Synthetic gauge potentials for ultracold neutral atoms

Experiment (II): spin-orbit coupled Bose-Einstein condensates

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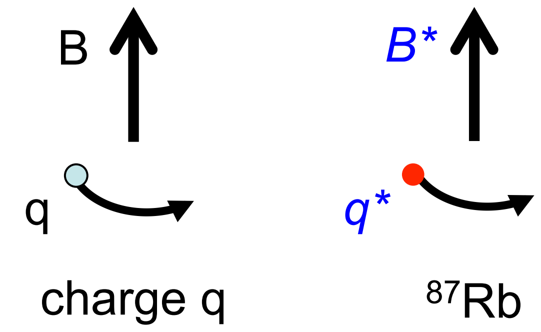
Introduction of gauge potential

- Optically induced vector gauge potential A^* for neutral atoms:

$$H = \frac{(p - q^* A^*)^2}{2m^*} + V(x)$$

$\rightarrow$ synthetic electric and magnetic fields $\quad E^* = -\frac{\partial A^*}{\partial t}, B^* = \nabla \times A^*$

- Create synthetic field B^* for neutral atoms:
  effective Lorentz force
  to simulate charged-particles in real magnetic fields



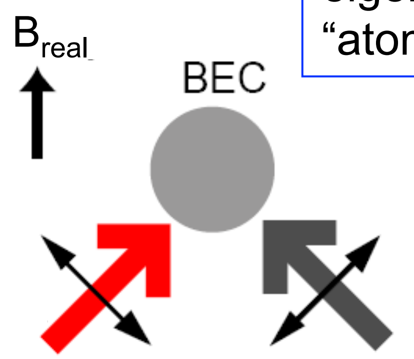
- New approach: light-induced generate B^* in lab frame, no rotation of trap:
  (1) steady B^*, not metastable
  (2) easy to add optical lattices

B* in rotating frame:
Coriolis force↔Lorentz force

Outline: synthetic gauge potentials A^*

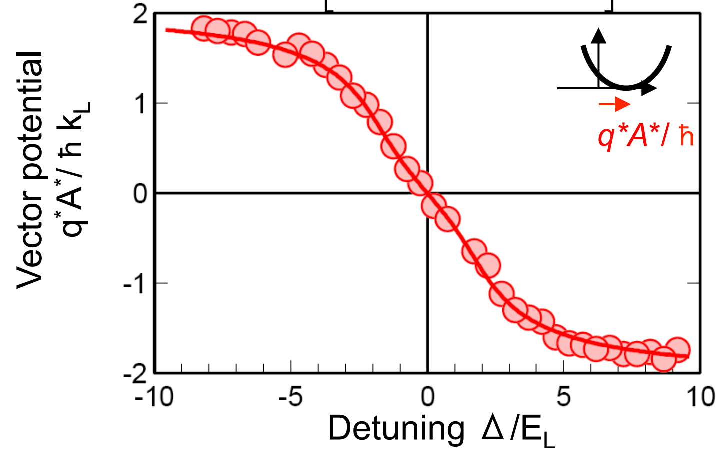
Raman-dressed BEC

eigenstate of
"atom+photon"

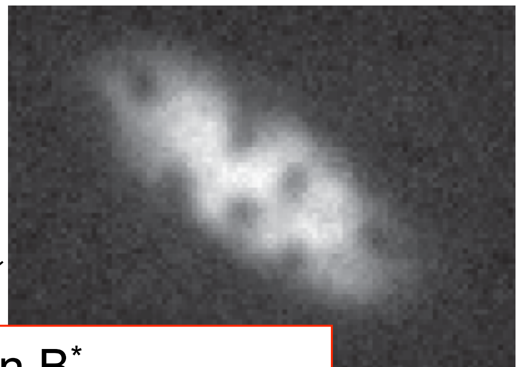


Synthetic Vector potential A^*

$$H = \frac{\hbar^2}{2m^*}\left[(k_x - \frac{q^* A_x^*}{\hbar})^2 + k_y{}^2\right]$$

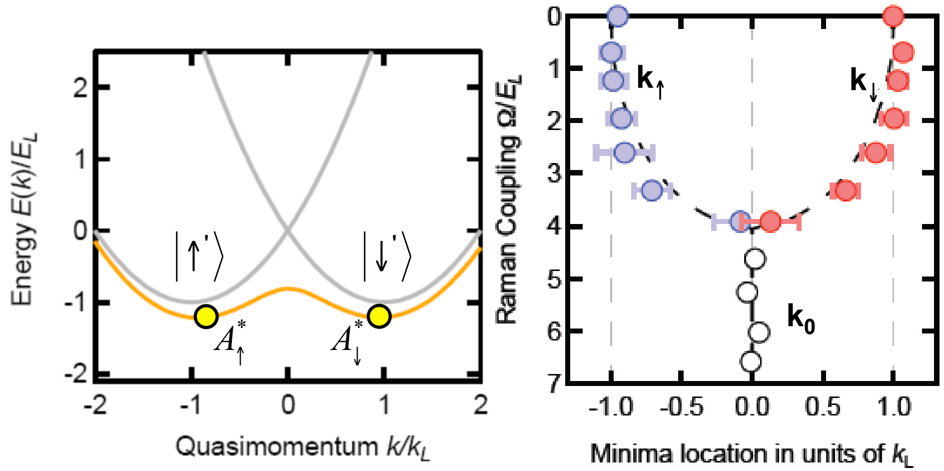


Magnetic field $B^* = \nabla \times A^*$



superfluid in B^*
(like superconductor in B)

Spin dependent A^*: spin-orbit coupling



Outline : spin-orbit coupling

- Spin-orbit (SO) coupling is important and widely studied in condensed matter physics

- Study SO coupled cold neutral atoms: clean and well controlled systems

- Our first observation of SO coupling of neutral atoms and SO coupling of bosons: atomic COM motion and spin. Scheme: Raman laser coupling.

- Quantum phase transition due to Raman laser dressing:
  spatially mixed → separated

  Ref. Y.-J. Lin, K.J.-Garcia and Ian Spielman, Nature, Mar 3, 2011.

- Some recent experiments

Introduction: spin-orbit coupling

Spin-orbit (SO) coupling: interaction between momentum *k* and spin σ

$$H_{SO} = -\vec{\mu}\cdot\vec{B}_{SO}(\vec{k}) \rightarrow \sum_{i,j}\sigma_j \cdot \gamma_{ij} k_i$$

$\vec{\mu} = g\mu_B\vec{\sigma}$: magnetic moment

$\vec{B}_{SO}(\vec{k})$: effective spin-orbit magnetic field

Connection of SO coupling and spin-dependent A*

$$H = \sum_i \frac{(\hbar k_i - q^* A_i^*)^2}{2m^*}$$

$A_i^*(\vec{\sigma}) \leftrightarrow$ SO coupling

Simple example for electrons in solids:

Rashba SO coupling in semiconductors

$$\vec{B}_{SO}(\vec{k}) = -\frac{\hbar}{m^* c^2}\vec{k}\times\vec{E}$$

$$H_{SO} = \alpha(-\sigma_x k_y + \sigma_y k_x)$$

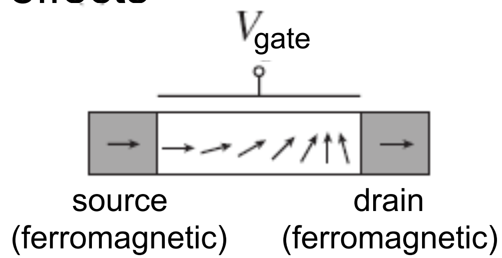
$\vec{E} = E_0\hat{z}$: static electric field in materials

$\hbar\vec{k}$: e$^-$ momentum

Importance of SO coupling

- Promising for making spintronics devices; spin-Hall effects
  Ex. Datta-Das spin field-effect transistor

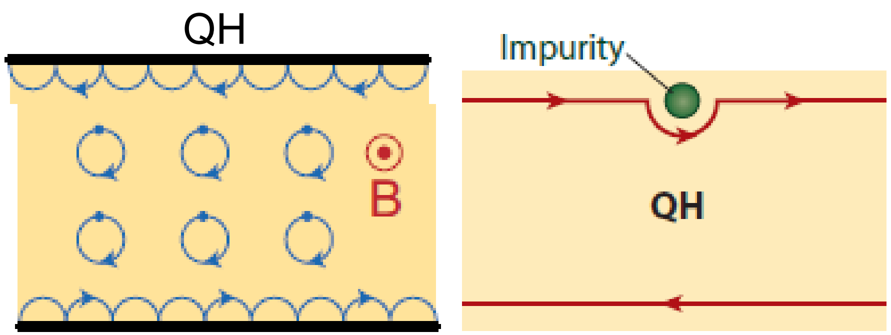
  Ref: Datta and Das (1990); Kato. et al. (2004)



