Ultracold Atoms in Controlled Disorder I:

Anderson Localization in Dimension d=1

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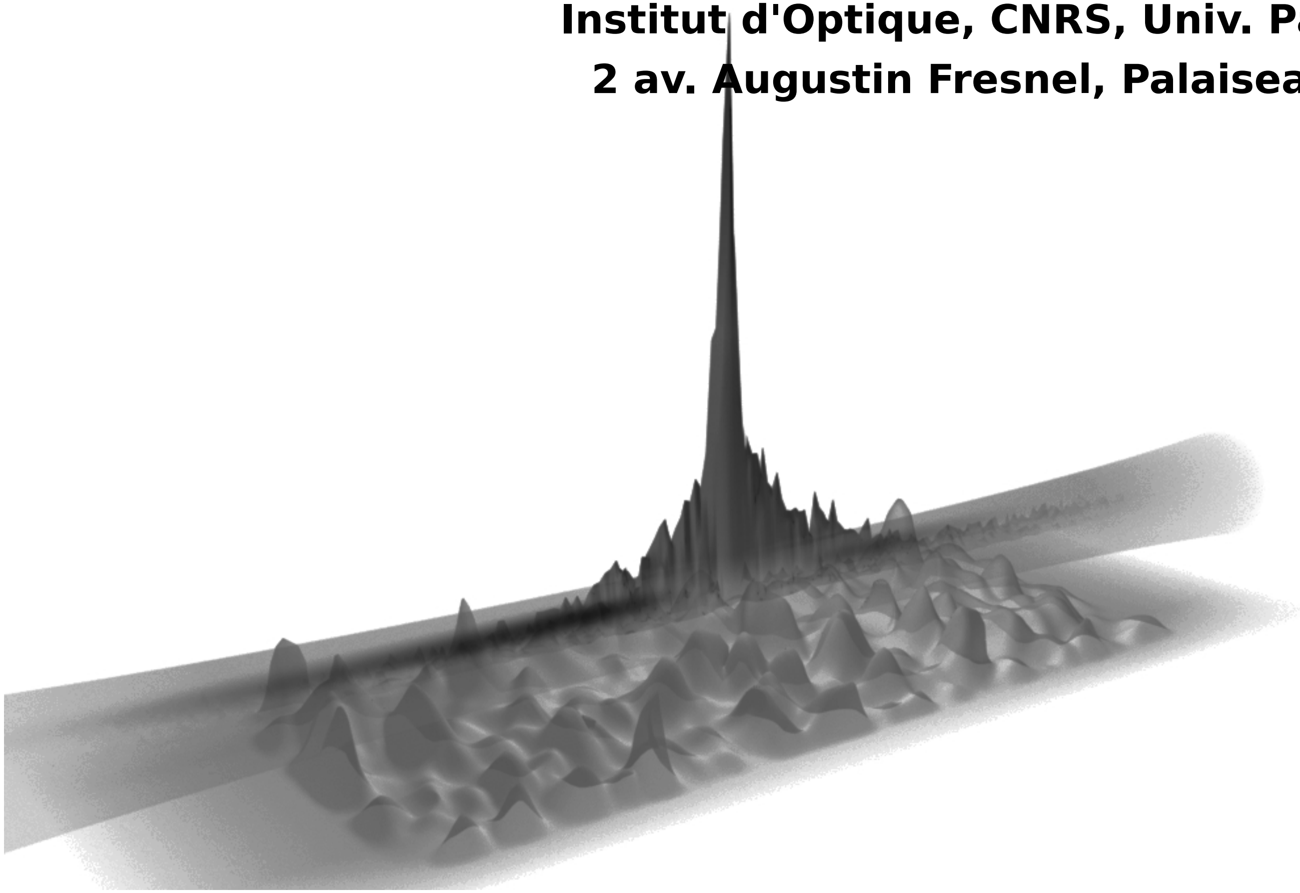
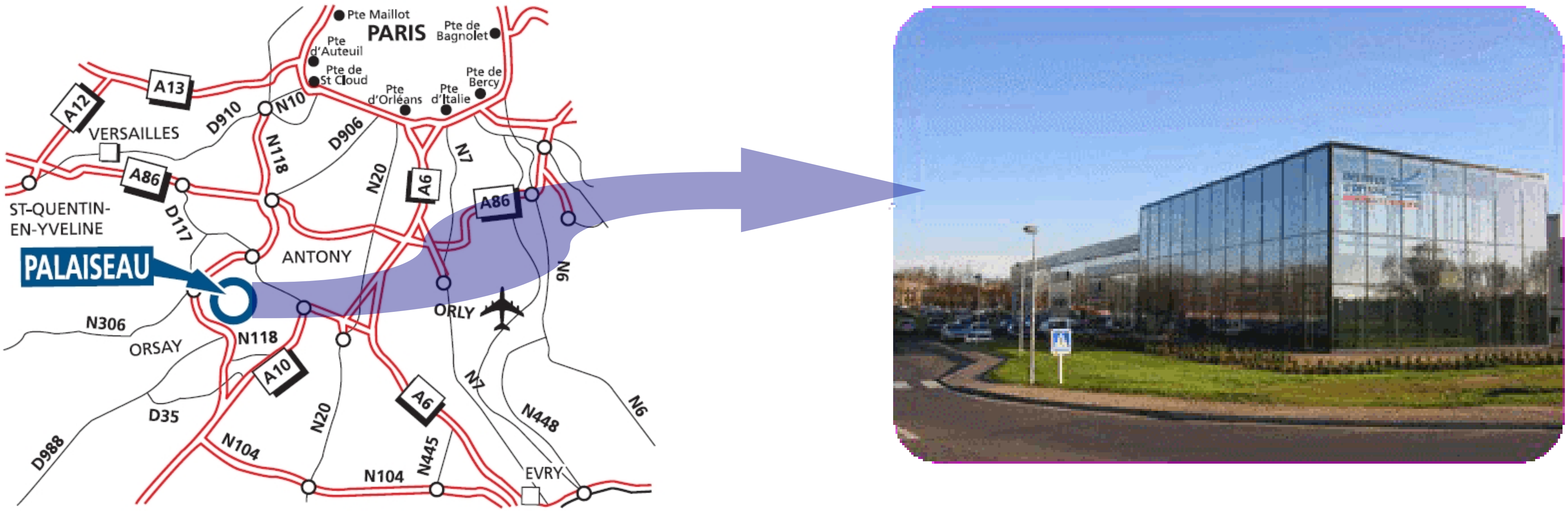


Acknowledgements



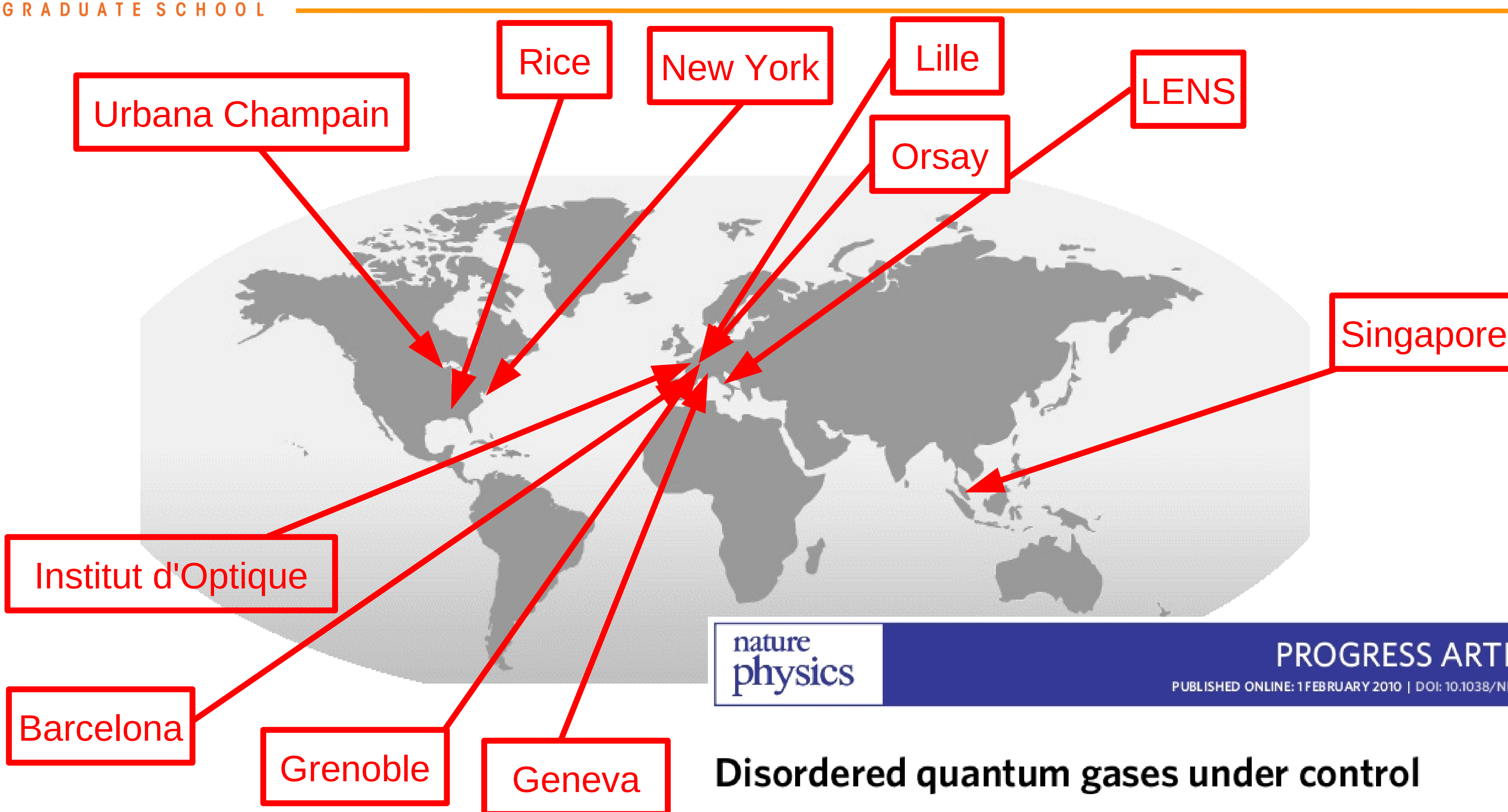
« Theory of Ultracold Quantum Gases » team

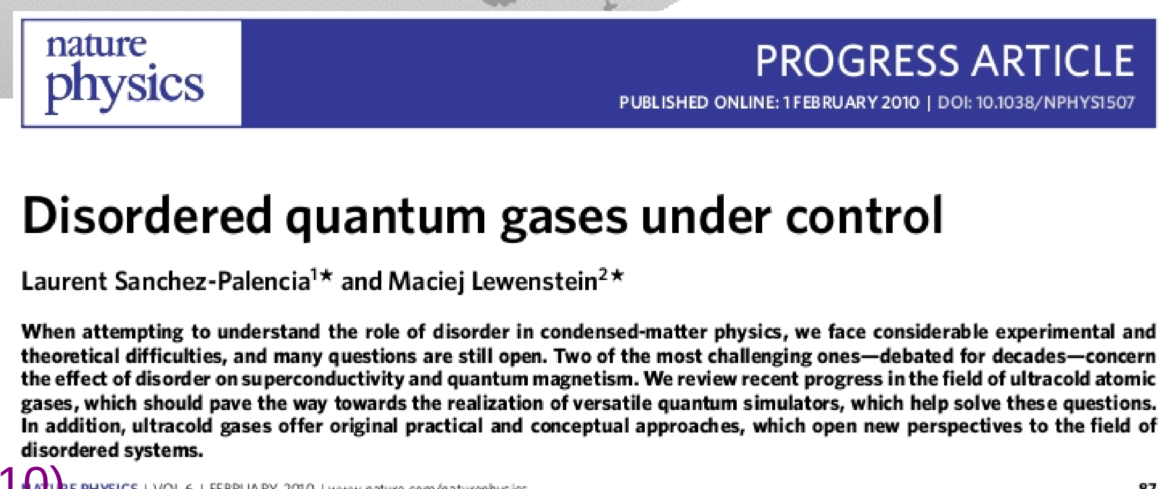
- M. Piraud, S. Lellouch, G. Boeris, G. Carleo, LSP
- former members : L. Pezzé, T.-L. Dao, B. Hambrecht, P. Lugan

Collaboration and stimulating discussions

- A. Aspect's experimental group, P. Bouyer, V. Josse, T. Bourdel *et al.*
- B. van Tiggelen, T. Giamarchi, M. Lewenstein, G.V. Shlyapnikov

Disordered Quantum Gases





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PROGRESS ARTICLE

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Disordered quantum gases under control

Laurent Sanchez-Palencia[1]* and Maciej Lewenstein[2]*

When attempting to understand the role of disorder in condensed-matter physics, we face considerable experimental and theoretical difficulties, and many questions are still open. Two of the most challenging ones—debated for decades—concern the effect of disorder on superconductivity and quantum magnetism. We review recent progress in the field of ultracold atomic gases, which should pave the way towards the realization of versatile quantum simulators, which help solve these questions. In addition, ultracold gases offer original practical and conceptual approaches, which open new perspectives to the field of disordered systems.

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LSP & Lewenstein, Nat. Phys. **6**, 87 (2010)
special issue in Phys. Today 2009
Modugno, Rep. Prog. Phys. **73**, 102401 (2010)
Fallani *et al.*, Adv. At. Mol. Opt. Phys. **56**, 119 (2008)
Shapiro, J. Phys. A: Math. Theor. **45**, 143001 (2012)

Outlook

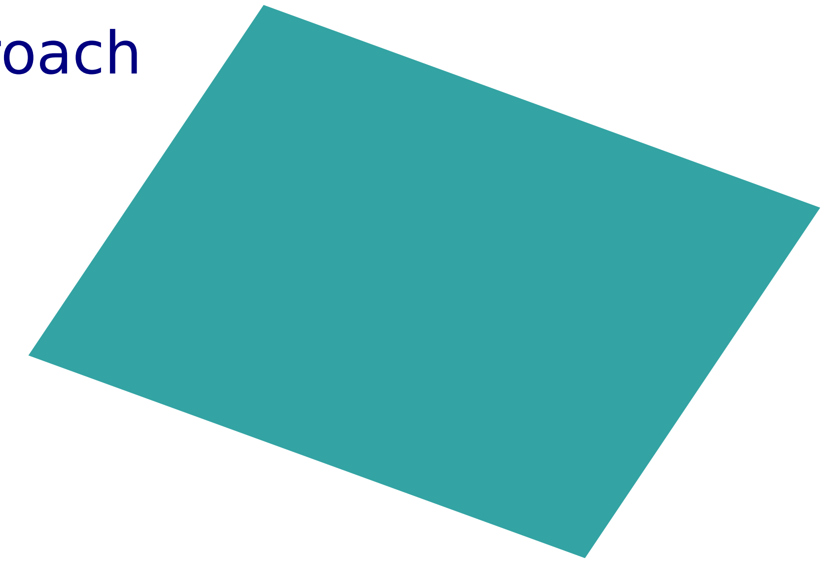
Lecture #1

1. Basics of quantum transport theory in disordered media

2. Ultracold atoms and speckle potentials
   Specific features of a controlled disorder

3. Anderson localization in 1D speckle potentials
   3.1 Effective mobility edges
   3.2 Expansion of Bose-Einstein condensates (exp. vs theory)

- Anderson localization : microscopic approach
  - Homogeneous (underlying) medium
    - $|\mathbf{k}\rangle$ states
    - characterized by $\varepsilon(\mathbf{k})$ and $\mathbf{v}=\partial\varepsilon(\mathbf{k})/\partial\hbar\mathbf{k}$



Anderson localization : microscopic approach

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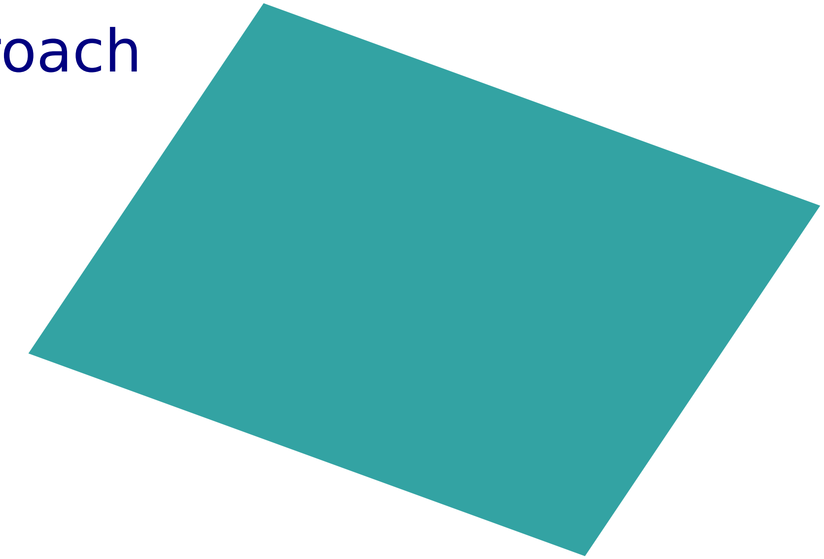

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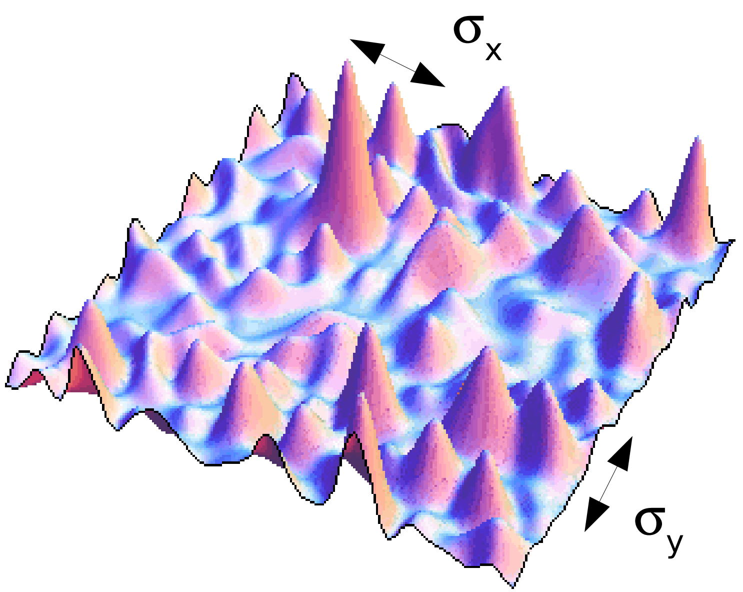
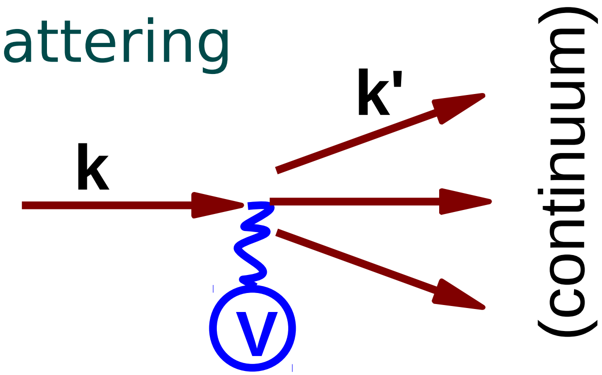
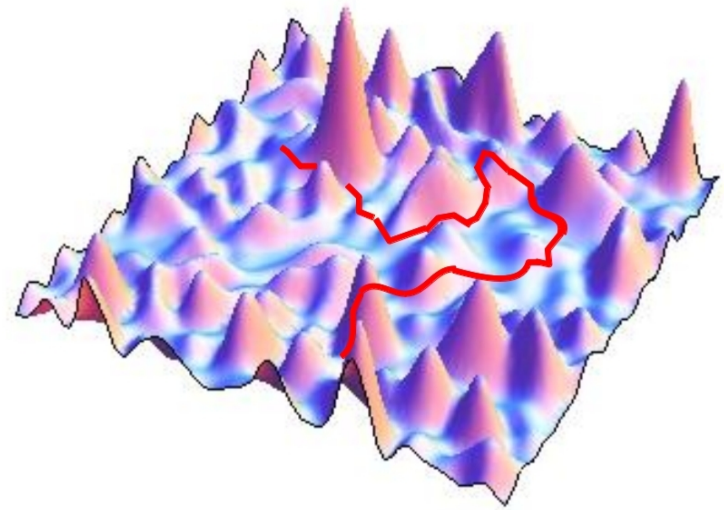
- Disorder
  - $\overline{V}=0$ (sets the zero of energy)
  - characterized by $C_n(\mathbf{r}_1, \ldots, \mathbf{r}_n)=\overline{V(\mathbf{r}_1) \ldots V(\mathbf{r}_n)}$
  - most importantly $C_2(\mathbf{r})=V_R{}^2c_2(\{\mathbf{z}_j/\sigma_j\})$ and its Fourier transform $C_2(\mathbf{k})$ (disorder power spectrum)
  - If c_2 is isotropic, $\zeta_{j,l}=\sigma_j/\sigma_l$ (anisotropy factors)



Basics of Transport Theory in Disordered Media

- Single scattering





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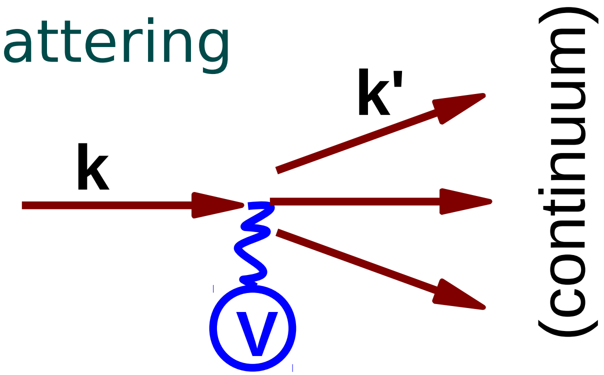
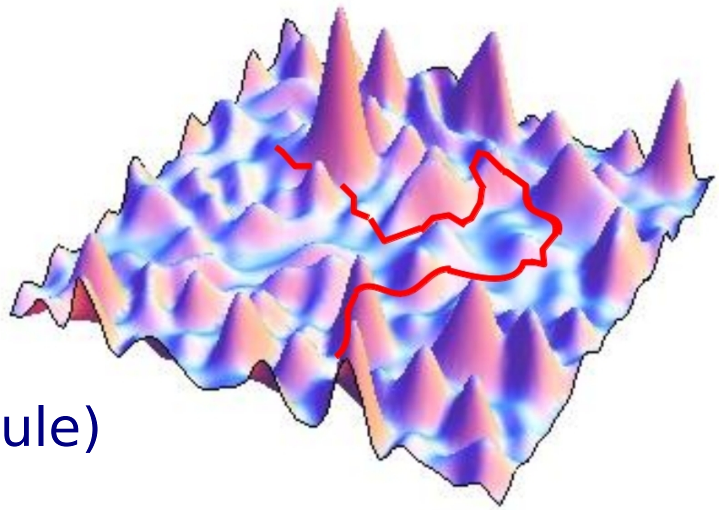


