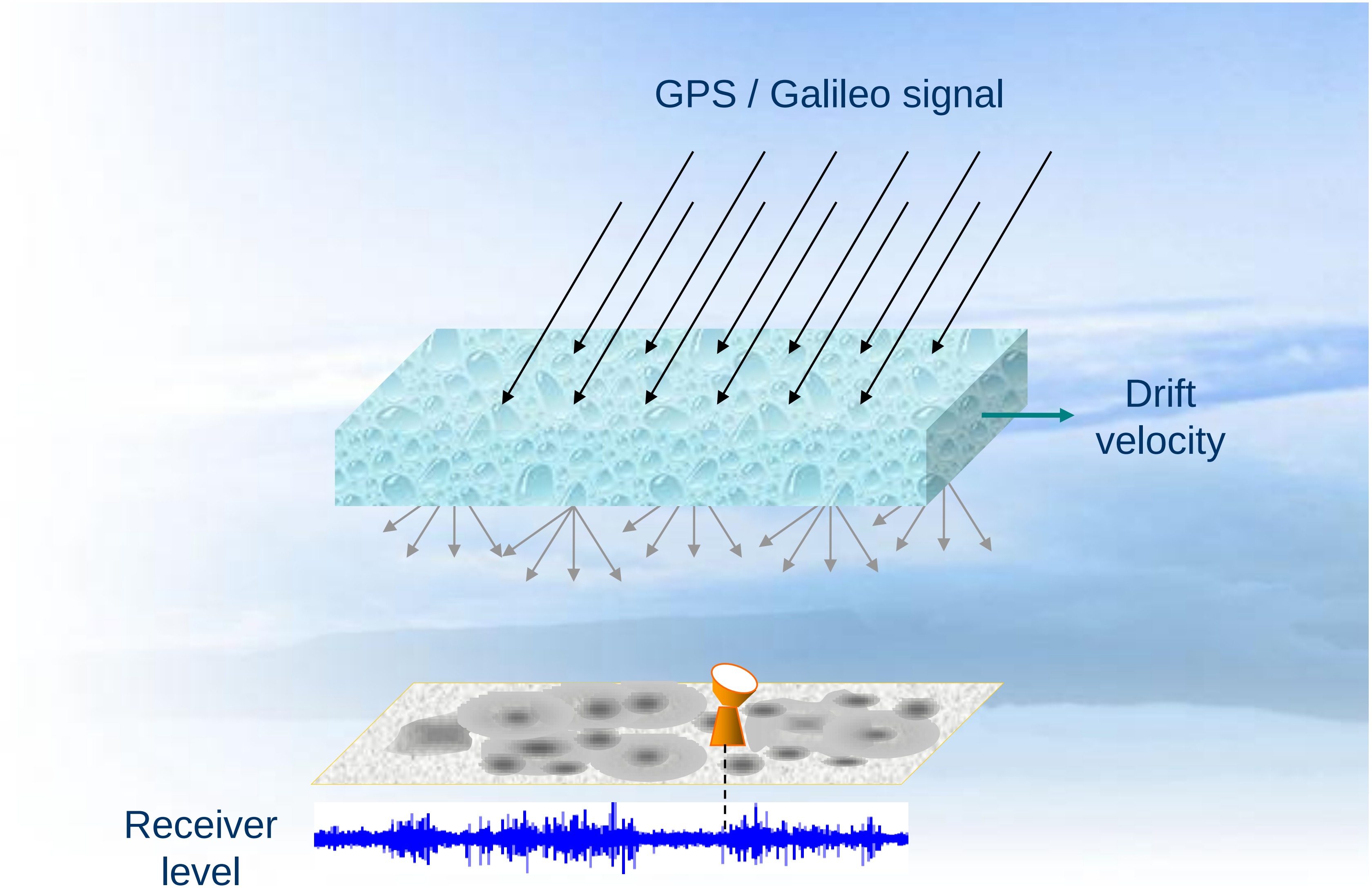
Low Latitudes Ionospheric Scintillation

Y. Béniguel, P. Hamel
IEEA, Paris, France

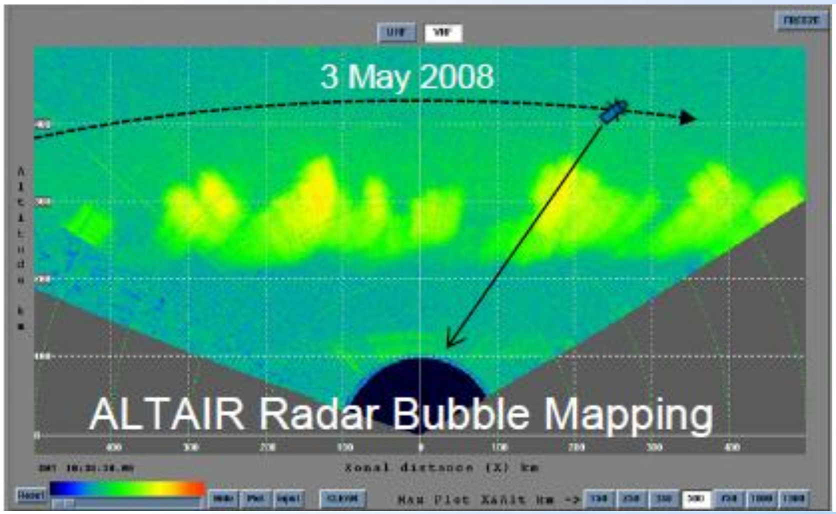
Outline

- Physical mechanism
- Observations
- Modelling & Comparisons
- Scattering function calculation (SAR observations)
- Conclusion

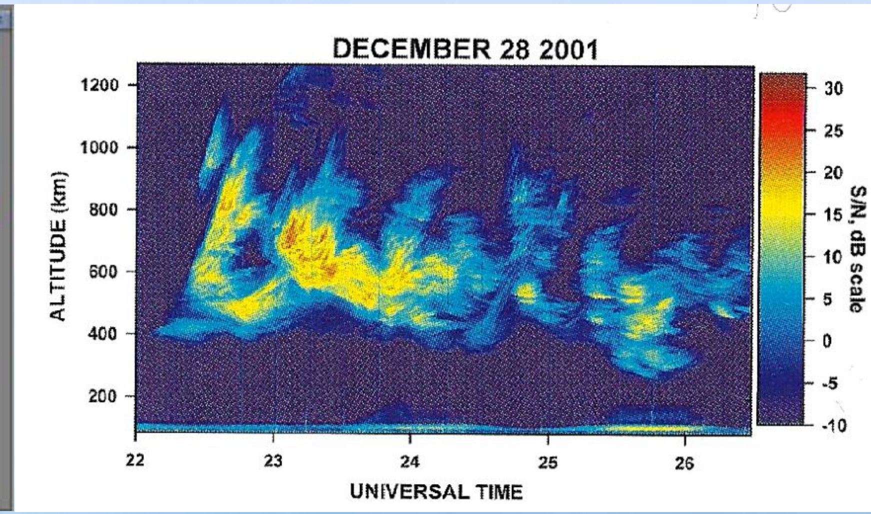
Physical Mechanism



Medium Radar Observations



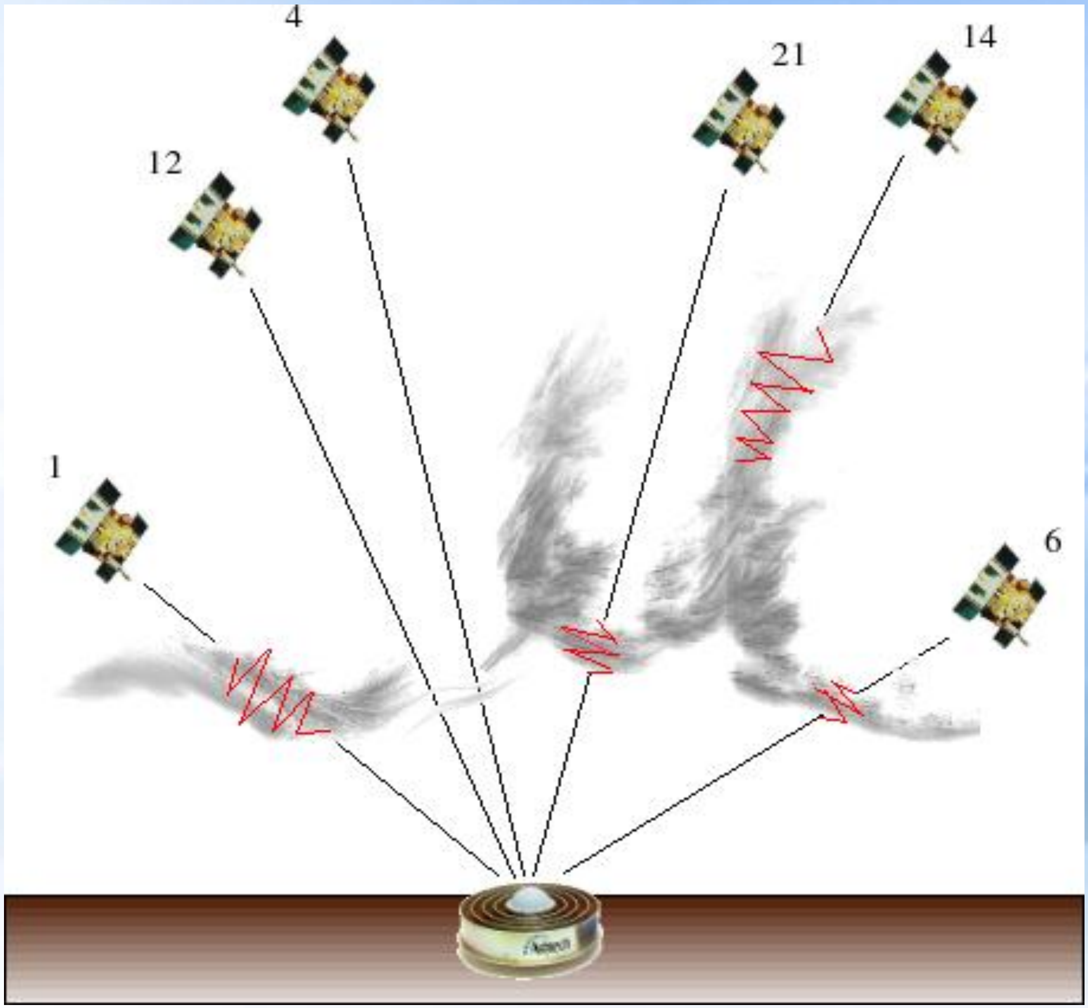
Observations at Kwajalen Islands
Courtesy K. Groves, AFRL



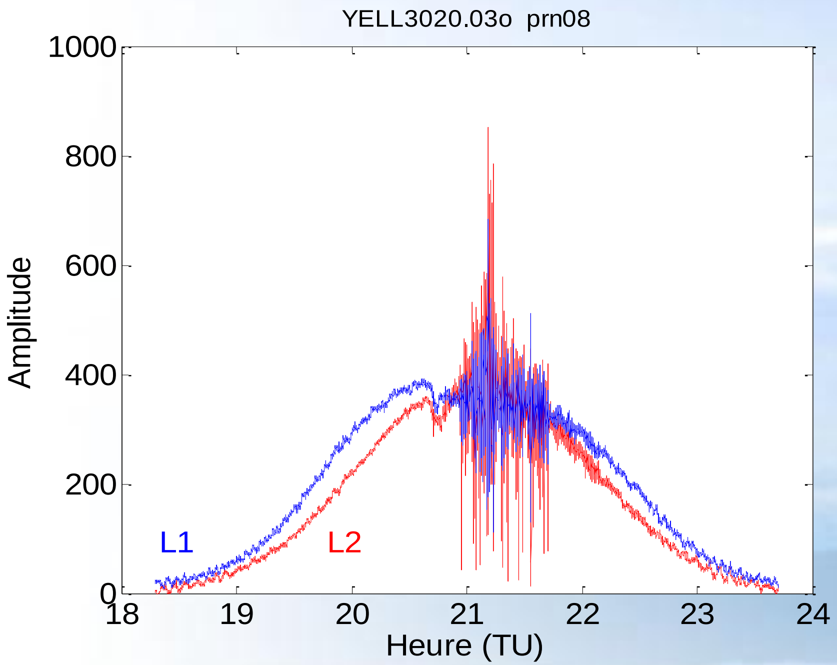
Observations in Brazil
Courtesy E. de Paula, INPE

The vertical extent may reach hundreds of kilometers

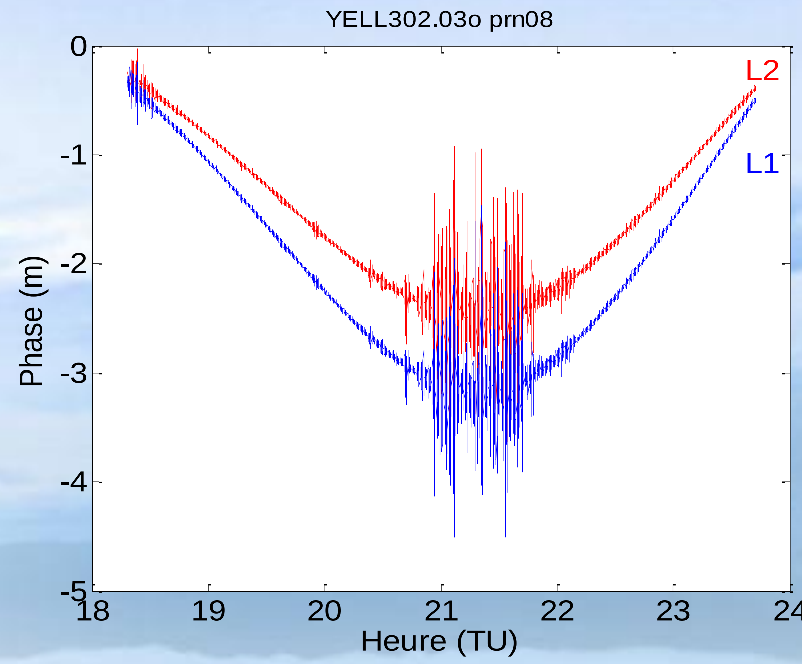
Medium Radar Observations



Signal at receiver level (Measurements)



