

ICTP Summer School, Trieste, 02-13 July 12

**Feshbach resonances,
ultracold molecules,
Efimov physics**

lecture #3: 05 July 2012

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University of Innsbruck
Austrian Academy of Sciences



<http://en.wikipedia.org/wiki/Trieste>

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Habsburg Empire

See also: *Austrian Empire and Austro-Hungarian Empire*

After two centuries of war against the nearby major power, the Republic of Venice (which occupied it briefly from 1369 to 1372), the main citizens of Trieste petitioned **Leopold II of Habsburg**, Duke of Austria, to become part of his domains. The agreement of cessation was signed in October 1382, in St. Bartholomew's church in the village of *Silka* (apud Sociam), today one of the city quarters of *Ljubljana*. The citizens, however, maintained a certain degree of autonomy up until the 17th century.

Following an unsuccessful Habsburg invasion of Venice in the prelude to the *War of the League of Cambrai*, the Venetians occupied Trieste again in 1585, and under the terms of the peace were allowed to keep the city. The Habsburg Empire recovered Trieste a little over one year later, however, when conflict resumed.

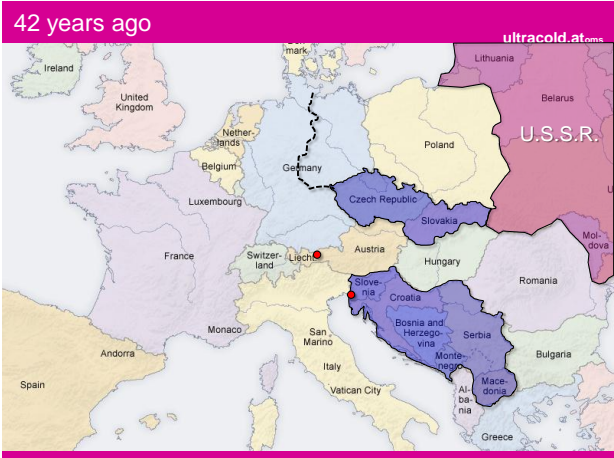
Trieste became an important port and trade hub. In 1719, it was made a free port within the Habsburg Empire by Emperor Charles VI, and remained a free port until 1 July 1891. The reign of his successor, *Maria Theresa of Austria*, marked the beginning of a flourishing era for the city.

In 1768, the German art historian *Johann Joachim Winckelmann* was murdered by a robber in Trieste, while on his way from Vienna to Italy.

Trieste was occupied by French troops three times during the *Napoleonic Wars*, in 1797, 1805 and in 1809. Between 1809 and 1813, it was annexed to the *Illyrian Provinces*, interrupting its status of free port and losing its autonomy. The municipal autonomy was not restored after the return of the city to the Austrian Empire in 1813. Following the Napoleonic Wars, Trieste continued to prosper as the *Free Imperial City of Trieste* (*Reichsunmittelbare Stadt Triest*), a status that granted economic freedom, but limited its political self-government. The city's role as main Austrian trading port and shipbuilding centre was later emphasized with the foundation of the merchant shipping line *Austrian Lloyd* in 1836, whose headquarters stood at the corner of the *Piazza Grande* and *Sanità*. By 1913 Austrian Lloyd had a fleet of 62 ships comprising a total of 236,500 tons.^[6] With the introduction of the constitutionalism in the Austrian Empire in 1860, the municipal autonomy of the city was restored, with Trieste became capital of the *Adriatic Coastland*, the *Austrian Littoral* region.

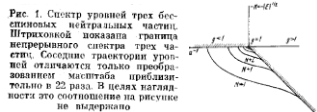
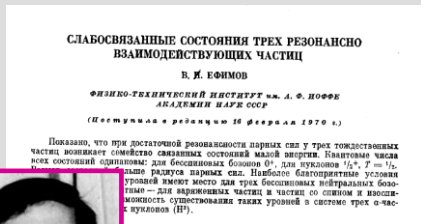
The particular *Friulan dialect*, called *Tergestino*, spoken until the beginning of the 19th century, was gradually overcome by the *Triestine dialect* of *Venetian* (a language deriving directly from vulgar Latin) and other languages, including German grammar. Slovene and standard Italian languages. While Triestine was spoken by the largest part of the population, German was the language of the Austrian bureaucracy and Slovene was predominant in the surrounding villages. From the last decades of the 19th century, Slovene language speakers grew steadily, reaching 25% of the overall population of the municipality of Trieste in 1911 (20% of the Austro-Hungarian citizens in Trieste).^[7]

According to the 1911 census, the proportion of Slovene speakers amounted to 12.6% in the city center, 47.6% in the suburbs, and 95.5% in the surroundings.^[8] They were the poorest ethnic group in 9% of the 19 urban



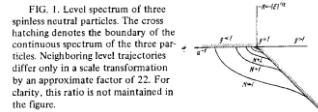
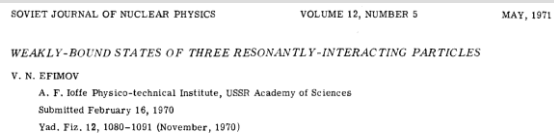
42 years ago

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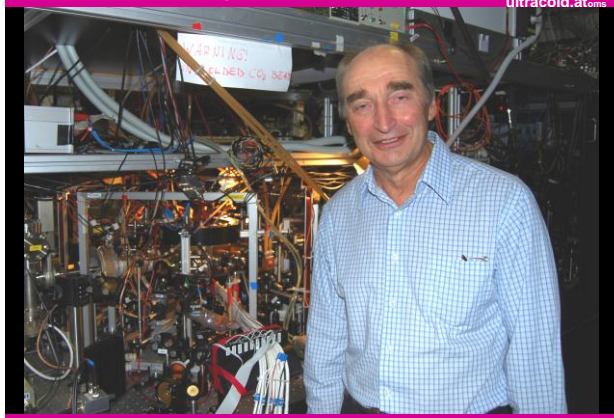
42 years ago

