Synthetic gauge potentials for ultracold neutral atoms
Experiment (I): synthetic magnetic fields

Yu-Ju Lin

Institute of Atomic and Molecular Sciences, Academia Sinica, Taiwan

Robert Compton, Karina Jimenez-Garcia, Trey Porto, William Phillips
and Ian Spielman

Joint Quantum Institute, National Institute of Standards and Technology, Gaithersburg
and University of Maryland

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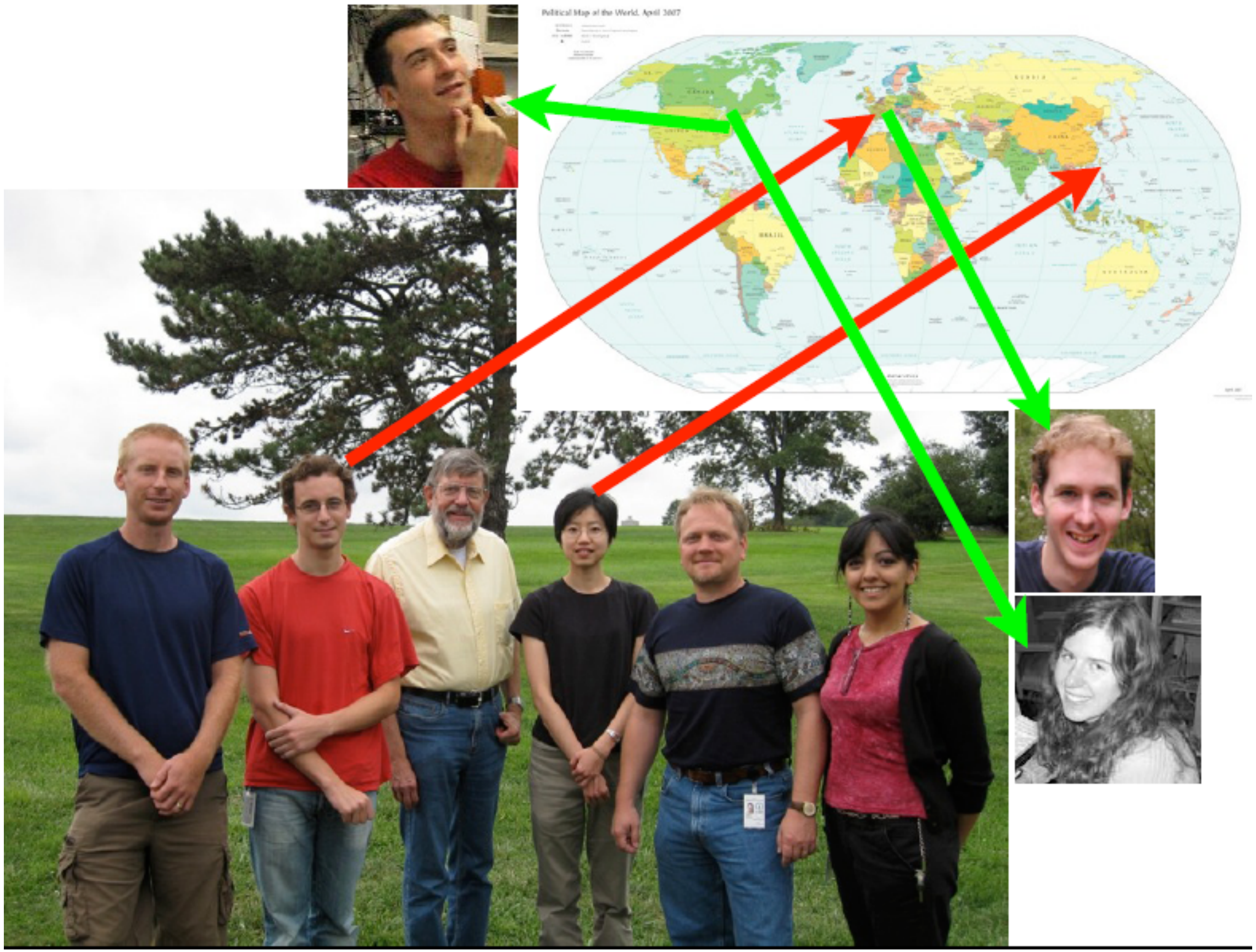
JQI/NIST group members



Berry's phase and gauge potential A*

want $H = \frac{(p - qA)^2}{2m}$

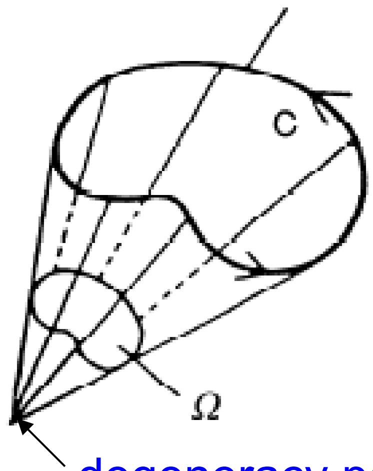
Berry's phase and gauge potential A*

$H(\boldsymbol{R}(t))$: Adiabatic evolution of an eigens $|n(\boldsymbol{R}(t))\rangle$

R(t)= external parameter

$$|n(\boldsymbol{R}_0), t_0 = 0; T\rangle = \exp(i\gamma_n(C))\exp\left\{-\frac{i}{\hbar}\int_0^T E_n(\boldsymbol{R}(t'))dt'\right\}|n(\boldsymbol{R}_0)\rangle$$

geometric phase   dynamic phase



$$\gamma_n(C) = i\int_{\boldsymbol{R}_0\ C}^{\boldsymbol{R}(t)} \langle n(\boldsymbol{R}(t'))\,|\,\nabla_{\boldsymbol{R}} n(\boldsymbol{R}(t'))\rangle\, d\boldsymbol{R}(t')$$

effective gauge potential A*

Ex: Spin m

spin aligned with B(t)=R(t)

$$H(\boldsymbol{B}) = -g\mu \boldsymbol{S}\cdot\boldsymbol{B}(t)/\hbar,$$

$$\gamma_m(C) = -m\iint_{S(C)} \frac{\mathbf{B}}{B^2}\cdot d\boldsymbol{B} = -m\int_C d\Omega = -m\Omega(C)$$

solid angle subtended in B space

From “Quantum mechanics”, Sakurai (based on Berry, 1984)

Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases $\frac{N}{V}\lambda_{dB}^3 \geq 1$

(1) cold and dilute:

λ_{dB}: de Broglie wave length
R_{vdw}: van der Waals potential range

cold: s-wave scattering dominated: $\lambda_{dB} > R_{vdw}$
dilute: inter-particle spacing $d > R_{vdw}$
cold and dilute: reduced to contact interaction

$$V(r-r') = \frac{4\pi\hbar^2 a}{m}\delta(r-r'), \quad a = \text{s-wave scattering length}$$

Ex. Bose-Einstein condensate (BEC) , Degenerate Fermi gas (DFG)

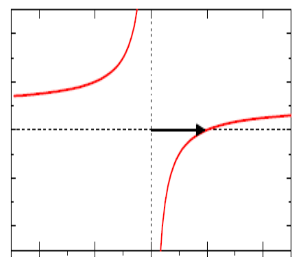
General references:
1. Many body physics with ultracold atoms, Rev. Mod. Phys. **80**, 885 (2008).
2. Making, probing, and understanding Bose-Einstein condensates, arxiv cond-mat 9904034

Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases

√ (1) cold and dilute: contact interaction $V(r-r^{'}) = \frac{4\pi\hbar^2 a}{m}\delta(r-r^{'})$

a vs. magnetic field B



(2) tunable interaction: Feshbach resonance

a= s-wave scattering length

* can achieve strong interaction limit a[illegible]d, even for a dilute gas

* can study strongly-correlated states with a simple model of interaction

Ex. BEC-BCS (Bardeen-Cooper-Schrieffer) crossover for fermions

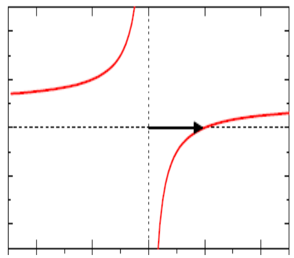
Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases

√ (1) cold and dilute: contact interaction $V(r-r') = \frac{4\pi\hbar^2 a}{m}\delta(r-r')$

√ (2) tunable interaction: Feshbach resonance

a vs. magnetic field B



(3) nearly disorder free

precisely controlled <u>magnetic</u> and <u>optical potentials</u>

(Zeeman shift) (AC stark shift)

Ultracold quantum gases

ultracold atoms: degenerate, non-classical gases

√ (1) cold and dilute: contact interaction $V(r-r') = \frac{4\pi\hbar^2 a}{m}\delta(r-r')$

√ (2) tunable interaction: Feshbach resonance

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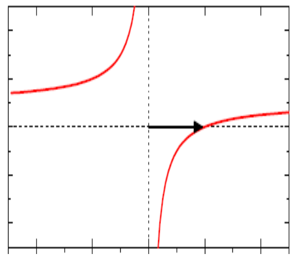


√ (3) nearly disorder free

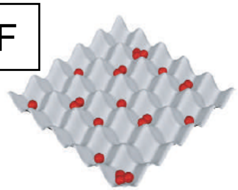
precisely controlled magnetic and optical potentials

→ ideal for quantum simulation:
model systems for condensed-matter physics

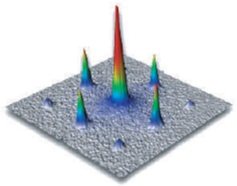
Ultracold atoms have realized iconic condensed matter systems