Intensity



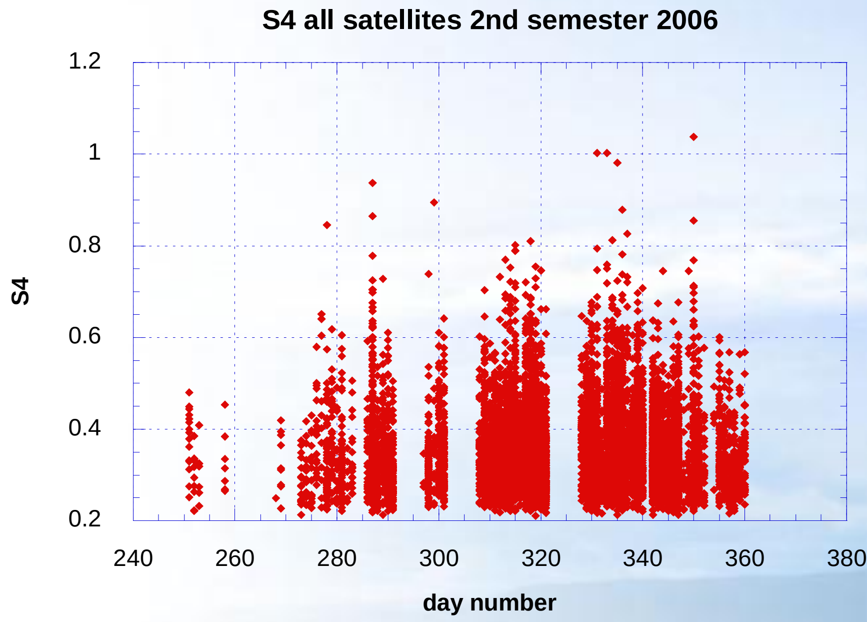
Phase

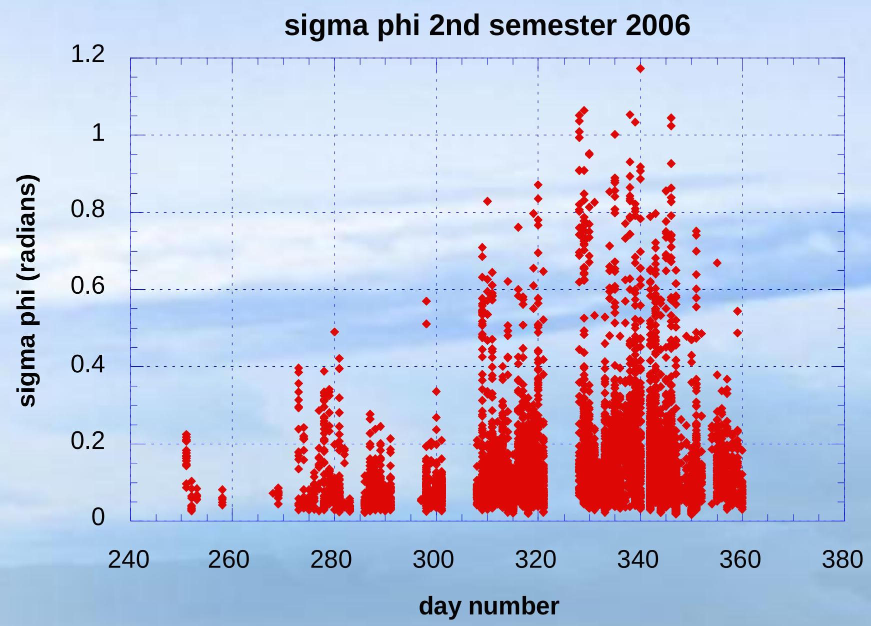
Observations
at Low Latitudes

Béniguel Y., J-P Adam, N. Jakowski, T. Noack, V. Wilken, J-J Valette, M. Cueto, A. Bourdillon, P. Lassudrie-Duchesne, B. Arbesser-Rastburg, Analysis of scintillation recorded during the PRIS measurement campaign, Radio Sci., Vol 44, (2009), doi:10.1029/2008RS004090

Prieto Cerdeira R., Y. Béniguel, "The MONITOR project: architecture, data and products", Ionospheric Effects Symposium, Alexandria VA, May 2011

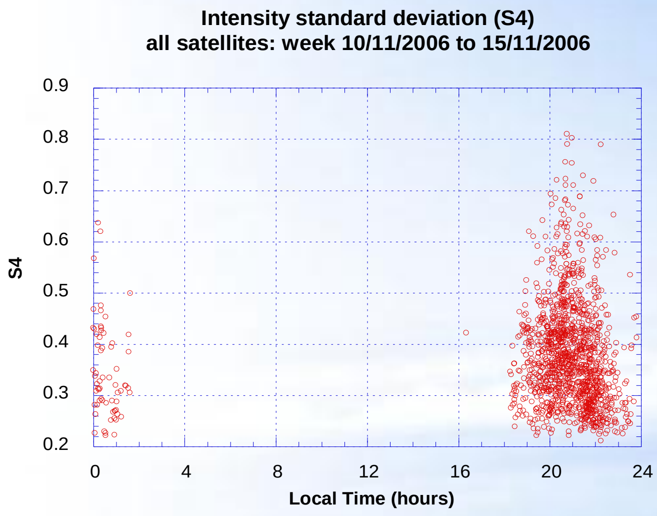
Seasonal Dependency

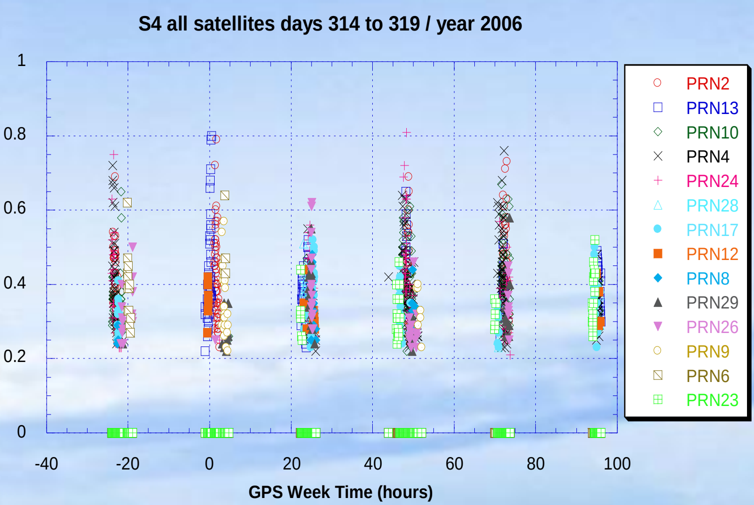


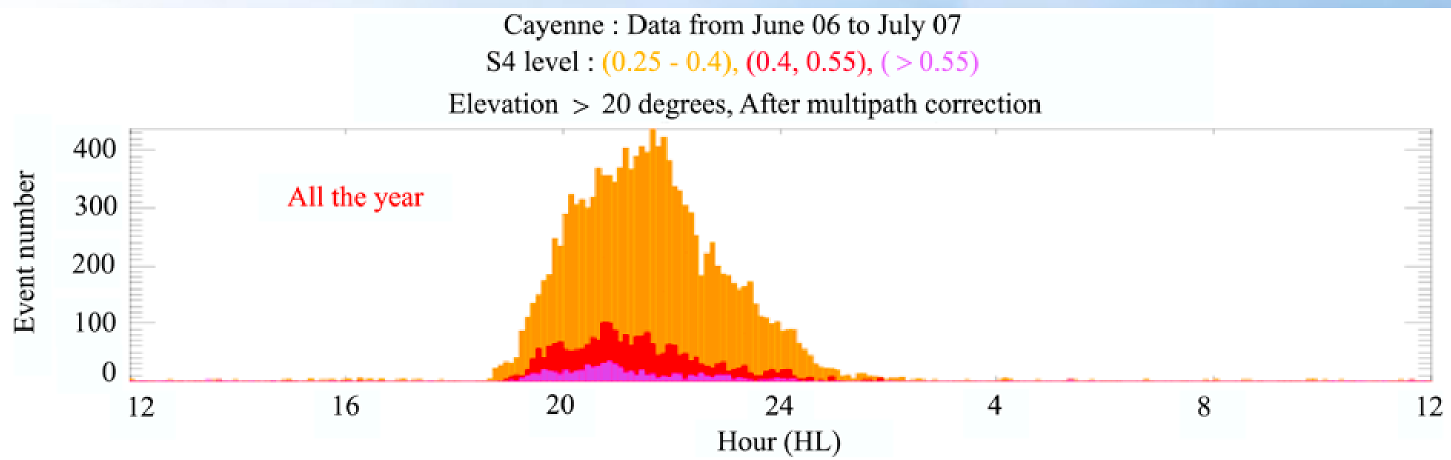


Measurements in Guiana (4 N)

One week of measurements close to the equator

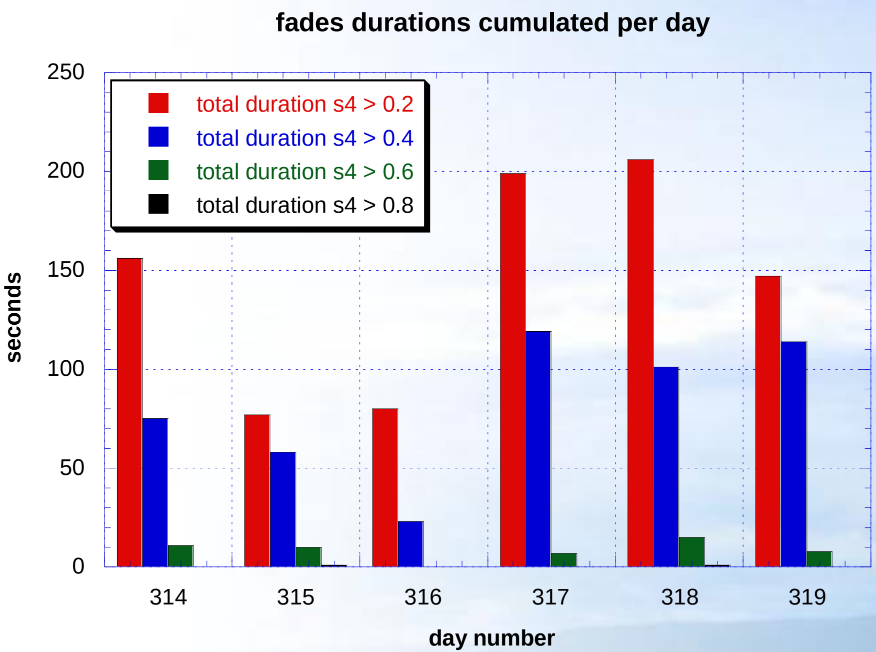


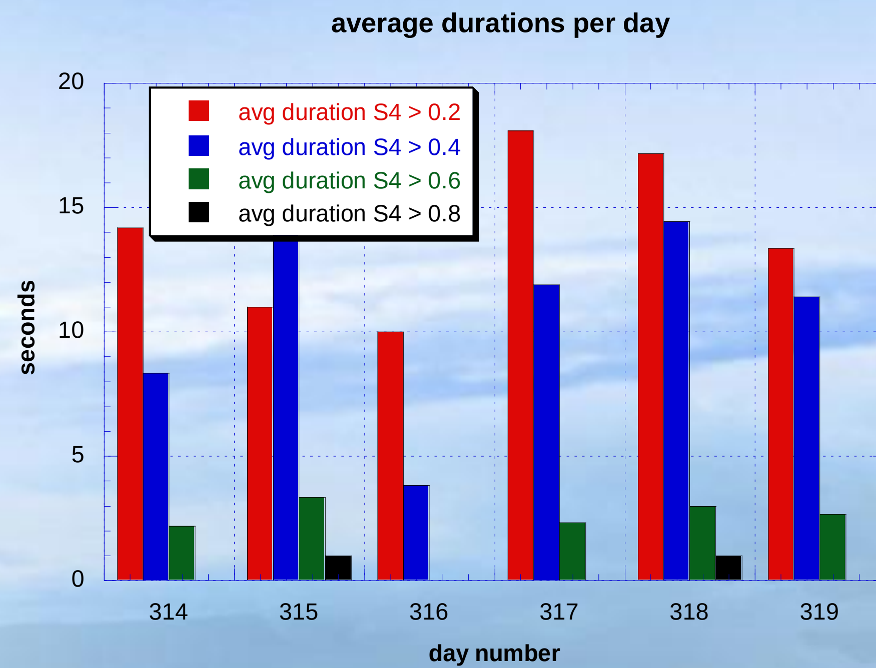




Local time : post sunset hours (CLS measurements, PRIS Campaign)

