One dimensional quantum systems

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Plan of the lectures (1)

- Lecture 1:
  - What are one dimensional systems
  - Universal physics in one dimension (Luttinger liquid)
  - Some realizations with cold atoms

Plan of the lectures (2)

- Lecture 2:
  - Effect of periodic potentials: Mott transition
  - Experimental tests of the Mot transition
  - Systems with internal degrees of freedom (spin)

Plan of the lectures (3)

- Lecture 3:
  - Disorder and 1D quantum systems (dirty bosons)
  - Disorder and quasiperiodicity
  - Loose ends and open questions

Lecture 1

References

TG, arXiv/0605472 (Salerno lectures)

TG, Quantum physics in one dimension, Oxford (2004)

M. Cazalilla et al.,
Rev. Mod. Phys.83 1405 (2011)

How to describe solids ?

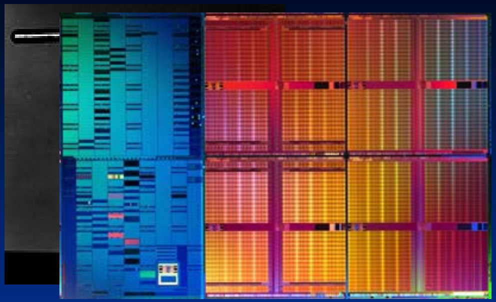
- Understood: free electrons
- Real systems : Coulomb interaction

E » 10 000 K !

- Properties of realistic systems ?

- Free electron theory works quite well : Landau Fermi liquid

• Transistor



1956

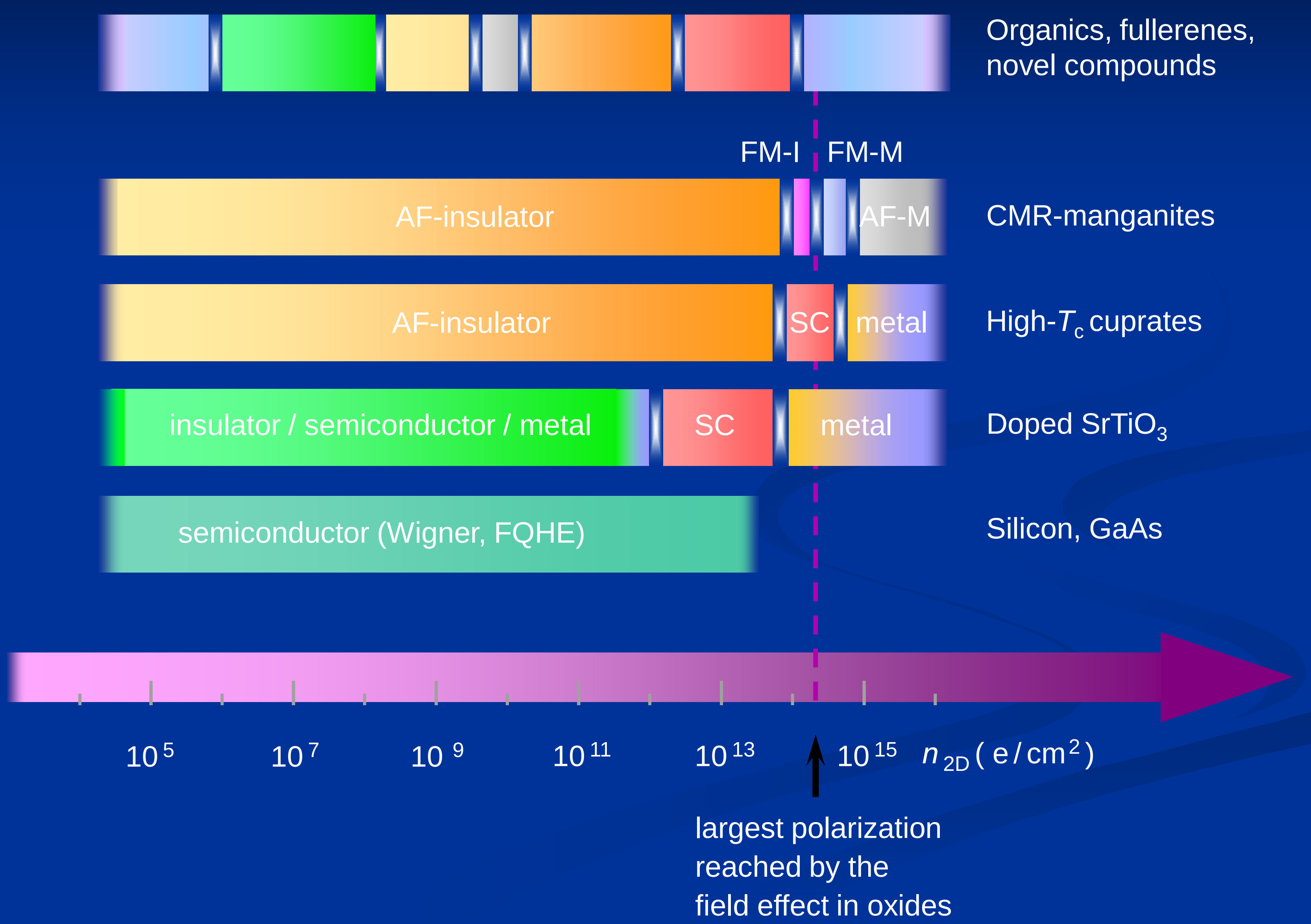
• Supraconductivité

(1913),1972,
1973,1987,2003

• Magnétorésistance géante

2007

Densities



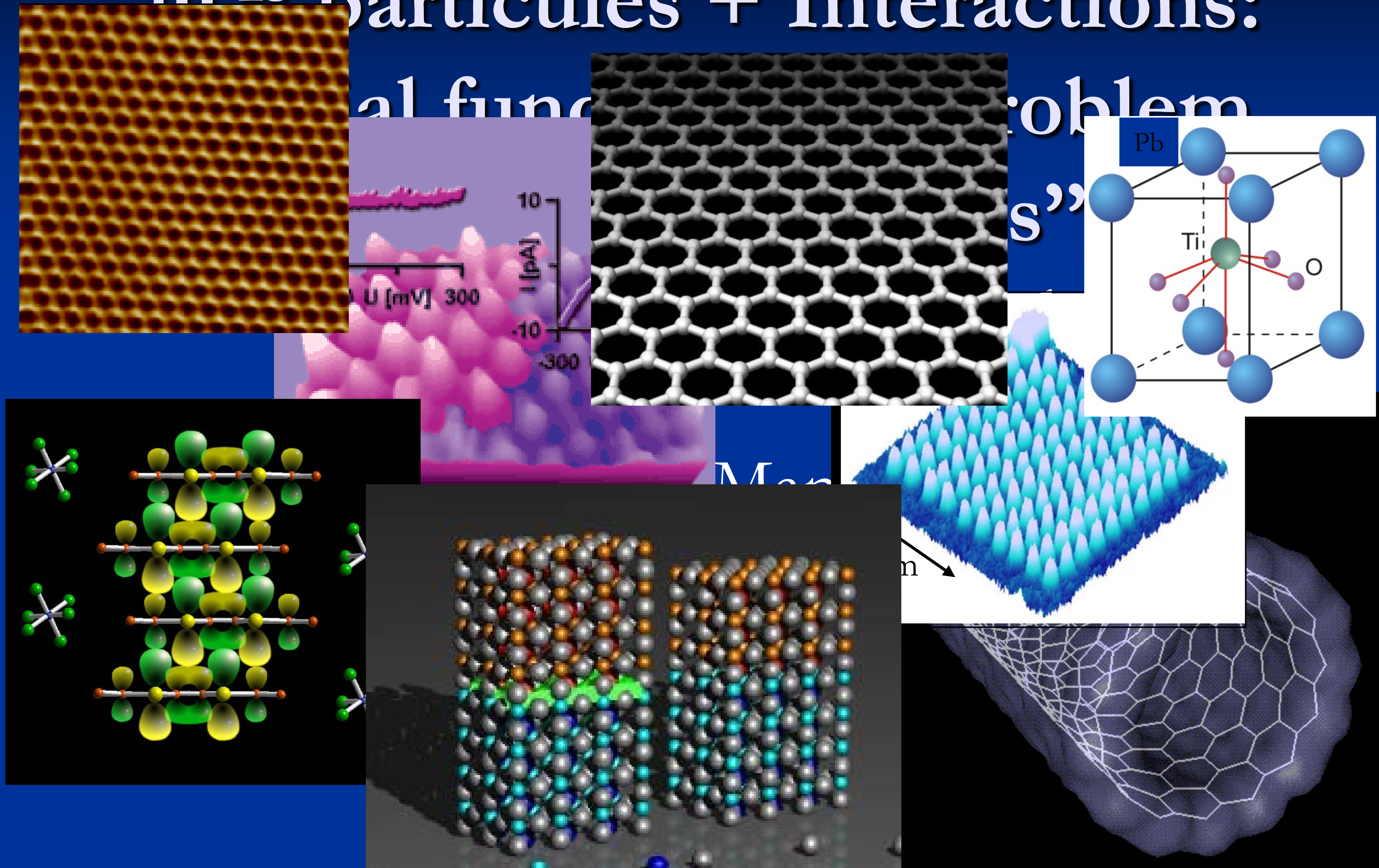
Organics, fullerenes, novel compounds
FM-I
FM-M
AF-insulator
AF-M
CMR-manganites
AF-insulator
SC
metal
High-T_c cuprates
insulator / semiconductor / metal
SC
metal
Doped $SrTiO_3$
semiconductor (Wigner, FQHE)
Silicon, GaAs
10^5
10^7
10^9
10^{11}
10^{13}
10^{15}
n_{2D} (e / cm^2)
largest polarization reached by the field effect in oxides

Need to understand interactions !

10^{23} particules + Interactions:

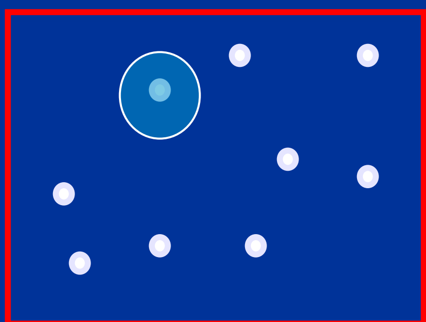
al func … roblem

s”



One dimension is specially interesting

- No individual excitation can exist (only collective ones)



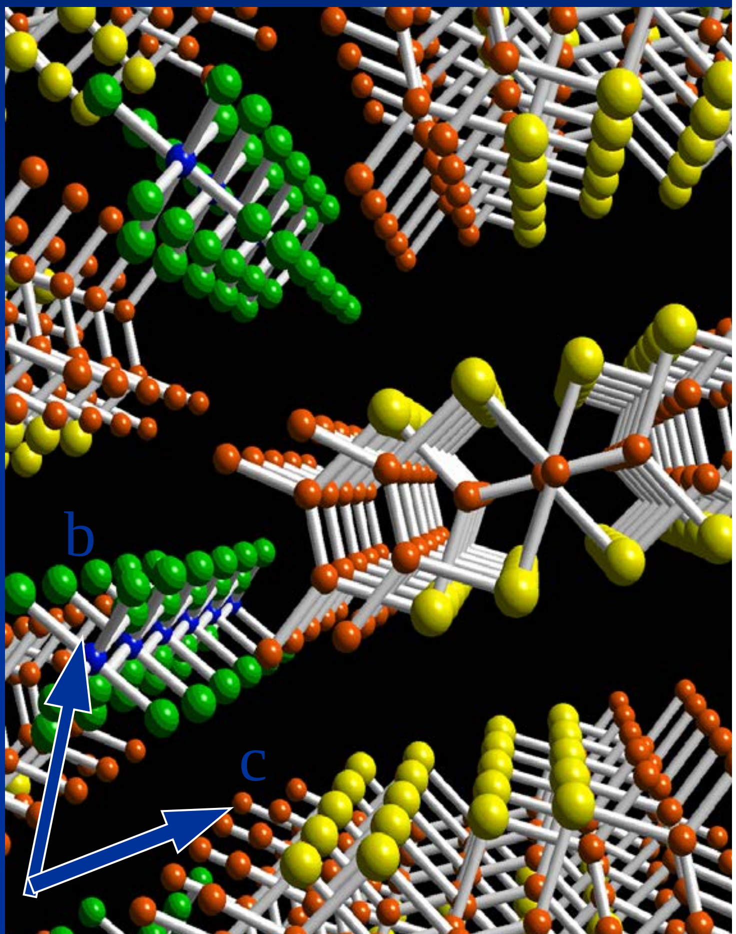
- Strong quantum fluctuations

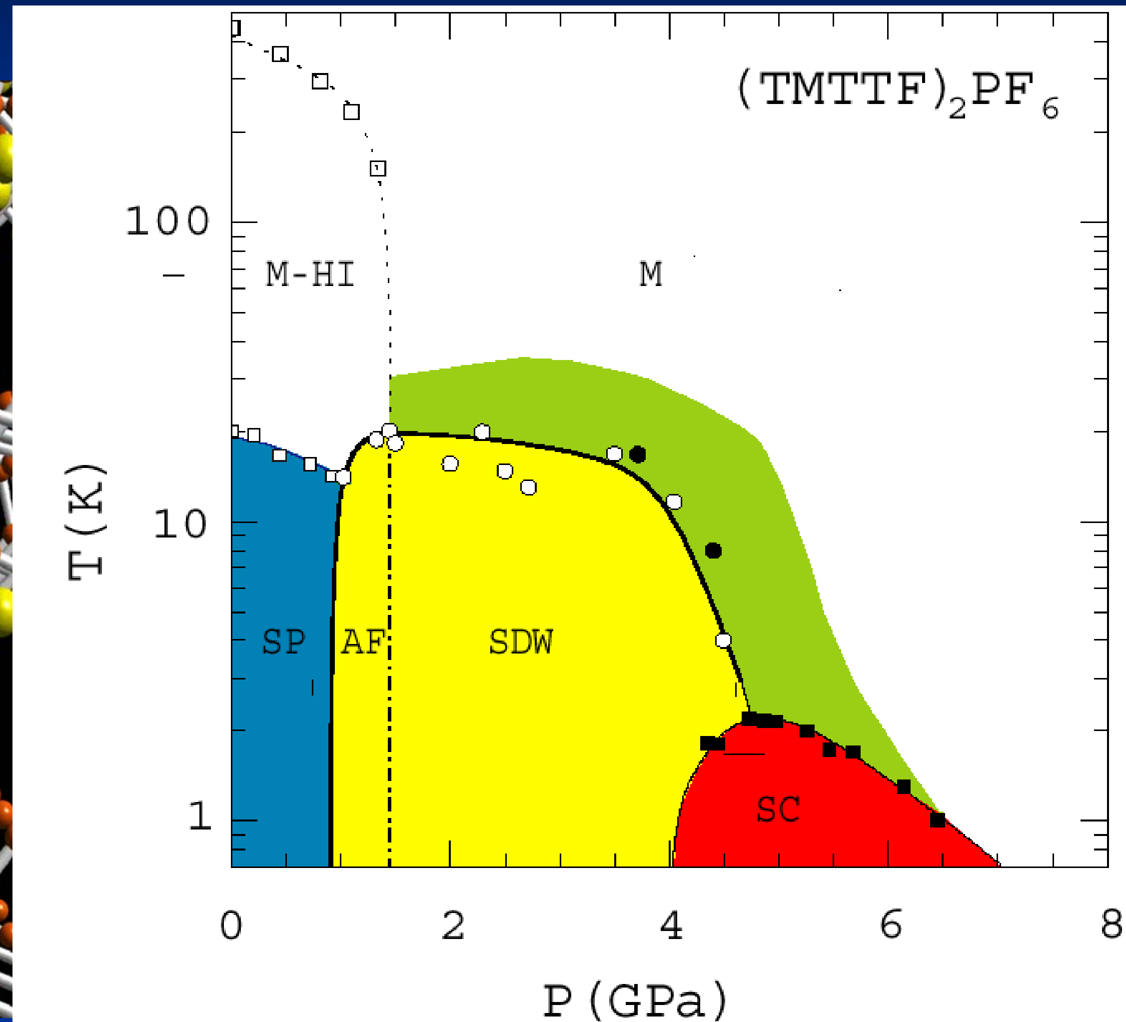
$$\psi = |\psi| e^{i\theta}$$

Difficult to order

How to realize

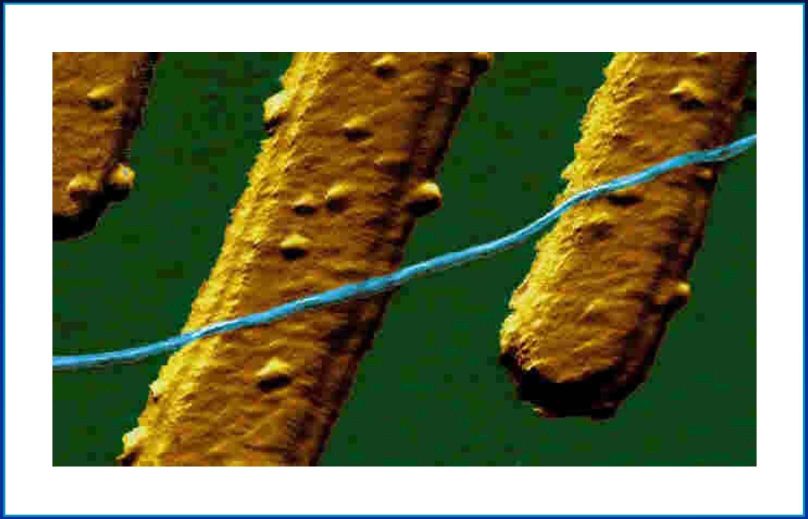
Organic conductors



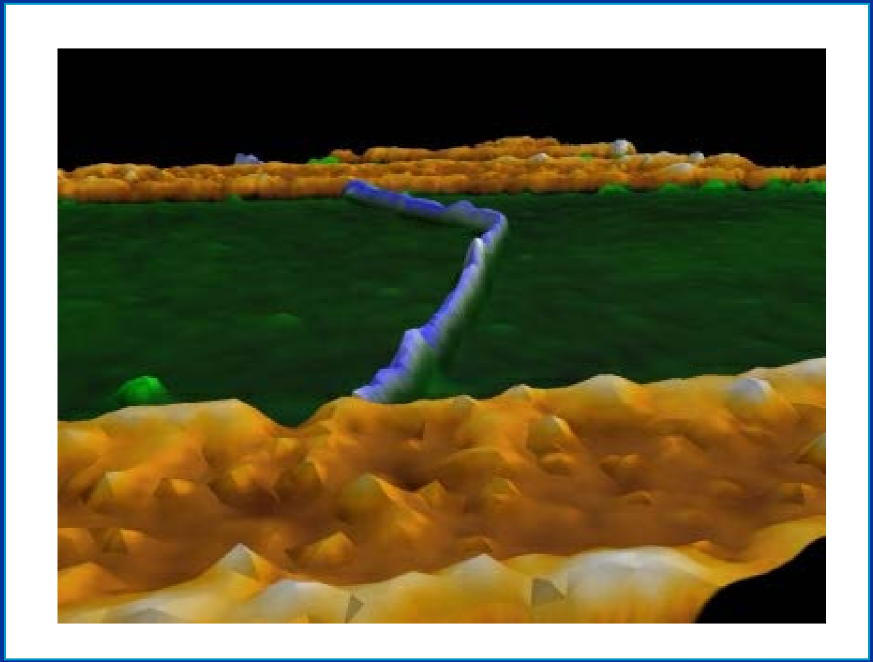


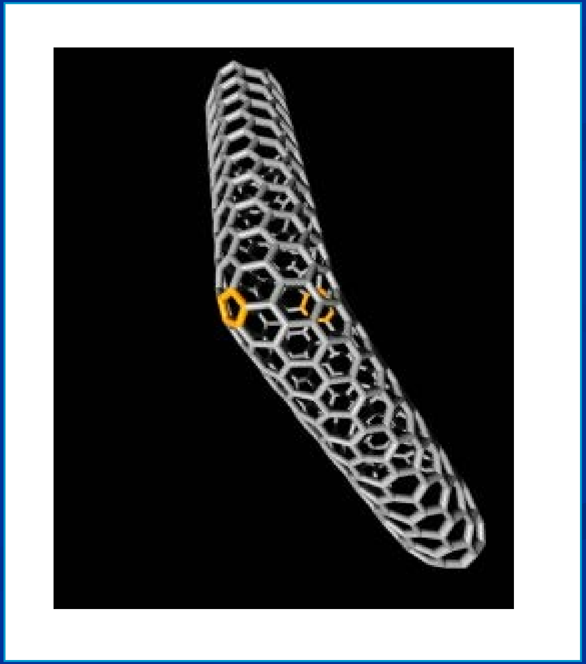
D. Jaccard et al., J. Phys. C, 13 L89 (2001)

CARBON NANOTUBES

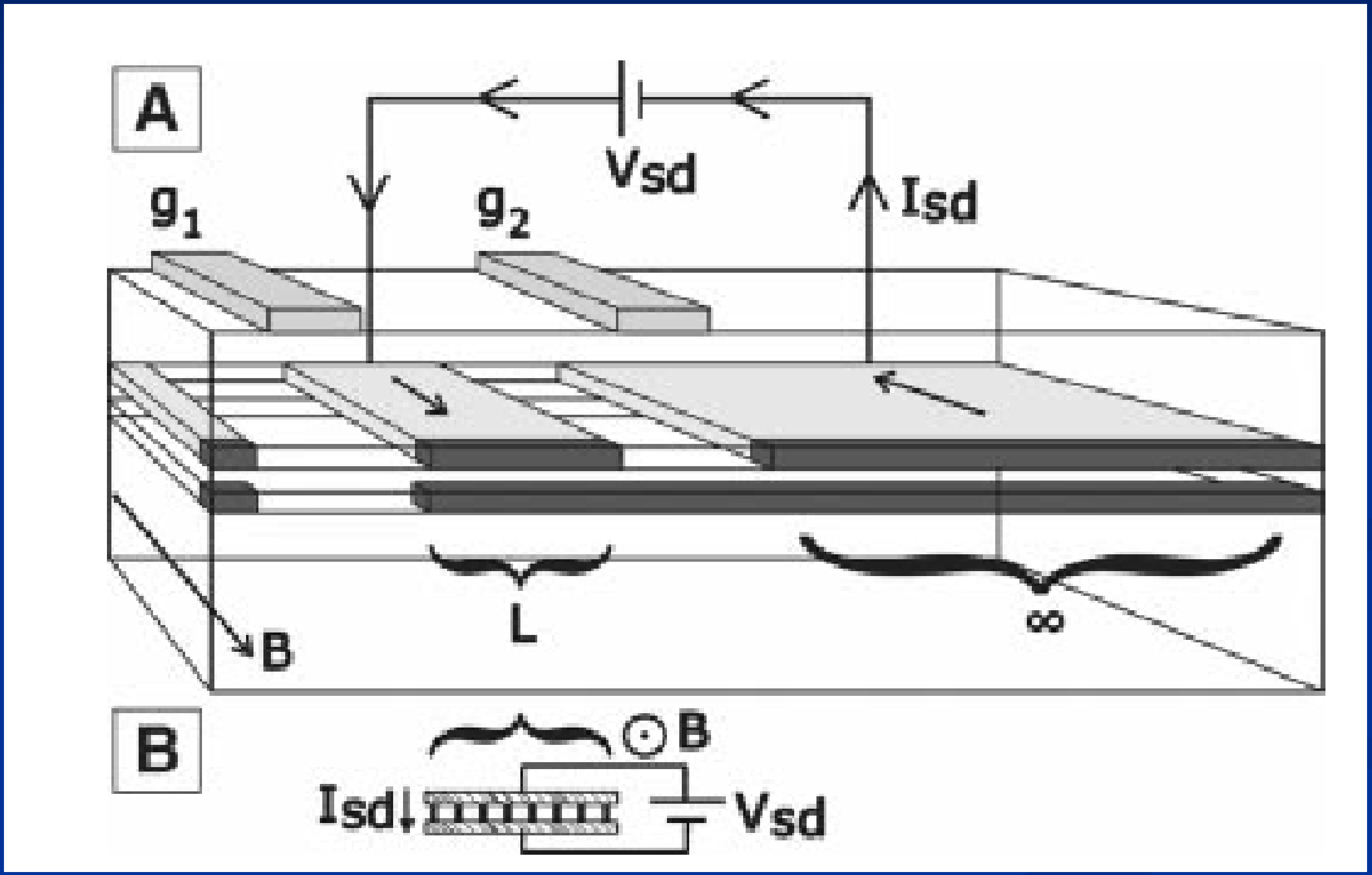


Cees Dekker





Quantum wires

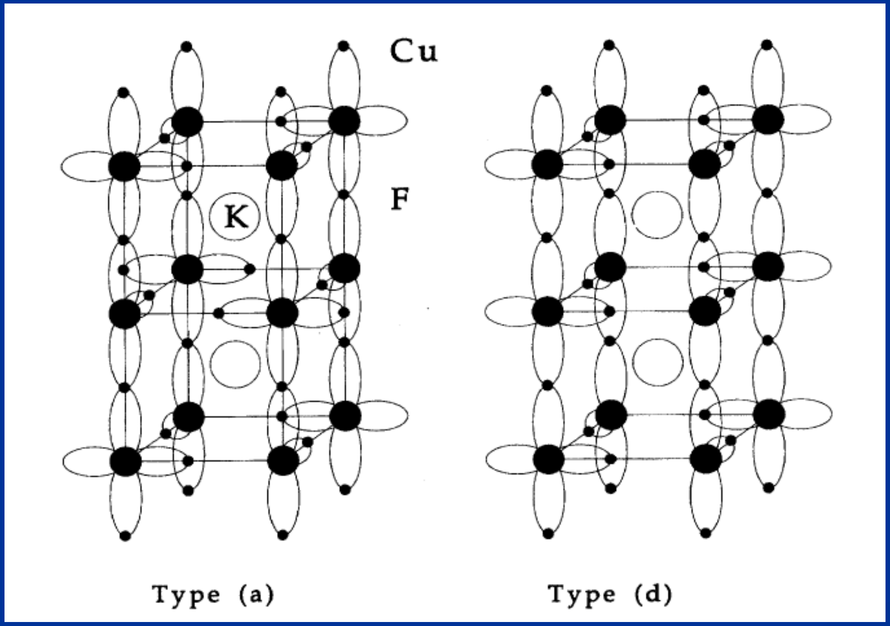


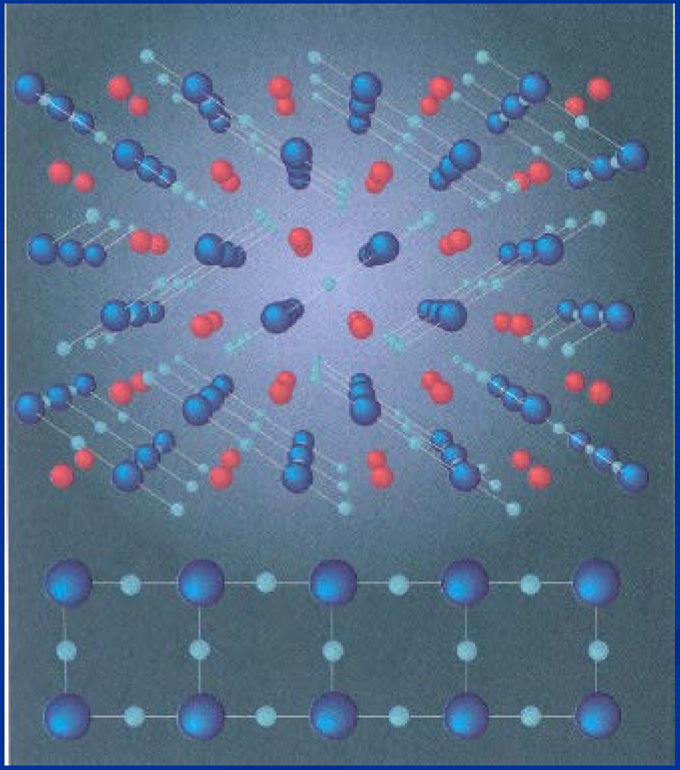
O.M Ausslander et al., Science **298** 1354 (2001)

