Title: Renormalization operator for multimodal maps.

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Abstract:

Renormalization theory in one-dimensional dynamics has been a hot topic along the years, specially after the seminal work of Douady-Hubbard and Sullivan. Perhaps one of the most striking developments is that a fine understanding of the renormalization operator can lead us a better knowledge of the behavior of "most" of one-dimensional dynamical systems. For instance, the work of Avila, Lyubich and de Melo on families of real analytic unimodal maps relays deeply on renormalization theory.

A similar approach for multimodal maps (many critical points) pose new difficulties. Mainly the parameter space is not one-dimensional. The parapuzzles, developed by Branner-Hubbard and applied successfully by Yoccoz and many others for unicritical maps, provided a very precise description of the parameter space of the quadratic family. The miraculous properties of codimension one holomorphic laminations were also a crucial tool to understand the space of quadratic-like maps. Both tools are no longer available in the multimodal case. In this work in progress our main result is as follows:

Main Theorem. *Let f_λ be a finite-dimensional family of real analytic multimodal maps and let Λ_b be the subset of parameters λ such that f_λ is infinitely renormalizable with bounded combinatorics (not all the critical points need to be involved in the renormalization). Then for a generic finite-dimensional family the set Λ_b has zero Lebesgue measure.*

One of the main steps of the proof is to show that the action of the renormalization operator on infinitely renormalizable multimodal maps with bounded combinatorics is hyperbolic. The contraction on the hybrid classes of infinitely renormalizable maps can be obtained using available methods (Sullivan, McMullen, Lyubich, S. , Lyubich and Avila). To show the expansion in the transversal direction we developed a new approach, based on the study of the derivative cocycle of the renormalization operator instead of the operator itself.