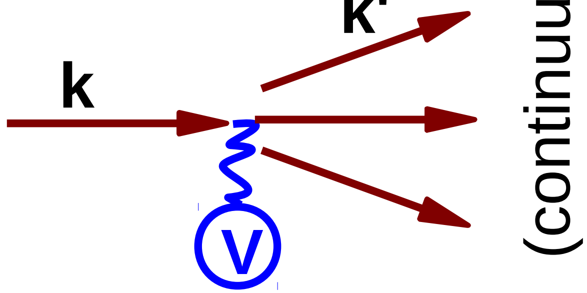


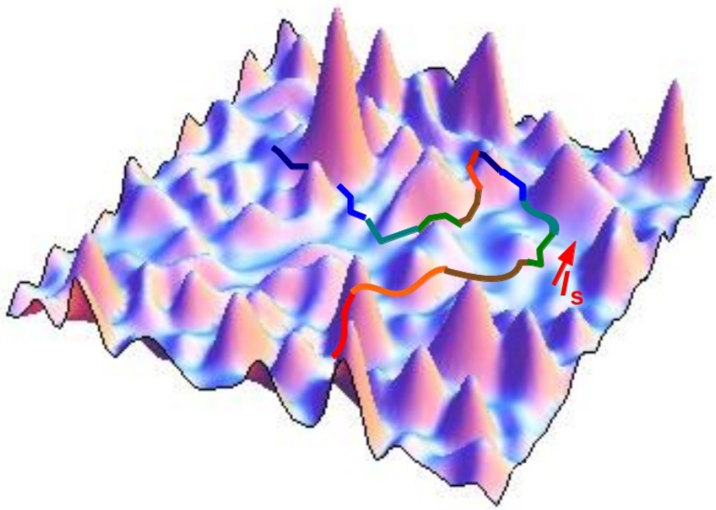
- finite life time τ_s of $|k\rangle$ states (Fermi golden rule)

$$\tau_s(E,\mathbf{k}) = \frac{\hbar}{2\pi} \frac{1}{\langle \tilde{C}(\mathbf{k}-\mathbf{k}') \rangle_{\mathbf{k}'|E}}$$

$$\langle ... \rangle_{\mathbf{k}'|E} = \int \frac{\mathrm{d}\mathbf{k}'}{(2\pi)^d} ... \delta\left[E - \epsilon(\mathbf{k}')\right]$$

- Single scattering





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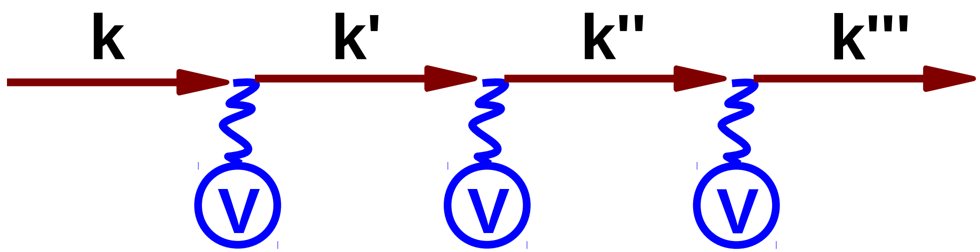
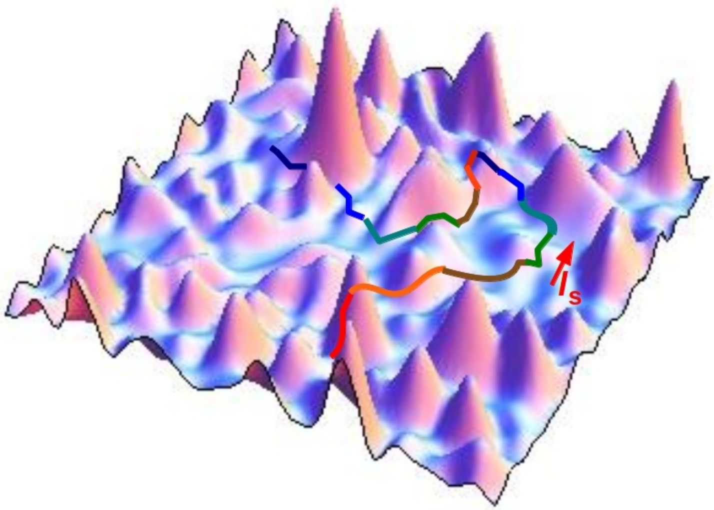
$$\langle\ldots\rangle_{\mathbf{k}'|E} = \int\frac{\mathrm{d}\mathbf{k}'}{(2\pi)^d}\ldots\delta\left[E-\epsilon(\mathbf{k}')\right]$$

scattering mean free path
$l_s(\mathbf{k}) = |\mathbf{v}|\,\tau_s(\mathbf{k})$

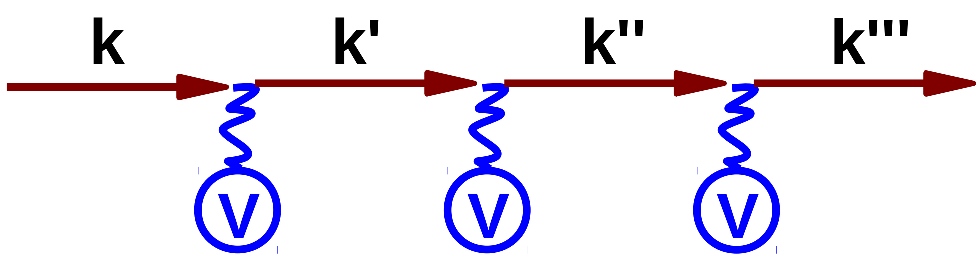
- In general C(**q**) decreases with |**q**| (maximum coupling **k**→**k**, *ie* forward scattering) ➔ τ_s : characteristic time of phase loss
- For white noise, C(**q**)=*cst* and the scattering is isotropic ➔ τ_s describes phase and direction loss

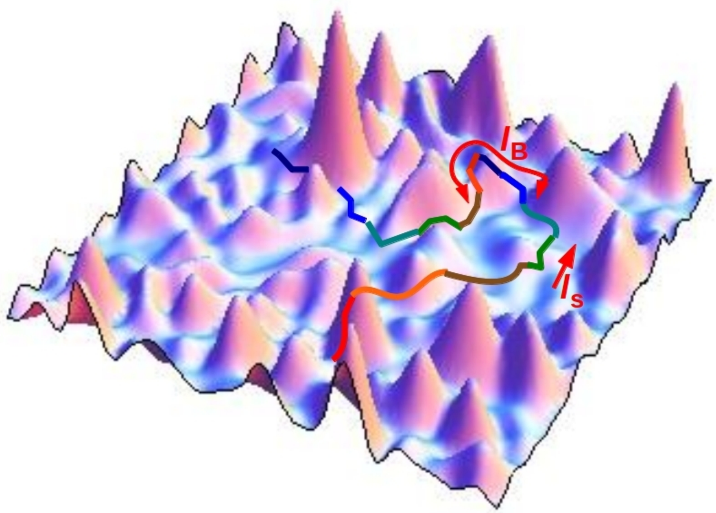
- Multiple scattering





- Multiple scattering



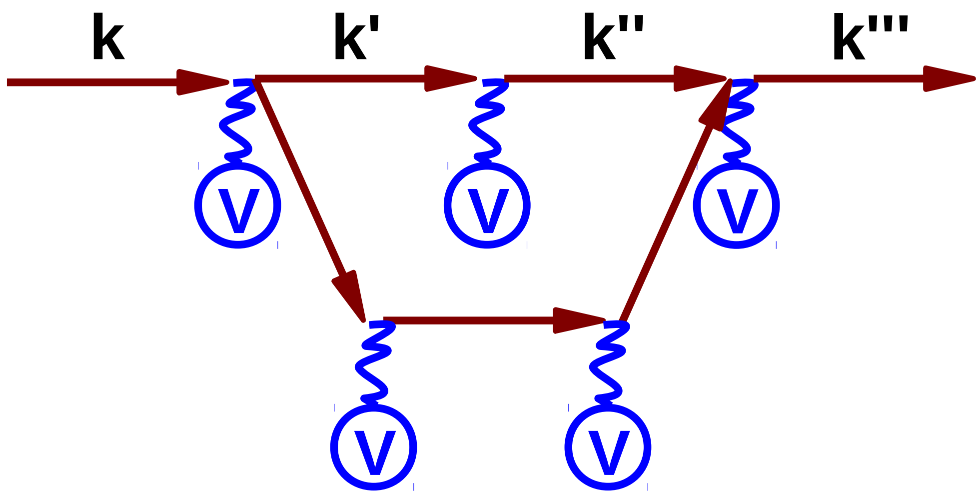


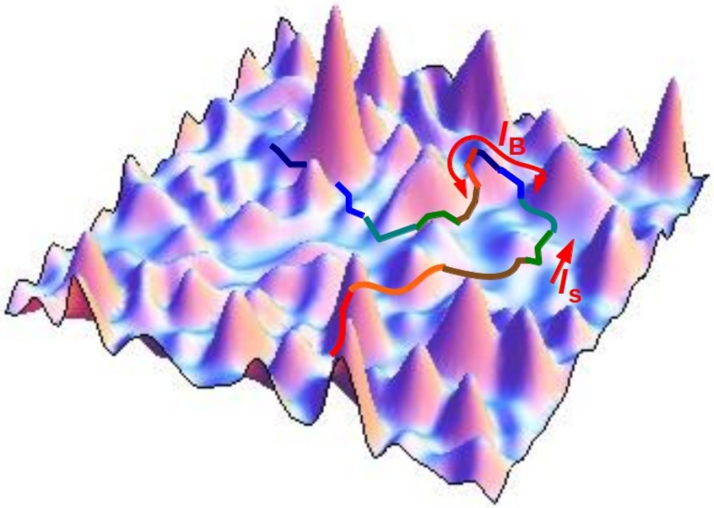
- deflection of of $|k\rangle$ states : random walk
  ➔ normal spatial diffusion, $D_B(E)$

transport mean free path
$D_B(E) = (1/d)\ |\mathbf{v}|\ l_B(\mathbf{E})$

- In general $C(\mathbf{q})$ decreases with $|\mathbf{q}|$ (maximum coupling $\mathbf{k}\rightarrow\mathbf{k}$, *ie* forward scattering) ➔ $l_B(E) > l_s(E)$
- For white noise, $C(\mathbf{q})=cst$ and the scattering is isotropic ➔ τ_s describes phase and direction loss, and $l_B(E) = l_s(E)$

- Interference effects





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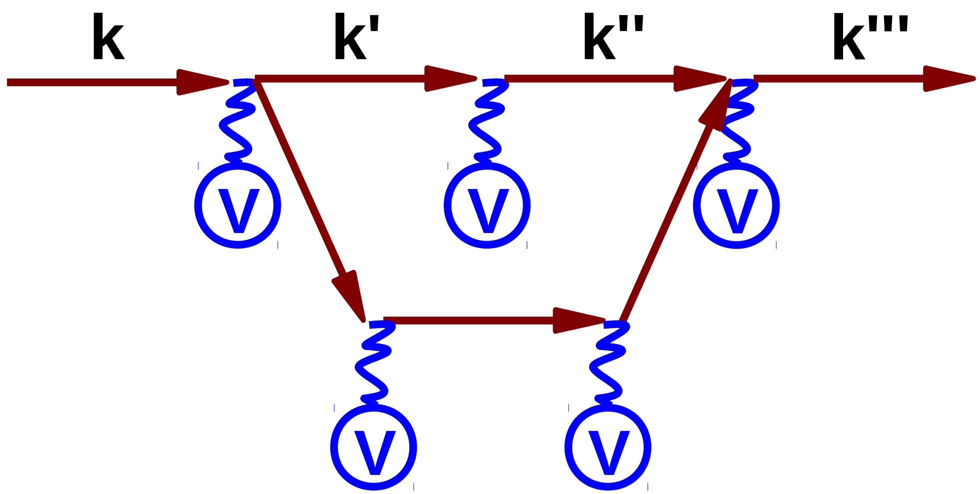
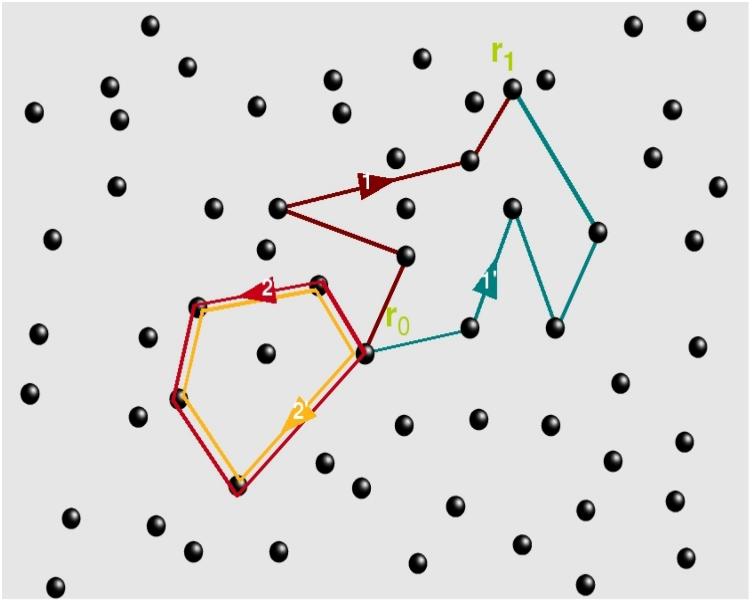
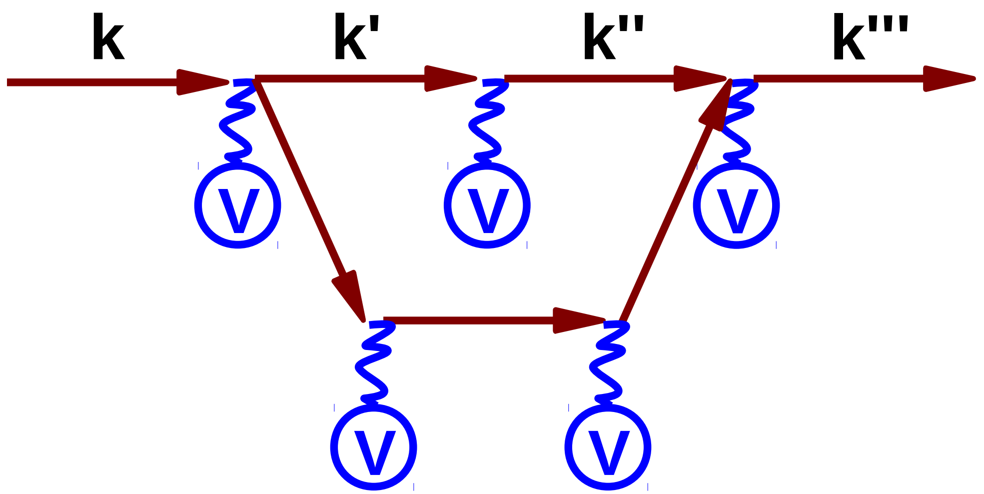
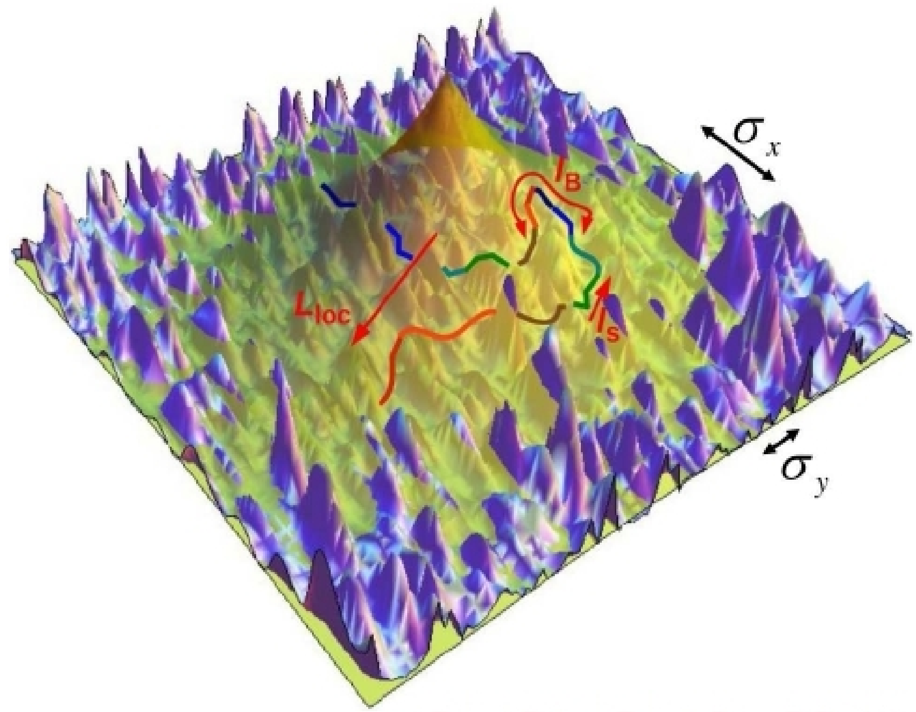




Basics of Transport Theory in Disordered Media

- Interference effects





decreases with
- d (return prob.)
- $k\, l_B$ (interf. param.)

localization length
$\gamma(E) = L_{loc}(E)^{-1} = l_B^{-1} F_d(k\, l_B)$

- weak localization ➔ $D_*(E) < D_B(E)$: diffusion
- strong (Anderson) localization ➔ $D_*(E) \sim -i\omega L_{loc}(E)^2$
  exponential localization

- Anderson localization : scaling theory
  - dimensionless conductance : g=0 (insulator) ; g= ∞ (metal)
  - RG based on the fundamental « on-parameter scaling » hypothesis : $d\ln(g)/d\ln(L)=\beta(g)$

P.W. Anderson, Phys. Rev. **109**, 1492 (1958)
Edwards and Thouless, J. Phys. C **5**, 807 (1972)
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Basics of Transport Theory in Disordered Media

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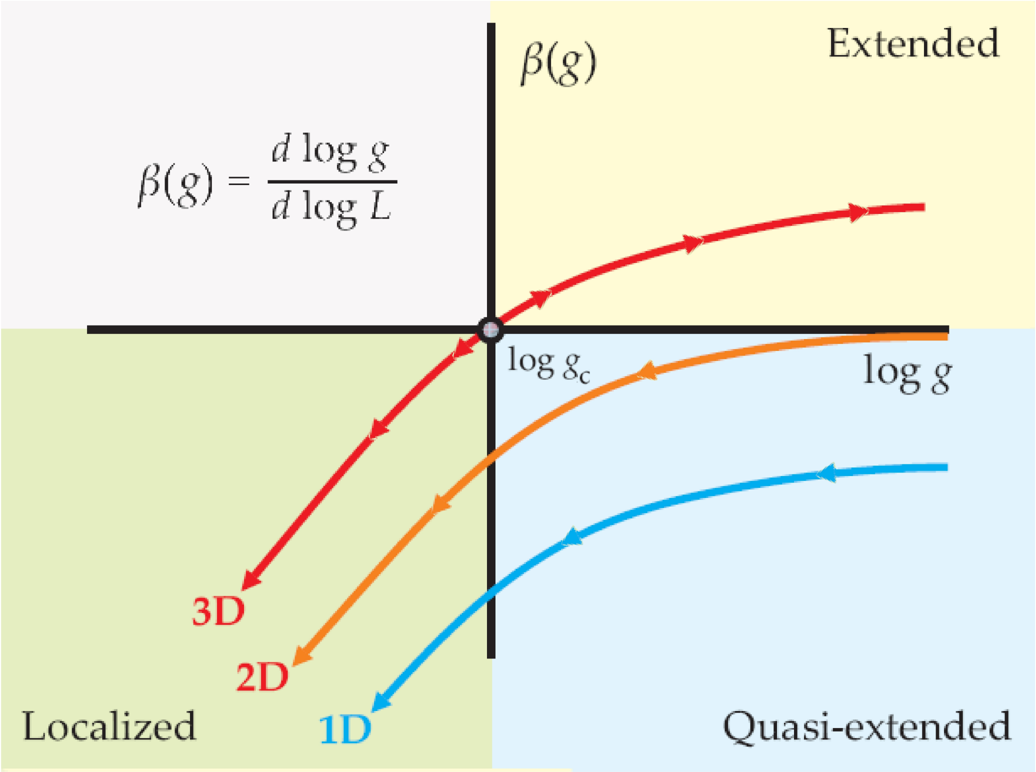
1D : no diffusion & all states localized
2D : all states localized
3D : mobility edge, kl*~1



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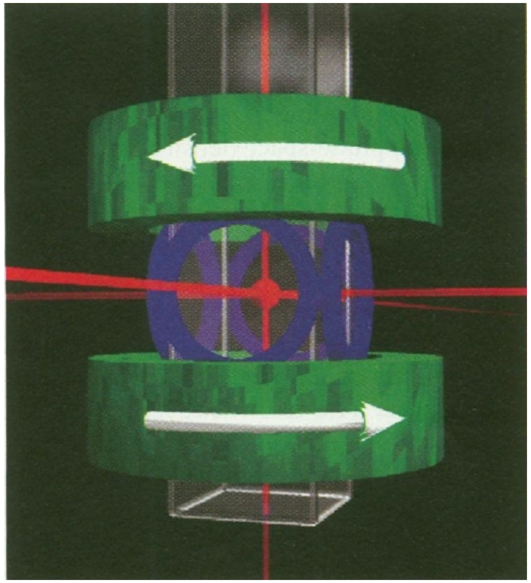
Condensed Matter with Ultracold Atoms

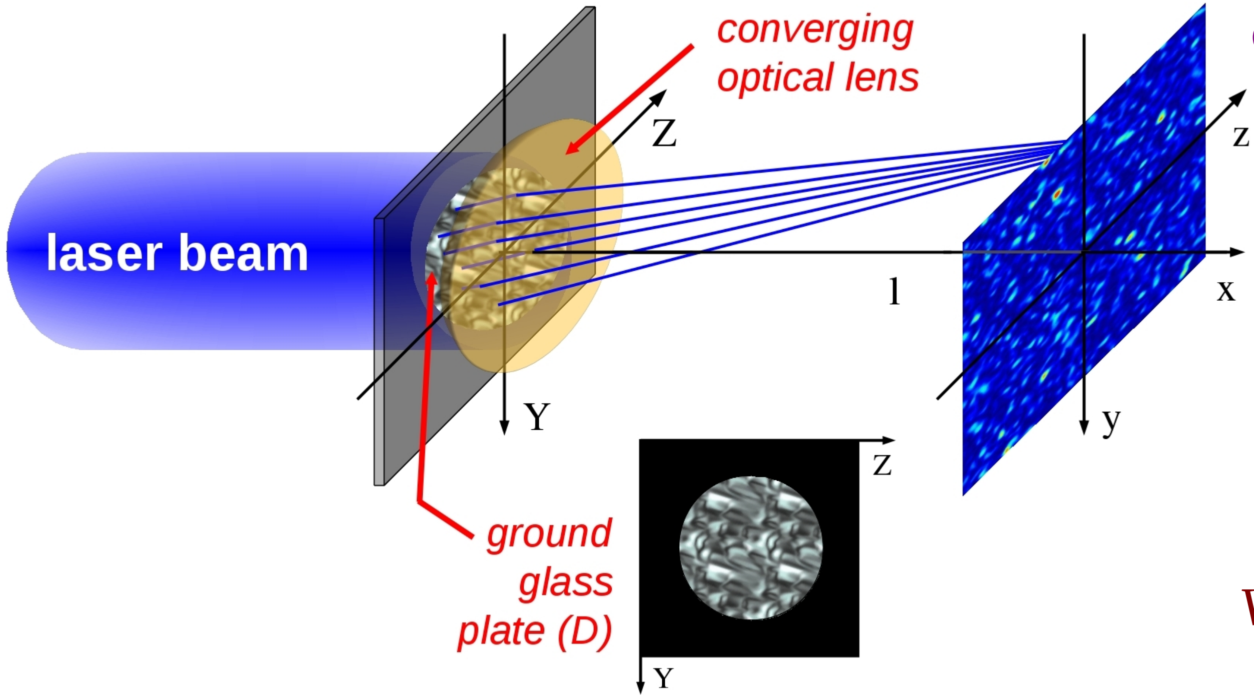


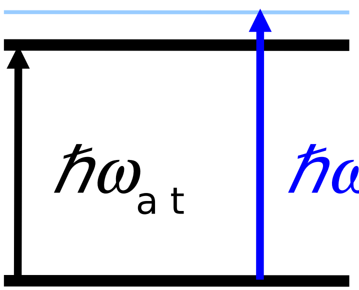
- Flexibility
  - control of parameters (dimensionality, shape, interactions, bosons/fermions/mixtures, ...)
- High-precision measurements
  - complementary to condensed-matter tools
  - variety of diagnostic tools (direct imaging, velocity distribution, oscillations, Bragg spectroscopy, ...)
- Model systems
  - parameters known ab-initio ⇒ direct comparison with theory
  - towards quantum simulators ⇒ artificial, controlled systems to study fundamental questions

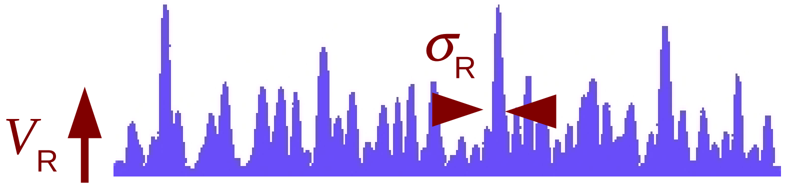
specific ingredients of ultracold atoms ...

