Summer School on Quantum Many-Body Physics of Ultra-Cold Atoms and Molecules

Lecture 1: Overview of tools in cold atom experiments
(11:30 am, July 12, 2012)

Lecture 2: In situ imaging: density, fluctuations, correlations and equation of state
(3:30 pm, July 12, 2012)

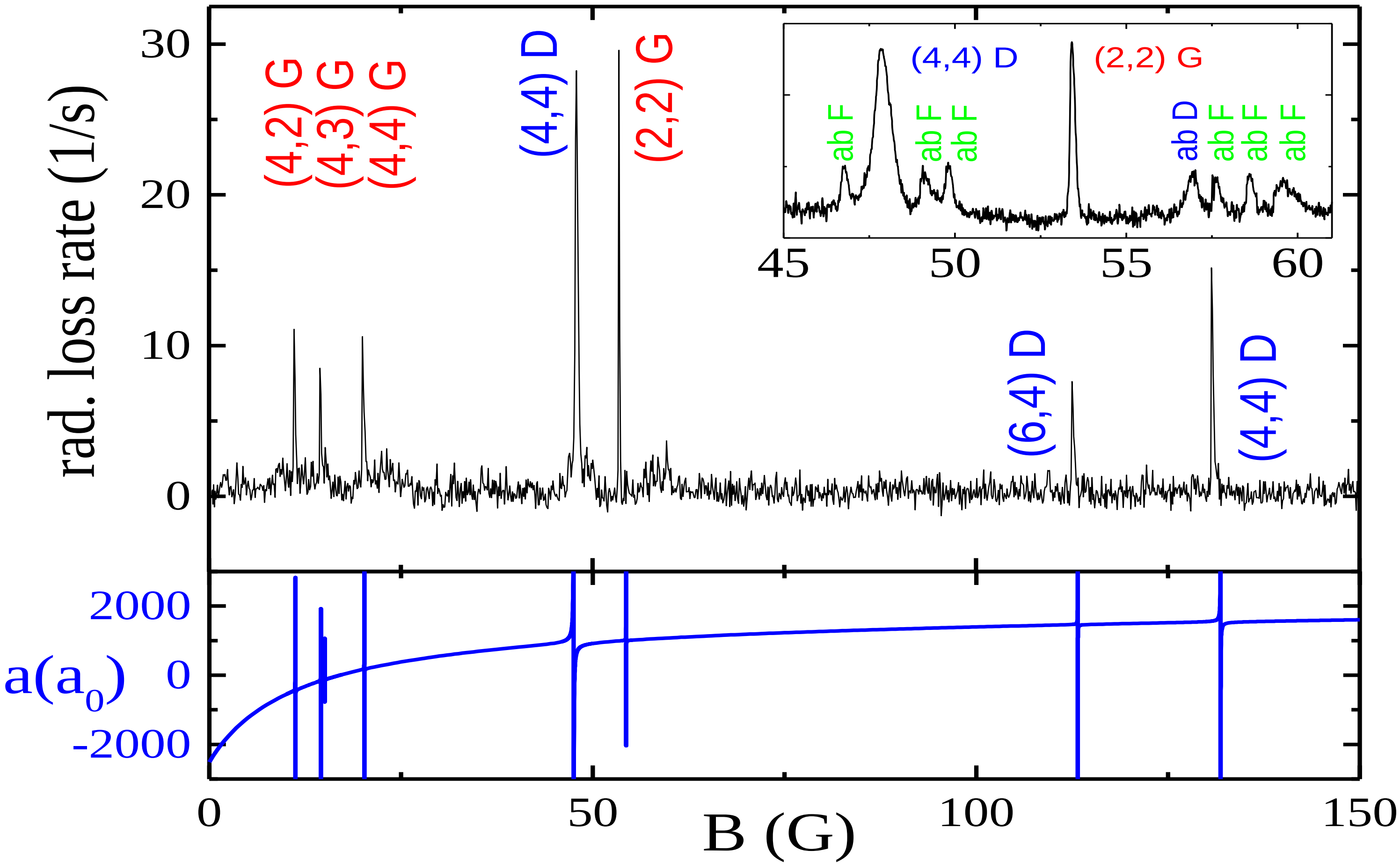
Lecture 3: Scale invariance, universality and quantum criticality of 2D quantum gases
(9 am, July 13, 2012)

WORKSHOP ON QUANTUM SIMULATIONS WITH ULTRACOLD ATOMS

Talk: Exploring Universal Quantum Physics in few- and many-body atomic systems
(9:40am, July 17, 2012)

Cheng Chin, James Franck institute and Department of Physics, University of Chicago

Feshbach resonances



Feshbach resonances and interactions: ***C. Chin et al., PRL `00,`03, PRA`04*** **(Theory: NIST)**

Creation of s-, d-, g-, l-wave molecules ***J. Herbig et al., Science ,03, C. Chin et al., PRL'05***