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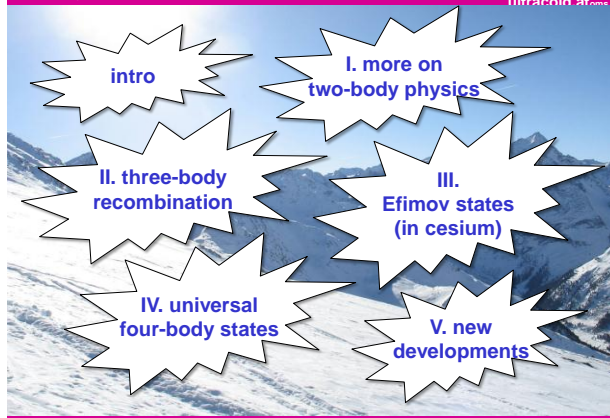
Vitaly Efimov visiting Innsbruck, 23 Oct 2009

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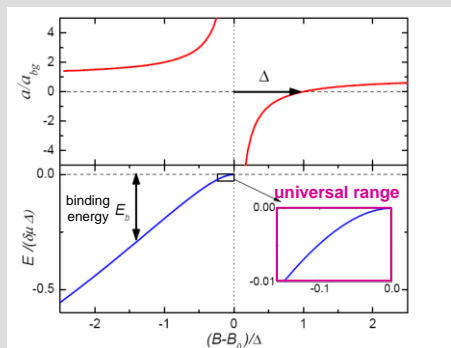
what you will hear in this lecture

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I. More on two-body physics

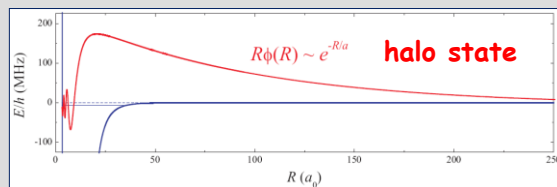
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Chin et al., Rev. Mod. Phys. **82**, 1225 (2010)

two-body universality

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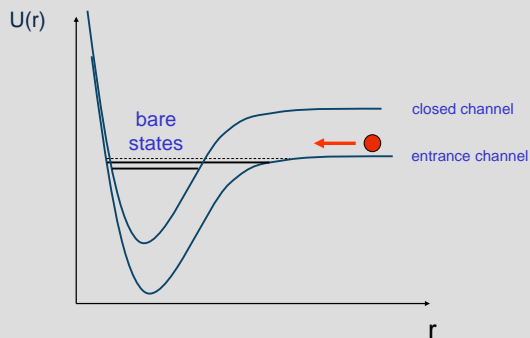
universal relation between
scattering length
and binding energy

$$E_b = \frac{\hbar^2}{ma^2}$$

one parameter tells you everything!

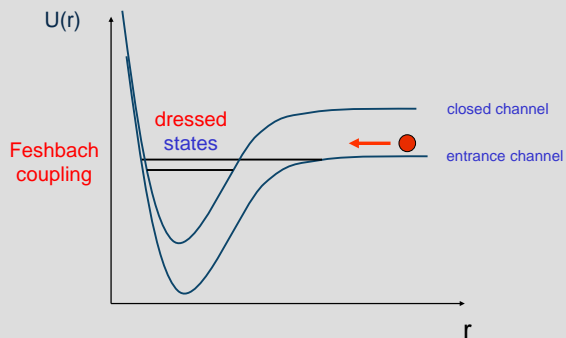
Feshbach physics in universal range

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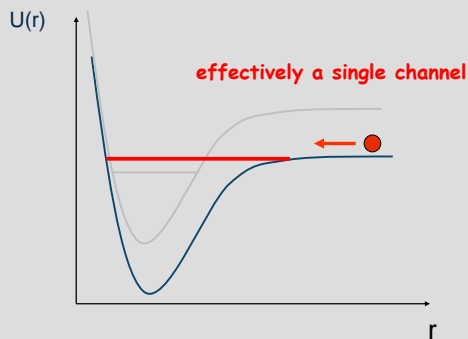
Feshbach physics in universal range

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Feshbach physics in universal range

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"broad" Feshbach resonances

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large universal range found for
"entrance-channel dominated resonances"
(often referred to as "broad" resonances)

what kind of resonances are strong ("broad") ?

resonance strength

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see review on Feshbach resonances

Chin et al., Rev. Mod. Phys. **82**, 1225 (2010)

mean scattering length of vdW potential

strength parameter

$$s_{\text{res}} = \frac{m\bar{a}}{\hbar^2} \delta\mu a_{\text{bg}} \Delta$$

differential magnetic moment

$$R^* = \bar{a}/s_{\text{res}}$$

relation to Petrov parameter

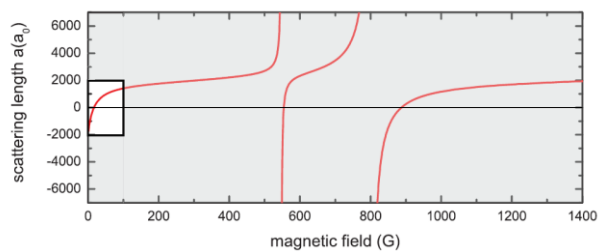
entrance-channel dominated character ($s_{\text{res}} \gg 1$)
usually found for FRs with

- large background scattering length
- large width

magnetic tunability of Cs

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three broad s-wave FRs



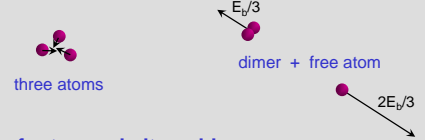
plus many d- and g-wave FRs (not shown)



III. Three-body recombination

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elementary process



consequences for trapped ultracold gas

large E_b / U_{trap} (for negative and moderately pos. scatt. length)
 → immediate loss of recombination products

small E_b / U_{trap} (for large positive scattering length !)