- Realizing topological insulators w/o breaking time reversal symmetry

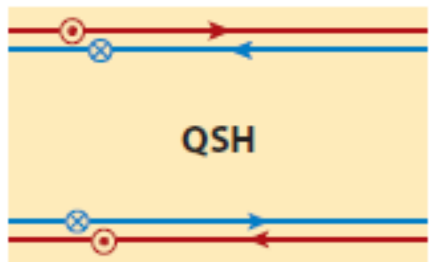
Topological insulators: fermionic, insulate in the bulk, conduct in the edge states

Ex.
B ≠ 0: integer quantum-Hall, chiral edge states



Ref: Klitzing et al. (1980)

Ex.
B=0: quantum spin-Hall, w/ SO coupling
(QSH)



Ref: Kane and Mele, PRL (2005)
Bernevig et al., Science (2006)

Importance of SO coupling

- Promising for making spintronics devices; spin-Hall effects

- Realizing topological insulators w/o breaking time reversal symmetry

- w/ interaction→ topological superconductors
  leading to anyons, Majorana fermions,
  non-Abelian statistics, topological quantum computing

Ref: Qi and Zhang, Physics Today (2009)
Fu and Kane, PRL (2008)
Sau et al., PRL (2010)
Nayak et al., Rev. Mod. Phys. (2008)

- Spin-dependent A*: $A_i^*(\vec{\sigma})$ non-abelian gauge potentials $\left[A_i^*, A_j^*\right] \neq 0$

SO coupling for electrons

kinetic — Zeeman — SO coupling

$$H=\frac{\hbar^2k^2}{2m}\otimes\hat{1}-\vec{\mu}\cdot\left[\vec{B}+\vec{B}_{SO}(\vec{k})\right]$$

$$=\frac{\hbar^2\hat{k}^2}{2m}\otimes\hat{1}+\frac{\Omega}{2}\cdot\sigma_z+\alpha(-\sigma_x k_y+\sigma_y k_x)+\beta(\sigma_x k_x-\sigma_y k_y)$$

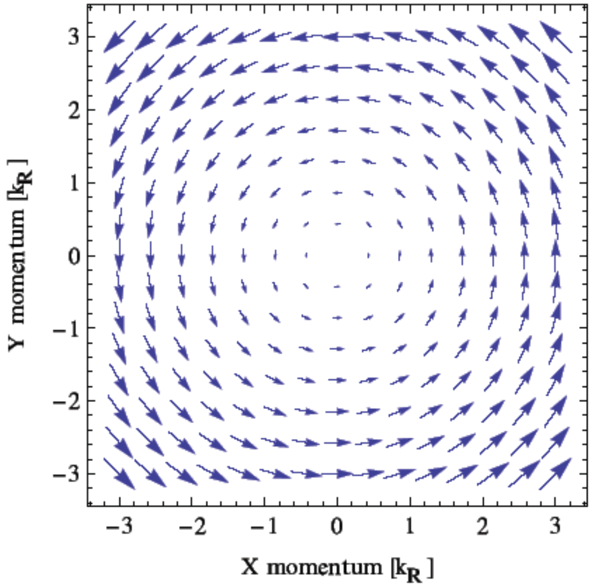
σ_i: spin ½ Pauli matrices
Ω: Zeeman magnetic field along z
α: Rashba SO coupling
β: Dresselhaus SO coupling

$\alpha=\beta\rightarrow H_{SO}=2\alpha k_x\sigma_y$

Electron spin helix
Ref: Koralek et al., 2009.

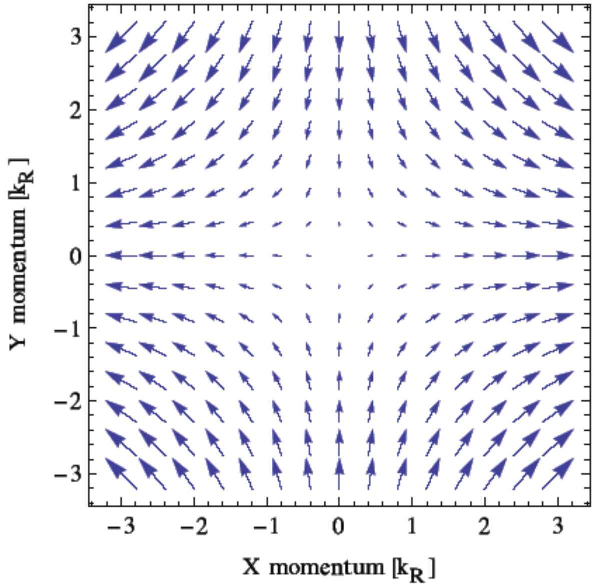
$$\vec{B}_{SO}(\vec{k})=(k_y,-k_x)$$

Rashba



$$\vec{B}_{SO}(\vec{k})=(-k_x,k_y)$$

Dresselhaus



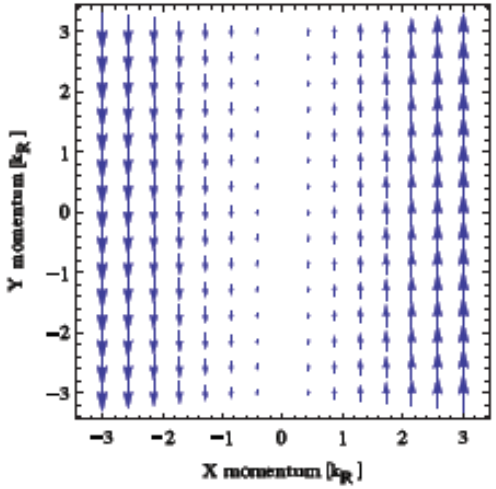
SO coupling for neutral atoms

Cold neutral atoms dressed by two Raman laser beams

$$H = \frac{\hbar^2 \hat{k}^2}{2m} \otimes \hat{1} + \frac{\Omega}{2} \cdot \sigma_z + \frac{\delta}{2} \cdot \sigma_y + 2\alpha k_x \sigma_y$$

Ω,δ: effective Zeeman field along z,y
$H_{SO}=2\alpha k_x \sigma_y$: SO coupling

For atoms : $\vec{B}_{SO}(\vec{k}) = (0, k_x)$



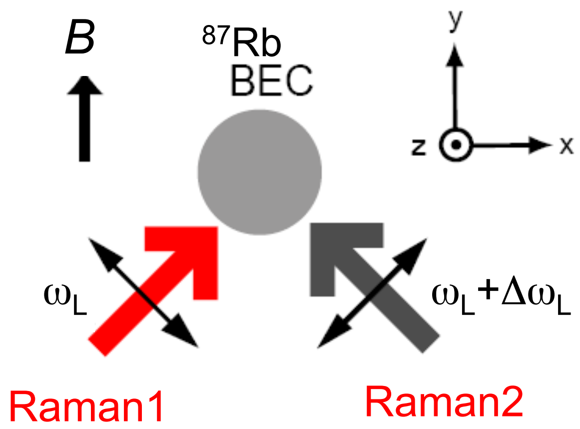
- H_{so} is equivalent to that for e-, $\alpha = \beta$

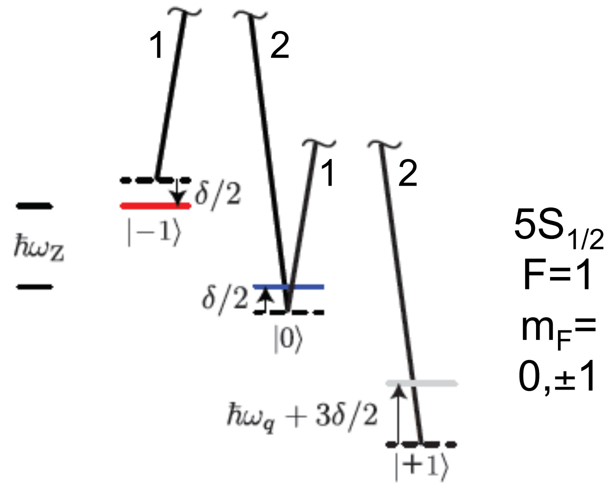
- $H_{SO}=2\alpha k_x \sigma_y$ term:
  coming from the Raman coupling $\Omega \cos(2k_L x)$

More general form:
(almost) independent control of R, D using multiple laser beams

Refs.: Campbell, Juzeliunas and Spielman, PRA (2011).