A tunable quantum system

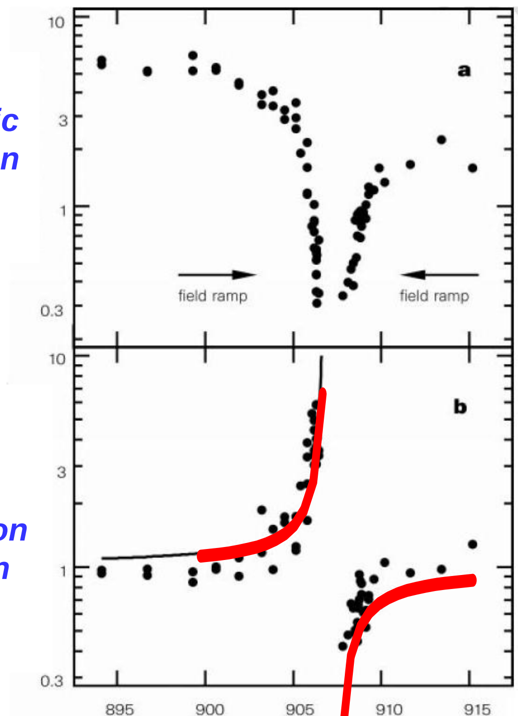
Feshbach resonance

Feshbach resonance in Na

Ketterle group, 1997

Inelastic collision rate

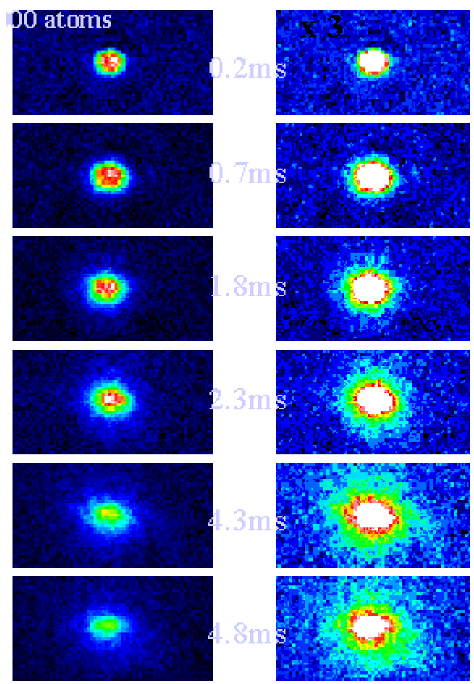
Interaction strength



magnetic field (G)

BEC implosion and explosion (bosenova)

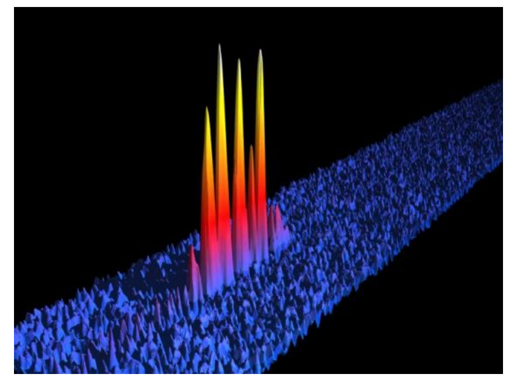
Wieman group, 2001



Bright Soliton

Hulet group, 2001

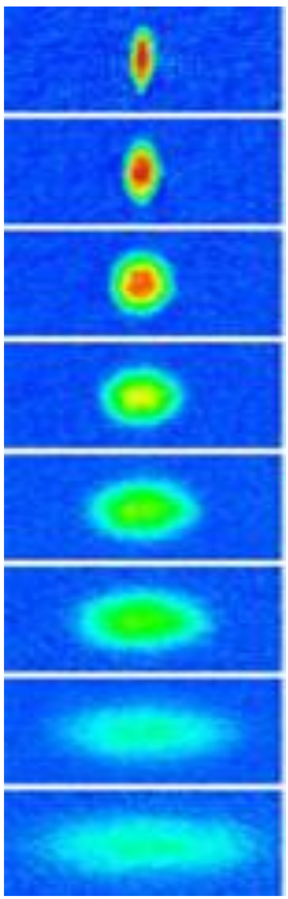
Salomon group, 2001



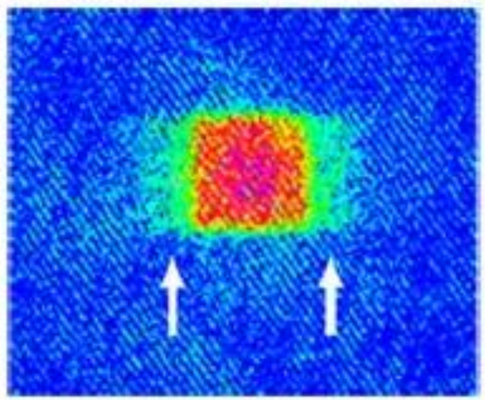
Feshbach resonances are found in all alkali species

Experiments on Fermionic condensates near a Feshbach resonance (02~05)

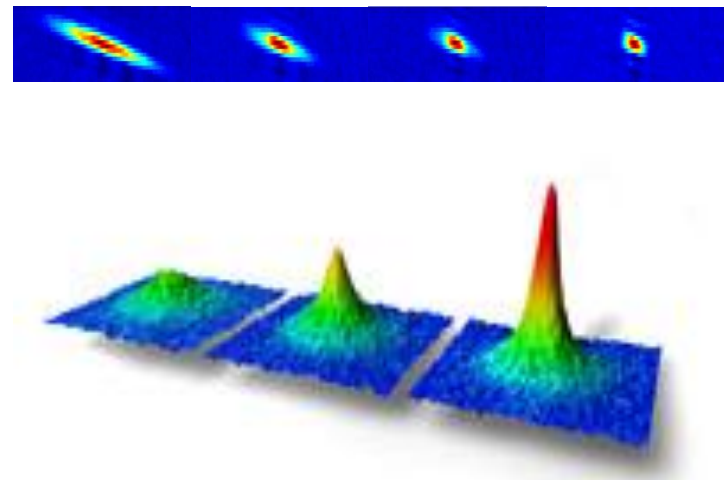
Hydrohynamic expansion
(Duke, ENS)



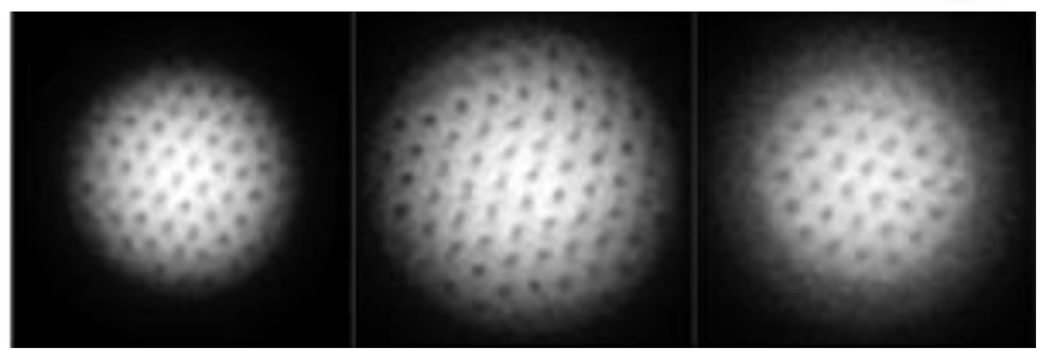
Band insulator
(ETH, Florence)