Spin chains and ladders

Spin chains

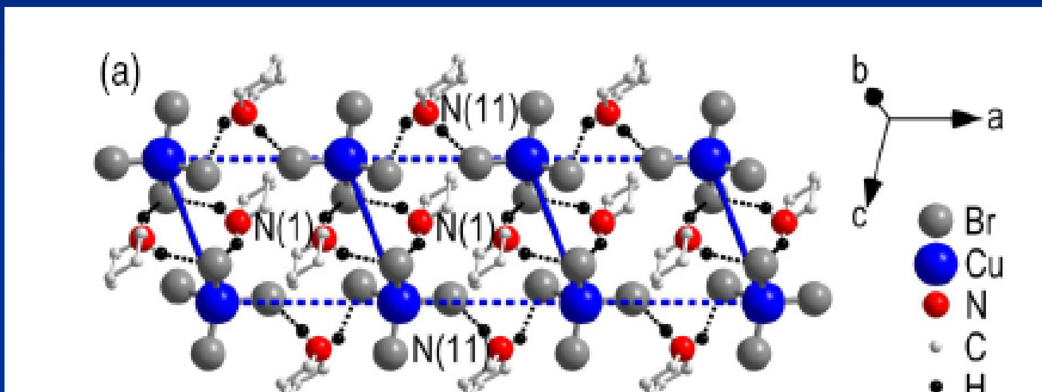
Ladder systems





Spin dimer systems

B. C. Watson et al., PRL 86 5168 (2001)

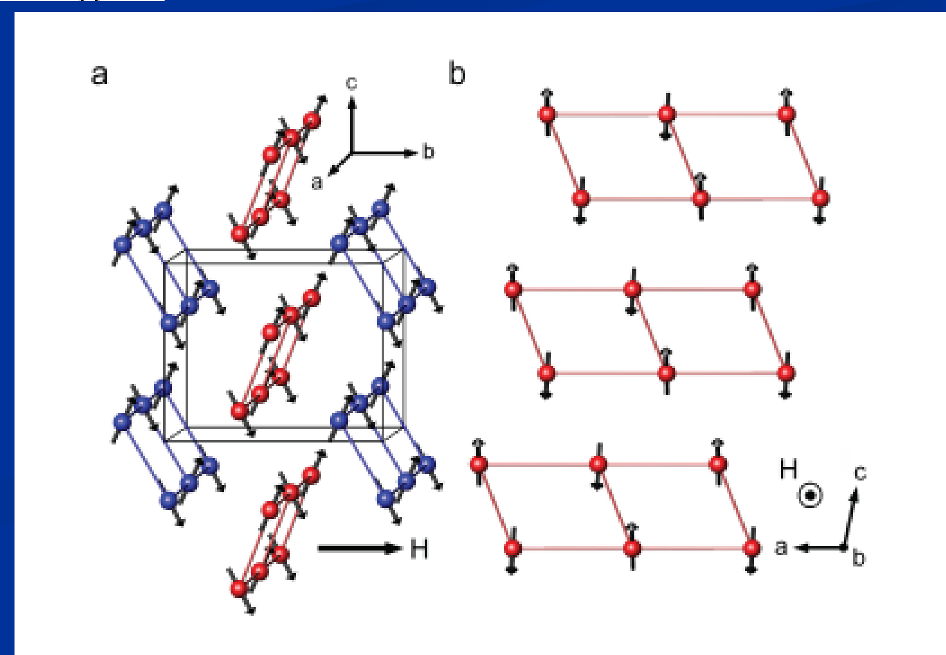


M. Klanjsek et al.,

PRL 101 137207 (2008)

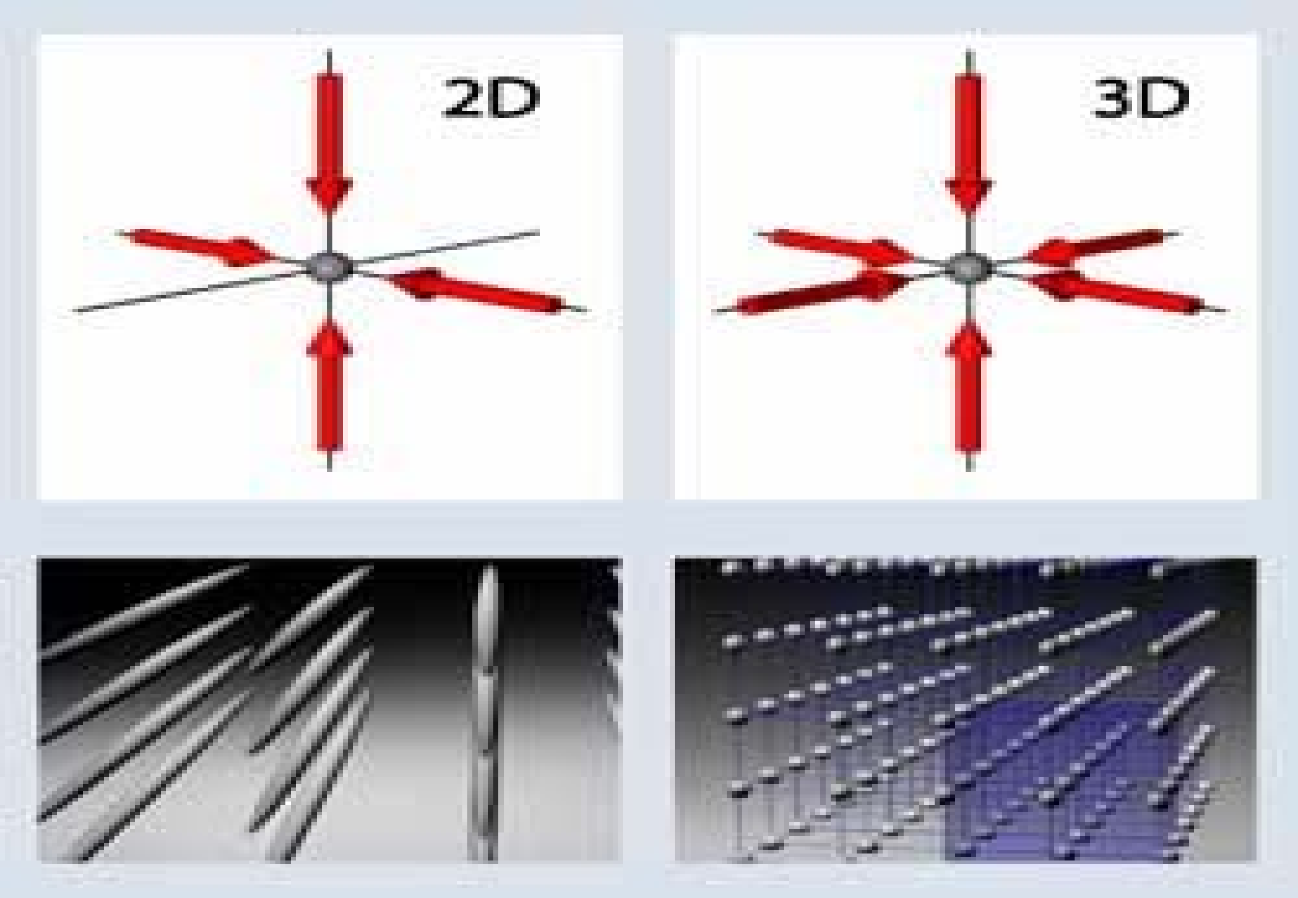
B. Thielemann et al.,

arXiv:0809.0440 (2008)



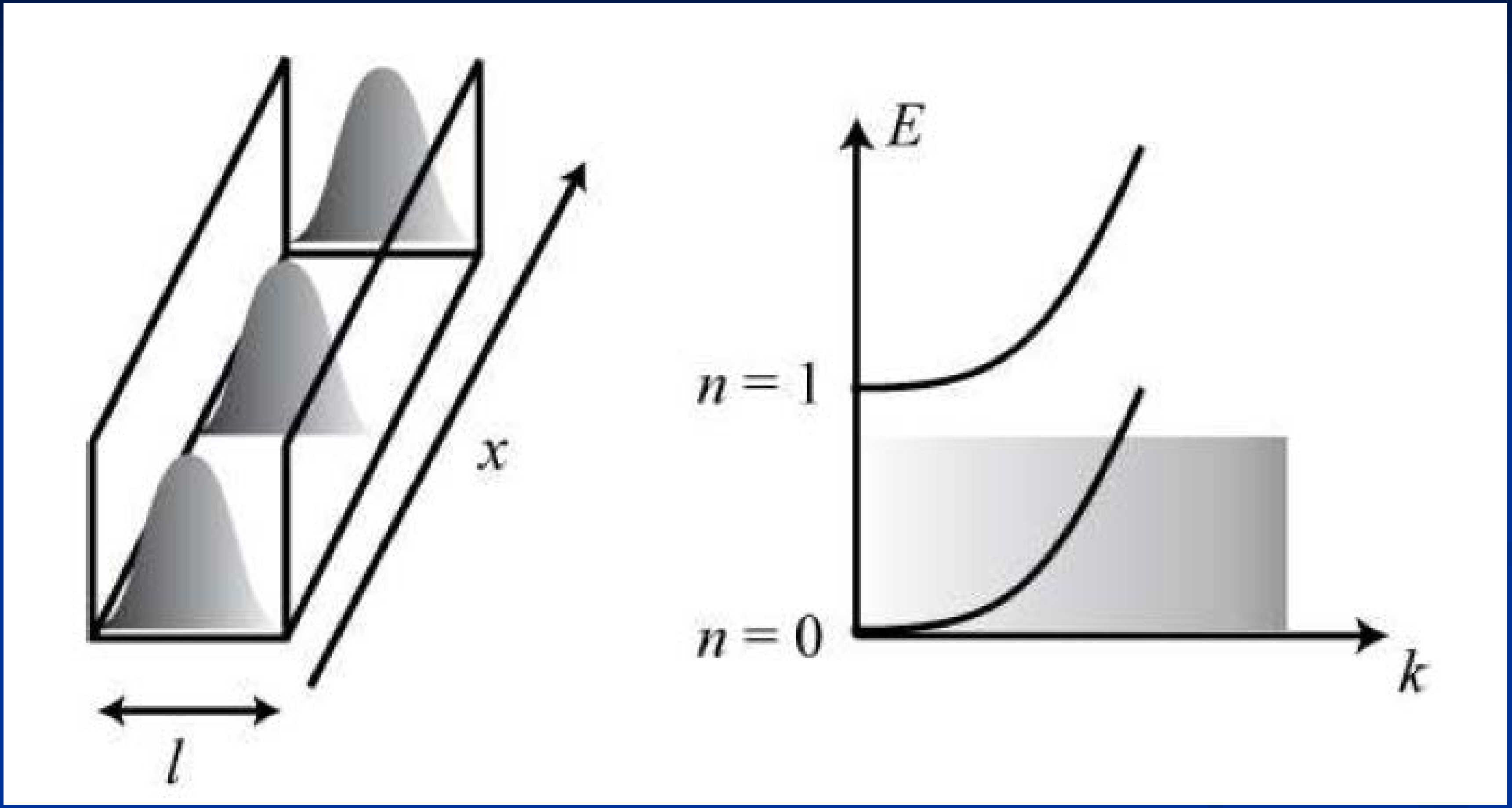
Cold atoms

Control on the dimension



I. Bloch, Nat. Phys 1, 23 (2005)

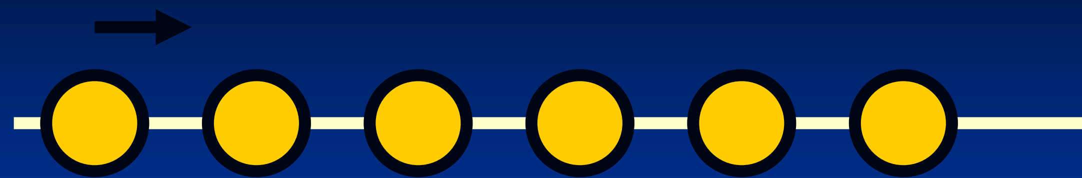
What does 1D mean



x
l
E
n = 1
n = 0
k

How to treat

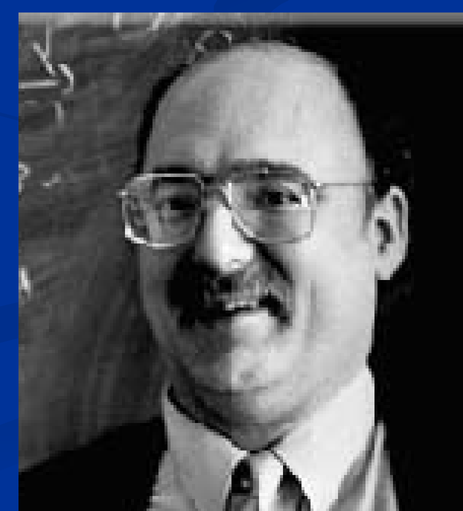
- Exact Solutions
  - Exact
  - Limited to some special models
  - Difficult to compute correlation functions
- Numerics
  - Very powerful
  - Difficult to reach low energy physics (space/time)
- Asymptotic methods
  - Physically transparent
  - Limited to low energy physics

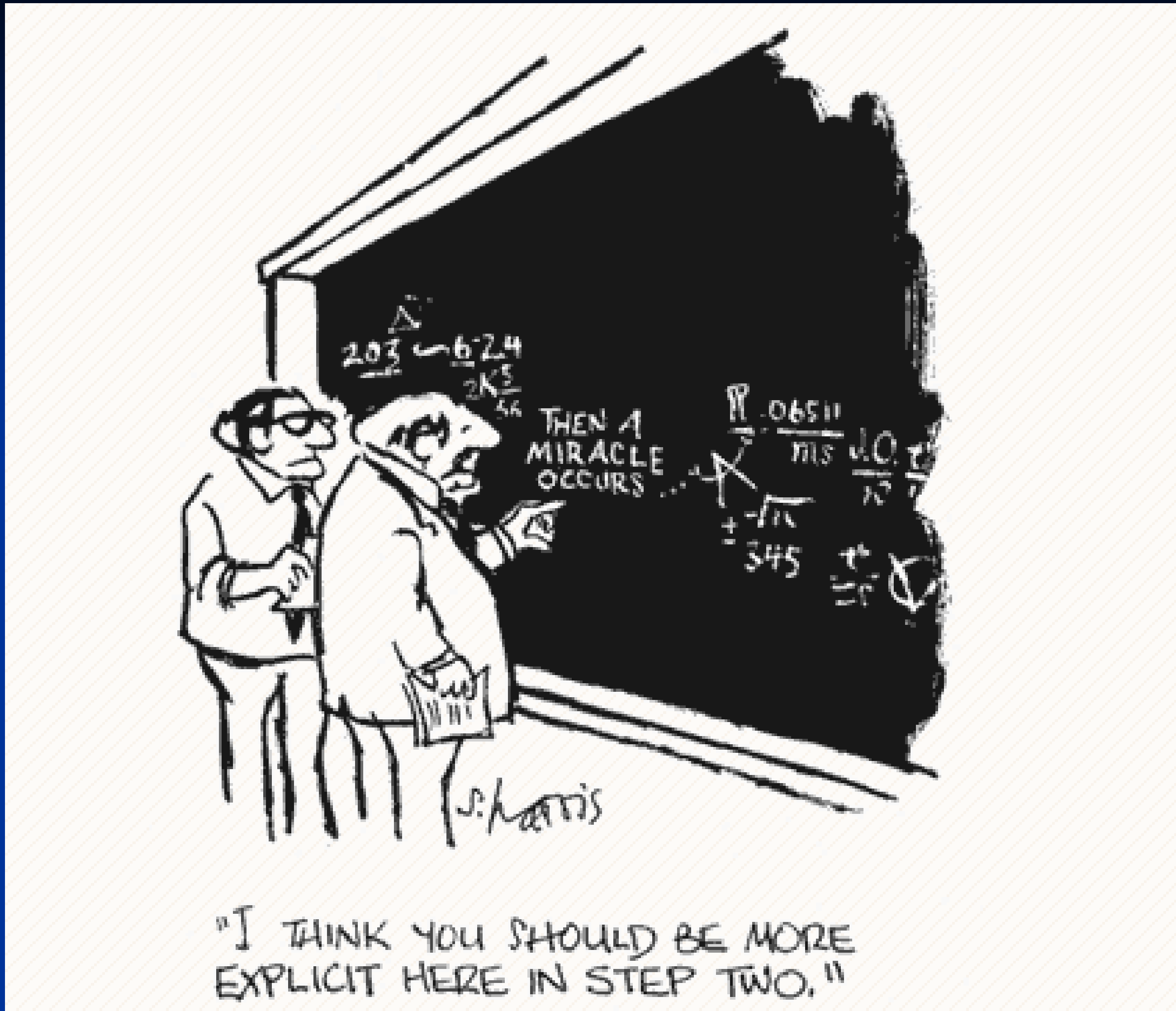


Good excitations: collective ones

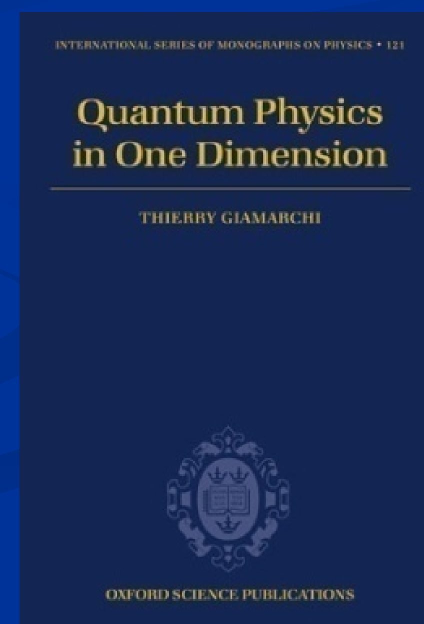
Tomonaga Luttinger

60': A. Larkin, I. Dzialoshinskii, L. Gorkov, Bichkov
E. Lieb, D. Mathis
70': A. Luther, V.J. Emery
80': F.D.M. Haldane





For more (among many refs) :

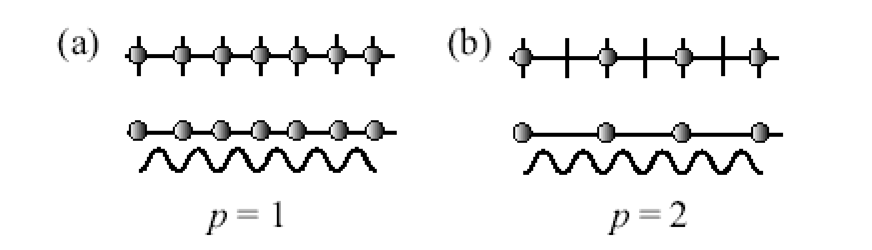


Bosonization

- Continuum:

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu \int dx\, \rho(x)$$

- Lattice:

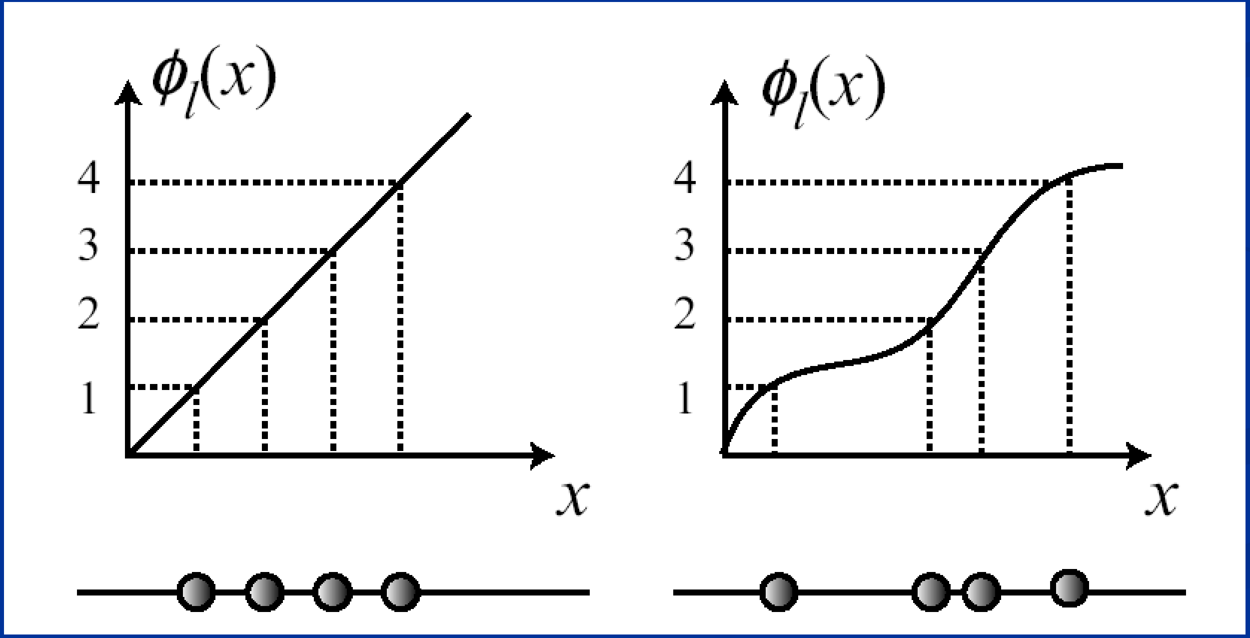


$$H = -J\sum_{\langle i,j\rangle} b_i^\dagger b_j + U\sum_i n_i(n_i-1) - \mu\sum_i n_i$$

Labelling the particles

$$\rho(x) = \sum_i \delta(x - x_i)$$
$$= \sum_n |\nabla \phi_l(x)| \delta(\phi_l(x) - 2\pi n)$$

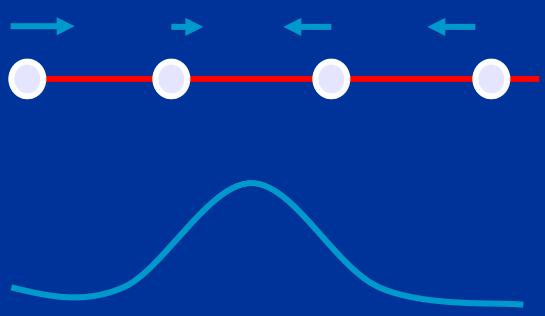
1D: unique way of labelling

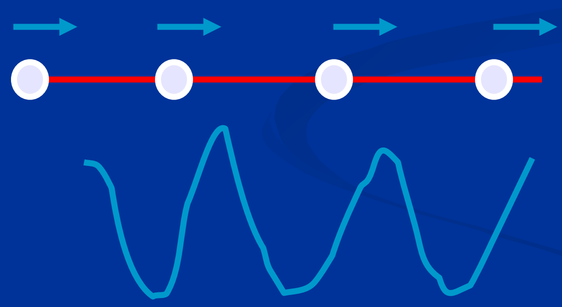


$$\phi_l(x) = 2\pi\rho_0 x - 2\phi(x)$$

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$\phi(x)$ varies slowly





CDW

$q \sim 0$ $\qquad$ $q \sim 2\pi\rho_0$

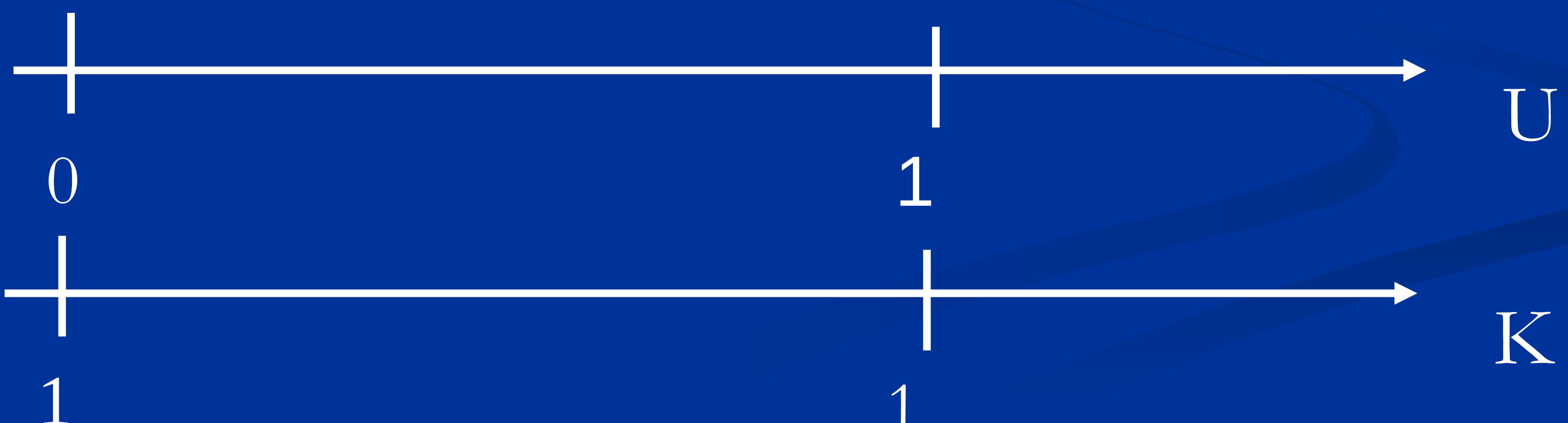
$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

θ: superfluid phase

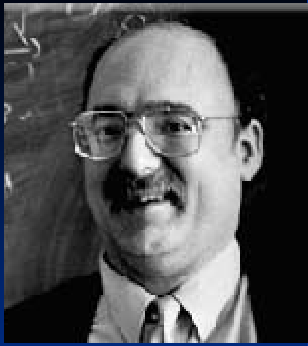
$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$



Luttinger liquid concept

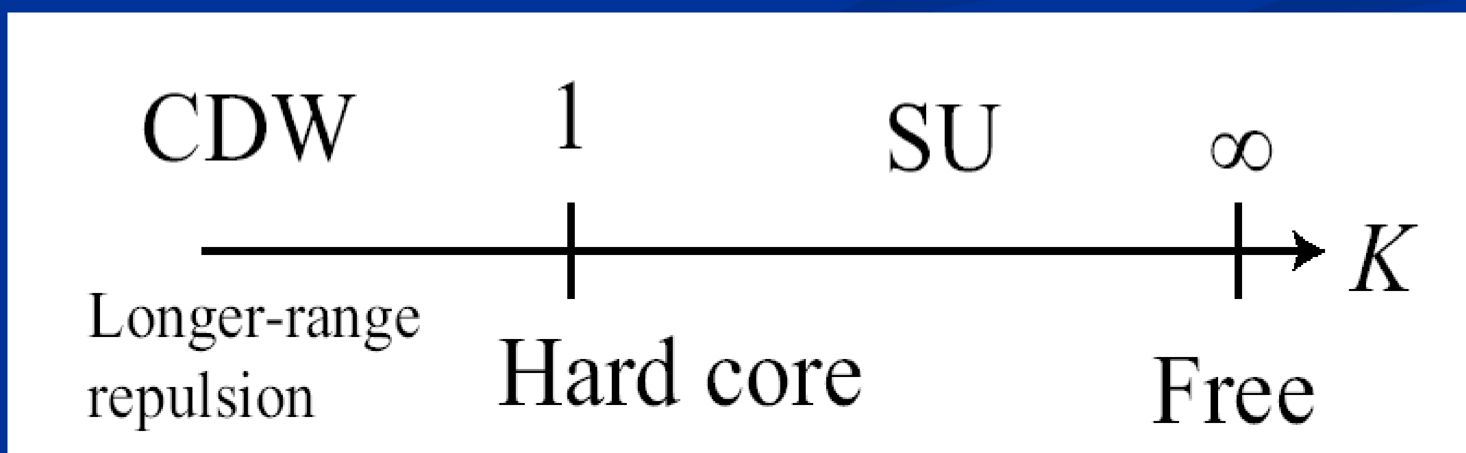


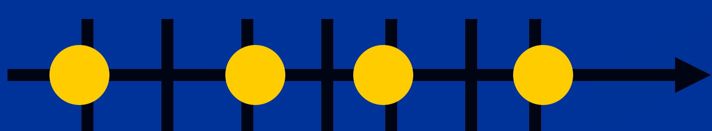
- How much is perturbative ?
- Nothing (Haldane):
  provided the correct u,K are used
- Low energy properties: Luttinger liquid (fermions, bosons, spins…)