- superfluid → Mott-insulator transition: BEC in optical lattices
  (SF) (MI)

SF

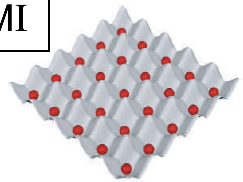


x-space

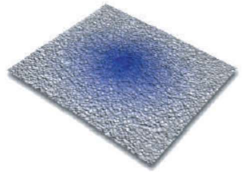


k-space (TOF)

MI



x-space

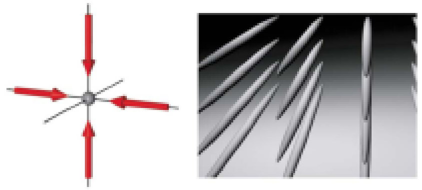


k-space (TOF)

Ref: Greiner et al., 2003

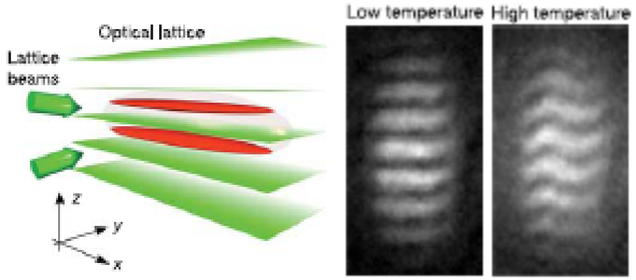
- low dimensional systems:

1D : Tonks-Giradeau gas



Ref: D. S. Weiss, 2004

2D : BKT superfluid



(Berezinskii-Kosterlitz-Thouless superfluid)

Ref: Jean Dalibard, 2006

Ultracold atoms have realized iconic condensed matter systems

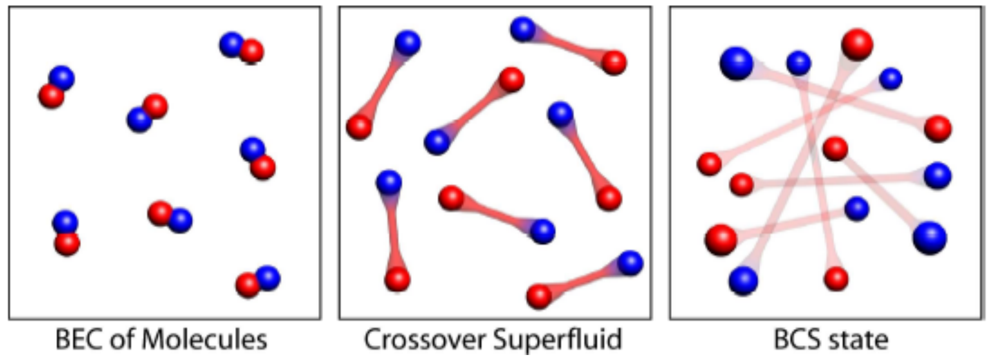
Use cold, degenerate gases: BEC (Bose-Einstein condensate) or degenerate Fermi gas

- superfluid → Mott-insulator transition: BEC in optical lattices

- low dimensional systems: 1D, 2D physics

- BEC-BCS (Bardeen-Cooper-Schrieffer) crossover:
  two-component Fermi gas, interaction tuned from repulsive →attractive

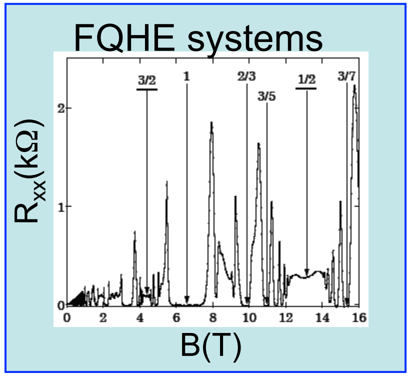
a>0 repulsive | $k_F a$ [illegible] 1 → | a<0 attractive



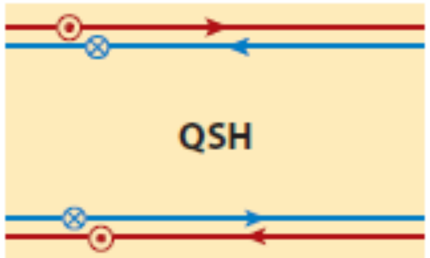
Ref: JILA, MIT, 2004

What we want to simulate with ultracold atoms

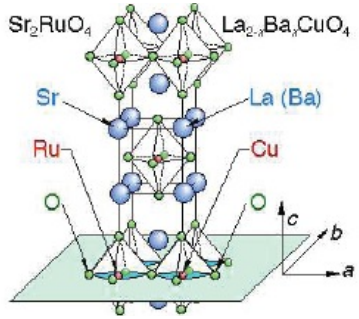
to "charge" neutral atoms by creating a " synthetic vector gauge potential A^* "



Topological insulators
(Quantum spin Hall effects)



p-wave superconductor



- new approach to generate large B* to study quantum-Hall physics

2D system and $\nu = N_{2D}/N_v \leq 1$
N_{2D}= atom#, N_v= # of flux quanta

- bosonic ν =1 state: w/ binary contact interaction,
nonabelian, for topological quantum computation

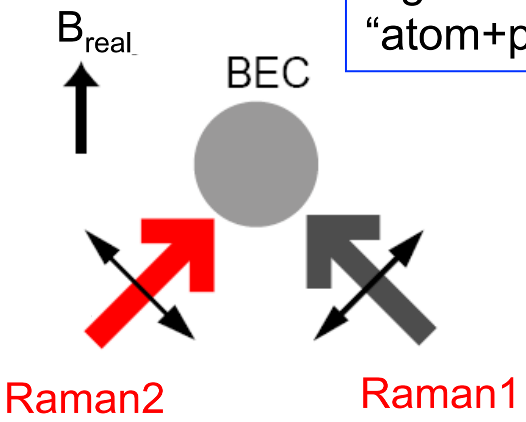
Ref: N. R. Cooper, 2008

- Spin-dependent A^*: spin-orbit coupling
TR preserved topological insulators, topological superconductors:
nonabelian gauge potentials $\longrightarrow \left[A_i^*, A_j^*\right] \neq 0$

Outline: synthetic gauge potentials A^*

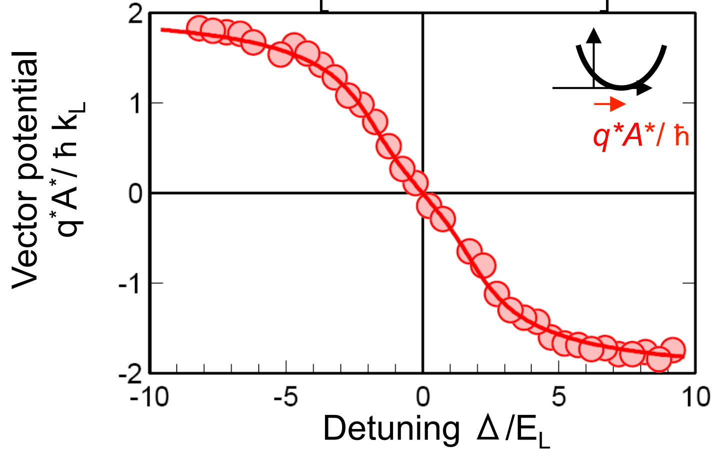
Raman-dressed BEC

eigenstate of
"atom+photon"

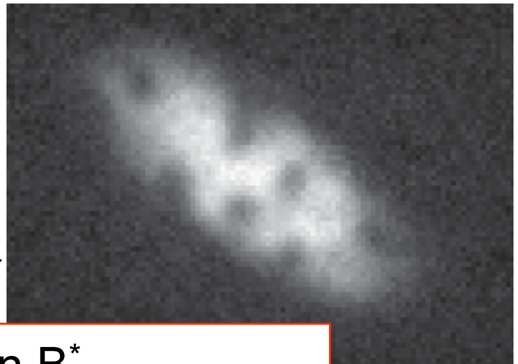


Synthetic Vector potential A^*

$$H = \frac{\hbar^2}{2m^*}\left[(k_x - \frac{q^* A_x^*}{\hbar})^2 + k_y{}^2\right]$$

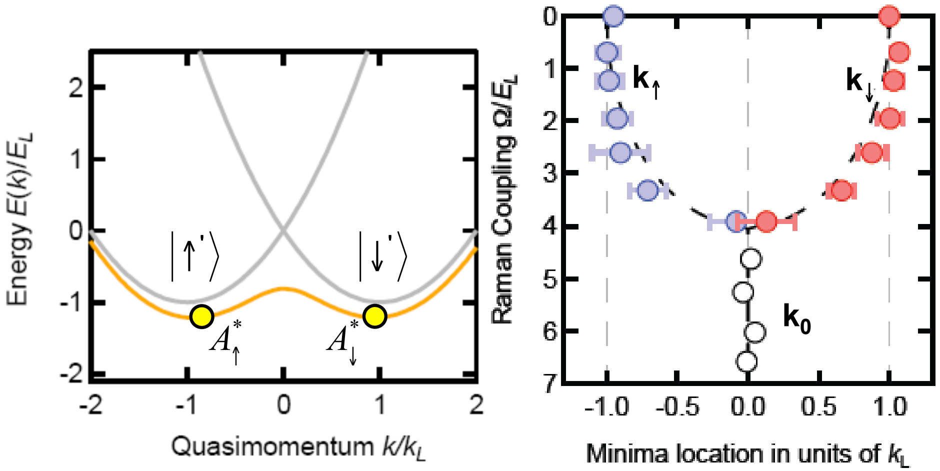


Magnetic field $B^* = \nabla \times A^*$



superfluid in B^*
(like superconductor in B)

