IST-solution of kdV for a single soliton

Maxim
BOS
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$u(x,0) = -2\,\mathrm{sech}^2(x)$

$b(k,0) = 0$

$\varkappa_1 = 1$

$\Psi_1 = \frac{1}{\sqrt{2}}\,\mathrm{sech}\,(x) \Rightarrow c_1(0) = \sqrt{2}$

$b(k,t) = 0$

$c_1(t) = \exp[4\varkappa_1^3 t]\, c_1(0) = \exp[4t]\sqrt{2}$

$F(\xi | t) = 2\exp[8t]\exp[-\xi]$

$K(x,z|t) + 2\exp[8t]\exp[-x-z] + \int_x^\infty dy\, K(x,y|t)\, 2\exp[8t]\exp[-y-z] = 0$

$K(x,z|t) = L(x,t)\exp[-z]$

$L(x,t)\exp[-z] + 2\exp[8t]\exp[-x-z] + \int_x^\infty dy\, L(x,t)\exp[-y]\,2\exp[8t]\exp[-y-z]$

$$L(x,t) + 2\exp[8t]\exp[-x] + \frac{\exp[-2x]}{\cancel{2}}\cancel{2}\exp[8t]\,L(x,t) = 0$$

$$L(x,t)\,(1 + \exp[-2x+8t]) = -2\exp[-x]\exp[8t)$$

$$L(x,t)\,(\exp[+(x-4t)] + \exp[-(x-4t)]\,) = -2\exp[4t]$$

$$L(x,t) = -\frac{\exp[4t]}{ch[x-4t]}$$

$$K(x,z|t) = -\frac{\exp[4t]\exp[-z]}{ch[x-4t]}$$

$$K(x,x/t) = -\frac{\exp[4t-x]}{ch[x-4t]}$$

$$u(x,t) = -2\frac{\partial}{\partial x}K(x,x|t) = -\frac{8\exp^2[x+4t]}{(\exp[8t]+\exp[2x])^2} =$$

$$= -\frac{8}{(\exp[-x+4t]+\exp[x-4t])^2} = -2\,\mathrm{sech}^2(x-4t)$$

$$\boxed{u(x,t) = -2\,\mathrm{sech}^2(x-4t)}$$