Bethe Ansatz for N_{at} bosons

Maxim
BOS
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$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{i=1}^{N_{at}}\frac{\partial^2}{\partial x_i^2} + g\sum_{i=1}^{N_{at}}\sum_{j=i+1}^{N_{at}}\delta(x_i - x_j)$$

$$a = -\frac{\hbar^2}{\mu g} = -\frac{2\hbar^2}{mg}$$

$$\boxed{\forall(i,j)}\ \Psi(x_1, x_2, \ldots, x_i, \ldots, x_j, \ldots, x_{N_{at}}) = \Phi(x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_{j-1}, x_{j+1}, \ldots, x_{N_{at}}) \times$$

$$\times \left\{1 - \frac{|x_i - x_j|}{a} + \mathcal{O}(|x_i - x_j|^2)\right\}$$

$$\Psi(x_1, x_2, \ldots, x_j, \ldots, x_i, \ldots, x_{N_{at}}) = \Psi(x_1, x_2, \ldots, x_i, \ldots, x_j, \ldots, x_{N_{at}})$$

Solutions:

$$\Psi(x_1, x_2, \ldots, x_{N_{at}}) = \text{const} \times \prod_{i=1}^{N_{at}}\prod_{j=i+1}^{N_{at}}\left(\frac{\frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_j}}{2} - \frac{\text{sign}(x_i - x_j)}{a}\right)\Psi_F(x_1, x_2, \ldots, x_{N_{at}})$$

$$-\frac{\hbar^2}{2m}\sum_{i=1}^{N_{at}}\frac{\partial^2}{\partial x_i^2}\Psi_F = E\Psi_F \qquad \Psi_F(x_1, x_2, \ldots, x_j, \ldots, x_i, \ldots, x_{N_{at}}) = -\Psi_F(x_1, x_2, \ldots, x_i, \ldots, x_j, \ldots, x_{N_{at}})$$

$$\Psi_F = |k_1, k_2, \ldots, k_{N_{at}}\rangle \longrightarrow \Psi_B \overset{\boxed{\text{locally}}}{=} |k_1, k_2, \ldots, k_{N_{at}}\rangle$$

← Bethe Ansatz

Remark: $(g \to \infty) \Rightarrow$ hard-core bosons $\Rightarrow \Psi(x_1, x_2, \ldots, x_{N_{at}}) = (-1)^{P(x_1, x_2, \ldots, x_{N_{at}})}\Psi_F(x_1, x_2, \ldots, x_{N_{at}})$

Remark: "soliton" $\underset{g<0}{\Longrightarrow} \boxed{\Psi \propto e^{-\sum_{i,j}|x_i - x_j|/a}}$