→ special situation

atom remains in trap,
 deposits energy into sample

dimer remains in trap,
 but lost by inelastic collisions
 with atoms

three-body recomb. theory basics

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$$\dot{n} = -L_3 n^3 \quad L_3: \text{three-body loss coefficient [cm}^6/\text{s]}$$

$$L_3 = 3C \frac{\hbar}{m} a^4$$

dimensionless

a^4 - scaling !
 very strong
 dependence on
 scattering length

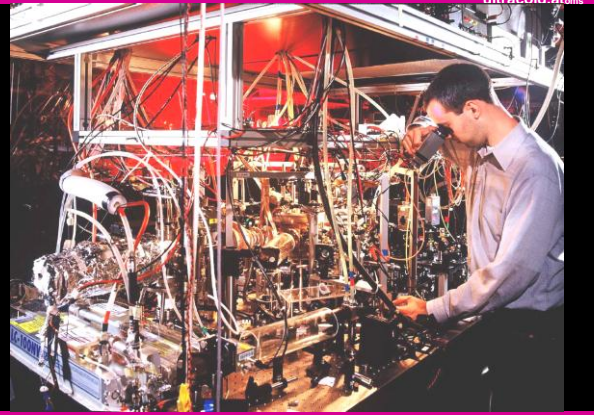
valid if a exceeds all other length scales
 (vdW length of molecular potential, typ. 30 – 100 a_0)

early theory paper:
 Fedichev *et al.*, PRL **77**, 2921 (1996)
 a^4 scaling with $C = 3.9$



Innsbruck Cs lab

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loss and heating curves

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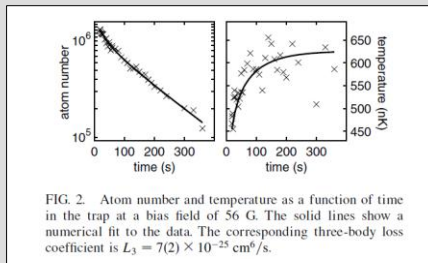


FIG. 2. Atom number and temperature as a function of time in the trap at a bias field of 56 G. The solid lines show a numerical fit to the data. The corresponding three-body loss coefficient is $L_3 = 7(2) \times 10^{-25} \text{ cm}^6/\text{s}$.

$$\frac{dN}{dt} = -\alpha N - \gamma \frac{N^3}{T^3} \quad \frac{dT}{dt} = \gamma \frac{N^2 (T + T_h)}{T^3}$$

measured $N(t)$ and $T(t)$ → 3-parameter fit yields α, γ, L_3, T_h

Weber *et al.*,
 PRL **91**, 123201
 (2003)

three-body loss vs. magnetic field

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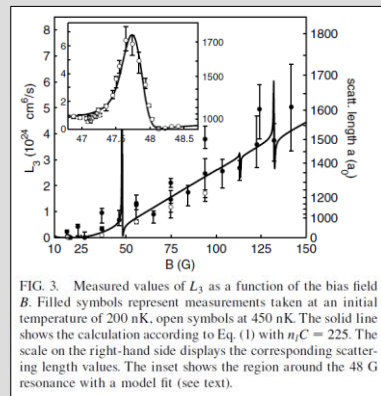


FIG. 3. Measured values of L_3 as a function of the bias field B . Filled symbols represent measurements taken at an initial temperature of 200 nK, open symbols at 450 nK. The solid line shows the calculation according to Eq. (1) with $n_i C = 225$. The scale on the right-hand side displays the corresponding scattering length values. The inset shows the region around the 48 G resonance with a model fit (see text).

Weber *et al.*,
 PRL **91**, 123201
 (2003)

verification of
 general a^4 -scaling

$$L_3 = n_i C \frac{\hbar}{m} a^4$$

fit yields **225**
 (too large ?)

is a^4 -scaling the whole story?

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n_l – number of atoms lost per recombination event

often $n_l=3$, but not always!
can vary between 2 and 4,
in some special cases even larger

$$L_3 = n_l C \frac{\hbar}{m} a^4$$

dimensionless, but not necessarily constant !
 $C(a)$ reveals very interesting physics

Nielsen & Macek, PRL 83, 1566 (1999)
Esry et al., PRL 83, 1751 (1999)
Bedaque et al., PRL 85, 908 (2000)
Braaten & Hammer, PRL 87, 160407 (2001)

$a > 0$:
 $C(a)$ upper limit ~ 70

log-periodic behavior

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$$C(22.7 a) = C(a)$$

three-body physics shows
log-periodic behavior with a
discrete **scaling factor of 22.7**



smells as if there is something interesting behind it !

effective field theory for $C(a)$

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Braaten and Hammer, Phys. Rep. 428, 259 (2006)

valid for large $|a|$

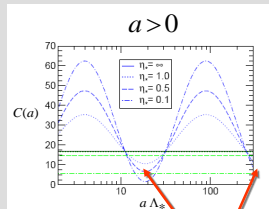
loss into shallow dimer

$$C(a) = 67.1 e^{-2\eta_+} (\cos^2(s_0 \ln(a/\Lambda_0) + 1.76) + \sinh^2 \eta_+) + 16.8 (1 - e^{-2\eta_+})$$

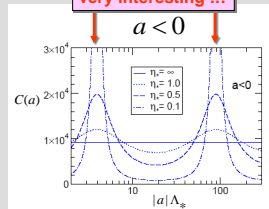
loss into deeply bound molecules

$$C(a) = \frac{4590 \sinh(2\eta_+)}{\sin^2(s_0 \ln(a/\Lambda_0) + 1.72) + \sinh^2 \eta_+}$$

very interesting !!!



very interesting !!!