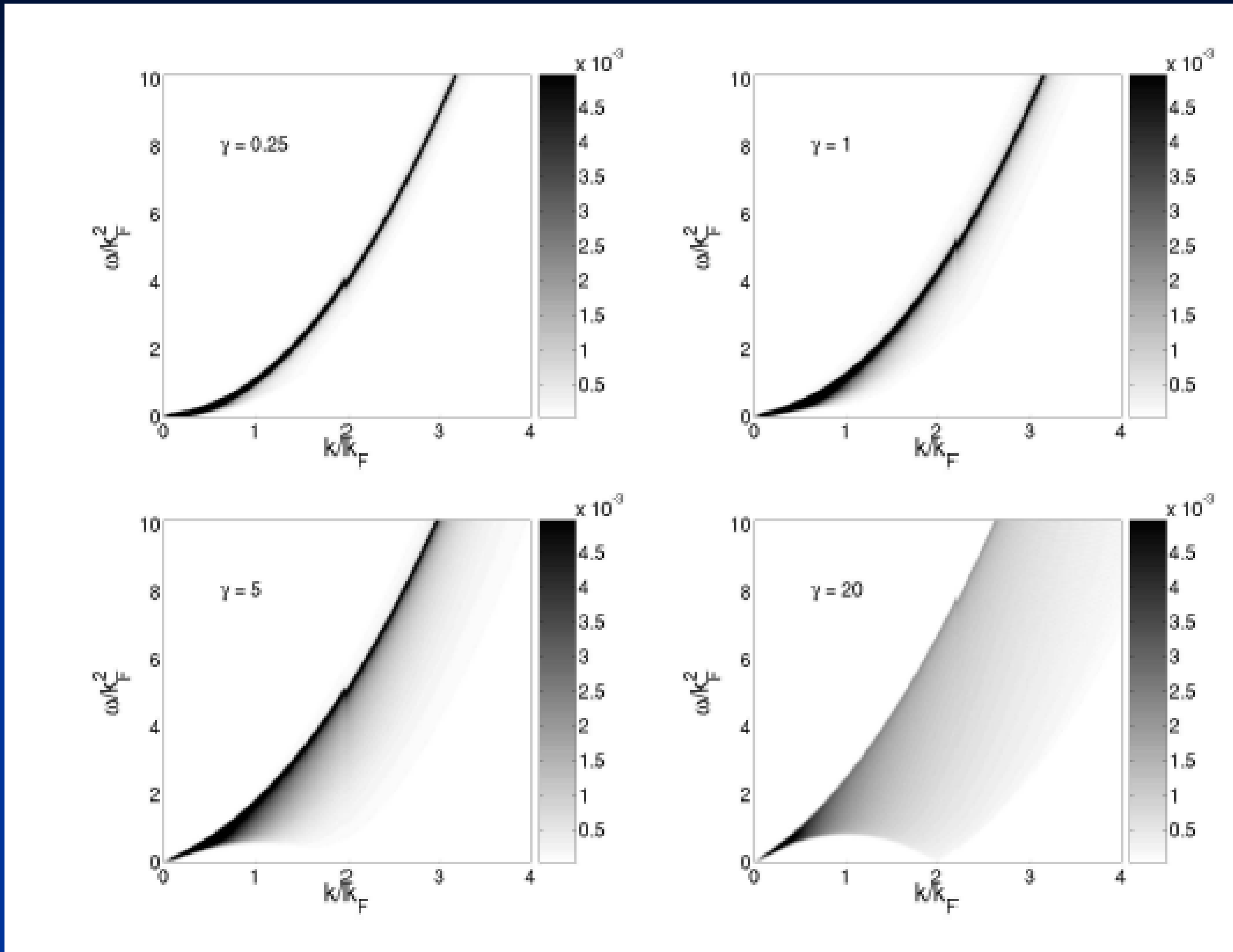
Correlations

$$\langle \psi(r)\psi^\dagger(0)\rangle = A_1 \left(\frac{\alpha}{r}\right)^{\frac{1}{2K}} + \cdots$$

$$\langle \rho(r)\rho(0)\rangle = \rho_0^2 + \frac{K}{2\pi^2}\frac{y_\alpha^2 - x^2}{(y_\alpha^2 + x^2)^2} + A_3 \cos(2\pi\rho_0 x)\left(\frac{1}{r}\right)^{2K} + \cdots$$



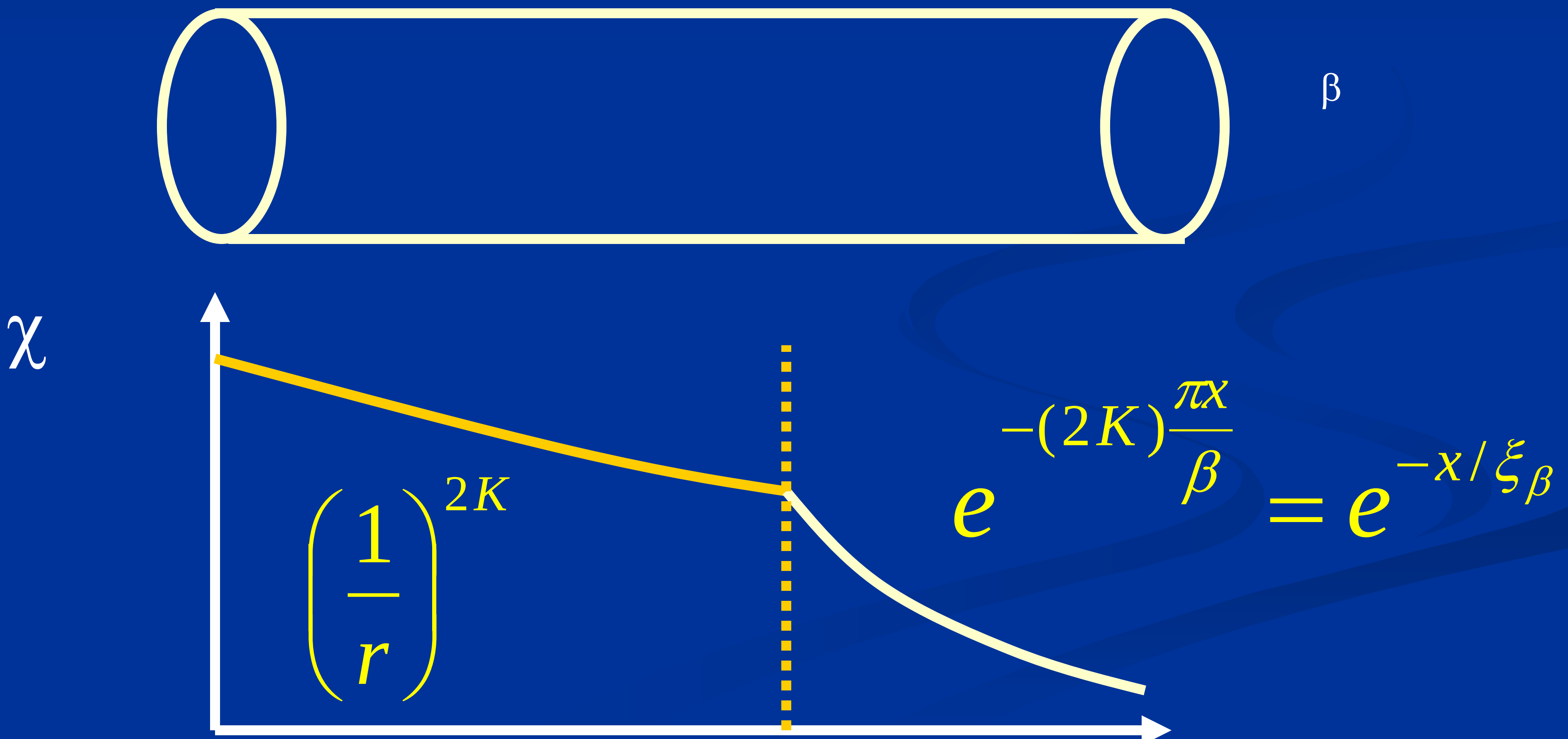




S(q,!) J.S. Caux et al PRA 74 031605 (2006)

Finite temperature

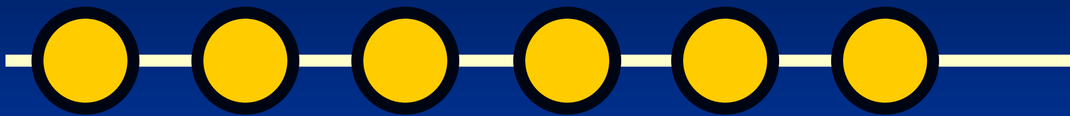
Conformal theory



Calculation of Luttinger parameters

- Trick: use thermodynamics and BA or numerics
- Compressibility: u/K
- Response to a twist in boundary: u K
- Specific heat : T/u
- Etc.

Tonks limit



U = 1 : spinless fermions

Not for n(k) : $\psi_F \neq \psi_B$

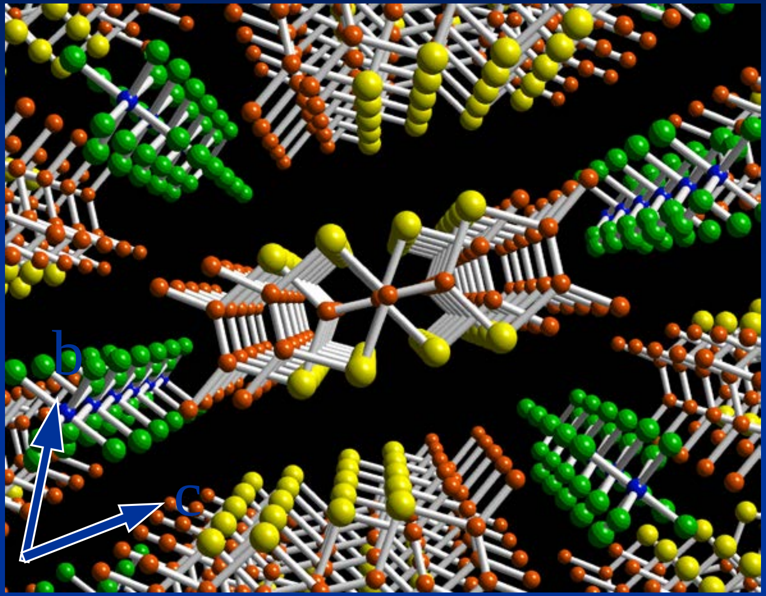
Free fermions: $$\langle \rho(x)\rho(0)\rangle \propto \cos(2k_F x)\left(\frac{1}{x}\right)^2$$

K=1

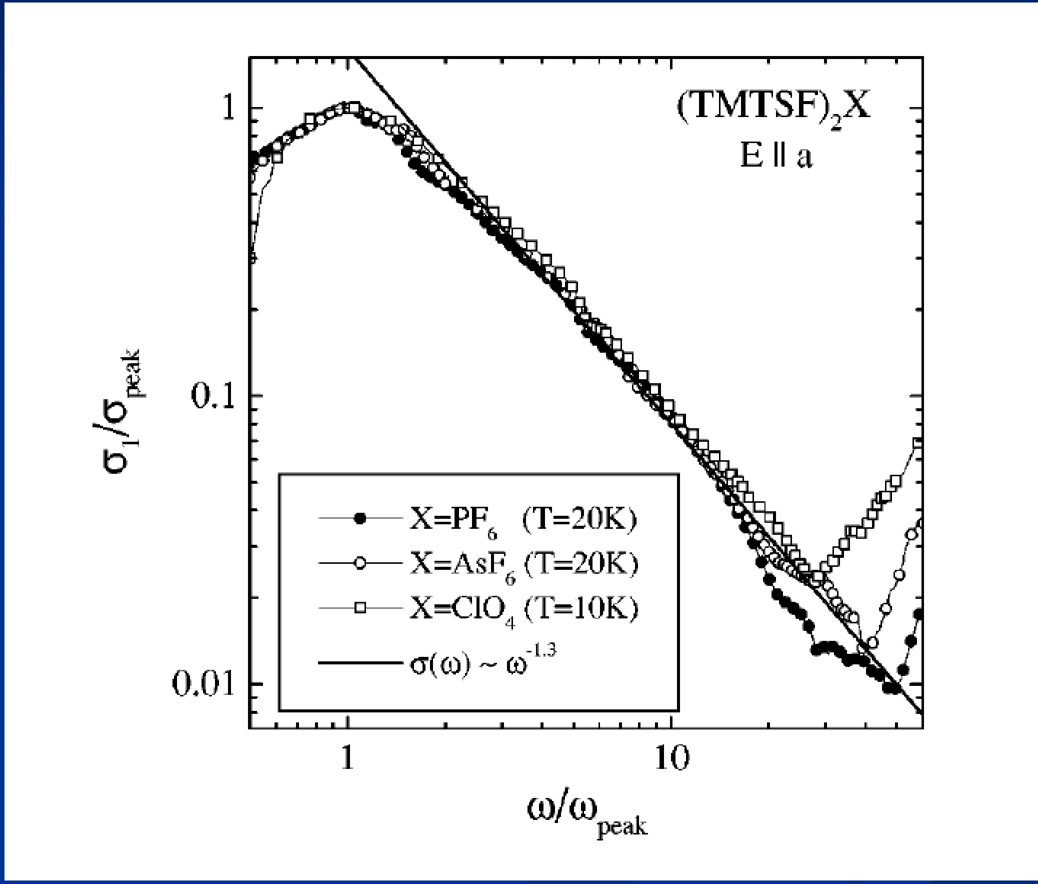
Note: $$\langle \psi_B(x)\psi_B(0)^\dagger\rangle \propto \left(\frac{1}{x}\right)^{1/2}$$

Tests of Luttinger liquids

Organic conductors



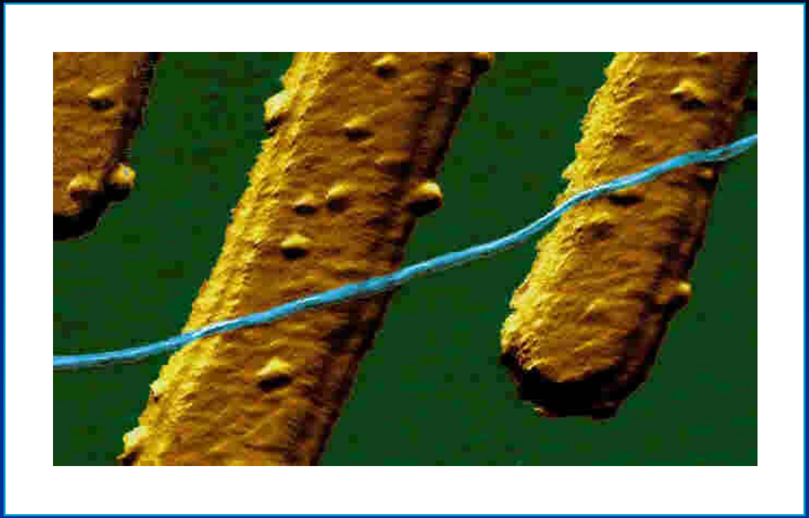
$$\sigma(\omega) \sim \omega^{-\nu}$$



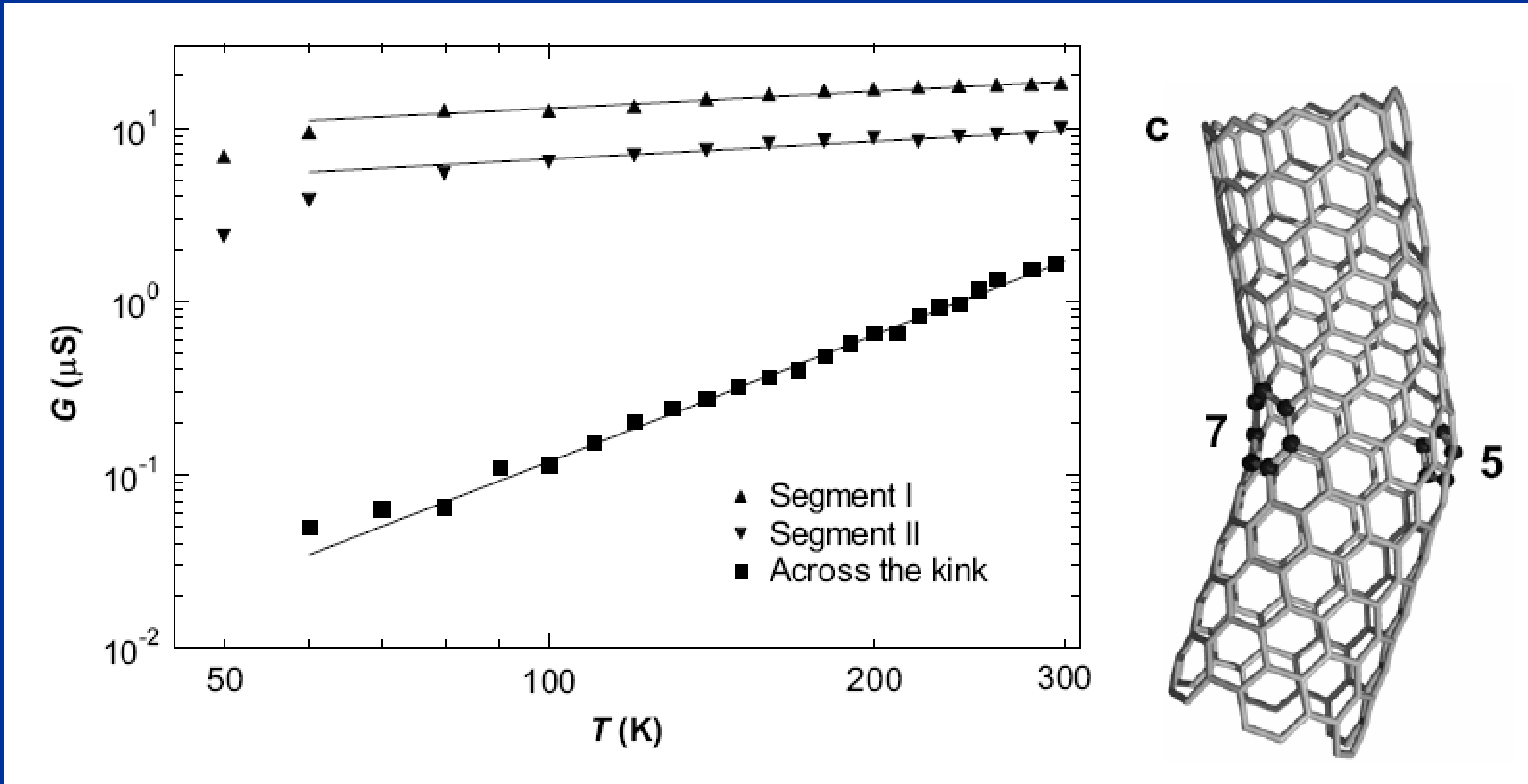
TG PRB (91) :
Physica B 230 (1996)

A. Schwartz et al. PRB 58 1261 (1998)

First observation of LL/powerlaw !!

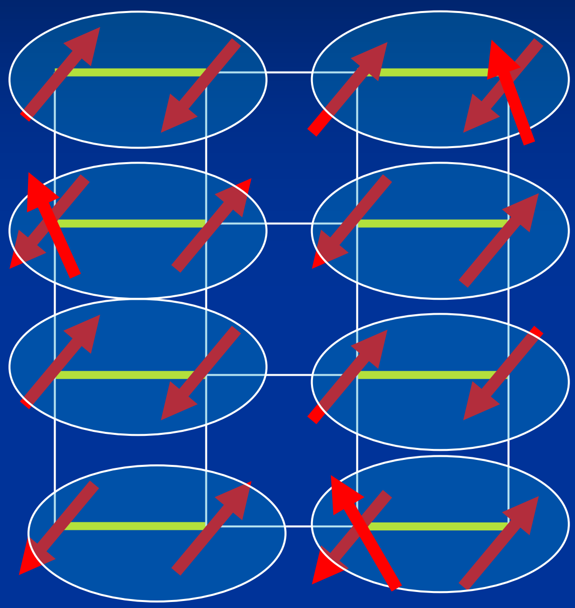


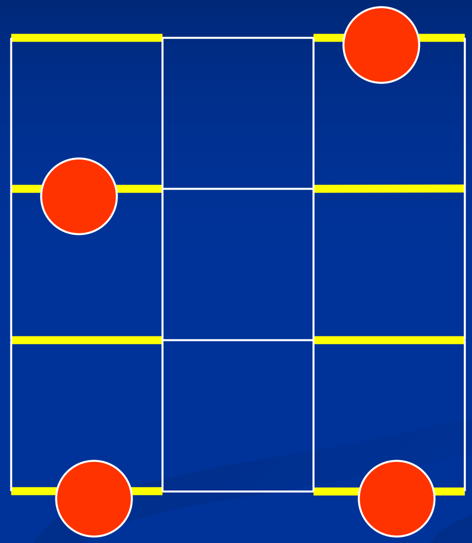
Z. Yao et al. Nature 402 273 (1999)



Magnetic insulators

triplon = hard core boson





$h \sim h_c$ dilute limit: « free » bosons

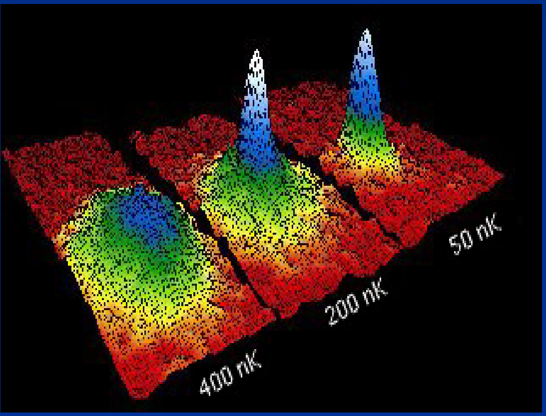
Bose Einstein Condensation

(TG and A. M. Tsvelik PRB 59 11398 (1999))

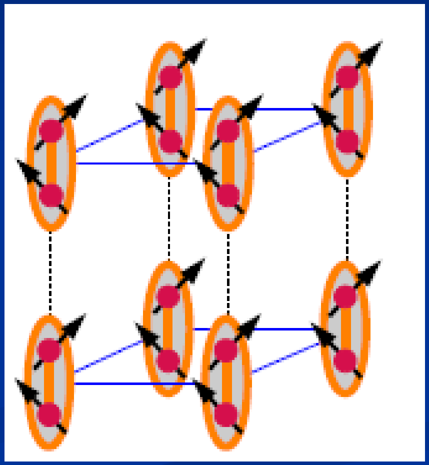
BEC vs BEC

Cold atoms

Dimers/Spins



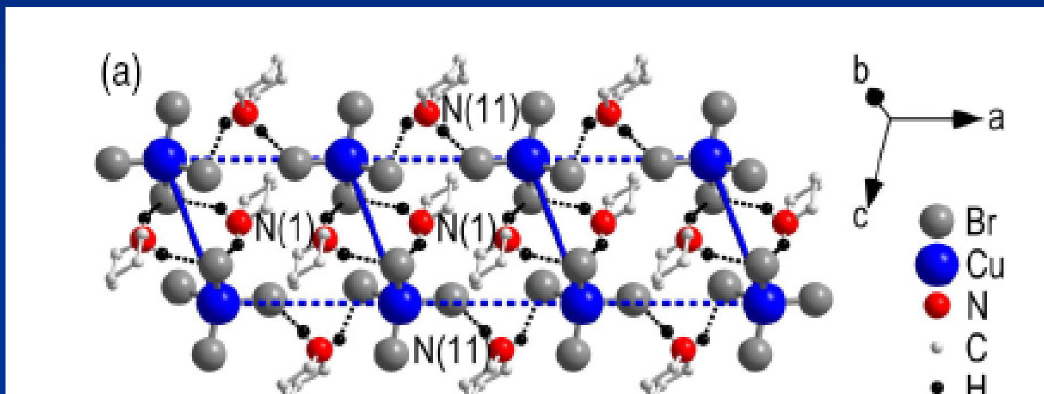
TG, Ch. Rüegg, O. Tchernyshyov, Nat. Phys. 4 198 (08)



Bose gas	Antiferromagnet
Particles	Spin excitations ($\Delta S^z = \pm 1$)
Boson number N	Spin component S^z
Charge conservation U(1)	Rotational invariance O(2)
Condensate wavefunction $\langle \psi(\mathbf{r}) \rangle$	Transverse magnetic order $\langle s_i^x + i s_i^y \rangle$
Chemical potential μ	Magnetic field B
Superfluid density ρ_s	Transverse spin stiffness
Mott insulating state	Integer magnetization plateau

Spin dimer systems

B. C. Watson et al., PRL 86 5168 (2001)

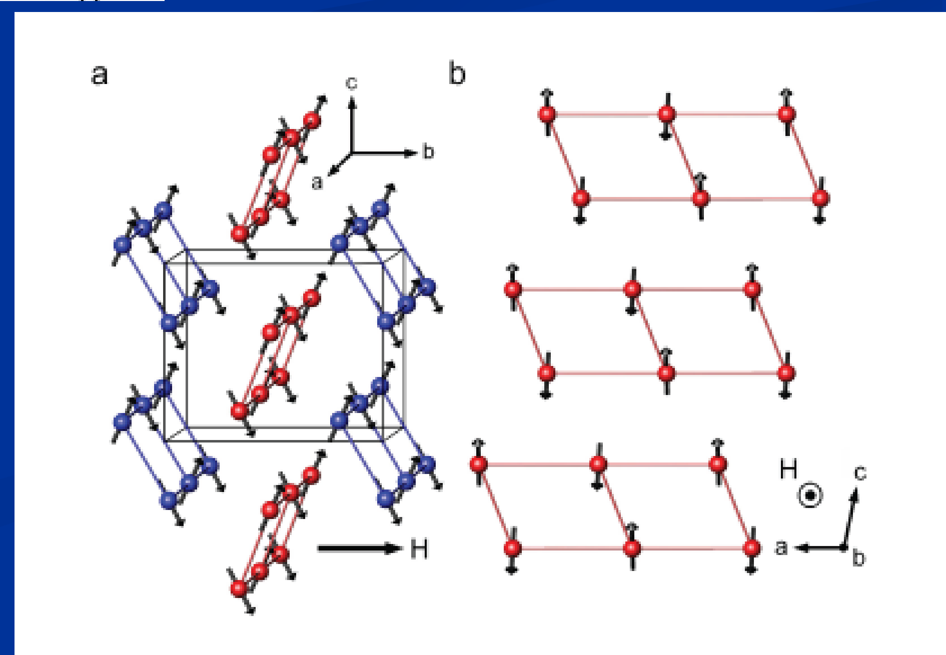


M. Klanjsek et al.,

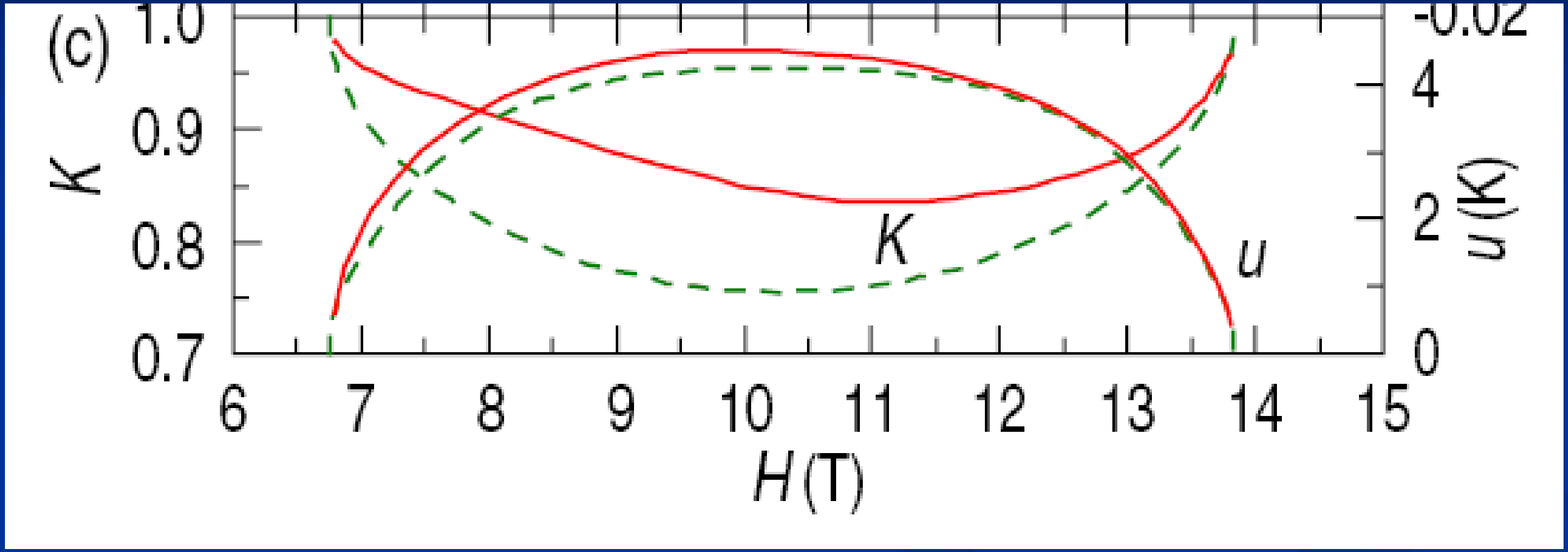
PRL 101 137207 (2008)

