Dipolar Quantum Gases

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Outline

- *Lecture 1*. Introduction. Dipolar Bose gases
- *Lecture 2*. Dipolar Fermi gases
- *Lecture 3*. Novel macroscopic quantum states

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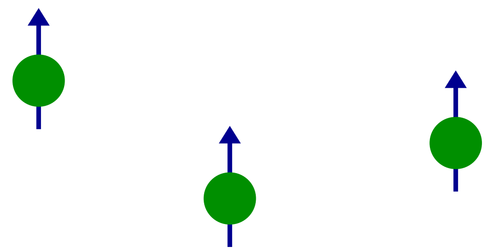
Lecture 1. **Introduction. Dipolar Bose gases**

Outline

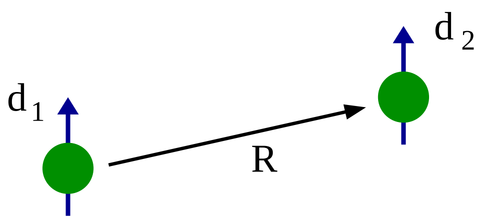
- Experiments with ultracold dipolar particles
- Scattering problem
- Dipolar BEC. Stability problem
- Roton-maxon spectrum and fluctuations in pancaked (2D) condensates

Novel object - Dipolar gas

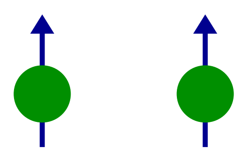
Polar molecules or atoms with a large magnetic moment



Dipole-dipole interaction $V_d = \frac{\vec{d}_1\vec{d}_2R^2 - 3(\vec{d}_1\vec{R})(\vec{d}_2\vec{R})}{R^5} \sim \frac{1}{R^3}$



long-range, anisotropic



repulsion

attraction

Different physics compared to ordinary atomic ultracold gases

Alkali-atom molecules d from $0.6\,D$ for KRb to $5.5\,D$ for LiCs

Atoms with large μ

Remarkable experiments with Cr atoms ($\mu = 6\mu_B \Rightarrow d \approx 0.05$ D)

T. Pfau group (Stuttgart)

Effects of the dipole-dipole interaction in the dynamics

Stability diagram of trapped dipolar BEC

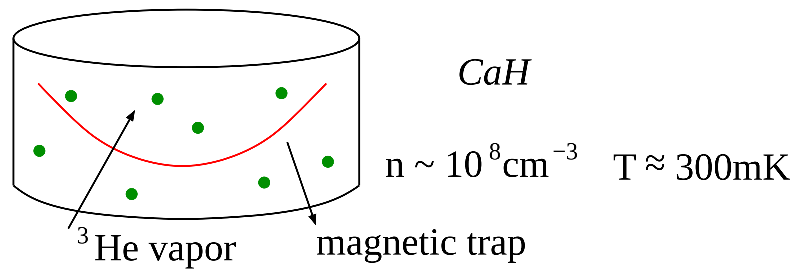
Spinor physics in chromium experiments at Villetaneuse, B. Laburthe-Tolra

Now disprosium ($\mu = 10\mu_B$, (B. Lev))

and erbium ($\mu = 7\mu_B$, (F. Ferlaino)) are in the game

Polar molecules. Creation of ultracold clouds

- Buffer gas cooling (Harvard, J. Doyle)



- Stark deceleration (Meijer, Berlin; JILA)

  H_3, ND_3, CO, etc. ; $T \sim 1mK$ and low density

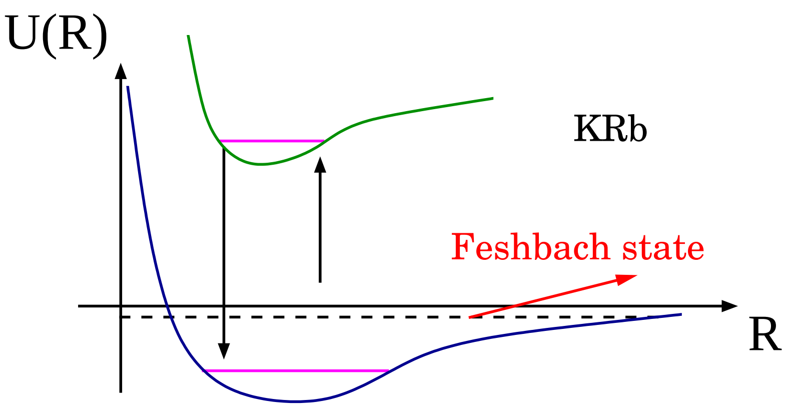
- Optical collisions (photoassociation, D. DMille group, JILA, elsewhere)

Polar molecules. Creation of ultracold clouds

Photoassociation

Transfer of weakly bound KRb fermionic molecules to the ground rovibrational state

JILA, D. Jin, J. Ye groups



$$n \sim 10^{12} - 10^{13} \text{ cm}^{-3}$$
$$T \approx 200nK \ \sim E_F$$

Ground-state LiCs molecules at Heidelberg

Ground-state RbCs molecules in Innsbruck

Ground-state KRb bosonic molecules in Tokyo

Experiments with NaK (MIT, MUnich, Trento) and KCs (Innsbruck) molecules

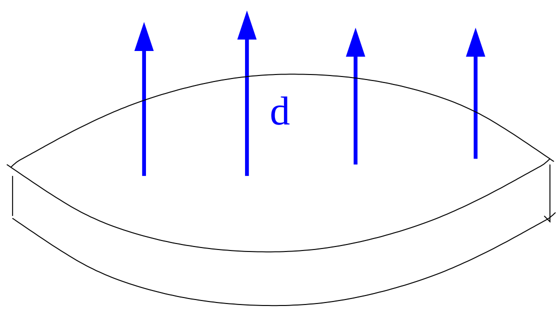
Ultracold chemistry

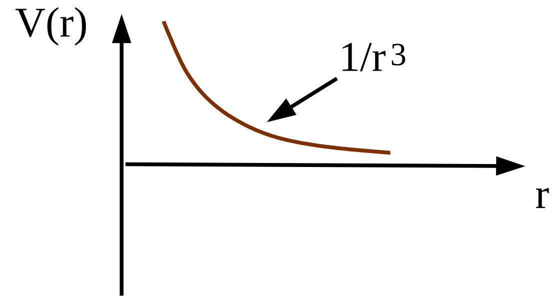
Ultracold chemical reactions $\mathrm{KRb} + \mathrm{KRb} \Rightarrow \mathrm{K}_2 + \mathrm{Rb}_2$

New trends in ultracold chemistry

Suppress instability $\rightarrow$ induce intermolecular repulsion

For example, 2D geometry with dipoles perpendicular to the plane





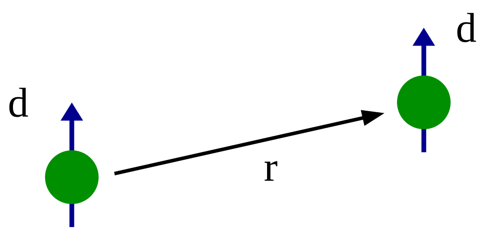
Reduction of the decay rate by 2 orders of magnitude at JILA

Select non-reactive molecules, like NaK, KCs, RbCs

Theoretical studies

- Innsbruck group (P. Zoller, G. Pupillo, M.A. Baranov et al). Large variety of proposals including bilayer systems, Rydberg atoms etc.
- Trento group (S. Stringari et al). Excitation modes etc
- Harvard group (E. Demler, M. Lukin et al). Multilayer systems etc
- Hannover group (L. Santos et al). Spinor and dipolar systems
- Tokyo group (M. Ueda et al) Spinor and dipolar systems
- Cambridge group (N.R. Cooper, Jesper Levinsen). Novel states
- Rice group (H. Pu et al). Excitations and stability etc
- Maryland group (S. Das Sarma et al) Fermi liquid behavior etc
- Taipei group (D.-W. Wang et al)
- Barcelona group (M. Lewenstein et al)

Dipole-diple scattering



$$V_d = \frac{d^2}{r^3} \underbrace{\left(1 - 3\cos^2\theta_{rd}\right)}_{\sim Y_{20}(\theta_{rd})}$$

Wave function of the relative motion

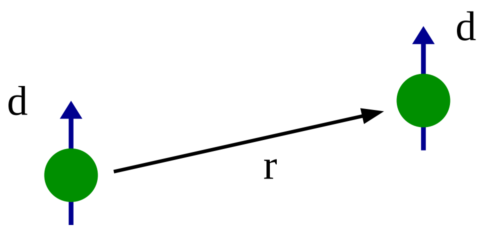
$$\psi_{in} \to \sum_{l,m} \psi_{kl}(r) i^l Y^*_{lm}(\theta_{kd}, \varphi_{kd}) Y_{lm}(\theta_{rd}, \varphi_{rd})$$

$$\psi_{out} \to \sum_{l',m'} \psi_{kl'}(r) i^{l'} Y^*_{l'm'}(\theta_{kd}, \varphi_{kd}) Y_{l'm'}(\theta_{rd}, \varphi_{rd})$$

Scattering matrix $\sim \int Y^*_{lm} Y_{l'm'} Y_{20} d\Omega_{rd}$

V_d couples all even l, and all odd l, but even and odd l are decoupled from each other

Radius of the dipole-dipole interaction



$$\left(-\frac{\hbar^2}{m}\Delta + V_d(\vec{r})\right)\psi(\vec{r}) = \frac{\hbar^2 k^2}{m}\psi(\vec{r})$$

$$\frac{\hbar^2}{m r_*^2} = \frac{d^2}{r_*^3} \Rightarrow r_* \approx \frac{m d^2}{\hbar^2}$$

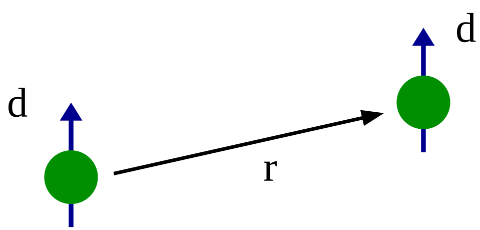
$r \gg r_*$ $\rightarrow$ free relative motion

$r_* \sim 10^6 \div 10^3 a_0$ polar molecules

$r_* \approx 50 a_0 \rightarrow$ chromium atoms

$$k r_* \ll 1 \quad \rightarrow \underbrace{\text{Ultracold limit}}_{T \ll 1mK \text{ for Cr}}$$

Scattering amplitude I



$$V(\vec{r}) = \mathcal{U}(\vec{r}) + V_d(\vec{r})$$

$$f = \int \psi^*_{k_i}(\vec{r})V(\vec{r})e^{i\vec{k}_f\vec{r}}d^3r$$

Ultracold limit $kr_* \ll 1$

$$V_d = 0 \Rightarrow f = g = \frac{4\pi\hbar^2}{m}a$$

What V_d does?

$$k = 0 \to g = \int \psi^*_0(\vec{r})(\mathcal{U}(\vec{r}) + V_d(\vec{r}))d^3r = \text{const}; \qquad r \lesssim r_*$$

g may depend on d and comes from all even l

Scattering amplitude II

$$k \neq 0$$

$$f = \int \psi^*_{k_i}(\vec{r}) V(\vec{r}) e^{i\vec{k}_f \vec{r}} d^3 r$$

$$r \lesssim r_* \rightarrow \text{put } k = 0 \rightarrow g$$

$$r \gg r_* \rightarrow \psi_{k_i} = e^{i\vec{k}_i \vec{r}}$$

$$f = \int V_d(\vec{r}) e^{i\vec{q}\vec{r}} d^3 r \longrightarrow \frac{4\pi d^2}{3}(3\cos^2\theta_{qd} - 1); \; \vec{q} = \vec{k}_f - \vec{k}_i$$

$$f = g + \frac{4\pi d^2}{3}(3\cos^2\theta_{qd} - 1)$$

Dipolar BEC I

Uniform gas

$$H = \int d^3 \Big[\psi^\dagger(\vec{r}) \left(-\frac{\hbar^2}{2m}\Delta\right)\psi(\vec{r}) + \frac{1}{2} g \psi^\dagger(\vec{r})\psi^\dagger(\vec{r})\psi(\vec{r})\psi(\vec{r})$$
$$+\frac{1}{2}\int d^3 r' \psi^\dagger(\vec{r})\psi^\dagger(\vec{r}')V_d(\vec{r}-\vec{r}')\psi(\vec{r}')\psi(\vec{r})\Big]$$