Raman-dressed BEC: light-atom coupling





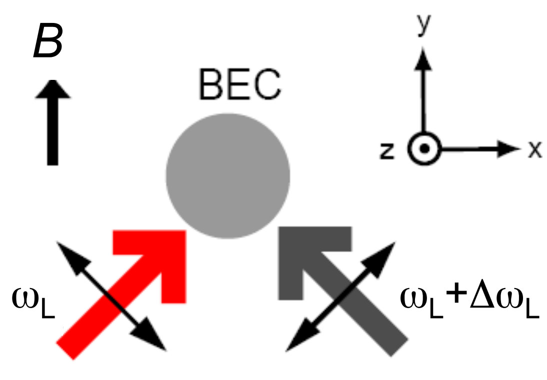
- Linear Zeeman shift $\omega_Z = g\mu_B B \equiv E_{-1} - E_0$

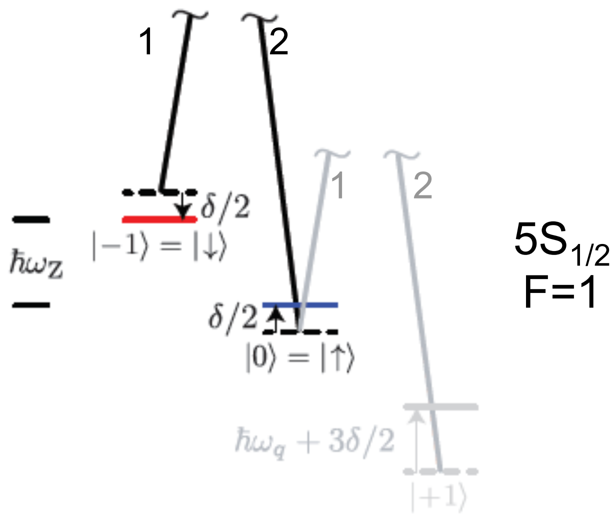
  Quadratic Zeeman shift ω_q

- Raman laser frequency difference: $\Delta\omega_L$
  Raman detuning $\delta = \Delta\omega_L - \omega_Z$

$\omega_Z \approx \Delta\omega_L \approx 2\pi$ x 4.81 MHz
$\omega_q \approx$ 6.8 kHz

Raman-dressed BEC: light-atom coupling





For large ω_q and $\delta \approx 0$, 3-level→ effective 2-level

$\omega_Z \approx \Delta\omega_L \approx 2\pi$ x 4.81 MHz
$\omega_q \approx$ 6.8 kHz [illegible] relevant energy

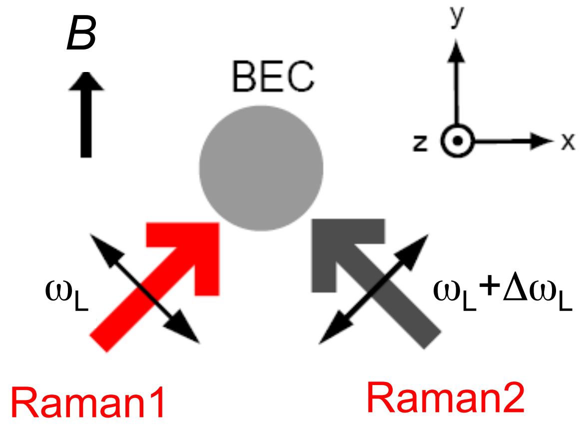
light-atom coupling

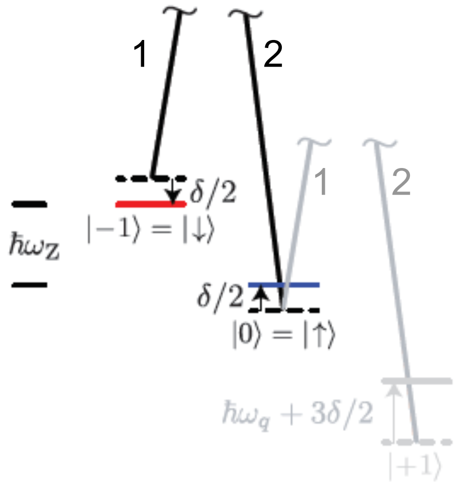
$$H = \frac{\hbar^2 \hat{k}^2}{2m} \otimes \hat{1} + \begin{pmatrix} \delta/2 & \Omega e^{i2k_L\hat{x}}/2 \\ \Omega e^{-i2k_L\hat{x}}/2 & -\delta/2 \end{pmatrix}$$

basis $= |\uparrow\rangle, |\downarrow\rangle$

Ω : Raman coupling
$2k_L\hat{e}_x = \vec{k}_1 - \vec{k}_2$

Raman-dressed BEC: light-atom coupling





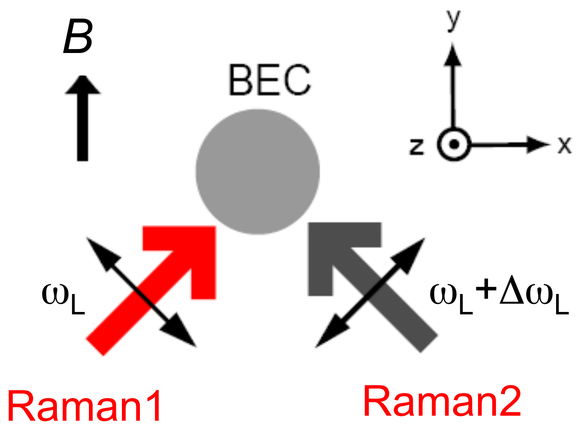
$$H = \frac{\hbar^2(\hat{k}_y^2+\hat{k}_z^2)}{2m}\otimes\hat{1}+\sum_k \begin{pmatrix} (k+1)^2+\delta/2 & \Omega/2 \\ \Omega/2 & (k-1)^2-\delta/2 \end{pmatrix}$$

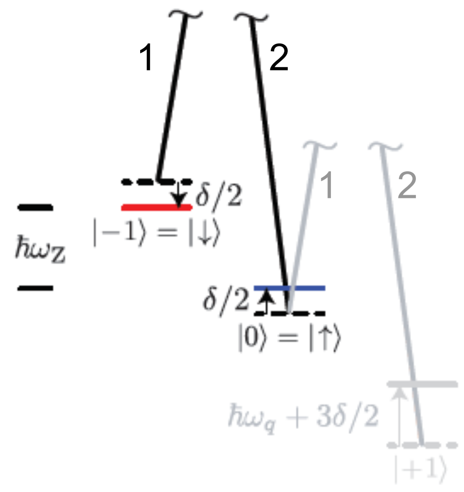
(columns labelled $|\uparrow, k+1\rangle$ and $|\downarrow, k-1\rangle$)

quasimomentum k $\leftrightarrow \hat{k}_x$,
labeling bare spin momentum

$2k_L\hat{e}_x = \vec{k}_1 - \vec{k}_2$
k (k_L), E(E_L)
$E_L = \hbar^2 k_L^2/2m$

Raman-dressed BEC: SO coupling





$|\uparrow, k+1\rangle \qquad |\downarrow, k-1\rangle$

$$H = \frac{\hbar^2(\hat{k}_y^2+\hat{k}_z^2)}{2m}\otimes\hat{1} + \sum_k \begin{pmatrix} (k+1)^2 + \delta/2 & \Omega/2 \\ \Omega/2 & (k-1)^2 - \delta/2 \end{pmatrix}$$

$$\downarrow$$

$$\frac{\hbar^2\hat{k}_x^{\,2}}{2m}\otimes\hat{1} + \frac{\hbar^2}{2m}2k_L\hat{k}_x\hat{\sigma}_z + E_L + \begin{pmatrix} \delta/2 & \Omega/2 \\ \Omega/2 & -\delta/2 \end{pmatrix}$$

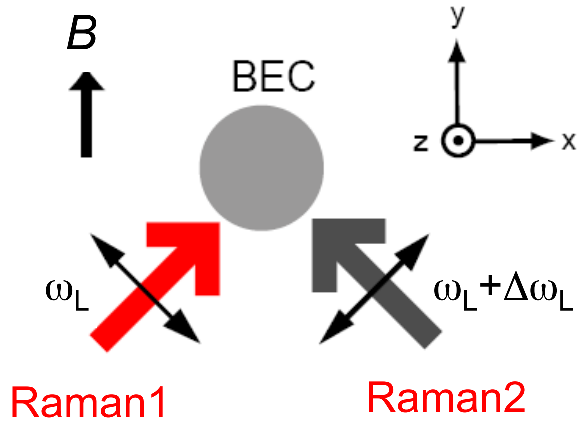
$$\rightarrow H = \frac{\hbar^2\hat{k}^2}{2m}\otimes\hat{1} + \frac{\hbar^2}{2m}2k_L\hat{k}_x\hat{\sigma}_y + E_L + \begin{pmatrix} \Omega/2 & -i\delta/2 \\ i\delta/2 & -\Omega/2 \end{pmatrix}$$

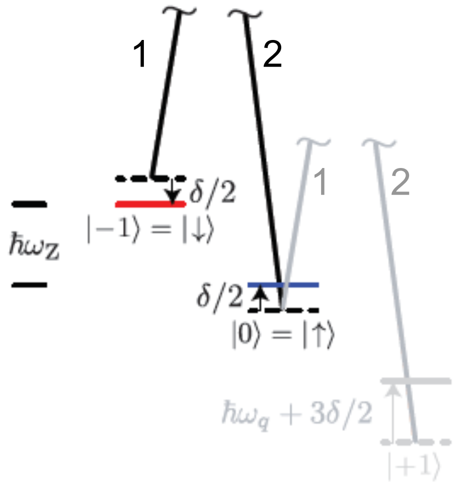
$$\frac{\Omega}{2}\cdot\sigma_z + \frac{\delta}{2}\cdot\sigma_y$$