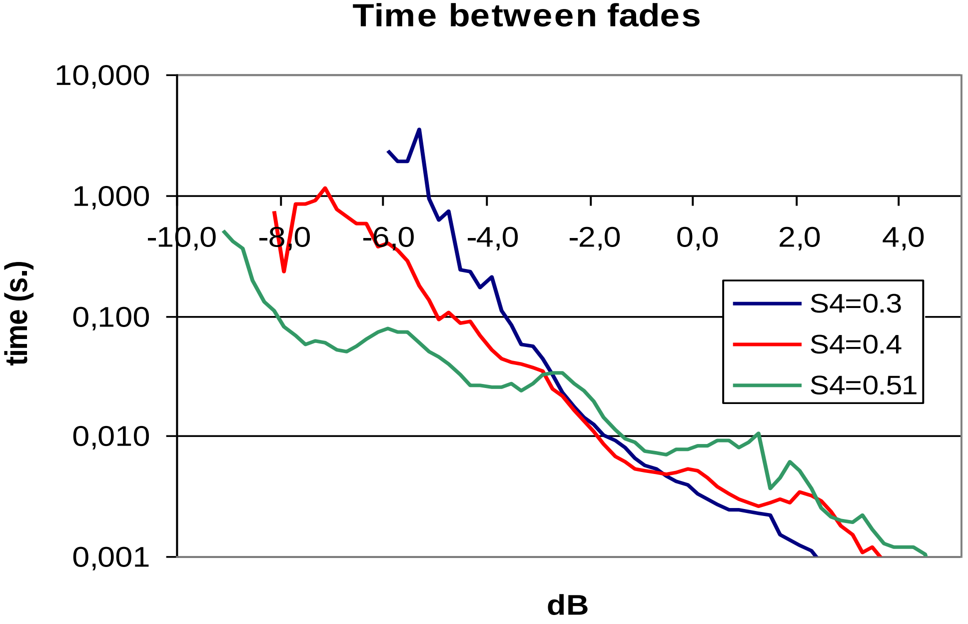
Fades Statistics

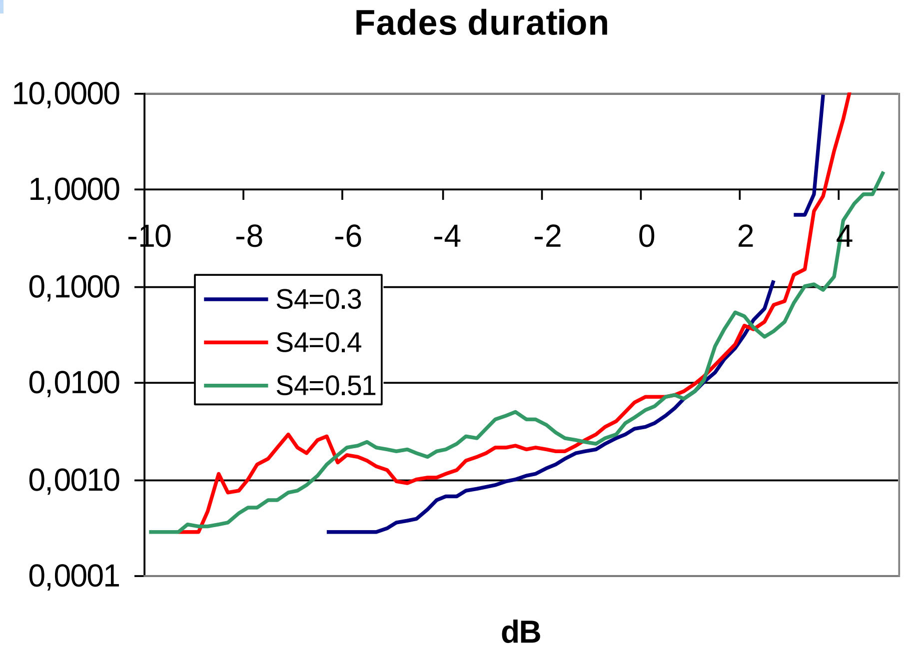




One week of measurements / november 2006

Fades Statistics

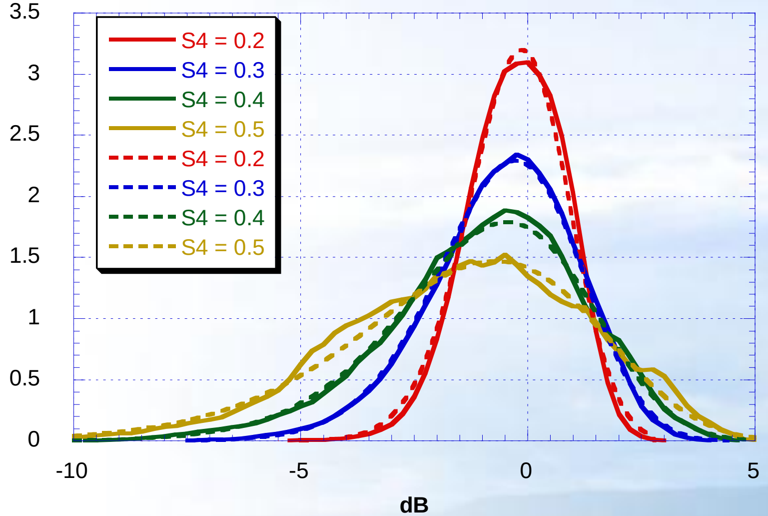




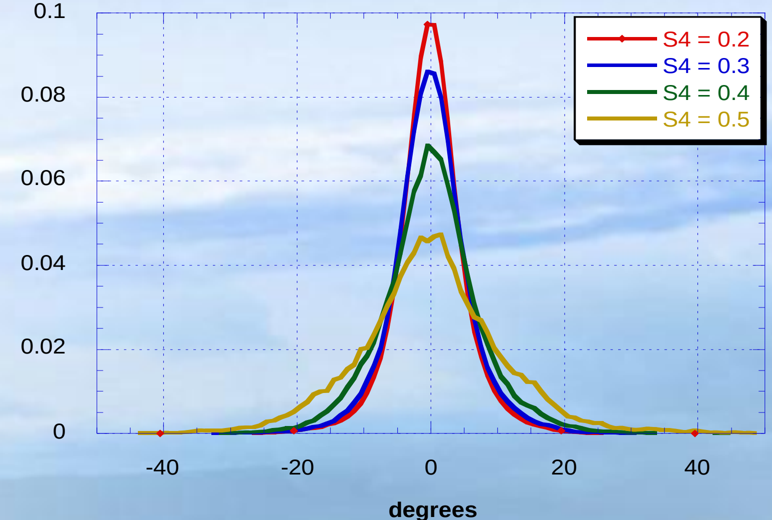
Intensity and Phase Distributions

one week of measurements

probability of intensity
Theoretical (Nakagami distribution) vs Measurements



phase probability (measurements)



$$p(A) = \frac{2\, m^m\, A^{2m-1}}{\Gamma(m)} \exp(-m A^2)$$

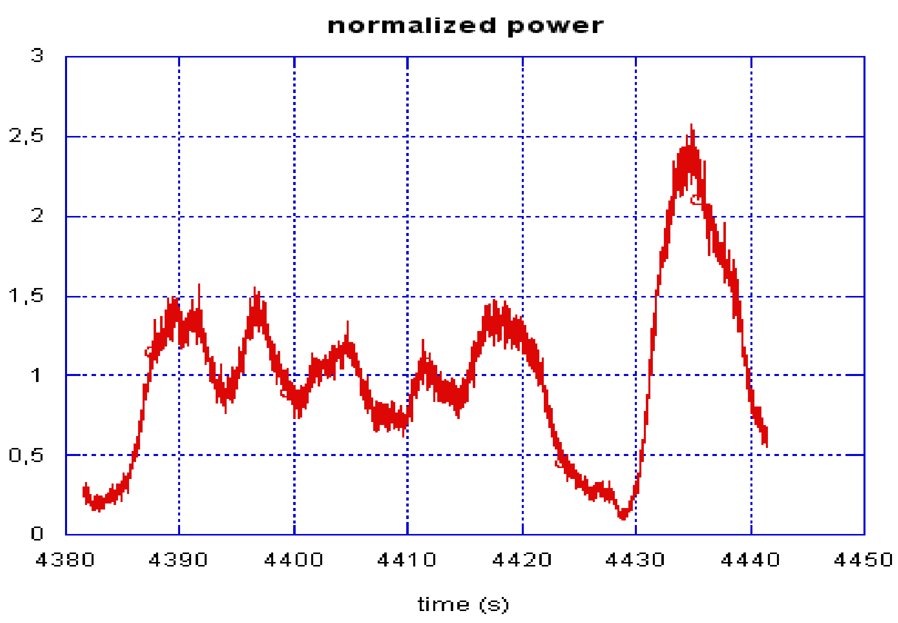
Gaussian distribution

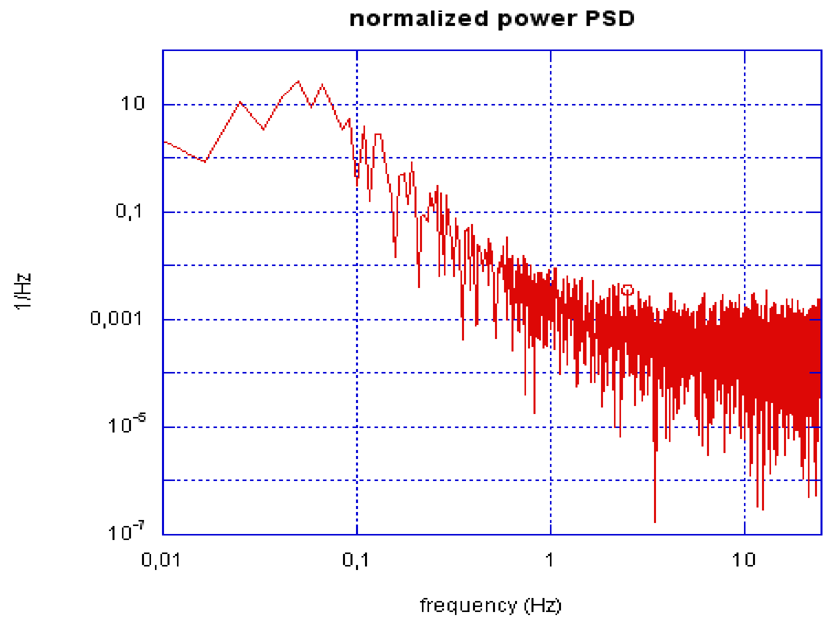
with $m = 1 / S_4^2$

Spectrum Analysis

One Sample : Intensity

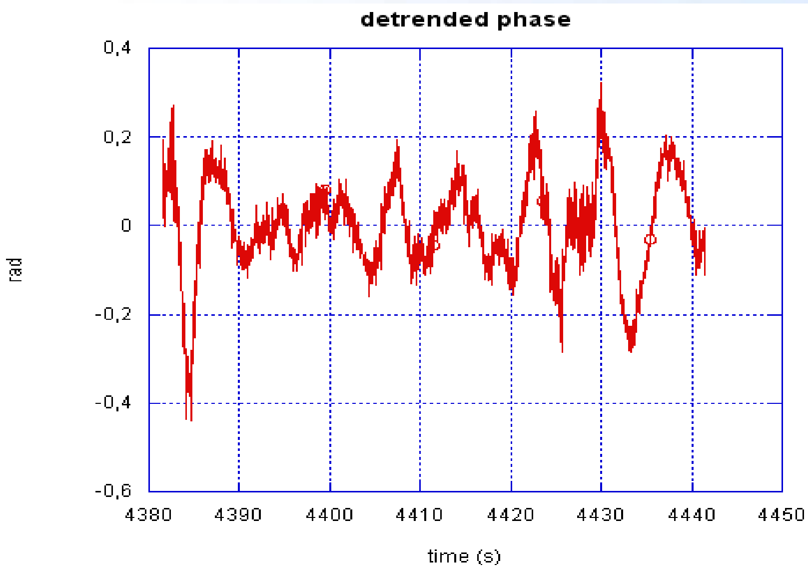
Sample characteristics : S4 = 0.51, sigma phi = 0.11

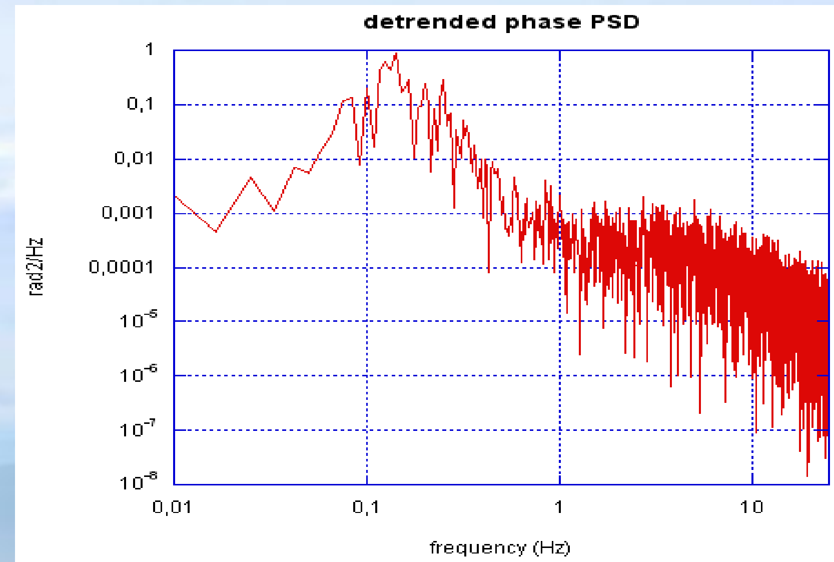




One Sample : Phase

Sample characteristics : S4 = 0.51, sigma phi = 0.11



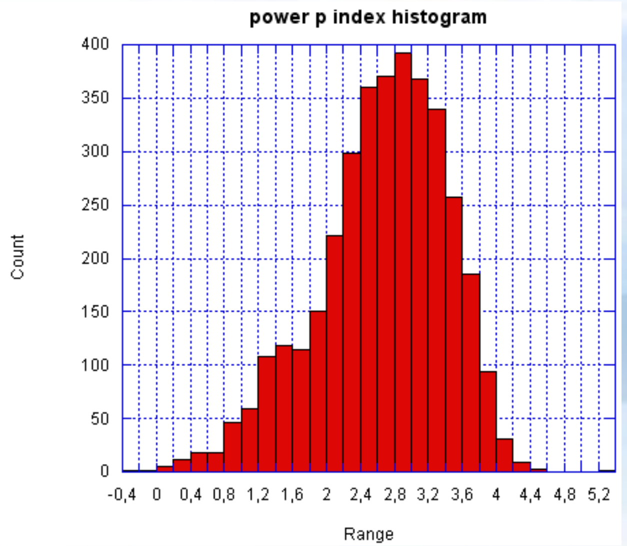


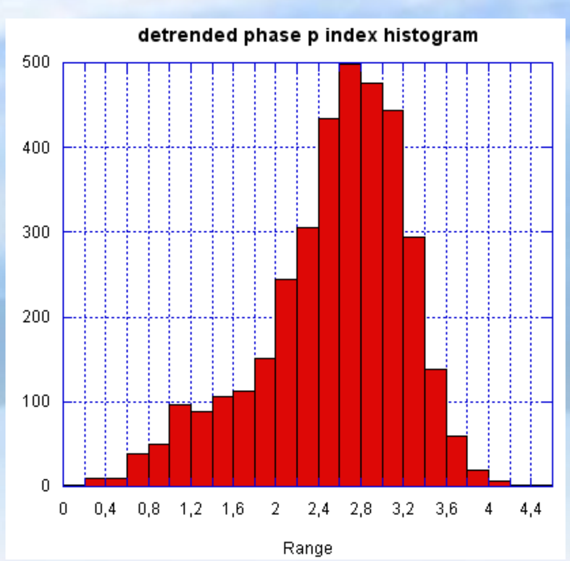
Spectrum Parameters

5 days RINEX files considered in the analysis

S4 > 0.2 & sigma phi < 2 (filter convergence)

2 parameters to define the spectrum : T (1 Hz value) & p

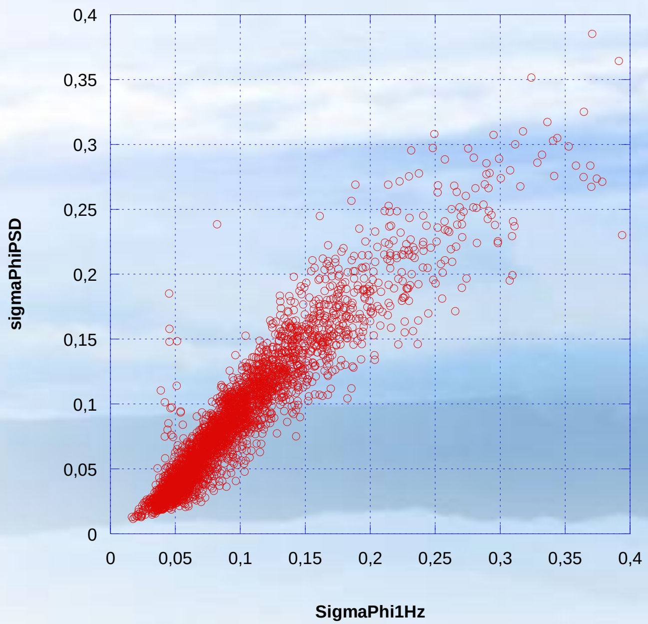




Phase Variance : Time Domain vs Frequency Domain

$$\sigma phi^2 = 2\int_{fc}^{\infty} PSD(f)df = 2\int_{fc}^{\infty} Tf^{-p}df = 2T\left[\frac{f^{-p+1}}{-p+1}\right]_{fc}^{\infty} = \frac{2T}{(p-1)fc^{p-1}} \qquad (\text{if } p>1)$$