Spin dependent A^*: spin-orbit coupling



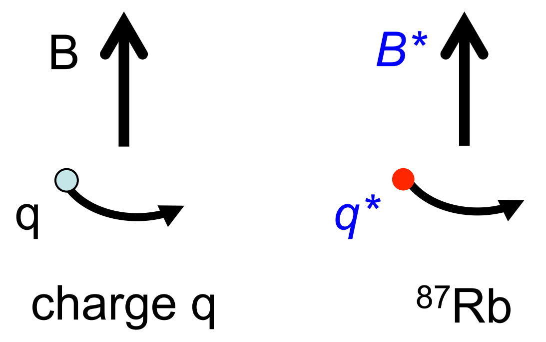
Introduction of synthetic gauge potentials

- Optically induced vector gauge potential A^* for neutral atoms:

$$H = \frac{(p - q^* A^*)^2}{2m^*} + V(x)$$

$\rightarrow$ synthetic electric and magnetic fields $\quad E^* = -\frac{\partial A^*}{\partial t}, B^* = \nabla \times A^*$

- Create synthetic field B^* for neutral atoms:
  effective Lorentz force
  to simulate charged-particles in real magnetic fields



- Light-induced potential to generate B^* in lab frame, no rotation of trap:
  (1) steady B^*, not metastable
  (2) easy to add optical lattices

B* in rotating frame:
Coriolis force↔Lorentz force

rotation: technical limit on B^*

Traditional methods to create B* : rotation

rotating neutral atoms

$F_{Coriolis} = 2m\Omega v_{rot}$

$\Omega \leftrightarrow qB/2m$

charge *q* in *B*

$F_{Lorentz} = qvB$

$$H_{rot} = \frac{\hbar^2}{2m}\left[(k_x - \frac{m\Omega y}{\hbar})^2 + (k_y + \frac{m\Omega x}{\hbar})^2\right] + V^{'}(r)$$

$$V^{'}(r) = \frac{1}{2}m(\omega^2 - \Omega^2)r^2$$

$$H_B = \frac{\hbar^2}{2m}\left[(k_x - \frac{qBy}{2\hbar})^2 + (k_y + \frac{qBx}{2\hbar})^2\right] + V(r)$$

$$V(r) = \frac{1}{2}m\omega^2 r^2$$

w/ mean field interaction ↓

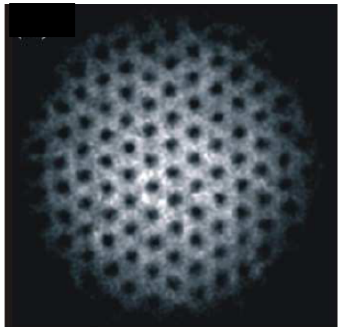
N_v vortices, $L/N = N_v/2$ (large N_v)

(one vortex $\leftrightarrow$ $\Phi_0 = h/q$)

↓

N_v flux quanta

rotating neutral BEC (experiment)



$\Omega/\omega = 0.975$, R~ 30μm

Coddington et al., JILA, 2004

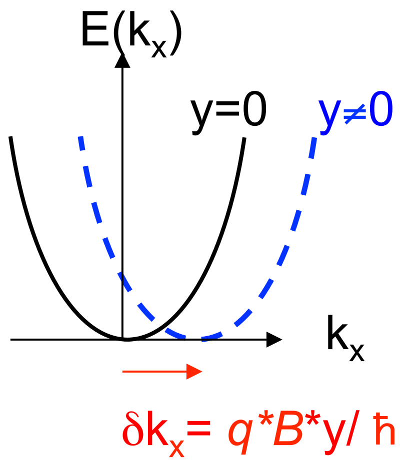
Principles (I)

- charged particle q in a real field $\vec{B} = B\hat{z}$, Landau gauge

$$H_B = \frac{\hbar^2}{2m}\left[(k_x - \frac{qA_x}{\hbar})^2 + k_y{}^2\right]$$

$$\delta k_x \equiv \frac{qA_x}{\hbar} = \frac{qBy}{\hbar}$$

$$\vec{B} = \nabla \times \vec{A}$$



- to simulate w/ laser-atom interaction

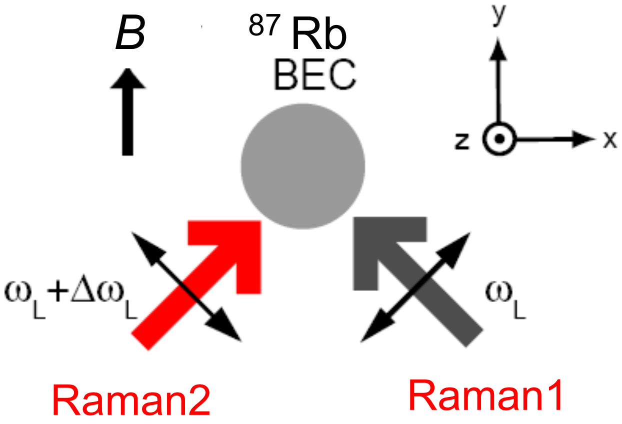
- laser photons : create δk_x= momentum shift along x

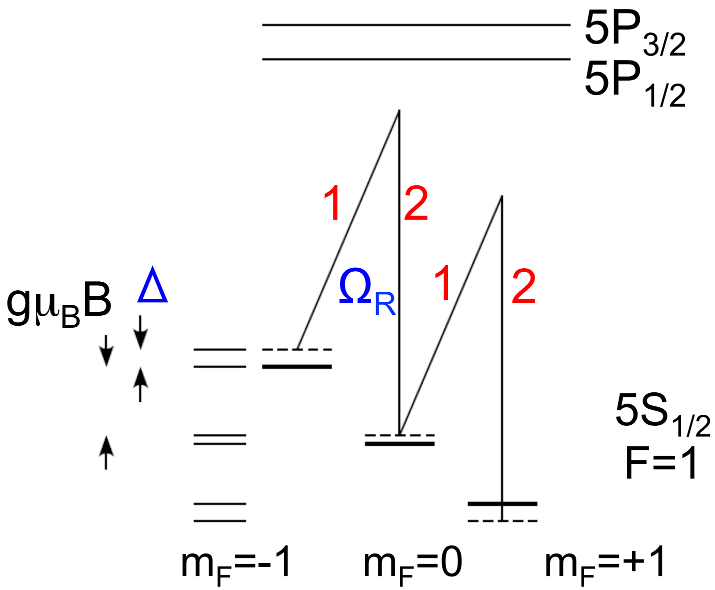
→ make $\delta k_x(\Delta)$ Δ=laser-atom detuning

→ make $\Delta=\Delta' y$: $\delta k_x(y)$

→ synthetic field $\frac{q^* B^*}{\hbar} = \frac{\partial(\delta k_x)}{\partial y}$ along z

Principles (II):
Formalism of Light-atom Coupling





- controlled Raman detuning $\Delta = \Delta\omega_L - g\mu_B B$, $B=B_0-b'y$ or $B=B_0(t)$

$2k_L\hat{e}_x = \vec{k}_1 - \vec{k}_2$

k (k_L), E(E_L)

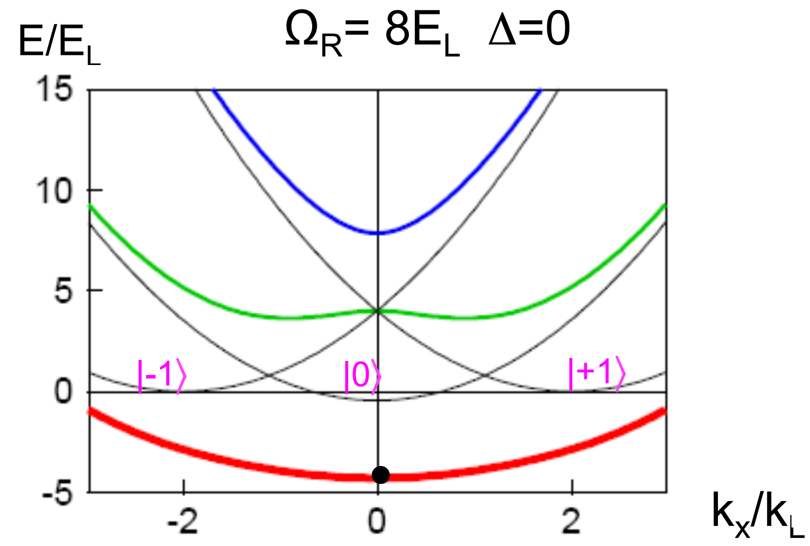
$E_L = \hbar^2 k_L^2/2m$

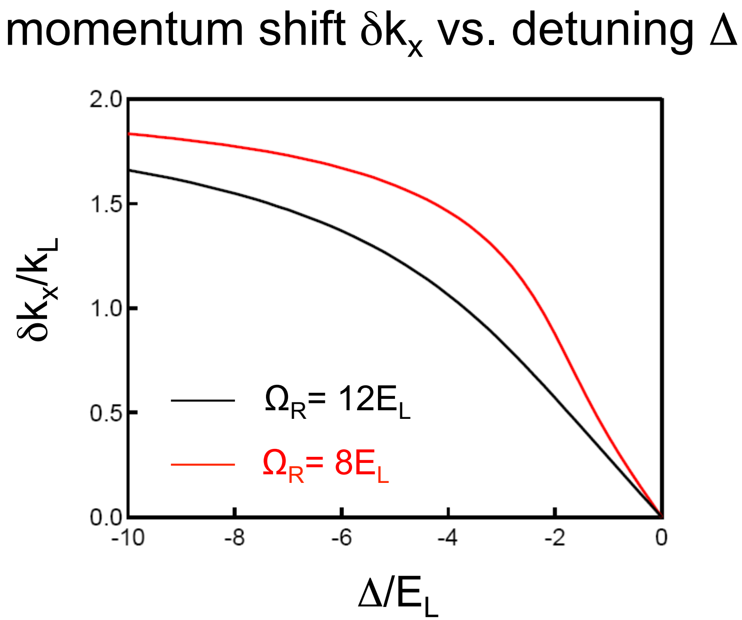
$|-1,k_x+2\rangle \quad |0,k_x\rangle \quad |+1,k_x-2\rangle$