Controlled Optical Disorder

J. W. Goodman, *Speckle Phenomena in Optics* (2007)
Clément *et al.*, New J. Phys. **8**, 165 (2006)







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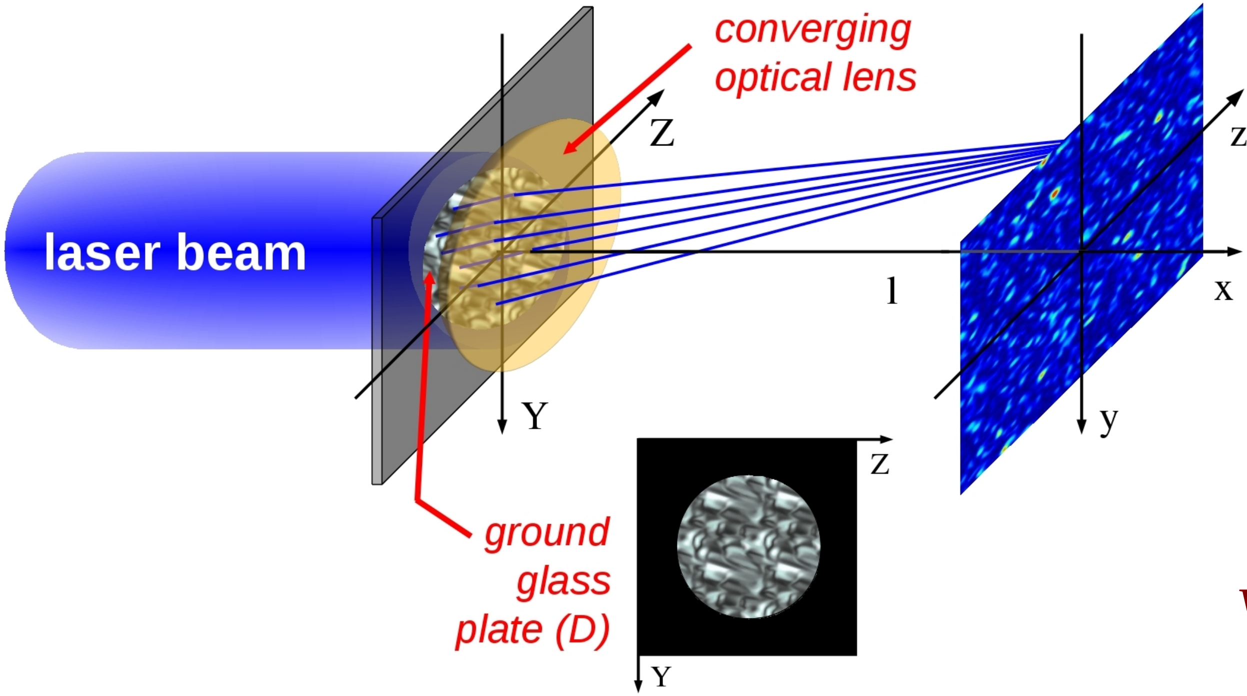

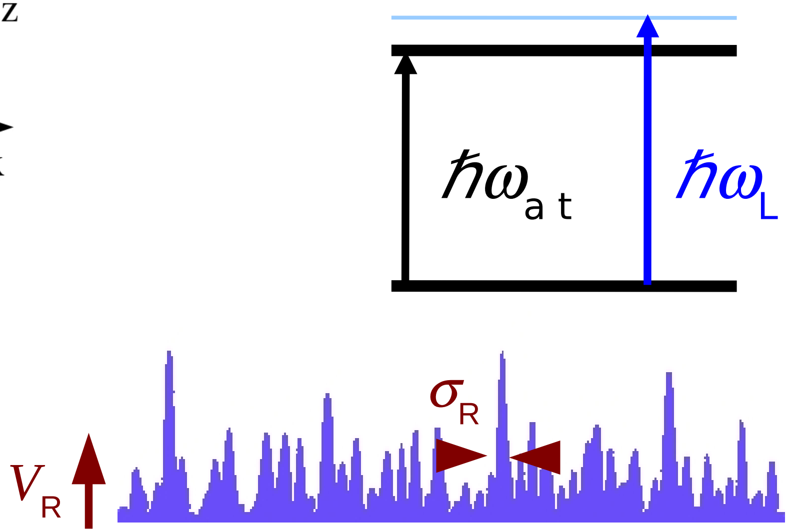

Speckles: an original class of disorder

- $\mathcal{E}(\boldsymbol{r})$ is a Gaussian random process
- $V(\boldsymbol{r}) \propto |\mathcal{E}(\boldsymbol{r})|^2 - \overline{|\mathcal{E}|^2}$ is not Gaussian
  is not symmetric
- $C_n(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_n) = \overline{V(\boldsymbol{r}_1) \ldots V(\boldsymbol{r}_n)}$ all
  determined by $C_{\mathcal{E}}(\boldsymbol{r}) = \overline{\mathcal{E}(\boldsymbol{r})^* \mathcal{E}(\boldsymbol{0})}$

Experimental control

$$C_2(\boldsymbol{r}) = \overline{V(\boldsymbol{r})V(0)} - \overline{V}^{\,2}$$

- $V_R \propto I_L / \Delta$
- σ_R: correlation length
  (depends only on the intensity
  profile of the ground-glass plate)
- $C_2(\mathbf{r})$ can be tailored almost at will

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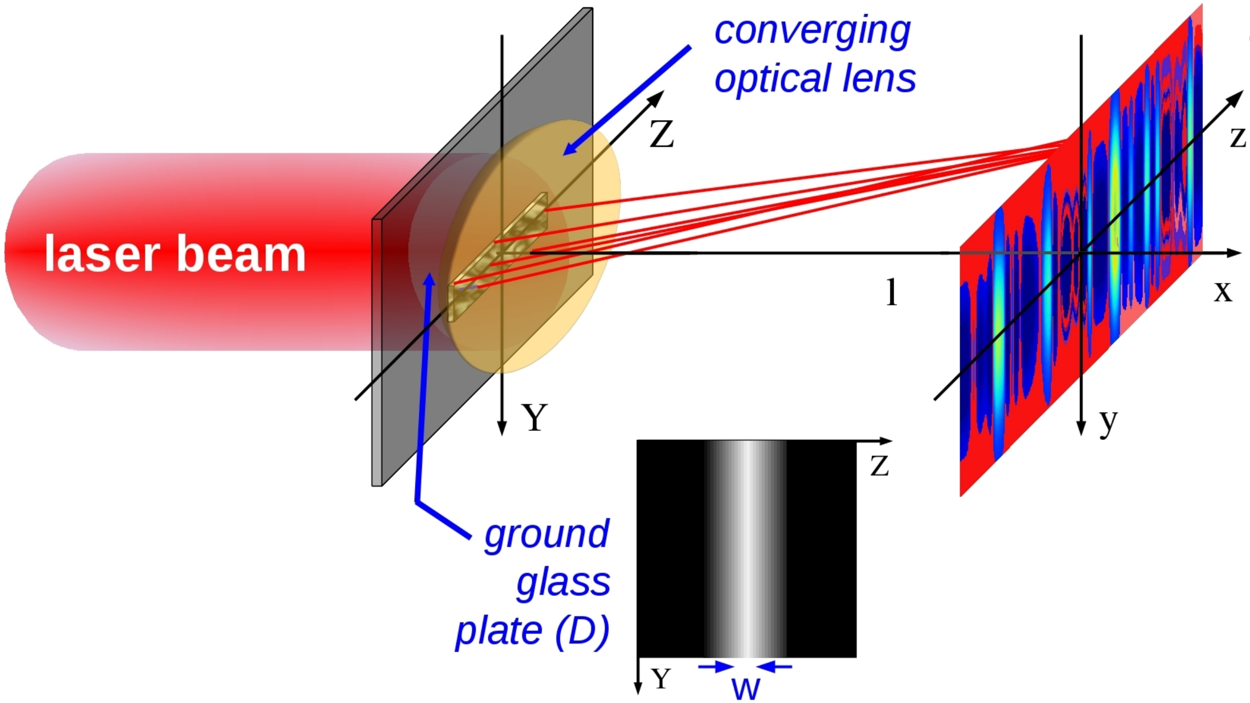

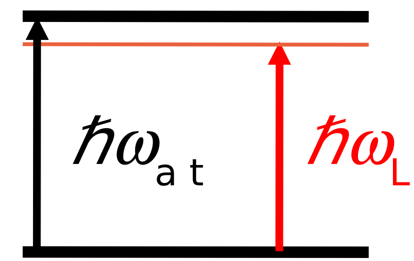

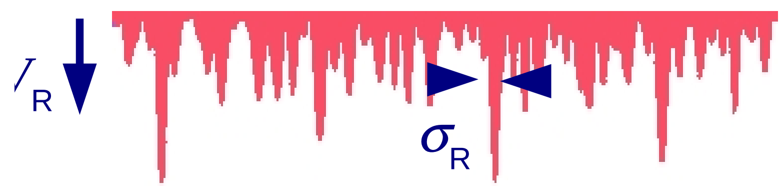

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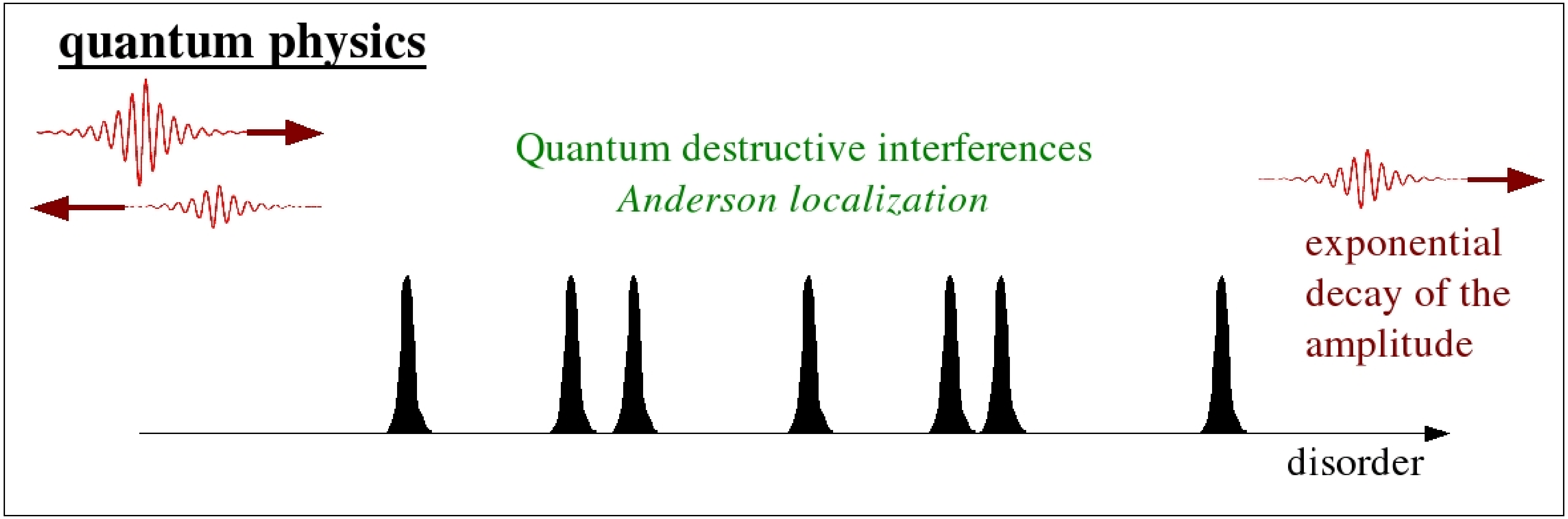

$$\ln(|\psi(z)|/|\psi(0)|) \sim -|z|/L_{loc}$$

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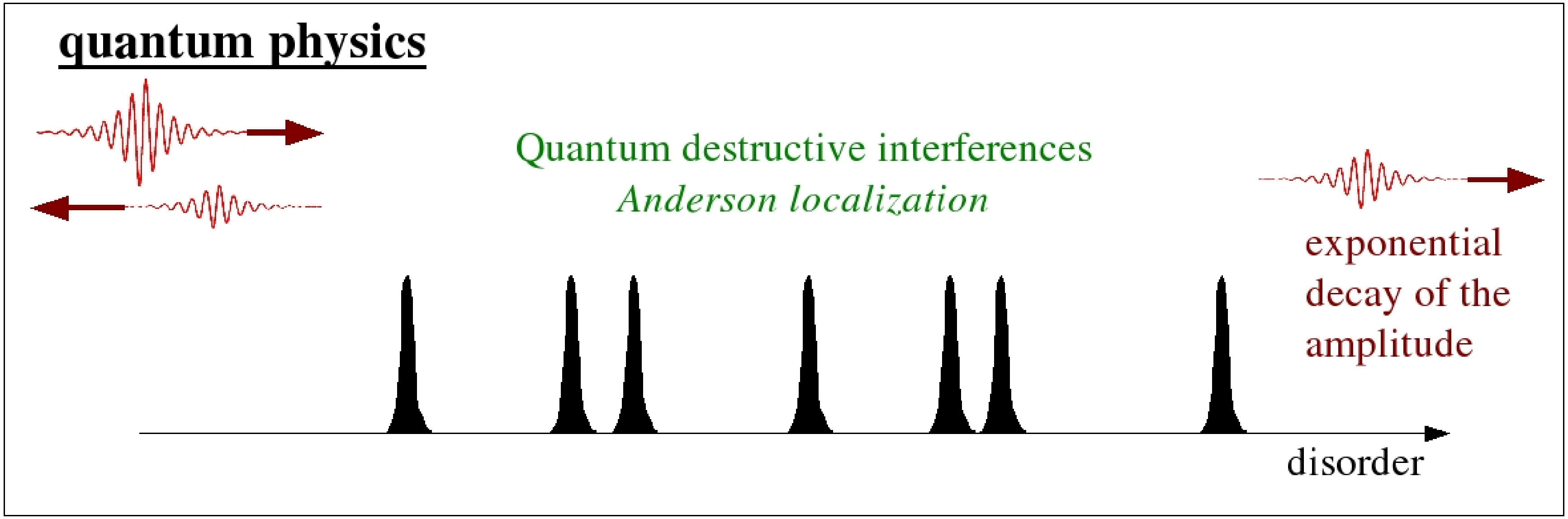

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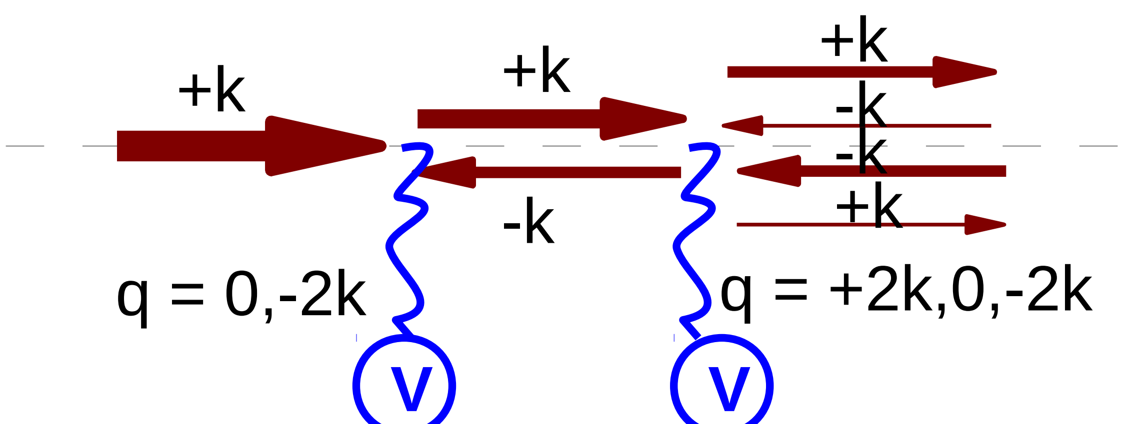

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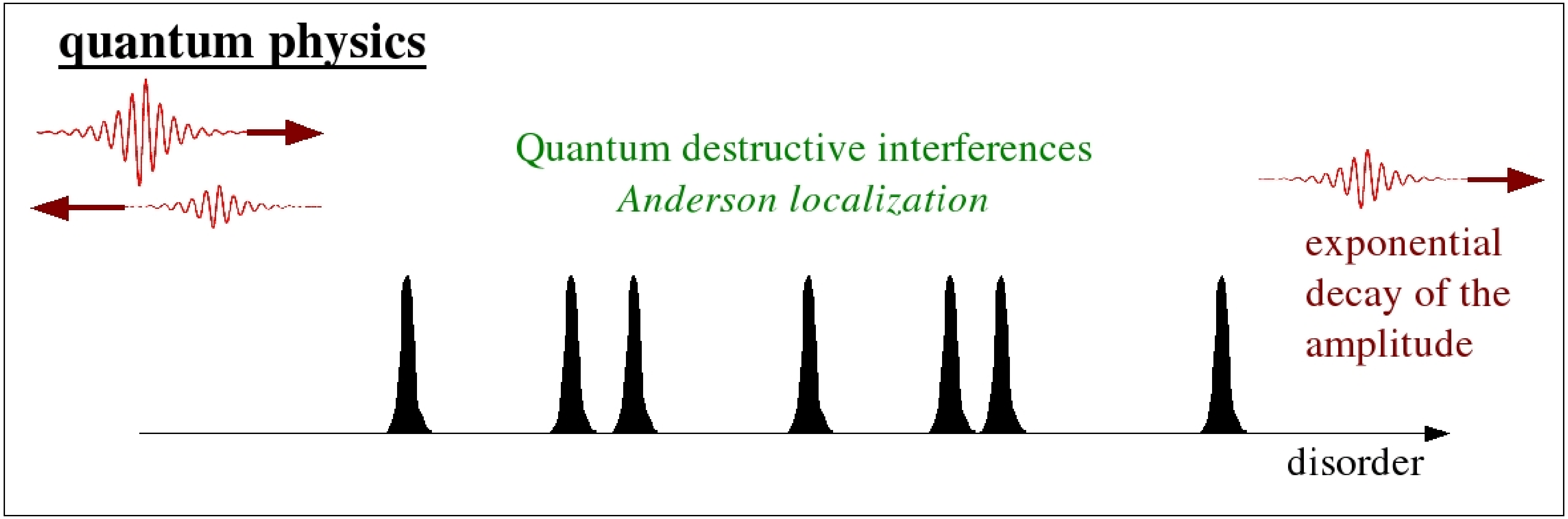
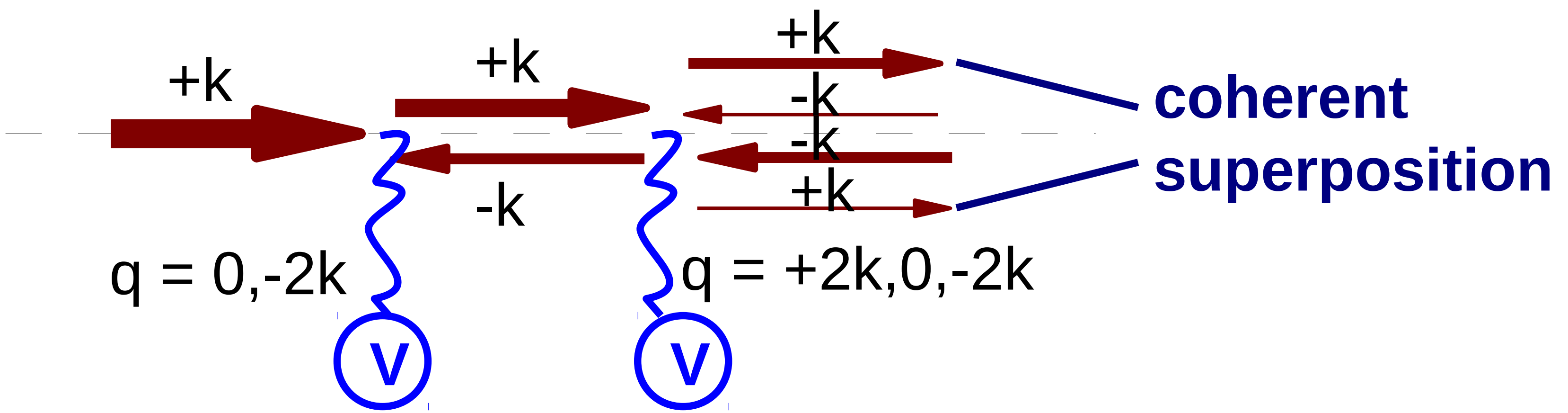