$H_{so} = 2\alpha k_x \sigma_y \quad \alpha=\beta$

Ω: Raman coupling
δ: Raman detuning

Dressed states: modified energy dispersion





$$H = \frac{\hbar^2(\hat{k}_y^2+\hat{k}_z^2)}{2m}\otimes\hat{1} + \sum_k \begin{pmatrix} (k+1)^2+\delta/2 & \Omega/2 \\ \Omega/2 & (k-1)^2-\delta/2 \end{pmatrix}$$

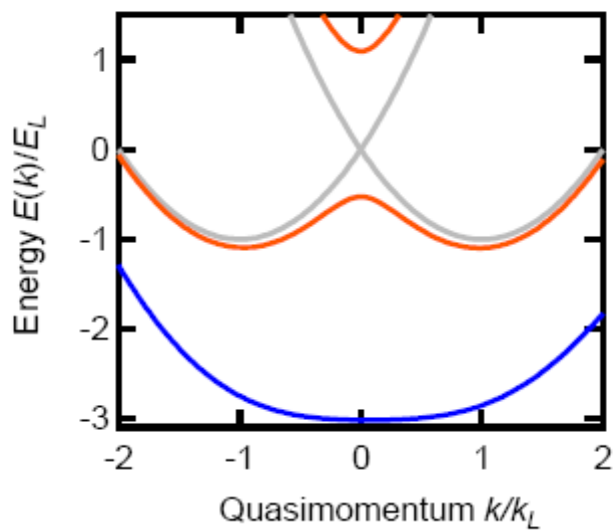
(basis: $|\uparrow, k+1\rangle$, $|\downarrow, k-1\rangle$)

quasimomentum k ↔ $\hat{k}_x$,
labeling bare spin momentum

$2k_L\hat{e}_x = \vec{k}_1 - \vec{k}_2$
k (k_L), E(E_L)
$E_L = \hbar^2 k_L^2/2m$

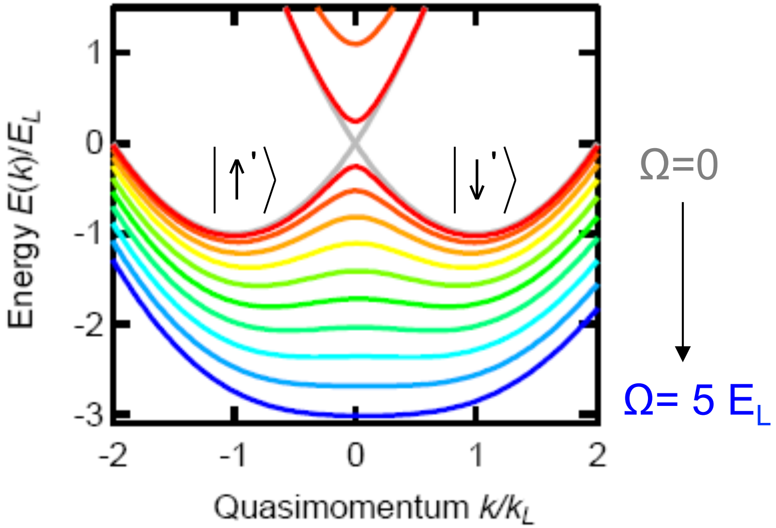
- diagonalize in *k* space
  → dressed state w/ modified E(k)

Energy dispersion E(k)
Ω= 0, 1 E_L, 5 E_L, δ=0



Energy dispersion at δ=0

2-level, $0<\Omega<5\ E_L$, δ=0



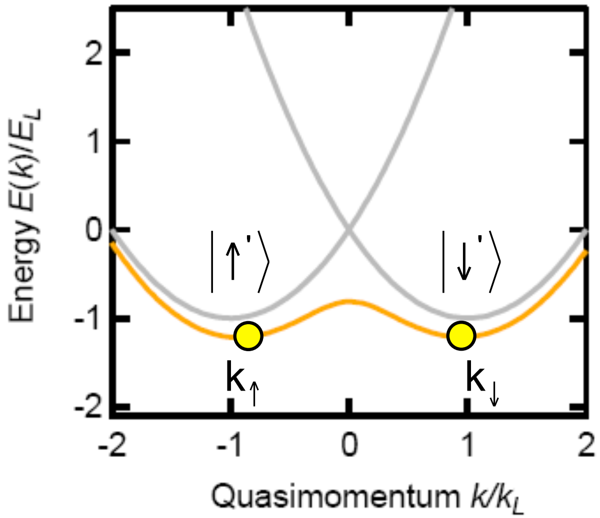
$0<\Omega< 4E_L$: double minimum, SO coupling limit

$H_{SO}=2\alpha k_x \sigma_y > \Omega$

$\Omega > 4E_L$: single minimum, vector potential limit

Ref: Y.-J. Lin et al., PRL (2009).

Ω=1.5 E_L, δ=0



- dressed spin states

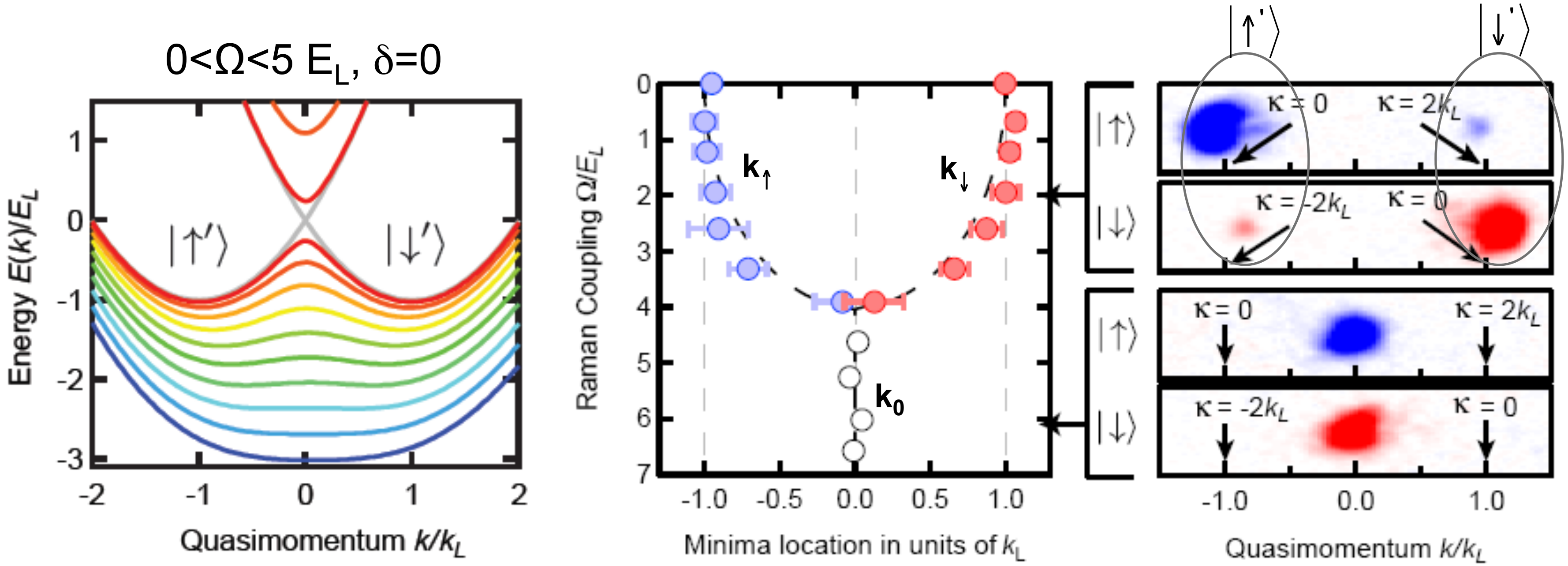
$|\uparrow'\rangle \approx |\uparrow, k+1\rangle - \varepsilon|\downarrow, k-1\rangle$ near $k = k_\uparrow$

$|\downarrow'\rangle \approx |\downarrow, k-1\rangle - \varepsilon|\uparrow, k+1\rangle$ near $k = k_\downarrow$