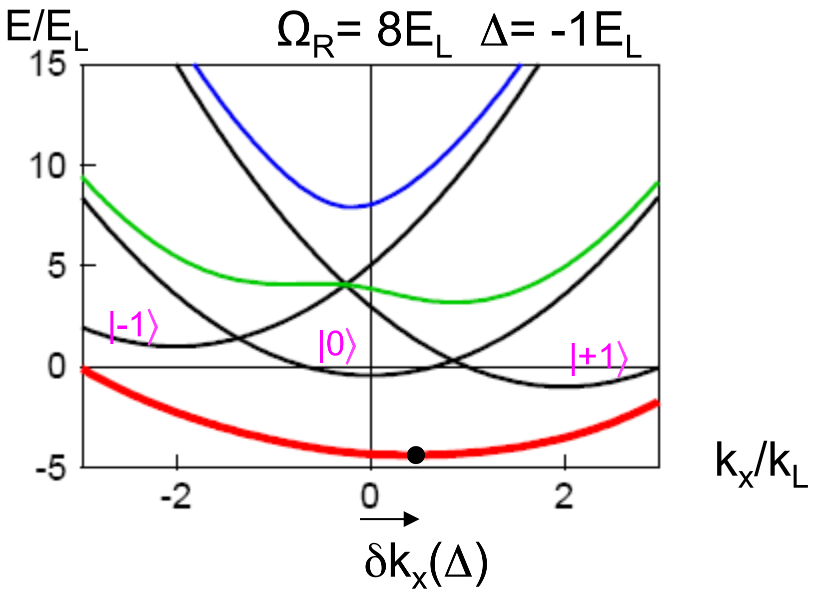
$$H = \frac{\hbar^2 \hat{k}_{y,z}^2}{2m} + V(x) + \underline{\frac{\hbar^2 \hat{k}_x^2}{2m}} + H_{\text{int}} \rightarrow \sum_{k_x} \begin{pmatrix} (k_x+2)^2 - \Delta & \boxed{\Omega_R/2} & 0 \\ \boxed{\Omega_R/2} & k_x^2\text{-}\varepsilon & \boxed{\Omega_R/2} \\ 0 & \boxed{\Omega_R/2} & (k_x-2)^2 + \Delta \end{pmatrix}$$

- diagonalize → Raman-dressed state : eigenvalue= modified $E_j(k)$

Principles (III):
Shift of momentum in dispersion relation





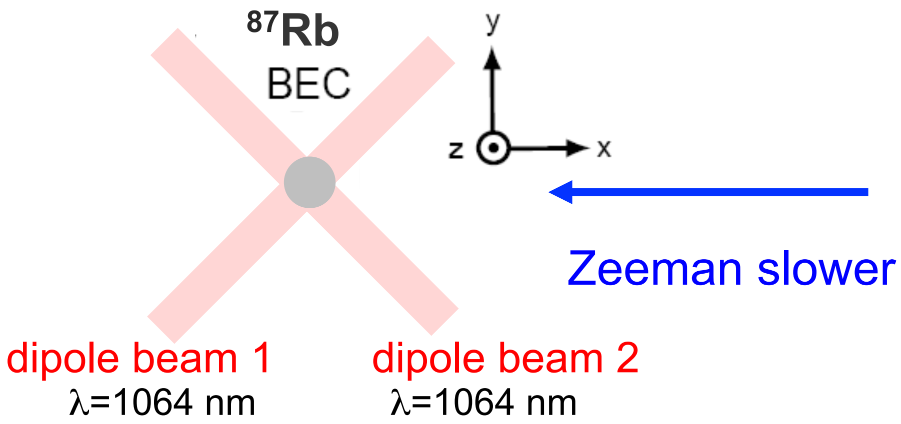


- the lowest energy dressed-state:

$$\delta k_x = \frac{q^* A_x^*}{\hbar} = \text{momentum shift}$$

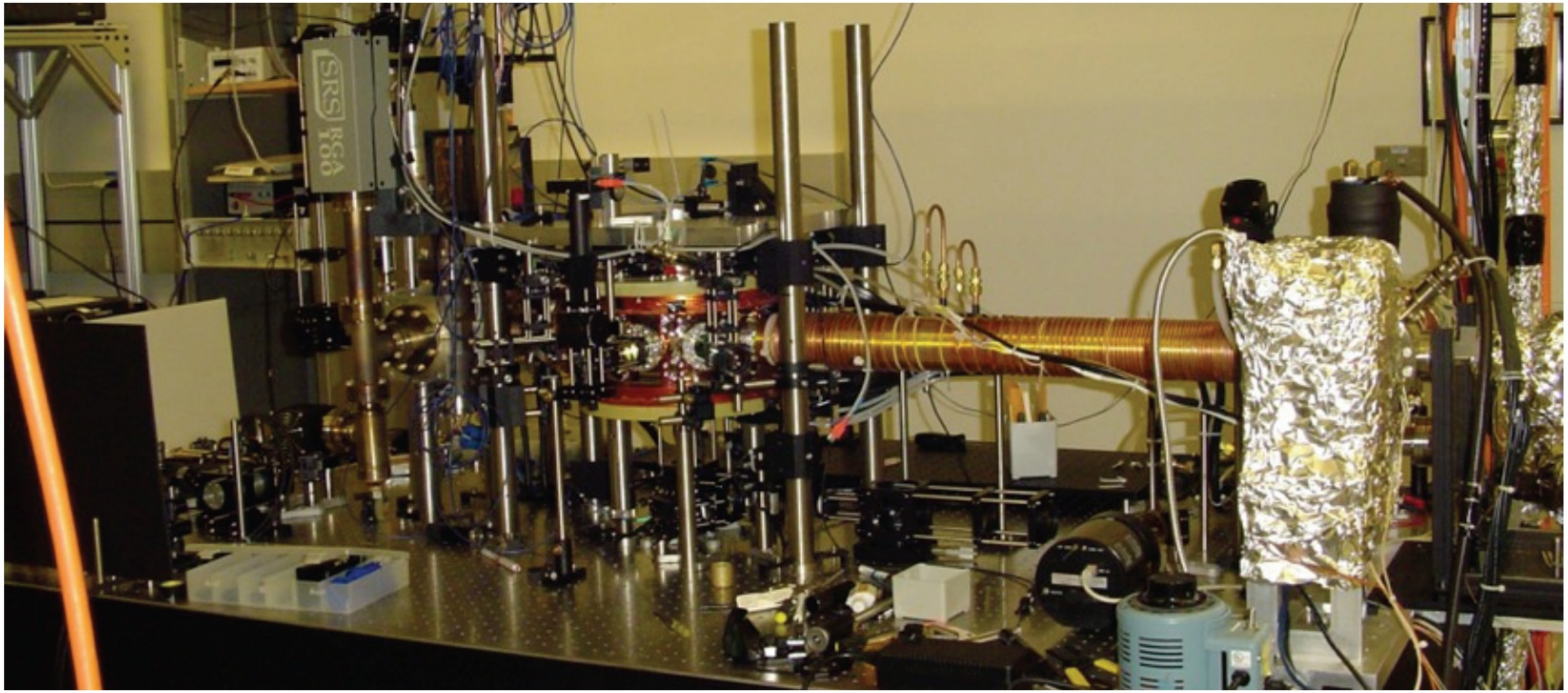
$$H_x = \frac{\hbar^2}{2m}\left[(k_x - \frac{q^* A_x^*}{\hbar})^2\right]$$

Setup: BEC production

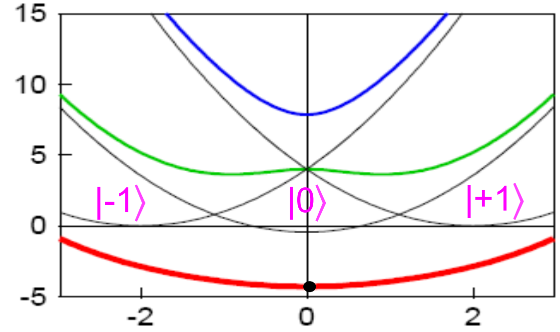


- load MOT from Zeeman slower: ~ 10^9 atoms in 3 s
- rf-evaporative cooling in a quadrupole magnetic trap for 3 s, $|F=1,m_F=-1\rangle$
- single beam optical dipole trap + weak magnetic trap:
  evaporate in hybrid potential for ~ 7 s→ 2 x10^6 atoms in BEC
- load the BEC into the crossed dipole trap: 5 x10^5 atoms
- total cycle time~ 15 s