III. Efimov states

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SOVIET JOURNAL OF NUCLEAR PHYSICS

VOLUME 12, NUMBER 5

MAY, 1971

WEAKLY-BOUND STATES OF THREE RESONANTLY-INTERACTING PARTICLES

V. N. EFIMOV

A. F. Ioffe Physico-technical Institute, USSR Academy of Sciences

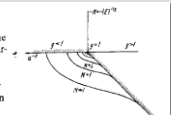
Submitted February 16, 1970

Yad. Fiz., 12, 1080-1091 (November, 1970)

It is shown that if the pair forces of three identical particles are sufficiently resonant, a family of bound states of low energy is produced. The quantum numbers of all the states are the same: for bosons 2π , $T = 1/2$. The dimension of the states is larger than the most favorable conditions for the appearance of a family of levels of bosons; the conditions are less favorable for charged particles and spin. The possibility of existence of such levels in a system of three and of three nucleons (H^3) is considered.

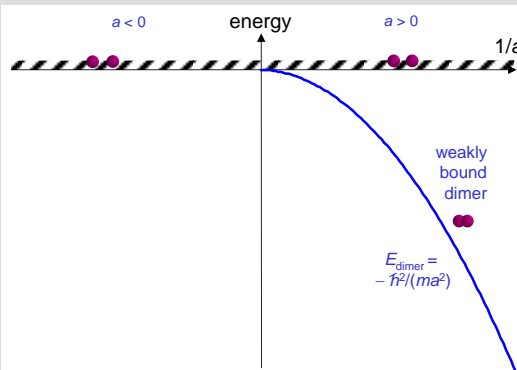


FIG. 1. Level spectrum of three spinless neutral particles. The cross hatching denotes the boundary of the continuous spectrum of the three particles. Neighboring level trajectories differ only in a scale transformation by an approximate factor of 22.7. For clarity, this ratio is not maintained in the figure.



quantum states near two-body resonance

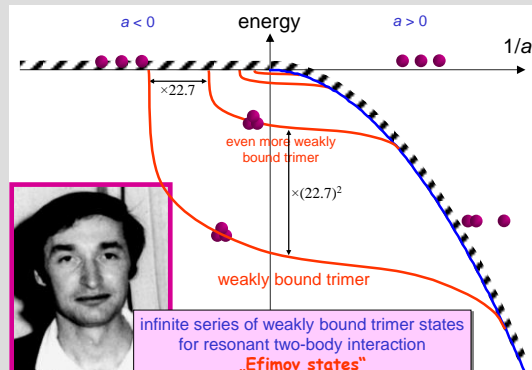
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$$E_{\text{dimer}} = -\frac{\hbar^2}{m a^2}$$

quantum states near two-body resonance

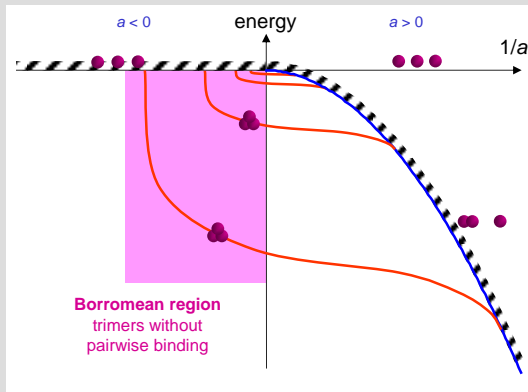
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infinite series of weakly bound trimer states for resonant two-body interaction
„Efimov states“

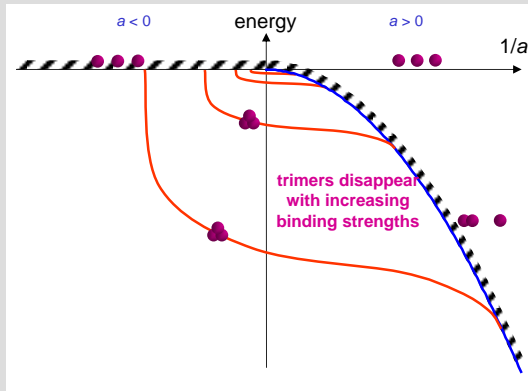
Efimov states: striking properties I

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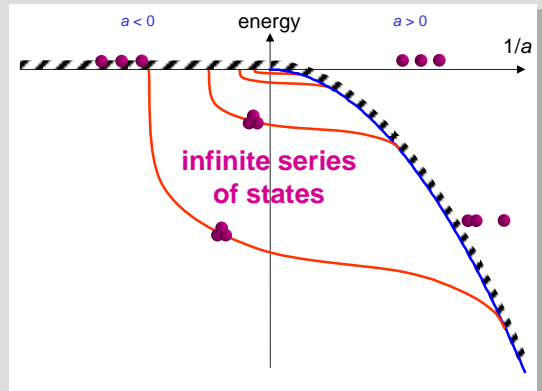
Efimov states: striking properties II