$\ln(|\psi(z)|/|\psi(0)|) \sim -|z|/L_{loc}$

Quantity of interest:

$C_2(2k)=\overline{V(+2k)V(-2k)} \neq 0$



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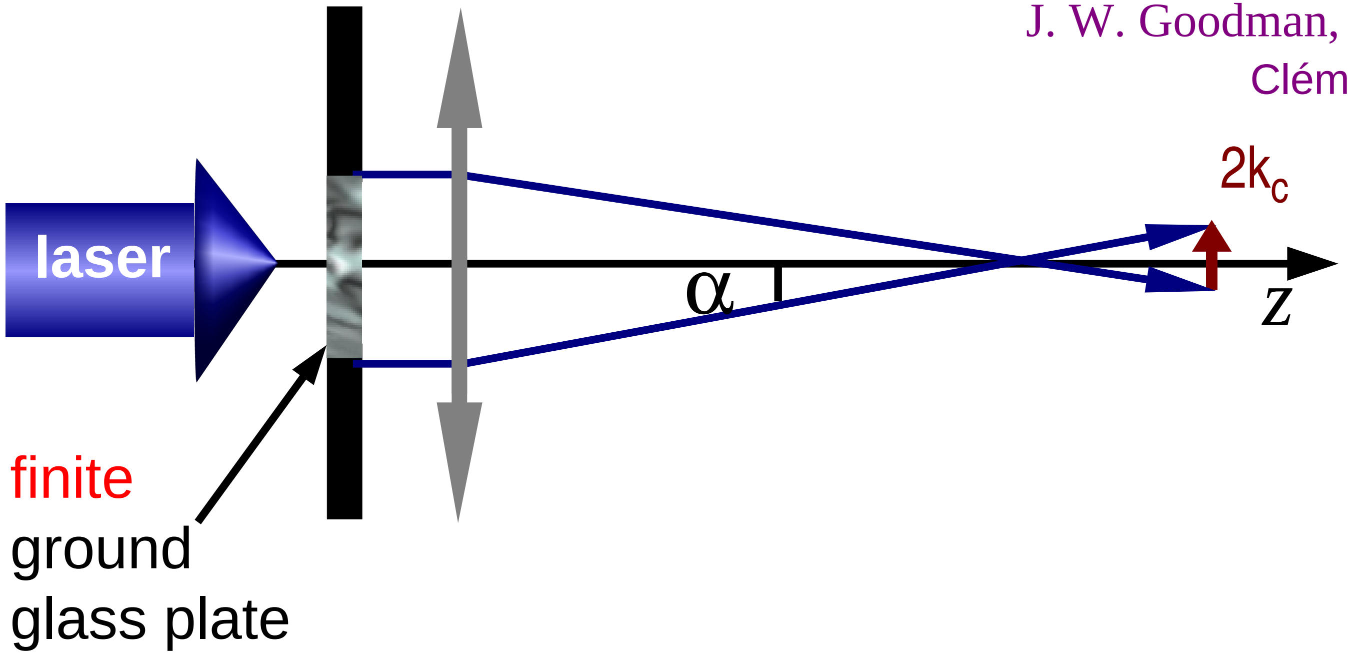
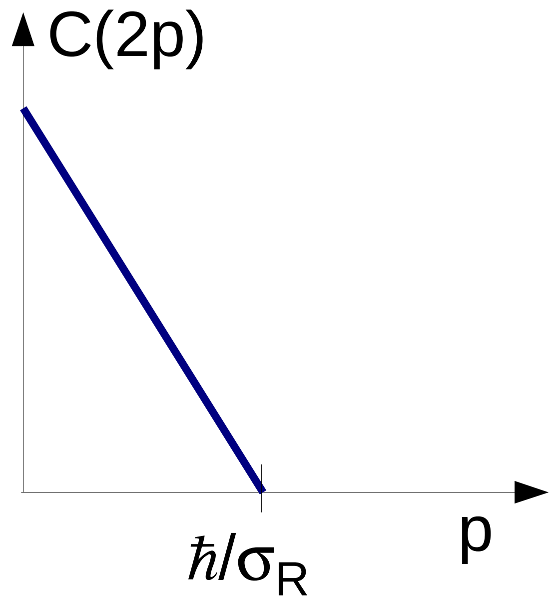




Potential cut-off:

$C_{\mathcal{E}}(k)=0$ for $k>k_c$

$C_2(2k)=0$ for $k>k_c$

$k_c=(2\pi/\lambda_L)\sin\alpha \equiv \sigma_R^{-1}$

$$\tilde{C}(p) = \pi V_R^2 \sigma_R [1 - |p|\sigma_R/2\hbar]_{\oplus}$$

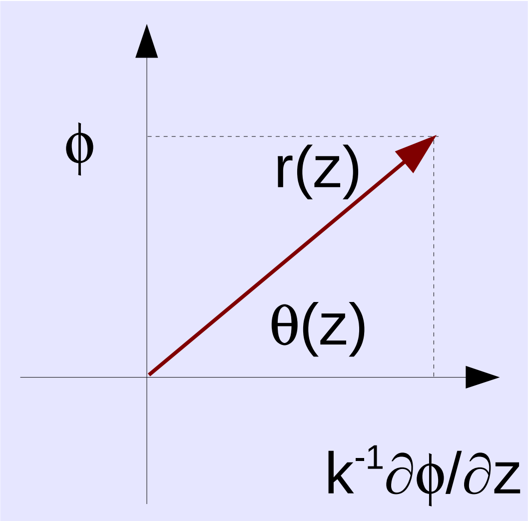
Anderson Localization in 1D Speckle Potentials

- Perturbation theory in the phase formalism

$$\phi(z) = r(z)\sin[\theta(z)]; \quad \partial_z\phi = kr(z)\cos[\theta(z)]$$

$$\partial_z\theta(z) = k\{1 - [V(z)/E]\sin^2[\theta(z)]\}$$

$$\ln[r(z)/r(0)] = k\int_0^z dz'[V(z')/2E]\sin[2\theta(z')]$$



k_E is defined by
$E = \hbar^2 k_E^2/2m = p_E^2/2m$

Lifshits, Gredeskul & Pastur, *Introduction to the Theory of Disordered Systems* (1988)

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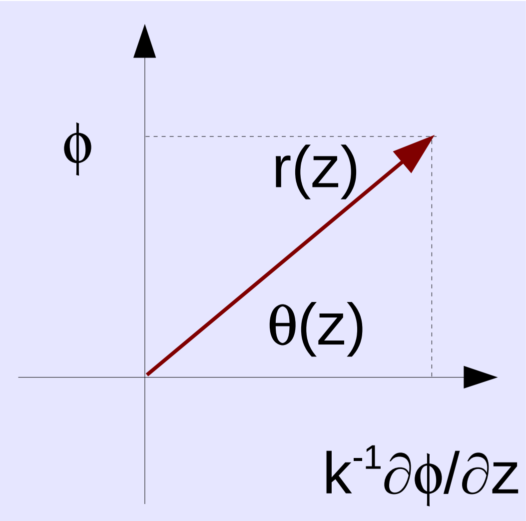

$$\partial_z\theta(z) = k\{1 - [V(z)/E]\sin^2[\theta(z)]\}$$

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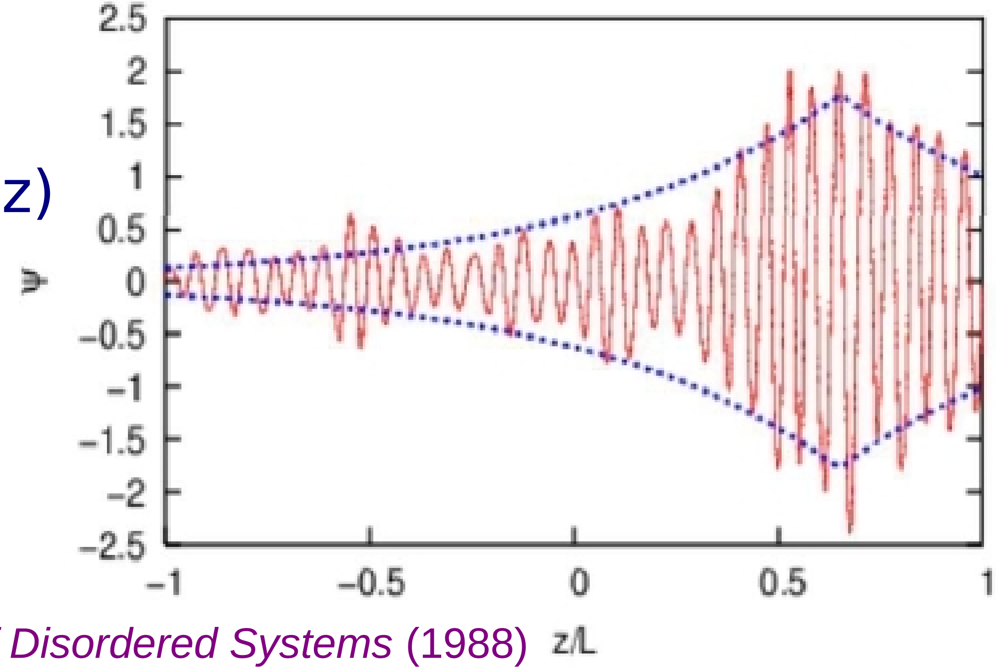
k_E is defined by
$E = \hbar^2 k_E^2/2m = p_E^2/2m$

- $\theta(z) = kz + \delta\theta^{(1)}(z) + \delta\theta^{(2)}(z) + \ldots$
- Include $\theta(z)$ into equation for $r(z)$

$$\gamma(k) = \lim_{|z|\to\infty}\frac{\langle\ln[r(z)]\rangle}{|z|} = \sum_{n\geq 2}\gamma^{(n)}(k)$$



Lifshits, Gredeskul & Pastur, *Introduction to the Theory of Disordered Systems* (1988)

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Anderson Localization in 1D Speckle Potentials

Perturbation theory

$$\gamma^{(n)}(k) = \sigma_{\mathrm{R}}^{-1} \left(\frac{\epsilon_{\mathrm{R}}}{k\sigma_{\mathrm{R}}} \right)^n f_n(k\sigma_{\mathrm{R}})$$

where $\epsilon_{\mathrm{R}} = 2m\sigma_{\mathrm{R}}^2 V_{\mathrm{R}}/\hbar^2$

determined by the
n-point correlation function

LSP *et al.*, Phys. Rev. Lett **98**, 210401 (2007)
P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)
E. Gurevich and O. Kenneth, Phys. Rev. A **79**, 063617 (2009)

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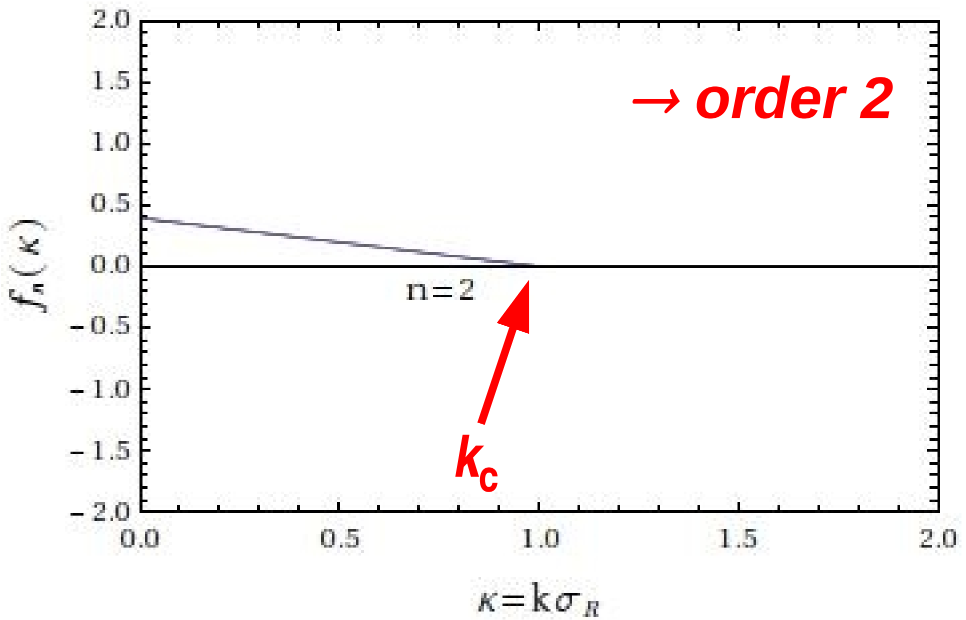

Perturbation theory

$$\gamma^{(n)}(k) = \sigma_R^{-1} \left(\frac{\epsilon_R}{k\sigma_R} \right)^n f_n(k\sigma_R)$$

where $\epsilon_R = 2m\sigma_R^2 V_R / \hbar^2$

determined by the
n-point correlation function

- Order 2
  - cut-off at k_c → defines an effective mobility edge
  - no difference between $V_R > 0$ and $V_R < 0$

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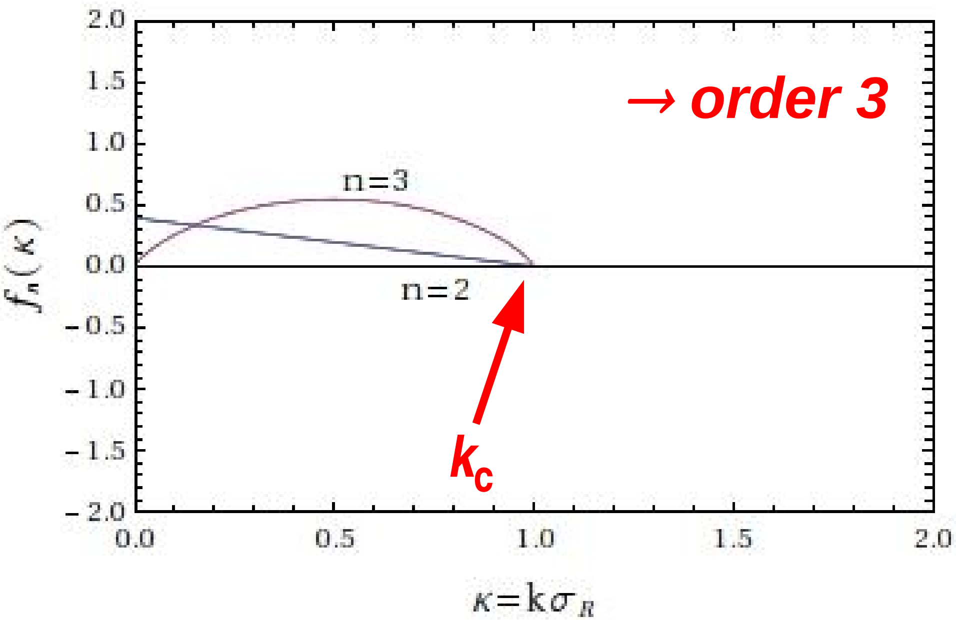

<u>*Perturbation theory*</u>

$$\gamma^{(n)}(k) = \sigma_R^{-1} \left(\frac{\epsilon_R}{k\sigma_R} \right)^n f_n(k\sigma_R)$$

where $\epsilon_R = 2m\sigma_R^2 V_R / \hbar^2$

determined by the n-point correlation function

- Order 3
  - same cut-off at k_c
  - opposite correction value of $\gamma^{(3)}$ for $V_R > 0$ and $V_R < 0$
  - this term can be relevant for experiments, see M. Piraud *et al.*, Phys. Rev. A 85, 063611 (2012)

LSP *et al.*, Phys. Rev. Lett **98**, 210401 (2007)
P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)
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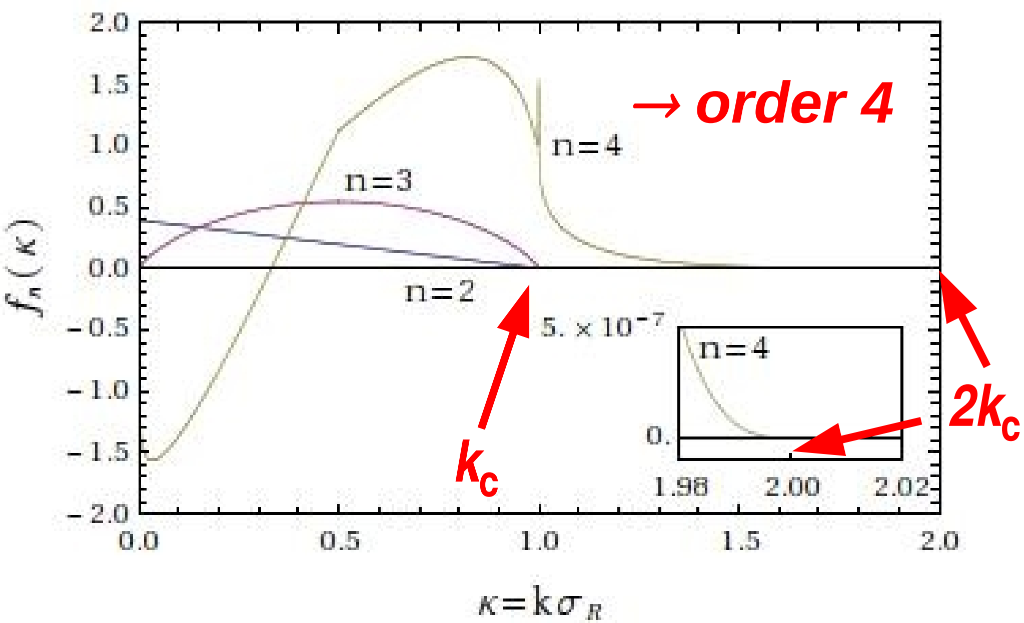


Perturbation theory

$$\gamma^{(n)}(k) = \sigma_{\mathrm{R}}^{-1} \left(\frac{\epsilon_{\mathrm{R}}}{k\sigma_{\mathrm{R}}}\right)^n f_n(k\sigma_{\mathrm{R}})$$

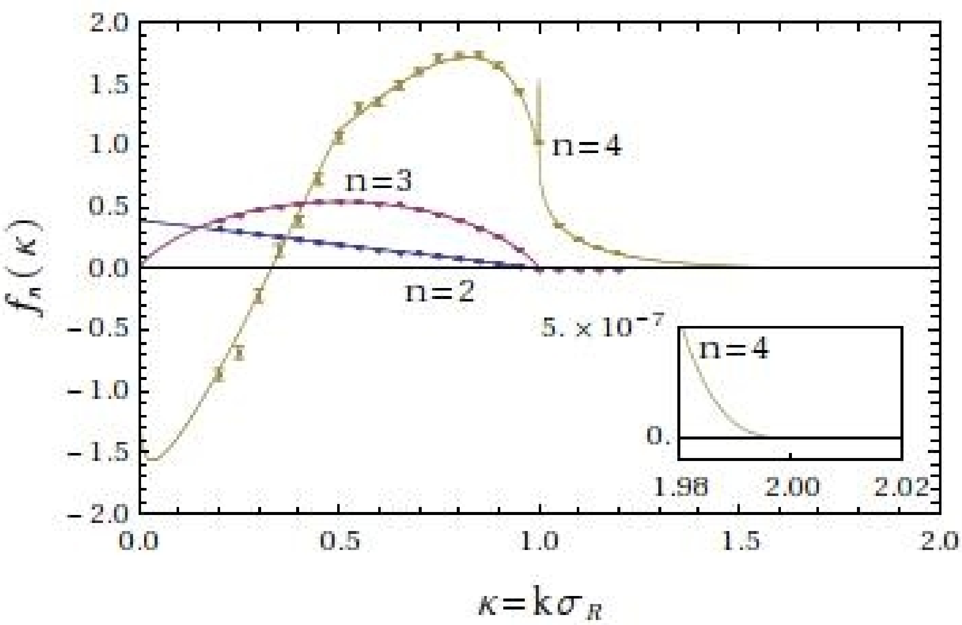
where $\epsilon_{\mathrm{R}} = 2m\sigma_{\mathrm{R}}^2 V_{\mathrm{R}}/\hbar^2$

determined by the n-point correlation function

- Order 4
  - new cut-off at $2k_c$ → defines a second effective mobility edge
  - no difference between $V_R > 0$ and $V_R < 0$
  - complicated behavior of $\gamma^{(4)}$

P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)
E. Gurevich and O. Kenneth, Phys. Rev. A **79**, 063617 (2009)

Anderson Localization in 1D Speckle Potentials



Perturbation theory

$$\gamma^{(n)}(k) = \sigma_R^{-1}\left(\frac{\epsilon_R}{k\sigma_R}\right)^n f_n(k\sigma_R)$$

where $\epsilon_R = 2m\sigma_R^2 V_R/\hbar^2$

determined by the n-point correlation function

- Orders 2 to 4
  - excellent agreement with exact numerical calculations (by D. Delande [P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)])

P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)
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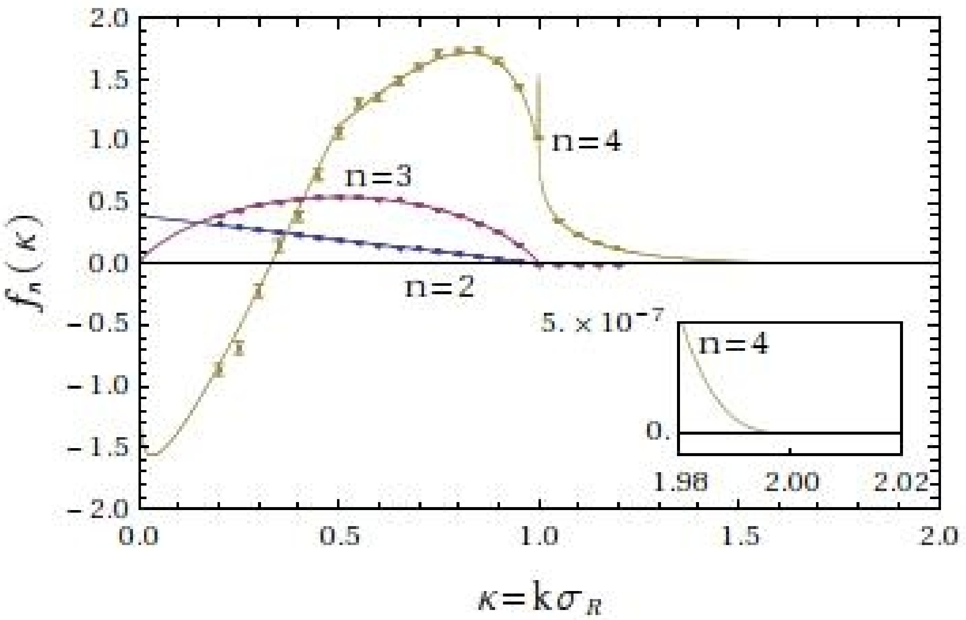