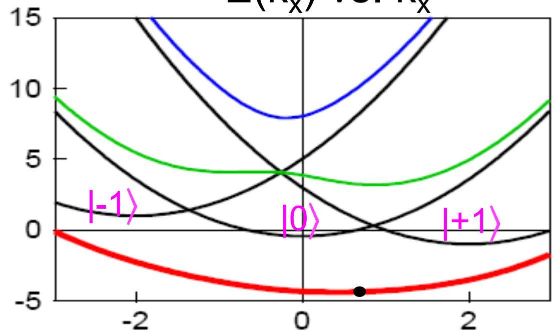
Actual experimental Setup



Adiabatic loading into the dressed state: uniform A*



measure δk_x from spin $|0, k_x = \delta k_x\rangle$



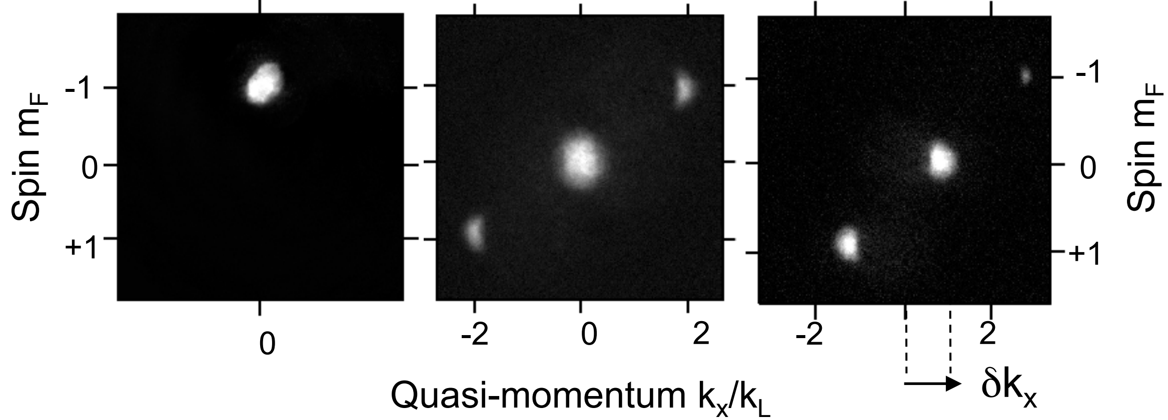
$\delta k_x(\Delta)$

P_{can} = [illegible] [illegible] $\delta k_x = q^* A^* \neq 0$

$$\langle \dot{x} \rangle_{m_F} = 0 \ !$$

Time-of-Flight images of $|-1, k_x+2\rangle$, $|0, k_x\rangle$, $|+1, k_x-2\rangle$

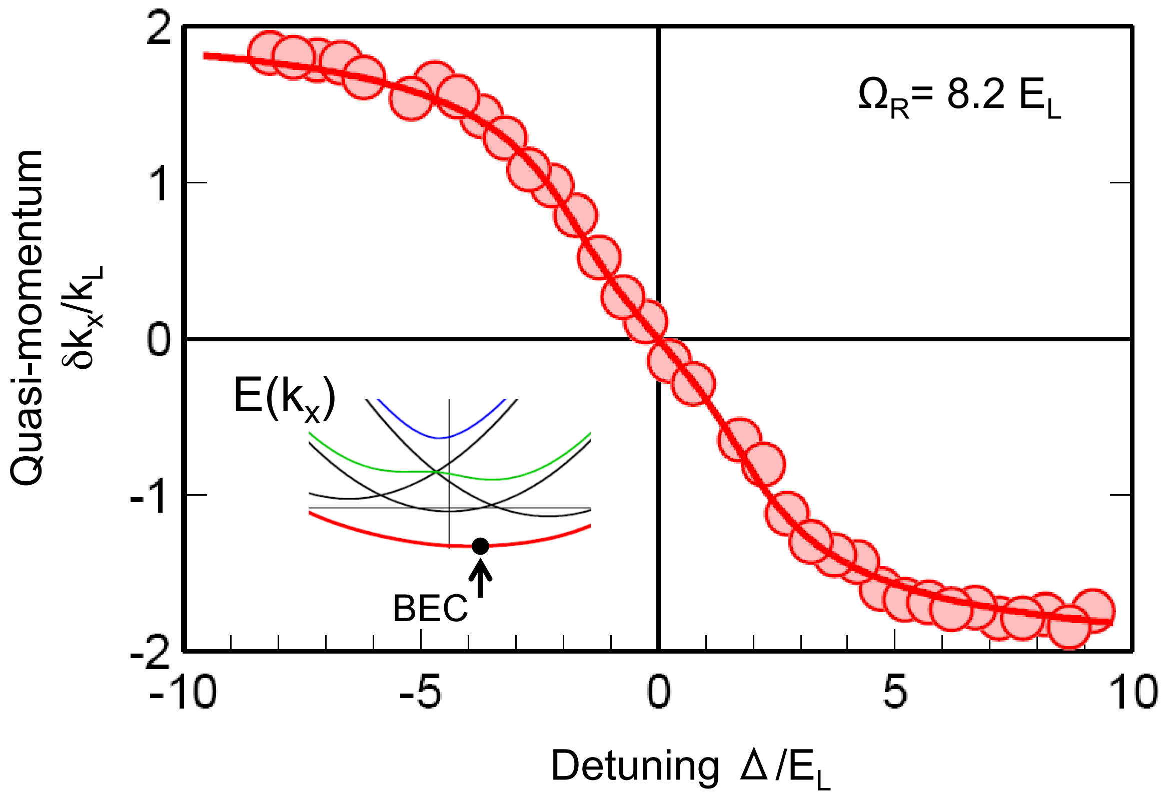
(a) → (b)(c) → (d) $\Delta = 0$, $\delta k_x = 0$ → (e) $\Delta \neq 0$, $\delta k_x \neq 0$



Uniform vector potential A* vs. detuning Δ

effective vector potential $q^*A^*/\hbar$ = measured quasi-momentum k_x
adiabatic loading at energy minimum → $k_x = q^*A^*/\hbar$

vector potential $q^*A^*/\hbar$ vs. detuning Δ

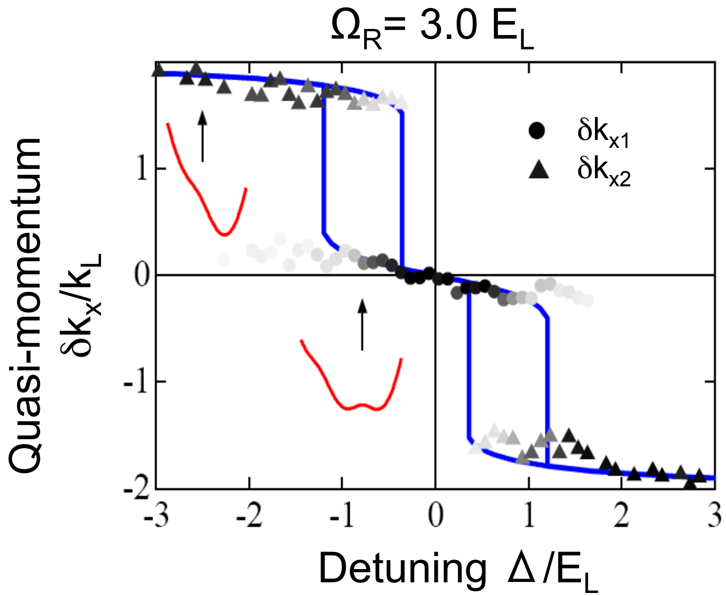


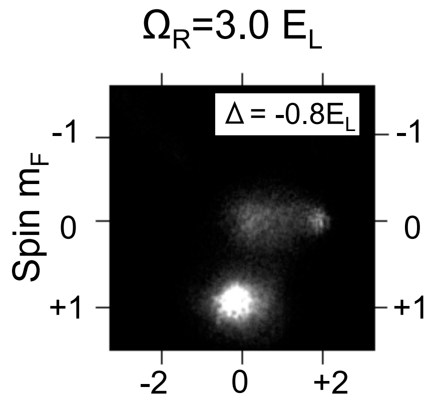
Ref: Y.-J. Lin et al., PRL **102**, 130401 (2009).

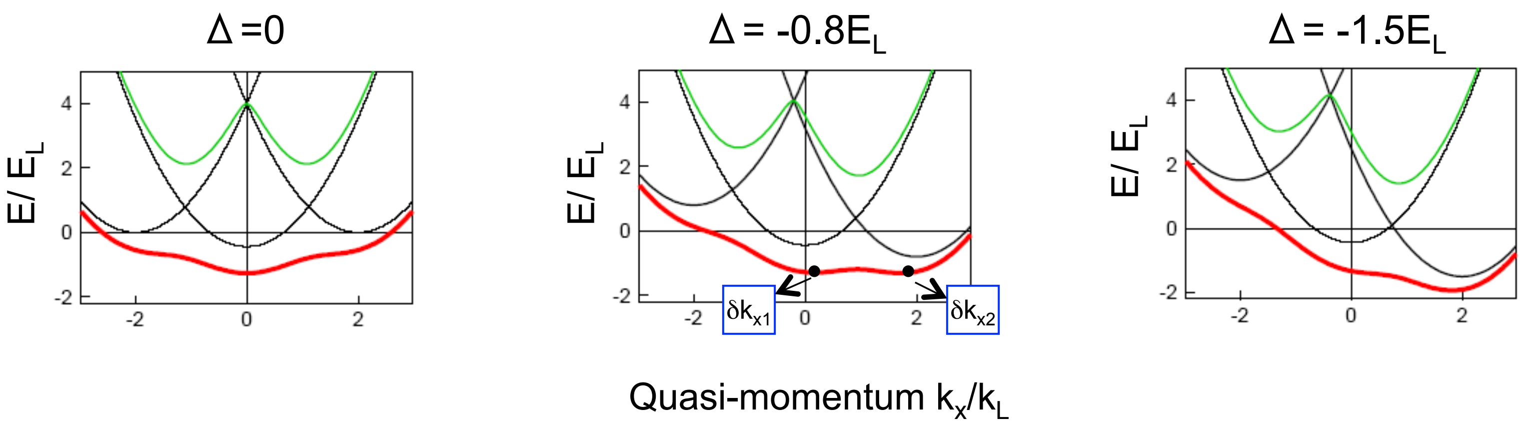
Uniform vector potential A* vs. detuning: small Ω_R