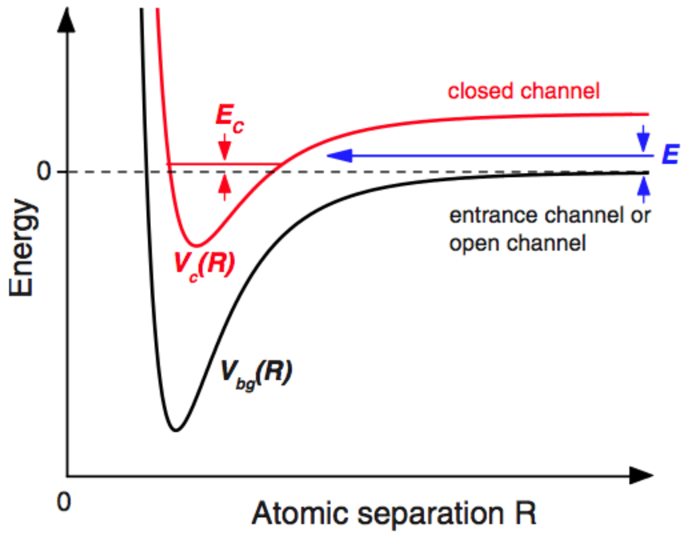
(BEC of Fermion pairs)
(Innsbruck, JILA)



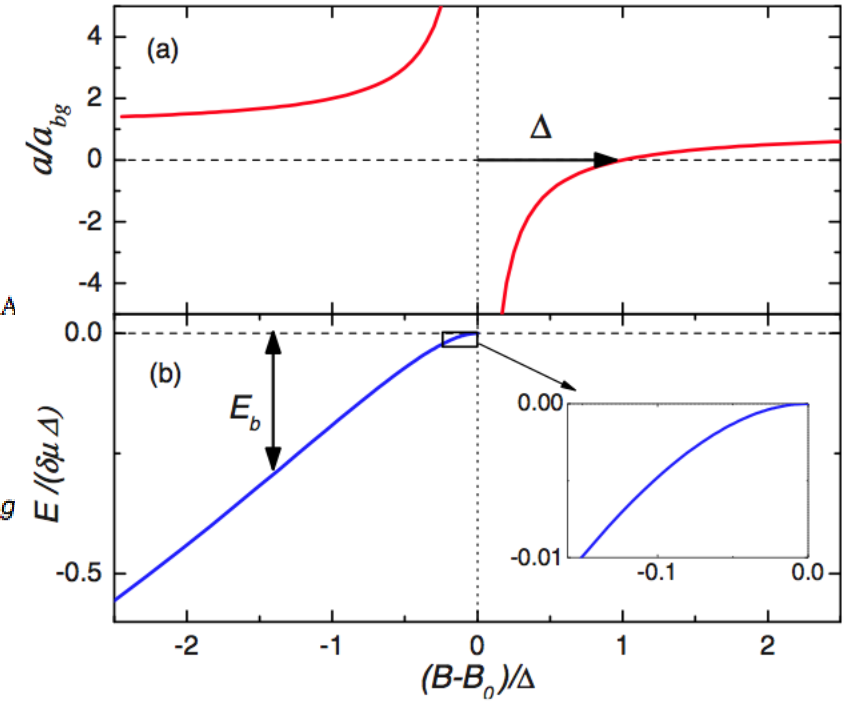
Fermionic superfluid
(MIT)



What is a Feshbach resonance?



$$a = a_{\rm bg}\left(1 - \frac{\Delta}{B - B_0}\right)$$



REVIEWS OF MODERN PHYSICS, VOLUME 82, A

Feshbach resonances in ultracold gases

Cheng Chin

Department of Physics and James Franck Institute, University of Chicag 60637, USA

Rudolf Grimm

Center for Quantum Physics and Institute of Experimental Physics, University of Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria and Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Otto-Hittmair-Platz 1, 6020 Innsbruck, Austria

Paul Julienne and Eite Tiesinga

Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, 100 Bureau Drive, Gaithersburg, Maryland 20899-8423, USA