Perturbation theory

$$\gamma^{(n)}(k) = \sigma_{\mathrm{R}}^{-1}\left(\frac{\epsilon_{\mathrm{R}}}{k\sigma_{\mathrm{R}}}\right)^n f_n(k\sigma_{\mathrm{R}})$$

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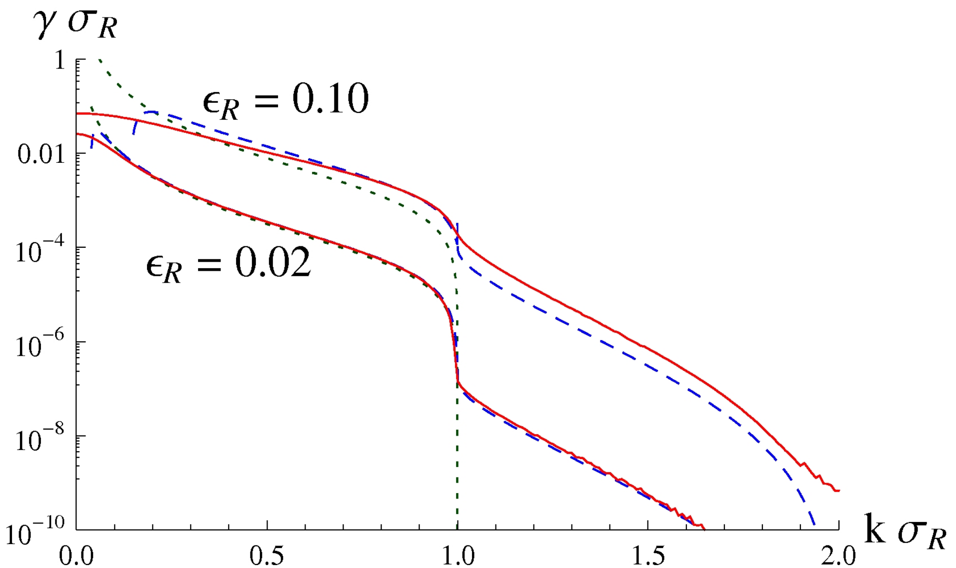

P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)
E. Gurevich and O. Kenneth, Phys. Rev. A **79**, 063617 (2009)

Outlook

Lecture #1

1. Basics of quantum transport theory in disordered media

2. Ultracold atoms and speckle potentials
   Specific features of a controlled disorder

3. Anderson localization in 1D speckle potentials
   3.1 Effective mobility edges
   3.2 Expansion of Bose-Einstein condensates (exp. vs theory)

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Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

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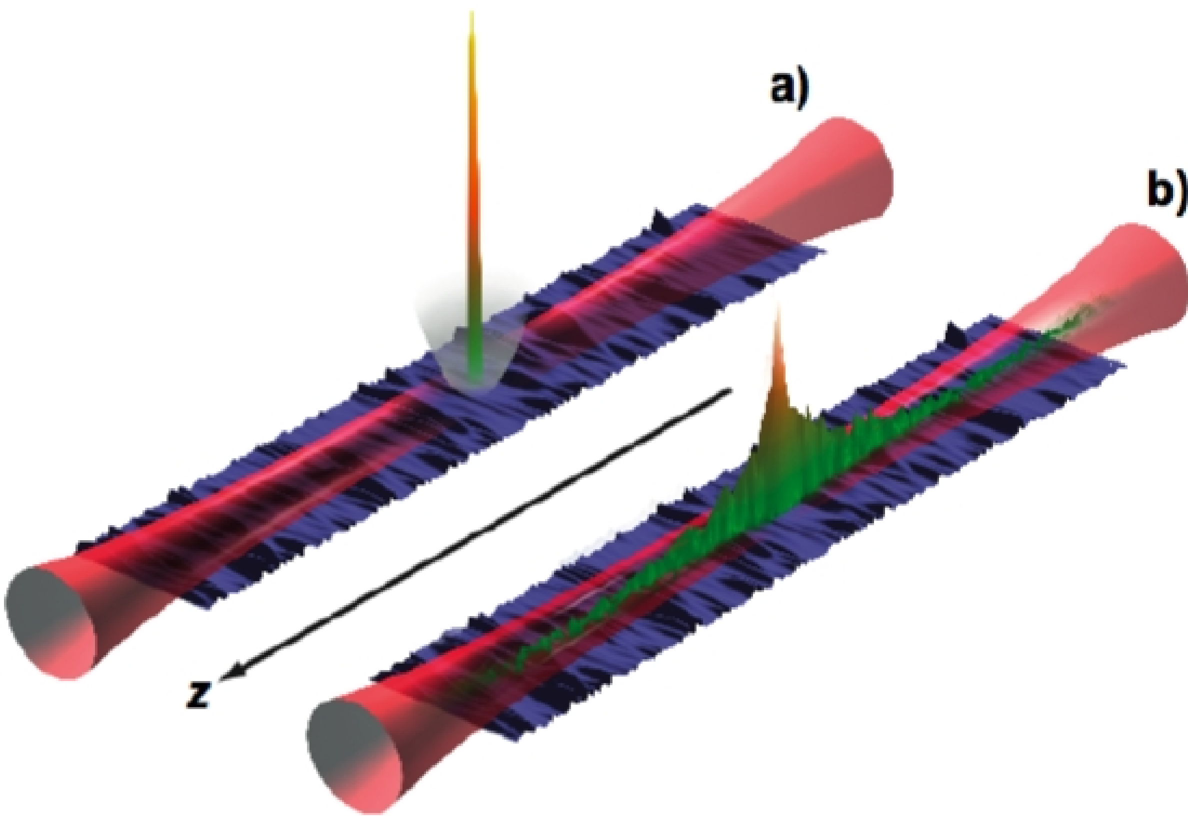
M. Piraud *et al.*, Phys. Rev. A **83**, 031603(R) (2011)
J. Billy *et al.*, Nature **453**, 891 (2008)
see also G. Roati *et al.*, Nature **453**, 895 (2008)



Superposition of matter waves with a broad energy distribution

$$n(z,t\to\infty) \simeq \int dE\ \mathcal{D}_E(E)\ P(z,t\to\infty|E)$$

Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

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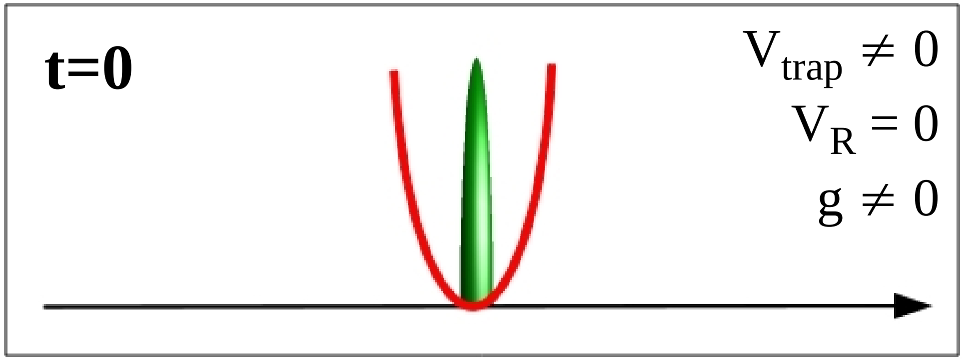

- Bose-Einstein condensate in a harmonic trap
  - Thomas-Fermi regime :

$$n_0(z) \propto 1 - (z/L_{\mathrm{TF}})^2$$

$$\theta(z) \equiv 0$$

Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

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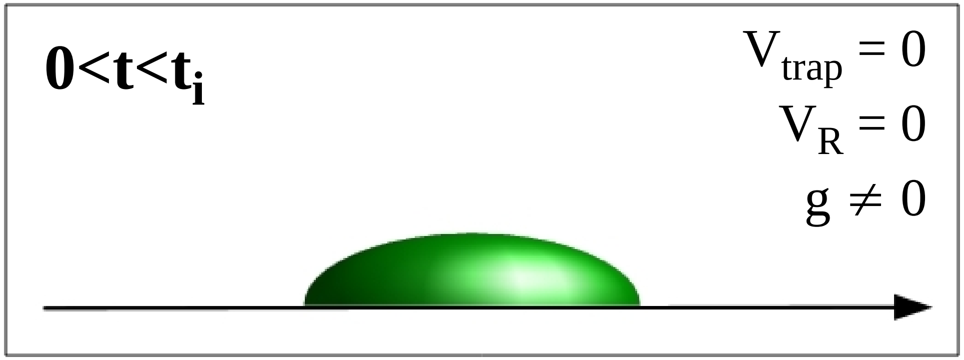
M. Piraud *et al.*, Phys. Rev. A **83**, 031603(R) (2011)



- Interaction-driven expansion of the BEC
  - Scaling theory of expanding BEC (in TF regime)
  - Momentum distribution :

$$\mathcal{D}_{\mathrm{i}}(p) \propto \left[1 - (p\xi_{\mathrm{in}}/\hbar)^2\right]_{\oplus}$$

$$\xi_{\mathrm{in}} \equiv \hbar/\sqrt{4m\mu}$$

Yu. Kagan, E. L. Surkov, and G.V. Shlyapnikov, Phys. Rev. A **54**, R1753 (1996)
Y. Castin and R. Dum, Phys. Rev. Lett. **77**, 5315 (1996)

Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

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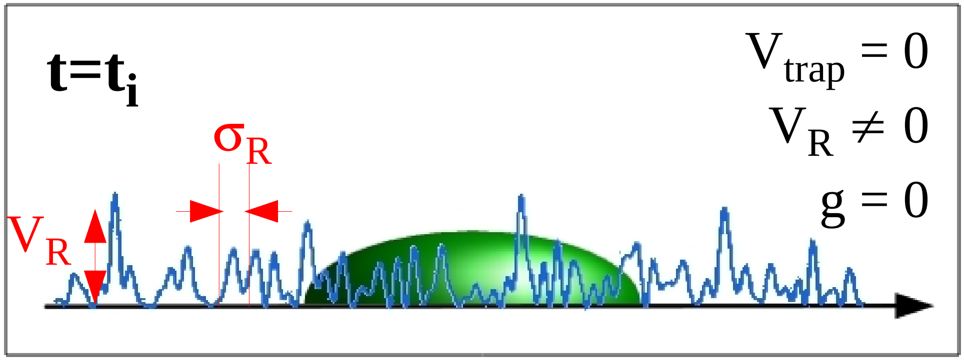

- Interactions off and disorder on
  - « p » is no longer a good quantum number but « E » is !

$$\mathcal{D}_E(E) = \int dp\ A(p,E)\mathcal{D}_{\rm i}(p)$$

Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

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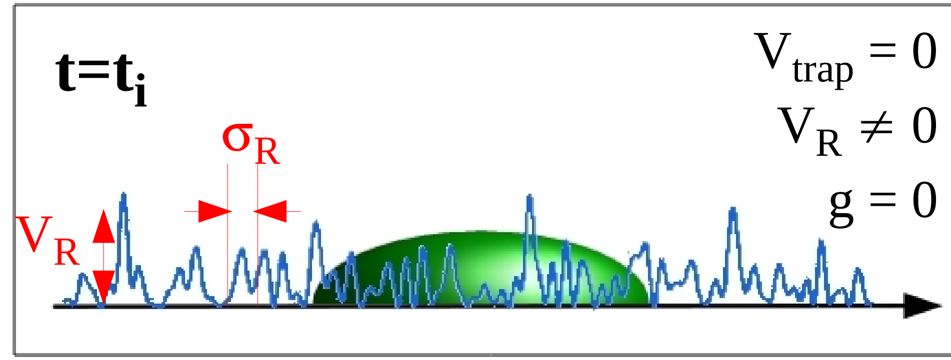
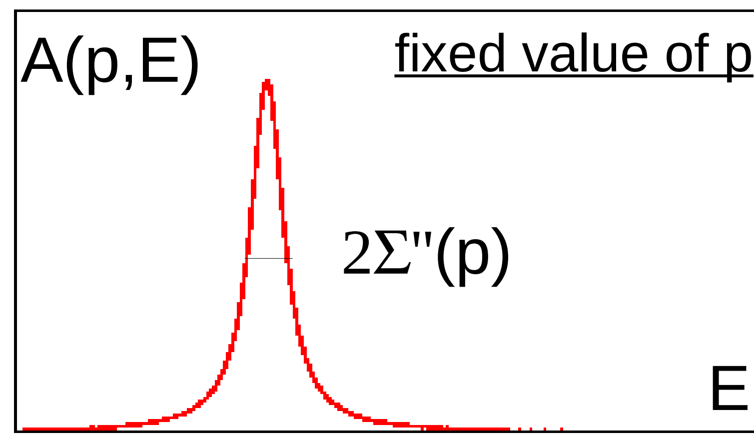


- Interactions off and disorder on
  - « p » is no longer a good quantum number but « E » is !

$$\mathcal{D}_E(E) = \int dp\, A(p,E)\mathcal{D}_{\rm i}(p)$$

spectral function



FT of the correlation function of the disorder

$$\Sigma''(p) \simeq -(m/2\hbar p)\{\tilde{C}(0) + \tilde{C}(2p)\}$$

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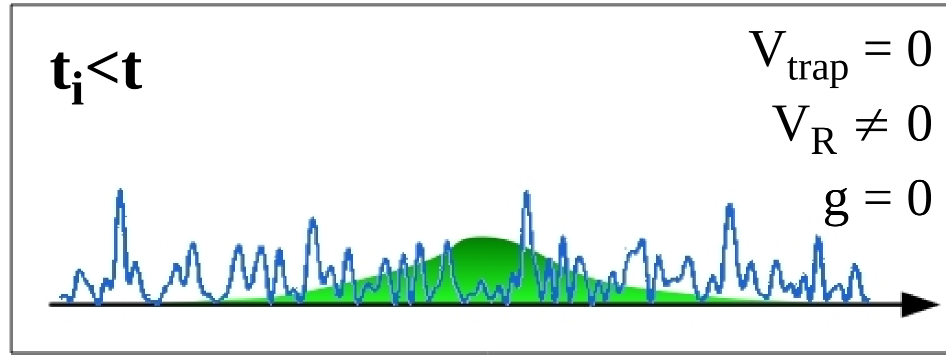

- Localization of the wave packet

$$P_\infty(z|E) = \frac{\pi^2\gamma(E)}{8}\int_0^\infty \mathrm{d}u\ u\sinh(\pi u)\left[\frac{1+u^2}{1+\cosh(\pi u)}\right]^2 \times\exp\left\{-(1+u^2)\gamma(E)|z|/2\right\},$$

<u>short distance :</u> ln(P) ~ $-2\gamma(E)|z|$
<u>long distance :</u> ln(P) ~ $-0.5\gamma(E)|z|$

V.L. Berezinskii, Sov. Phys. JETP **38**, 620 (1974)
A.A. Gogolin *et al.*, Sov. Phys. JETP **42**, 168 (1976)

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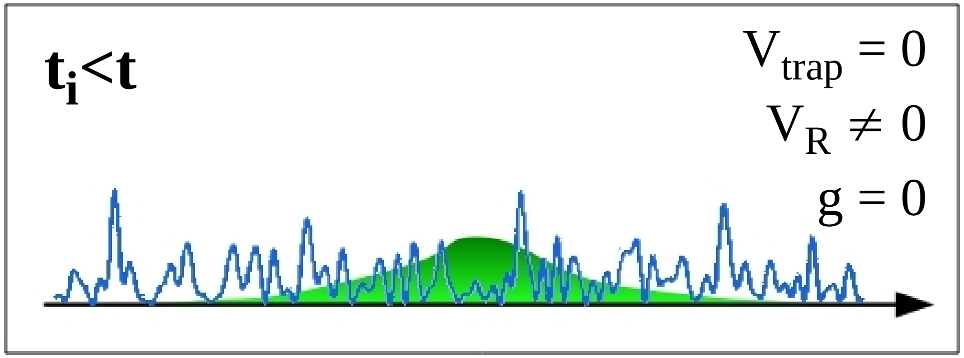

- Localization of the wave packet

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$$\gamma(E) \simeq (m^2/2\hbar^2 p_E^2)\; \tilde{C}(2p_E)$$

FT of the correlation function of the disorder

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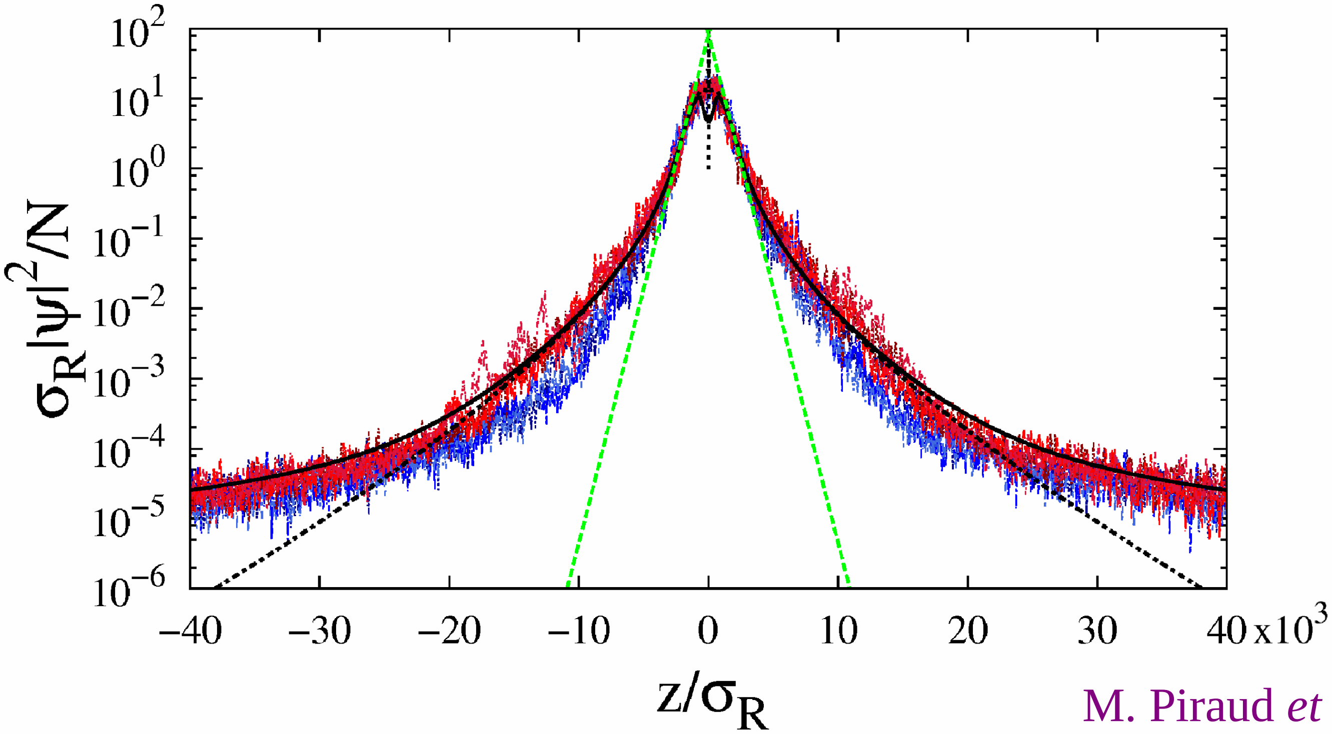
A.A. Gogolin *et al.*, Sov. Phys. JETP **42**, 168 (1976)

Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

$$n(z, t \to \infty) \simeq \int dE\, \mathcal{D}_E(E)\, P(z, t \to \infty | E)$$

versus numerics



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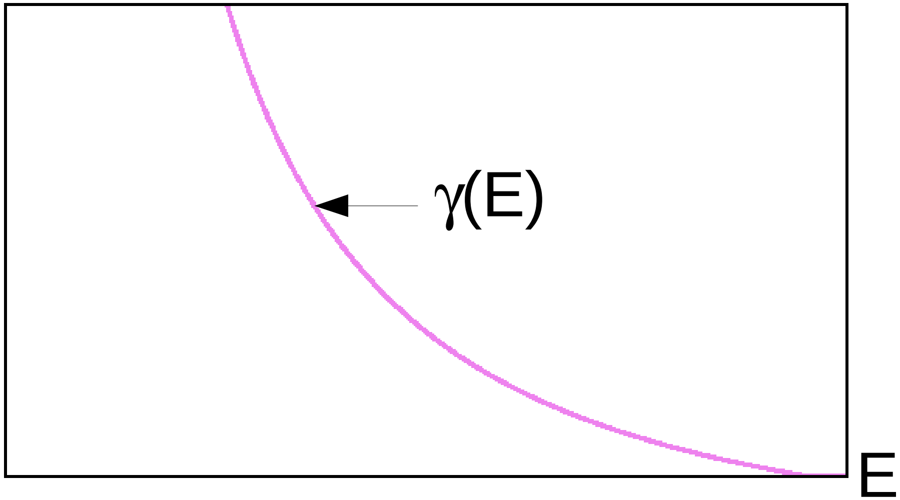