Bogoliubov approach $\psi = \psi_0 + \delta\Psi \rightarrow$ biliniear Hamiltonian

$$H_B = \frac{N^2}{2V} g + \sum_k \Big[\frac{\hbar^2 k^2}{2m} a_k^\dagger a_k + n\left(g + \frac{4\pi d^2}{3}\left(3\cos^2\theta_k - 1\right)\right) a_k^\dagger a_k$$
$$\frac{n}{2}\left(g + \frac{4\pi d^2}{3}\left(3\cos^2\theta_k - 1\right)\right)\left(a_k^\dagger a_{-k}^\dagger + a_k a_{-k}\right)\Big]$$

Dipolar BEC II

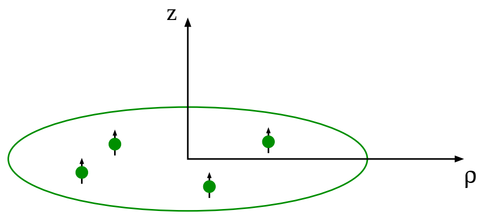
Excitation spectrum

$$\epsilon_k = \sqrt{E_k^2 + 2E_k n \left(g + \frac{4\pi d^2}{3}\left(3\cos^2\theta_k - 1\right)\right)}$$

$g > \frac{4\pi d^2}{3} \rightarrow$ dynamically stable BEC

$g < \frac{4\pi d^2}{3} \rightarrow$ complex frequencies at small k
and $\cos^2\theta_k < \frac{1}{3} \rightarrow$ collapse

Trapped dipolar BEC



Cylindrical trap

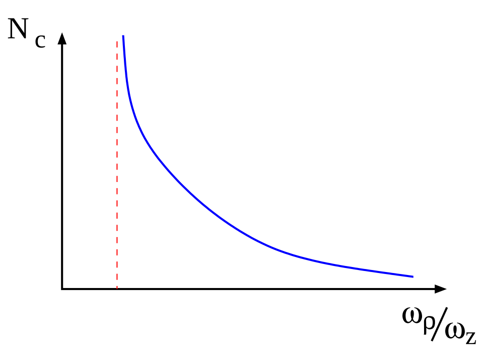
$V_h = \frac{m}{2}\left(\omega_\rho^2\rho^2 + \omega_z^2 z^2\right)$

Gross-Pitaevskii equation

$$\left[-\frac{\hbar^2}{2m}\Delta + V_h(\vec{r}) + g\psi_0^2 + \int \psi_0(\vec{r}')^2 V_d(\vec{r}-\vec{r}')d^3r'\right]\psi_0(\vec{r}) = \mu\psi_0(\vec{r})$$

Important quantity

$$V_{eff} = g\int \psi_0^4(\vec{r})d^3r + \int \psi_0^2(\vec{r}')V_d(\vec{r}-\vec{r}')\psi_0^2(\vec{r})d^3r d^3r'$$



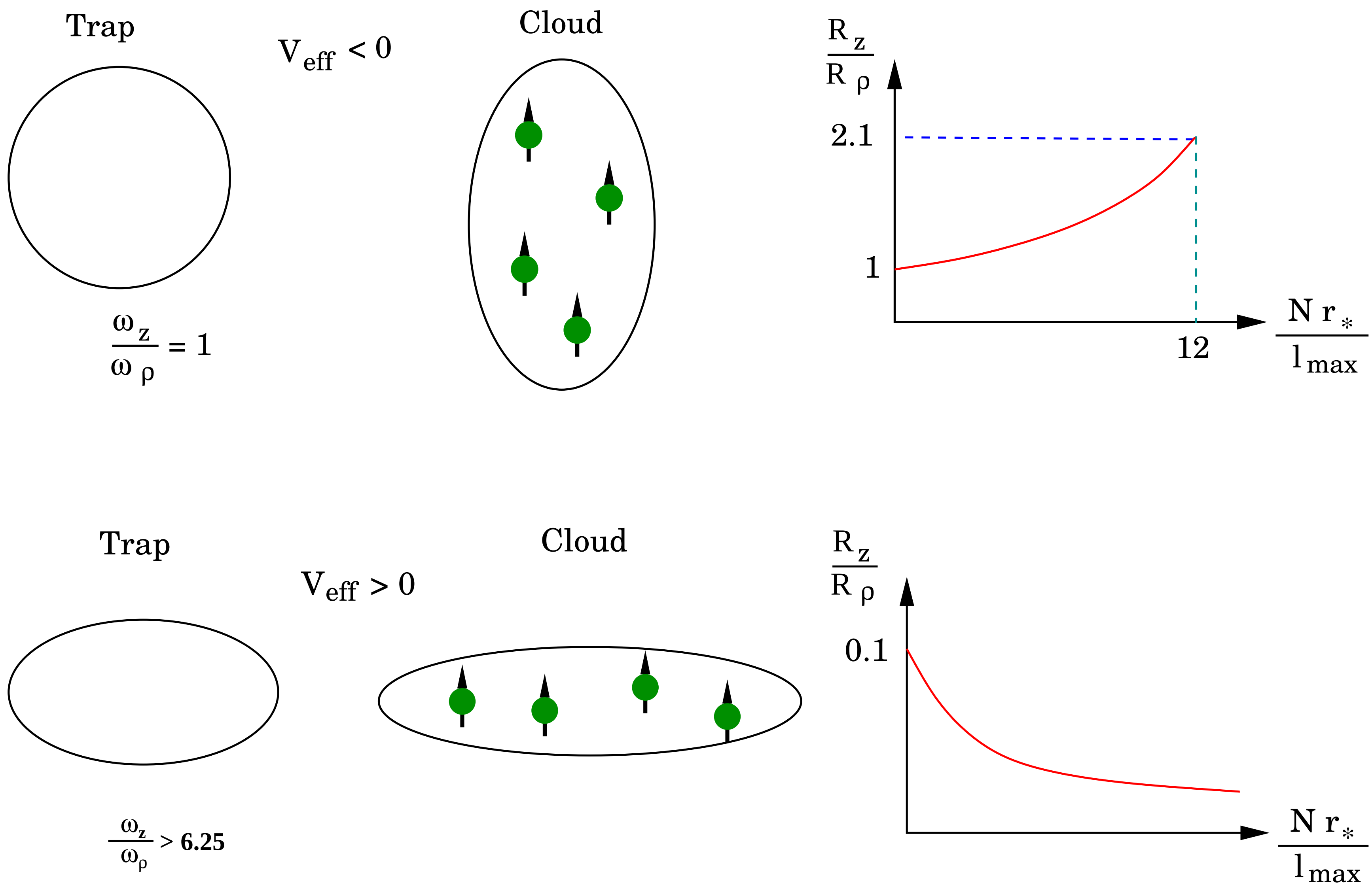
$V_{eff} > 0$ or $V_{eff} < 0$ and $|V| < \hbar\omega$

$g = 0 \to N < N_c \to$ suppressed

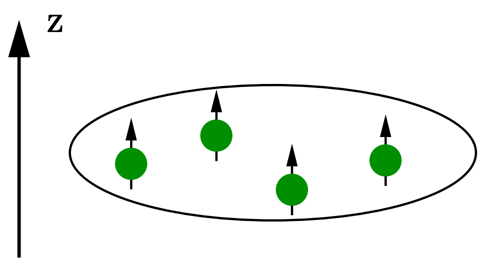
low k instability

(Santos et.al, 2000)

It is sufficient?



Stability problem



Dipolar BEC

$$\langle V_d \rangle = \int n_0(\vec{r}') V_d(\vec{r}' - \vec{r}) d^3 r = -d^2 \frac{\partial^2}{\partial z^2} \int \frac{n_0(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 r$$

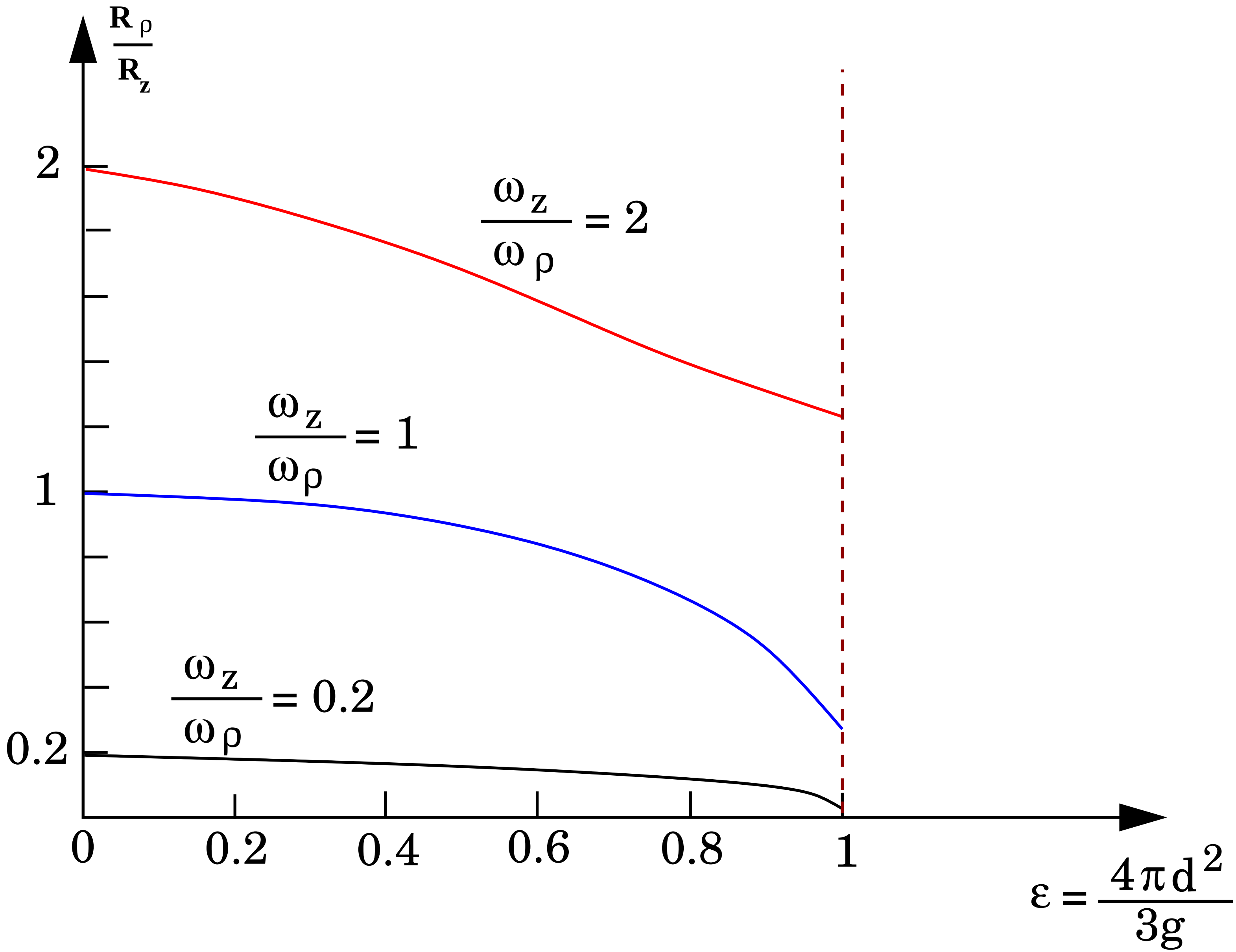
$$V_d = -d^2 \frac{\partial^2}{\partial z^2} \frac{1}{|\vec{r} - \vec{r}'|} - \frac{4\pi d^2}{3} \delta(\vec{r} - \vec{r}')$$

Large $N \Rightarrow$ Thomas-Fermi BEC

$$n_0 = n_{0\,max} \left(1 - \frac{z^2}{R_z^2} - \frac{\rho^2}{R_\rho^2} \right)$$ Eberlein et. al (2005)

$g > \frac{4\pi d^2}{3} \rightarrow$ stable at any N

Example



$\frac{R_\rho}{R_z}$
2
1
0.2
$\frac{\omega_z}{\omega_\rho} = 2$
$\frac{\omega_z}{\omega_\rho} = 1$
$\frac{\omega_z}{\omega_\rho} = 0.2$
0
0.2
0.4
0.6
0.8
1
$\varepsilon = \frac{4\pi d^2}{3g}$