Slope set to 2.8

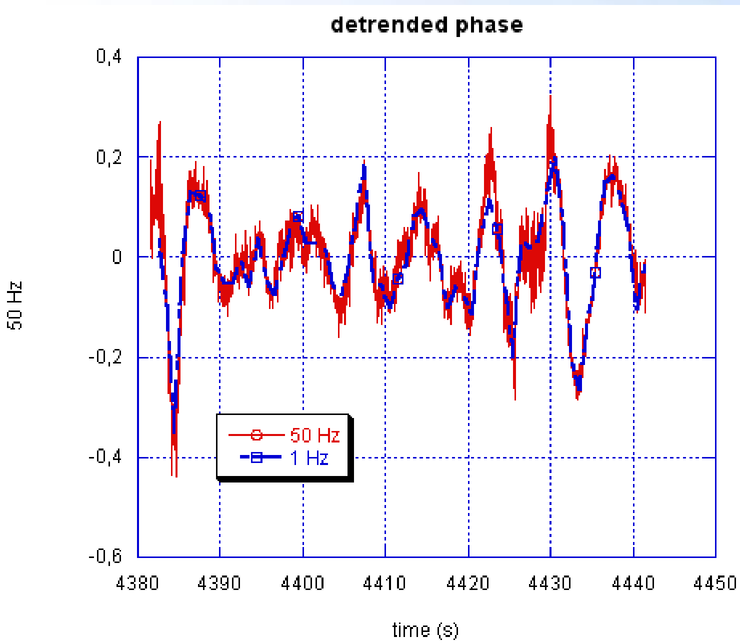


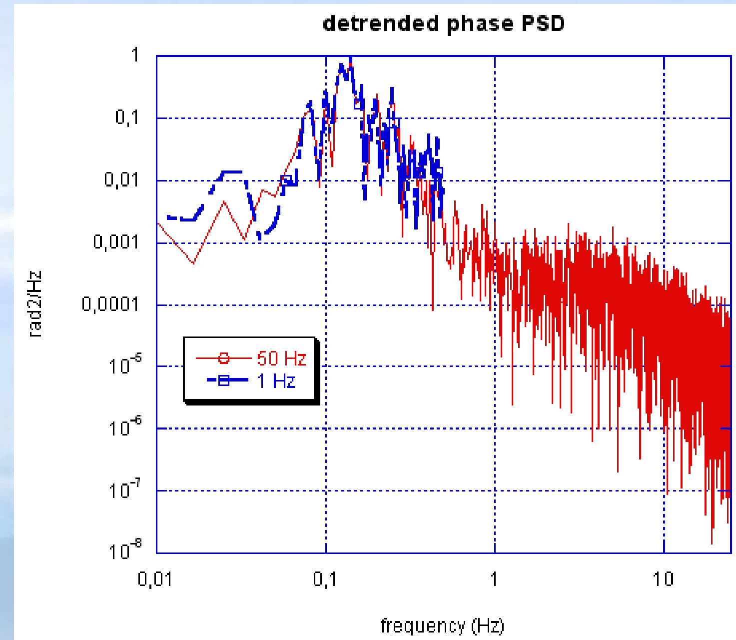
linear relationship

Under Sampling

1 Hz instead of 50 Hz

Cut off frequency → 0.5 Hz

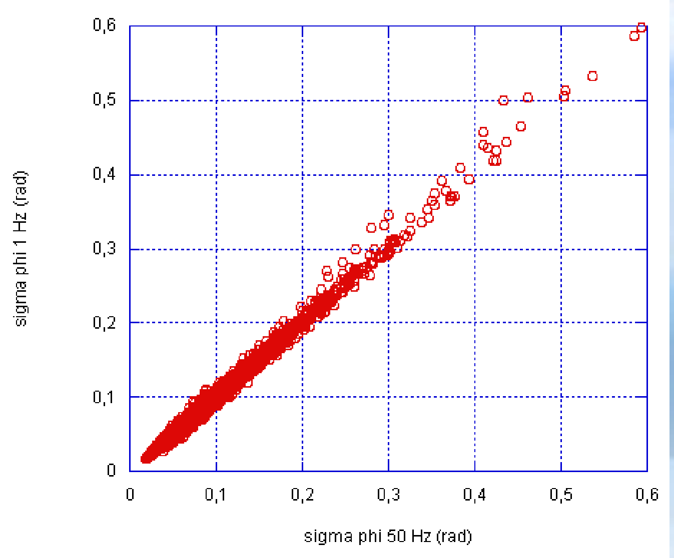




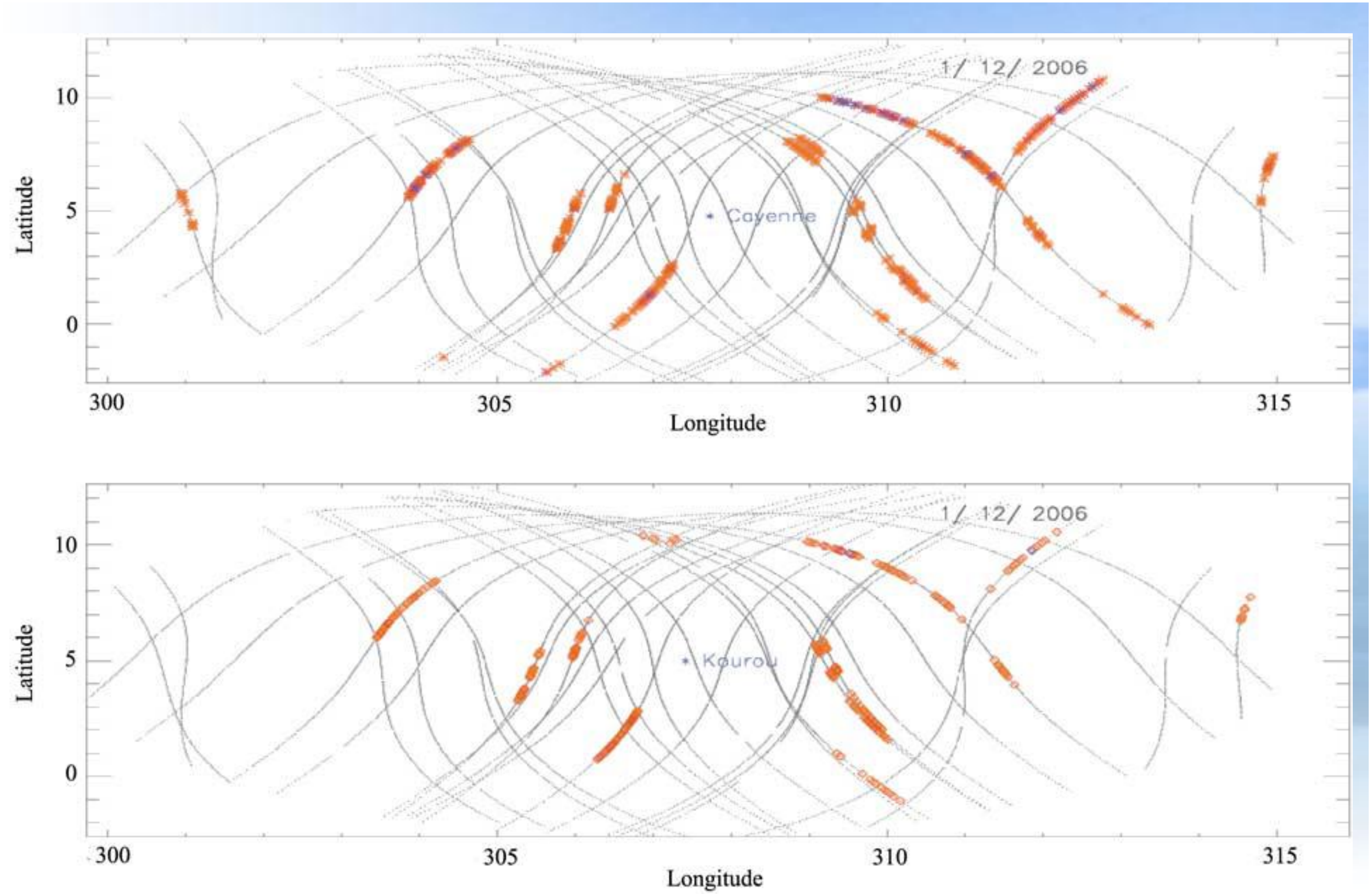
$$\sigma phi^2 = 2T \int_{0.1Hz}^{\infty} f^{-p} df \cong 2T \int_{0.1Hz}^{25Hz} f^{-p} df = 2T \left(\int_{0.1Hz}^{0.5Hz} f^{-p} df + \int_{0.5Hz}^{25Hz} f^{-p} df \right)$$

$$\int_{0.1Hz}^{0.5Hz} f^{-2.8} df = 33$$

$$\int_{0.5Hz}^{25Hz} f^{-2.8} df = 1.9$$



Scintillation indices at 50 Hz (top panel) vs 1 Hz (IGS data: bottom panel)



Modelling

Global Ionospheric Scintillation Model (GISM)

http://www.ieea.fr/en/gism-web-interface.html

Field received at ground level

Solution of the parabolic equation

Field Propagation Equation

Solution of the parabolic equation

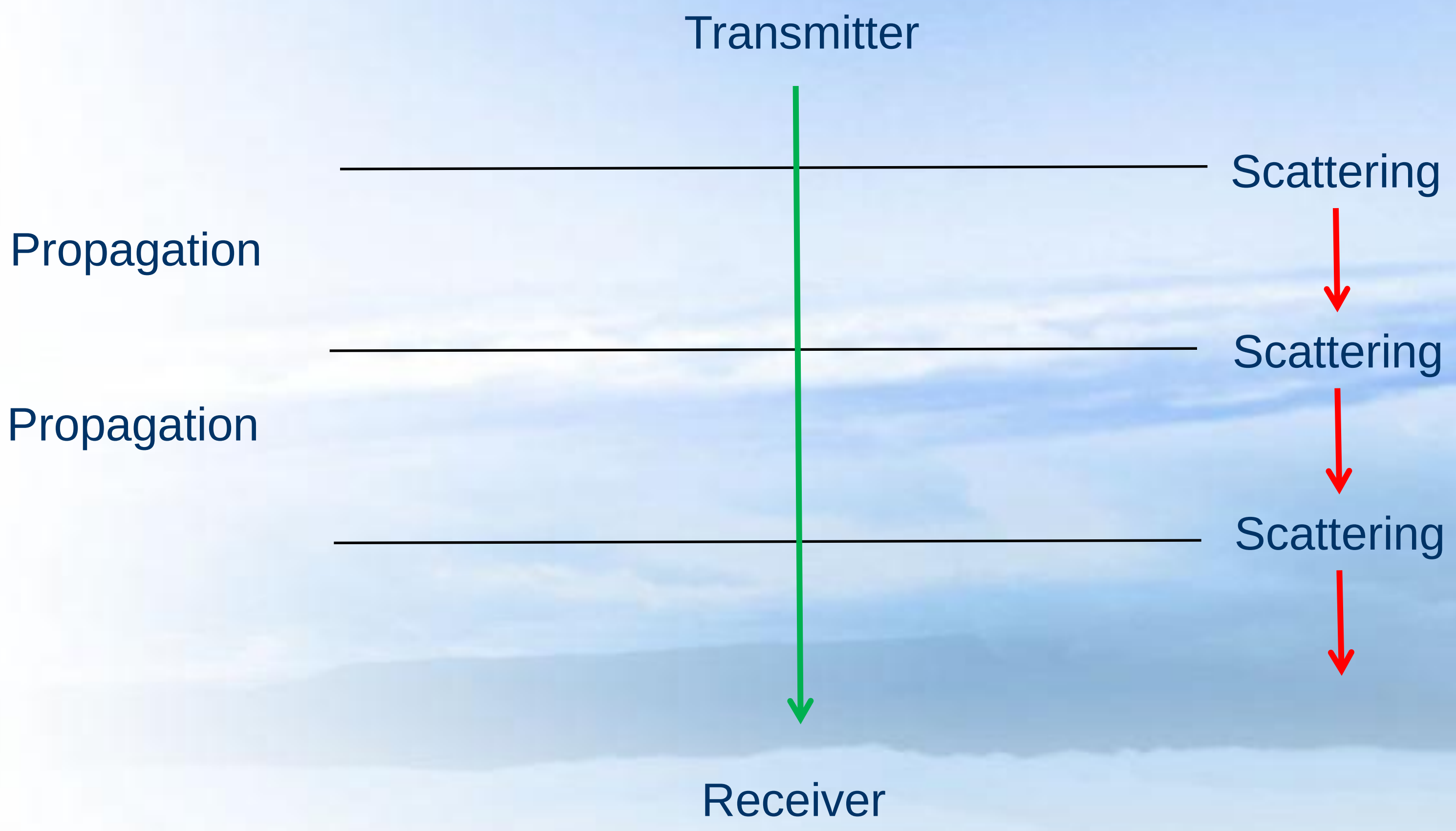
$$2jk\frac{\partial}{\partial z}\langle U(r)\rangle + \nabla_t^2\langle U(r)\rangle + k^2\langle \varepsilon(r)\, U(r)\rangle = 0$$

$$2jk\frac{\partial}{\partial z}\langle U(r)\rangle + \nabla_t^2\langle U(r)\rangle + j\,\frac{k^3}{4}\,A(0)\,\langle U(r)\rangle = 0$$

Using the phase index autocorrelation function

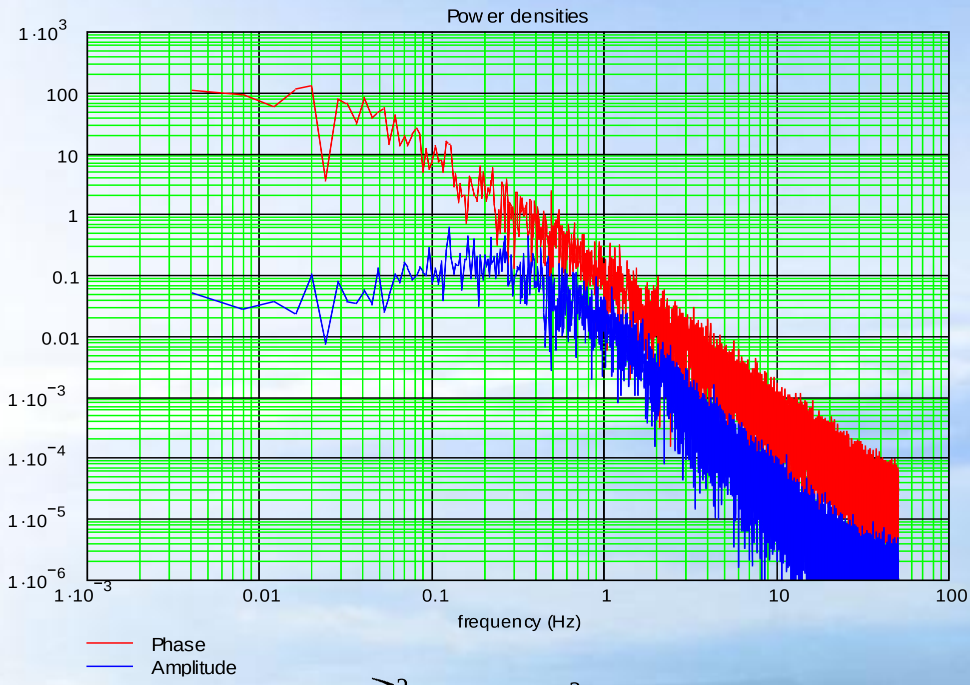
$$B(z,\rho) = \langle \varepsilon(\rho_1)\;\varepsilon(\rho_2)\rangle \qquad A(\rho) = \int B(z,\rho)\,dz$$

Phase Screen Technique*



* Béniguel Y., P. Hamel, “A Global Ionosphere Scintillation Propagation Model for Equatorial Regions”, Journal of Space Weather Space Climate, 1, (2011), doi: 10.1051/swsc/2011004