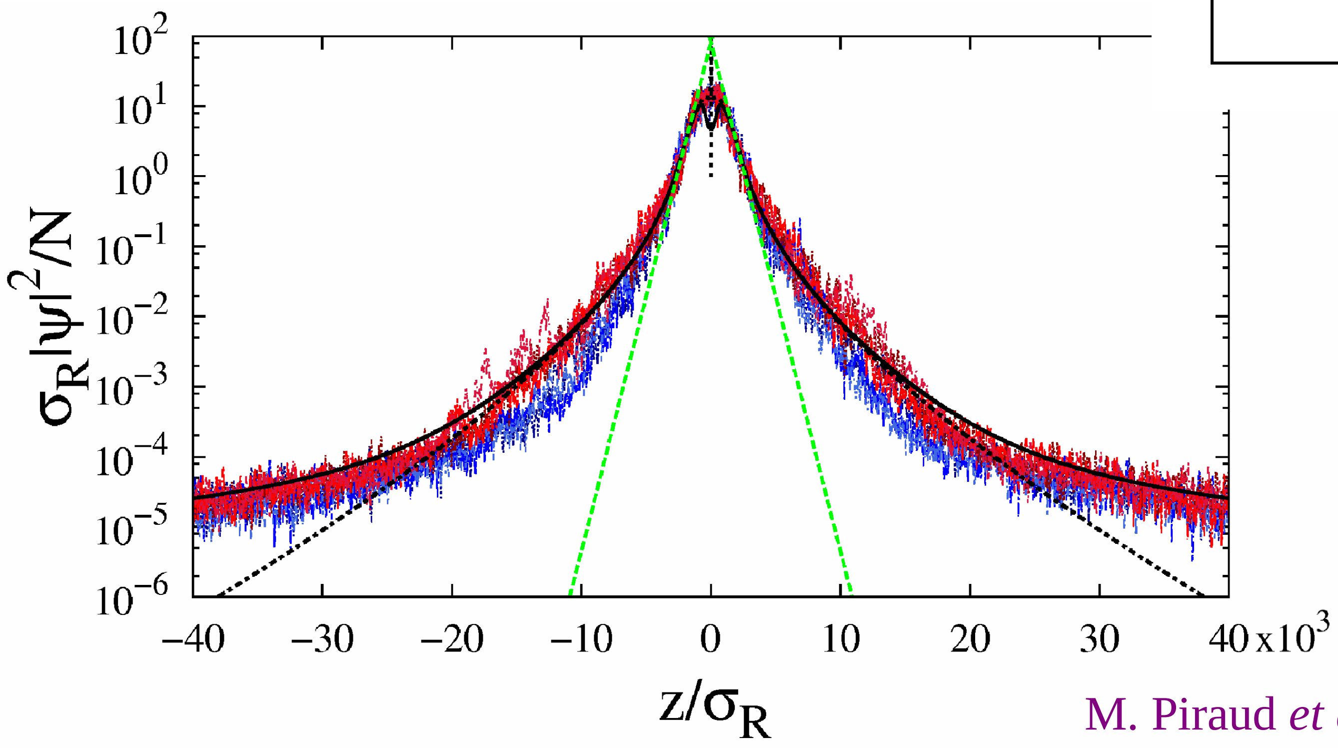

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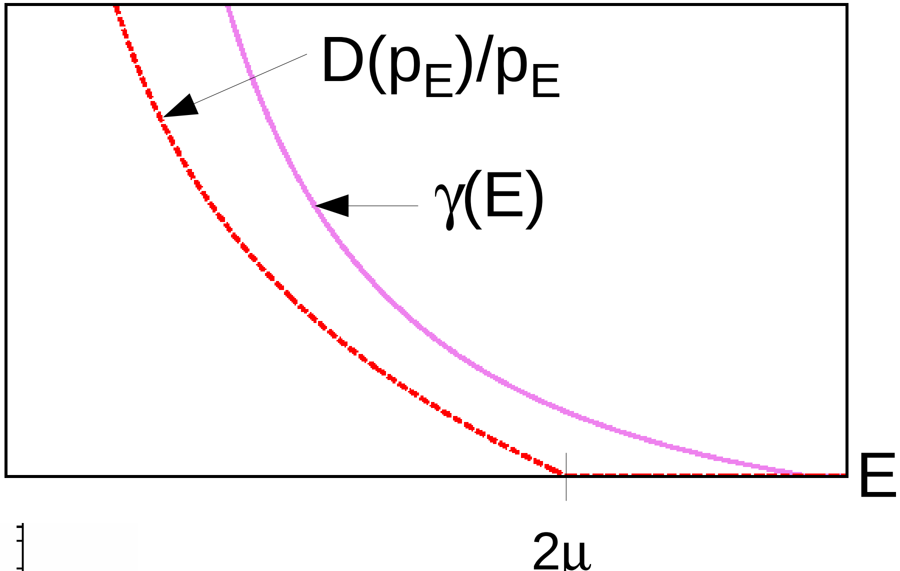
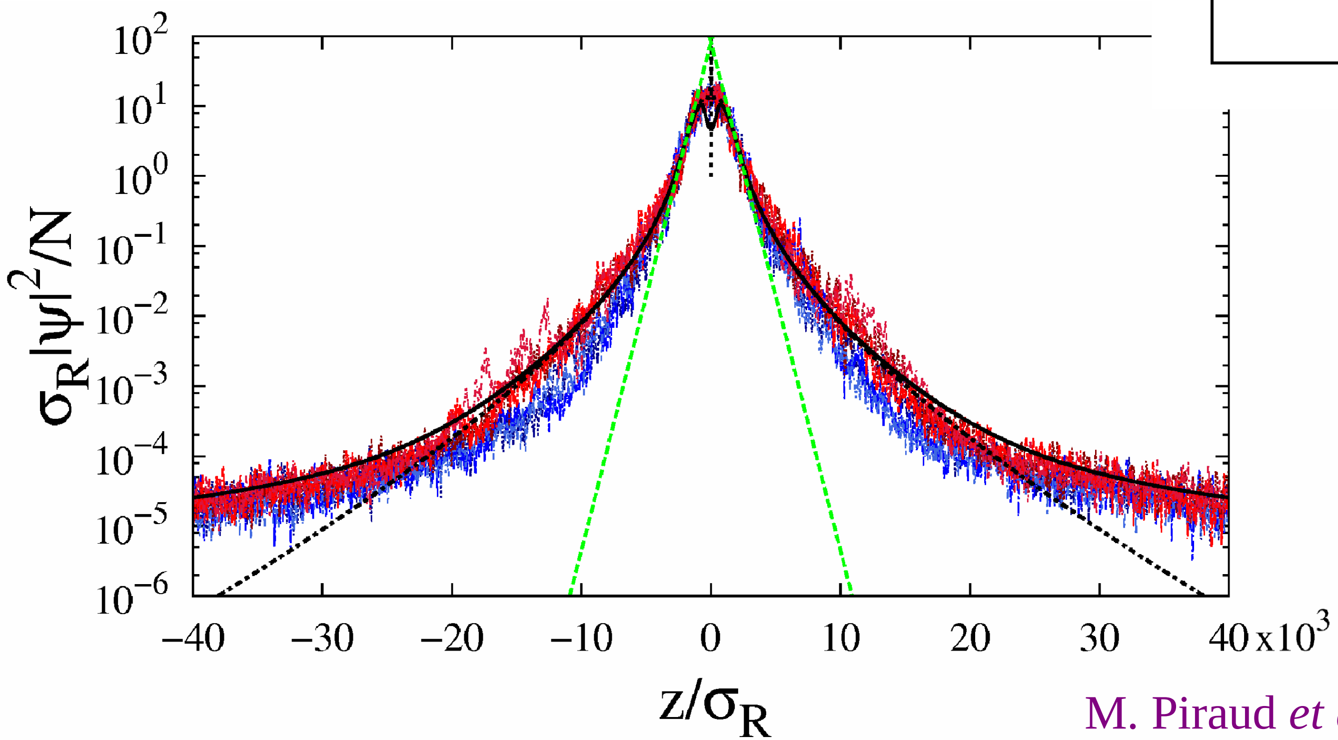
versus numerics

$$\mathcal{D}_E(E) = \int dp \, A(p, E) \, \mathcal{D}_\mathrm{i}(p) \simeq \frac{m}{p_E} \mathcal{D}_\mathrm{i}(p_E)$$

(«on-shell» approximation)





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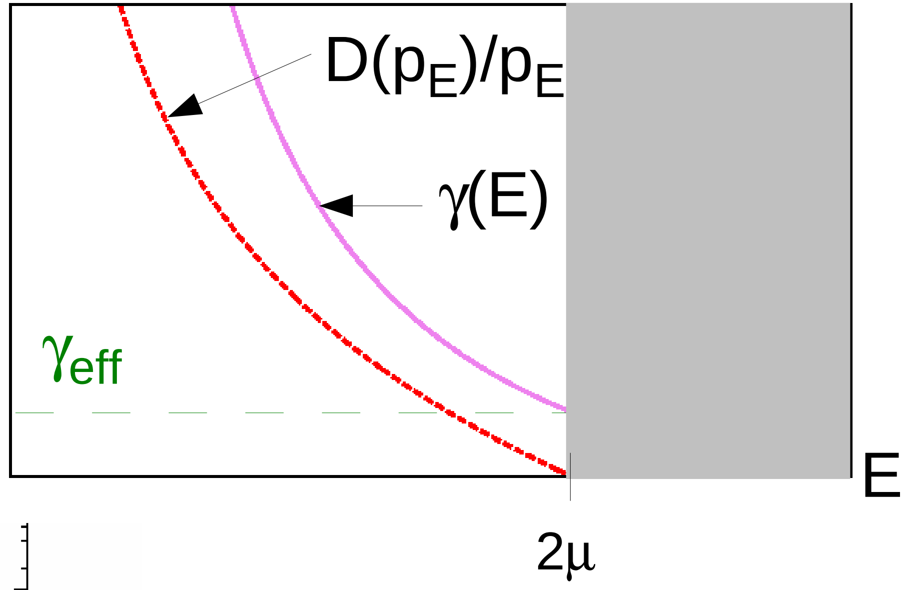

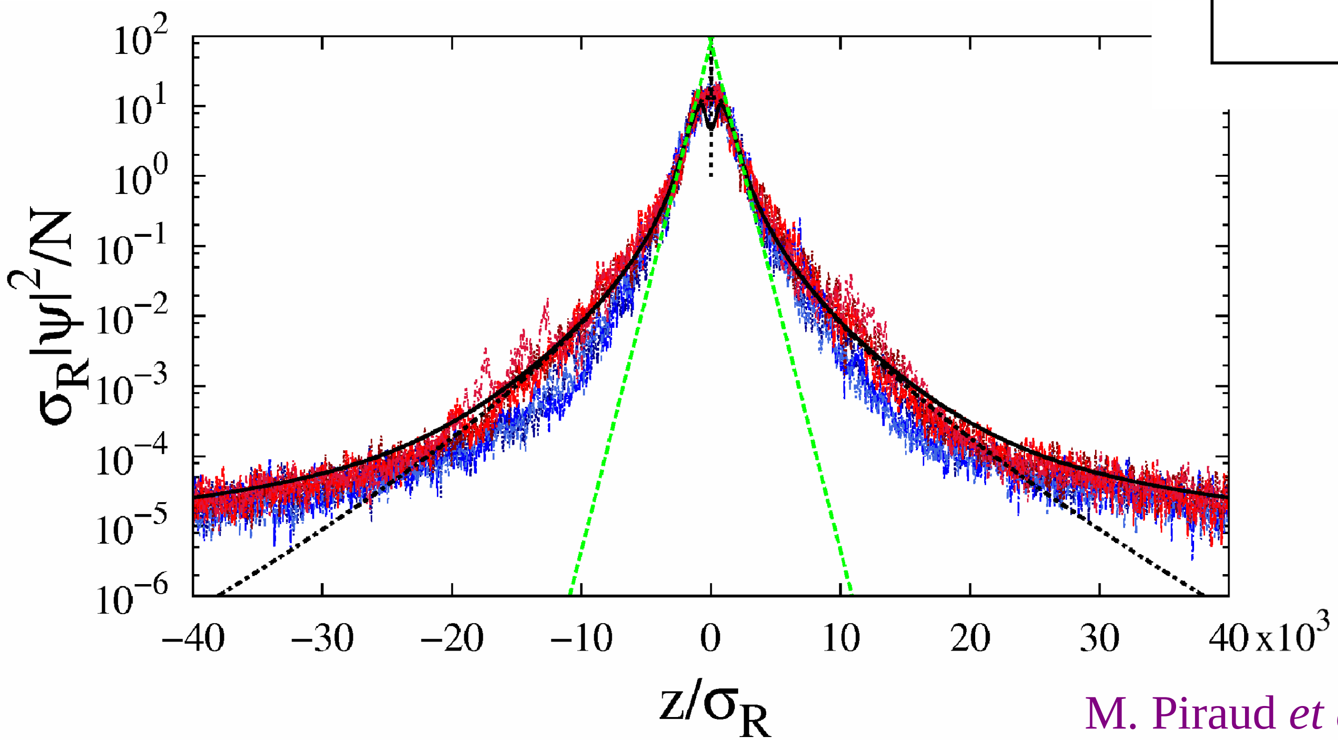

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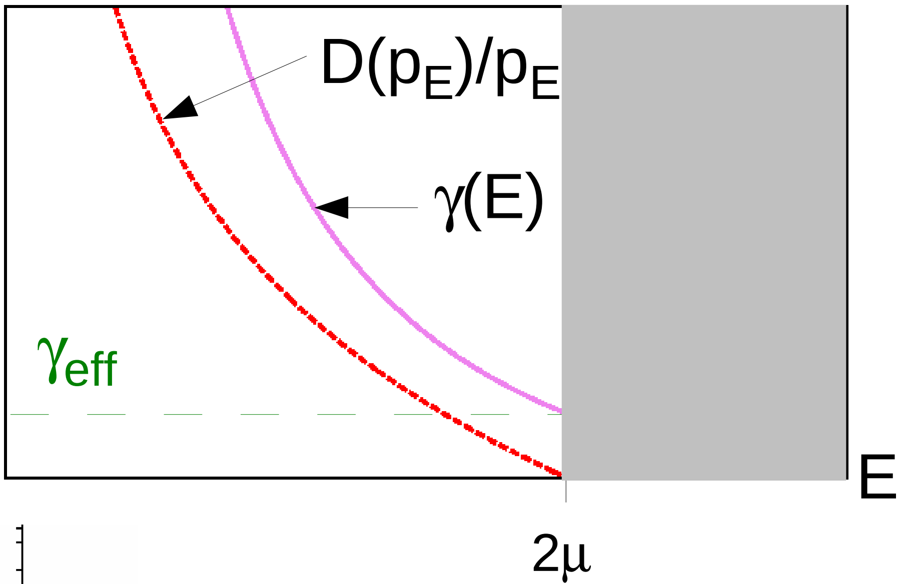

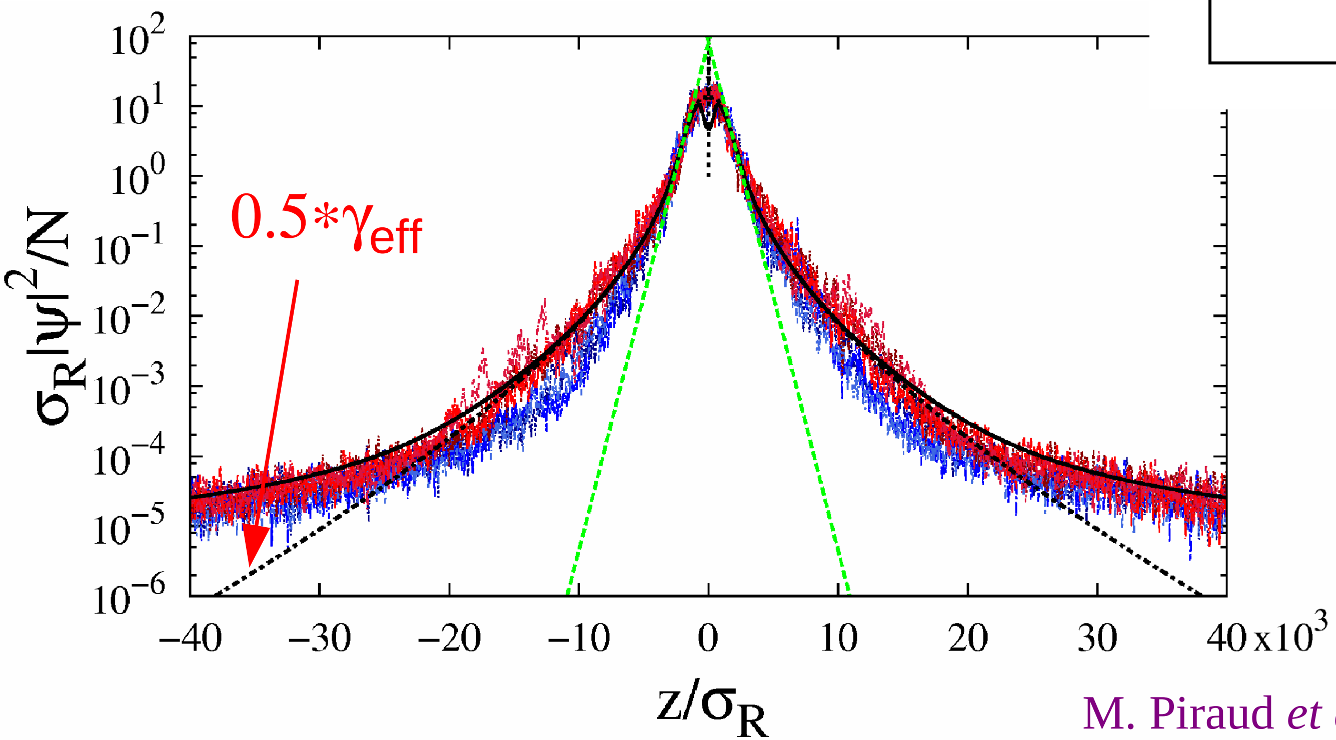

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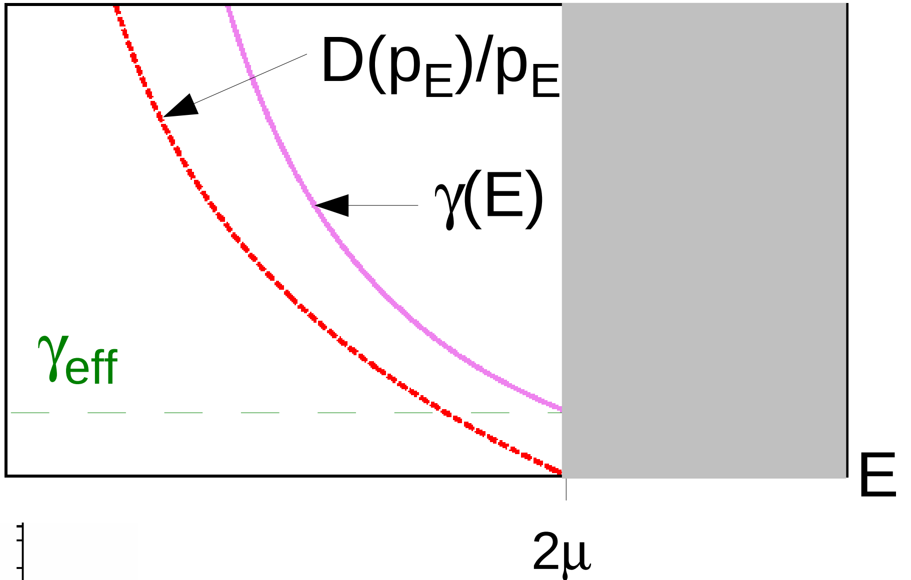

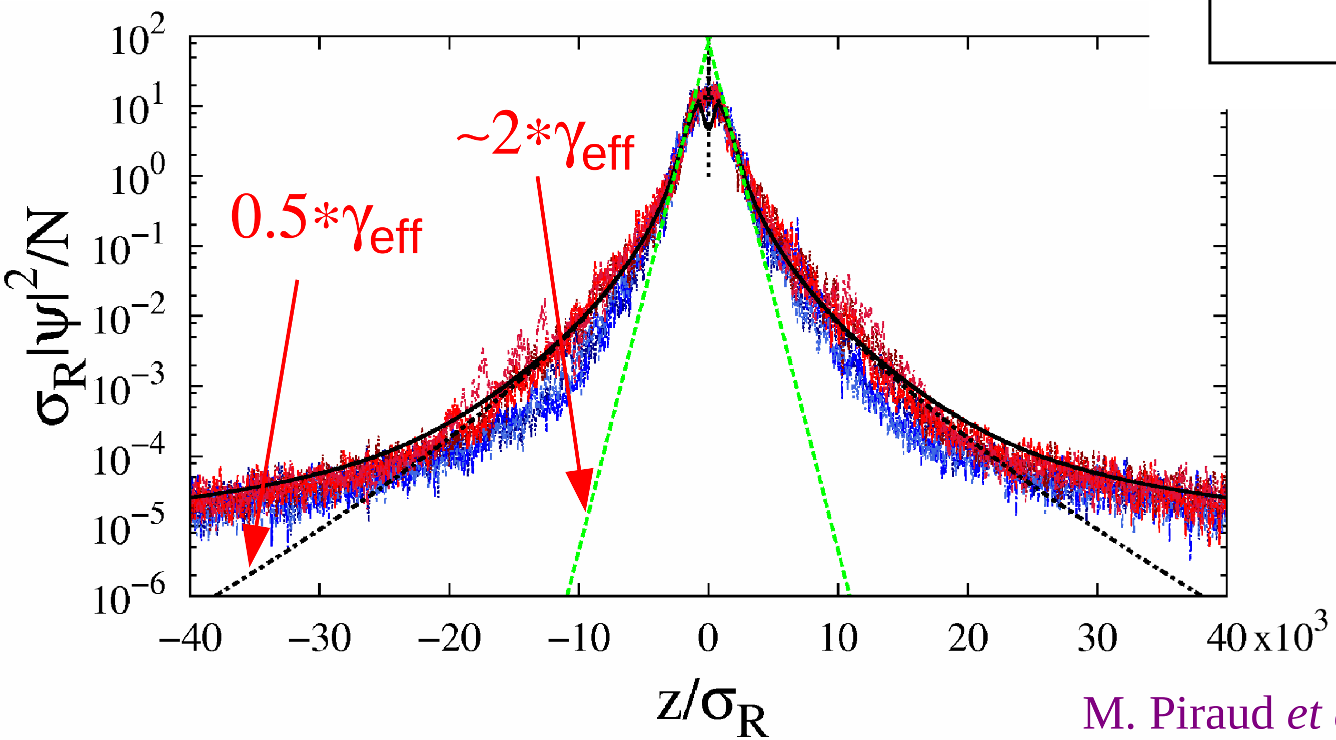

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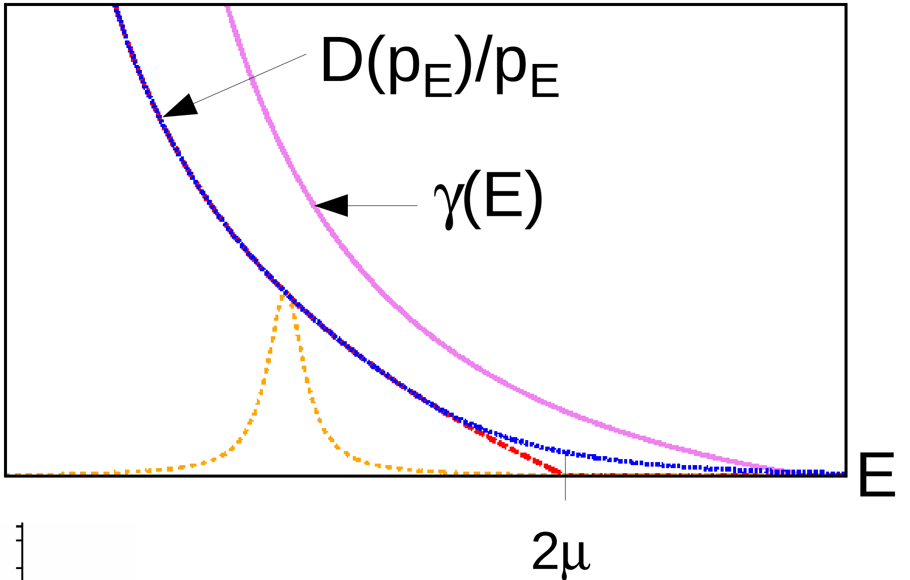