B. Thielemann et al.,

arXiv:0809.0440 (2008)



Luttinger parameters



M. Klanjsek et al., PRL 101 137207 (2008)

Red : Ladder (DMRG)

Green: Strong coupling ($J_r \to 1$) (BA)

Correlation functions

M. Klanjsek et al., PRL 101 137207 (2008)

R. Chitra, TG PRB 55 5816 (97); TG, AM Tsvelik PRB 59 11398 (99)

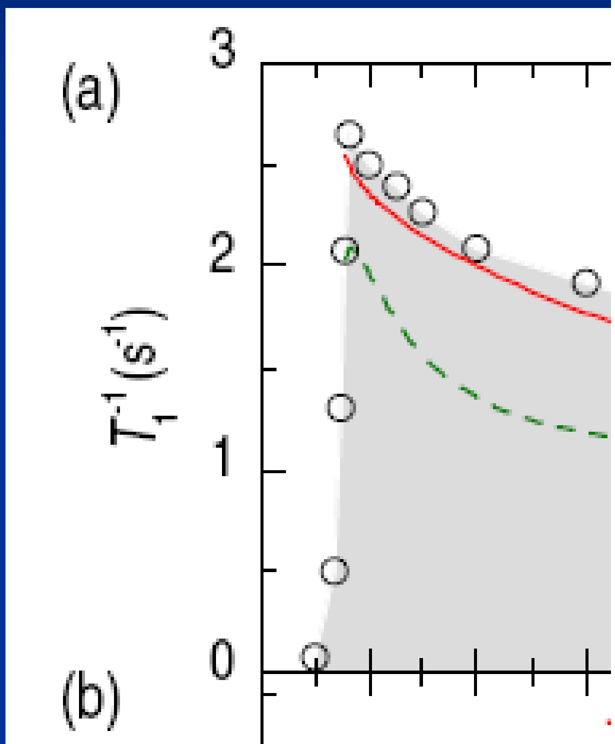
- NMR relaxation rate:

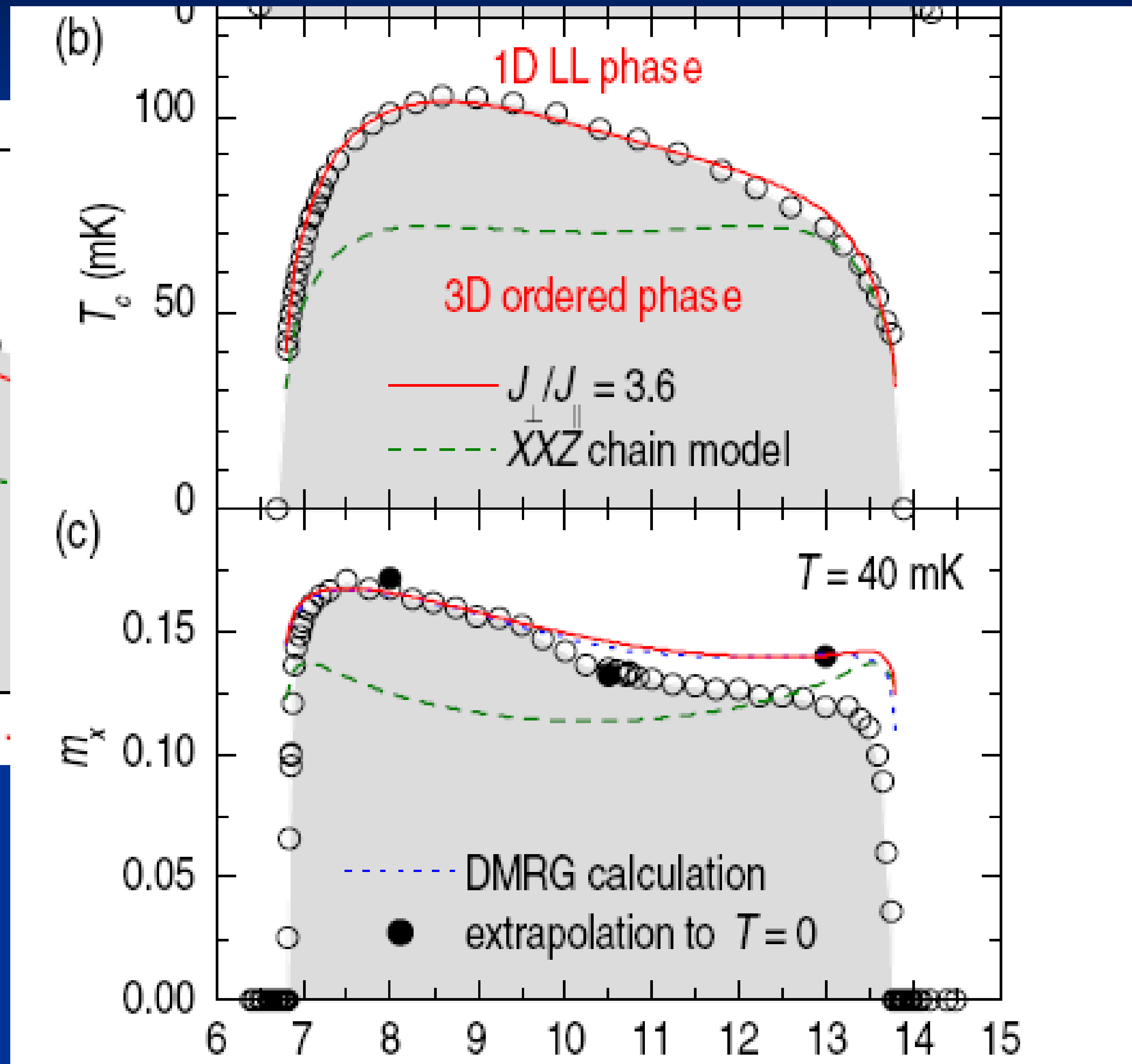
$$T_1^{-1} = \frac{\hbar\gamma^2 A_\perp^2 A_0^x}{k_B u} \cos\left(\frac{\pi}{4K}\right) B\left(\frac{1}{4K}, 1 - \frac{1}{2K}\right)\left(\frac{2\pi T}{u}\right)^{(1/2K)-1},$$

- Tc to ordered phase: $1/J' = \chi_{1D}(Tc)$

$$T_c = \frac{u}{2\pi}\left[\sin\left(\frac{\pi}{4K}\right) B^2\left(\frac{1}{8K}, 1 - \frac{1}{4K}\right)\frac{zJ'A_0^x}{2u}\right]^{2K/(4K-1)}.$$

NMR





M. Klanjsek et al.,

PRL 101 137207 (2008)

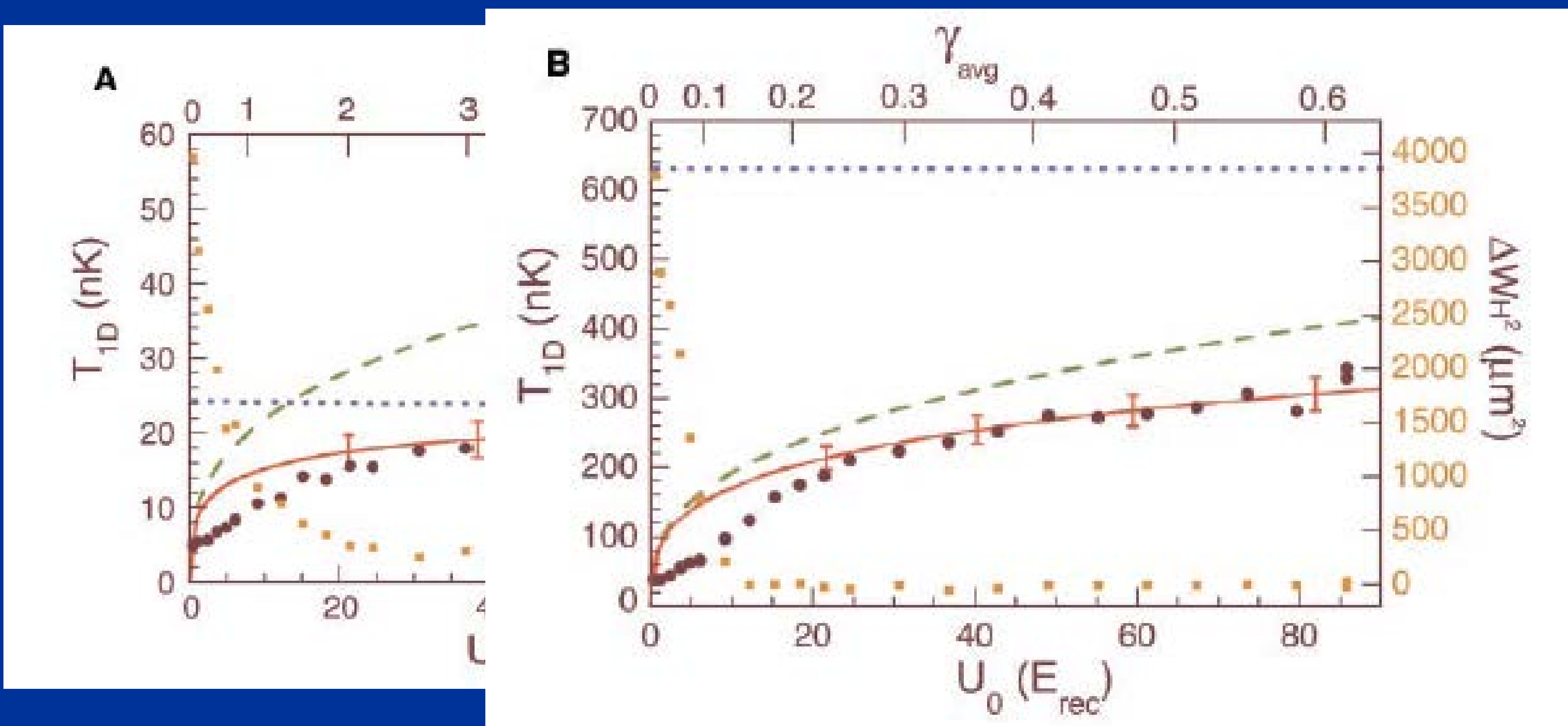
Cold atoms

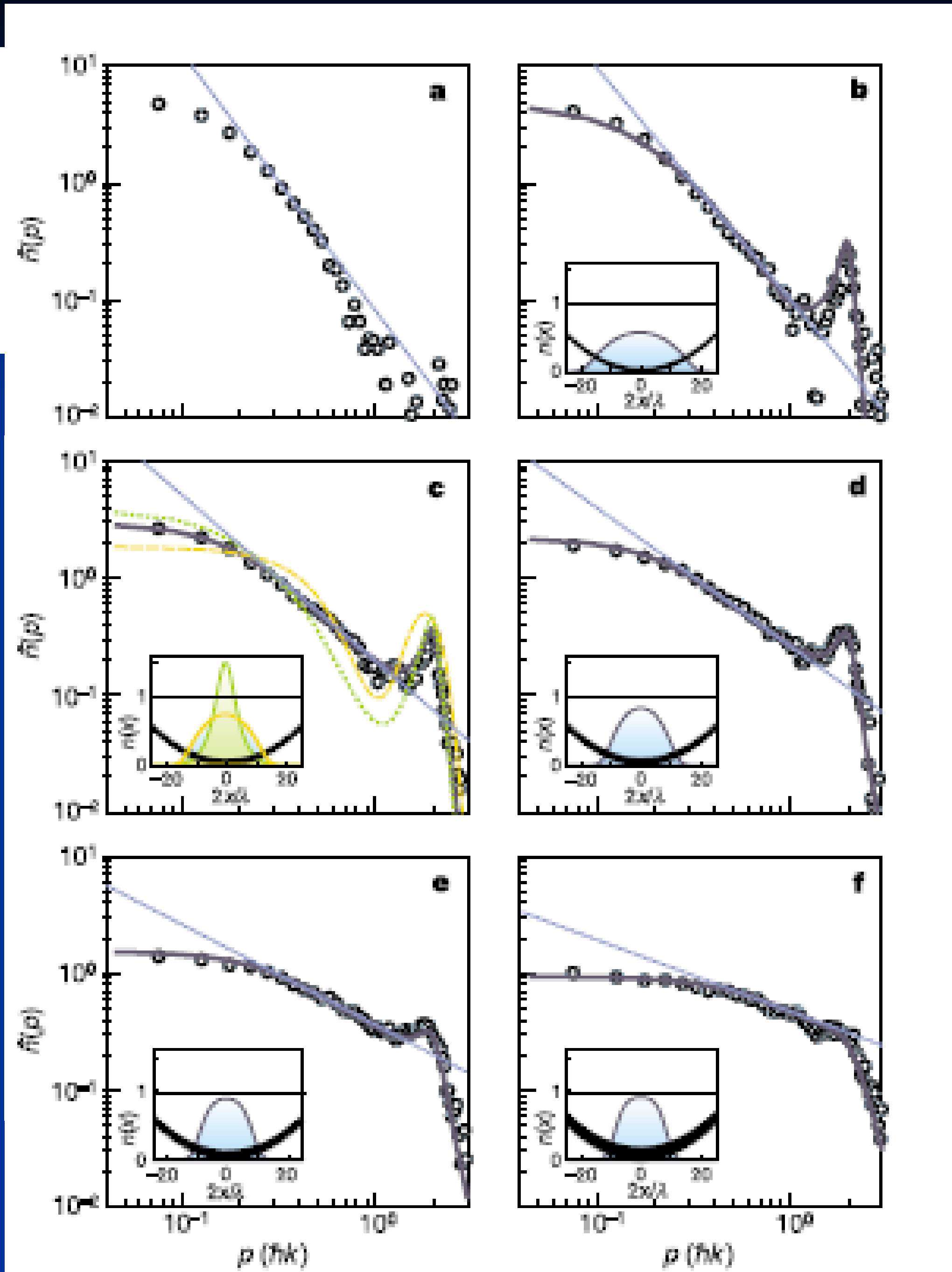
Observation of a One-Dimensional Tonks-Girardeau Gas

Toshiya Kinoshita, Trevor Wenger, David S. Weiss*

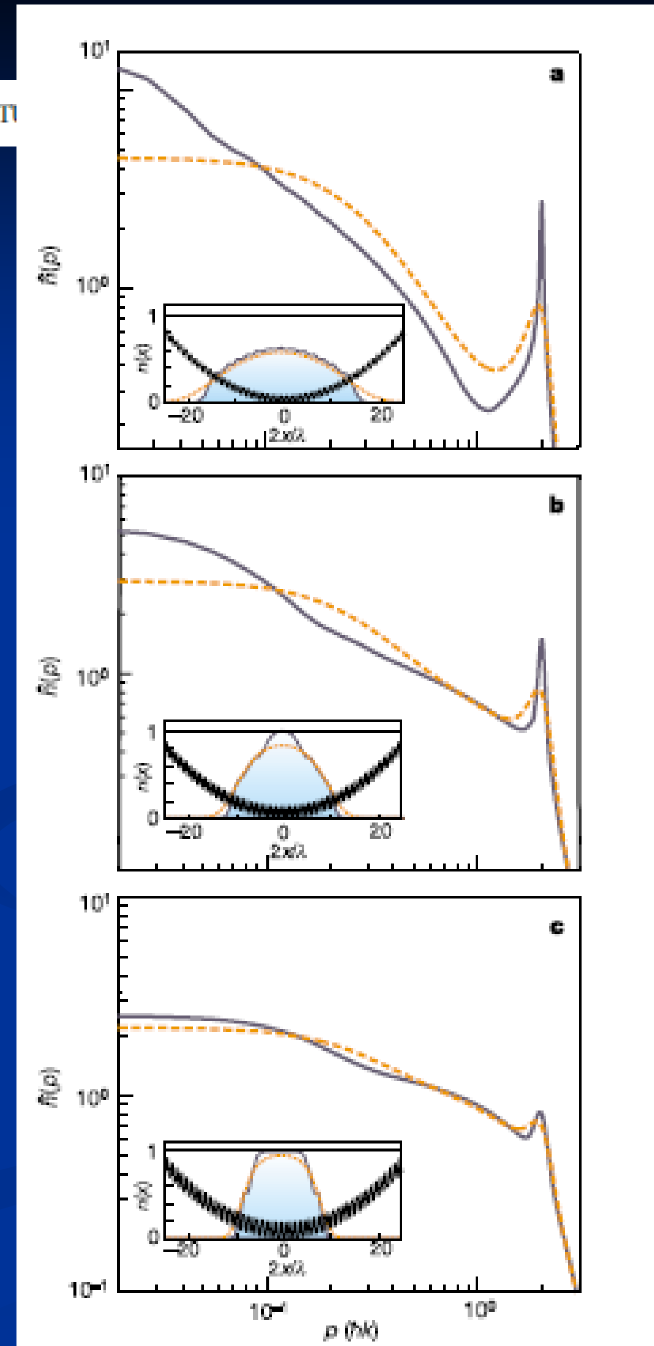
SCIENCE VOL 305 20 AUGUST 2004

1125





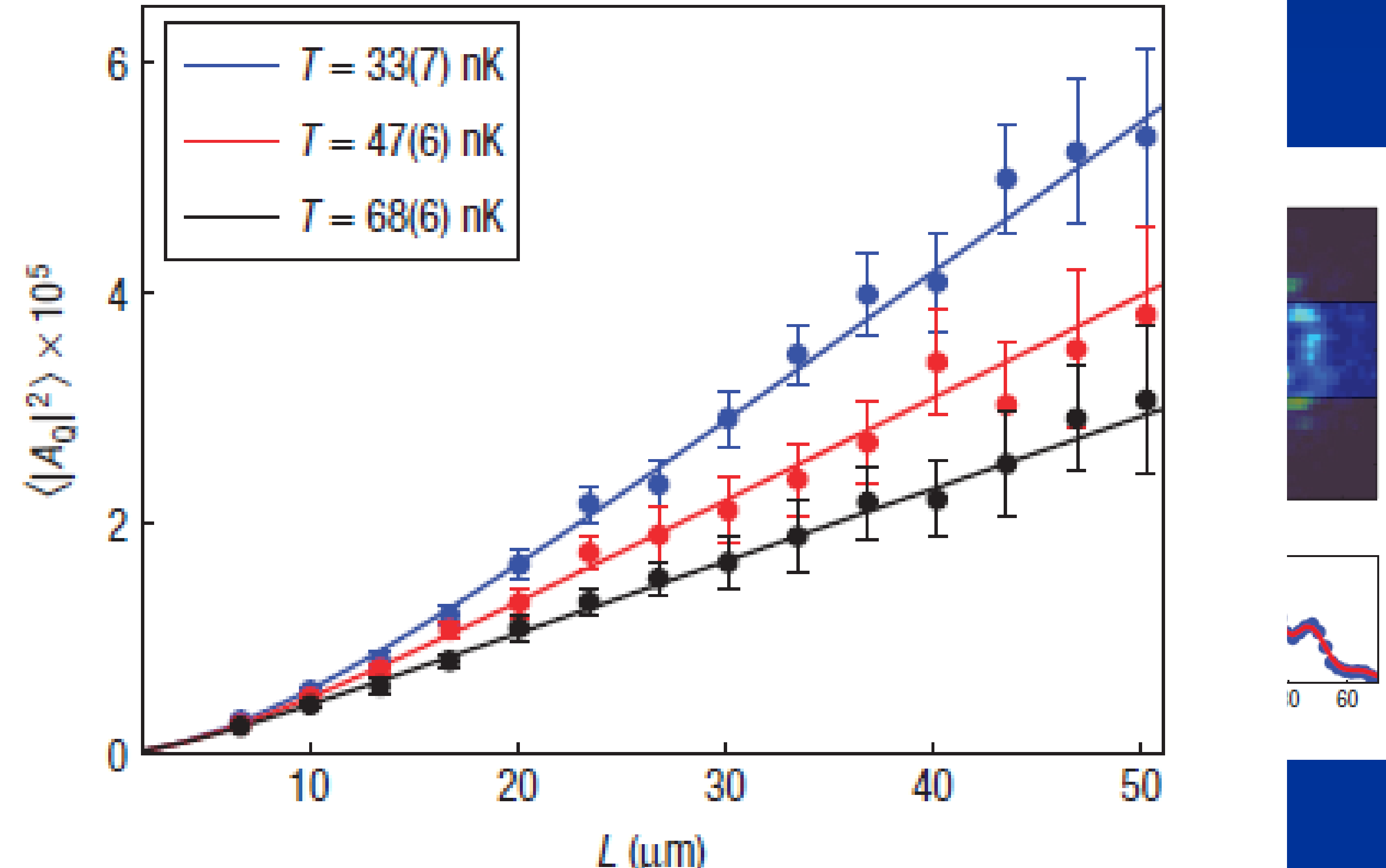
a
b
c
d
e
f
ñ(p)
n(x)
2x/λ
p (ħk)
10¹
10⁰
10⁻¹
10⁻²



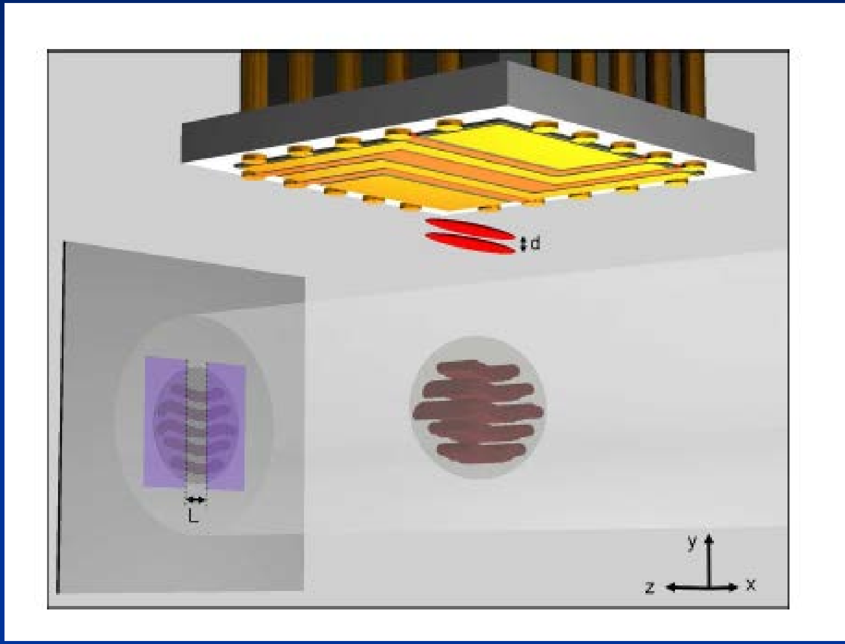
a
b
c
ñ(p)
n(x)
2x/λ
p (ħk)
10¹
10⁰
10⁻¹

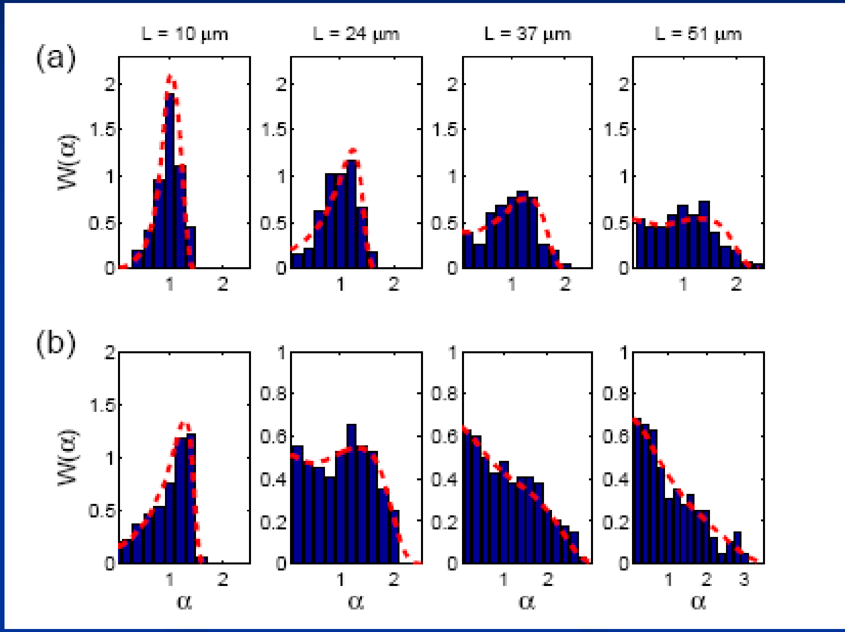
Probing quantum and thermal noise in an interacting many-body system

S. HOFFERBERTH[1,2,3], I. LESANOVSKY[2,4], T. SCHUMM[1], A. IMAMBEKOV[3,5], V. GRITSEV[3], E. DEMLER[3] AND J. SCHMIEDMAYER[1,2*]



Cold atoms: Interferences





$$\int_0^L dr \langle \psi(r) \psi^\dagger(0) \rangle$$

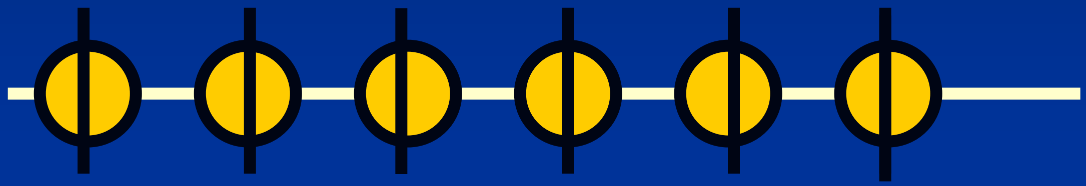
S. Hofferberth et al. Nat. Phys 4 489 (2008)

but K large (42)

hydrodynamic ?