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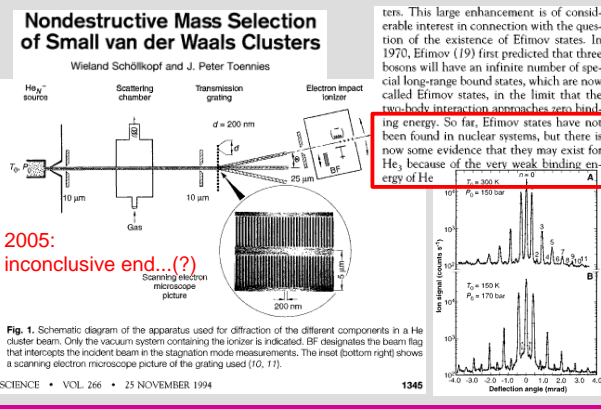
Efimov states: striking properties III

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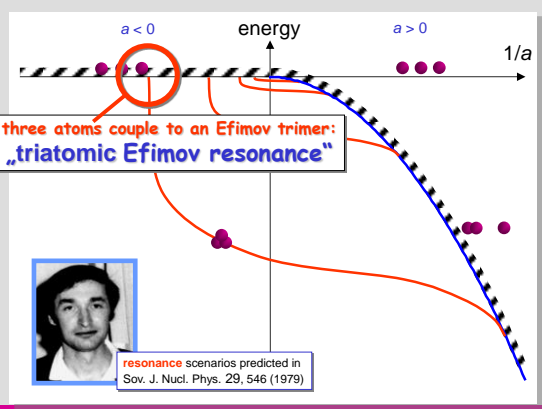
search for Efimov states in He (1994-2005)

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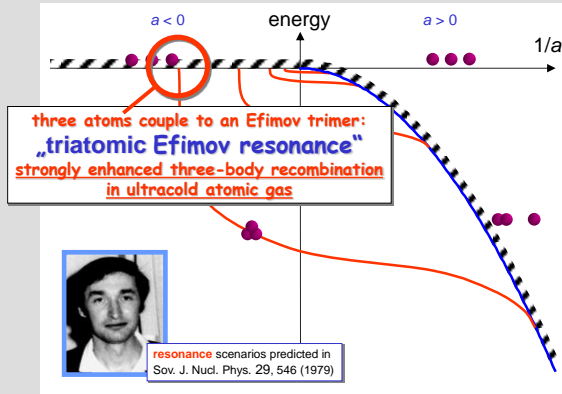
Efimov resonances

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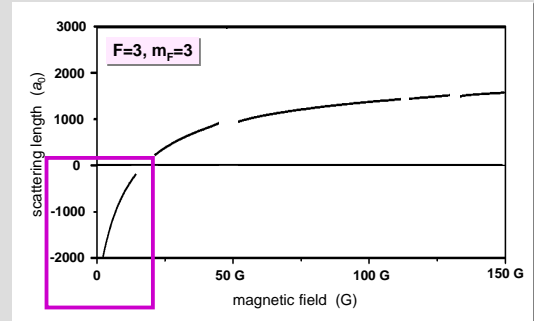
Efimov resonances

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magnetic tunability of Cs

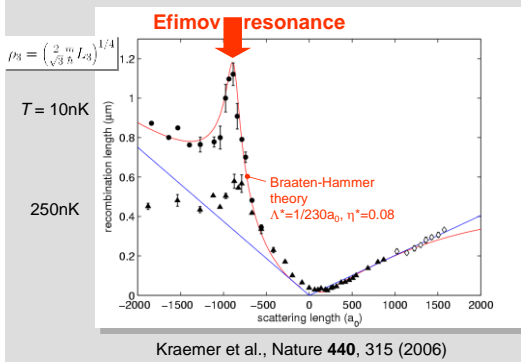
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there should be an Efimov resonance !

exp. results (2005) !

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temperature limitation

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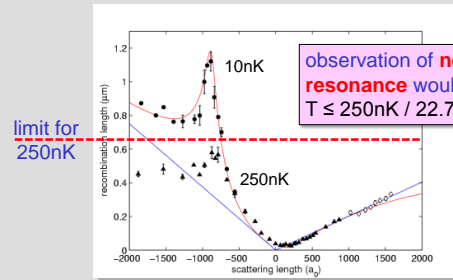
maximum recombination length
temperature-limited

$$\rho_3^{\text{max}} \approx 2 \Lambda$$

D'Incao, Suno, Esry,
PRL 93, 123201 (2004)

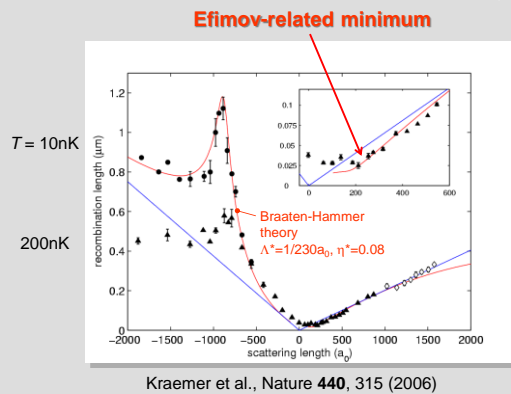
$$\Lambda = h/\sqrt{2\pi m k_B T}$$

thermal de Broglie wavelength



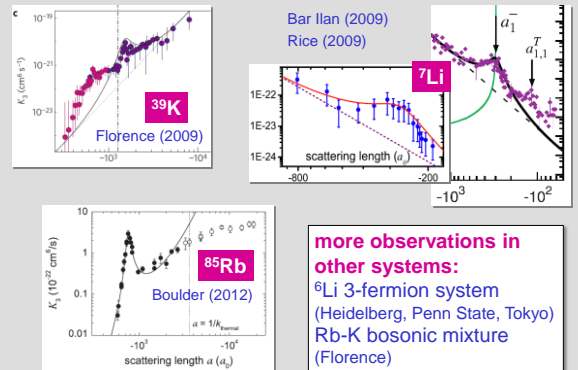
exp. results (2005)