Questions that puzzled me…

1. How does interaction suddenly change sign at a resonance?

2. Isn't van der Waals interaction always attractive?

3. How does molecular state approach zero quadratically?

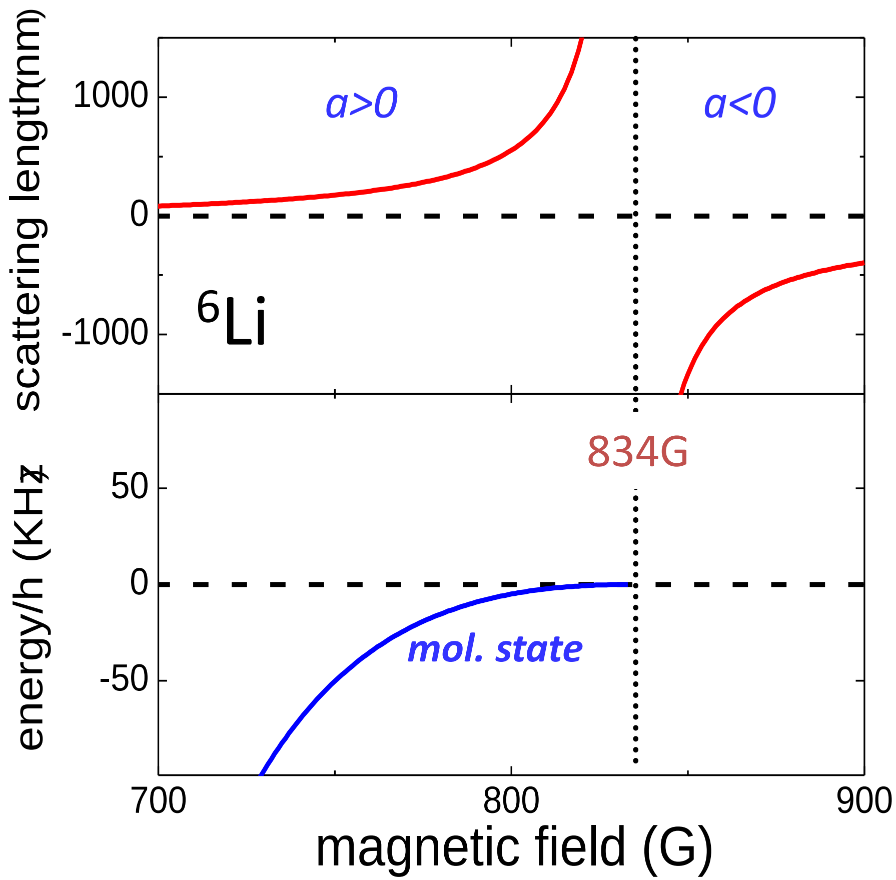
4. Why attraction means no bound state, while repulsion means bound state? Shouldn't it be the opposite?

5. How does a bound state disappear above resonance?

6. What does divergence of atomic interaction really mean?

…

Universal behavior in the threshold regime

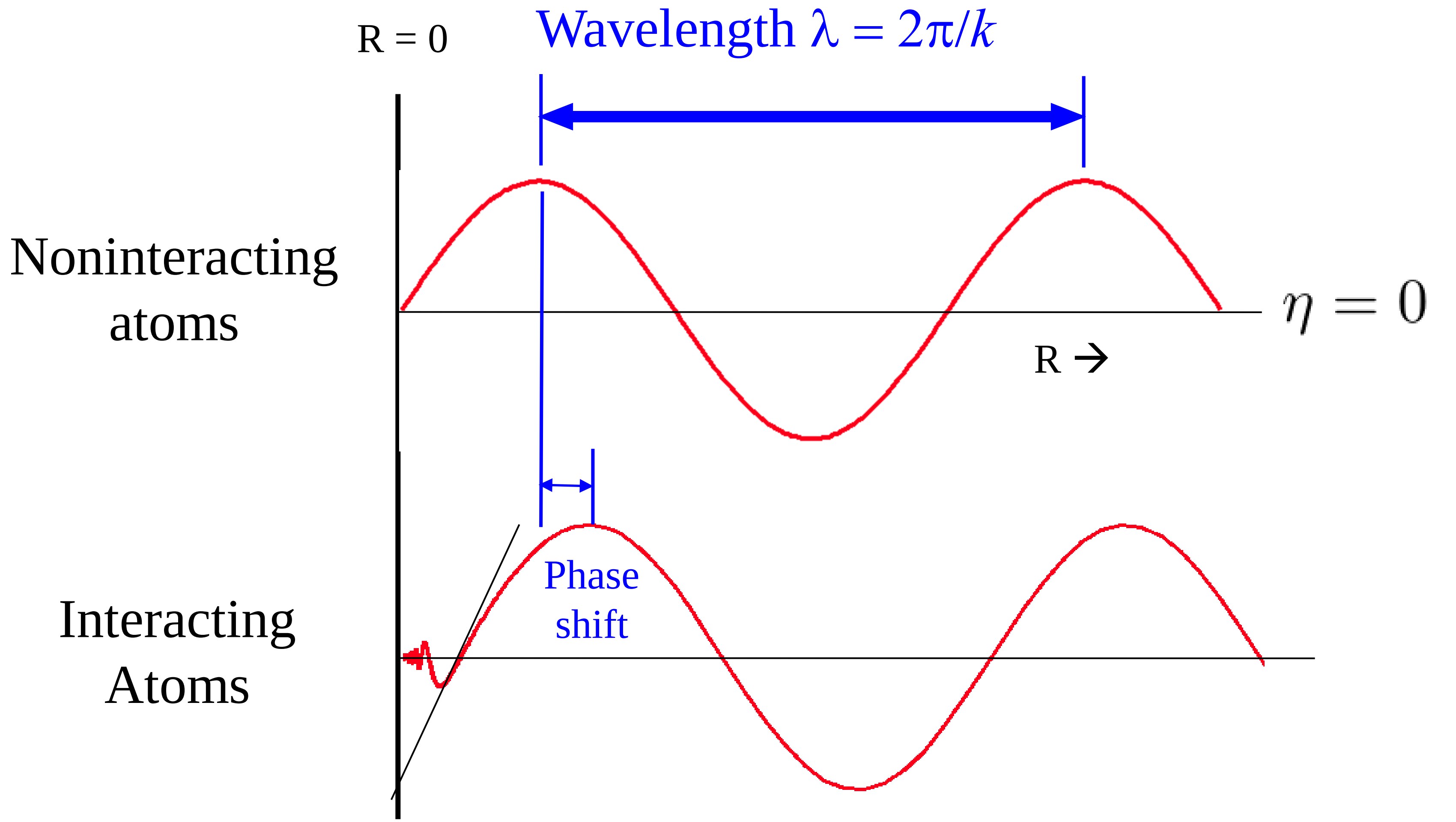


$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

$$E_b = \frac{\hbar^2}{m(a - r_0)^2}$$

Why does molecular state approach the continuum non-linearly?