Mott transition

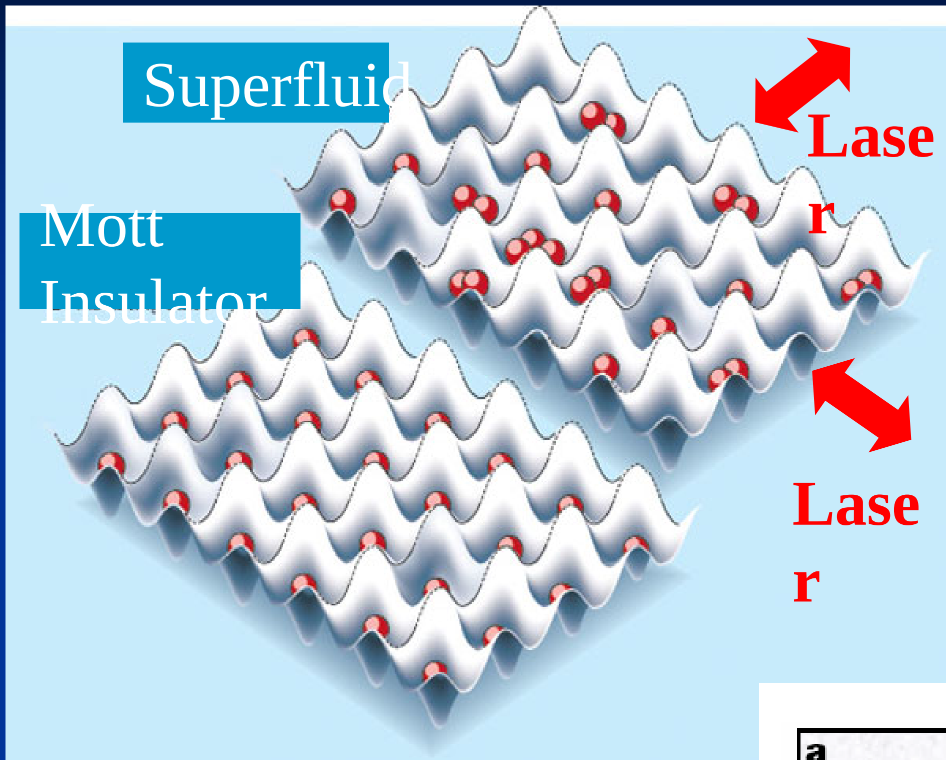
Lattice: Mott transition



Costs U

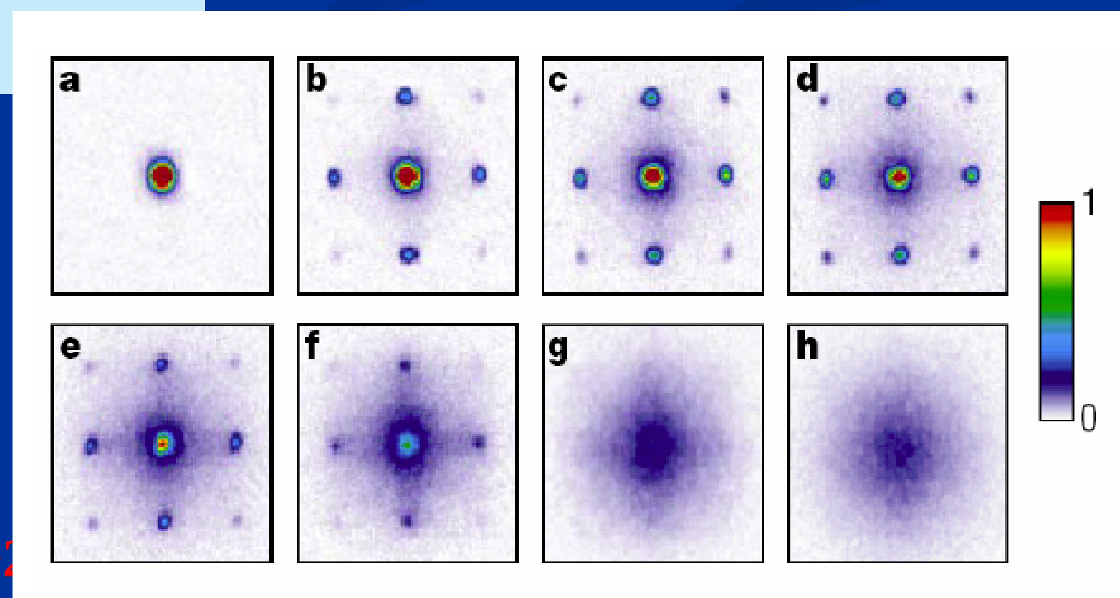
Quantum phase transition

Mott transition and cold atoms



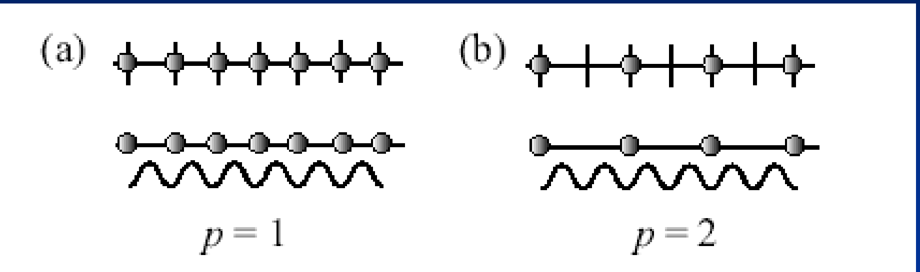
Superfluid to Mott insulator transition in a 3D optical lattice

[D. Jaksch et al. PRL 81 (1998)]
[M Greiner *et al.* Nature, 415 (2002




Seminar BINGO!
To play, simply print out this bingo sheet and attend a departmental seminar.
Mark over each square that occurs throughout the course of the lecture.
The first one to form a straight line (or all four corners) must yell out BINGO!! to win!
SEMINAR BINGO
Speaker bashes previous work
Repeated use of "um…"
Speaker sucks up to host professor
Host Professor falls asleep
Speaker wastes 5 minutes explaining outline
Laptop malfunction
Work ties in to Cancer/HIV or War on Terror
"…et al."
You're the only one in your lab that bothered to show up
Blatant typo
Entire slide filled with equations
"The data clearly shows…"
FREE
Speaker runs out of time
Use of Powerpoint template with blue background
References Advisor (past or present)
There's a Grad Student wearing same clothes as yesterday
Bitter Post-doc asks question
"That's an interesting question"
"Beyond the scope of this work"
Master's student bobs head fighting sleep
Speaker forgets to thank collaborators
Cell phone goes off
You've no idea what's going on
"Future work will…"
Results conveniently show improvement
JORGE CHAM © 2007
WWW.PHDCOMICS.COM

How to treat?



$$H=\int dx V_0 \cos(Qx)\rho(x)$$

$$H=\int dx V_0 \cos(Qx)\rho_0 e^{i(2\pi\rho_0 x-2\phi(x))}$$

- Incommensurate: $Q \neq 2\pi\rho_0$

$$H=\int dx \cos(2\phi(x)+\delta x)$$

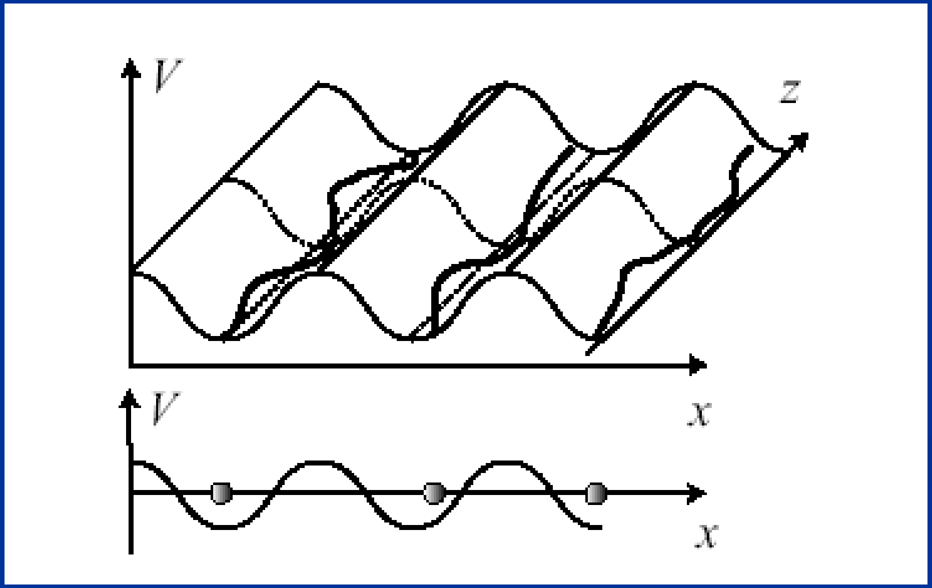
- Commensutate: $Q = 2\pi\rho_0$

$$H=\int dx \cos(2\phi(x))$$

Competition

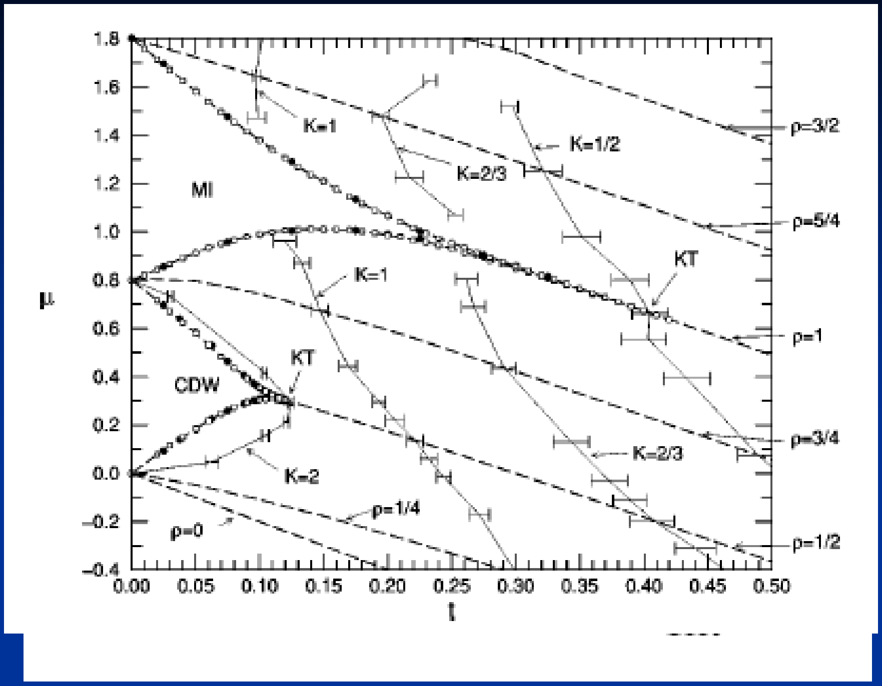
$$S_0 = \int \frac{dxd\tau}{2\pi K}[\tfrac{1}{u}(\partial_\tau \varphi(x,\tau))^2 + u(\partial_x \varphi(x,\tau))^2]$$

$$S_L = -V_0 \rho_0 \int dxd\tau \cos(2\phi(x))$$



Beresinski-Kosterlitz-Thouless transition

K=2

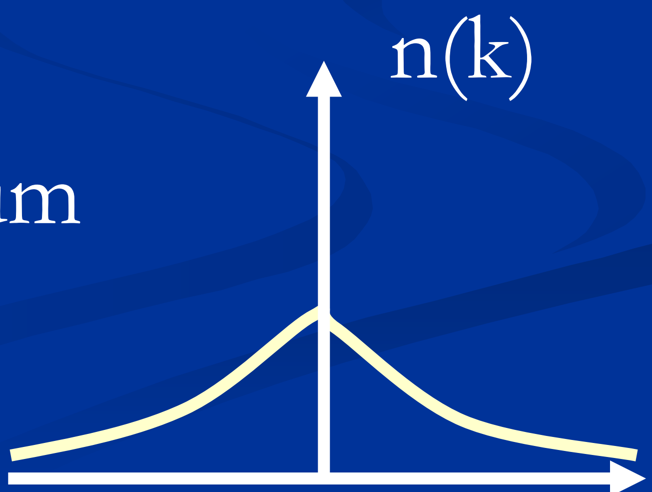


Mott insulator:
ϕ is locked

Density is fixed

TG, Physica B 230 975 (97)

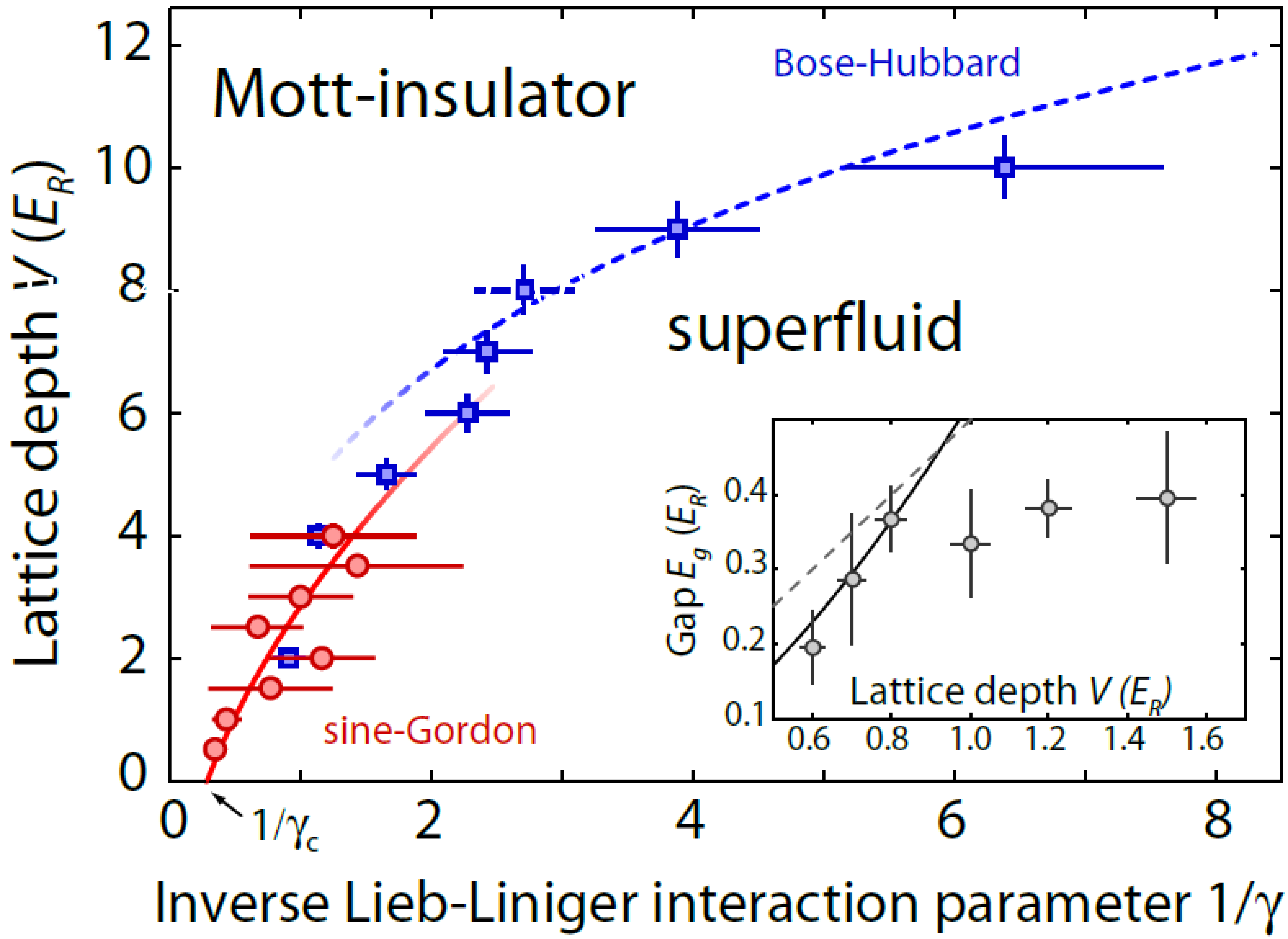
T. Kuhner et al. PRB 61 12474 (2000)



Gap in the excitation spectrum

$$G(r) \propto e^{-r/\xi}$$

Pinning quantum phase transition for a Luttinger liquid of strongly



Other 1D systems

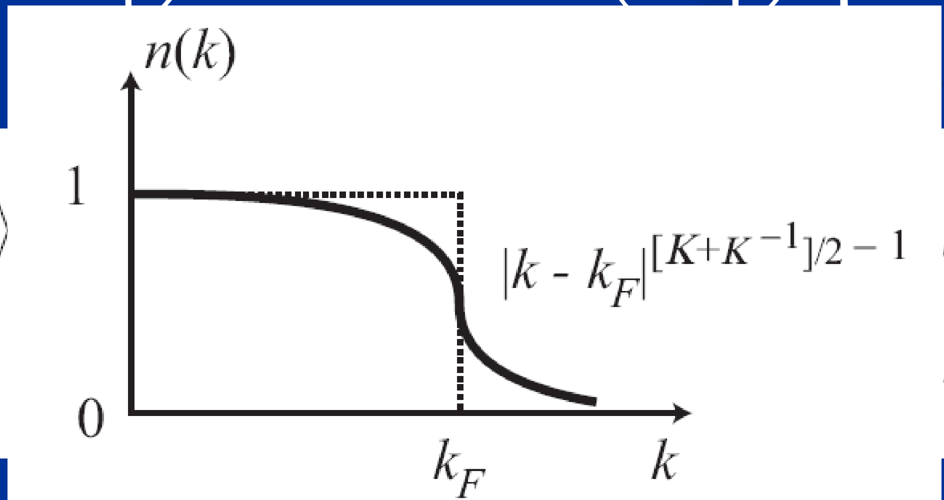
Fermions

$$\psi_F^\dagger(x) = \psi_B^\dagger(x) e^{i\frac{1}{2}\phi_l(x)}$$

$$\psi_F^\dagger(x) = [\rho_0 - \frac{1}{\pi}\nabla\phi(x)]^{1/2} \sum_p e^{i(2p+1)(\pi\rho_0 x - \phi(x))} e^{-i\theta(x)}$$

Right ($+k_F$) and left ($-k_F$) particles

$\langle\rho(x,\tau)\rho(0)\rangle$



$\rho_0 x) \left(\frac{\alpha}{r}\right)^{2K}$

$\pi\rho_0 x) \left(\frac{\alpha}{r}\right)^{8K} + \cdots$

Spins

Use boson or fermions mapping

$$S^{+} = (-1)^{i} e^{i\theta} + e^{i\theta} \cos(2\phi)$$

$$S^{z} = \frac{-1}{\pi} \nabla \phi + (-1)^{i} \cos(2\phi)$$

Powerlaw correlation functions

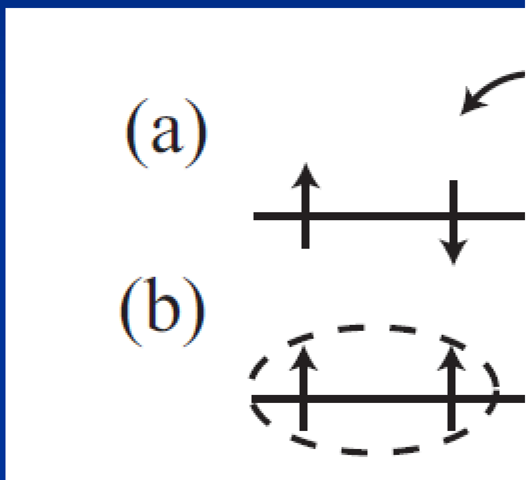
$$\langle S^z(x,0) S^z(0,0) \rangle = C_1 \frac{1}{x^2} + C_2 (-1)^x \left(\frac{1}{x}\right)^{2K}$$

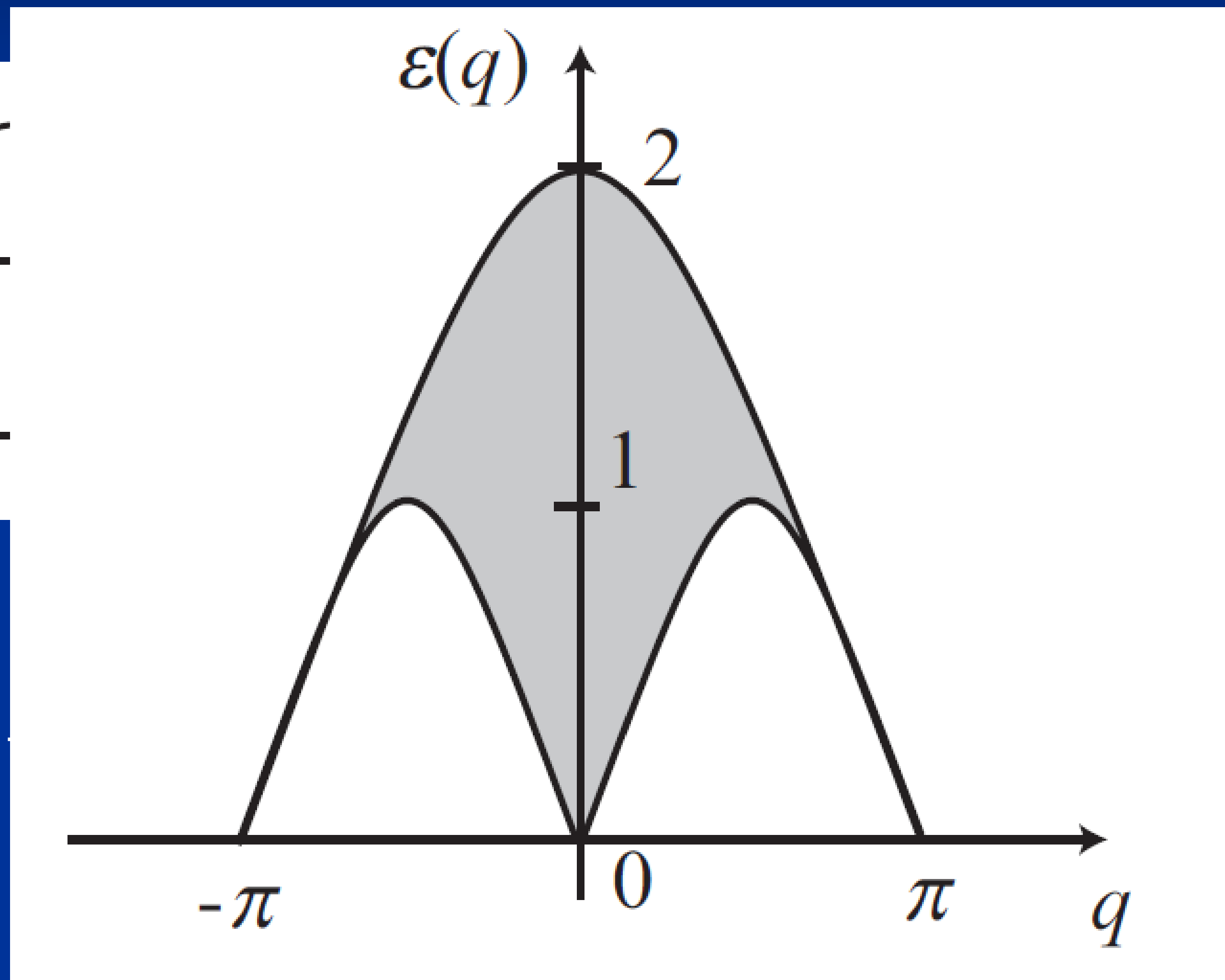
$$\langle S^+(x,0) S^-(0,0) \rangle = C_3 \left(\frac{1}{x}\right)^{2K+\frac{1}{2K}} + C_4 (-1)^x \left(\frac{1}{x}\right)^{\frac{1}{2K}}$$

Non universal exponents K(h,J)

Fractionalization of excitations

1 = ½ + ½

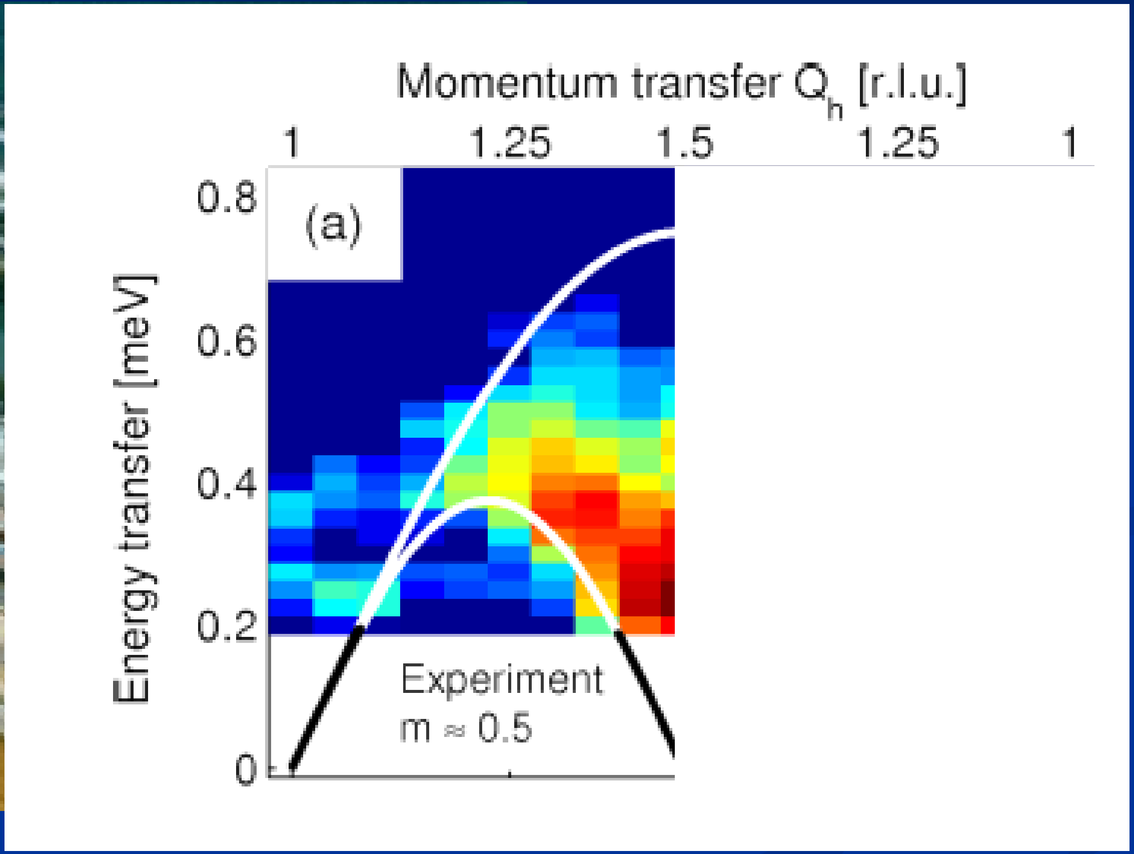




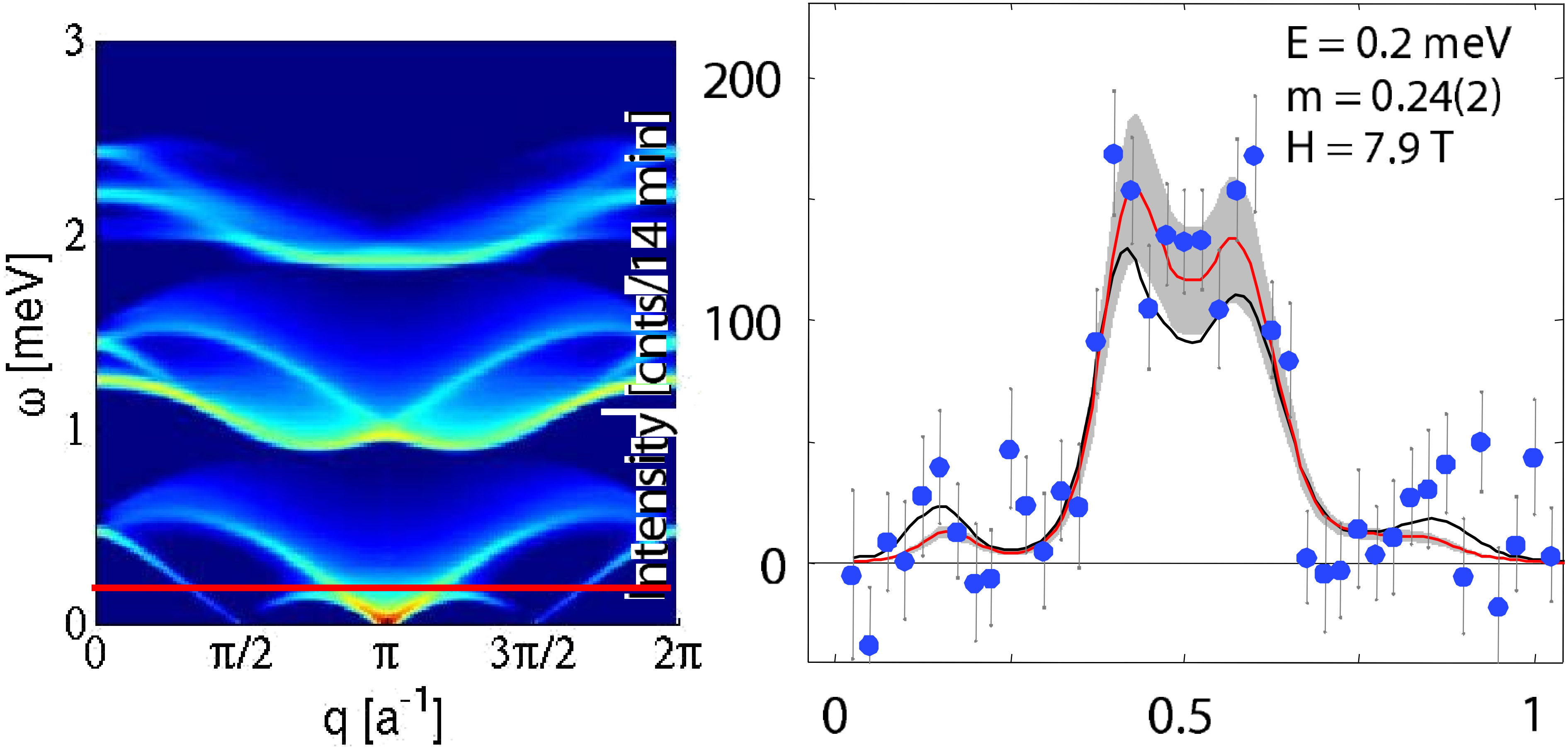
$E(k) = \cos(k_1)$

$k = k_1 + k_2$

Neutron scattering



B. Thieleman et al. PRL 102, 107204 (2009)



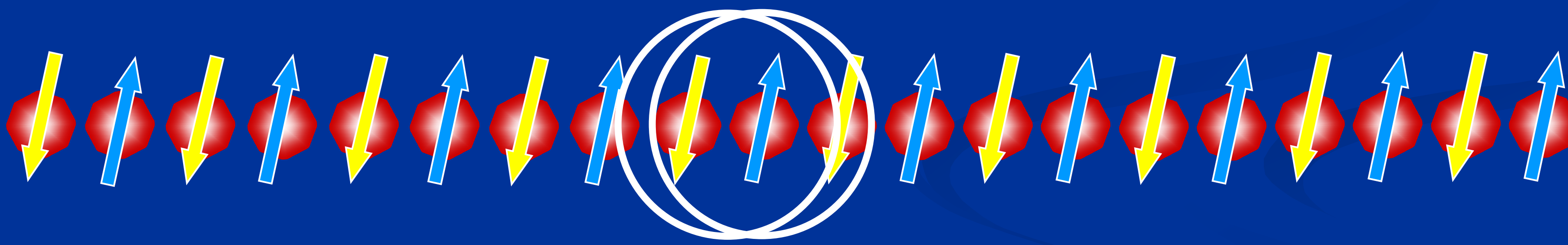
P. Bouillot et al. PRB 83, 054407 (2011)