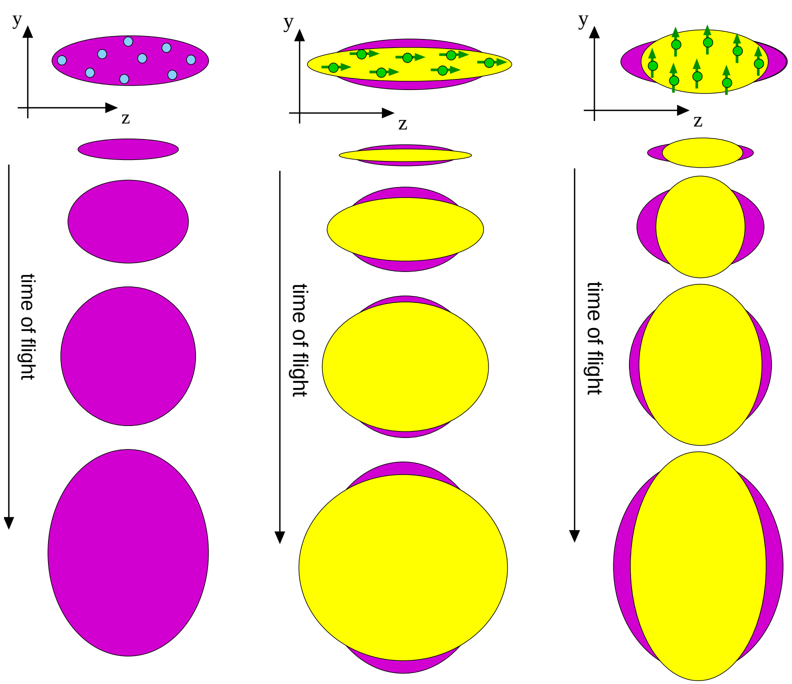
Experiment with Cr

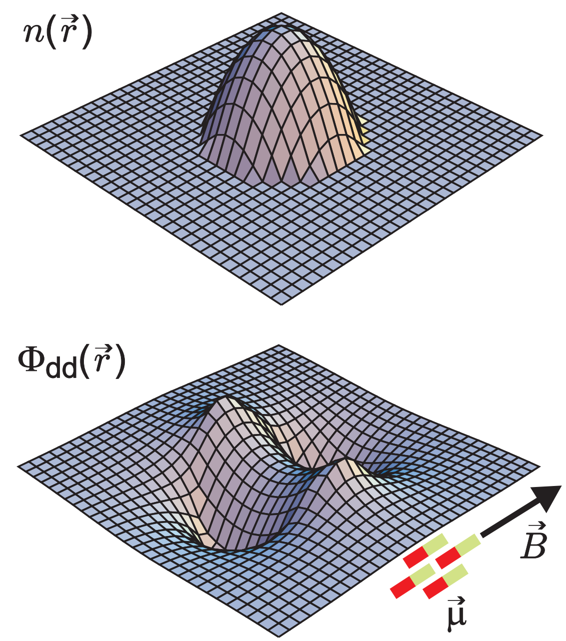
$$g > \frac{4\pi d^2}{3}$$

($\mu = 6\mu_B$!)

(T. Pfau, Stuttgart) BEC ($n \sim 10^{14} cm^{-3}$)

effect of the dipole-dipole interaction (small)



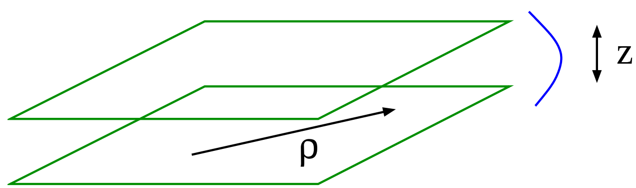


Pancake dipolar BEC

$$g < g_d = \frac{4\pi d^2}{3} \quad ?$$

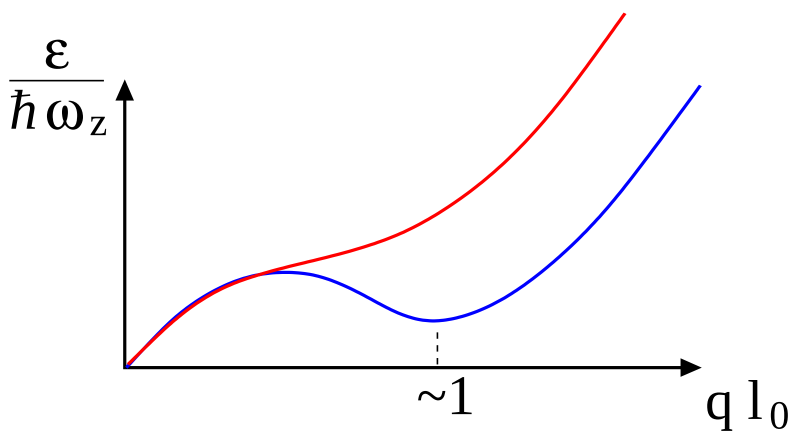
Thomas-Fermi in the z direction

Extreme pancake ($\omega_\rho = 0$)



$$l_0 = \left(\frac{\hbar}{m\omega_z} \right)^{1/2}$$

$V_d + g$ (short-range) $\quad g > 0$



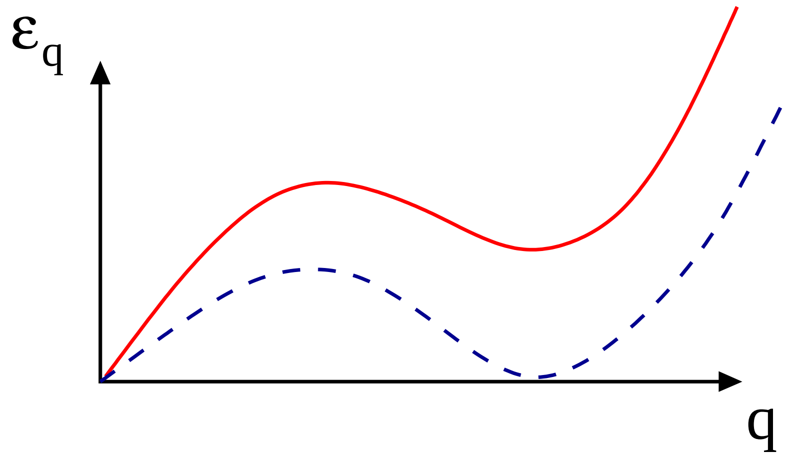
$g/g_d = 1.06 \qquad \mu/\hbar\omega_z = 46$

$g/g_d = 0.94 \qquad \mu/\hbar\omega_z = 53$

The reason for the roton structure

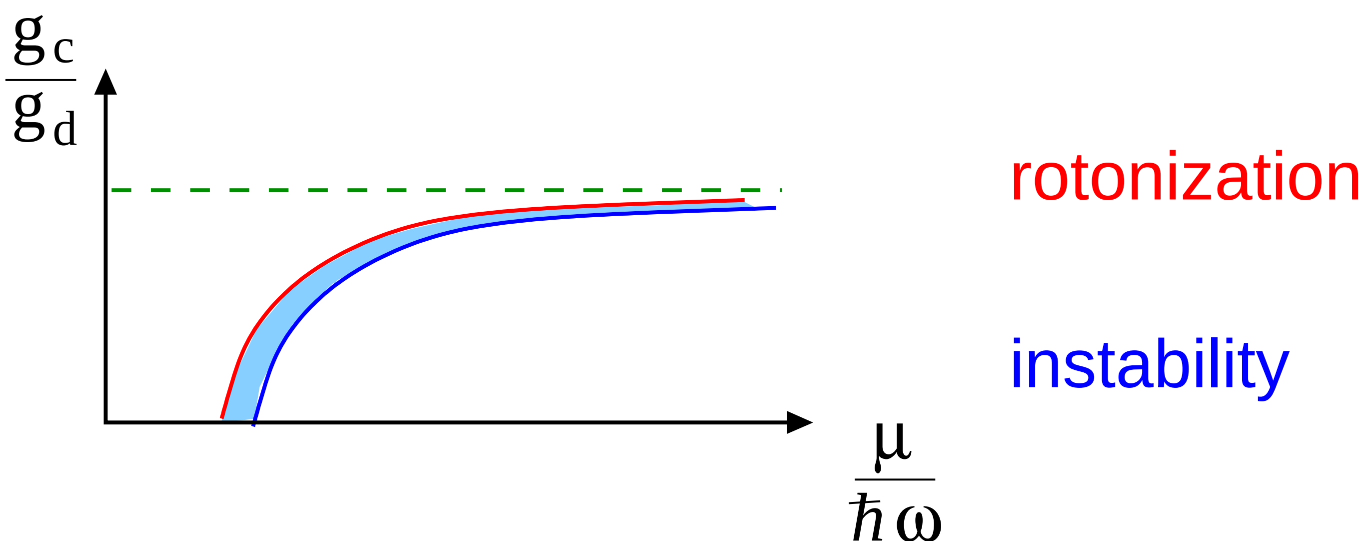
$\rightarrow$ decrease of the interaction amplitude

Roton-maxon structure



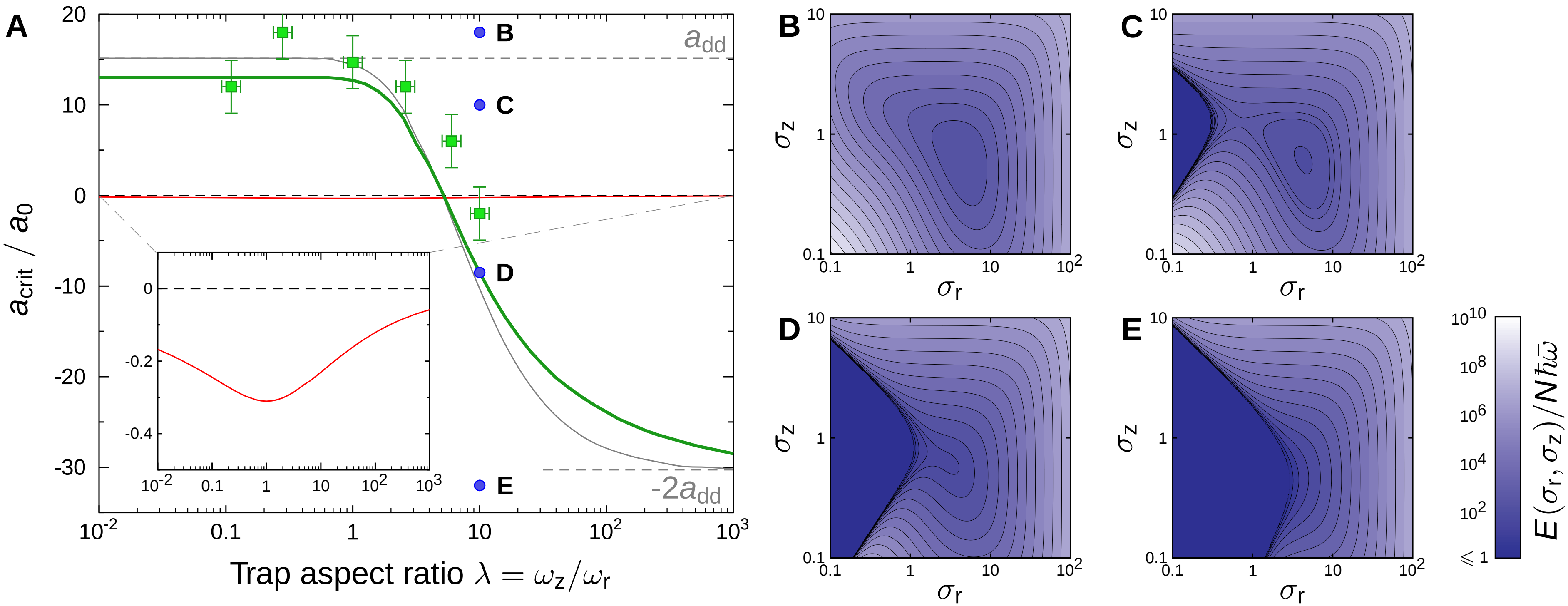
Roton minimum can be but at zero

Instability!