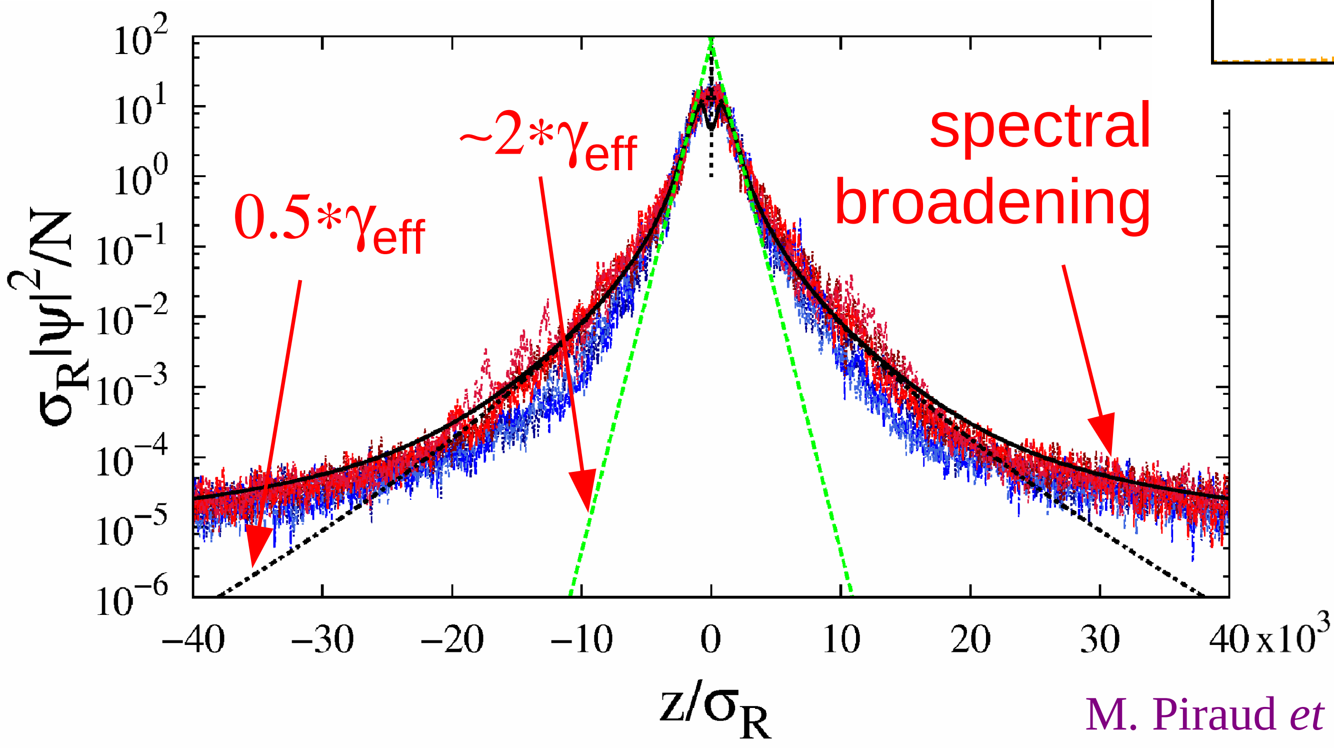

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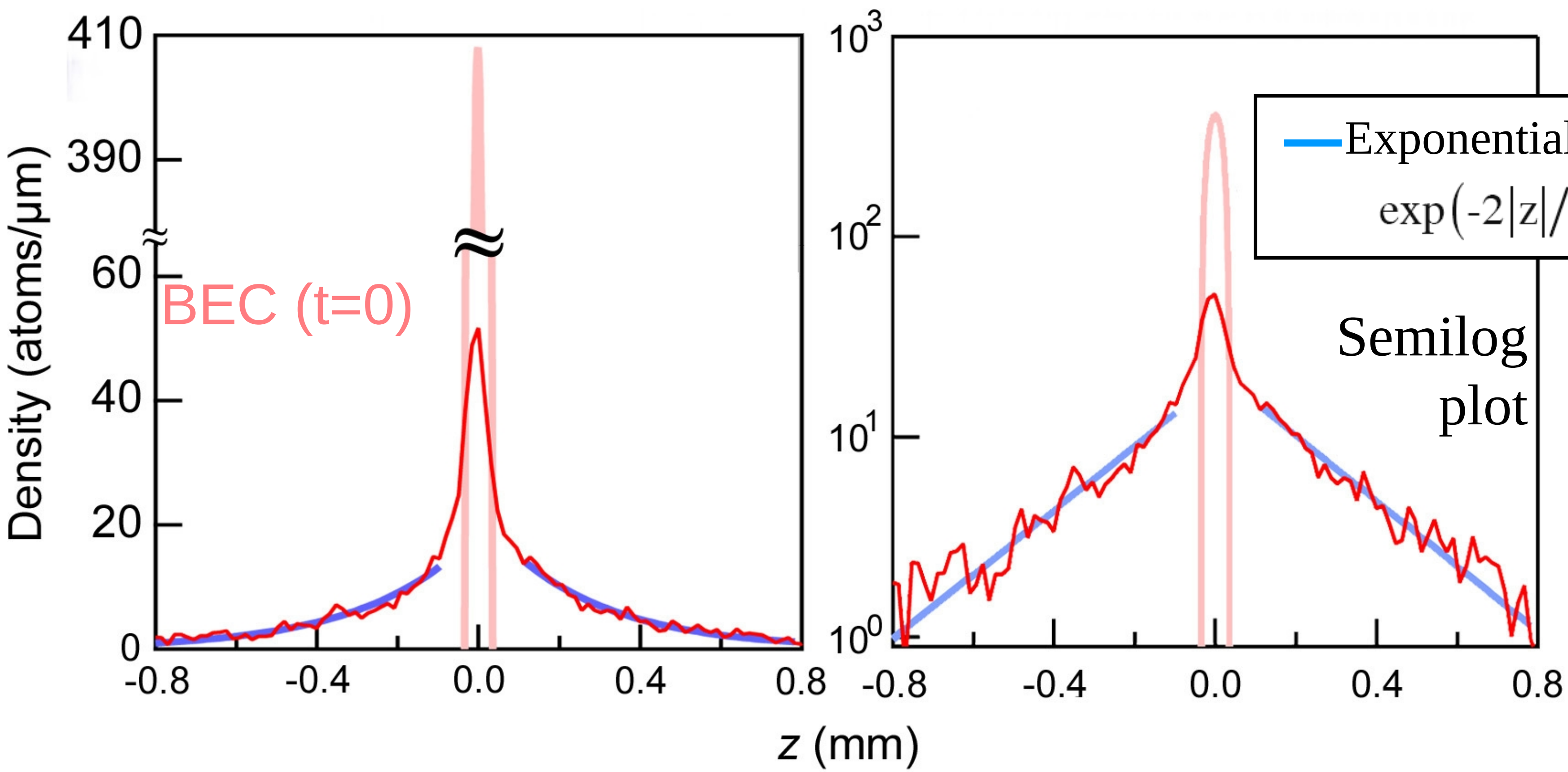
Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

J. Billy *et al.*, Nature **453**, 891 (2008)
see also G. Roati *et al.*, Nature **453**, 895 (2008)

BEC parameters : N=1.7 10^4 atoms, ($\xi_{in} \cong 1.5\sigma_R$)
Weak disorder : V_R/μ_{in} =0.12 << 1



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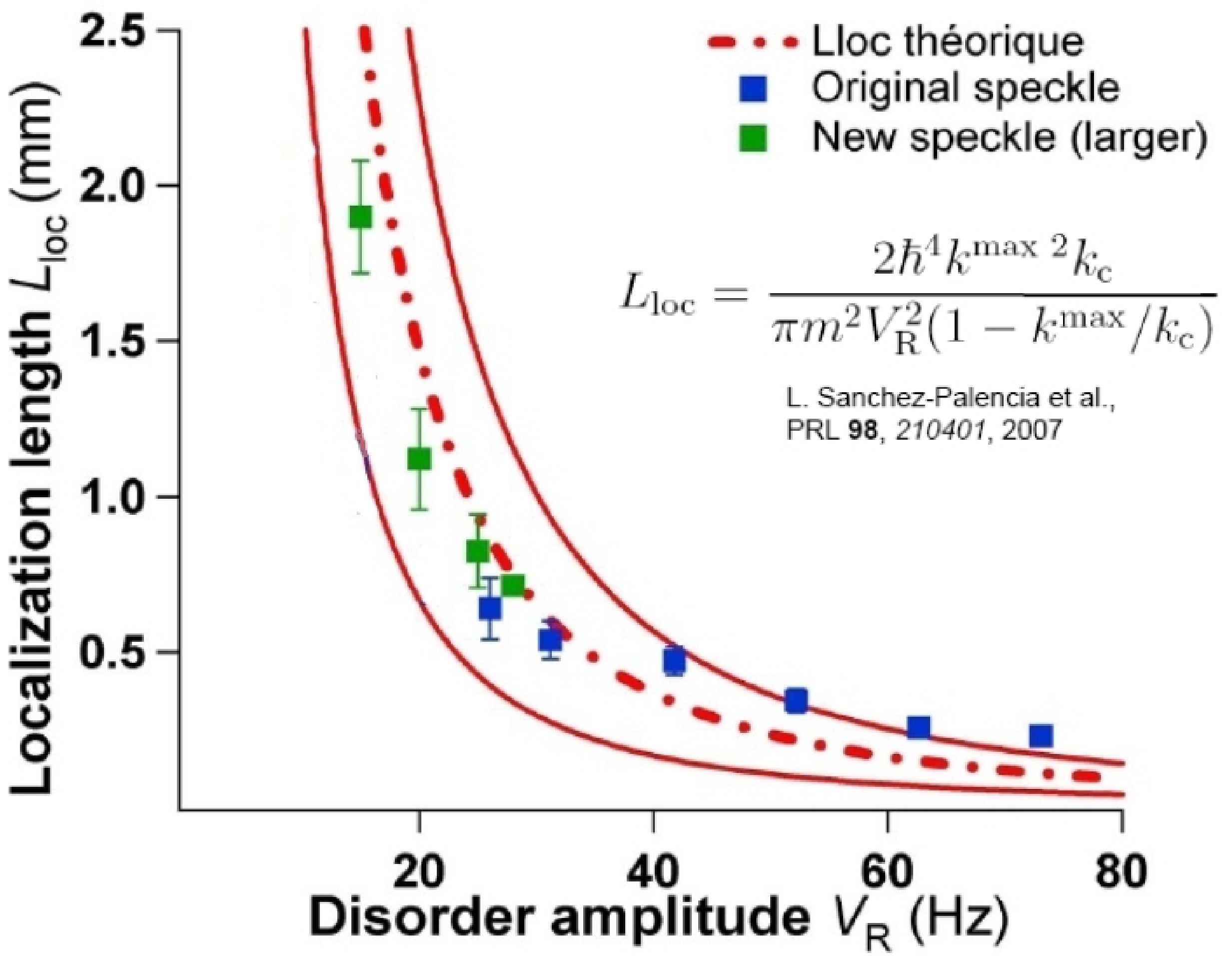


Fair agreement with theoretical prediction

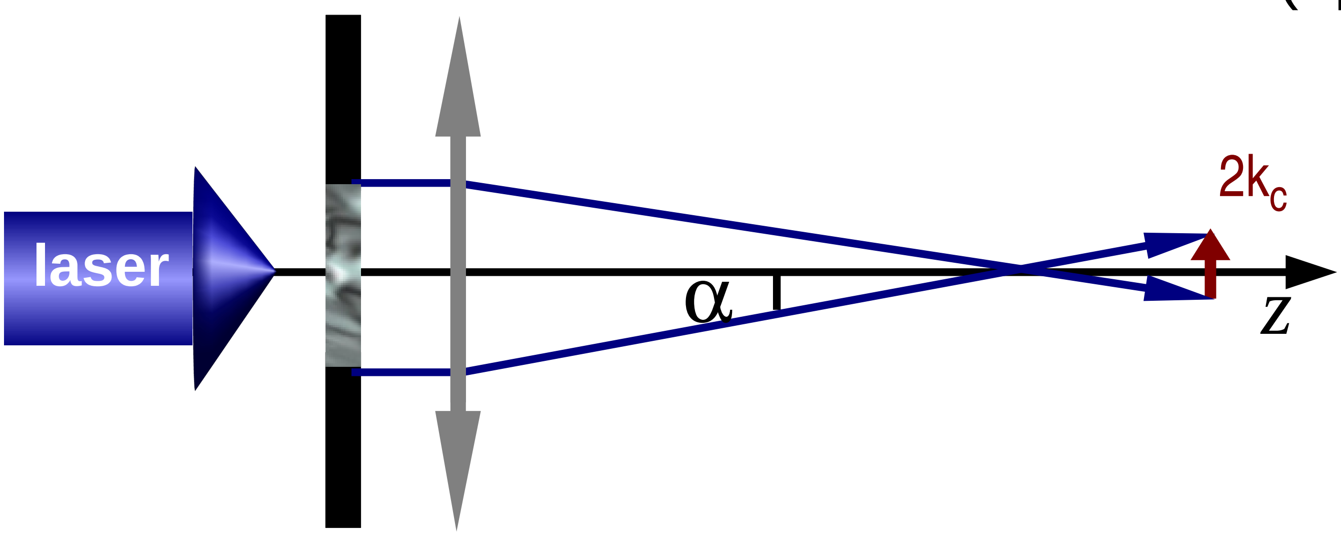
Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

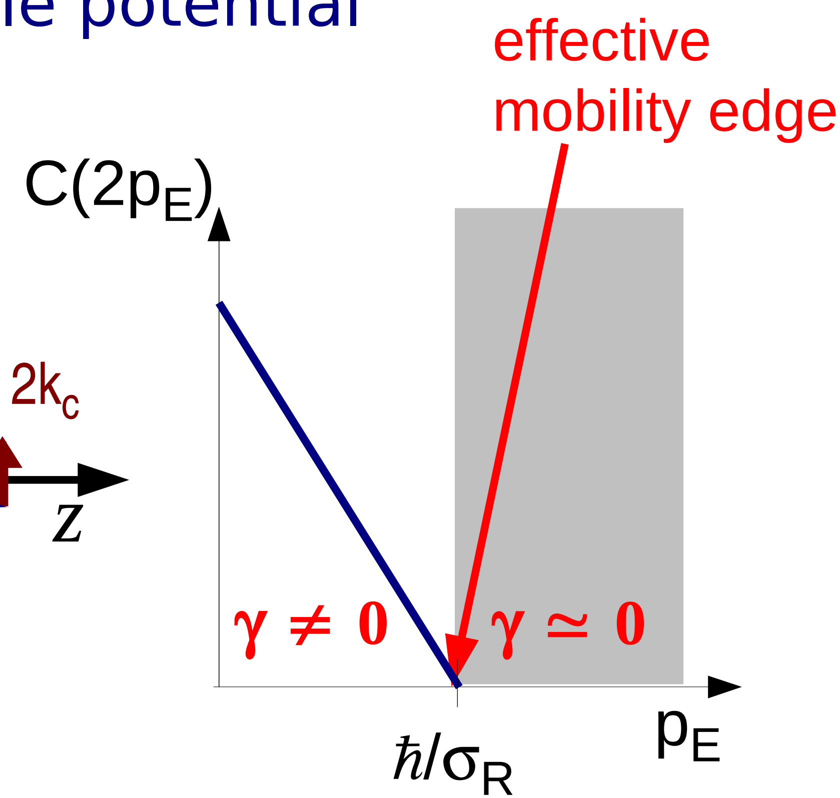
$$\gamma(E) \simeq (m^2/2\hbar^2 p_E^2)\ \tilde{C}(2p_E)$$

FT of the correlation function of the disorder

- Correlation function of a 1D speckle potential

$$\tilde{C}(p) = \pi V_{\mathrm{R}}^2 \sigma_{\mathrm{R}} [1 - |p|\sigma_{\mathrm{R}}/2\hbar]_{\oplus}$$





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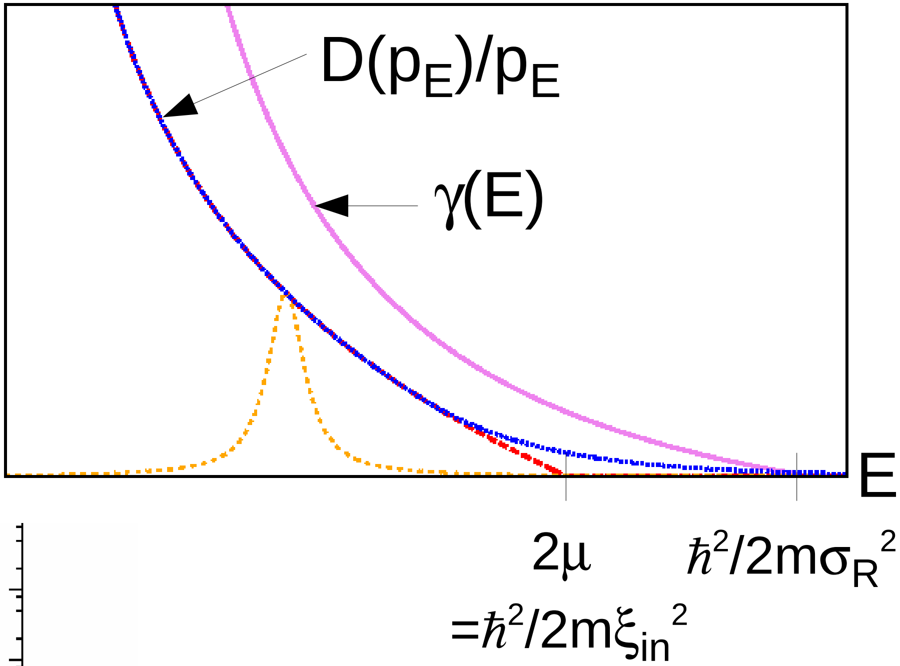
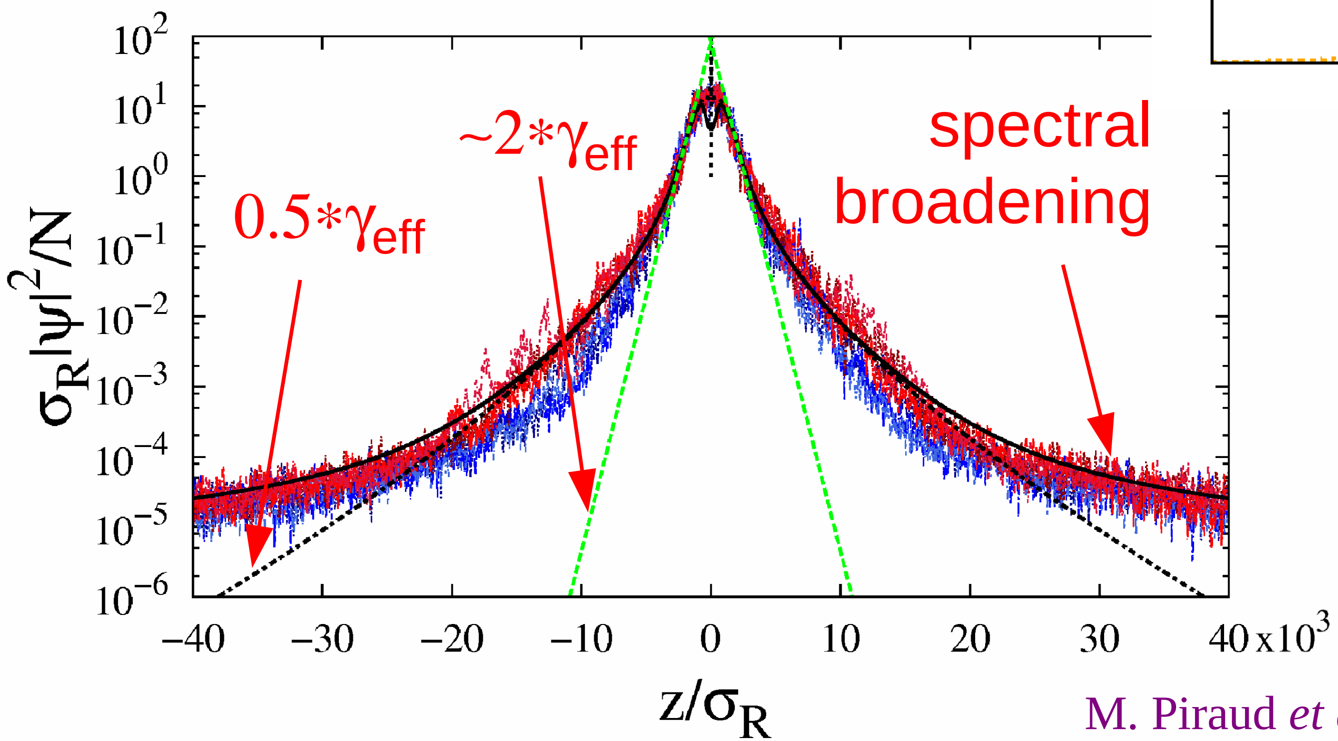