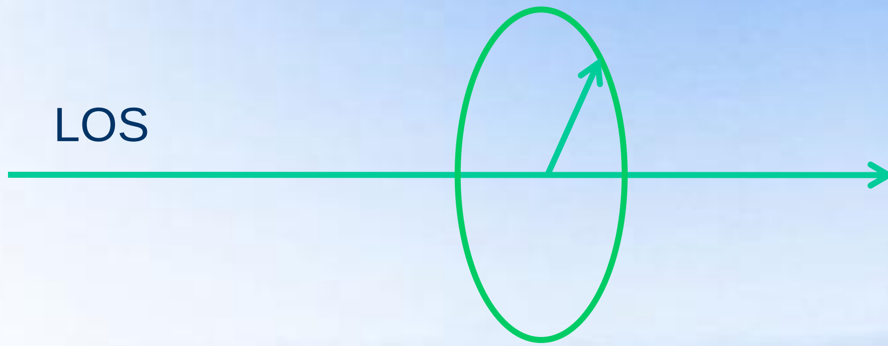
Medium's Characterisation*



$$\gamma_{\Phi}(K) = \frac{(\lambda\, r_e)^2\, L\, C_S\, \sigma_{Ne}^2}{(K^2 + q_0^2)^{p/2}} = \frac{C_P}{(K^2 + q_0^2)^{p/2}}$$

3 parameters : σ_{Ne} ; q_0 ; p

2D Analysis : Isotropic transverse medium

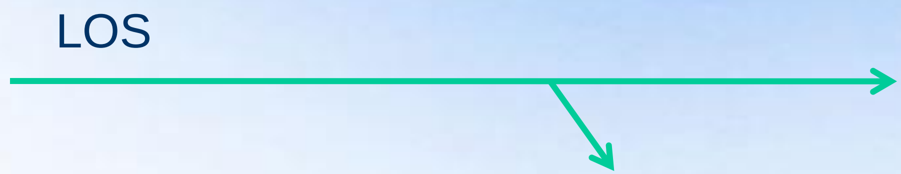


$$B_\Phi(\rho) = \frac{C_P}{(2\pi)^2} \iint \gamma_\Phi(K)\, \exp(-\, j\bar{K}.\bar{\rho})\, dK$$

$$\longrightarrow \quad [B_\Phi(\rho)]_{iso} = \frac{\sigma_\Phi^2}{2^{(p-4)/2}\, \Gamma((p-2)/2)} (\rho q_0)^{((p-2)/2)}\, K_{((p-2)/2)}(\rho q_0)$$

$$\sigma_\Phi^2 = B_\Phi(0) = (\lambda r_e)^2\, L\, L_0\, \sigma_{Ne}^2$$

1D Analysis : Isotropic transverse medium



$$B_{\Phi}(\rho) = \frac{C_p}{2\pi} \int \gamma_{\Phi}(k) \exp(-jk\rho)\, dk$$

$$\left[B_{\Phi}(\rho)\right]_{1D} = \frac{\sigma_{\Phi}^2}{2^{(p-3)/2}\, \Gamma\left((p-1)/2\right)} \left(\rho q_0\right)^{((p-1)/2)} K_{((p-1)/2)}\left(\rho q_0\right)$$

$$\sigma_{\Phi}^2 = B_{\Phi}(0) = \left(\lambda r_e\right)^2 L\, L_0\, \sigma_{Ne}^2$$

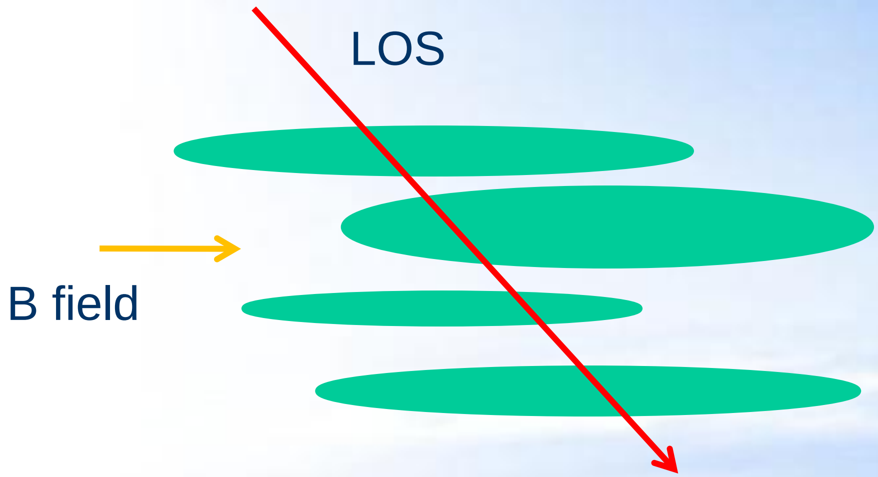
1D vs Isotropic (2D)

$$\left[B_\Phi(\rho)\right]_{iso} = \frac{\sigma_\Phi^2}{2^{(p-4)/2}\,\Gamma\left((p-2)/2\right)} \left(\rho q_0\right)^{((p-2)/2)} K_{((p-2)/2)}\left(\rho q_0\right)$$

$$\left[B_\Phi(\rho)\right]_{1D} = \frac{\sigma_\Phi^2}{2^{(p-3)/2}\,\Gamma\left((p-1)/2\right)} \left(\rho q_0\right)^{((p-1)/2)} K_{((p-1)/2)}\left(\rho q_0\right)$$

Slope p → p - 1

Anisotropic vs Isotropic*



$$\gamma_\Phi(K) = \frac{(\lambda r_e)^2 L\, C_S\, \sigma_{Ne}^2\, a\, b}{(q_\perp^2 + q_0^2)^{p/2}}$$

$$\gamma_\Phi(K) = \frac{a\, b\, C_P}{\left(\left(A K_{x\perp}^2 + B K_{x\perp} q_{y\perp} + C K_{y\perp}^2\right)^2 + q_0^2\right)^{p/2}}$$

Additional geometric factor with respect to the 2D case

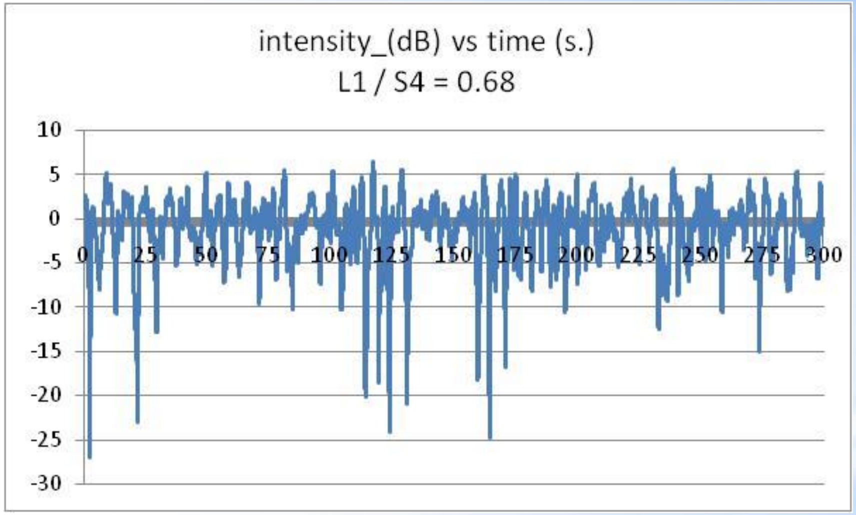
$$G = \frac{a\, b}{(A C - B^2/4)^{1/2}}$$

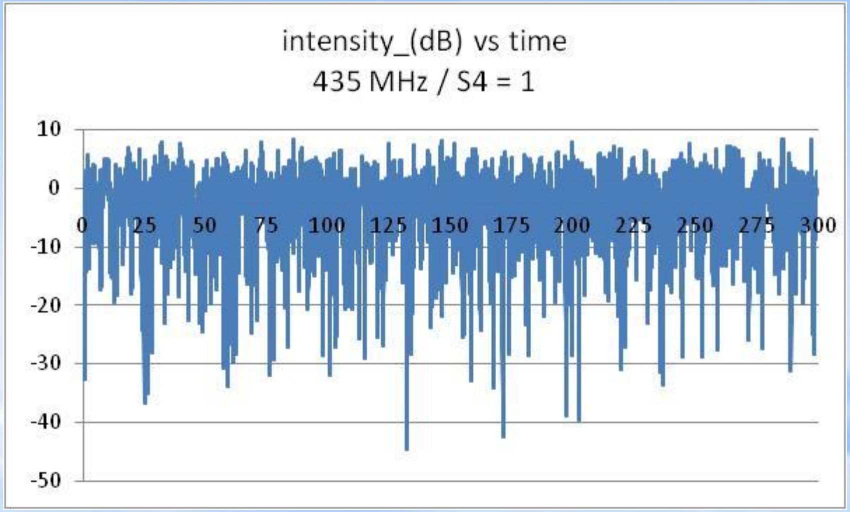
a, b ellipses axes

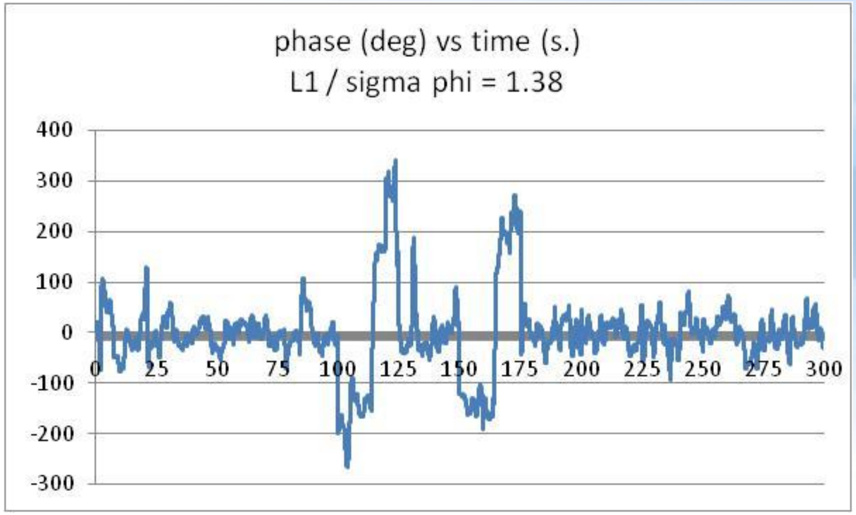
A, B, C trigonometric terms resulting from rotations related to variable changes

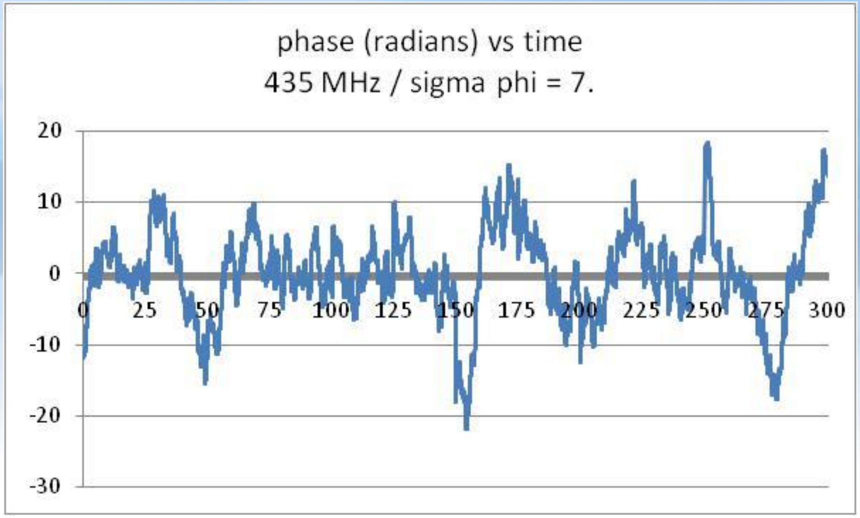
* Rino, Radio Sci. 1979

Signal at receiver level

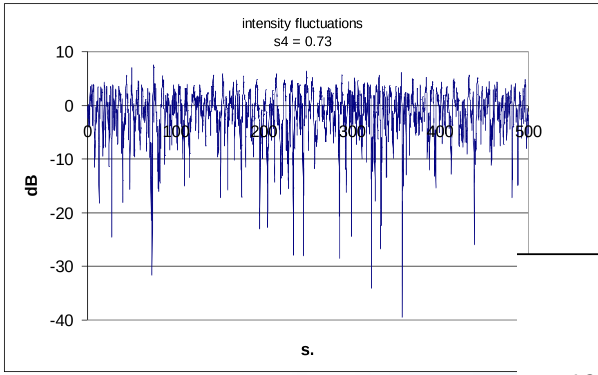




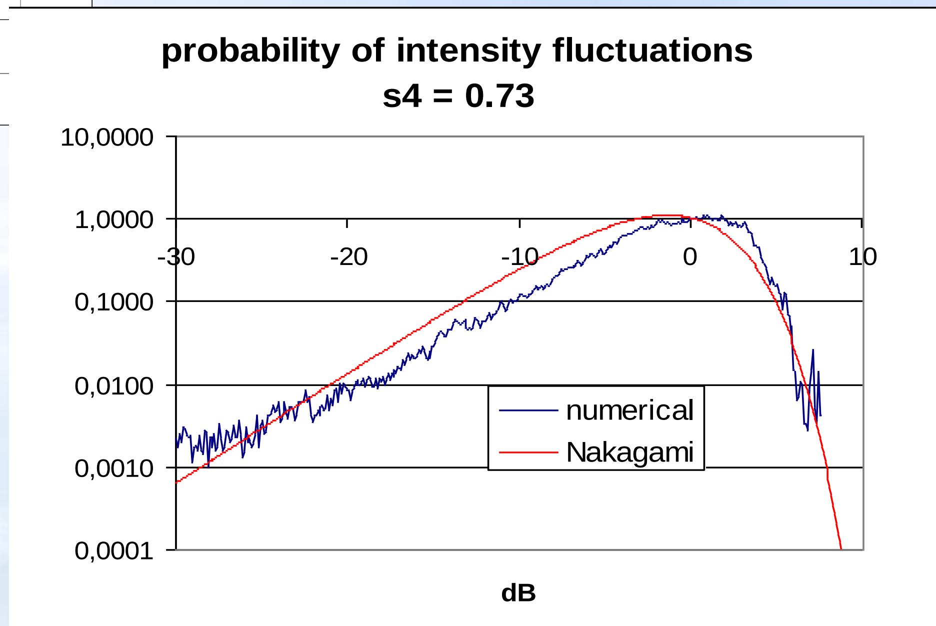




Probability of intensity Modelling



GISM output



Nakagami law

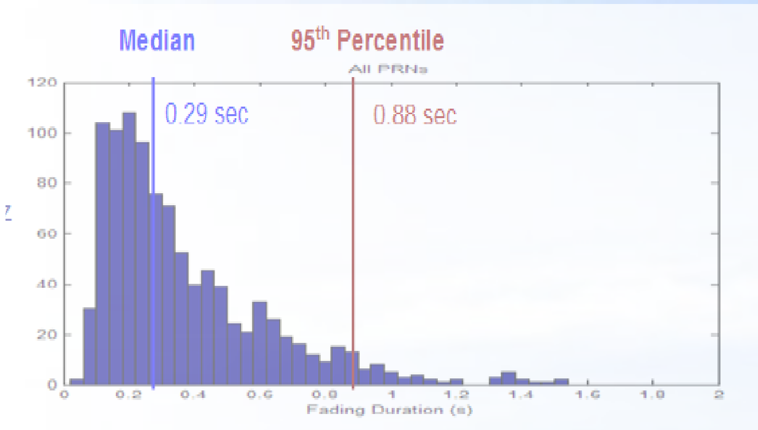
$$p(A) = \frac{2\ m^m\ A^{2m-1}}{\Gamma(m)}\ \exp(-m A^2)$$