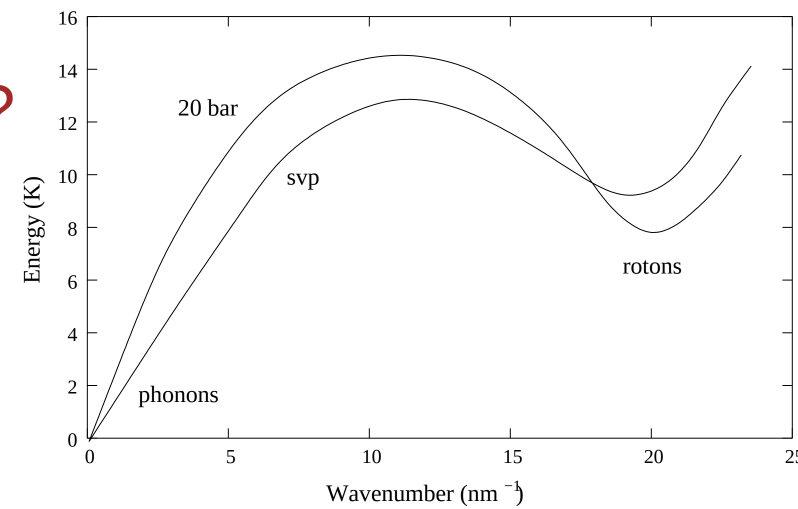


(L. Santos et al., 2003)

Stuttgart experiment



Ideas from superfluid ^4He



Why the roton-maxon structure?

R. Feynman

$$\epsilon_q = \frac{q^2}{2mS(q,\epsilon)}$$

How to put the roton minimum higher (*) or lower (**)?

What happens? (S.Balibar, P. Nozieres, L. Pitaevskii)

(*) negative pressure (acoustic pulses)

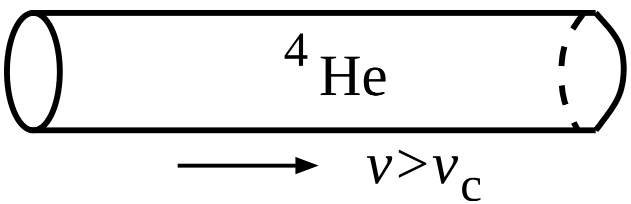
(**) increase the pressure

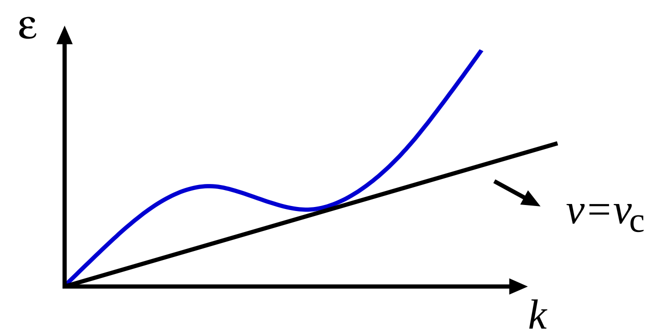
Metastable liquid

(**) $\Rightarrow$ supersolid (density wave), loss of superfluidity, or?

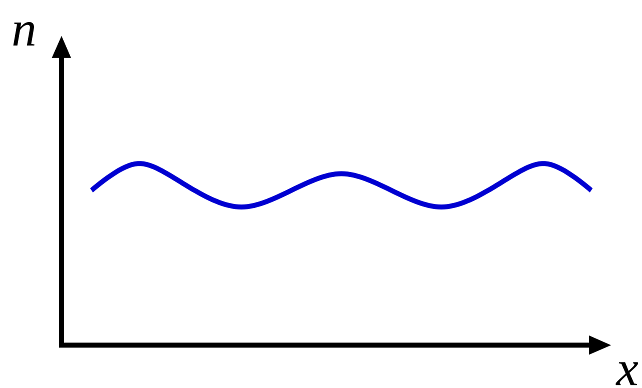
Prehistory

L.P. Pitaevskii(1981)

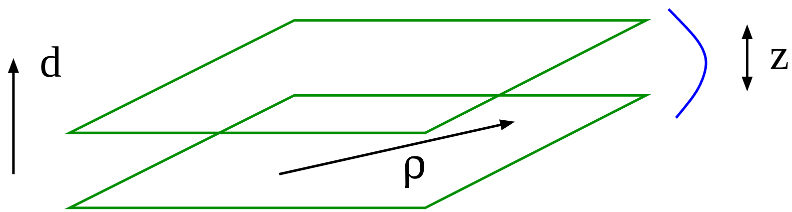




$v > v_c \quad \Rightarrow$ Density wave



Quasi2D dipolar BEC at $T = 0$



$$l_0 = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$$

$$\varphi_0(z) = \frac{1}{\pi^{1/4} l_0^{1/2}} \exp\left\{\frac{-z^2}{2l_0^2}\right\}$$

short range interaction (g) + dipole-dipole

Consider $0 < g \ll \frac{4\pi d^2}{3}$. Then, for $qr_* \ll 1$

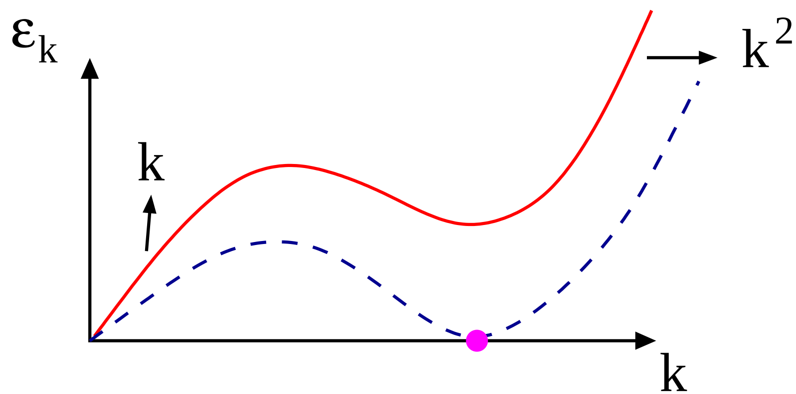
$$V_{\vec{q}\vec{p}} = g(1 - C|\vec{q} - \vec{p}|)$$

where

$$C = \frac{2\pi d^2}{g}$$

Spectrum

$$\hat{H} = \sum_{\vec{k}} \frac{\hbar^2 k^2}{2m} a^\dagger_{\vec{k}} a_{\vec{k}} + \frac{g}{2} \sum_{\vec{k},\vec{q},\vec{p}} (1 - C|\vec{q} - \vec{p}|) a^\dagger_{\vec{k}+\vec{q}} a^\dagger_{\vec{k}-\vec{q}} a_{\vec{k}+\vec{p}} a_{\vec{k}-\vec{p}}$$



$$\epsilon_k^2 = E_k^2 + 2\mu E_k (1 - Ck)$$

$$k_r = \frac{3}{2}\left(1 + \sqrt{\frac{C^2}{\xi^4} - \frac{8}{9\xi^2}}\right) \quad \xi = \frac{\hbar}{mng}.$$

Rotonization

$$\xi \geq C \geq \frac{\sqrt{8}}{3}\xi$$

The roton minimum touches zero for

$$C = \xi \Rightarrow \quad k_r = \frac{2C}{\xi}$$

For $C > \xi$ we have collapse. No stable supersolid state

Pedri/Shlyapnikov; Cooper/Komineas (2007)

Fluctuations I

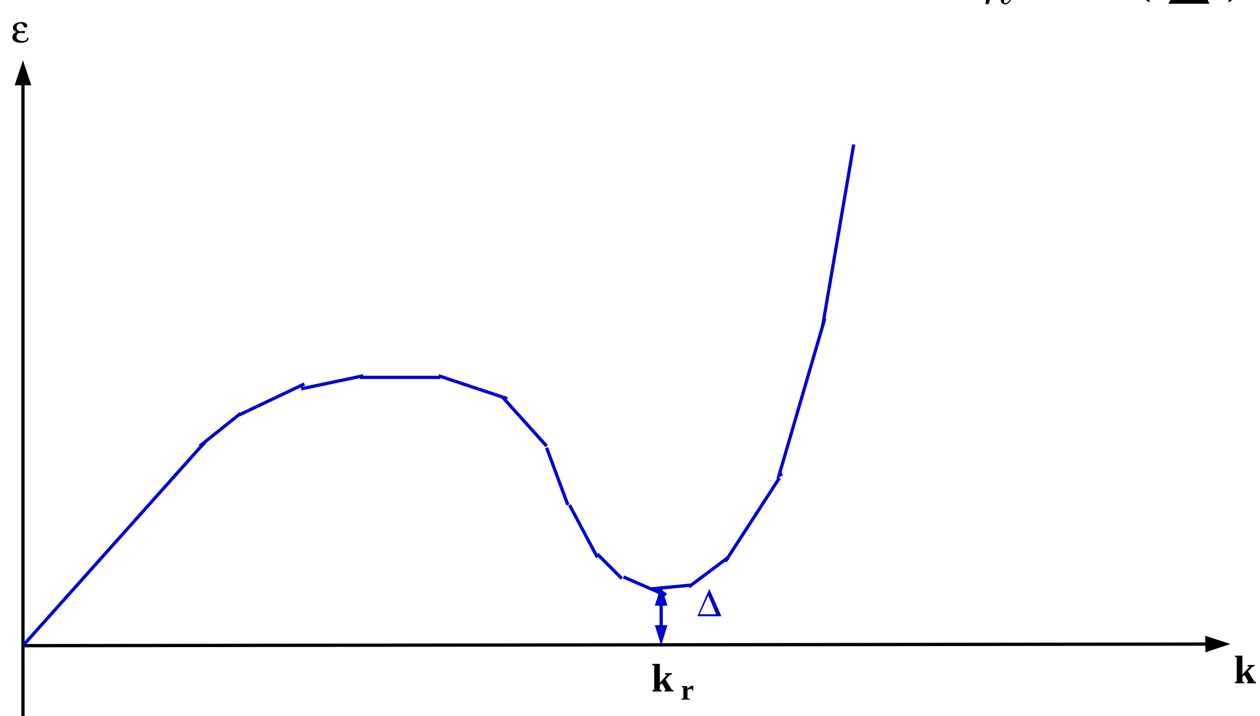
Uniform BEC. Roton close to zero (Δ small) $\Rightarrow$ strong density fluctuations

$$\frac{\langle \delta\hat{n}(0)\delta\hat{n}(r)\rangle}{n^2} \propto \frac{2mg}{\pi\hbar^2}\ln\left(\frac{2ng}{\Delta}\right); \quad \frac{mg}{2\pi\hbar^2} \ll 1$$

non-condensed fraction $\frac{n'}{n} \simeq \frac{mg}{\pi\hbar^2}\ln\left(\frac{2ng}{\Delta}\right)$

Chemical potential $\mu = ng\left[1 + \frac{2mg}{\pi\hbar^2}\ln\left(\frac{2ng}{\Delta}\right)\right]$

Validity of the Bogoliubov approach? Likely $\frac{mg}{\hbar^2}\ln\left(\frac{ng}{\Delta}\right) \ll 1$



Fluctuations II

Finite T (small Δ)

Normal fraction $\frac{n_T}{n} \simeq \frac{2mg}{\hbar^2}\frac{T}{\Delta}$

Significant reduction of the superfluid fraction

The bosons can remain only slightly condensed even at very T, such as $\mu \simeq ng$

Interesting questions

How to describe the emerging state with strong fluctuations ?

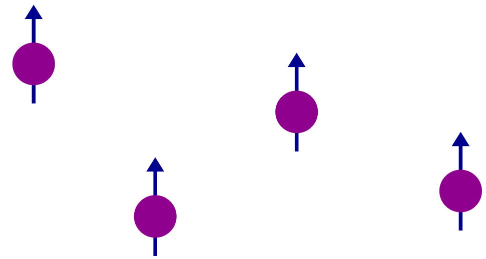
How to manipulate the short-range physics for obtaining a stable supersoliod?

Lecture 2. Dipolar Fermi gases

Outline

- Introduction. Scattering amplitude
- Superfluid pairing in a single-component Fermi gas
- BCS transition temperature
- 2D dipolar Fermi gas. Superfluid-non-superfluid quantum transition
- 1D dipolar Fermi gas. Quantum transition.

Dipolar Fermi gas



What does th dipole-dipole interaction do in a Fermi gas?