For $\xi_{in} > \sigma_R$



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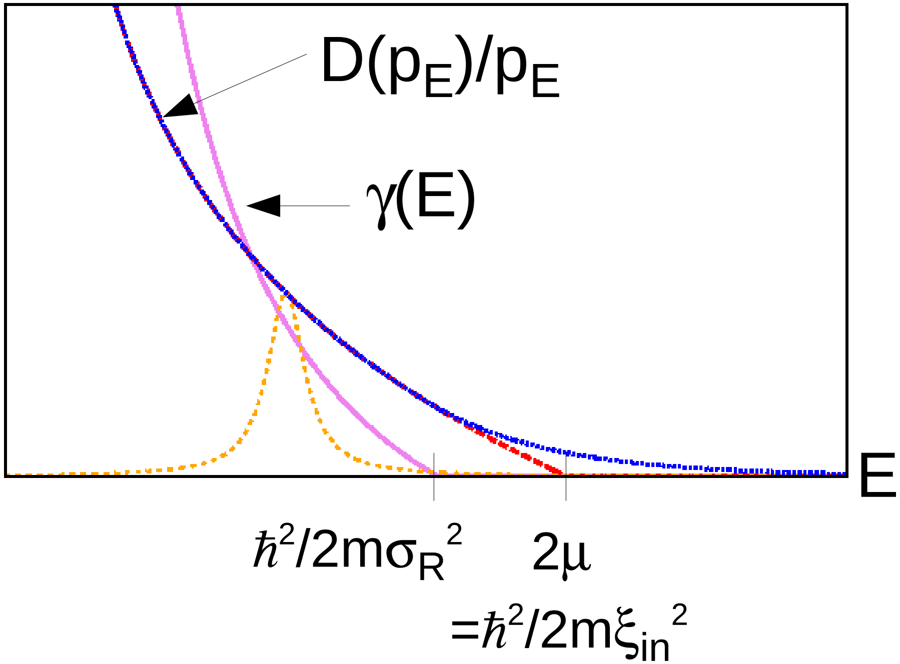


For $\xi_{in} < \sigma_R$

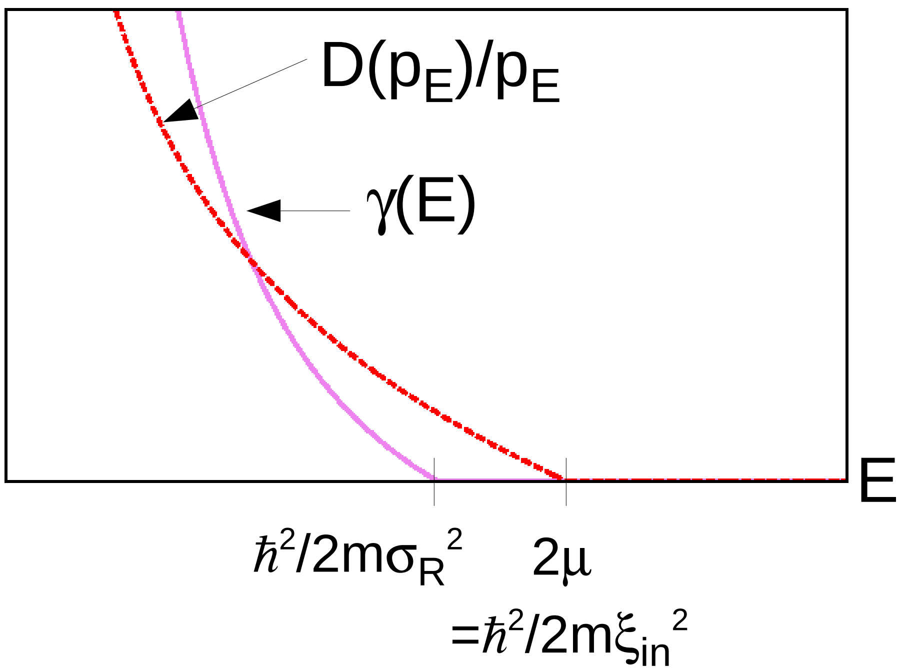
M. Piraud *et al.*, Phys. Rev. A **83**, 031603(R) (2011)

Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

$$n(z, t \to \infty) \simeq \int dE \, \mathcal{D}_E(E) \, P(z, t \to \infty | E)$$

versus numerics

$$\mathcal{D}_E(E) = \int dp \, A(p, E) \, \mathcal{D}_\mathrm{i}(p) \simeq \frac{m}{p_E} \mathcal{D}_\mathrm{i}(p_E)$$



algebraic localization $n(z) \propto 1/|z|^2$

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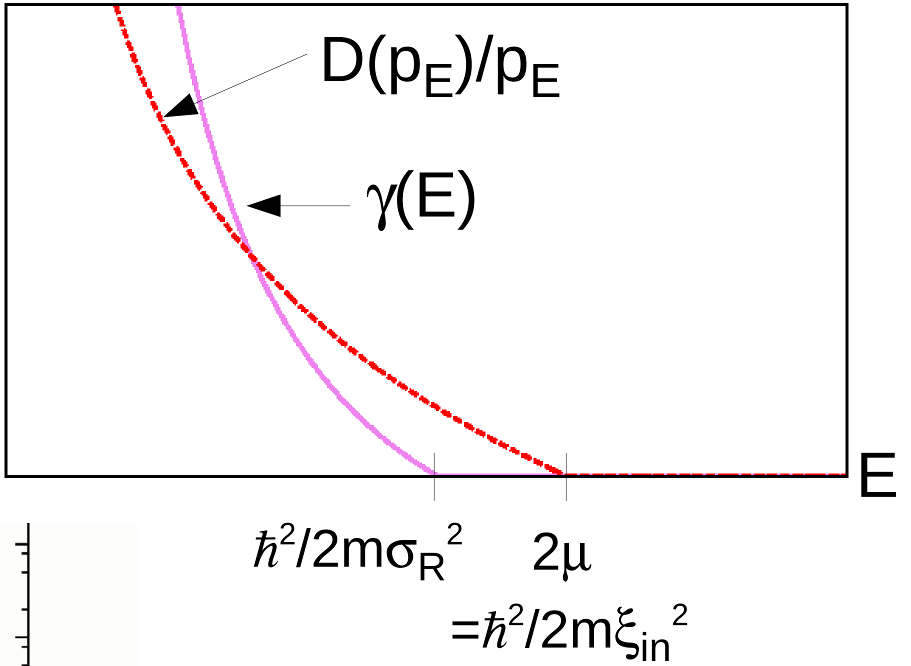
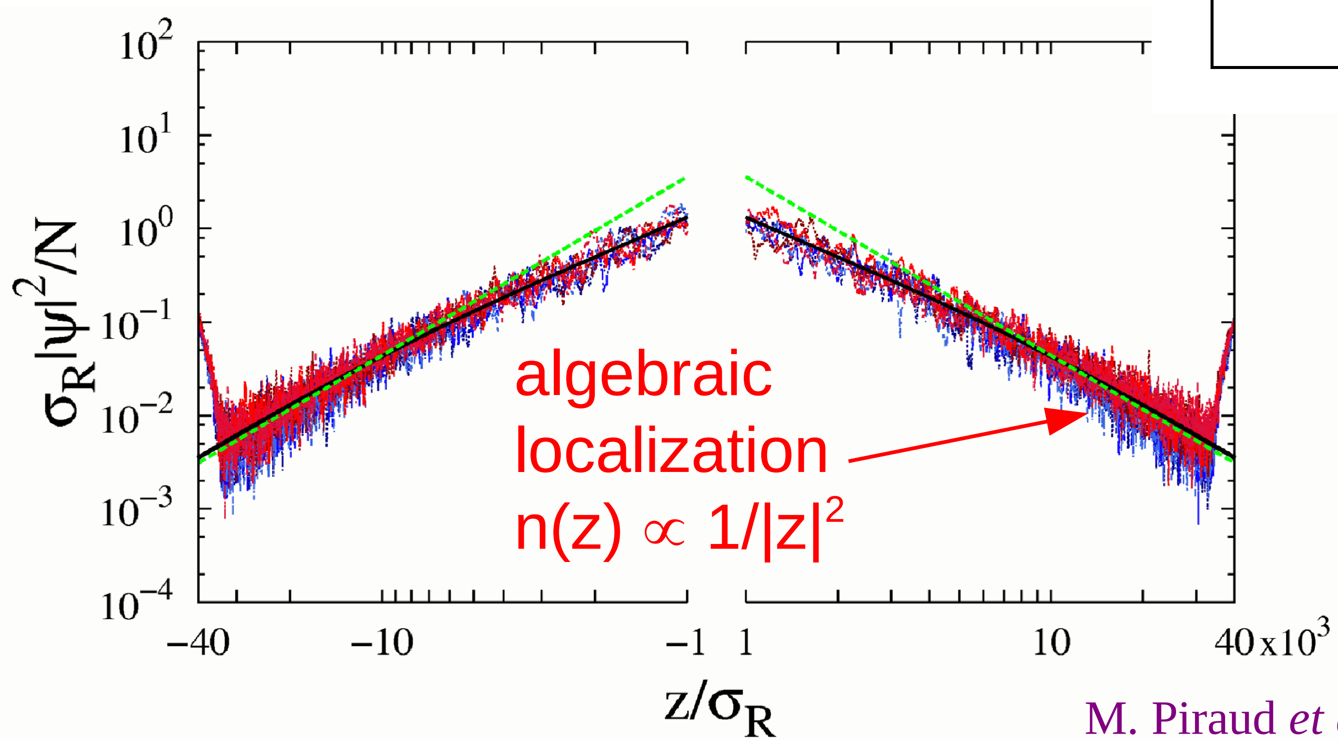




For $\xi_{in} < \sigma_R$

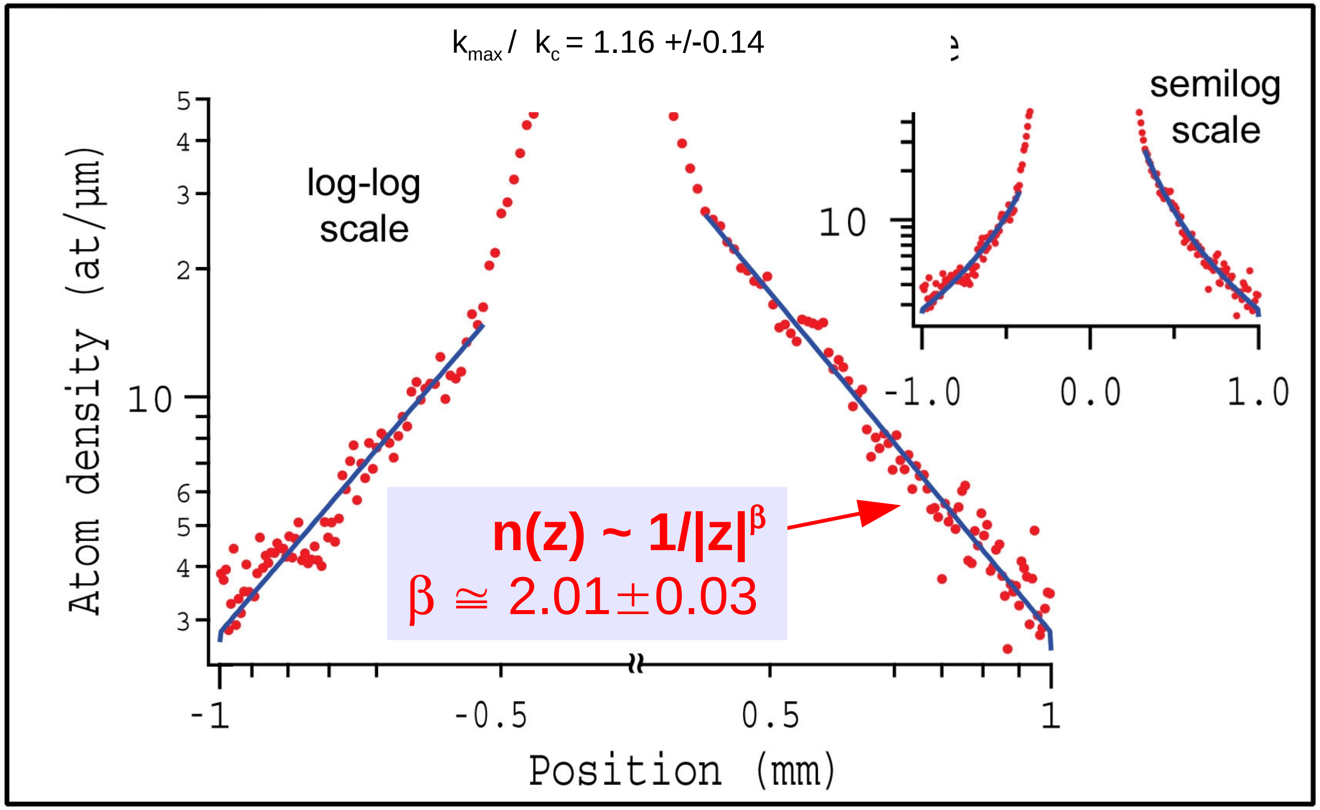
M. Piraud *et al.*, Phys. Rev. A **83**, 031603(R) (2011)

Anderson Localization of an Expanding Matterwave in a 1D Speckle Potential

J. Billy *et al.*, Nature **453**, 891 (2008)

BEC parameters : N=1.7 10^5 atoms, ($\xi_{in} \cong 0.8\sigma_R$)

Weak disorder : V_R/μ_{in} =0.15 << 1



Anderson Localization in a 1D Speckle Potential : Conclusions

- Model of 1D Anderson localization

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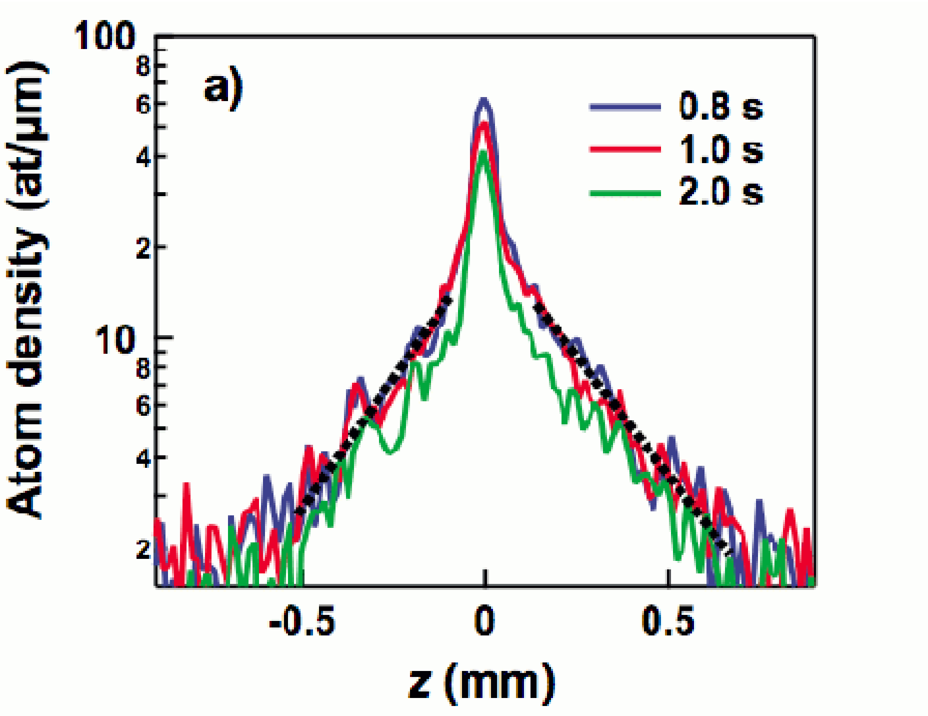

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Anderson Localization in a 1D Speckle Potential : Conclusions

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  - For $\xi_{in} > \sigma_R$,
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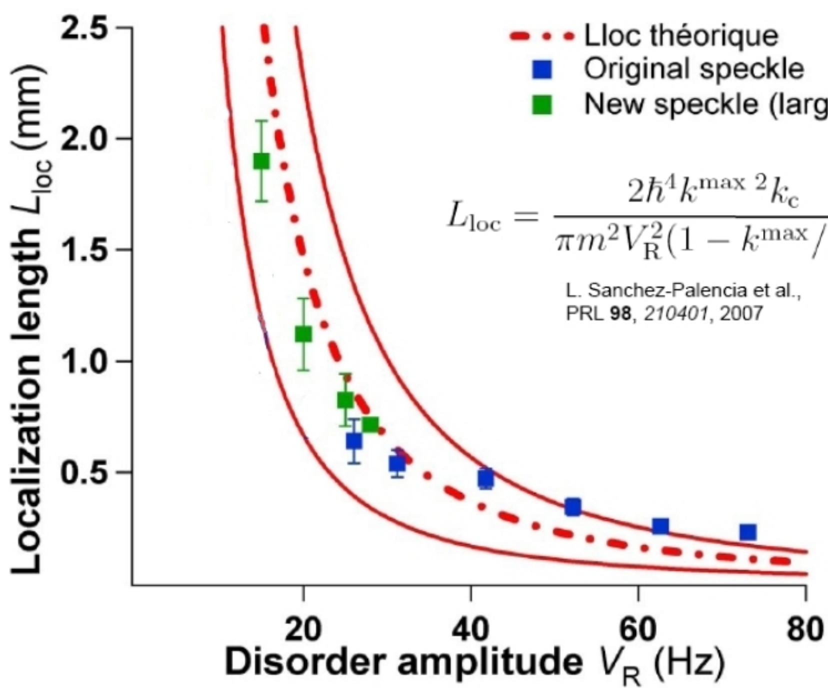
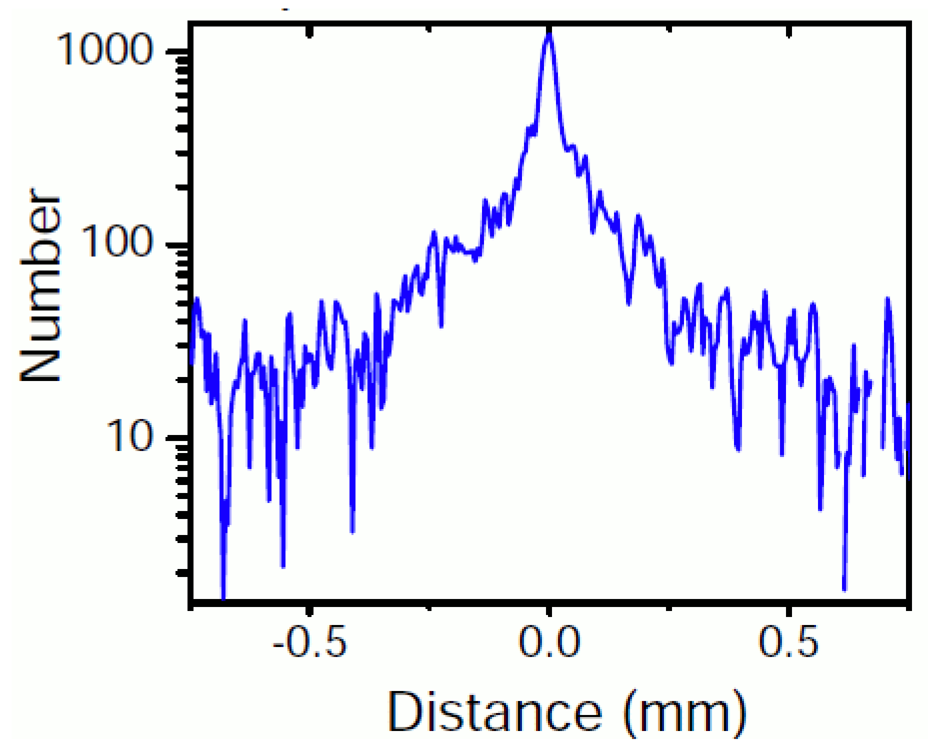
    - large distance : ln[n(z)] ~ $-0.5\gamma_{eff}$ ➔ possible signatures (?)

      \+ deviations from exponential decay



J. Billy *et al.*, Nature **453**, 891 (2008)



Courtesy of R. Hulet, *unpublished* (2009)

Anderson Localization in a 1D Speckle Potential : Conclusions

- Model of 1D Anderson localizatio
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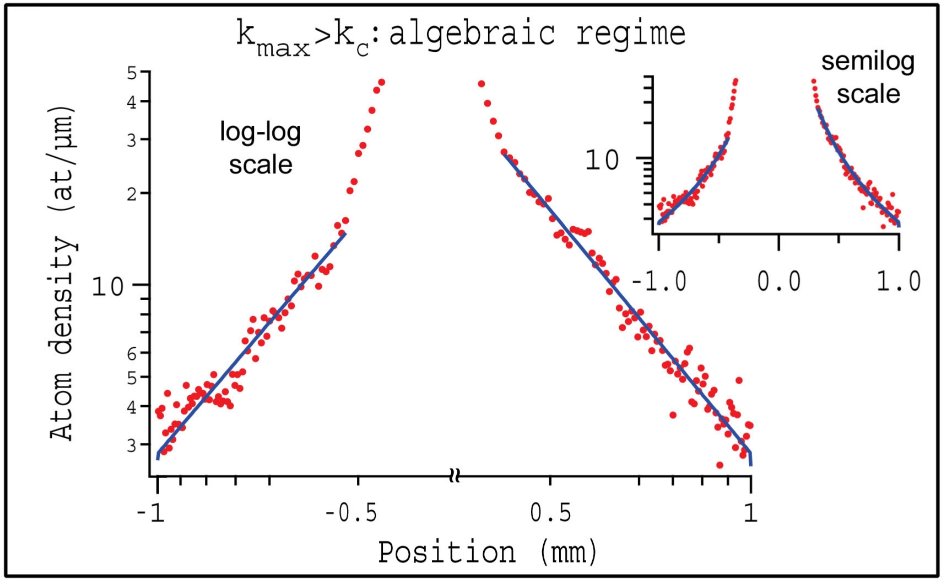
    - large distance : ln[n(z)] ~ -0.5γ
      + deviations fr



J. Billy *et al.*, Nature **453**, 891 (2008)

  - For $\xi_{in} < \sigma_R$,
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Anderson Localization in a 1D Speckle Potential : Conclusions

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- Perspectives
  - tailored correlations [M. Piraud *et al.*, Phys. Rev. A **85**, 063611 (2012)]
  - extension to many-body localization

Bibliography (I)

- Anderson localization in 1D disorder
  - Lifshits, Gredeskul & Pastur, *Introduction to the Theory of Disordered Systems* (1988)
  - V.L. Berezinskii, Sov. Phys. JETP **38**, 620 (1974)
  - A.A. Gogolin *et al.*, Sov. Phys. JETP **42**, 168 (1976)
- Speckle potentials in ultracold atom systems
  - J. W. Goodman, *Speckle Phenomena in Optics* (2007)
  - D. Clément *et al.*, New J. Phys. **8**, 165 (2006)
  - B. Shapiro, J. Phys. A: Math. Theor. **45**, 143001 (2012)
- Anderson localization in 1D speckle potentials
  - LSP *et al.*, Phys. Rev. Lett. **98**, 210401 (2007)
  - J. Billy *et al.*, Nature **453**, 891 (2008)
  - P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)
  - E. Gurevich and O. Kenneth, Phys. Rev. A **79**, 063617 (2009)
  - M. Piraud *et al.*, Phys. Rev. A **83**, 031603(R) (2011)
- Localization in bichromatic lattices (not discussed here)
  - S. Aubry and G. André, Ann. Israel Phys. Soc. **3**, 133 (1980)
  - G. Roati *et al.*, Nature **453**, 895 (2008)
  - G. Modugno, Rep. Prog. Phys. **73**, 102401 (2010)