$\varepsilon = \Omega / 8E_L << 1$

k = quasi momentum

Data: SO coupled BEC, δ=0

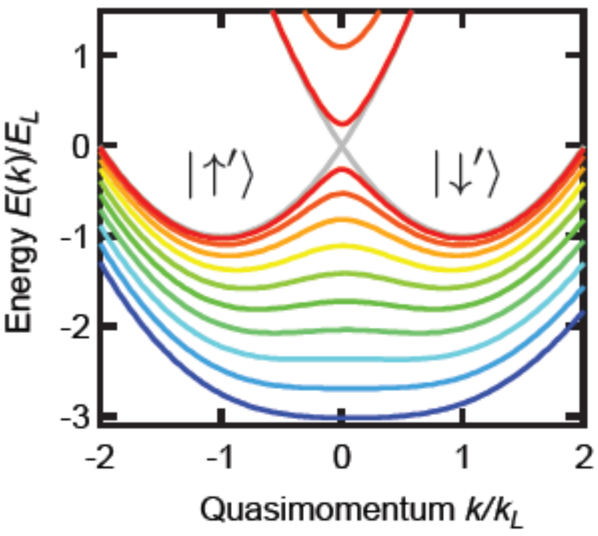


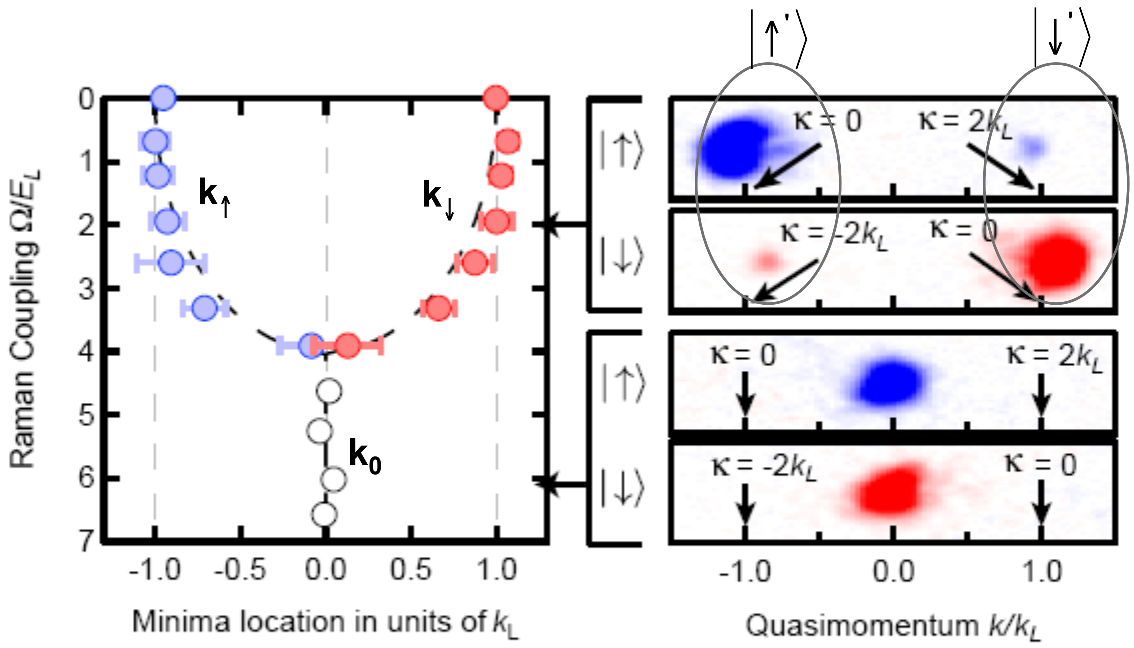
- TOF images: measure quasi-momentum k
- $\Omega < 4\ E_L$: double well regime, measure $k_\uparrow$, $k_\downarrow$ $\leftrightarrow$ $|\uparrow'\rangle, |\downarrow'\rangle$
- $\Omega > 4\ E_L$: single minimum, measure k_0

Ref. Y.-J. Lin, K.J.-Garcia and Ian Spielman, Nature, Mar 3, 2011.

Results: double well, δ=0

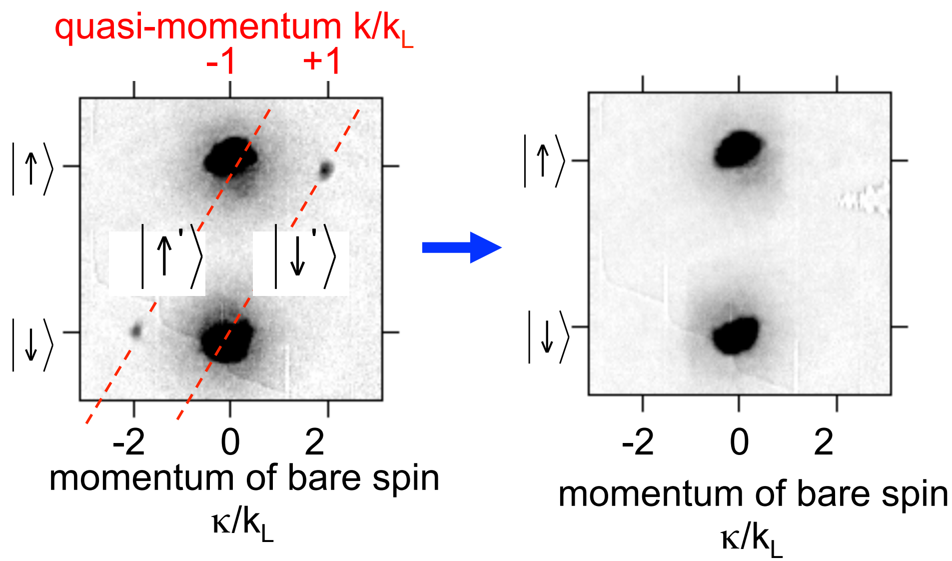
$0<\Omega<5\ E_L,\ \delta=0$





- measure $k_\uparrow$, $k_\downarrow$: projection of $|\uparrow'\rangle, |\downarrow'\rangle$

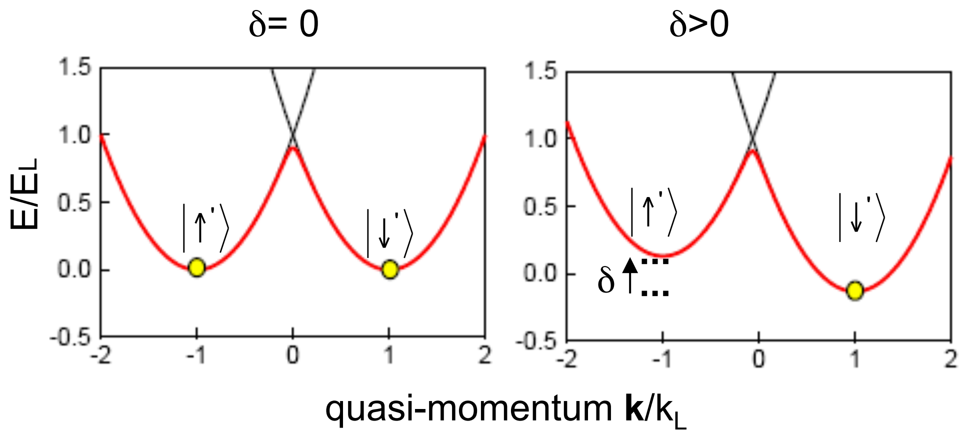
- band mapping: $|\uparrow'\rangle \to |\uparrow\rangle, |\downarrow'\rangle \to |\downarrow\rangle$
  easy detection of number N and temperature T of $|\uparrow'\rangle, |\downarrow'\rangle$



Ref. Y.-J. Lin, K.J.-Garcia and Ian Spielman, Nature, Mar 3, 2011.

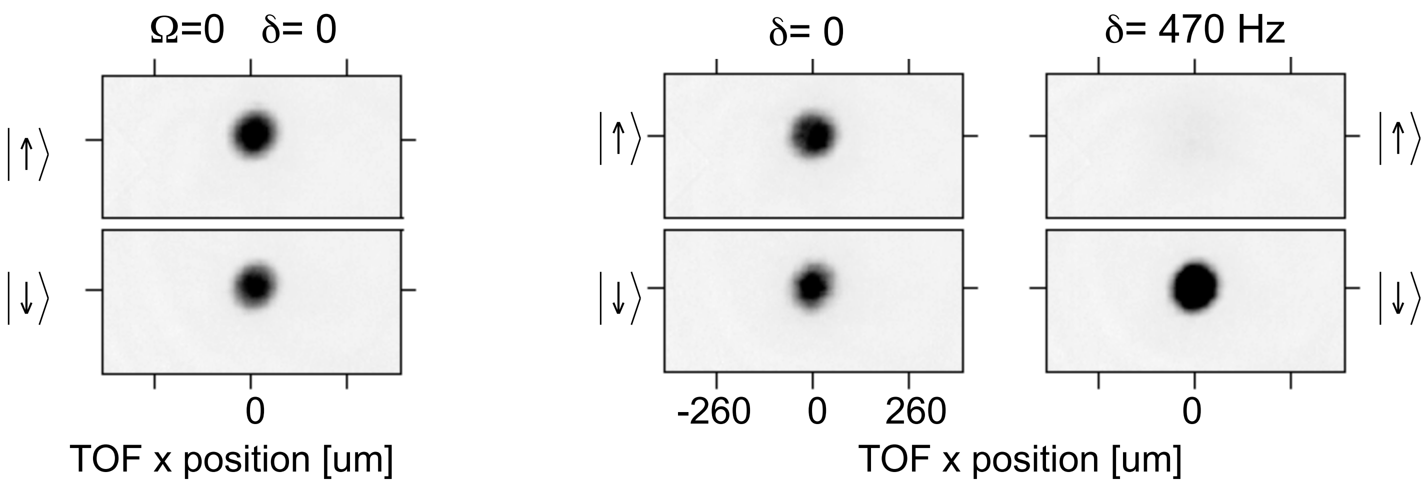
SO coupled BEC, $\delta \neq 0$

Ω=0.19 E_L, hold time t_h=3s



Detection: band mapping $|\uparrow'\rangle \rightarrow |\uparrow\rangle, |\downarrow'\rangle \rightarrow |\downarrow\rangle$