- Single component gas

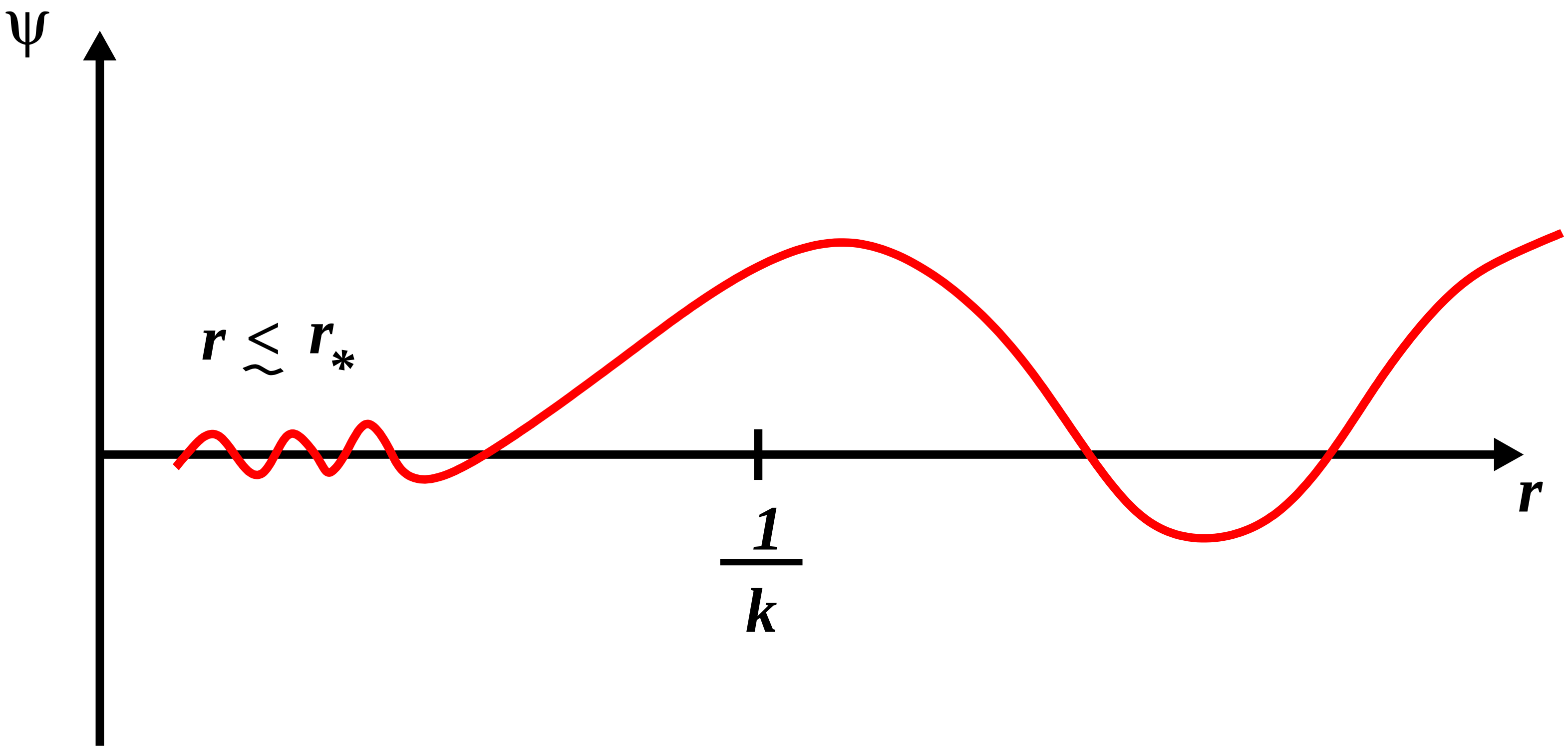
The dipole-dipole scattering amplitude is independent of $|k|$ at any orbital angular momenta alowed by the selection rules.

Long-range cotribution $\sim d^2$

$$\underbrace{\text{Short-range cotribution} \sim k^2}_{\text{omit}}$$

Universal result for f

Physical picture. Stability



nd^2 significantly larger than $E_F = \dfrac{(6\pi^2 n)^{2/3}}{2m}$,

which is $k_F r_*$ significantly larger than 1, leads to collapse

Odd-l scattering amplitude

$$f = \frac{1}{2}\int \left(e^{i\vec{k}_i\vec{r}} - e^{-i\vec{k}_i\vec{r}}\right) V_d(\vec{r}) \left(e^{-i\vec{k}_f\vec{r}} - e^{i\vec{k}_f\vec{r}}\right) d^3r =$$
$$= 4\pi d^2(\cos^2\theta_{dq_-} - \cos^2\theta_{dq_+})$$

$$\vec{q}_\pm = \vec{k}_i - \vec{k}_f$$

$$e^{i\vec{k}\vec{r}} = 4\pi \sum_{l=0}^{\infty}\sum_{m=-l}^{l} j_l(kr) i^l Y_{lm}(\hat{r}) Y^*_{lm}(\hat{k})$$

Partial amplitude $f(l_i m_i; l_f m_f)$

$$f(10;10) = -\frac{6d^2}{5}\cos\theta_{dk_i}\cos\theta_{dk_f}$$

Superfluid p-wave pairing

$$nd^2 \ll E_F$$

Analog of $a \to \sim \frac{md^2}{\hbar^2}$ $(r_*!)$

$$\Delta = g\langle\psi_\uparrow\psi_\downarrow\rangle \sim E_F \exp\left(-\frac{1}{\lambda}\right); \; \lambda \sim \frac{1}{k_F r_*};$$

$$\Delta \propto E_F \exp\left\{-\frac{\pi E_F}{12nd^2}\right\}$$

Cooper pairs are superpositions

of all odd angular momenta (for $m_l = 0$)

Transition temperature

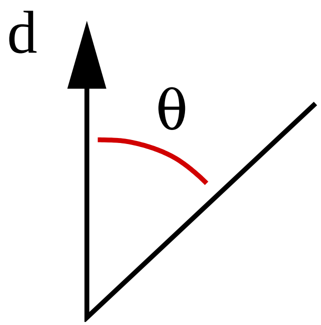
$$nd^2 \ll E_F$$

$$T_c = 1.44 E_F \exp\left\{-\frac{\pi E_F}{12nd^2}\right\}$$

Baranov et.al (2002)

GM correction included

$\Delta \rightarrow$ anisotropic $\propto \sin\left(\frac{\pi}{2}\cos\theta\right)$



- Maximum in the direction of the dipoles.
- Vanishes in the direction perpendicular to the dipoles

Distiguished features

Anisotropic gap $\rightarrow$ gapless excitations

in the direction perpendicular to

the dipoles. Damping rates etc.

Anisotropy $\rightarrow$ different from p-wave

superfluid B and A phases of ^{3}He. In B

Δ is isotropic, and in A it vanishes only at

2 points on the Fermi shpere ($\theta = 0$ and $\theta = \pi$)

Value of T_c

$$d \sim 0.1 \div 1\ \mathcal{D}$$

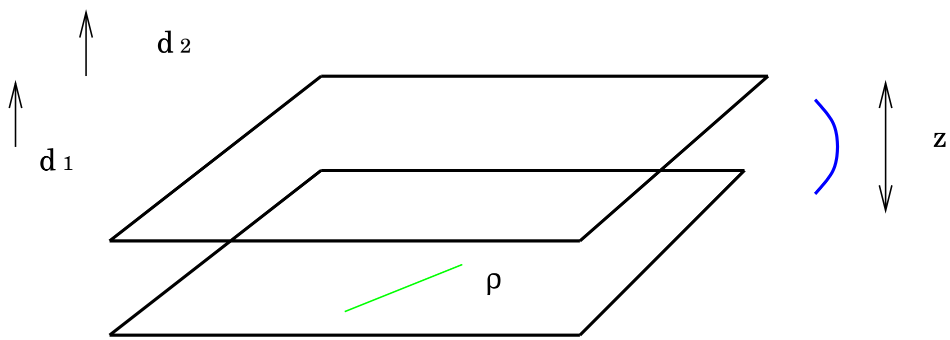
NaK $\Rightarrow$ $d = 2.7\mathcal{D}$ and can be made $0.5\mathcal{D}$ ($r_* = 2500\mathring{A}$) in a certain electric field

$\frac{T_c}{E_F} \to 0.025$ at $n \to 6 \times 10^{12}\mathrm{cm}^{-3}$ ($E_F \approx 400$ nK

($T_c \to 10nK$)

One easily achieves the strongly interacting regime

2D dipolar Fermi gas



$$l_0 = \left(\frac{\hbar}{m\omega_z}\right)^{1/2}$$

$$\varphi_0(z) = \frac{1}{\pi^{1/4} l_0^{1/2}} \exp\left\{\frac{-z^2}{2l_0^2}\right\}$$

2-component Fermi gas (↑ and ↓). Short range $g > 0$

$$H = \sum_k \frac{\hbar^2 k^2}{2m}\left(a^\dagger_{k\downarrow} a_{k\downarrow} + a^\dagger_{k\uparrow} a_{k\uparrow}\right) +$$
$$+|g| \sum_{k,p,q} \left(1 - C|\vec{q} - \vec{p}|\right) a^\dagger_{k+q\uparrow} a^\dagger_{k-q\downarrow} a_{k-p\downarrow} a_{k+p\uparrow}$$

$$C = \frac{2\pi d^2}{|g|}$$

Interesting problem

s-wave interaction on the Fermi surface

$$|g|(1 - 4Ck_F/\pi)$$

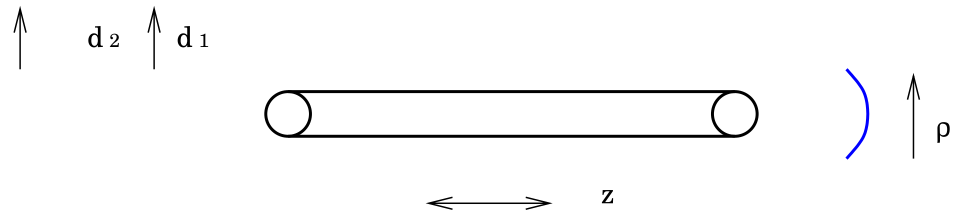
$k_F C > \pi/4 \rightarrow 8d^2 k_F > |g| \rightarrow$ superfluidity

$k_F C < \pi/4 \rightarrow 8d^2 k_F < |g| \rightarrow$ no superfluidity

Quantum transition to a normal state with decreasing density

Superfluid pairing for tilted dipoles $\rightarrow$ Baranov/Sieberer (2011)

1D dipolar Fermi gas



$$l_0 = \left(\frac{\hbar}{m\omega_\rho}\right)^{1/2}$$

$$\varphi_0(\rho) = \frac{1}{\pi^{1/2} l_0} \exp\left\{\frac{-\rho^2}{2l_0^2}\right\}$$

short range $\quad g > 0$

$$H = \sum_k \frac{\hbar^2 k^2}{2m} \left(a^\dagger_{k\downarrow} a_{k\downarrow} + a^\dagger_{k\uparrow} a_{k\uparrow}\right) +$$

$$+|g| \sum_{k,p,q} \left(1 + B|\vec{q} - \vec{p}|^2 \ln(|\vec{q} - \vec{p}| l_0)\right) a^\dagger_{k+q\uparrow} a^\dagger_{k-q\downarrow} a_{k+p\downarrow} a_{k-p\uparrow}$$

$$B = \frac{d^2}{|g|}$$

Quantum transition

Interaction at the Fermi points

$$g_{eff} = |g|[1 + 2B(k_F l_0)^2 \ln(k_F l_0)]$$

$g_{eff} < 0 \rightarrow$ superfluid

$g_{eff} > 0 \rightarrow$ ordinary Luttinger liquid

Quantum transition superfluid-Luttinger liquid with decreasing density

Lecture 3. Novl macroscopic quantum statea

Outline

- RF-dressed polar molecules
- Topologcal $p_x + ip_y$ phase in 2D
- Bilayer systems of dipolar fermions. BCS-BEC crossover
- p-wave interlayer superfluids

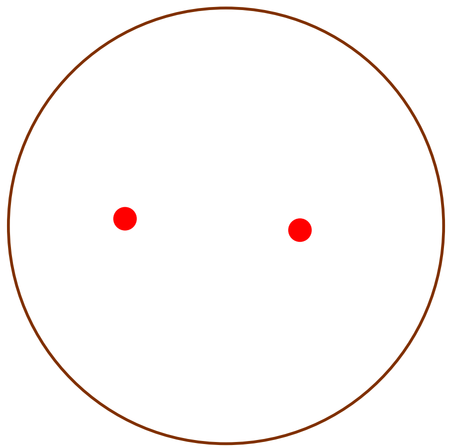
Are there novel states with single-component fermions?