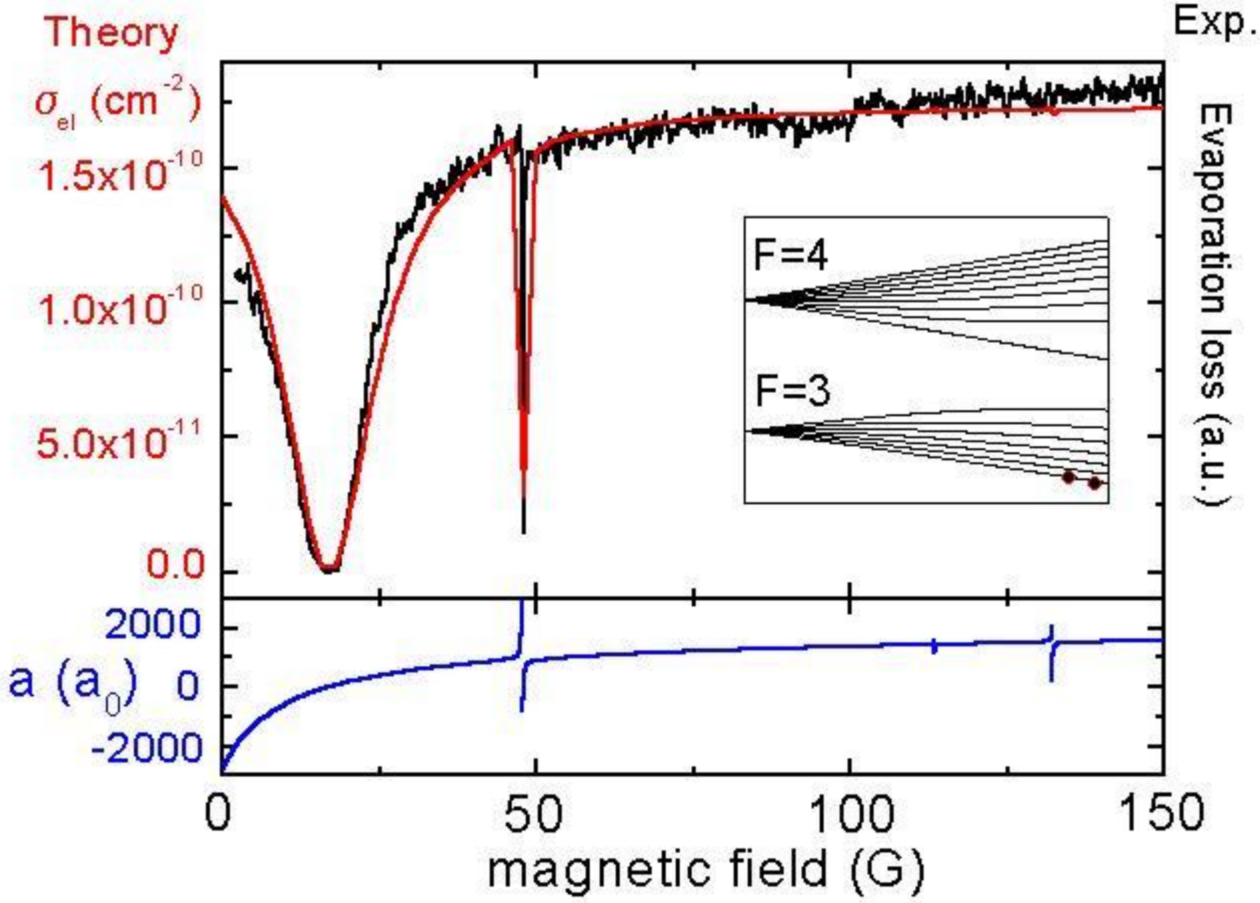
s-wave scattering phase shift $\Psi(R) \rightarrow \sin(kR + \eta)$



$$a = \lim_{\substack{k\to 0 \\ r>>r_0}} -\frac{\psi(r)}{\psi'(r)} = -\lim_{\substack{k\to 0 \\ r>>r_0}} \frac{\sin(kr+\eta)}{k\cos(kr+\eta)} \Rightarrow k\cot\eta = -\frac{1}{a} + O(k^2)$$