Systems with spins

Bosonize each spin separately

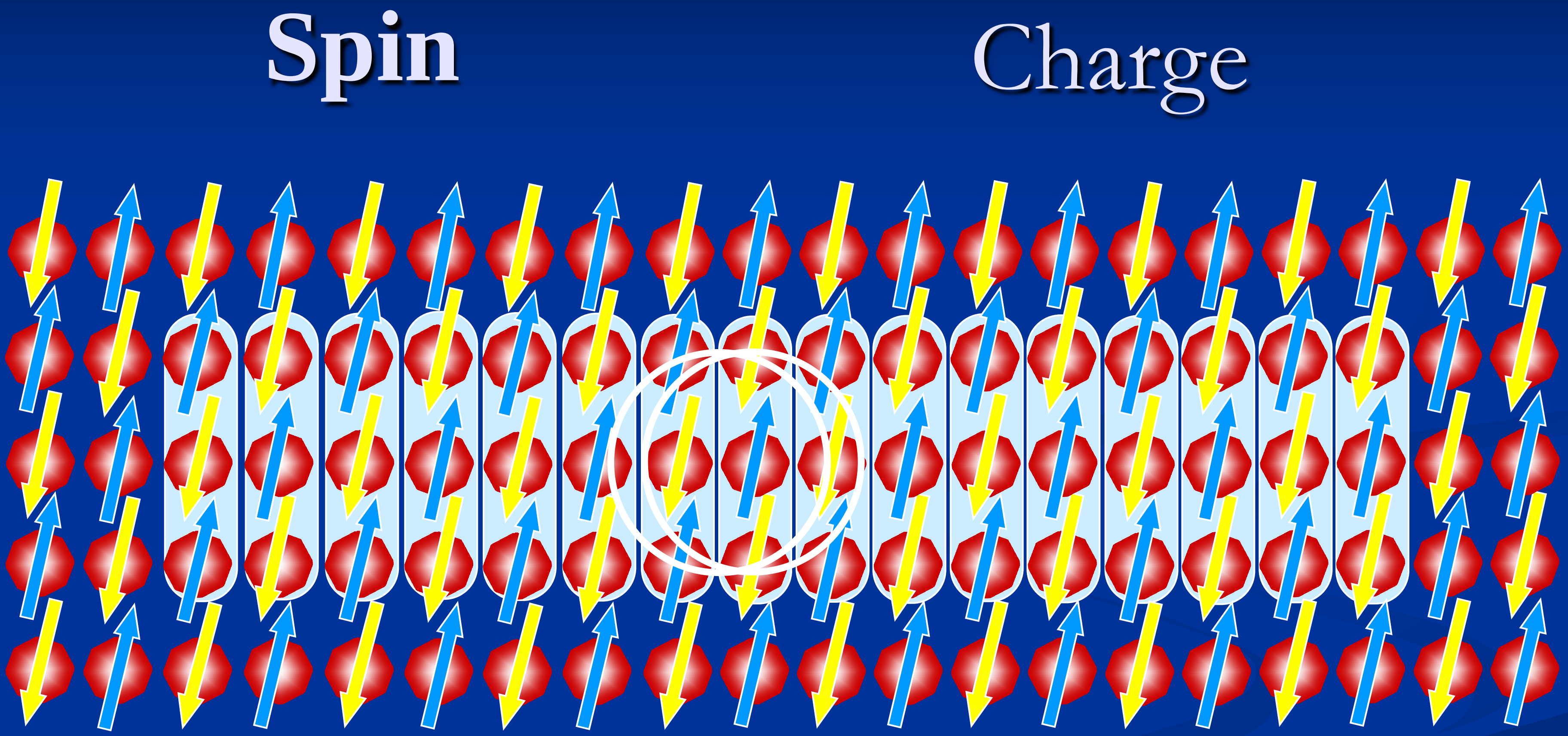
Spin-Charge Separation

Spin Charge



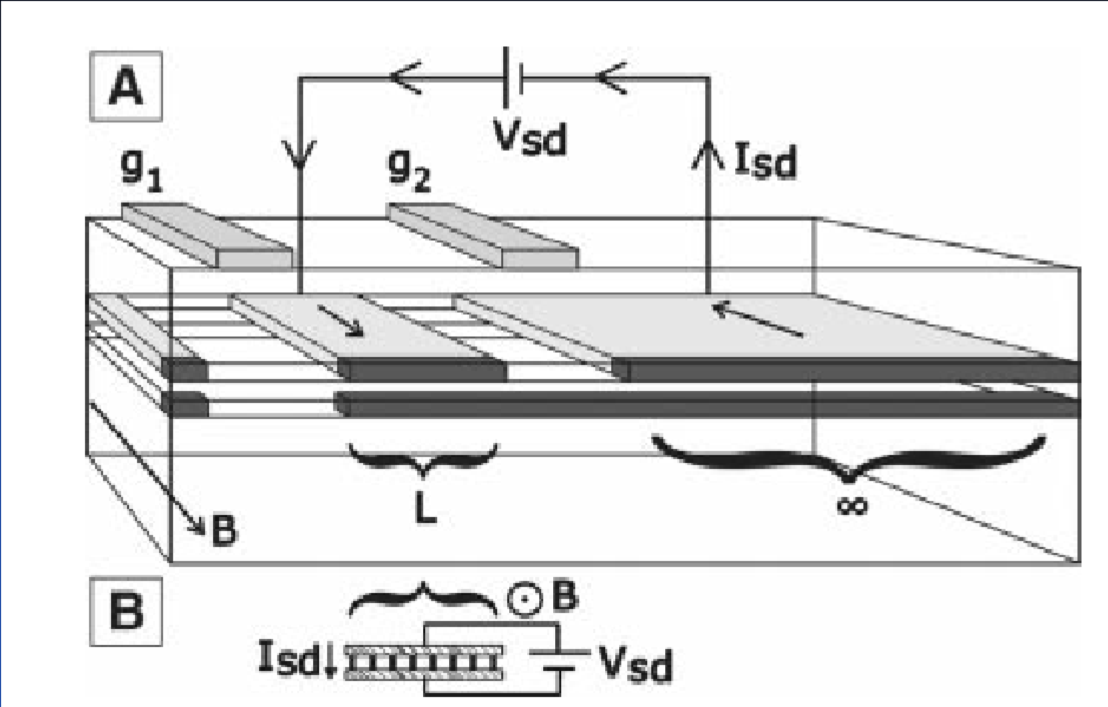
Spinon Holon

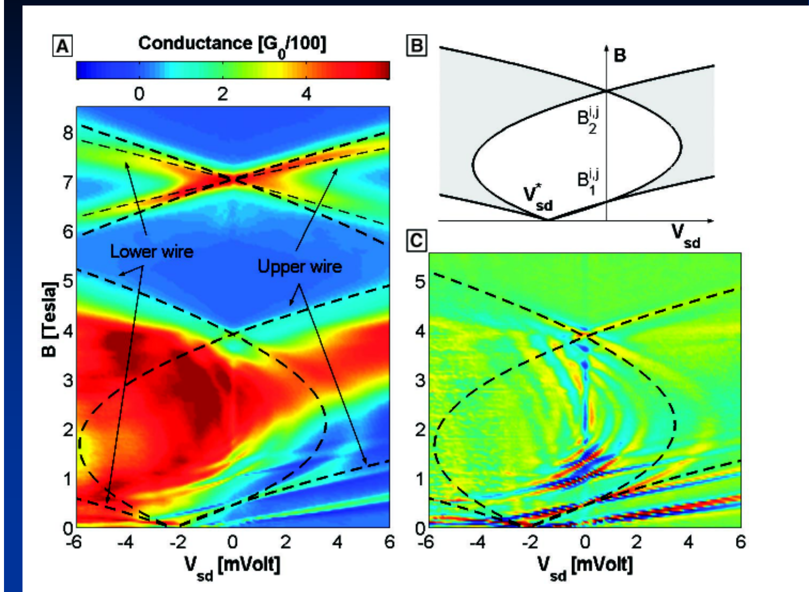
Spin-Charge Separation higher D ?



Energy increases with spin-charge separation

Confinement of spin-charge: « quasiparticle »

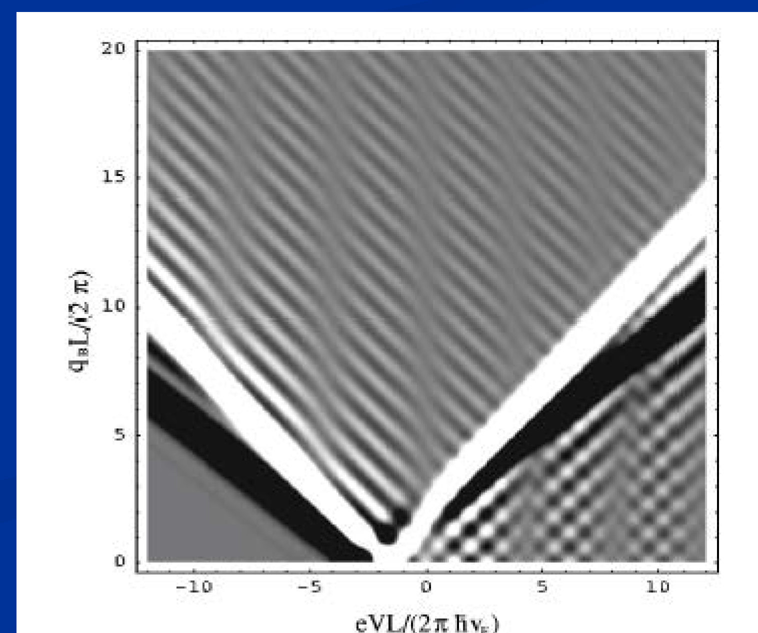




O.M Ausslander et al., Science **298** 1354 (2001)

Y. Tserkovnyak et al., PRL **89** 136805 (2002)

Y. Tserkovnyak et al., PRB **68** 125312 (2003)



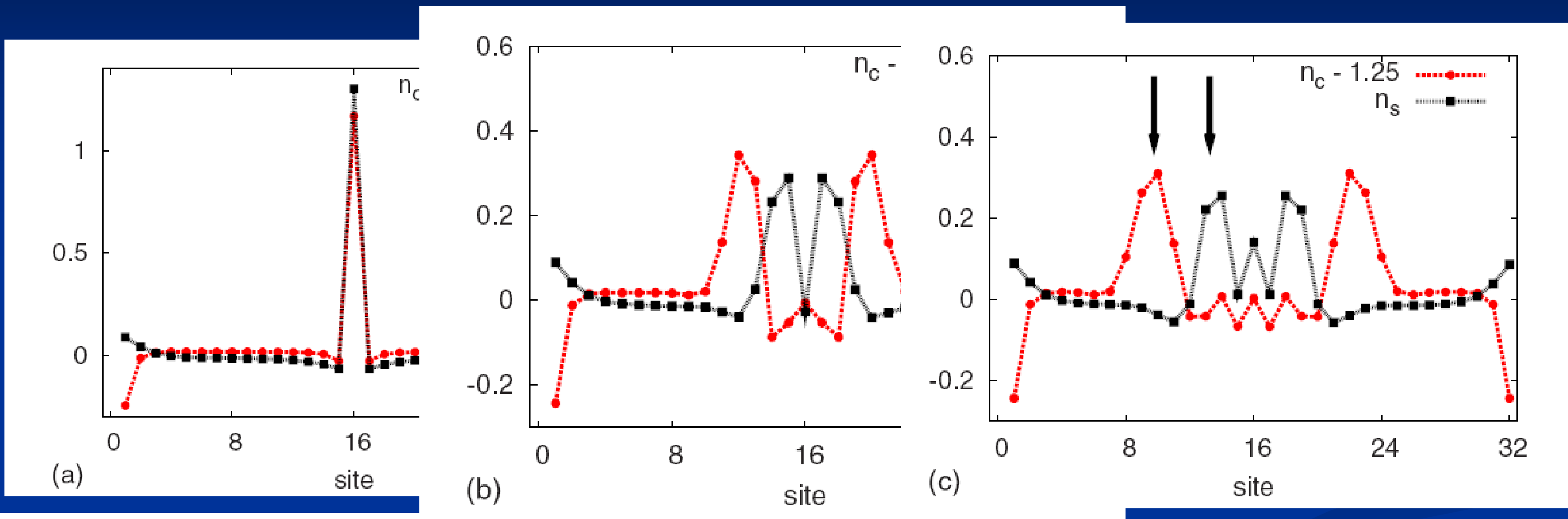
Proposal

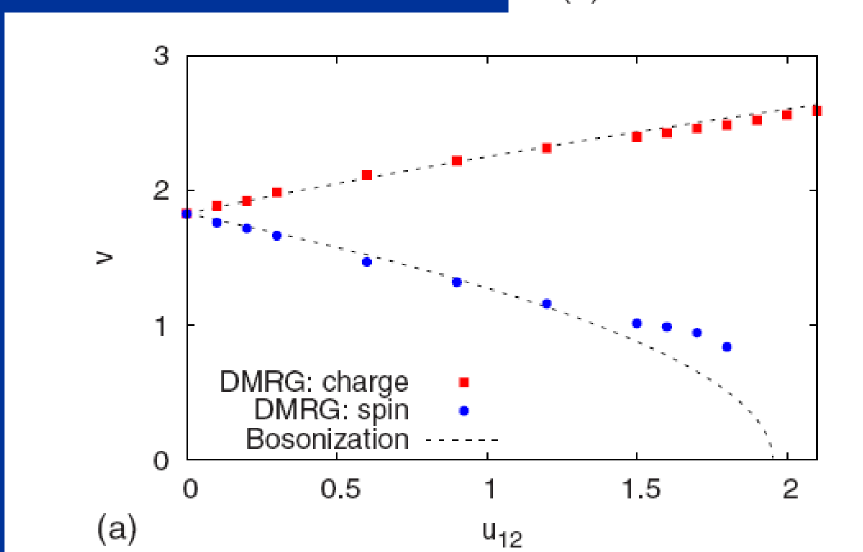
- ^{87}Rb $|F=2,m_F=-1\rangle$ and $|F=2,m_F=1\rangle$ good potential candidates to observe spin-charge separation

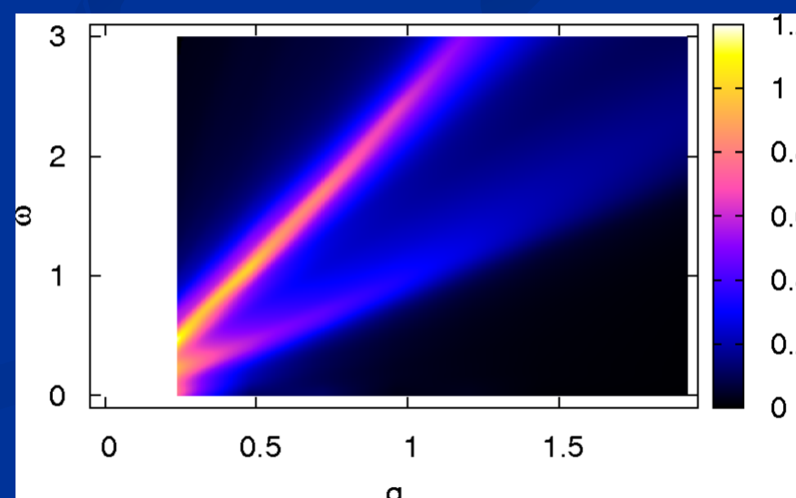
- No problem of temperature (≠ fermions)

- Phase separation

A. Kleine, C. Kollath et al. PRA 77 013607 (2008); NJP 10 045025 (2008)







Beyond Luttinger liquids

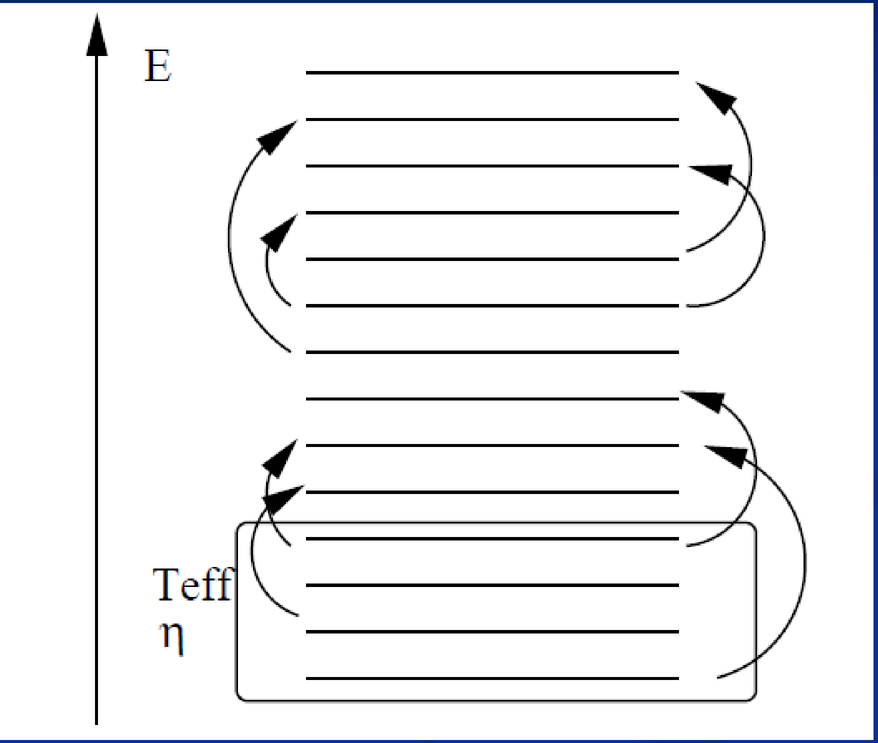
To boldly go where no theorist has gone before….

Out of equilibrium Luttinger liquids

Quenches and thermalization in LL

A. Mitra, TG,
Phys. Rev. Lett. 107, 150602 (2011)
Phys. Rev. B 85, 075117 (2012)

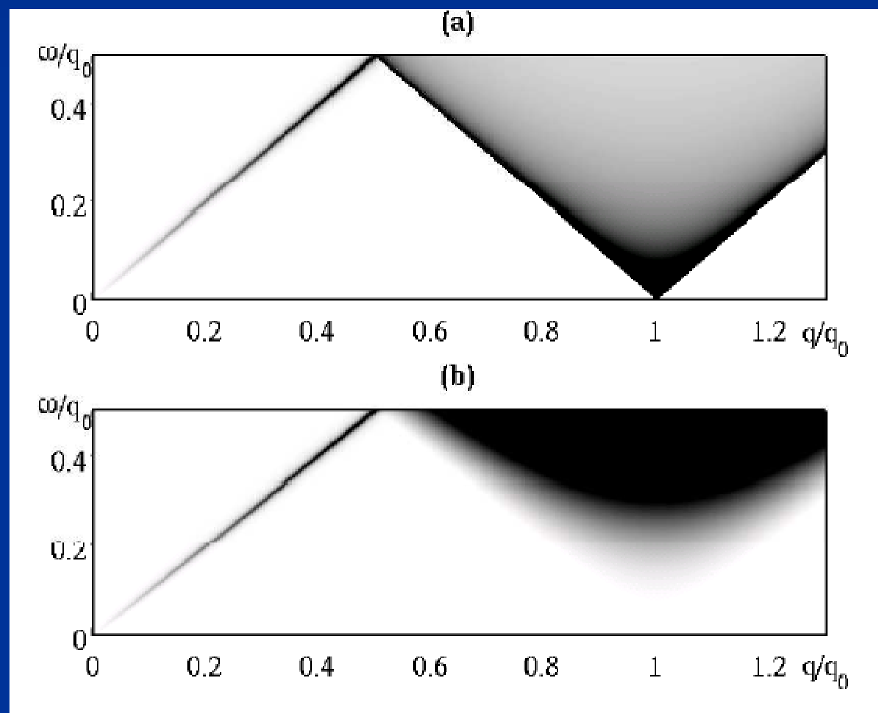
J. Lancaster, TG, A Mitra,
Phys. Rev. B 84, 075143 (2011)



Out of equilibrium sine-Gordon

Thermalization for the low energy part

LL + external noise



E. Dalla Torre, E. Demler,TG, E. Altman, Nat. Phys. 6 806 (2010); PRB 85 184302 (2012)

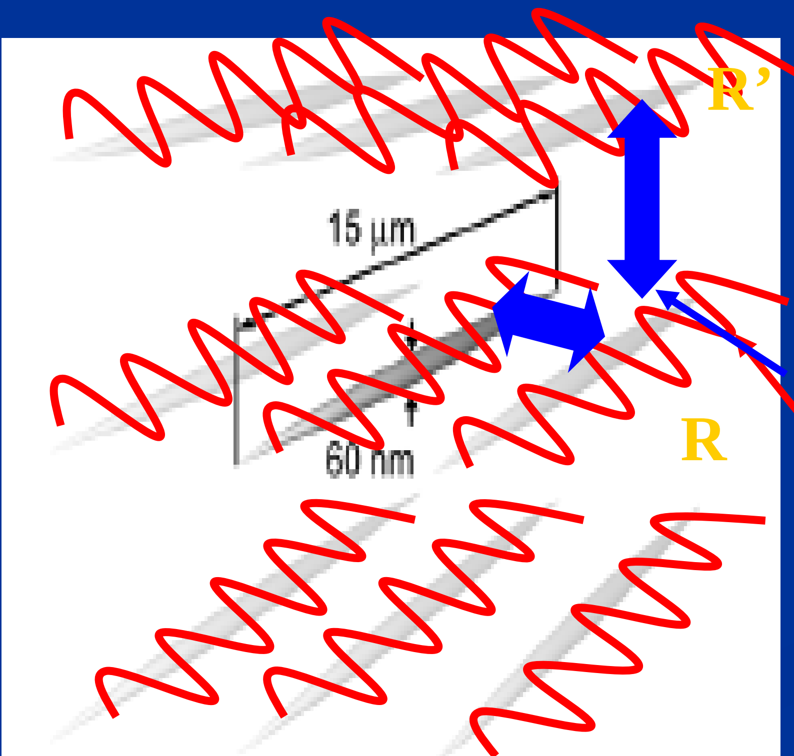
Coupled 1D chains: deconfinememt

Mott vs. Josephson

$$H_{\rm eff} = \frac{\hbar v_s}{2\pi}\sum_{\mathbf{R}}\int_0^L dx\,\left[\frac{1}{K}\left(\partial_x\phi_{\mathbf{R}}(x)\right)^2 + K\left(\partial_x\theta_{\mathbf{R}}(x)\right)^2\right]$$

$$- \frac{\hbar v_s g_J}{2\pi a^2}\sum_{\langle \mathbf{R},\mathbf{R}'\rangle}\int_0^L dx\,\cos\left(\theta_{\mathbf{R}}(x) - \theta_{\mathbf{R}'}(x)\right)$$

$$+ \frac{\hbar v_s g_u}{2\pi a^2}\sum_{\mathbf{R}}\int_0^L dx\,\cos\left(2\phi_{\mathbf{R}}(x) + \delta\pi x\right)$$

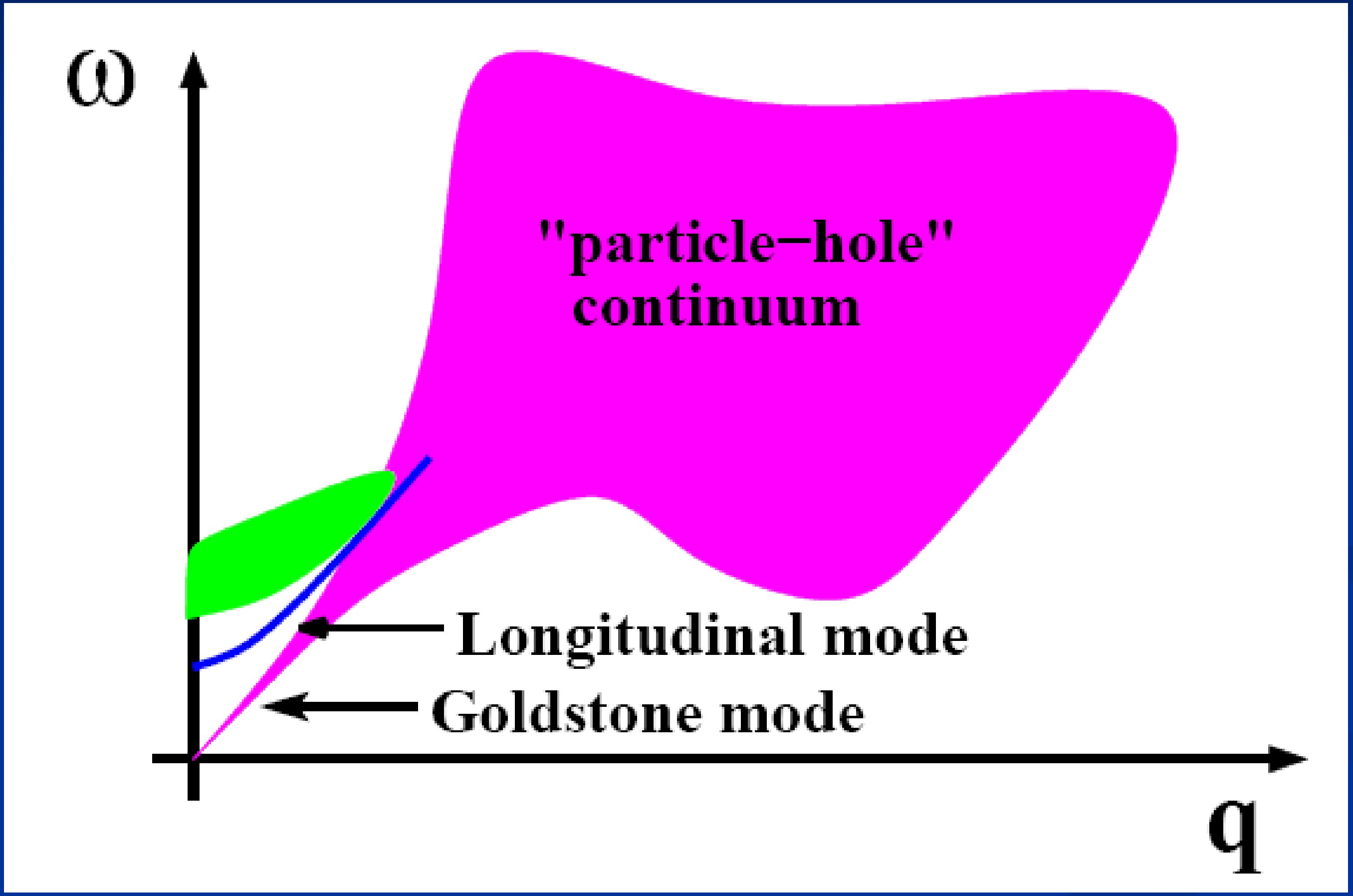


Josephson coupling: delocalizes atoms

"Mott" potential: localizes atoms

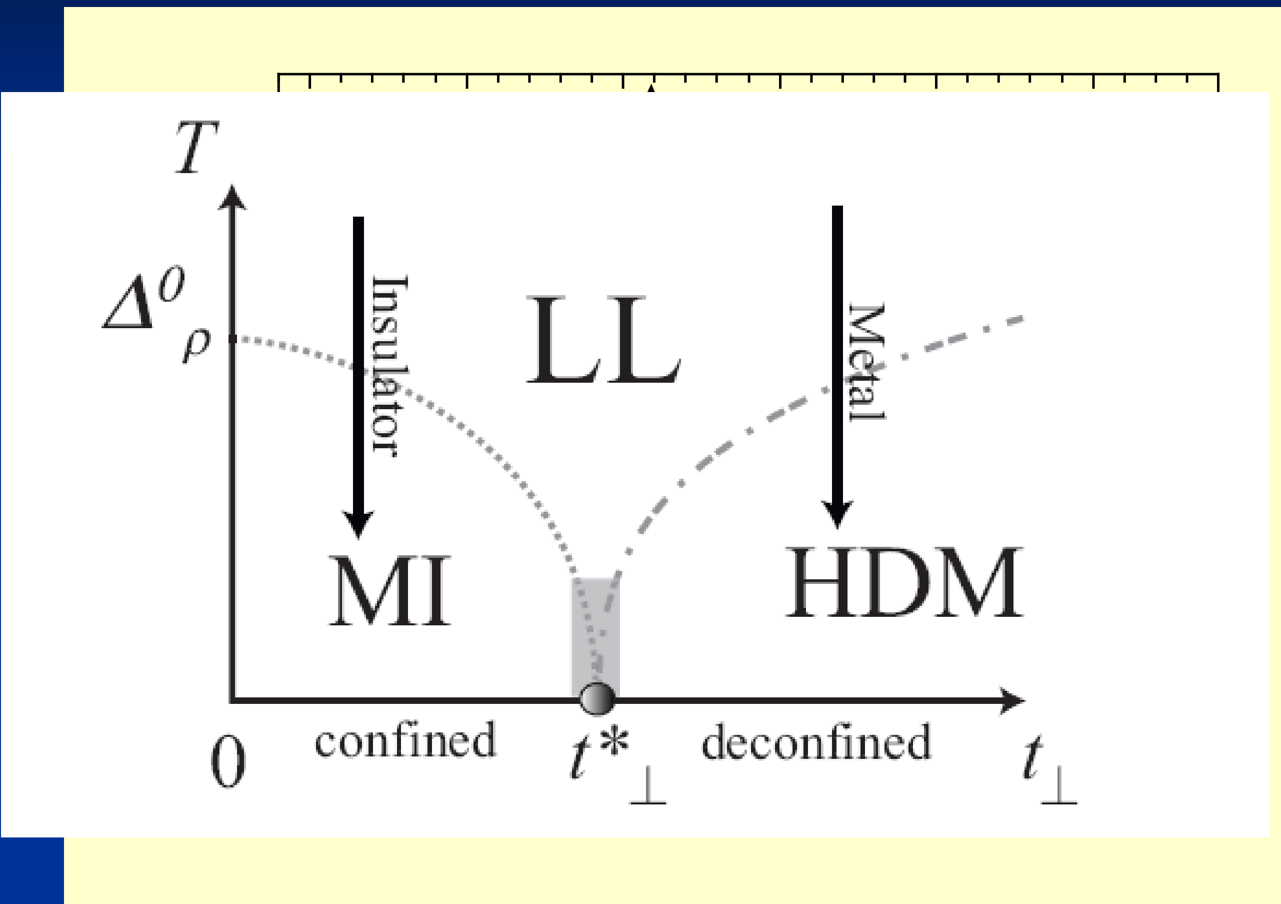
AF Ho, M.A. Cazalilla, TG, PRL **92** 130403 (2004); NJP 8 158 (2006).