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3-boson resonance gallery

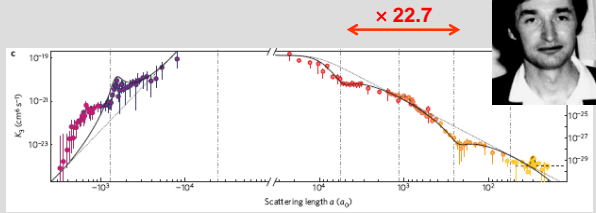
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³⁹K experiments in Florence

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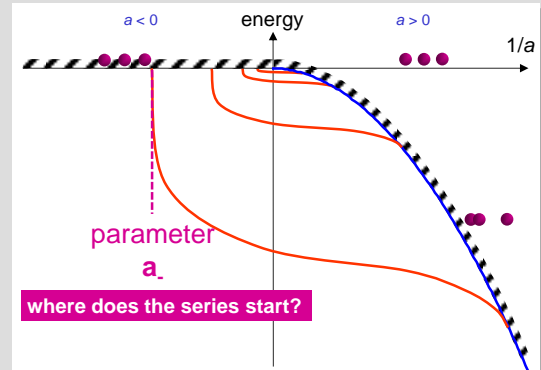
discrete scaling invariance observed !



Zaccanti et al., Nature Phys. 5, 586 (2009)

three-body parameter

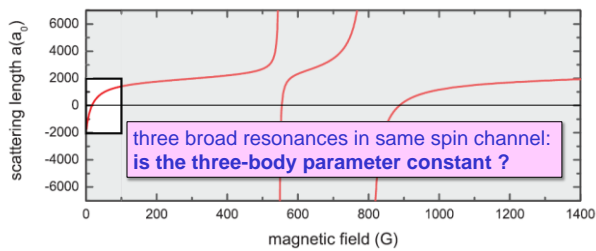
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magnetic tunability of Cs (high-field range)

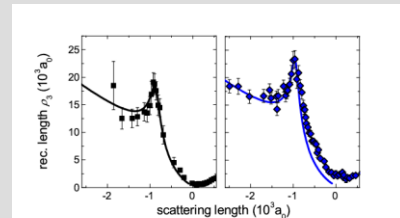
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three broad s-wave FRs



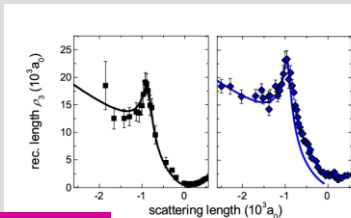
variations of the three-body parameter?

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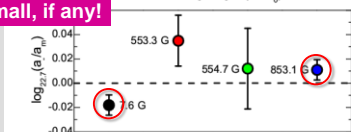


variations of the three-body parameter?

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very small, if any!



one tenth of Efimov period

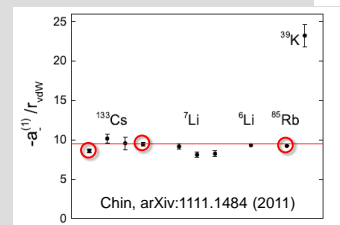
strongly entrance-channel dominated FRs

can this be an accidental coincidence?

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experiments on ⁶Li, ⁷Li, ⁸⁵Rb, ¹³³Cs show

$$a_3 \approx -9 R_{\text{vdW}} \quad R_{\text{vdW}} = \frac{1}{2} \left(\frac{2\mu C_6}{\hbar^2} \right)^{1/4}$$



van der Waals length

only ³⁹K does not fit into the picture...

... quite interesting, indeed!

three answers from theory

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Chin, arXiv:1111.1484

"quantum reflection of the Efimov wavefunction"

analytical model:

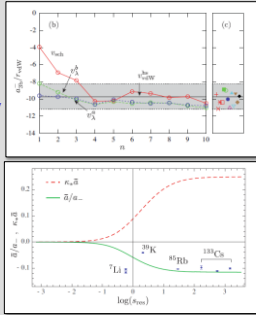
$$|a_s|/R_{\text{vdW}} = 9.48$$

Wang, D'Incao, Esry, Greene,
arXiv:1201.1176

"A sharp cliff of attraction in the two-body interactions produces a strongly repulsive universal barrier in the effective three-body interaction potential."

Schmidt, Rath, Zwerger,
arXiv:1201.4310

solution of a two-channel model incl. character of the Feshbach resonance (open vs. close-channel dominated)



conclusions on 3BP

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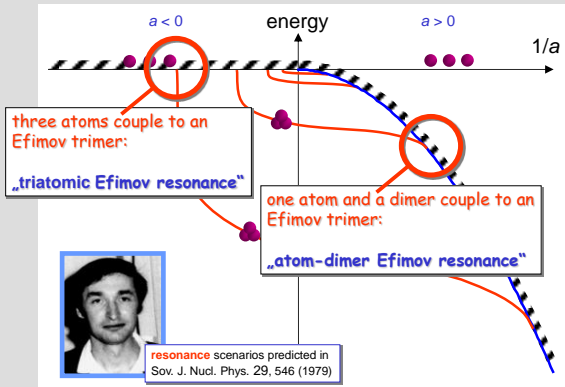
"3BP universality"

exists for interacting atoms with vdW potentials (not for other particles, like nuclei) and entrance-channel dominated resonances (^{133}Cs , ^{85}Rb)

situation of intermediate cases (^{39}K , ^7Li) still needs more investigations

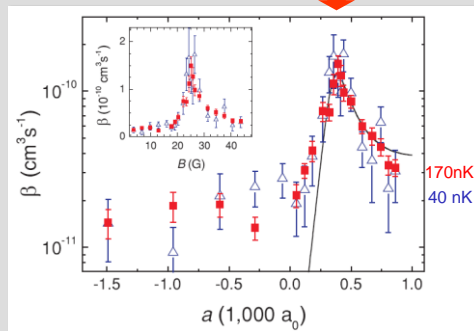
Efimov resonances

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atom-dimer "Efimov" resonance