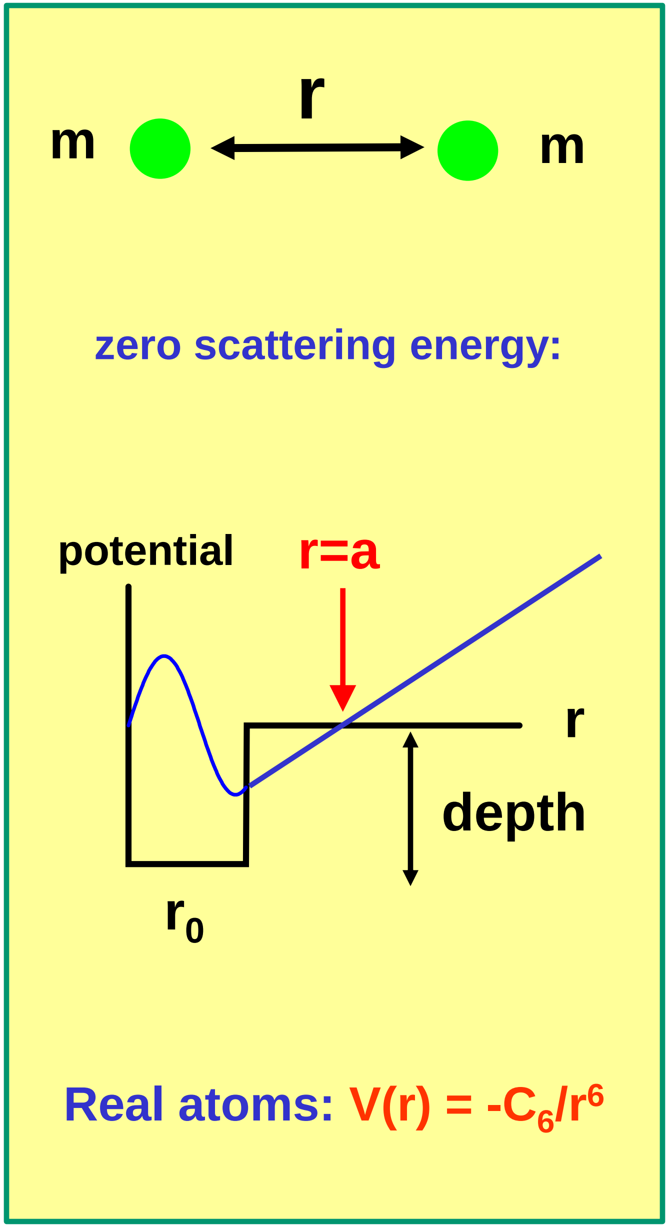
Experiments on Feshbach resonances

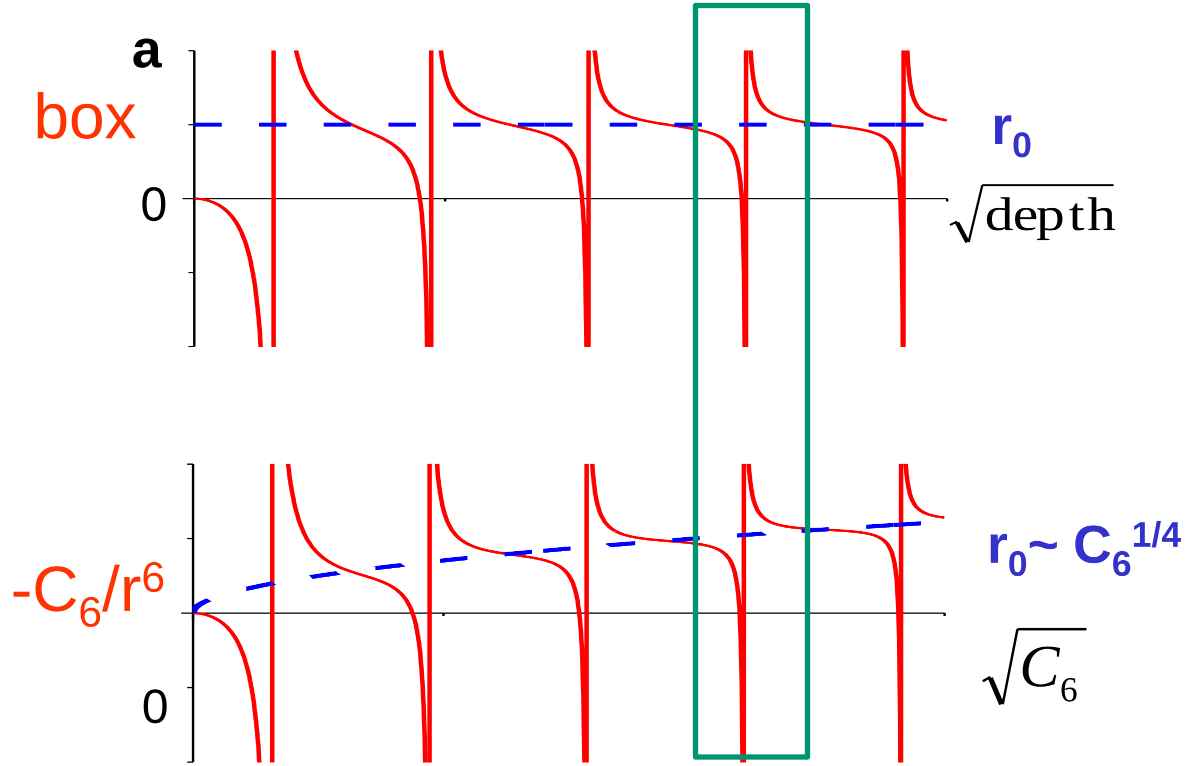
Elastic cross section $\sigma_{el} = \frac{\sin^2 \delta}{k^2} = \frac{4\pi a^2}{1+k^2 a^2}$



Chin et al., PRL ‘00

Single channel (potential resonance)