Ω=0.19 E_L t_h=3s

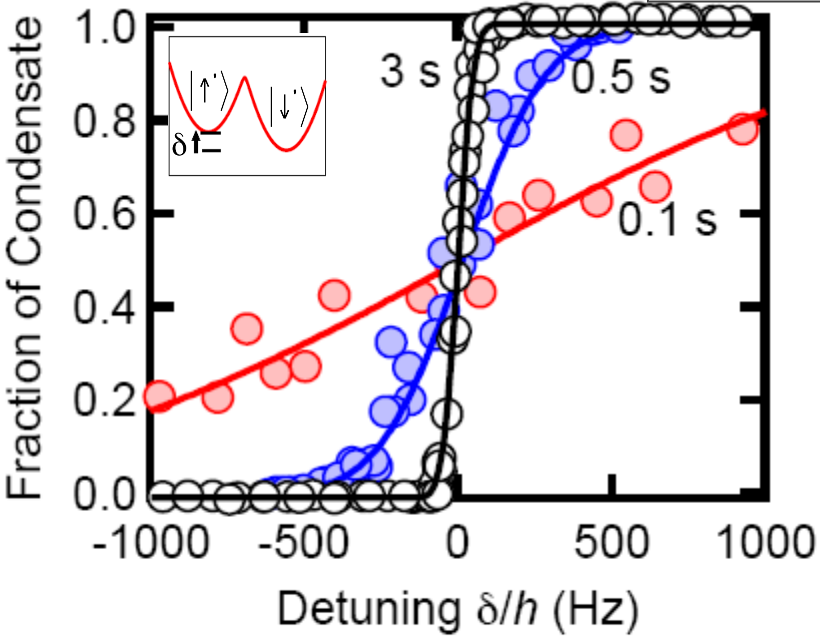


Results: double well, $\delta \neq 0$

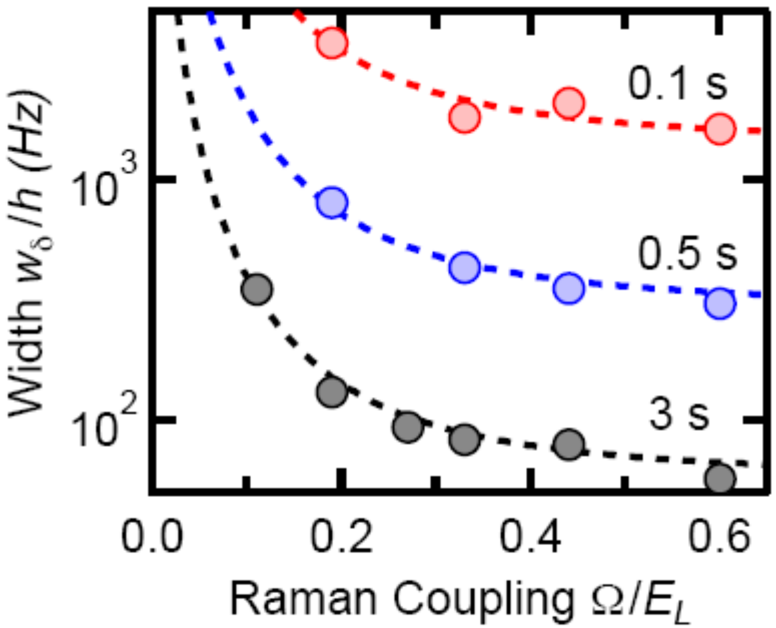
- measure $N_{\uparrow'}$, $N_{\downarrow'}$ at (Ω,δ) and $t=t_h$
- number is metastable up to t_h=3s: not ground state yet
  detuning width w_δ decreases with t_h
  $f_{\downarrow'}$ =0.50 ±0.16 ⟺ $\delta = \pm w_\delta$
- detuning width w_δ decreases with Ω

$|\downarrow'\rangle$ fraction vs. δ

Ω=0.6 E_L
t_h=0.1,0.5,3s



Detuning width w_δ vs. Ω



Mean field phase diagram vs. (Ω,δ)

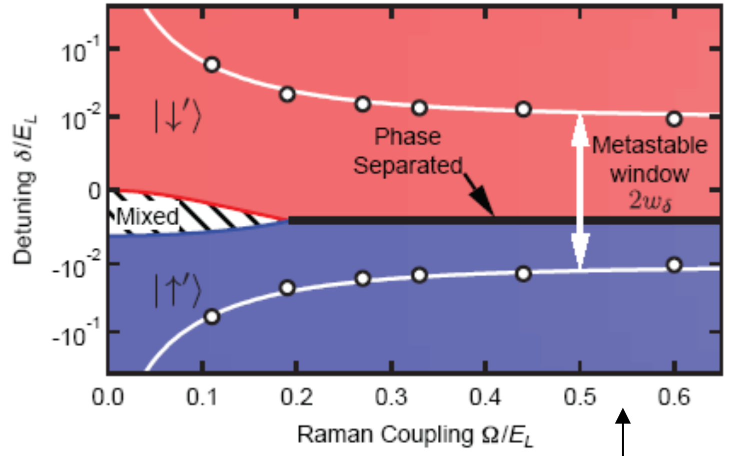
Theory: $H=H_{int}+(N_{\uparrow'}-N_{\downarrow'})\delta/2$

- $H_{int}=0$, zero transition width in δ
- $H_{int}>0$: finite transition width in δ;
  miscible to immiscible due to Raman dressing

Experiment

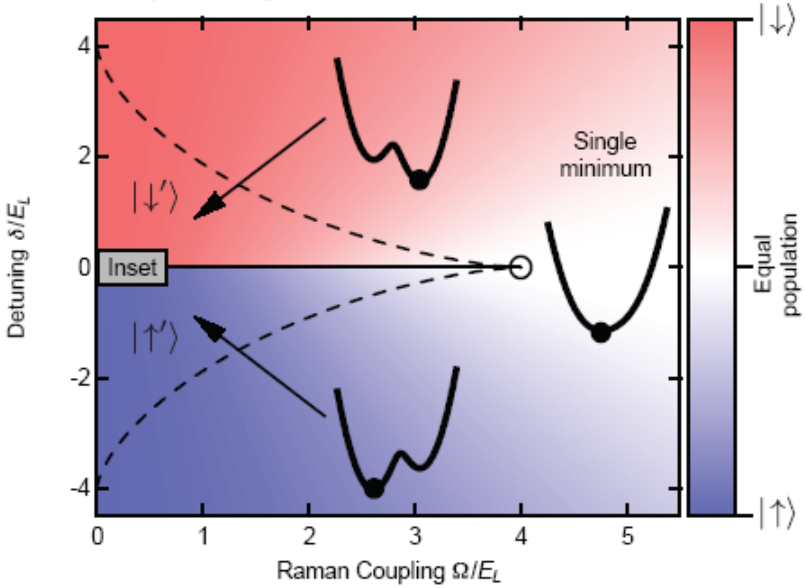
- population is metastable, up to t_h=3s
- observe transition of miscibility

Inset

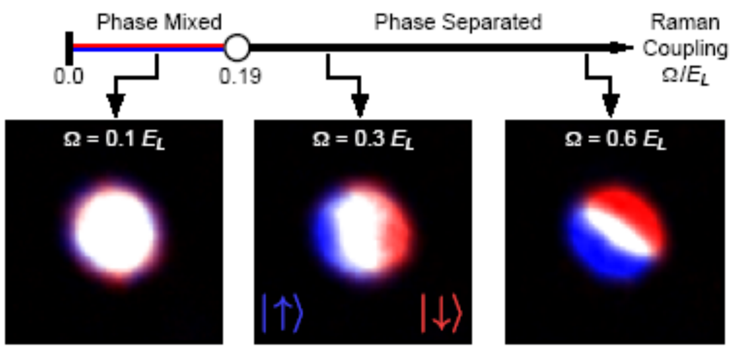


Theory: T.-L. Ho (2011), Y. Li (2012)
Rashba: C. Wang et al. (2010), S.-K Yip (2011).