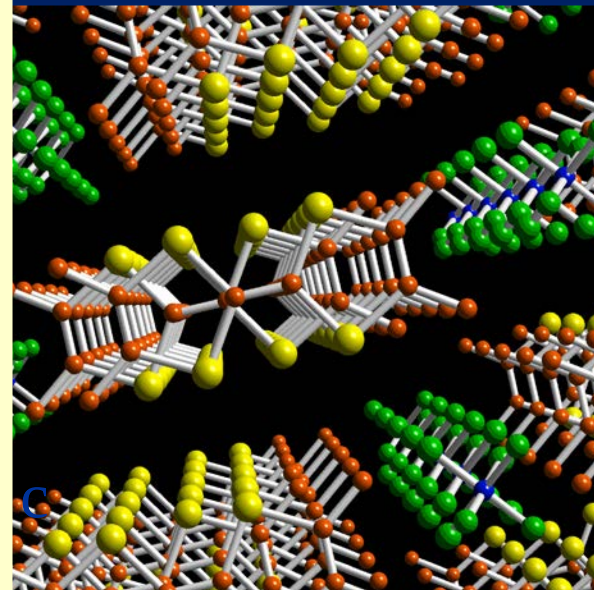
AF Ho, M.A. Cazalilla, TG,NJP 8 158 (2006).



Higgs “amplitude” mode

Deconfinement





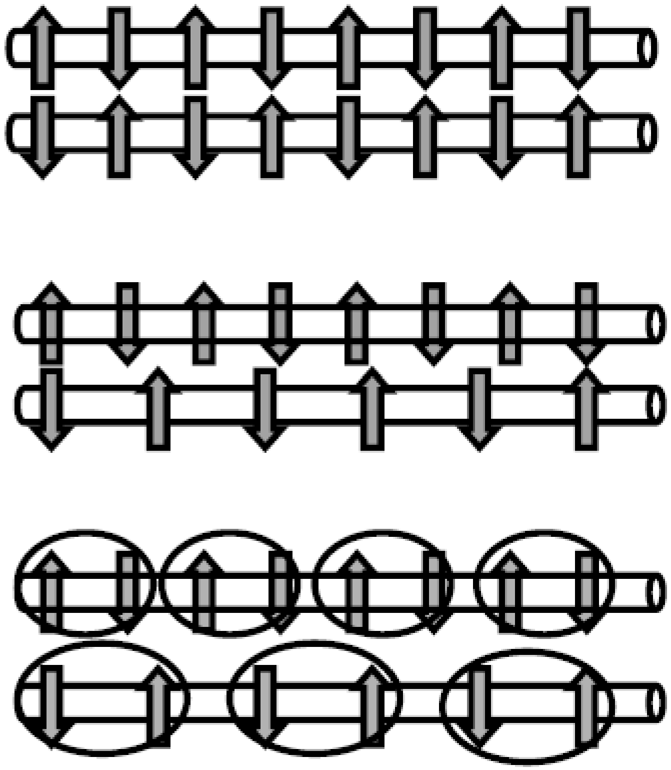
TG Chemical Review 104 5037 (2004)

P. Auban-Senzier, D. Jérome, C. Carcel and J.M. Fabre J de Physique IV, (2004)
A. Pashkin, M. Dressel, M. Hanfland, C. A. Kuntscher, PRB 81 125109 (2010)

D. Jaccard et al., J. Phys. C, 13 L89 (2001)

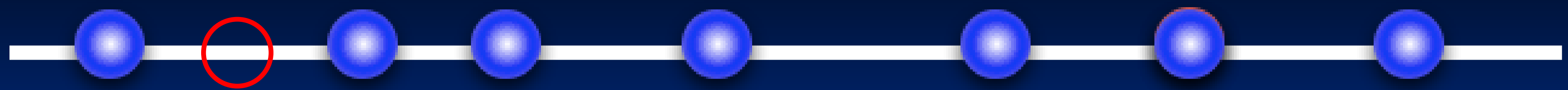
Cold atoms

AF Ho, M.A. Cazalilla, TG, PRL **95,** 226402 (2005); Arxiv/0604525



Triplet superconductor (repulsive interactions)

Non-LL and impurity problems



- Interacting 1D system (Luttinger liquid): e.g. bosons -- ``spin up”

- Add one particle of ``spin down”, or flip a spin

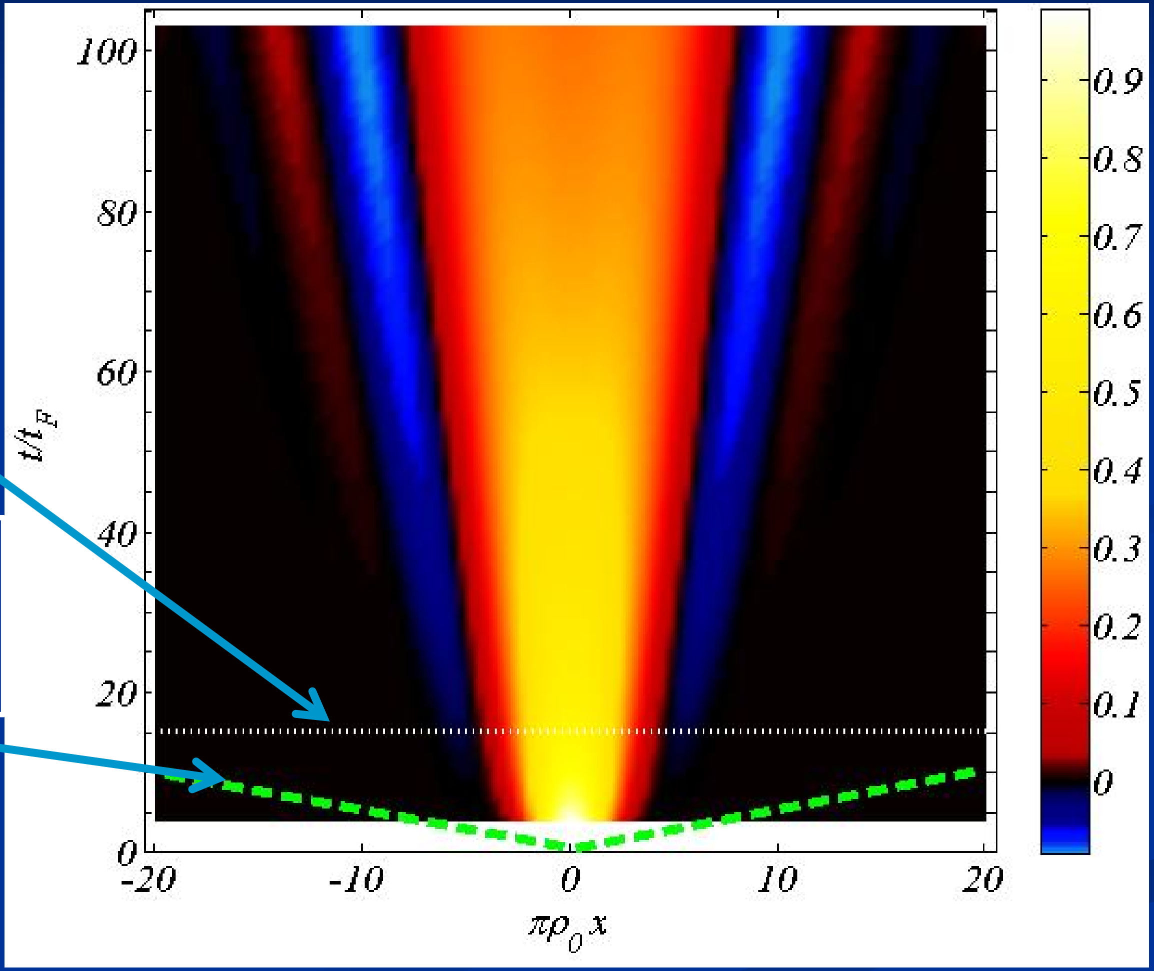
$$G(x,t) = \langle S^{+}(x,t) S^{-}(0,0) \rangle$$

- Example: two component bose-Hubbard model

M. B. Zvonarev, V. V. Cheianov, TG, PRL 99 240404 (2007)

Trapped/open regimes

Light cone of spinless bosons



$$G_\perp(x,t) \simeq \frac{1}{\sqrt{\ln(t/t_F)}} \exp\left\{-\frac{1}{K}\frac{(\pi\rho_0 x)^2}{2\ln(t/t_F)}\right\}.$$

$$G_\perp \simeq e^{-(x^2/2\ell^2)} t^{-\alpha} G_\perp^H, \qquad \ell(t) = \frac{2K^{-(1/2)}}{\pi\rho_0}\frac{t/t_F}{\sqrt{\ln t/t_F}}\frac{m}{m_*}.$$

Theories … Matveev,
Furusak … ngardt,
Lamacra …

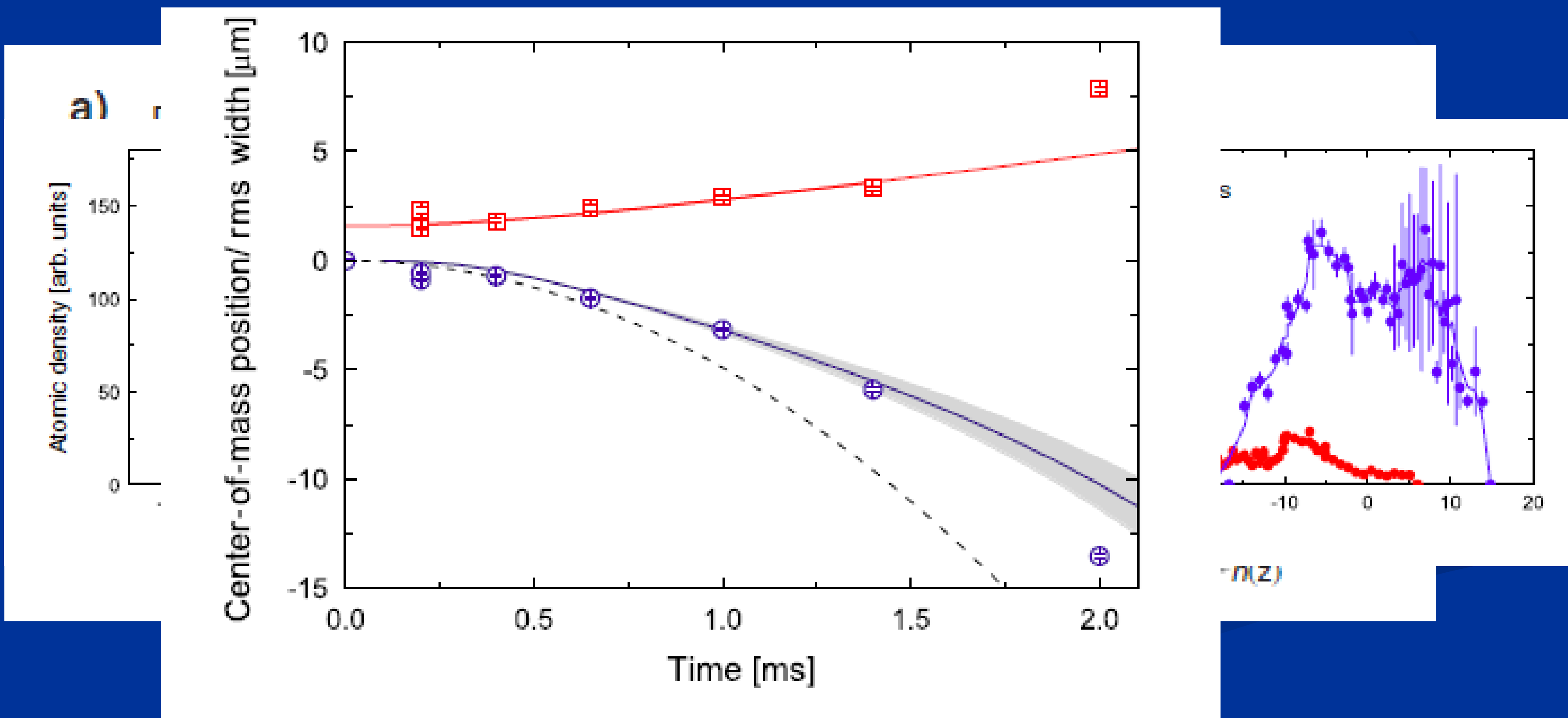
Experin … e,
Innsbru …

Driven impurity

PRL 103, 150601 (2009) PHYSICAL REVIEW LETTERS week ending 9 OCTOBER 2009

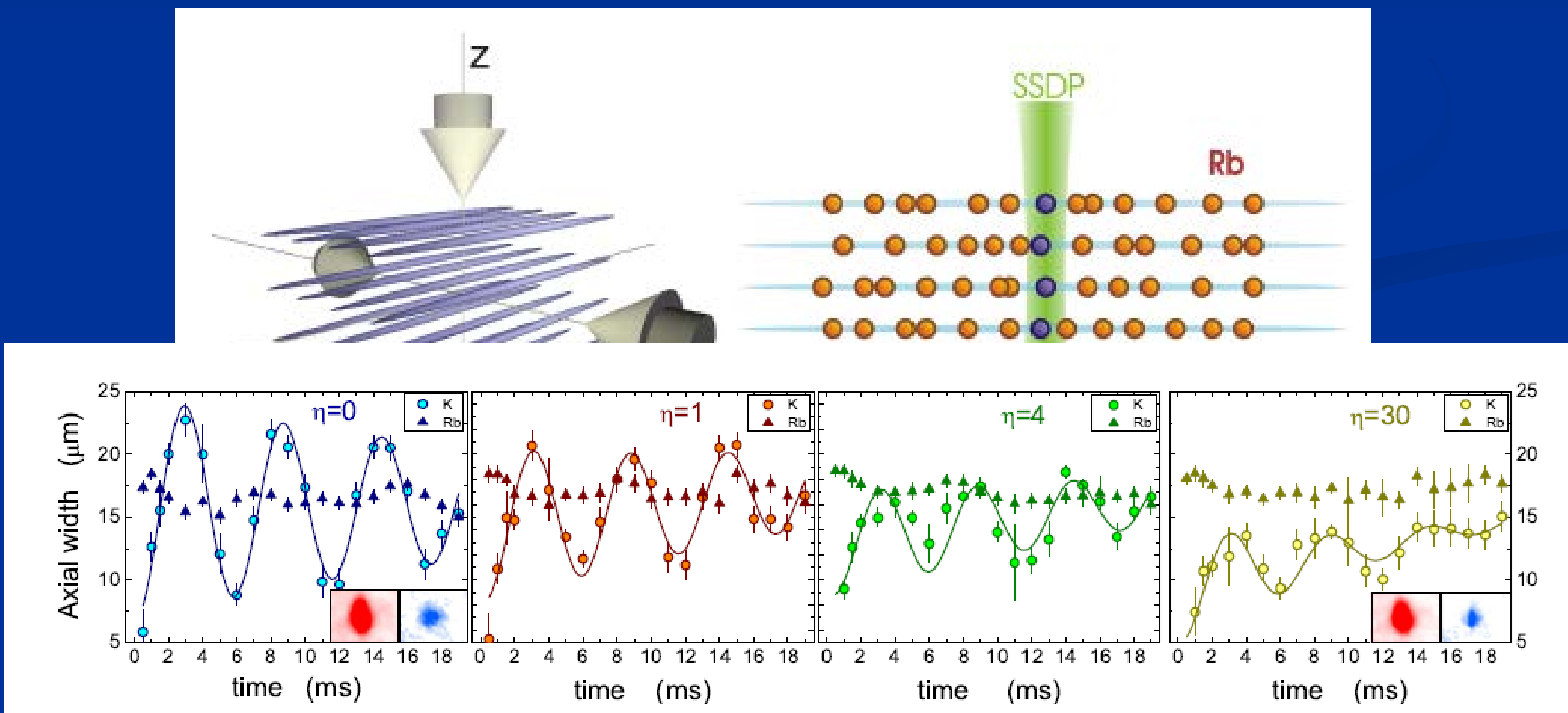
Quantum Transport through a Tonks-Girardeau Gas

Stefan Palzer, Christoph Zipkes, Carlo Sias,* and Michael Köhl



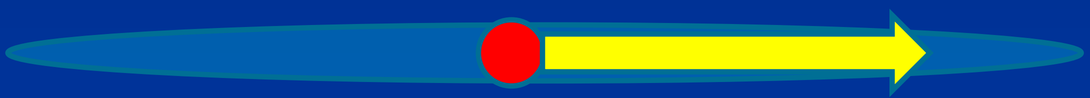
Diffusive impurity

J. Catani et al. PRA **85**, 023623 (2012)



Driven impurity vs diffusion

- Normal transport



$$v = \mu F \qquad v = f(F)$$

$$\langle x^2 \rangle \sim Dt \qquad \langle x^2 \rangle \sim \log(t)$$

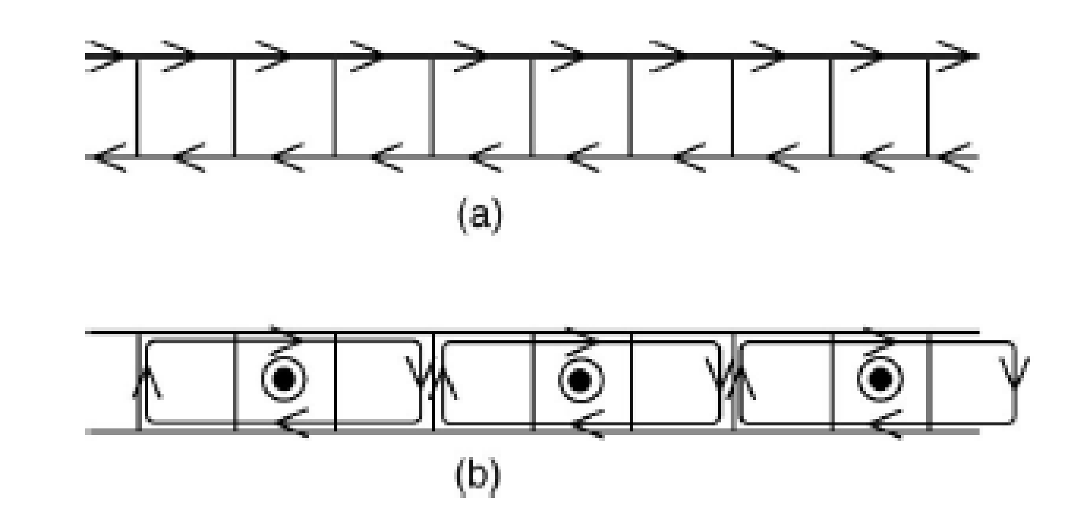
- Einstein relation: μ = D

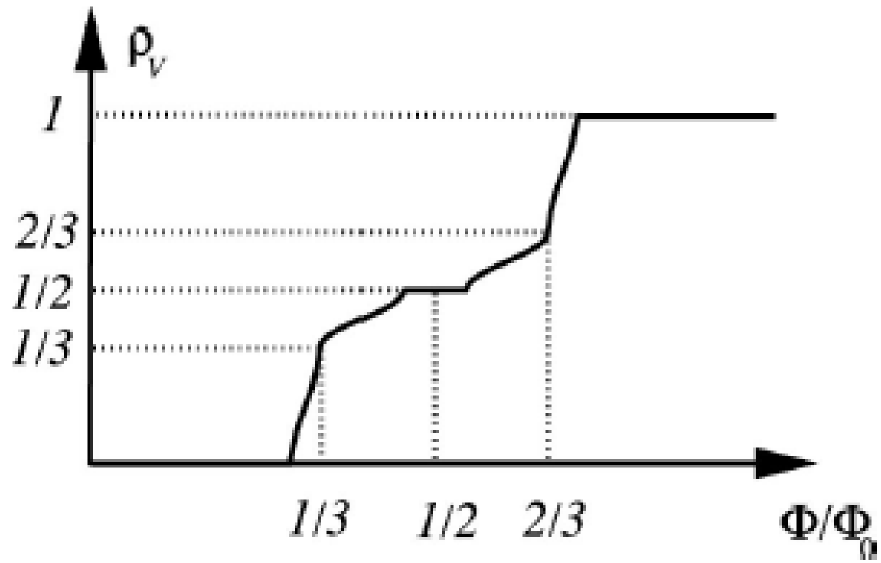
Artificial Gauge fields

PHYSICAL REVIEW B, VOLUME 64, 144515

Meissner effect in a bosonic ladder

E. Orignac + TG





Disorder and interaction effects

Dirty bosons

TG + H. J. Schulz EPL 3 1287 (1987); PRB 37 325 (1988)

$$H_{\rm dis} = \int dx\, V(x)\rho(x)$$

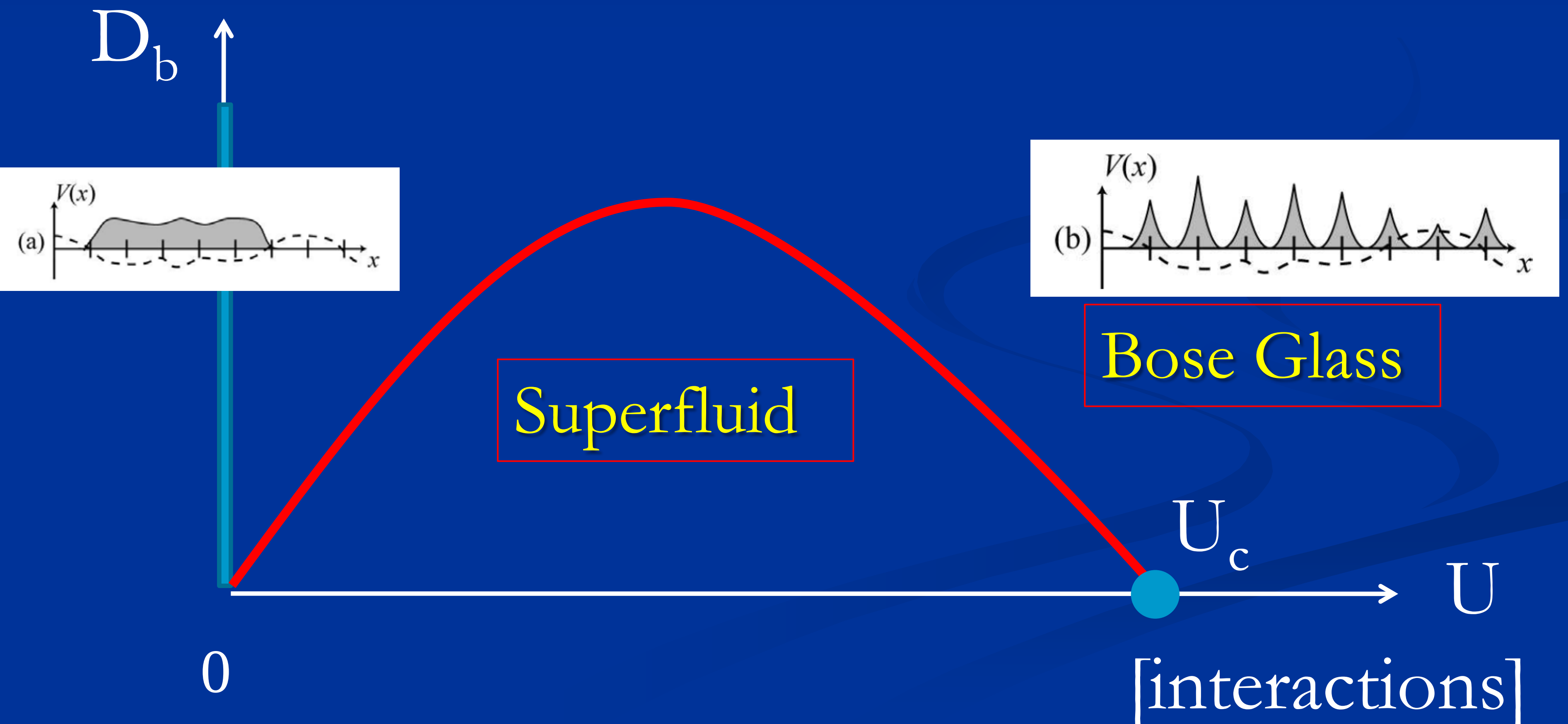
$$H_{\rm dis} = \int dx\, V(x)[-\frac{1}{\pi}\nabla\phi(x) + \rho_0(e^{i(2\pi\rho_0 x - 2\phi(x))} + {\rm h.c.})]$$

``Two" fourier components of disorder

Bose glass phase

TG + H. J. Schulz EPL 3 1287 (87); PRB 37 325 (1988);

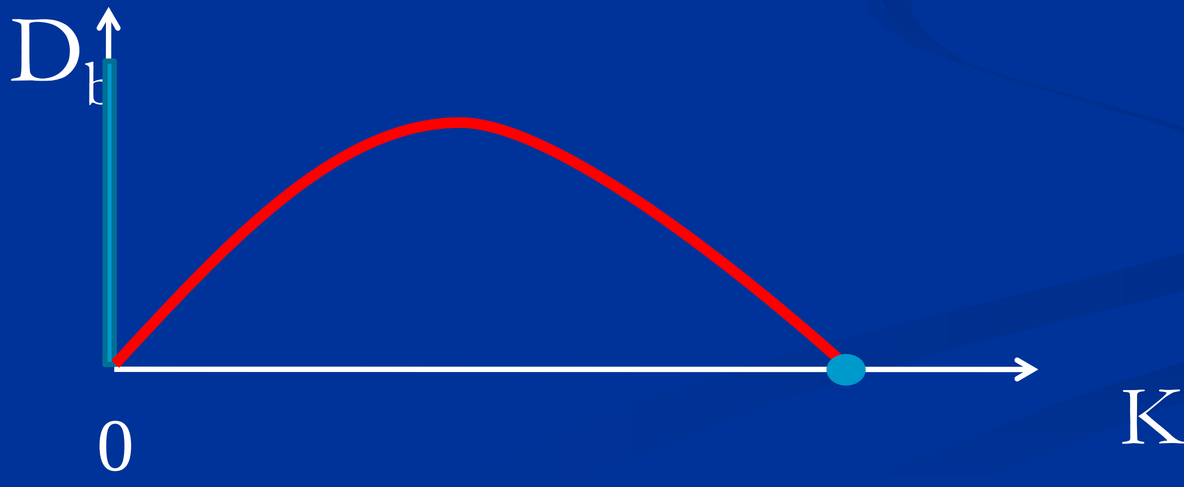
M.P.A. Fisher et al. PRB 40 546 (1989)



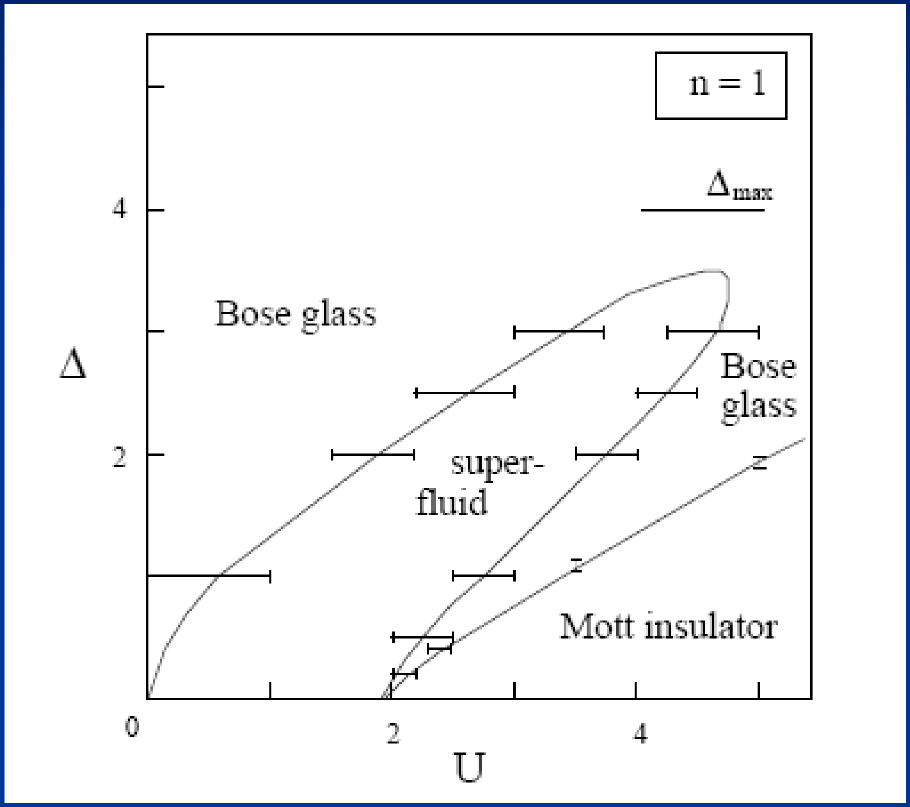
SU-BG transition in d=1

$$\langle \psi(r)\psi^\dagger(0)\rangle \sim \left(\frac{1}{r}\right)^{\frac{1}{2K}} \qquad K \to 3/2$$

Universal exponent at the SU-BG transition !