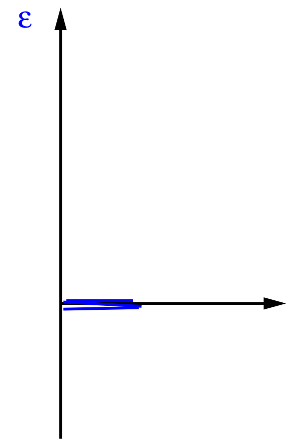
Does the dipole-dipole interaction lead to the emergence of novel phases for identical fermions?

Why single-component fermions are interesting?

Topological aspects of $p_x + ip_y$ state in 2D

Vortices. Zero-energy mode related to two vortices. (Read/Green, 2000)





The number of zero-energy states exponentially grows with the number of vortices $2^{(N_v/2-1)}$

Non-abelian statistics $\Rightarrow$ Exchanging vortices creates a different state!

Non-local character of the state. Local perturbation does not cause decoherence

Topologically protected state for quantum information processing

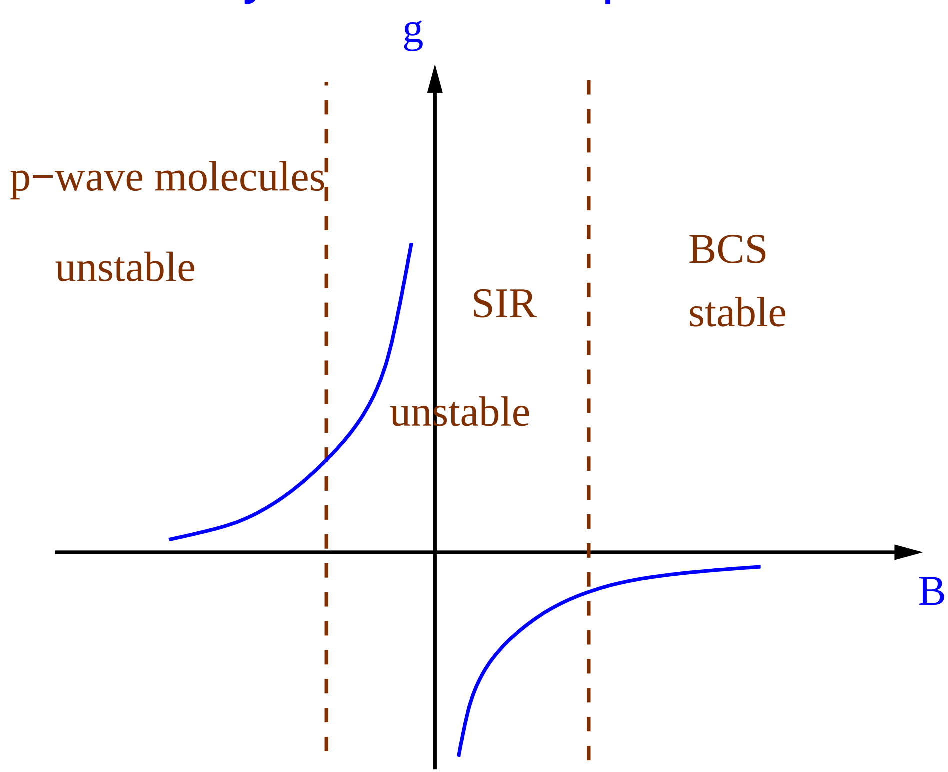
p-wave resonance for fermionic atoms

p-wave resonance   Experiments at JILA, ENS, Melbourne, Tokyo, elsewhere

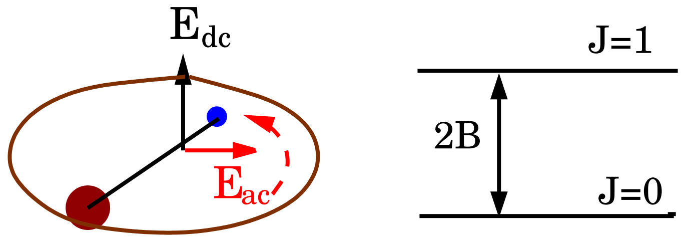
BCS $\Rightarrow$ $\quad T_c \sim \exp\left(-\frac{1}{(k_F b)^2}\right)$ practically zero

Molecular and strongly interacting regimes $\Rightarrow$ rather high T_c, but collisional instability

Gurarie/Radzihovsky; Gurarie/Cooper; Castin/Jona-Lazinio



RF-dressed polar molecules in 2D. Innsbruck idea



σ^+ polarized RF E_{ac}

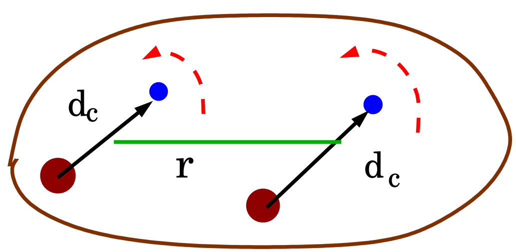
+ constant $E_{dc} \longrightarrow 0$

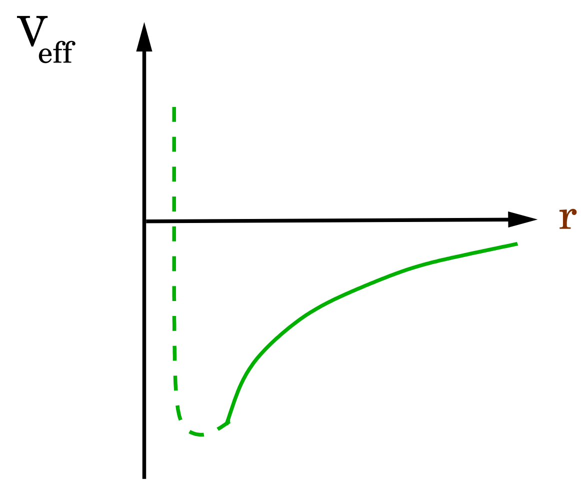
Dressed states $|+\rangle = \alpha|0,0\rangle + \beta|1,1\rangle; \quad , |-\rangle = \beta|0,0\rangle - \alpha|1,1\rangle$

$$\alpha = -\frac{A}{\sqrt{A^2+\Omega^2}}; \quad \beta = \frac{\Omega}{\sqrt{A^2+\Omega^2}}; \quad A = \frac{1}{2}(\delta + \sqrt{\delta^2 + 4\Omega^2})$$

Two RFD molecules in 2D. The dipole moment is rotating with RF frequency

$d_c = 2^{1/2} \alpha \beta d$





Large $r \rightarrow \quad V_{eff} = \langle (1 - 3\cos^2\phi) \rangle \frac{d_c^2}{r^3} = -\frac{d_c^2}{2r^3}; \quad r_* = m d_c^2 / 2\hbar^2$

Fermionic RFD molecules. Superfluid transition

Fermionic RFD molecules in a single quantum state in 2D

Attractive interaction for the p-wave scattering ($l=\pm 1$)

$$\hat{H}=\int d^2r\,\hat{\Psi}^{\dagger}(\mathbf{r})\{-(\hbar^2/2m)\Delta+\int d^2r'\hat{\Psi}^{\dagger}(\mathbf{r}')V_{eff}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r}')-\mu\}\hat{\Psi}(\mathbf{r})$$

$$\Delta(\mathbf{r}-\mathbf{r}')=\langle V_{eff}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}')\rangle$$

Gap equation $\Delta(\mathbf{k})=-\int\frac{d^2k}{(2\pi)^2}V_{eff}(\mathbf{k}-\mathbf{k}')\Delta(\mathbf{k}')\frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')}$

$$\epsilon(k)=\sqrt{(\hbar^2k^2/2m-\mu)^2+|\Delta(k)|^2};\quad \mu\approx E_F$$

$$T_c\approx E_F\exp(-3\pi/4k_Fr_*)$$

$\Delta(\mathbf{k})=\Delta\exp(i\phi_k)$ $\quad p_x+ip_y$ state ($l=\pm 1$)

Superfluid transition. Role of anomalous scattering

For short-range potentials should be $V_{eff} \propto k^2$ and $T_c \propto \exp(-1/(k_F b)^2)$
This is the case for the atoms

Anomalous scattering in $1/r^3$ potential $\rightarrow$ Contribution from $r \sim 1/k$

$$V_{eff}(k) = -\frac{8\hbar^2}{3m}(kr_*); \qquad |k| = |k'|$$

$$T_c \propto \exp\left(-\frac{1}{\nu(k_F)|V_{eff}(k_F)|}\right); \qquad \nu = \frac{m}{2\pi\hbar^2}$$

$$T_C \propto \exp\left(-\frac{3\pi}{4k_F r_*}\right)$$

Transition temperature

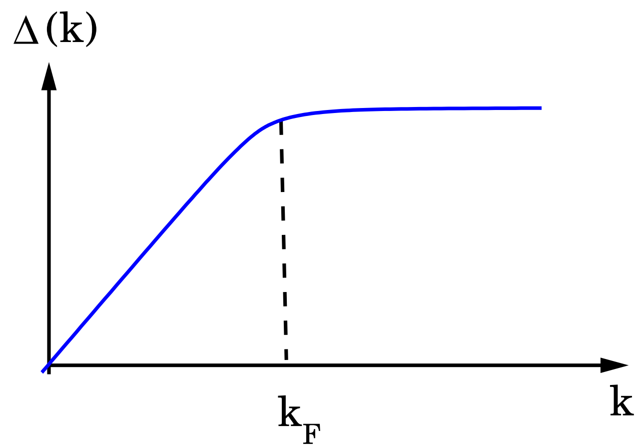
Do better than simple BCS. Reveal the role of short-range physics

Renormalized gap equation

$$\Delta(\mathbf{k}') = -\int f(\mathbf{k}', \mathbf{k})\Delta(\mathbf{k}) \left\{ \frac{\tanh[\epsilon(k)/2T]}{2\epsilon(k)} - \frac{1}{(E_k - E_{k'} - i0)} \right\} \frac{d^2k}{(2\pi)^2}$$

$\Delta(\mathbf{k}) = \Delta(k)\exp(i\phi_k)$; $f(\mathbf{k}', \mathbf{k}) = f(k', k)\exp[i(\phi_k - \phi_{k'})]$ scattering amplitude

$$\Delta(k) = \Delta(k_F) f(k, k_F)/f(k_F, k_F)$$



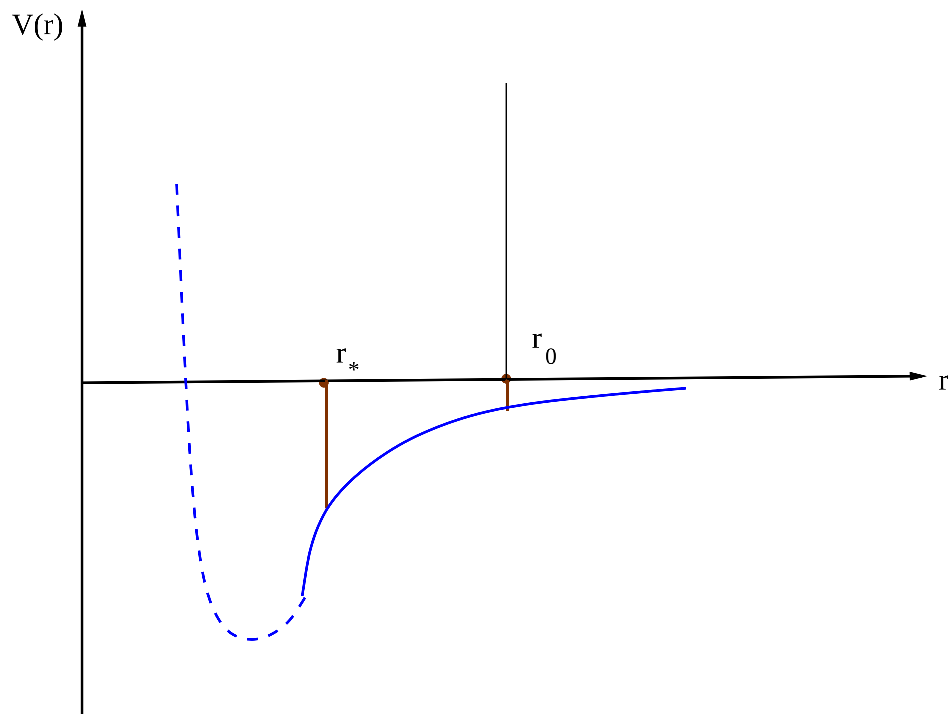
2D scattering in the potential with a $1/r^3$ tail