with

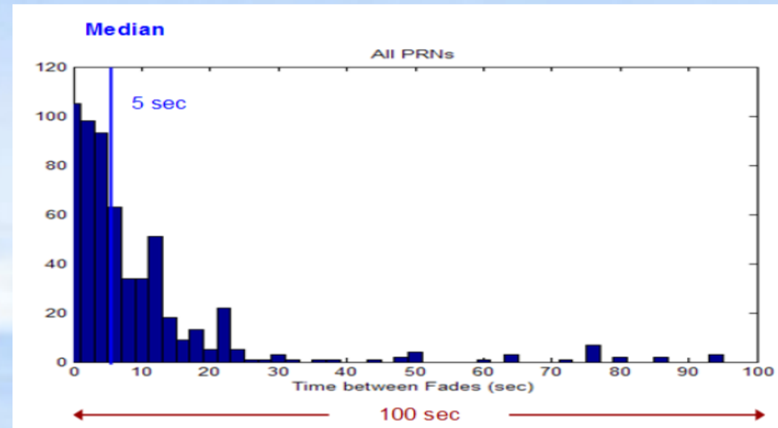
$$m = 1 / S_4^2$$

Fades Statistics

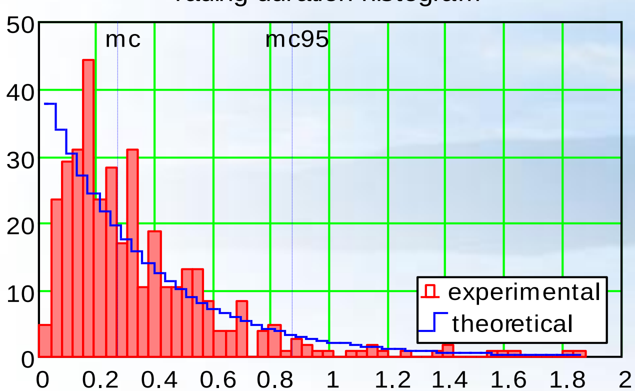
Example of equatorial scintillation in Ascension Island, in solarmax conditions (2001)



•*Real data*

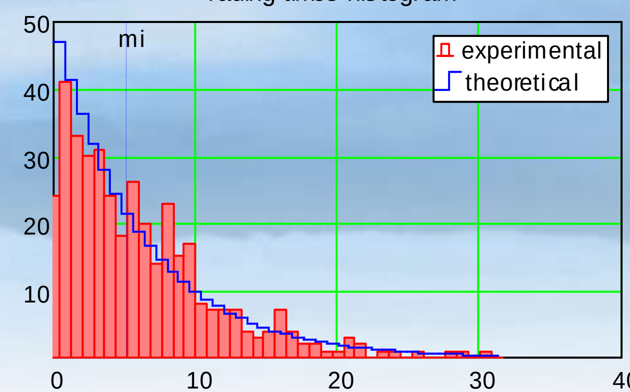


fading duration histogram



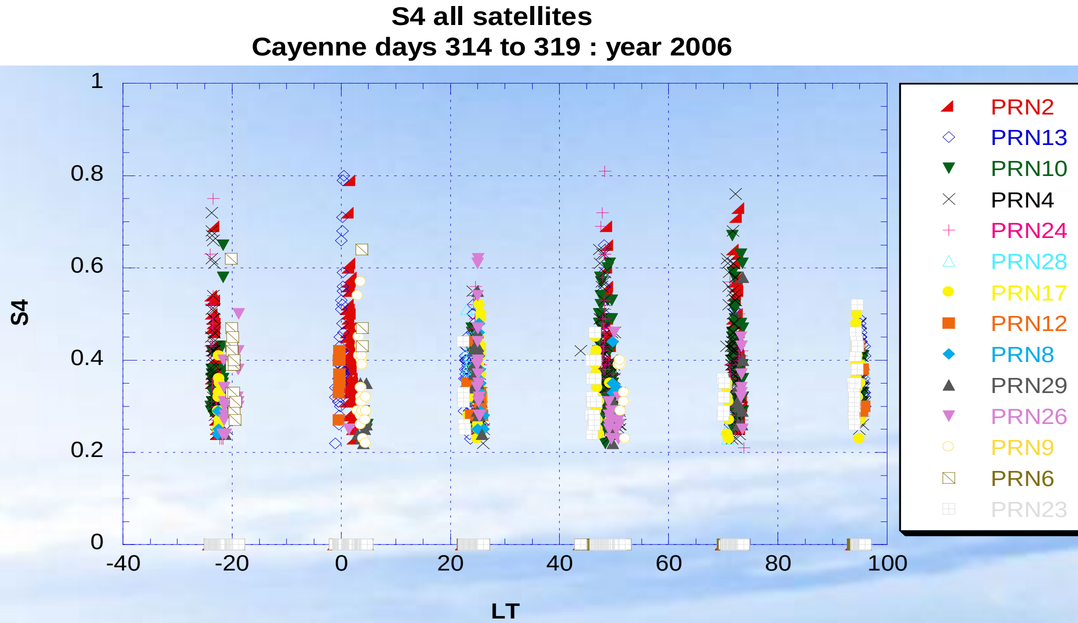
•*GISM simulation*

fading times histogram



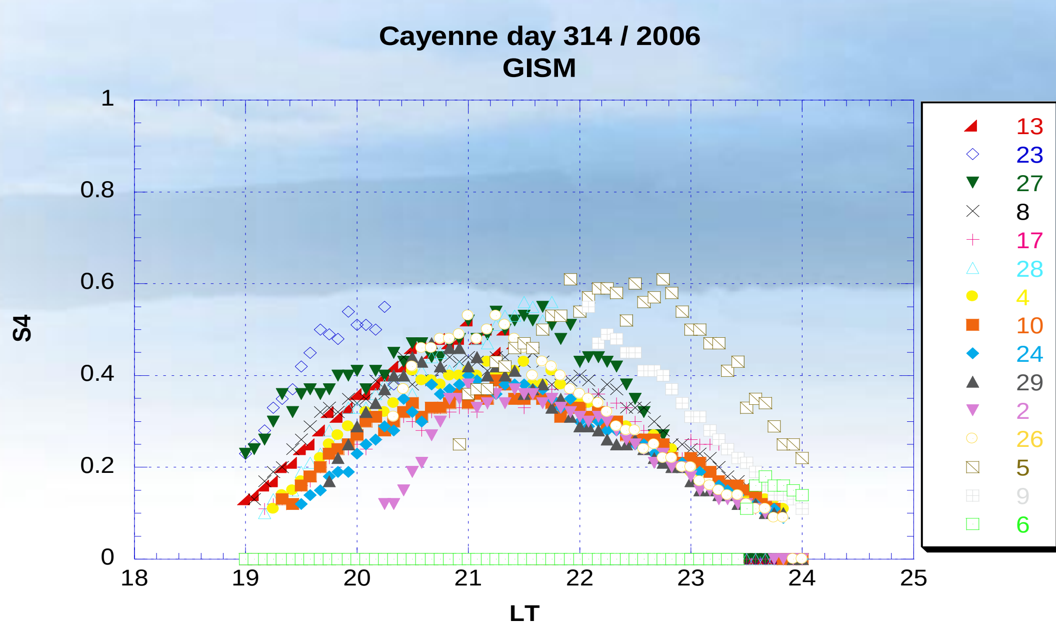
Modelling vs Measurements (Intensity)

Measurements →



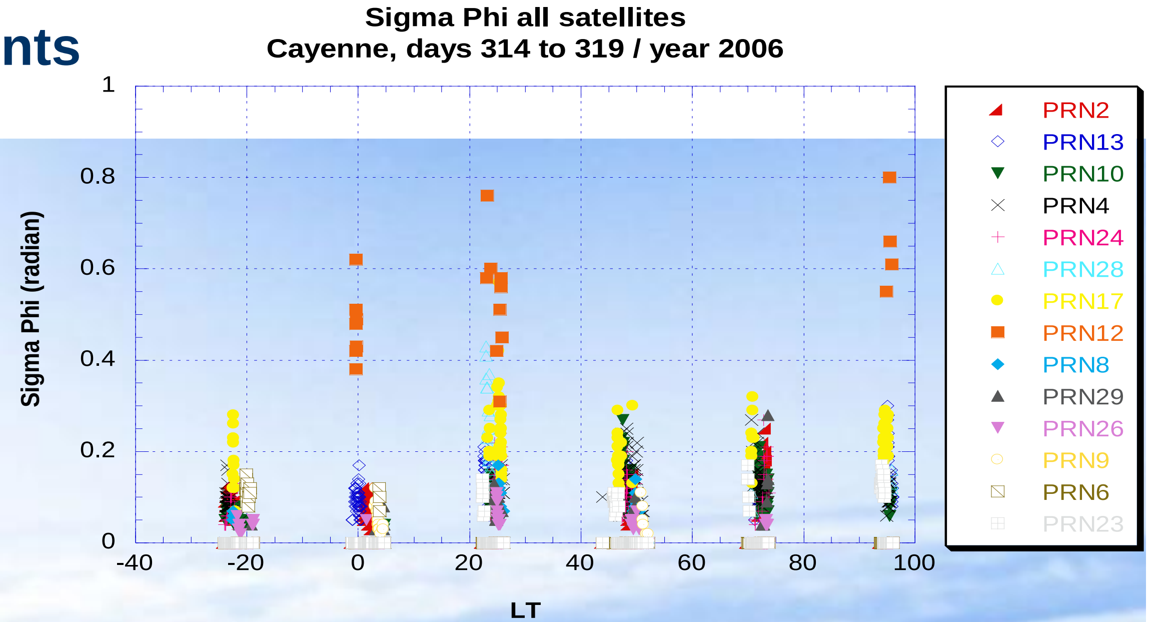
Samples with S4 < 0.2 were ignored (noise level)

Modelling →



Modelling vs Measurements (Phase)

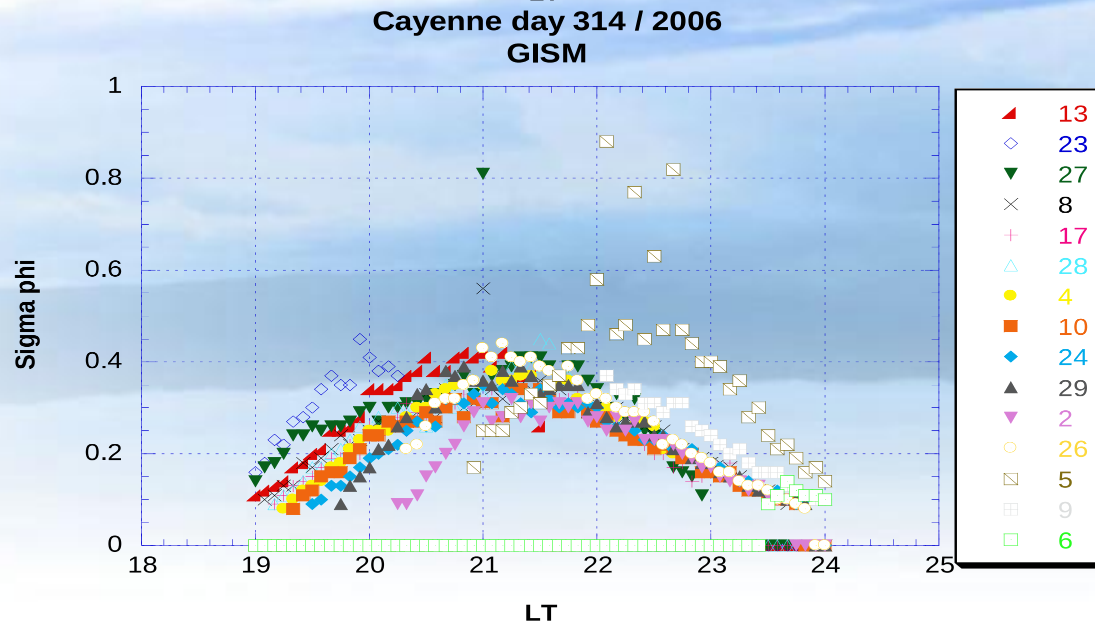
Measurements →



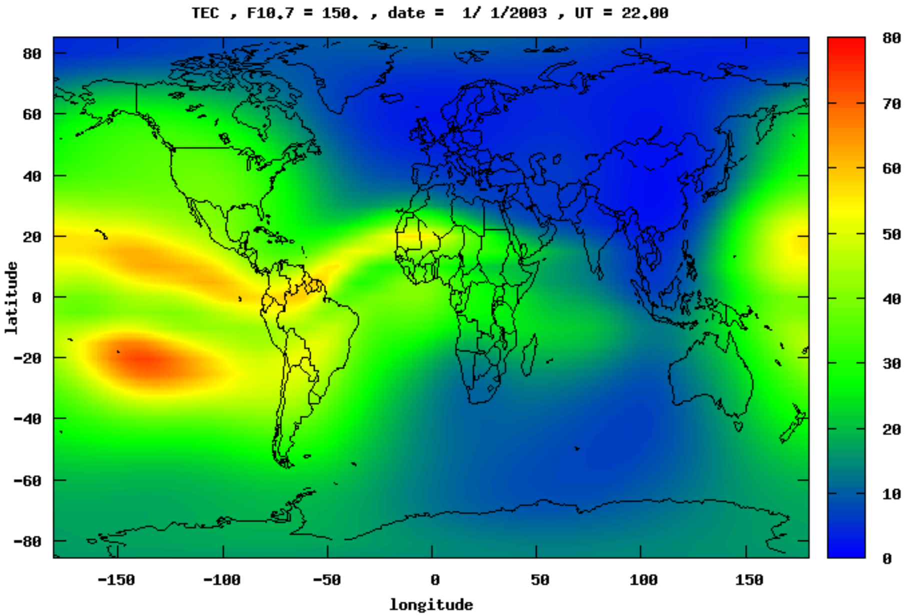
The phase RMS value is slightly lower than the S4 value