Numerics



S. Rapsch, U. Schollwoeck, W. Zwerger EPL 46 559 (1999);

G. Batrouni et al. PRL 65 1765 (90);
N. Prokofev et al. PRL 92 015703 (04);
O. Nohadani et al. PRL 95, 227201 (05)
K. G. Balabanyan et al. PRL 95, 055701 (05);
L. Pollet et al. PRL 103, 140402 (2009)
…..

Two loops RG for Bose glass

Z. Ristivojevic, A. Petkovic, P. Le Doussal, TG PRL (2012)

$$\frac{\mathrm{d}\mathscr{D}_R}{\mathrm{d}\ell} = -2\mathscr{D}_R\delta_R + A\mathscr{D}_R^2 + \mathcal{O}(\mathscr{D}_R^2\delta_R),$$

$$\frac{\mathrm{d}\delta_R}{\mathrm{d}\ell} = -9\mathscr{D}_R + B\mathscr{D}_R\delta_R + \mathcal{O}(\mathscr{D}_R^2),$$

$$\frac{\mathrm{d}}{\mathrm{d}\ell}(aD_{fR}) = 0, \quad \frac{\mathrm{d}}{\mathrm{d}\ell}\left(\frac{v_R}{K_R}\right) = 0,$$

$\Gamma_2^{(1)} = -$ … , (10)

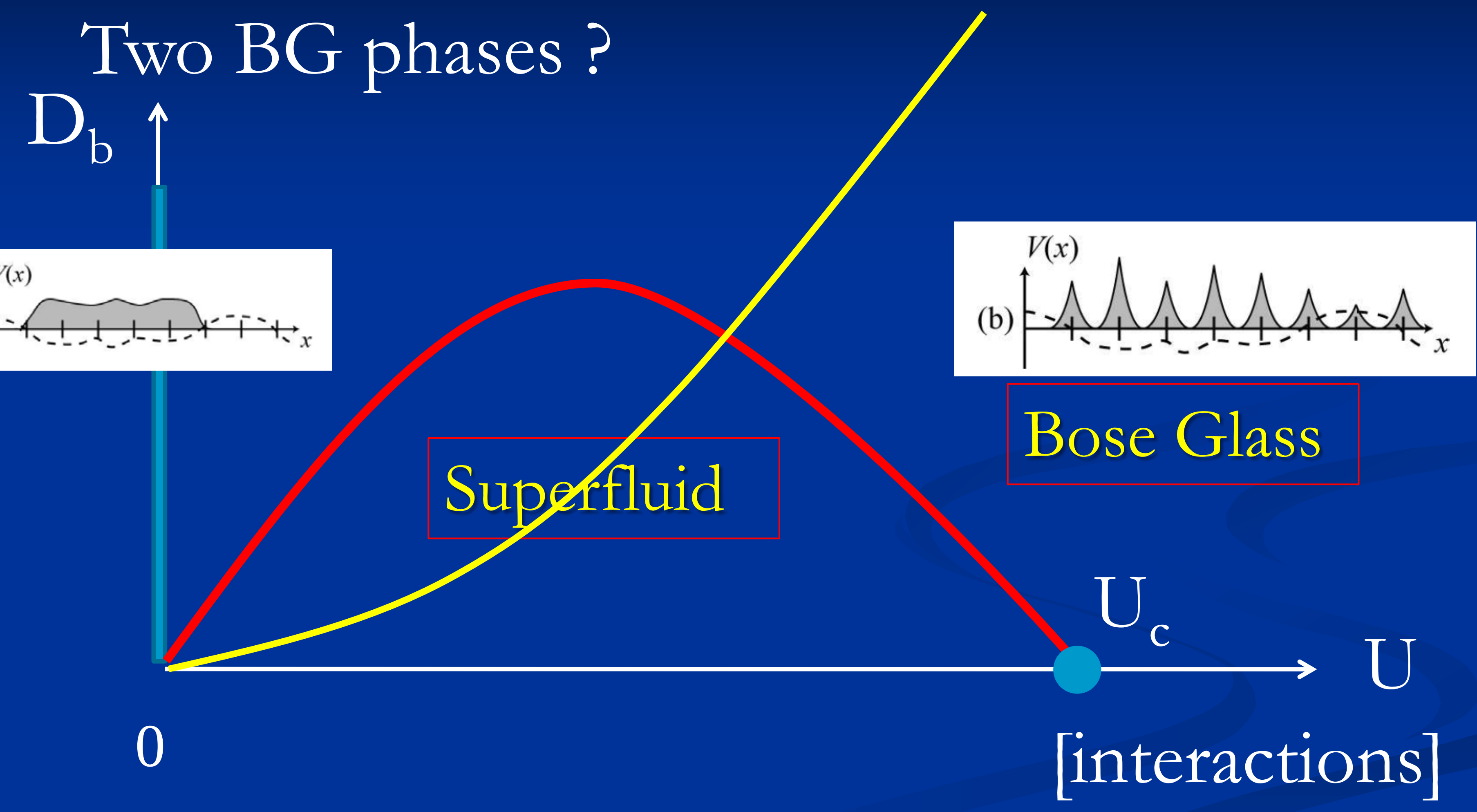
$\Gamma_2^{(2)} = -$ … $_\beta(x,\tau')]$

… $)]\Big\}$, (11)

$\Gamma_2^{(3)} = -$ … $_1')]$. (12)

$$K = \frac{3}{2} + \delta_R$$

Phase diagram



Order parameter ? Moments of distribution ?

Experiments

Cold atomic gases

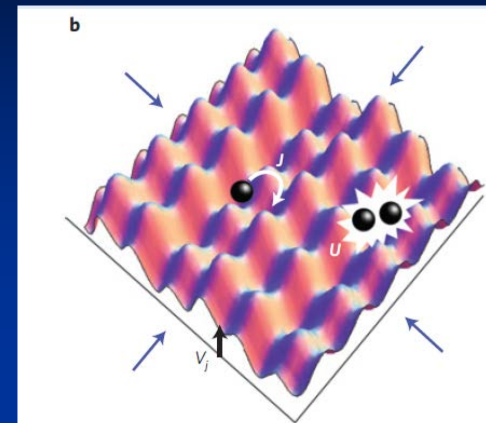

nature physics

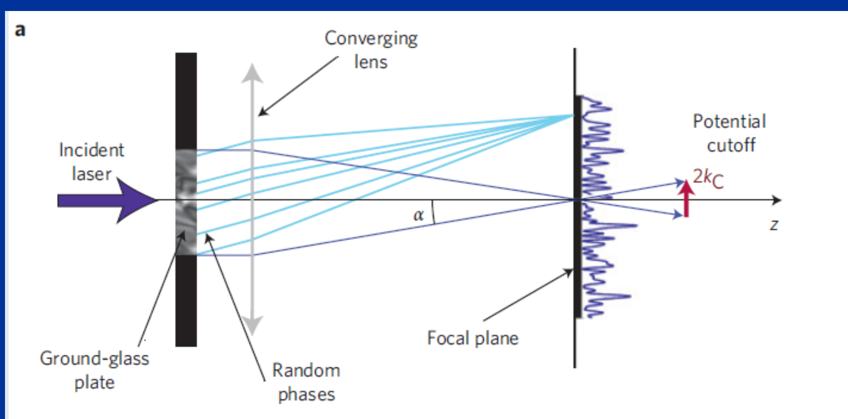
PROGRESS ARTICLE

PUBLISHED ONLINE: 1 FEBRUARY 2010 | DOI: 10.1038/NPHYS1507

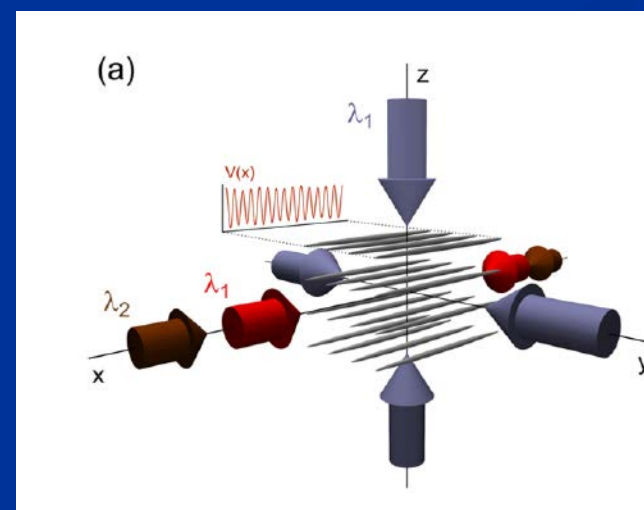
Disordered quantum gases under control

Laurent Sanchez-Palencia[1]* and Maciej Lewenstein[2]*





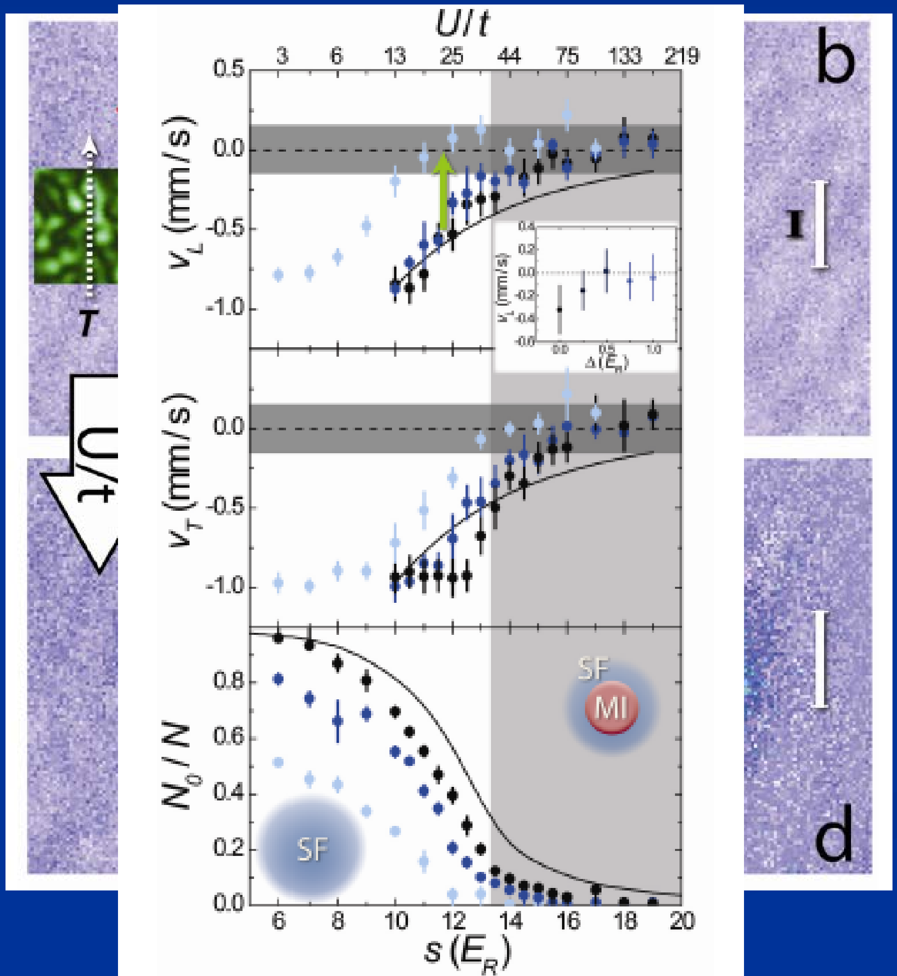
Speckle

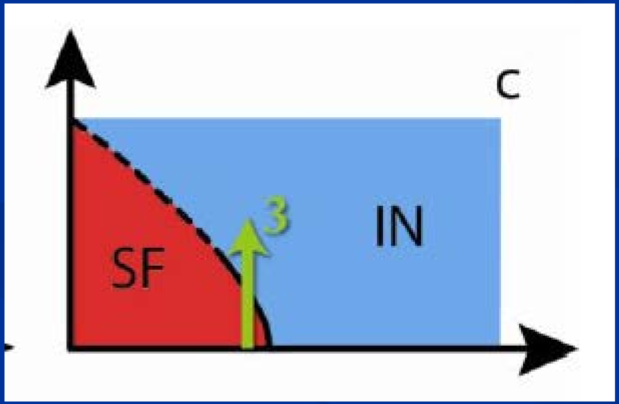


Biperiodic lattices

Speckle

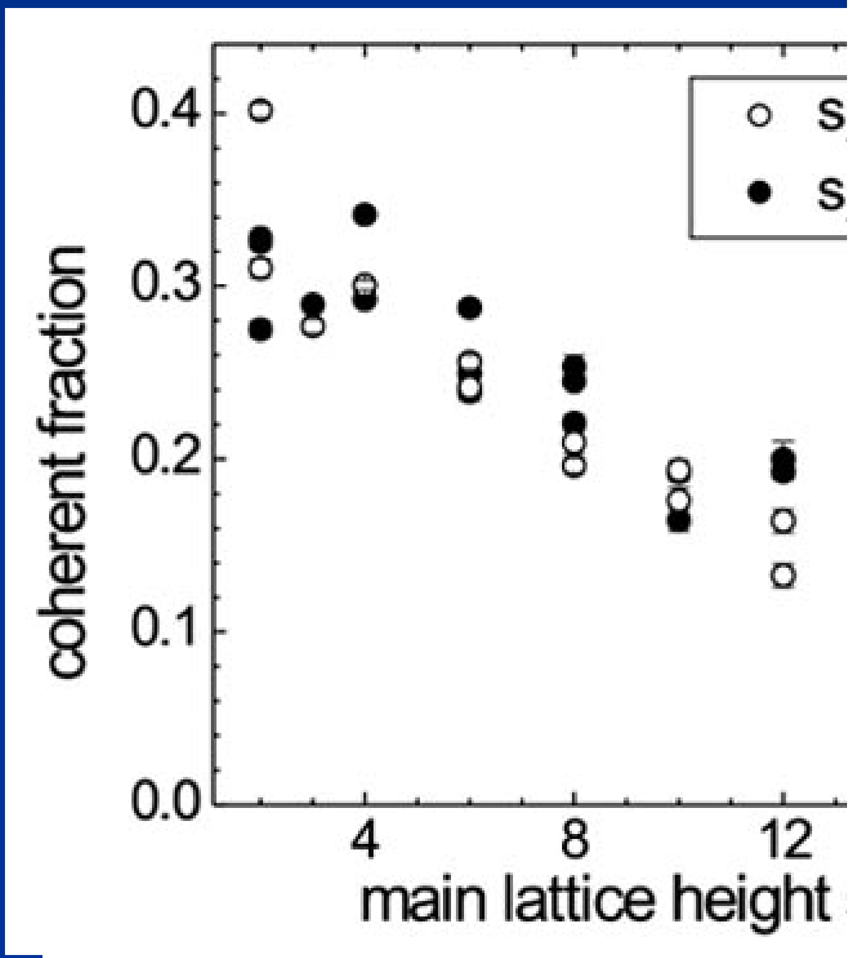
Pasienski et al. Nat. Phys 6 677 (2010)

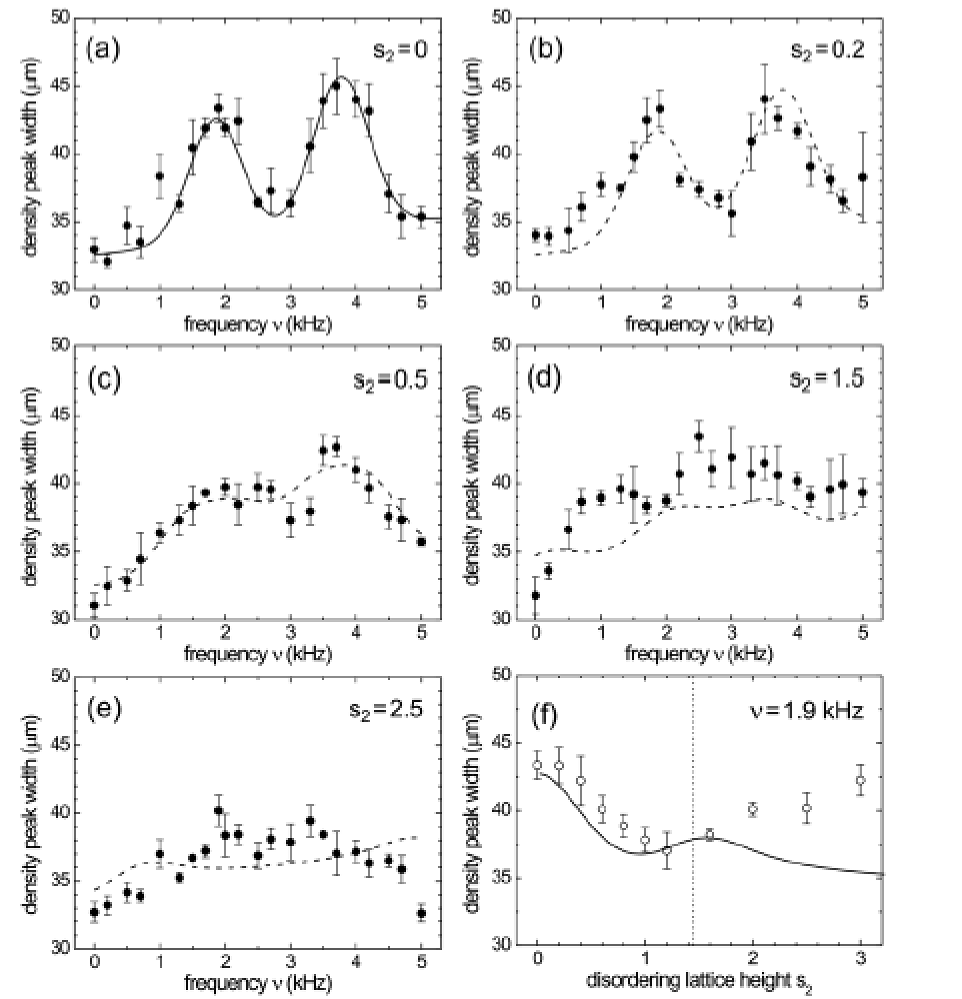




Quasi-periodic

Quasiperiodic (1D):





Same as true disorder ?

Renormalization treatment

J. Vidal, D. Mouhanna, TG PRL 83 3908 (1999); PRB 65 014201 (2001)

$$\frac{dK}{dl} = -K^2 \Xi(l),$$

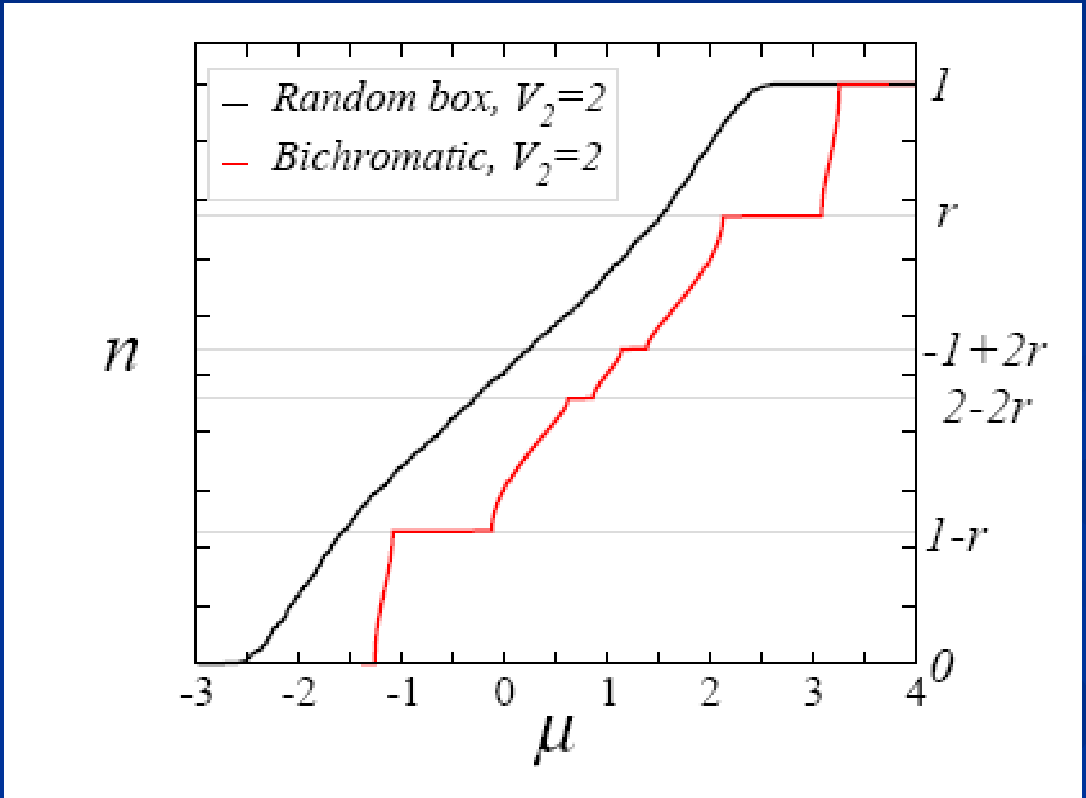
$$\frac{dy_Q}{dl} = (2-K) y_Q,$$

y_Q : Fourier components of potential

$$\Xi(l) = \frac{1}{2} \sum_Q y_Q^2 [\mathrm{J}(Q^+ \alpha(l)) + \mathrm{J}(Q^- \alpha(l))],$$

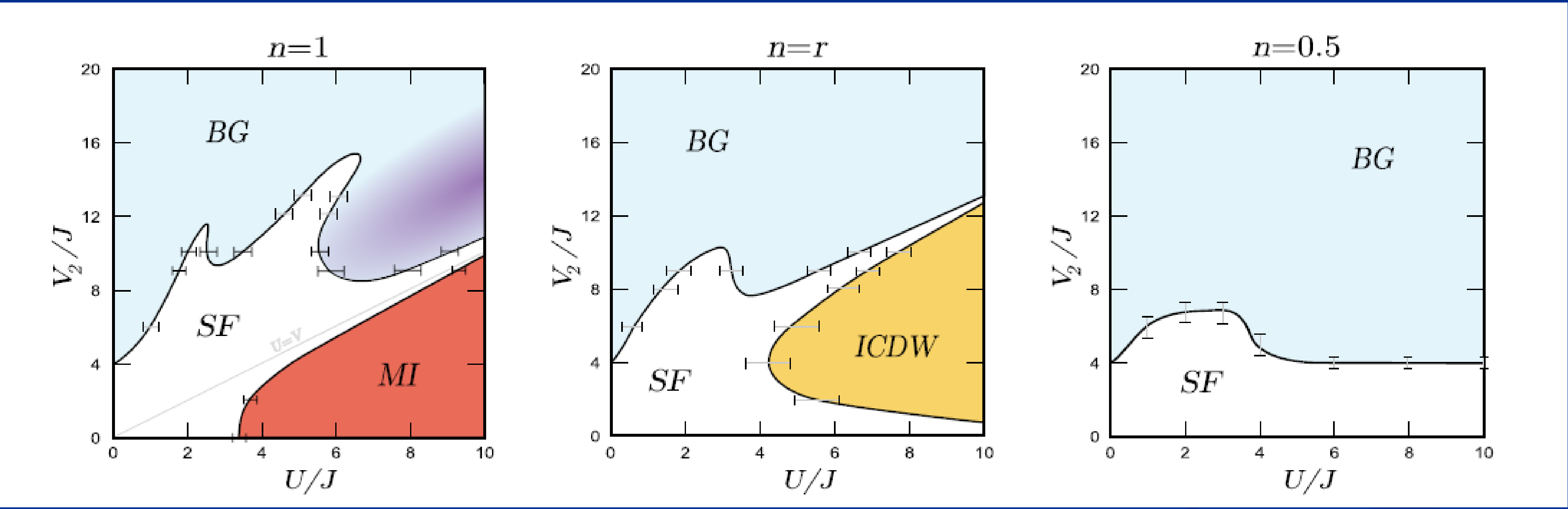
Numerics: DMRG

G. Roux et al. PRA 78 023628 (2008)



Related works: T. Roscilde, Phys. Rev. A 77, 063605 2008; X. Deng et al PRA 78, 013625 (2008)

Phase diagram



Conclusions

- Tour of one dimensional physics
- Luttinger liquid theory provides a framework to study this physics, and to go beyond
- Many effects: lattice, disorder, long range forces, competition between orders, out of equilibrium physics, gauge fields,…..
- Cold atoms give access to novel one/quasi-one dimensional physics


Calvin and Hobbes
WATTERSON
WOW, IT REALLY SNOWED LAST NIGHT! ISN'T IT WONDERFUL?
EVERYTHING FAMILIAR HAS DISAPPEARED! THE WORLD LOOKS BRAND-NEW!
A NEW YEAR... A FRESH, CLEAN START!
IT'S LIKE HAVING A BIG WHITE SHEET OF PAPER TO DRAW ON!
A DAY FULL OF POSSIBILITIES!
IT'S A MAGICAL WORLD, HOBBES, OL' BUDDY...
...LET'S GO EXPLORING!