- for weak coupling $\Omega_R < 4.5\ E_L$, double energy minimum in k space

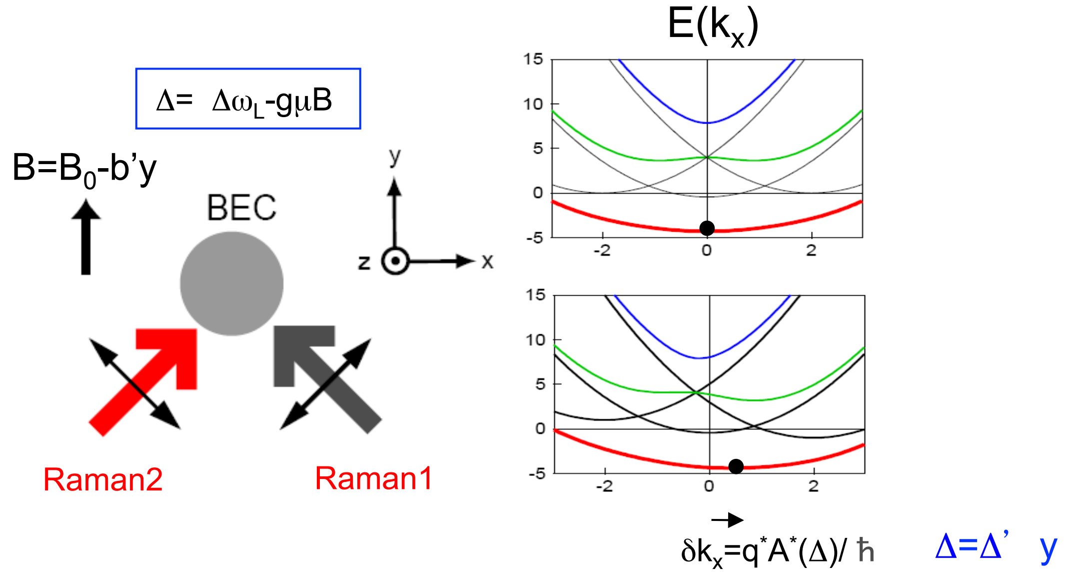
→ spin-orbit coupling



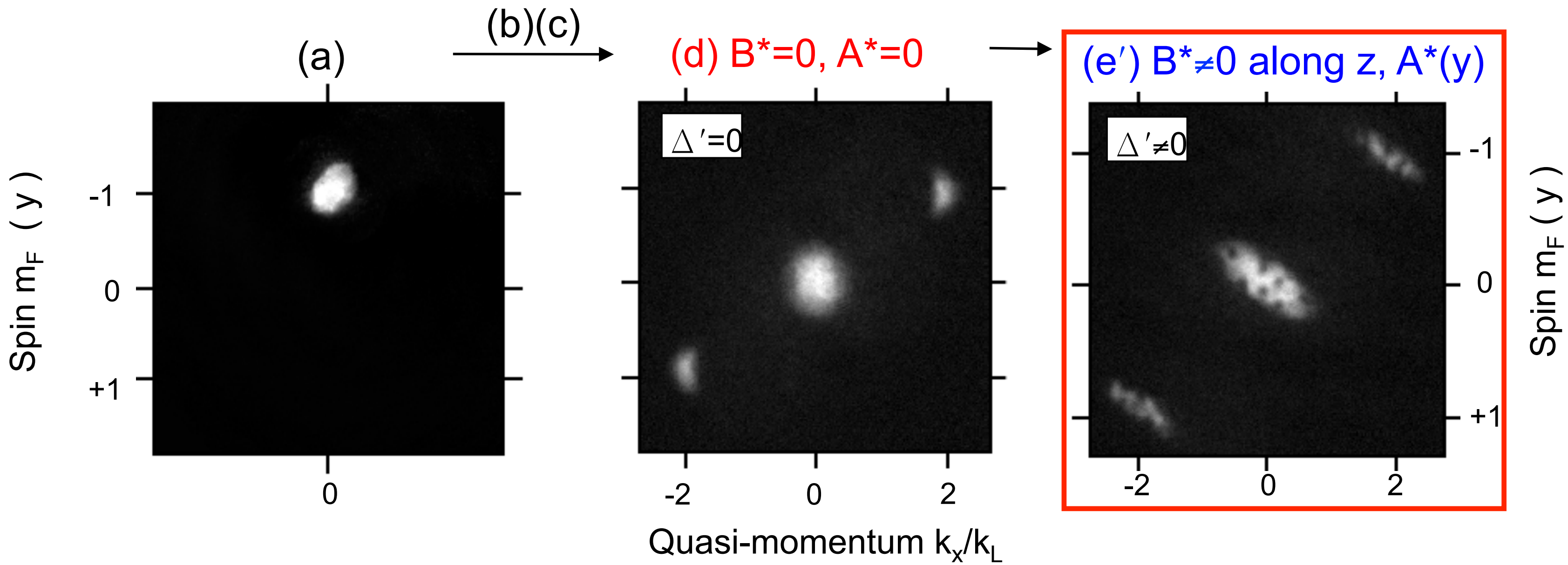




Synthetic field $B^* = \nabla \times A^*$

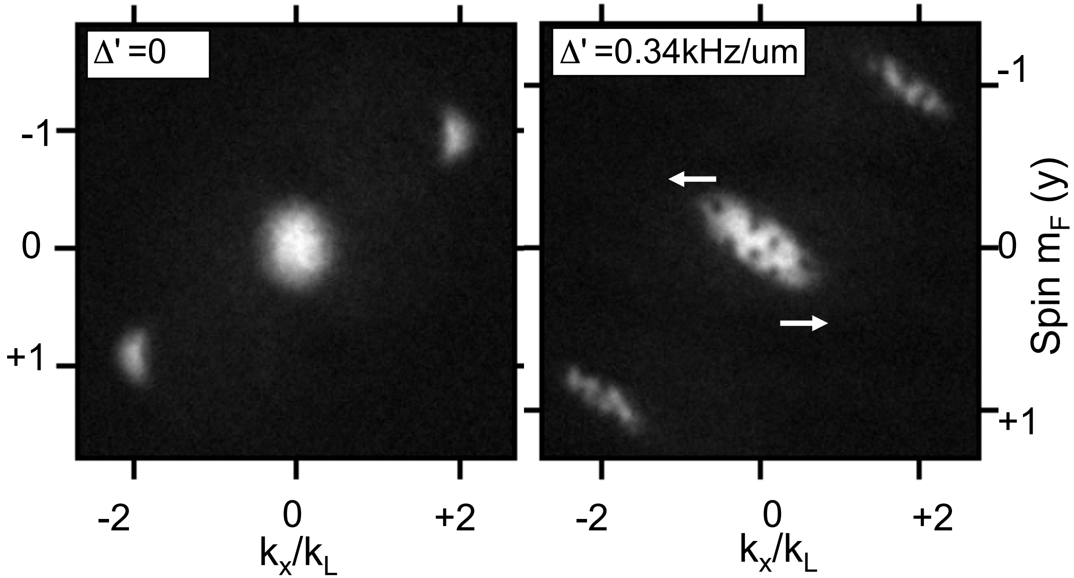


Time-of-Flight images of $|-1, k_x+2\rangle$, $|0, k_x\rangle$, $|+1, k_x-2\rangle$



Raman-dressed BEC in synthetic B*

Spin and momentum projection, TOF=25.1ms



$$H_B = \frac{\hbar^2}{2m^*}\left(k_x - \frac{q^* B^* y}{\hbar}\right)^2 + \frac{\hbar^2 k_y^{\,2}}{2m} + V(x,y)$$

- vector potential $\delta k_x \equiv \frac{q^* A^*}{\hbar} = \frac{q^* B^* y}{\hbar}$