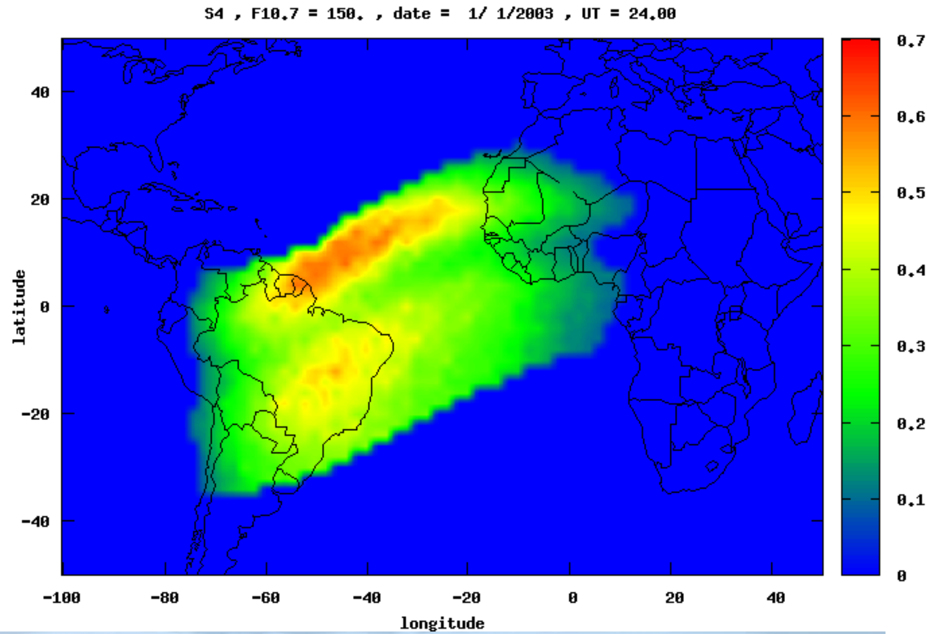
Some samples exhibit high values (both measurements and modelling) due to the phase jumps

Modelling →



Global Maps





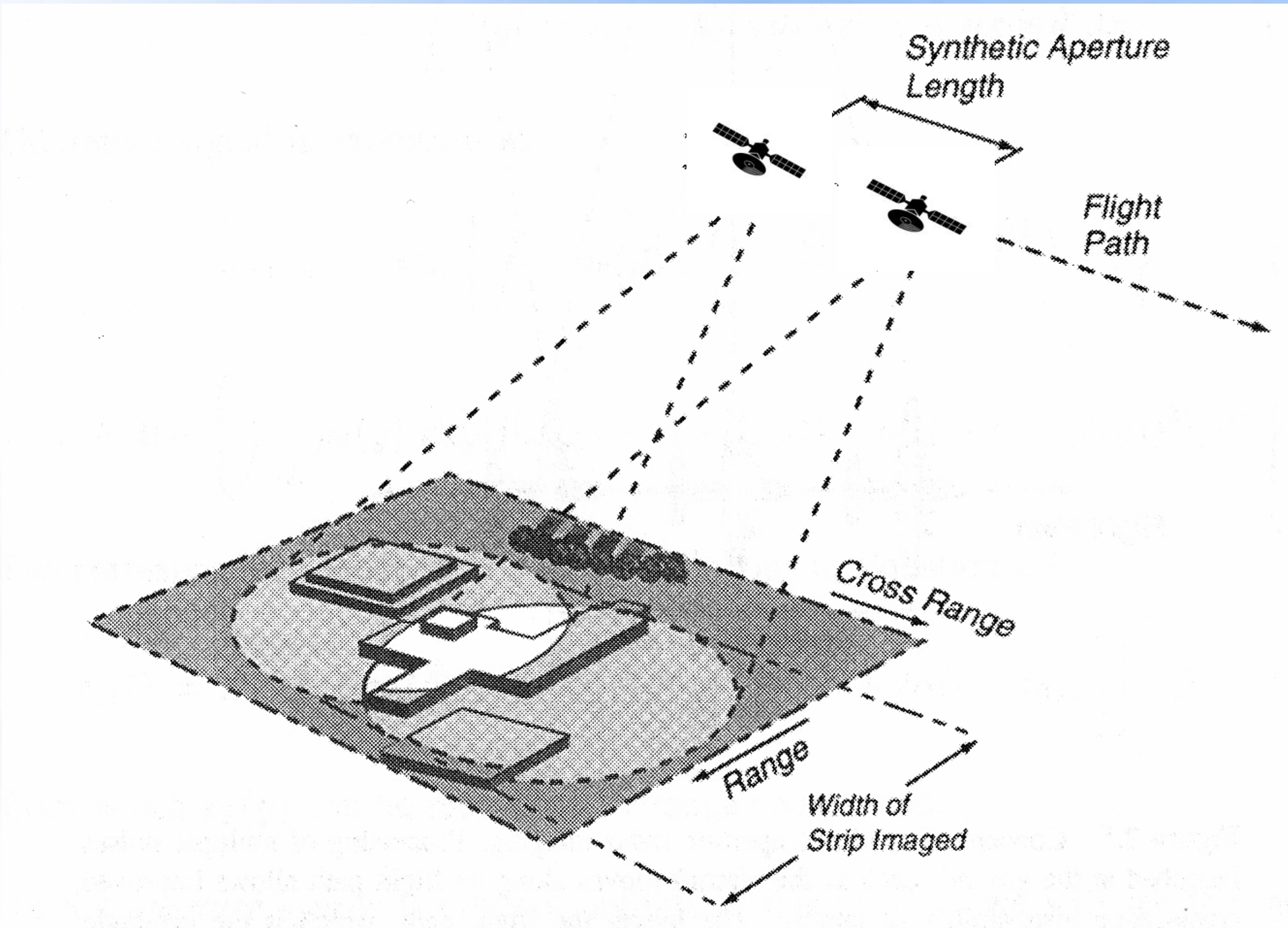
TEC Map Modelling

Scintillation Map Modelling

Radar Observations

Mutual Coherence Function

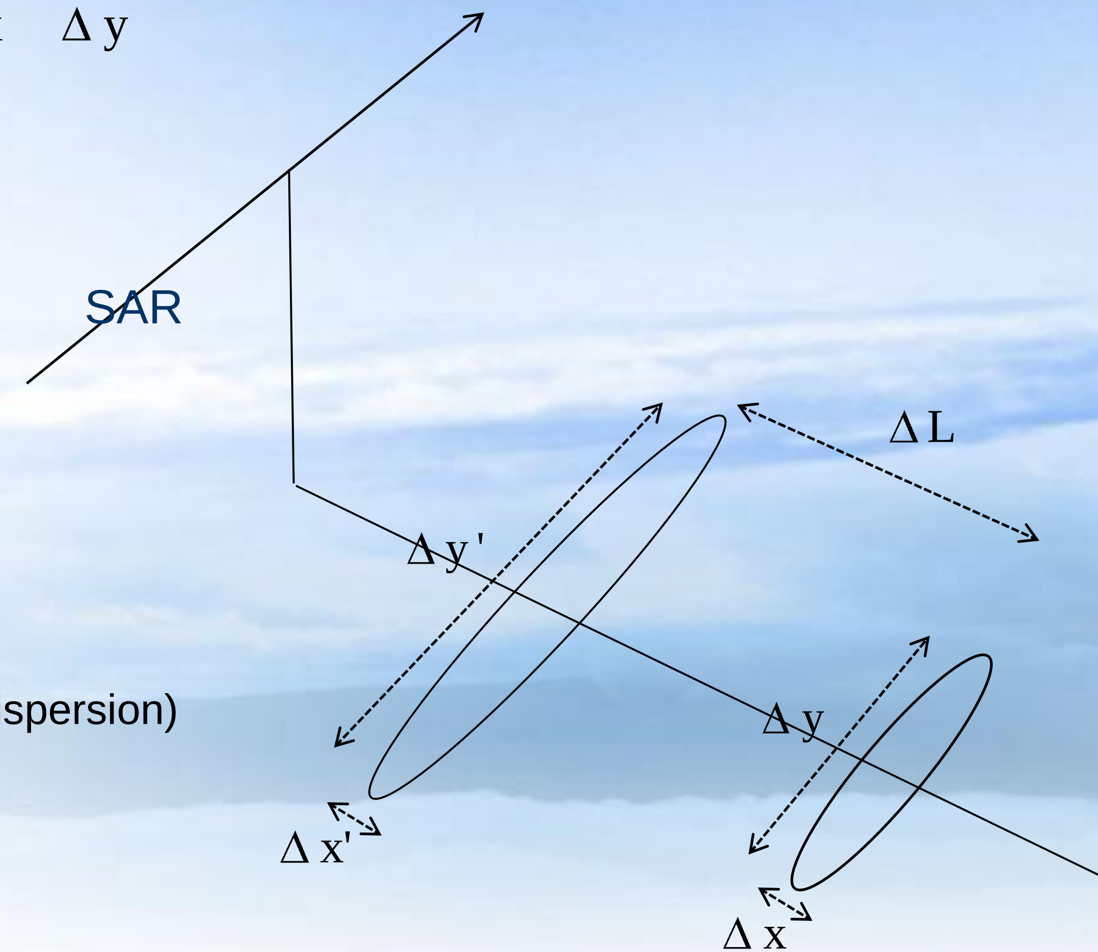
Correlation distance vs L_{SAR}



L_e Synthetic Aperture Length → 10 km

Ionosphere Effects

Free Space Resolution Δx Δy



Ionosphere Effects

$\Delta x'$ Pulse broadening (dispersion)

$\Delta y'$ Turbulence Effect

ΔL Group Delay

Two Points - Two Frequencies Coherence Function

$$\Gamma(z, k_1, k_2, \rho_1, \rho_2) = \left\langle U_1(z, k_1, \rho_1)\; U_2^*(z, k_2, \rho_2) \right\rangle$$

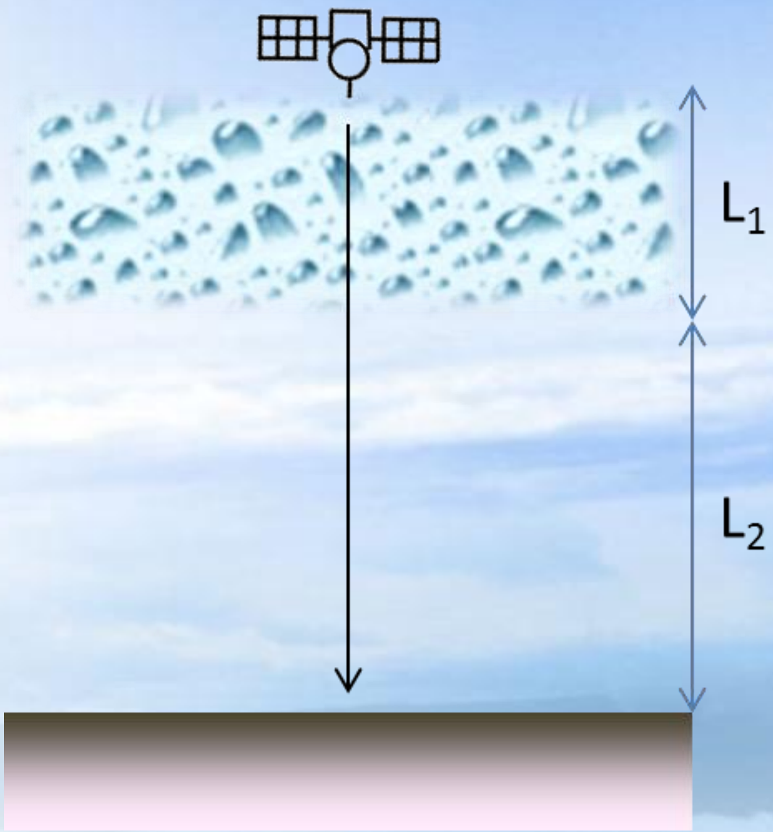
Using the parabolic equation

$$\left[\frac{\partial}{\partial z} - \frac{j}{2} \frac{k_d}{k_0^2} \nabla_d^2 + \frac{k_p^4}{8\, k_0^2} \left[\frac{k_d^2}{k_0^2} A_\xi(0) + D_\xi(\rho) \right] \right] \Gamma(k_d, z, \rho) = 0$$

The structure function $D_\Phi(z,\rho) = 2\left[B_\Phi(0) - B_\Phi(\rho)\right]$ is quadratic with respect to the distance

Same process than previously : propagation 1st & 3rd terms ; diffraction : 2nd & 3rd terms

Two Points - Two Frequencies Coherence Function



$$\Gamma(z, k_1, k_2, \rho_1, \rho_2) = \left\langle U_1(z, k_1, \rho_1) \; U_2^*(z, k_2, \rho_2) \right\rangle$$

Solution (one single screen)

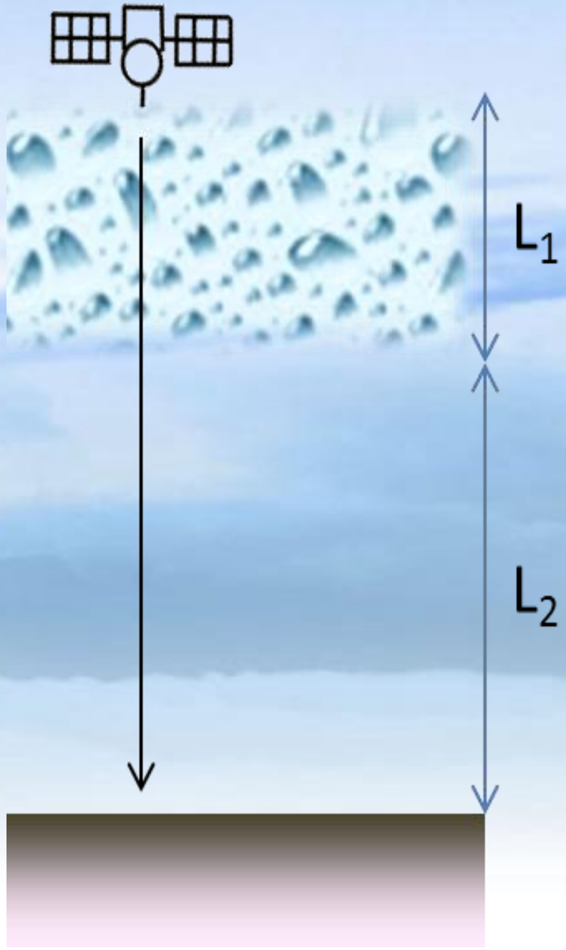
Scattering → 2 constants

$$B = \frac{\sigma_\Phi^2}{2\,\omega^2}$$

$$S = \sigma_\Phi^2 \left(L_1\right)^2 \frac{\mathrm{Log}\,(L_0 / \ell_i)}{6\,L_0^2}$$

Propagation → 1 constant

$$P = \frac{1}{2\ c\ k^2}\left(\frac{1}{L_1 + L_2} - \frac{1}{L_1}\right)$$



* Nickisch, RS 92, Knepp & Nickisch, RS, 2010

One single Screen

Analytical Solution

$$\Gamma(\tau, K_x, z) = \frac{z}{2\sqrt{BS}} \exp\left(-\frac{z^2 K_x^2}{4S}\right) \exp\left(-\frac{\left(+ z^2 K_x^2 P\right)^2}{4B}\right)$$