Ground state mean-field phase diagram



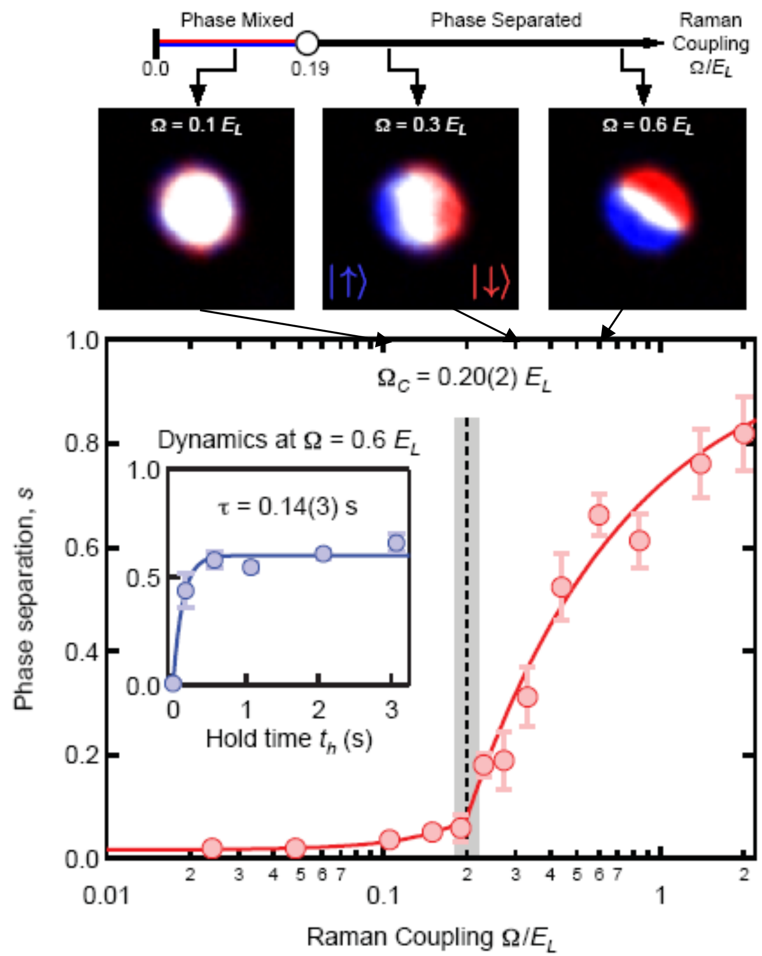
Miscible to immiscible transition



Observing quantum phase transition: spatially mixed to separated

- phase separation $s = 1 - \langle n_{\uparrow'} n_{\downarrow'} \rangle \Big/ \sqrt{\langle n^2_{\downarrow'} \rangle \langle n^2_{\uparrow'} \rangle}$

- t_h=3s □ phase separation time τ



Ref. Y.-J. Lin, K.J.-Garcia and Ian Spielman, Nature, Mar 3, 2011.

Origin of phase separation

Effective repulsive interaction between dressed spins $c_{\uparrow'\downarrow'}>0$

$$H_{\text{int}} = \frac{1}{2}\int d^3r\left[c_0\hat{\rho}_{\uparrow}{}^2 + (c_0+c_2)\hat{\rho}_{\downarrow}{}^2 + 2(c_0+c_2)\hat{\rho}_{\uparrow}\hat{\rho}_{\downarrow}\right]$$

$$= \frac{1}{2}\int d^3r\left[(c_0+\frac{c_2}{2})(\hat{\rho}_{\downarrow}+\hat{\rho}_{\uparrow})^2 + \frac{c_2}{2}(\hat{\rho}_{\downarrow}^2-\rho_{\uparrow}^2) + c_2\hat{\rho}_{\downarrow}\hat{\rho}_{\uparrow}\right]$$

in $|\uparrow'\rangle, |\downarrow'\rangle$ basis: $\hat{\psi}_{\uparrow}(r) \approx \hat{\psi}_{\uparrow'}(r) + \varepsilon\, e^{i2k_Lx}\hat{\psi}_{\downarrow'}(r),\ \hat{\psi}_{\downarrow}(r) \approx \hat{\psi}_{\downarrow'}(r) + \varepsilon\, e^{-i2k_Lx}\hat{\psi}_{\uparrow'}(r)$

$$\rightarrow H_{\text{int}} = \frac{1}{2}\int d^3r\left[(c_0+\frac{c_2}{2})(\hat{\rho}_{\downarrow'}+\hat{\rho}_{\uparrow'})^2 + \frac{c_2}{2}(\hat{\rho}_{\downarrow'}^2-\hat{\rho}_{\uparrow'}^2) + (c_2+\underline{c_{\uparrow'\downarrow'}})\hat{\rho}_{\downarrow'}\hat{\rho}_{\uparrow'}\right]$$

$c_{\uparrow'\downarrow'} = c_0\Omega^2/(8E_L^2)$ effective interaction

$c_{\uparrow'\downarrow'}=0$, $c_2<0$: miscible (spatially mixed)

$c_{\uparrow'\downarrow'}>0$

miscible, at $\Omega<\Omega_c$

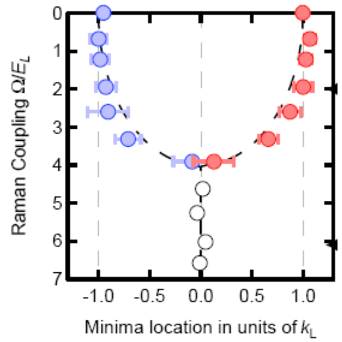
immiscible (spatially separated), at $\Omega>\Omega_c$

transition at $c_0 + c_2 + c_{\uparrow'\downarrow'}/2 = \sqrt{c_0(c_0+c_2)}$, at $\Omega = \Omega_c$

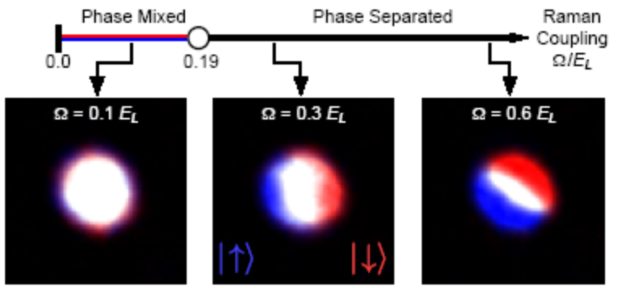
Continued work.: Williams et al., Science **335**, 314-317 (2012).

Conclusion

- First experimental realization of SO coupling for neutral atoms, and for bosons



- Quantum phase transition due to Raman dressing: spatially mixed → separated

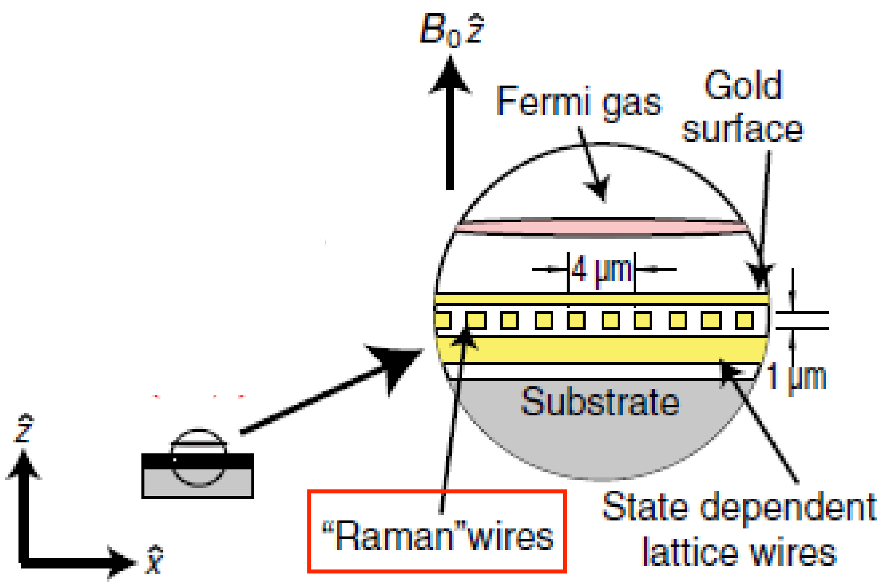


Ref. Y.-J. Lin, K.J.-Garcia and Ian Spielman, Nature, Mar 3, 2011.

Some recent work in NIST group:

- New results (May 2012): Spin-Hall effect $B^*_{\uparrow'} = -B^*_{\downarrow'}$
- on going: Create SO coupling in fermionic ^{40}K: towards topological insulators

NIST atom chip experiment: towards topological insulators



"Raman coupling": moving magnetic lattice
not optical, no spontaneous emission: work for alkali atoms

Theory: Goldman et al., PRL **105**, 255302 (2010).

Other recent experiments

boson:

• S. Chen group, China, arxiv 1201.6018
collective dipole oscillations of atoms in H_{so}
Theory: Y. Li et al. arxiv 1205.6398

fermion

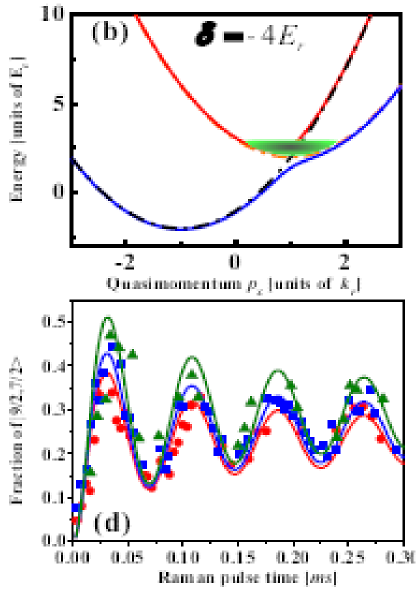
- J. Zhang group, China, arxiv 1204.1887
- M. Zwierlein group, MIT, arxiv 1205.3483
- To induce p-wave coupling between spin-polarized fermions
  make p-wave superfluids: non-Abelian statistics, Majorana fermions

  Ref: Zhang et al., PRL (2008).