Scattering amplitude. No transparent exact solution for a finite k

Asymptotic method for slow scattering ($kr_* \ll 1$)

Divide the range of distances into two parts, $r < r_0$ and $r > r_0$

The distance r_0 is such that $r_0 \gg r_*$, but $kr_0 \ll 1$



$r < r_0$ Match exact zero-energy with free finite-k solution at $r = r_0$: $f \Rightarrow (\pi/2)d^2 r_* k^2 \ln k$

$r > r_0$ interaction as perturbation: $f = -(8\pi/3)d^2 k + (\pi/2)d^2 r_* k^2 \ln k$

Related results for the off-shell scattering amplitude

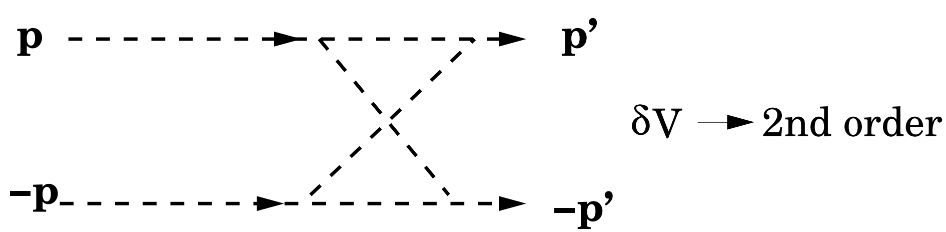
Manipulate T_c?

$$f(k',k) = -\pi d^2 k F\left(\frac{1}{2}, -\frac{1}{2}, 2, \frac{k^2}{k'^2}\right); \;\; k \le k'; \;\; kr_* \ll 1$$

Include k^2-term $f = \frac{1}{2}\pi d^2 r_* k^2 \ln[kr_* u]$

$$T_c = \frac{2e^C}{\pi} E_F \exp\left\{-\frac{3\pi}{4k_F r_*} - \frac{9\pi^2}{64}\ln[k_F r_* u]\right\}$$

Take into account second-order Gor'kov-Melik-Barkhudarov processes



$$\Delta(\mathbf{k}) = -\int \frac{d^2k'}{(2\pi)^2} f(\mathbf{k},\mathbf{k}') \left\{\frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')} - \frac{1}{2(E_{k'} - E_k)}\right\} \Delta(\mathbf{k}')$$
$$- \int \frac{d^2k'}{(2\pi)^2} \delta V(\mathbf{k},\mathbf{k}') \frac{\tanh(\epsilon(k')/T)}{2\epsilon(k')} \Delta(\mathbf{k}')$$

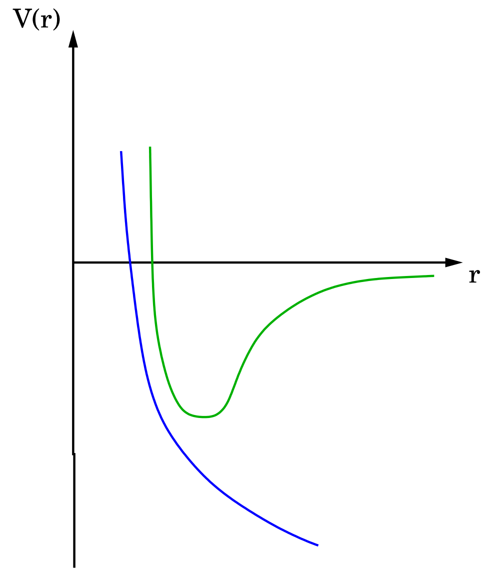
$$T_c = \kappa E_F^{0.3} E_*^{0.7} \exp\left\{-\frac{3\pi}{4k_F r_*}\right\}; \quad E_* = \frac{\hbar^2}{2mr_*^2} \gg E_F$$

κ depends on short-range physics and can be varied within 2 orders of magnitude

Collisional stability and T_c

p-wave atomic superfluids: $\mathrm{BCS} \Rightarrow T_c \to 0$ Resonance $\Rightarrow$ collisional instability

Polar molecules $\Rightarrow$ sufficiently large T_c and collisional stability

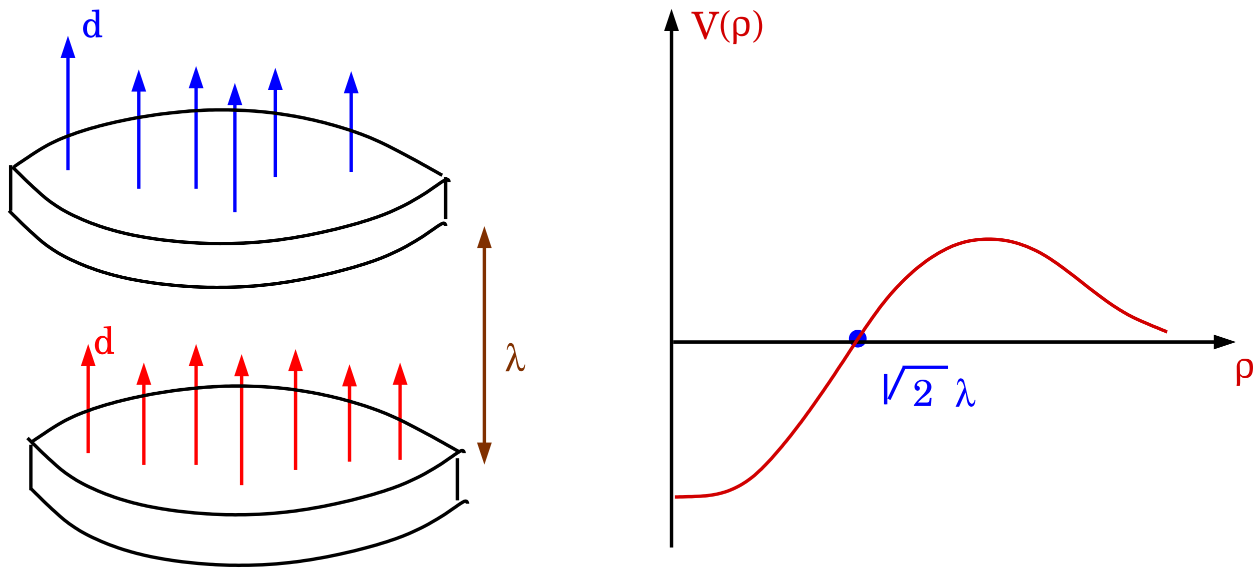


$\alpha_{in} = A\frac{\hbar}{m}(kr_*)^2;$ $A \Rightarrow 10^{-3} - 10^{-4}$ $\alpha_{in} \to (10^{-8} - 10^{-9})$ cm^2/s

LiK molecules $\to$ $d \simeq 3.5$ D $r_* \approx 4000a_0$

$n = 2 \times 10^8$ cm$^{-2} \Rightarrow$ $E_F = 2\pi\hbar^2 n/m = 120$ nK $T_c \approx 10$ nK; $\tau \sim 2$s

Bilayered dipolar fermionic systems



$$V(\rho) = d^2 \left\{ \frac{1}{(\rho^2+\lambda^2)^{3/2}} - \frac{3\lambda^2}{(\rho^2+\lambda^2)^{5/2}} \right\}$$

Dipole-dipole length $r_* = md^2/\hbar^2$ Dipole-dipole strength $\beta = r_*/\lambda$.

$$V_{min} = -\frac{2d^2}{\lambda^3}; \quad \int_0^\infty V(r) r dr = 0$$

Always a bound state of $\uparrow$ and $\uparrow$ dipoles $\rightarrow$ B. Simon, 1974

$$\beta \lesssim 1 \Rightarrow \quad \epsilon_b \simeq \frac{\hbar^2}{m\lambda^2} \exp[-8/\beta^2 + 8/\beta - (5 + 2C - 2\ln 2)]$$

BCS-BEC crossover

$\epsilon_b \ll E_F (r_b \gg n^{-1/2}) \Rightarrow f < 0 \rightarrow$ s-wave BCS pairing

$\epsilon_b \gg E_F \Rightarrow$ Molecules of $\uparrow$ and $\uparrow$ dipoles (interlayer dimers). Molecular BEC

New BCS-BEC crossover (Pikovski, Klawunn, Santos, GS), 2010

Baranov et al, 2011, Zinner et al, 2011

Transition temperature. BCS regime

Kosterlitz-Thouless transition $\epsilon_b \ll E_F \to T_{KT}$ is close to T_{BCS}

$$\Delta(\mathbf{k}) = -\int \frac{dk'^2}{(2\pi)^2} \frac{V(\mathbf{k}-\mathbf{k}')\Delta(\mathbf{k}')}{2\epsilon_{k'}} \tanh\left(\frac{\epsilon_{k'}}{2T}\right)$$

$$\epsilon_k = \sqrt{(E_k-\mu)^2 + |\Delta(k)|^2}; \quad E_k = \hbar^2 k^2/2m$$

$$\Delta(k) = -\frac{\hbar^2}{2m}\int \frac{d^2k'}{(2\pi)^2} f(k,k')\Delta(k')\left\{\frac{\tanh(\epsilon_{k'}/2T)}{\epsilon_{k'}} - \frac{1}{E_{k'}-E_k-i0}\right\}$$

$$f(\mathbf{k},\mathbf{k}') = (m/\hbar^2)\int \exp(-i\mathbf{k}'\mathbf{r})V(r)\psi_{\mathbf{k}}(\mathbf{r})d^2r$$

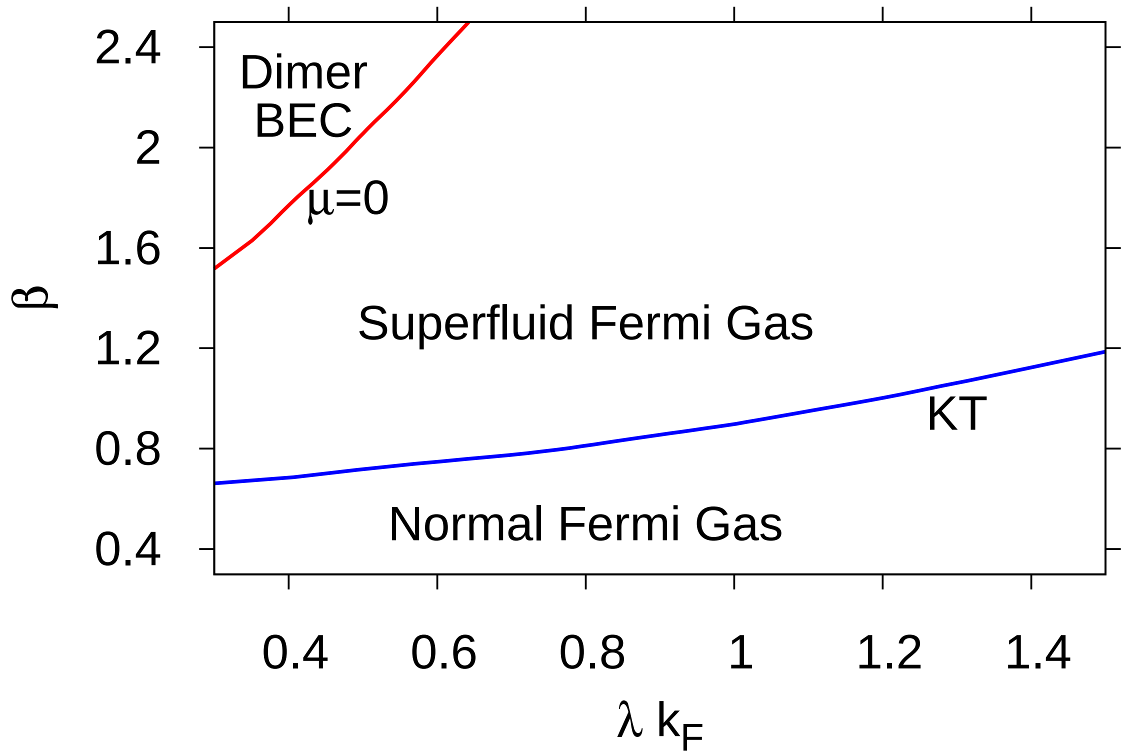
$$f(k,k') = \frac{2\pi}{\ln(\kappa/k)+i\pi/2} - 2\pi k r_* F_1\left(\frac{k'}{k}\right) + k^2\text{terms}$$

$$T_c = \frac{\exp C}{\pi}\Delta_0(k_F)$$

Strongly interacting and BEC regimes

Leggett model. Gap equation plus number equation

$$n = \frac{1}{2}\int \frac{d^2k}{(2\pi)^2}\left\{1 - \frac{E_k - \mu}{\epsilon_k}\tanh\left(\frac{\epsilon_k}{2T}\right)\right\}$$