$\frac{z}{2\sqrt{BS}}$ → Maximum

$\exp\left(-\frac{z^2 K_x^2}{4S}\right)$ → K_x

$\exp\left(-\frac{\left(+ z^2 K_x^2 P\right)^2}{4B}\right)$ → τ and K_x

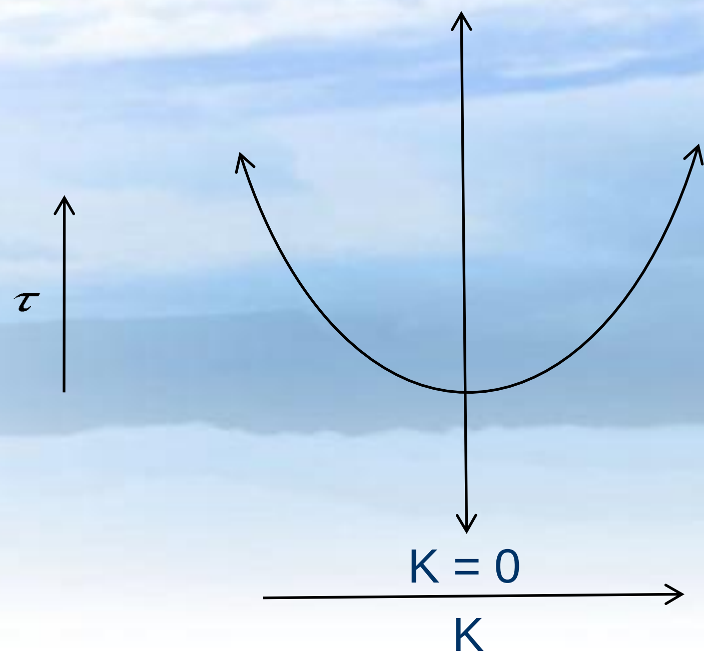
Spreading Extent

$$K = 0 \rightarrow \tau_1 = 2A\sqrt{B}$$

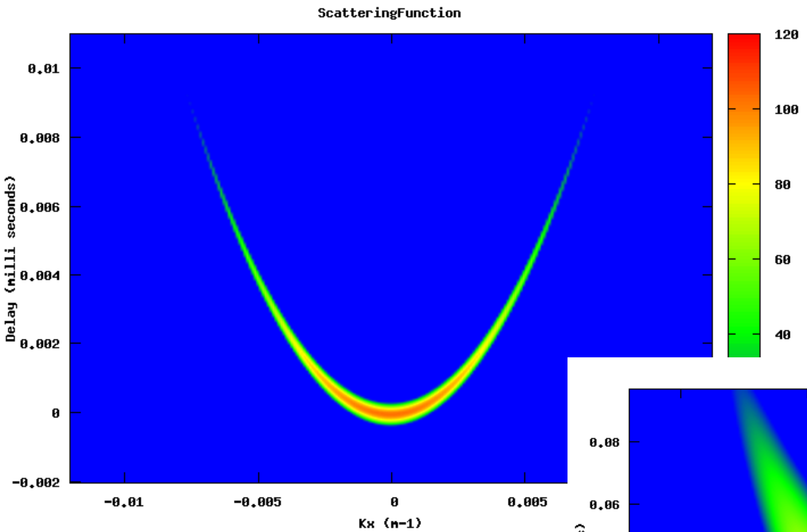
$$K = KMax \rightarrow \tau_2 = 4SPA$$

Spread factor

$$Q = \frac{\tau_2}{\tau_1} = \frac{2\sqrt{A}\,SP}{\sqrt{B}}$$

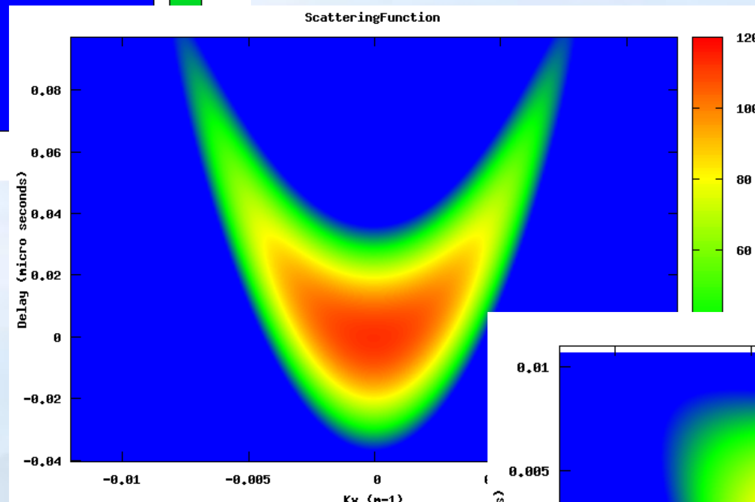


Spreading Extent



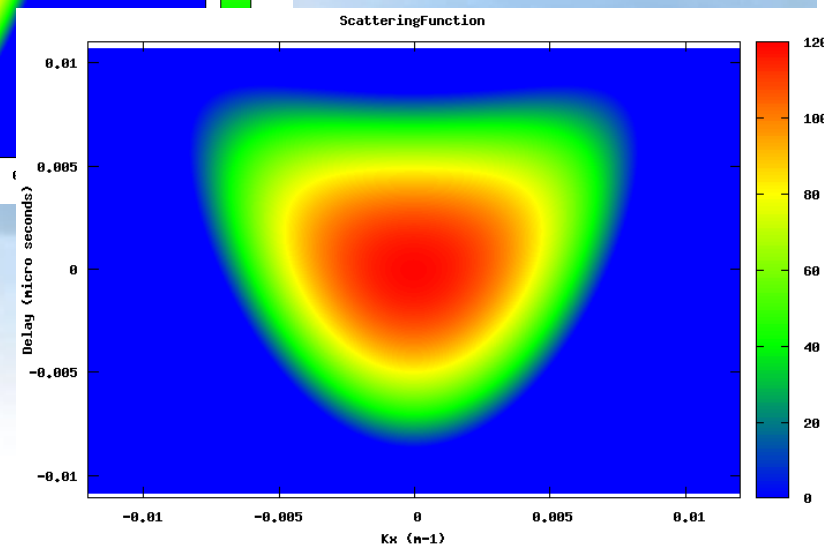
10 MHz

Very large value of Q



100 MHz

Large value of Q



435 MHz

Q around 1

Several Screens / Refined Analysis

- The algorithm can easily be generalized
- Different statistical properties may be assigned to the different layers
- Numerical FFT (1D) shall be performed to get the coherence function

Ambiguity Function

$$\chi(r, r_0) = \sum_n \int g_n(t, r_n)\, f_n^*(t, r_{0n})\, dt$$

g_n is the received signal and f_n is the matched filter

$$\chi_n(r_n, r_{0n}) = \frac{1}{(4\pi r_n)^2} \exp(j\Phi_0) \int \exp\left(-j\left[(\omega-\omega_0)\Phi_1 - (\omega-\omega_0)^2\Phi_2\right]\right) d\omega$$

Value of Coherent field received

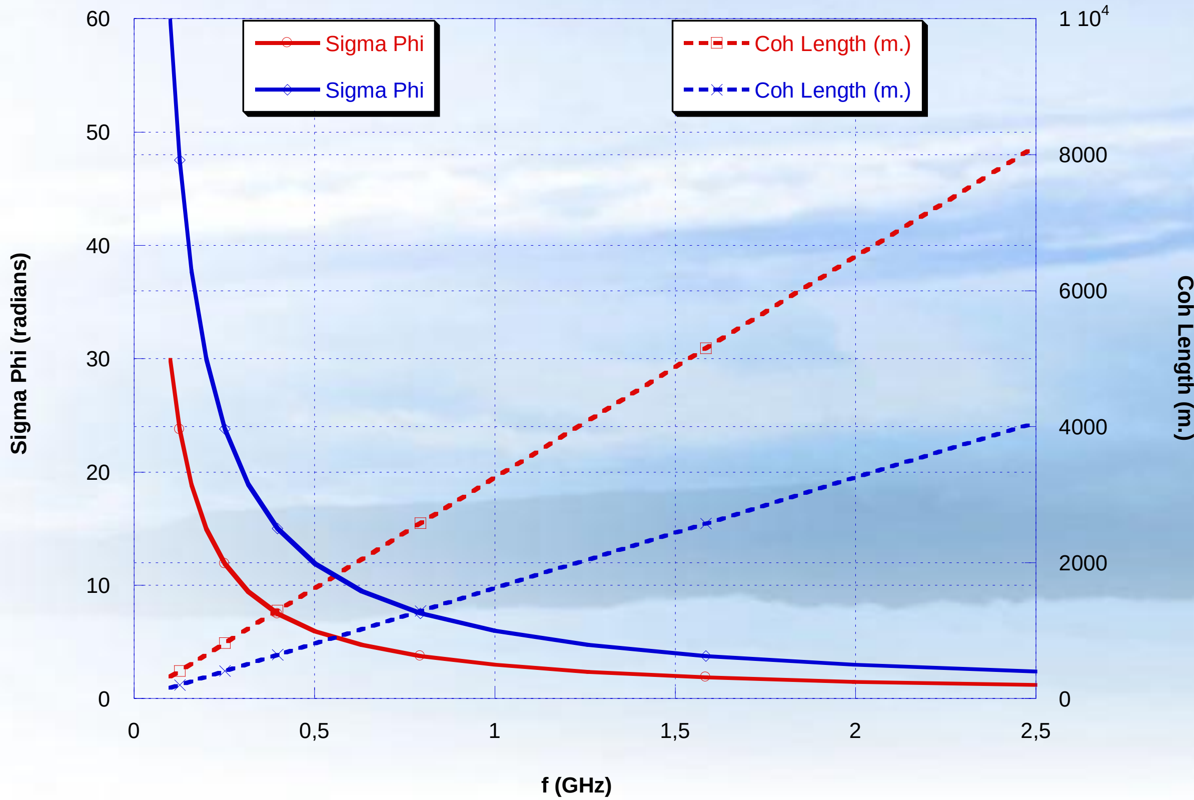
$$\langle \chi(r, r_0) \rangle = \frac{\exp(-\sigma_\Phi^2)}{(4\pi r_0)^2} \int_{-Le/2}^{Le/2} \exp\left(2jk_0\left(r_0 + \frac{\rho^2}{2r_0}\right)\right) d\rho = \frac{\exp(-\sigma_\Phi^2)}{(4\pi r_0)^2} \operatorname{sinc}(k_0 \rho L_e / r_0)$$

An attenuation factor on the coherent component has been included

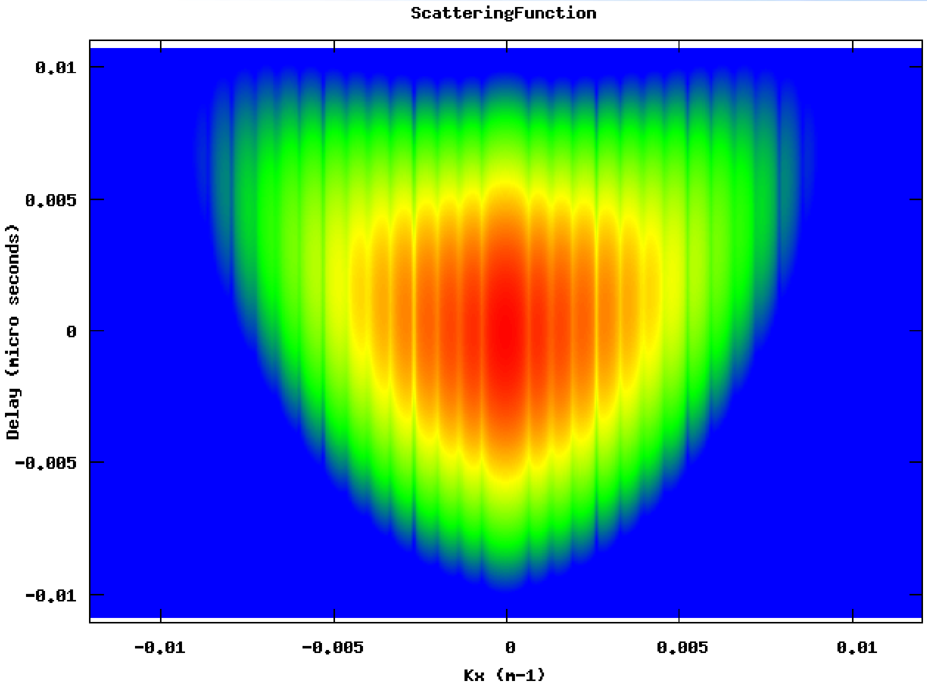
Coherent Length

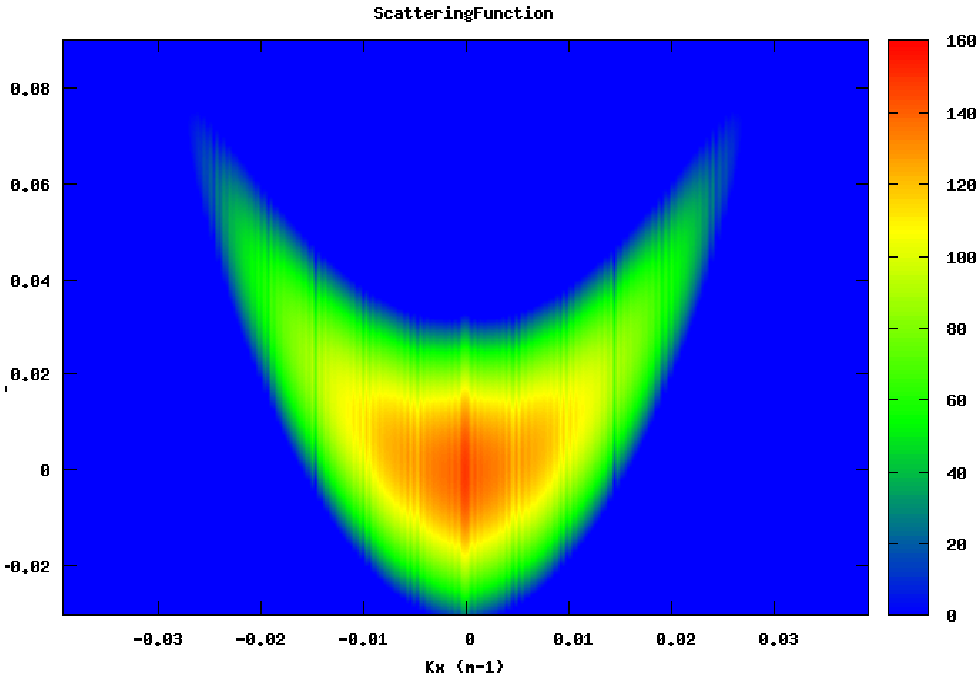
It is given by function $\Gamma(\omega_d, \rho, r) = \sqrt{\frac{D}{S}} \exp\left(-B\,\omega_d^2 - D\left(\rho / r_0\right)^2\right)$

Phase Standard Deviation and related Coherent Length
in the case of strong fluctuations (red lines) & very strong fluctuations (blue lines)



Scattering Function
400 MHz





Sigma Phi = 3.15 **Sigma Phi = 10.**

The Fourier transform (Ky space) of the antenna pattern is included in the calculation

Conclusion

- Reasonable agreement between the model (GISM) and the measurements

- The Mutual Coherence function has an analytical solution for one screen

- The algorithm can be extended for an arbirary number of screens

- The azimuthal resolution of a SAR may be significantly decreased

Run GISM

http://www.ieea.fr/en/gism-web-interface.html