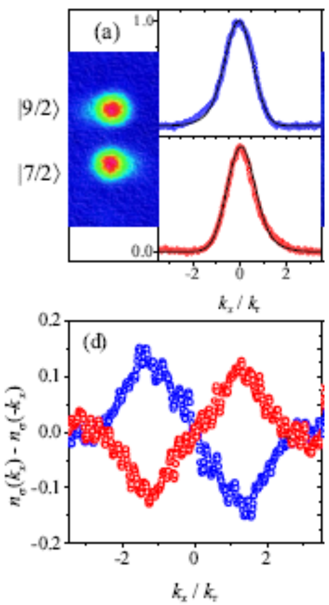
Fermions with SO coupling

^{40}K: Wang et. al, J. Zhang, 2012. arxiv 1204.1887

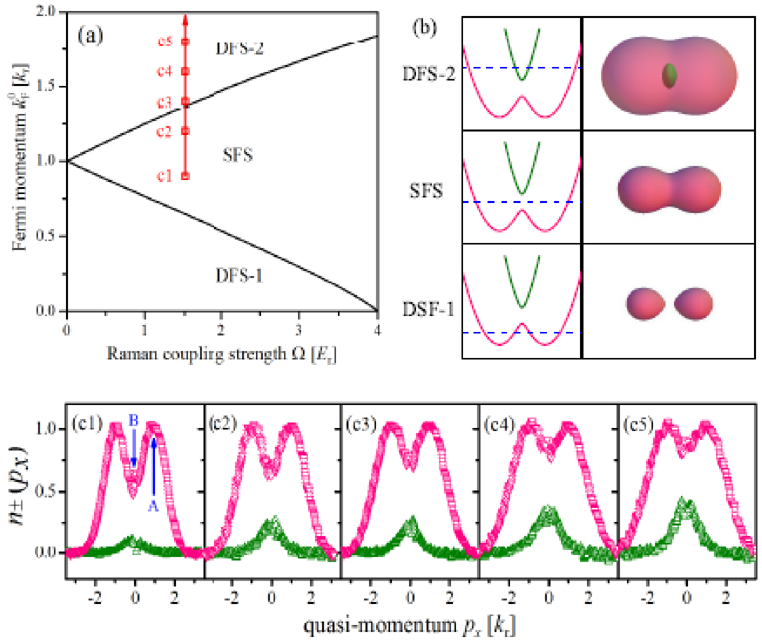
(I)



(II)

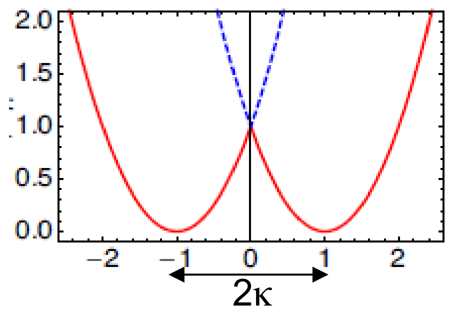


(III)



$$H = \frac{\hbar^2}{2m}(k - \kappa\sigma_z)^2$$

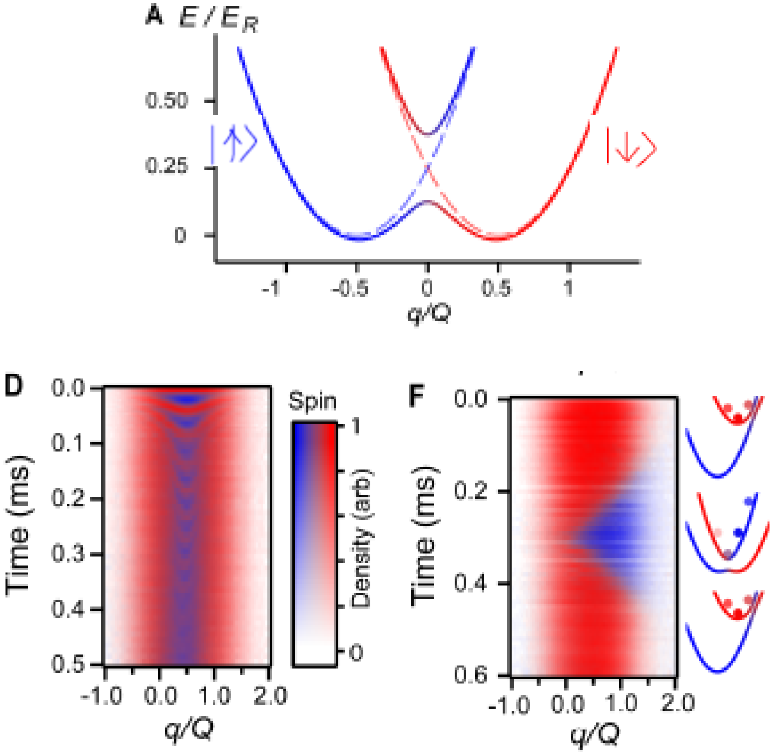
- (I) momentum-dependent Raman Rabi oscillation
- (II) asymmetric momentum distribution: signature of H_{so}
- (III) probe Fermi surface



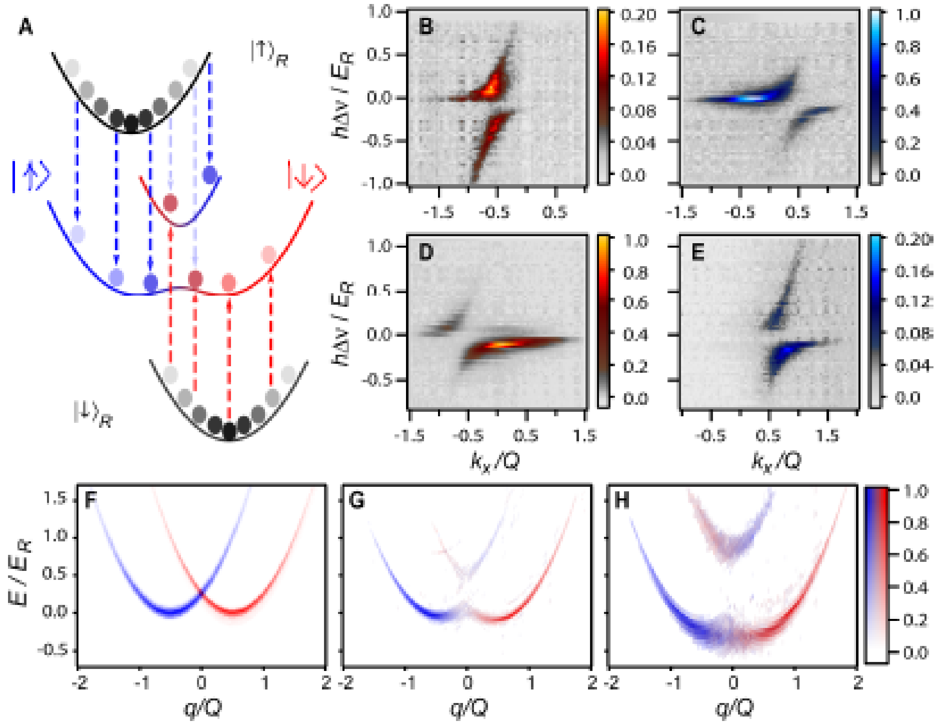
Fermions with SO coupling

^{6}Li, Cheuk et. al, M. Zwierlein, 2012. arxiv 1205.3483

Spin-orbit gap characterization

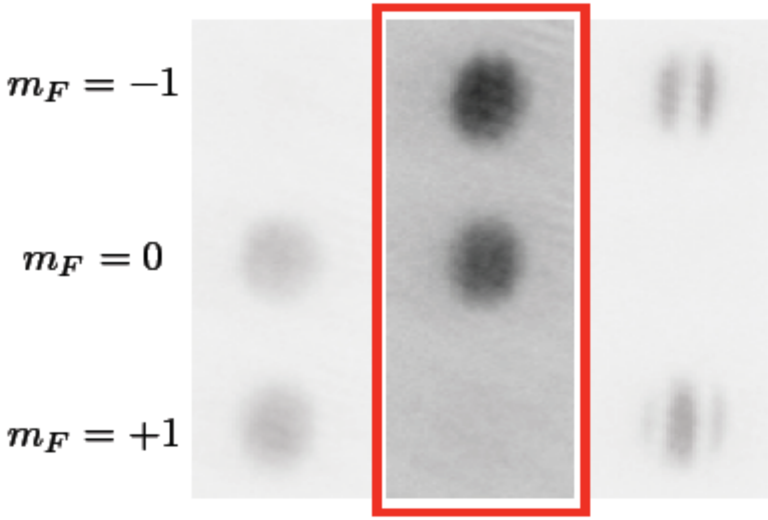


Measure E(q) w/ spin texture



* may be useful for measuring topological invariant : probe TI and Majorana fermions

|-1>, |0> mixture: $c_2<0$, miscible for ^{87}Rb



Ph. D thesis of M.-S. Chang,
Chapman group