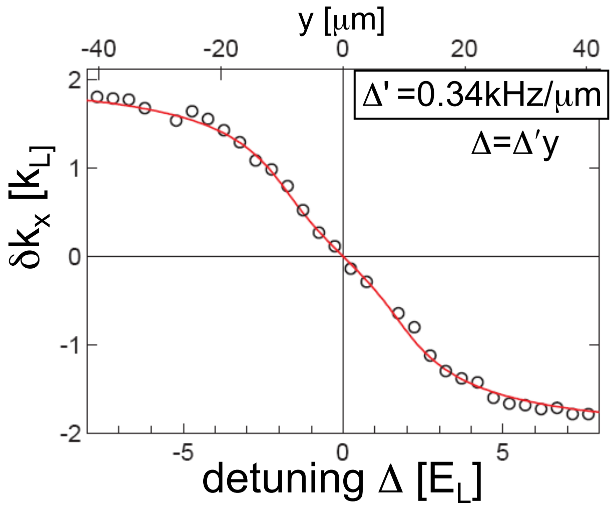
$$\vec{B}^* = \nabla \times \vec{A}^*$$

- magnetic field B*

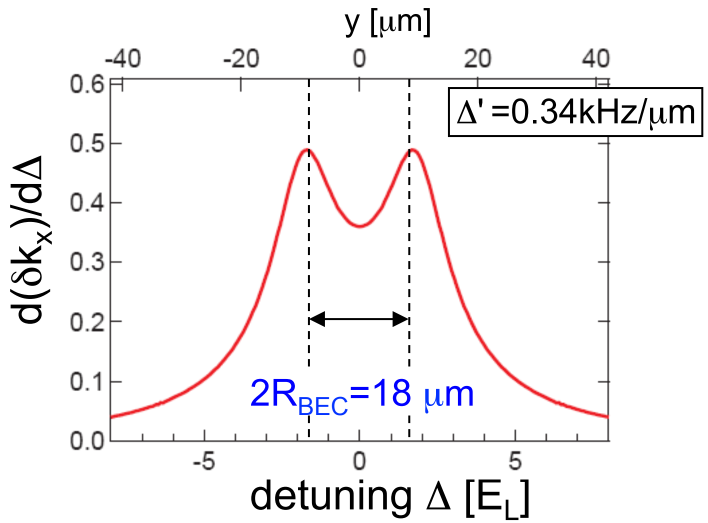
$$\frac{q^* B^*}{\hbar} = \frac{\partial(\delta k_x)}{\partial y} = \Delta' \frac{\partial(\delta k_x)}{\partial \Delta}$$

detuning gradient $\Delta' = \frac{d\Delta}{dy}$

A^* vs. Δ or y



B^* vs. Δ or y



Vortex number N_v vs. B^*

- For infinite system size:
  $N_v = \text{area} \times B^*/(h/q^*)$
  $= \Phi_{B^*} /(h/q^*)$

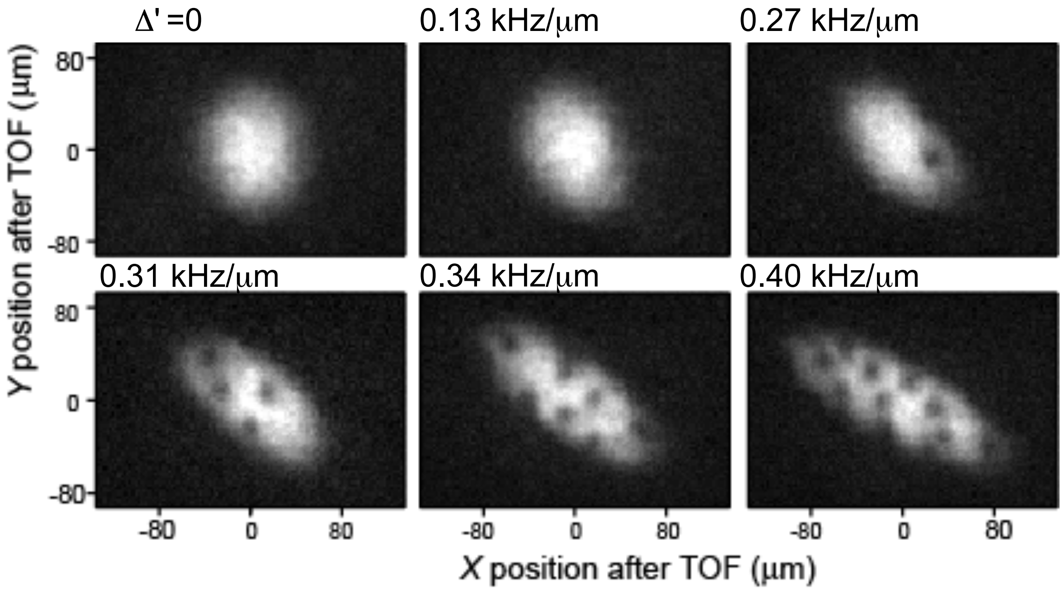
- threshold energy E_v to create N_v=1
  for finite system radius R: $E_v \propto 1/R^2$

R_{BEC}=9 μm (in situ)

$$\frac{q^* B^*}{\hbar} = \Delta' \frac{\partial(\delta k_x)}{\partial \Delta}$$

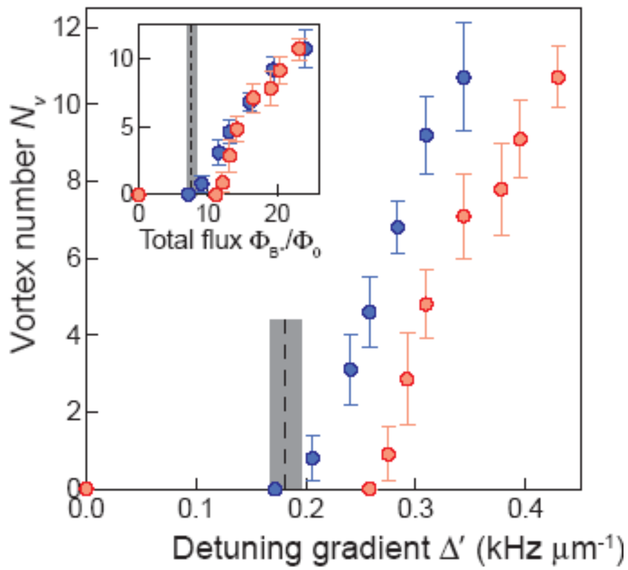
TOF images vs. gradient $\Delta' \propto B^*$

Ω_R= 8.2 E_L, t_h=0.6s



Vortex number N_v vs. gradient Δ'

● Ω_R= 5.9 E_L ● Ω_R= 8.2 E_L



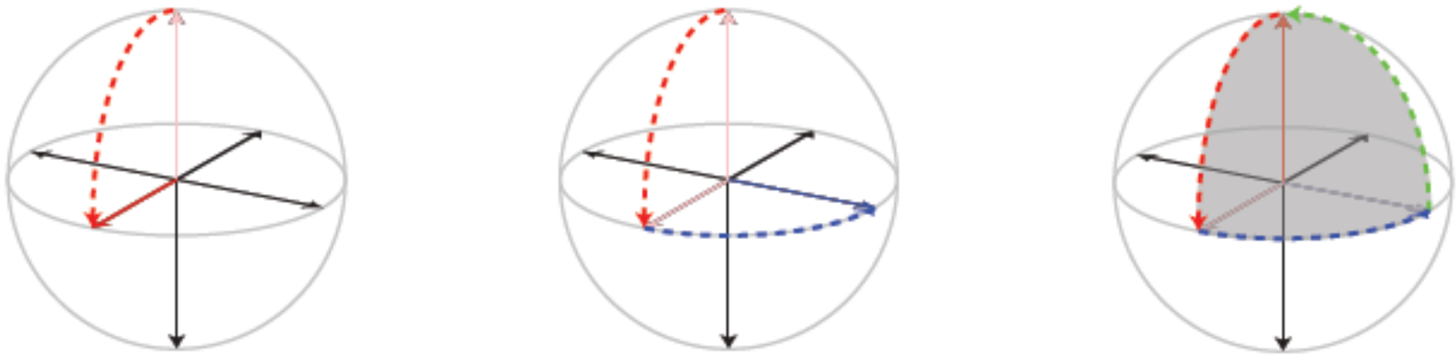
Ref: Y.-J. Lin et al., Nature **462**, 628 (2009).

Berry's phase

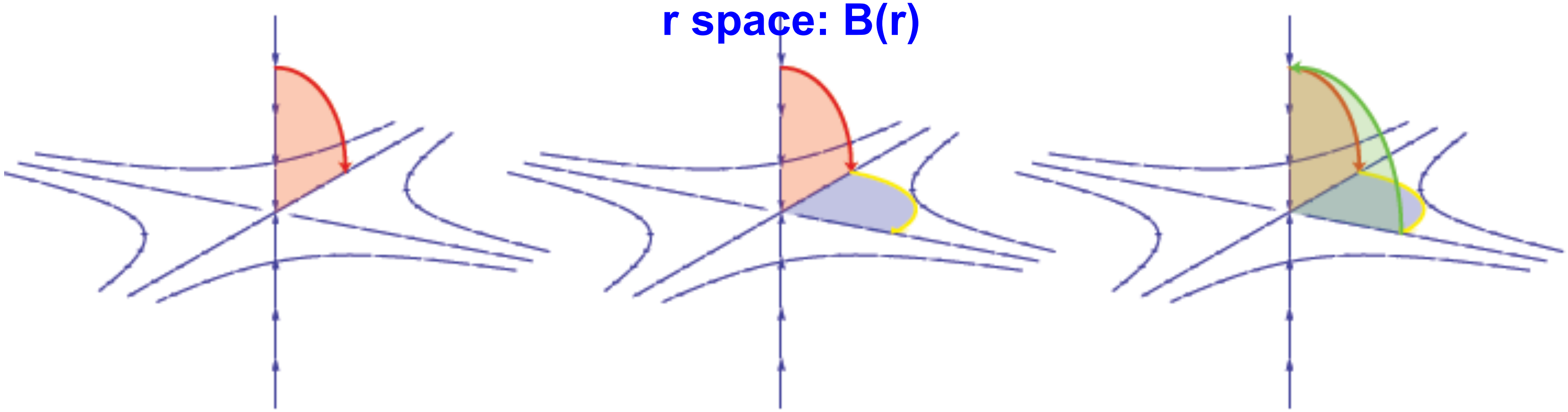
Atoms: S=1/2 in quadrupole B(r)

phase= -1/2* solid angle

B space



r space: B(r)



Berry's phase

Atoms dressed in spatially-dependent Raman coupling (Ω_R, Δ)