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Knoop et al., Nature Phys. 5, 227 (2009)



new insights from theory

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Hammer and Platter, EPJA 32,113 (2007)

conjecture of two universal four-body states attached to an Efimov state

von Stecher, D'Incao, and Greene
Nature Phys. 5, 417 (2009)

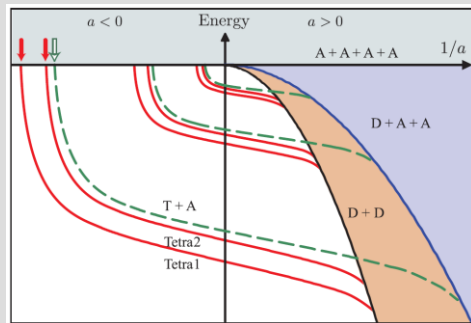
confirmation and extension of Hammer-Platter conjecture

prediction of universal relations

suggesting 4-body recombination as a probe

extended Efimov scenario (4 bosons)

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Efimov states and universal 4-body states
 Hammer and Platter, EPJA (2007)
 von Stecher, D'Incao, and Greene, Nature Phys. (2009)

trios and quartets

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<http://www.musik-base.de/images/groups/Alpentrio-Tirol.jpg>
http://www.volksmusikfan.com/var/.../Alpentrio-Tirol_fullscreen.jpg

where do the 4-body resonances occur?

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universal predictions

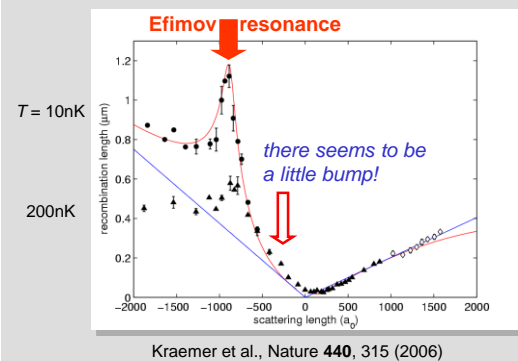
$$a_{4b1}^* \approx 0.43a_{3b}^*, \text{ and } a_{4b2}^* \approx 0.90a_{3b}^*.$$

von Stecher, D'Incao, Greene, Nature Phys. 5, 417 (2009)

-375 a_0 **-783 a_0**
expectations for Cs
(3-body resonance at -870(20) a_0)

exp. results revisited by C. Greene et al. (2008)

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Kraemer et al., Nature **440**, 315 (2006)

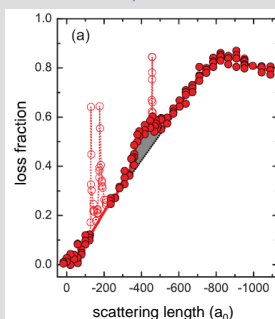
new set of experiments

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prepare sample very close to BEC

Ferlaino et al., PRL **102**, 140401 (2009)

$T = 50\text{ nK}$, hold time 250ms



new set of experiments

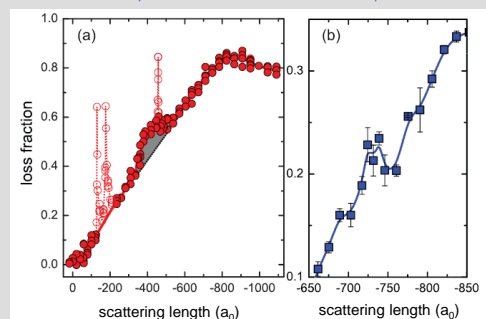
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prepare sample very close to BEC

Ferlaino et al., PRL **102**, 140401 (2009)

$T = 50\text{ nK}$, hold time 250ms

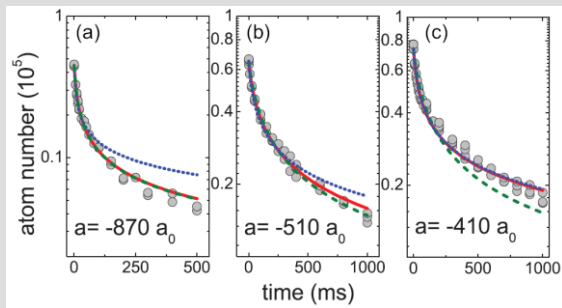
$T = 30\text{ nK}$, hold time 8ms



three- or four-body loss ?