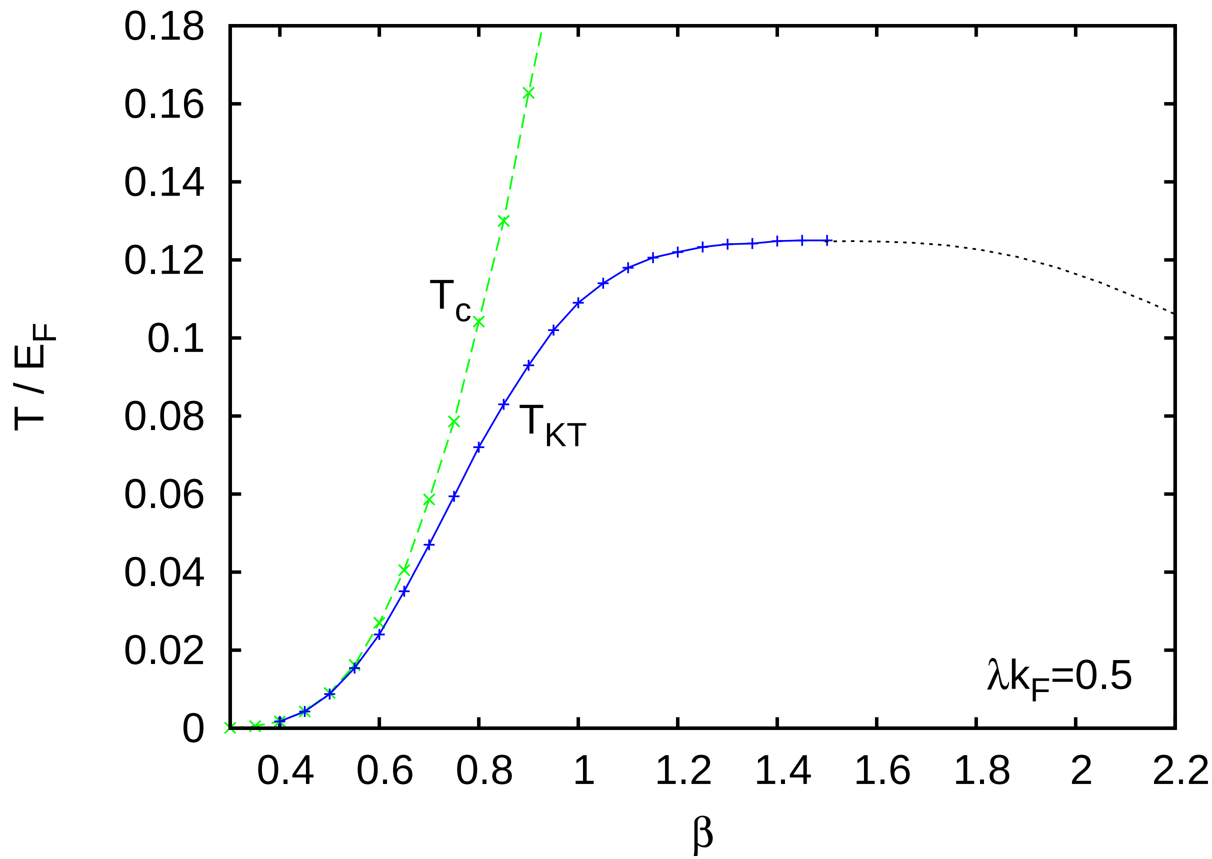


$$T_{KT} = \pi\hbar^2\rho_s(T_{KT})/2M^2; \quad M = 2m$$

$E_F \ll \epsilon_b$ $\rightarrow$ Formation of bound pairs by fermions of different layers

T_{KT} of a weakly interacting Bose gas

Transition temperature



LiCs and KRb molecules $\lambda \simeq 250$ nm, $n \simeq 5\,10^8$ cm^{-2}, $k_F\lambda \simeq 2$, $E_F \simeq 110$ nk

$\Rightarrow T_{KT}$ up to ~ 10 nK

Interlayer superfluids

Multilayer system $\rightarrow$ Harvard group of E. Demler

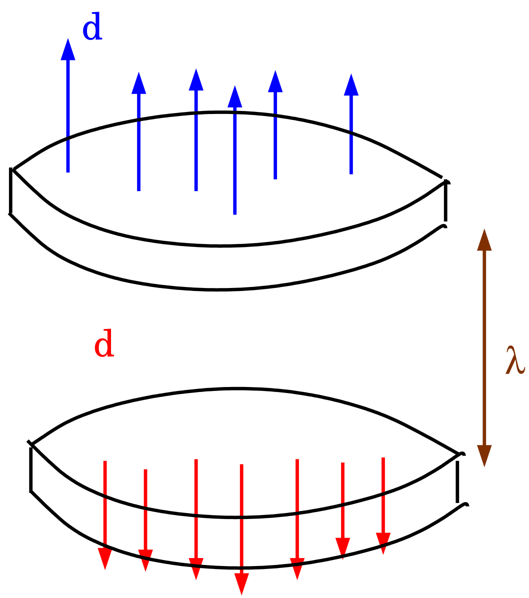
Bilayer systems of $\uparrow$ and $\downarrow$ dipoles

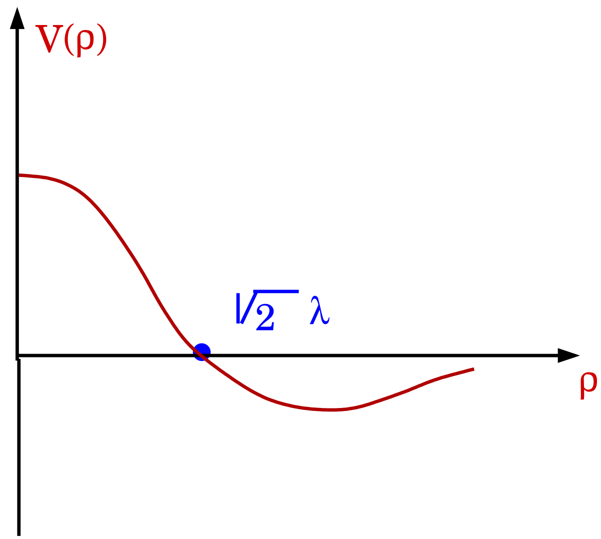
Bilayer system of $\uparrow$ and $\downarrow$ dipoles

Put $J = 0$ molecules in one layer and $J = 1$ in the other

Apply an electric field perpendicularly to the layers

Slightly non-uniform to prevent resonant dipolar flips leading to a rapid decay





Always a bound state of $\uparrow$ and $\downarrow$ dipoles

$$\beta \lesssim 1 \Rightarrow \quad \epsilon_b \simeq \frac{\hbar^2}{m\lambda^2} \exp[-8/\beta^2 - 8/\beta - (5 + 2C - 2\ln 2)]$$

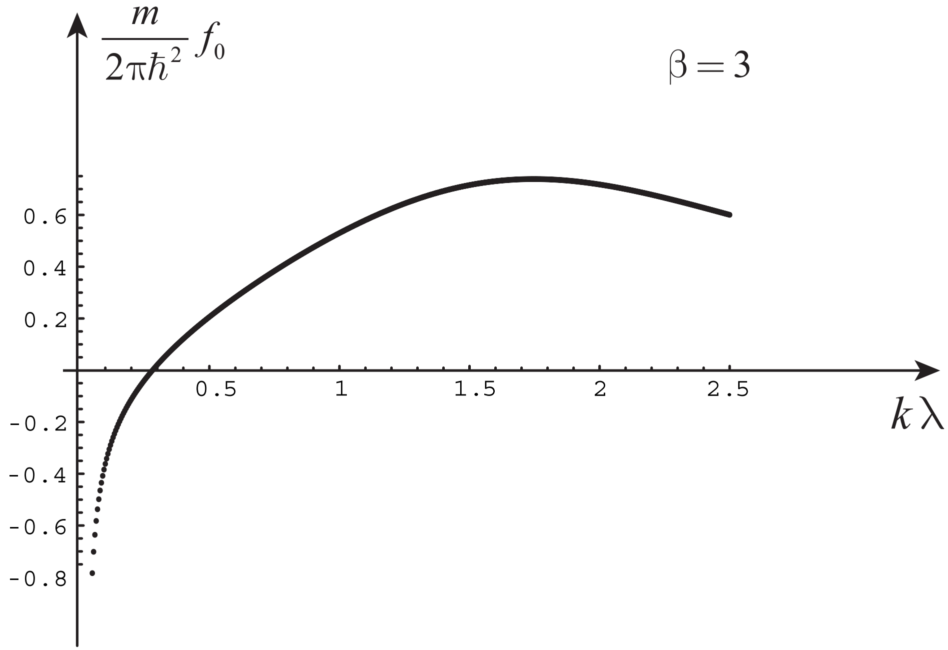
$$\beta = r_*/\lambda$$

Interlayer interaction. Scattering amplitudes

s-wave amplitude $k \to 0$ $\quad f_0(k) = \dfrac{4\pi\hbar^2}{m \ln(\epsilon_b/\epsilon)} + \dfrac{8\hbar^2}{m} k r_*$

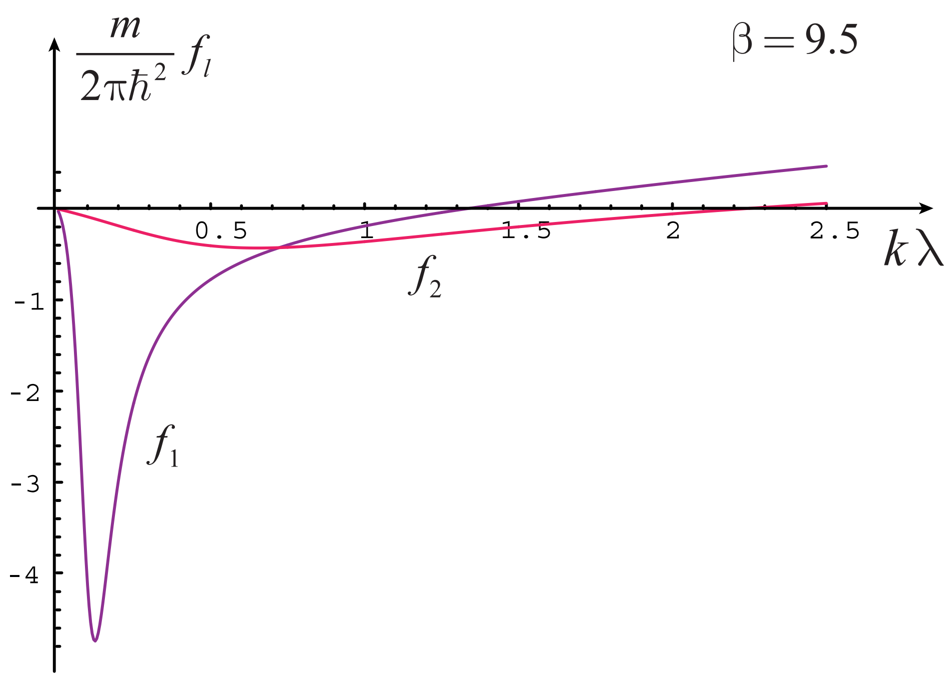
$$\epsilon = \hbar^2 k^2/m \quad r_* = md^2/\hbar^2$$

$f_0 > 0$ for reasonable k. No interlayer superfluid pairing



Interlayer interaction. Scatering amplitudes

p-wave and d-wave amplitudes are < 0



Interlayer p-wave and d-wave pairing

For $k_F r_* \gtrsim 1$ the effective mass significantly decreases

Transition temperature $T_c \sim E_F^* \exp\left(\frac{2\pi\hbar^2}{m_*|f(k_F)|}\right)$

The quasiparticle Fermi energy increases

Compensate the decrease of m_* in the exponent by increasing d^2 and, hence, f

p-wave interlayer superfluid with $T_c \sim$ tens of nK

d-wave superfluids with $T_c \sim$ nK. Analogy with high-temperature superconductors

LiCs with $n > 10^9$ cm^{-2}