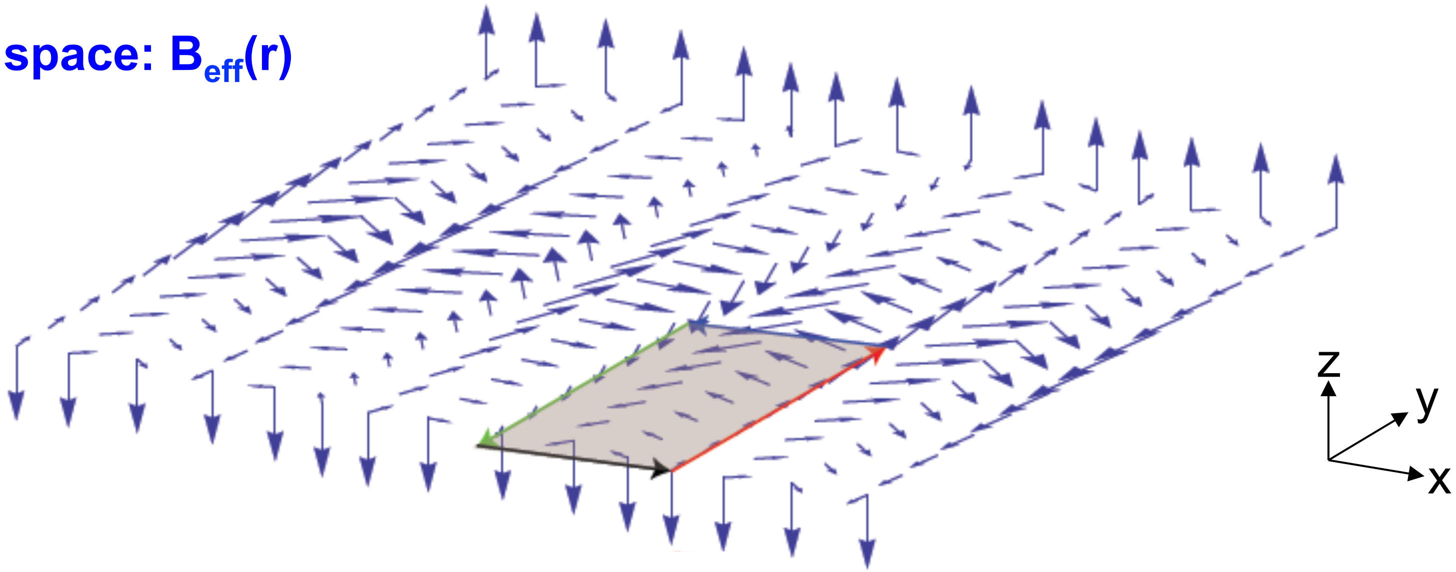
$$H=\frac{\hbar^2\hat{k}^2}{2m}\otimes\hat{1}+\begin{pmatrix}\Delta/2 & \Omega_R e^{i2k_L\hat{x}}/2\\ \Omega_R e^{-i2k_L\hat{x}}/2 & -\Delta/2\end{pmatrix}$$

$$=\frac{\hbar^2\hat{k}^2}{2m}\otimes\hat{1}+\frac{\Delta}{2}\sigma_z+\frac{\Omega_R}{2}\cos(2k_Lx)\sigma_x-\frac{\Omega_R}{2}\sin(2k_Lx)\sigma_y$$

$\Delta = cy$

$-\vec{\sigma}\cdot\vec{B}_{eff}(r)$

r space: B_{eff}(r)



Berry's phase

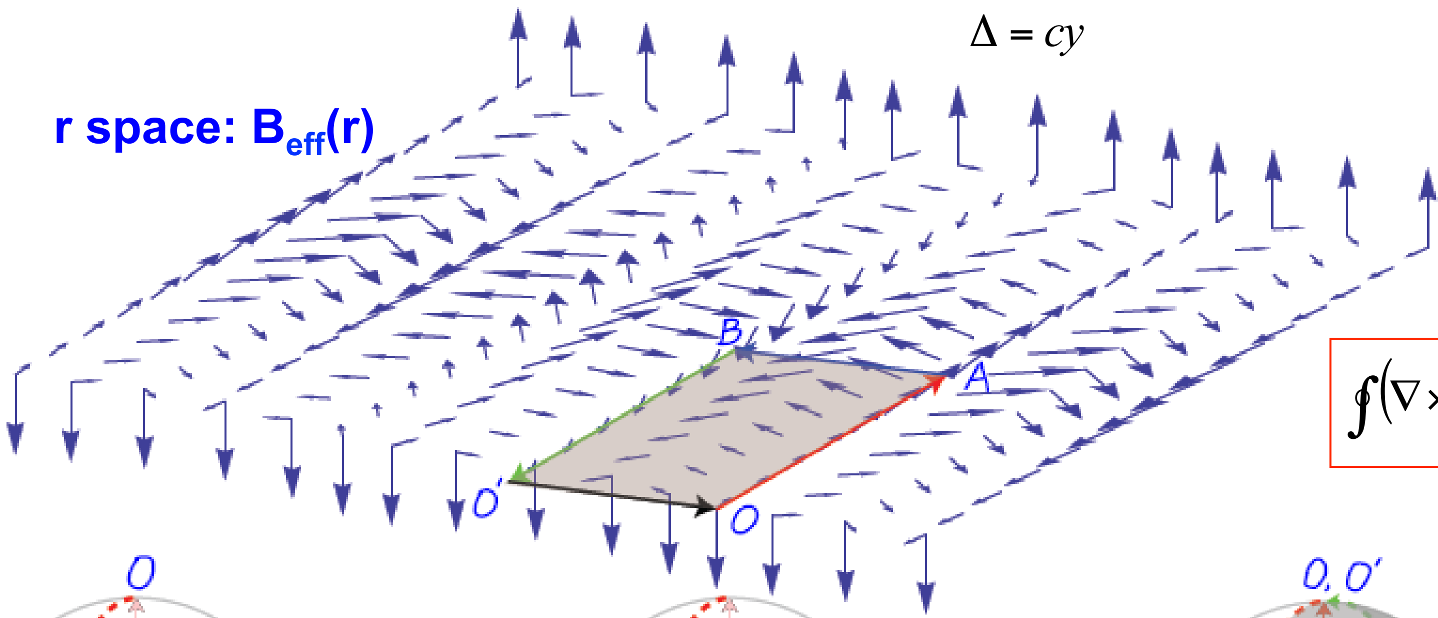
Atoms dressed in spatially-dependent Raman coupling (Ω_R, Δ)

$$H = \frac{\hbar^2 \hat{k}^2}{2m} \otimes \hat{1} + \frac{\Delta}{2}\sigma_z + \frac{\Omega_R}{2}\cos(2k_L x)\sigma_x - \frac{\Omega_R}{2}\sin(2k_L x)\sigma_y$$

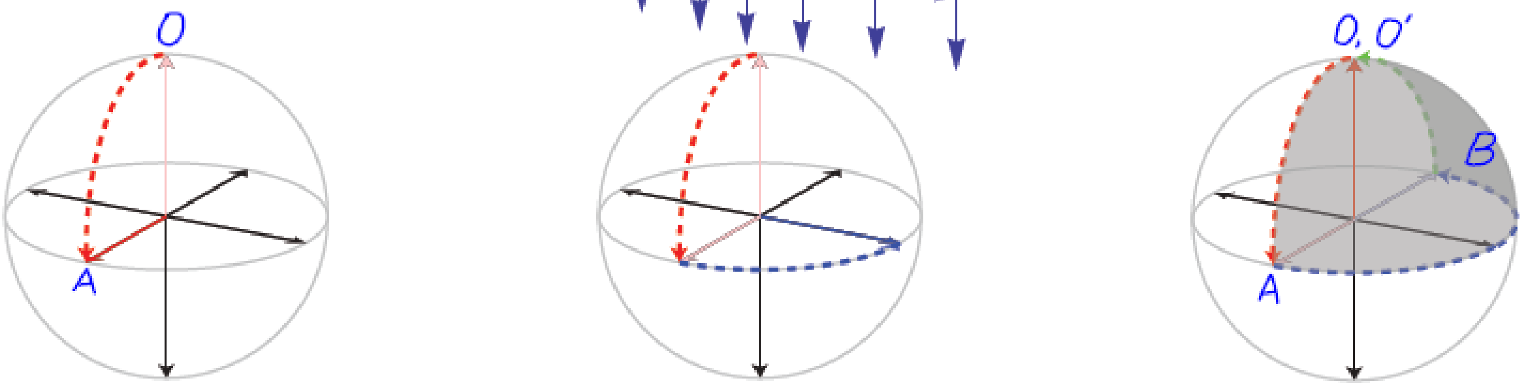
$$\Delta = cy$$

$$-\vec{\sigma} \cdot \vec{B}_{eff}(r)$$

r space: B_{eff}(r)



$$\oint \left(\nabla \times A^*\right)_z \, dxdy = \frac{\gamma_n}{2\pi} \cdot \frac{h}{q^*}$$



B_{eff} space

(=Bloch sphere for Δ[illegible]Ω_R)

Conclusions: synthetic magnetic field B^*

- Observing vortices in a Raman-dressed BEC: superfluid in a synthetic magnetic field B^*
- stable B* in lab frame, easy to add optical lattices
- outlook: Hofstadter butterfly: add 2D lattices
- long term: large B^* in quantum Hall regime
  prepare 2D systems w/ 1D lattice, w/ filling factor $\nu = N_{2D}/N_v \leq 1$

1D lattice along z, compress along y, relax along x
(small N_{2D},large B* and N_v)

Recent related works: towards large B^* not limited by system size

- Experiment: strong B* field in optical lattice, I. Bloch (2011).
- Proposals: optical flux lattice, N. R. Cooper and J. Dalibard (2011).