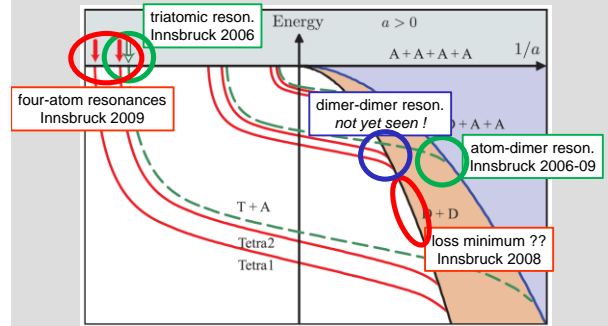
ultracold.atoms

$$\dot{N}/N = -L_3 \langle n^2 \rangle - L_4 \langle n^3 \rangle$$



extended Efimov scenario (4 bosons)

ultracold.atoms



Efimov states and universal 4-body states

as predicted by Hammer and Platter, EPJD (2007), von Stecher, D'Incao, and Greene, Nature Phys. (2009)



prediction of bosonic cluster states

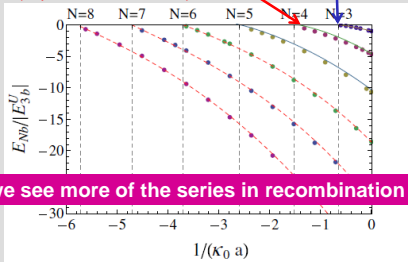
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J. von Stecher, JPB 43, 101002 (2010)



universal four-body state
(expt. confirmed, 2009)

Efimov state
(expt. confirmed, 2005)



only
ground
states
shown

can we see more of the series in recombination loss?

5-body recombination

ultracold.at.oms

Resonant Five-Body Recombination in an Ultracold Gas

A. Zenesini, B. Huang, M. Berninger, S. Besler, H.-C. Nägerl, F. Ferlaino, and R. Grimm
Institut für Experimentalphysik und Zentrum für Quantenphysik, Universität Innsbruck, and
Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria

Chris H. Greene and J. von Stecher*

arXiv:1205.1921

Department of Physics and JILA, University of Colorado, Boulder, CO 80309, USA
(Dated: May 10, 2012)

We combine theory and experiment to investigate five-body recombination in an ultracold gas of atomic cesium. A refined theoretical model, in combination with extensive laboratory tunability of the scattering length, enables the five-body resonant recombination rate to be calculated and measured. The position of the new observed recombination maximum agrees with a recent theoretical prediction and strongly supports the prediction of a family of universal cluster states tied to an Efimov trimer.

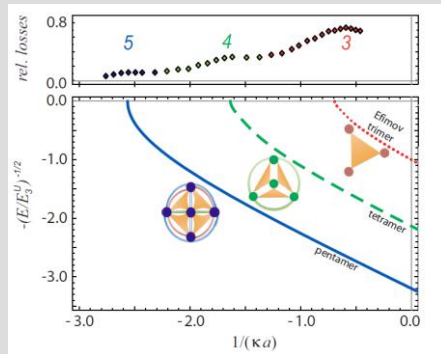
very challenging for both theory and experiment

$N > 4$ not tractable with standard numerical tools

background from 3- and 4-body processes

encouraging observations at high B-fields

ultracold.at.oms



loss rate equations

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sum of all losses ($L_1=L_2=0$)

$$\dot{\mathcal{N}}/\mathcal{N} = - \sum_{N=1}^{+\infty} L_N \langle n^{N-1} \rangle,$$

$$\dot{T}/T = \sum_{N=1}^{+\infty} \varepsilon_N L_N \langle n^{N-1} \rangle,$$

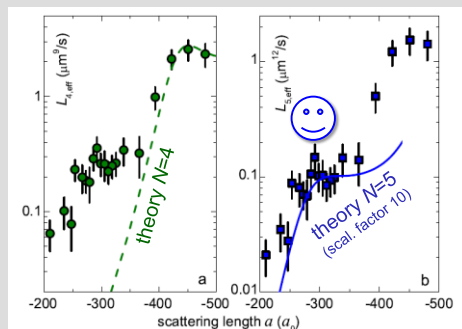
anti-evaporation heating

$$2\varepsilon_N \equiv 1 - 1/N$$

our expt. $L_1=L_2=0$, L_3 from eff. field theory

strong evidence for 5-body state

ultracold.at.oms



$N > 5$?

ultracold.at.oms

enough confidence
to trust in theory

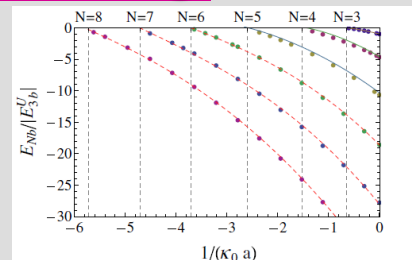


observations in Cs

2009

2012

2005/6

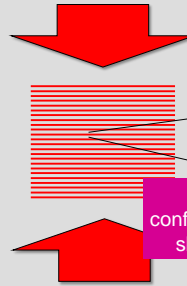


reduced dimensionality
(optical lattices) \longleftrightarrow heteronuclear
mixtures

some new directions

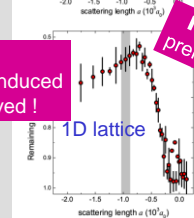
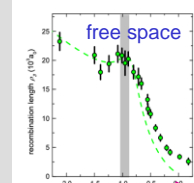
strong dipolar interactions

1D lattice



lattice depth $35E_{\text{rec}}$
harm. osc. length $a_{\text{ho}} = 1350 a_0$

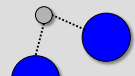
strong
confinement-induced
shift observed !



**NEW &
preliminary**



three bosons: Efimov factor 22.7

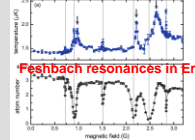


two bosons and
a boson/fermion Efimov factor ≈ 4

very interesting for experimental realizations:
mixtures of Li with lanthanides (Er, Dy...)



two fermions and
a boson/fermion
Efimov factor ≈ 8



thank you for your attention !