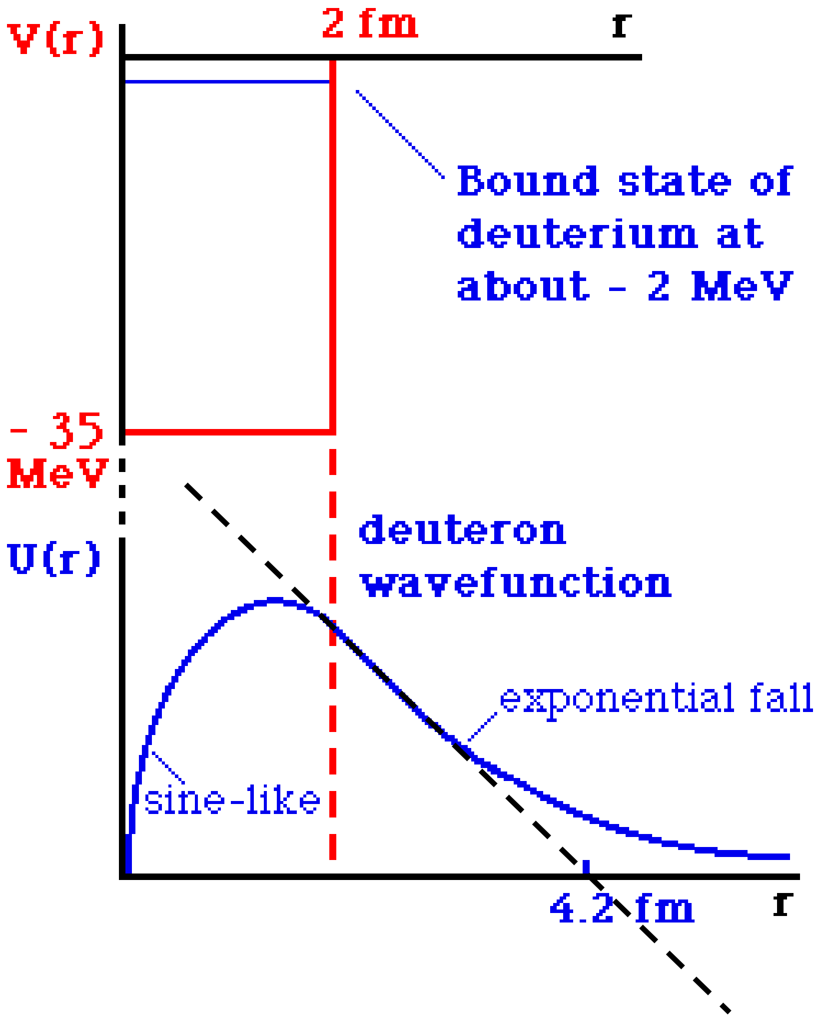


Scattering length *a* is a measure of

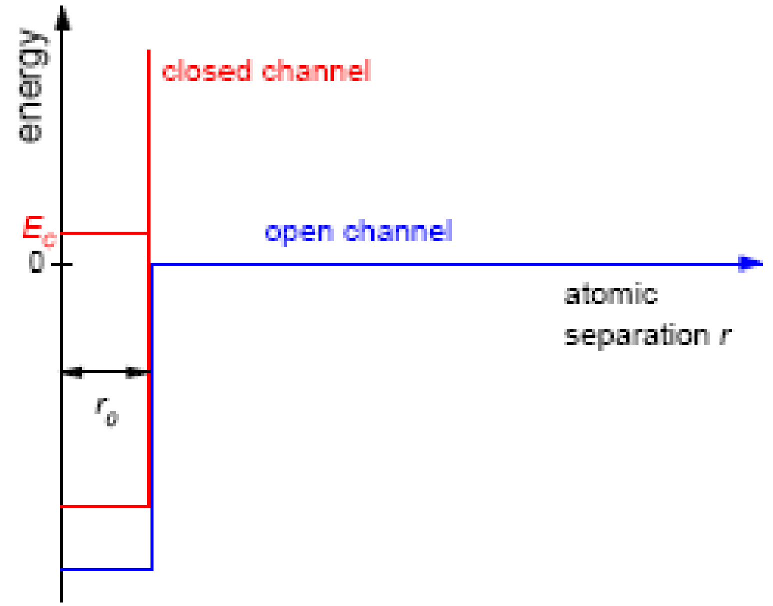
- How far a new bound state is near the continuum
- How much η deviates from π/2, 3π/2…
- How strongly 2 particles interact

Gribakin and Flambaum PRA 48 563

Large positive scattering length ⇒ Bound state
Example: deuterium nucleus



Two-channel model for Feshbach resonance



Parameterization

Open channel $V_o \Rightarrow a_{bg}$

Closed channel $V_c \Rightarrow E_c(B)$

Coupling $\Omega \Rightarrow \Delta B$

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \hat{V}\right)|\psi\rangle = E|\psi\rangle$$

$$\hat{V} = \begin{pmatrix} -V_c & \hbar\Omega \\ \hbar\Omega & -V_o \end{pmatrix}$$ **Short range**

$$= \begin{pmatrix} \infty & 0 \\ 0 & 0 \end{pmatrix}$$ **Long range**

Result:

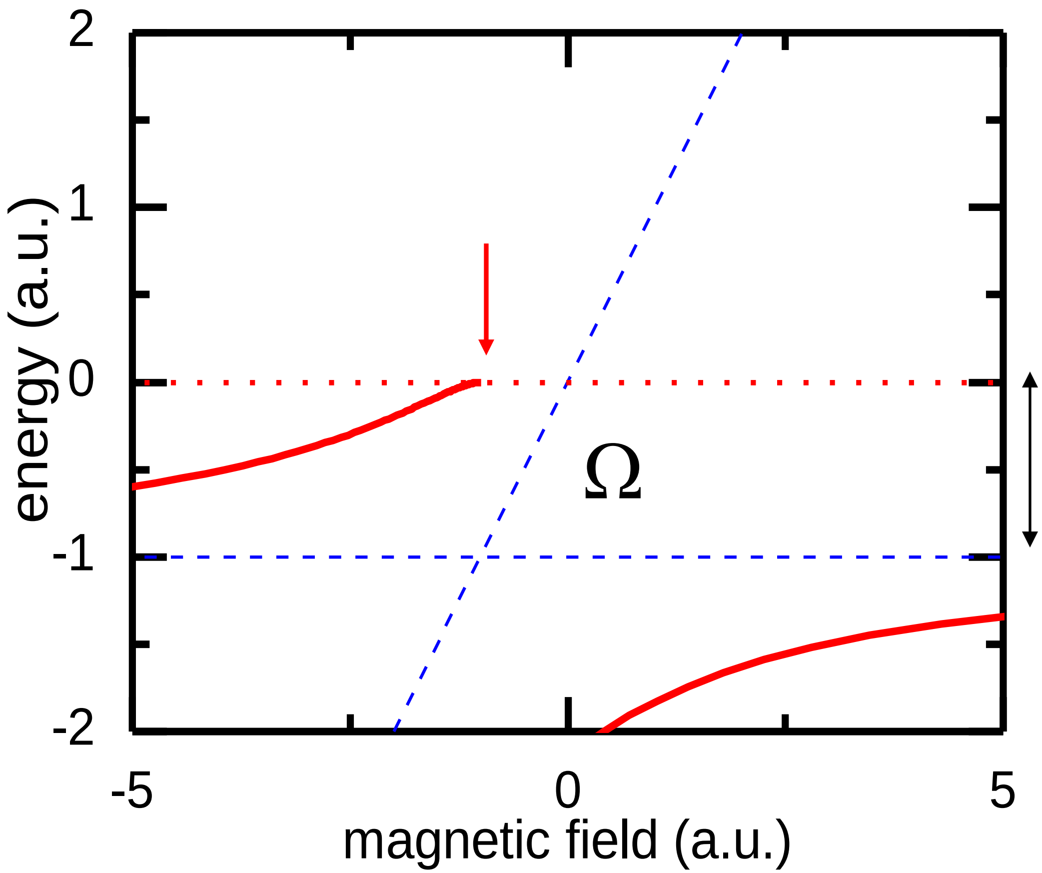
Single channel:

$$k\cot\eta = -\frac{1}{a_{bg} - r_0}$$

Two channel:

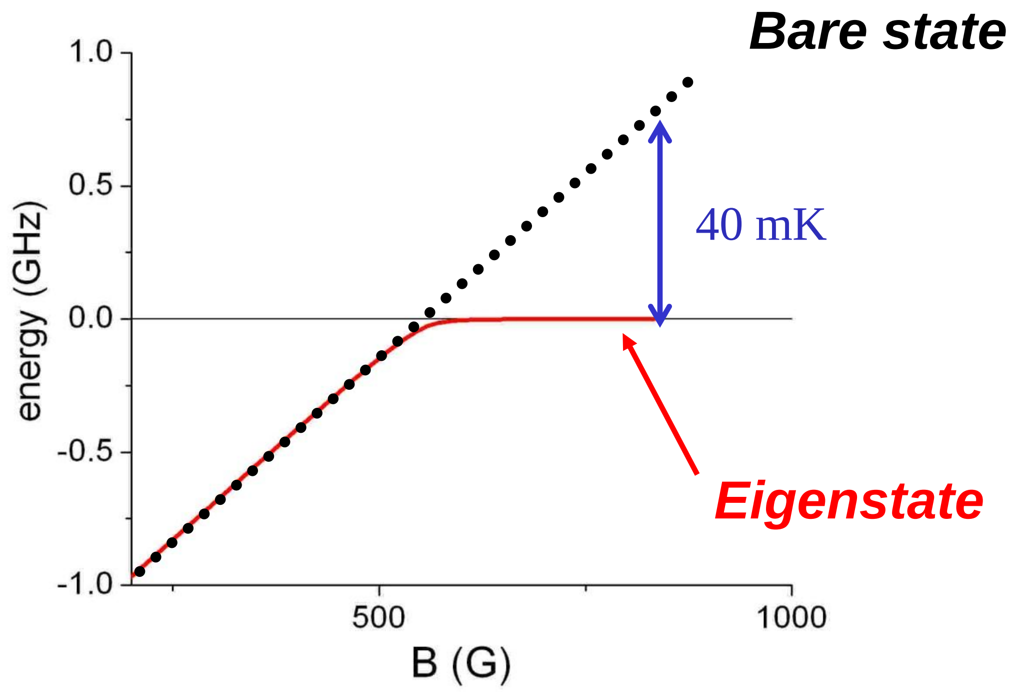
$$k\cot\eta = -\frac{1}{a_{bg} - r_0} + \frac{\Gamma/2r_0}{E_i}$$

Molecular Energy for $a_{bg} > 0$: Na, K, Rb-87, Cs-133



$$\frac{\hbar^2}{m(a_{bg} - r_0)^2}$$

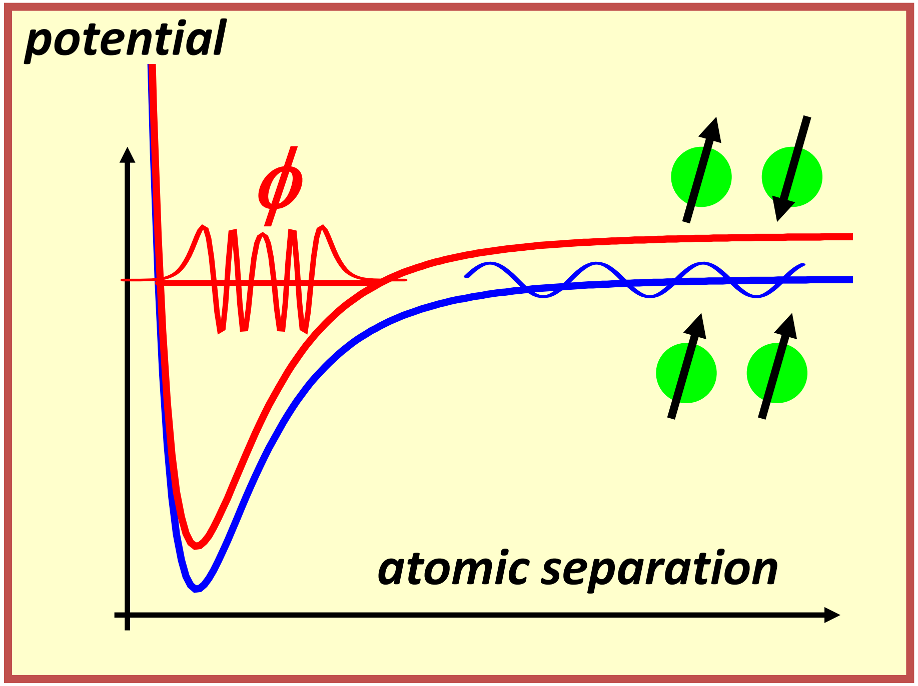
^{6}Li Broad Feshbach resonance with a_{bg}= -1700 a_0



$$k \cot \eta = -\frac{1}{a_{bg} - r_0} + \frac{\Gamma / 2r_0}{E_i}$$

r_0=30 a_0
E_i=bare state energy
Γ=Feshbach coupling

Scattering channels and Feshbach resonance



Open channel

(typically) Triplet potential

Closed channel

(typically) Singlet potential

Feshbach tuning

External magnetic field

Transition matrix

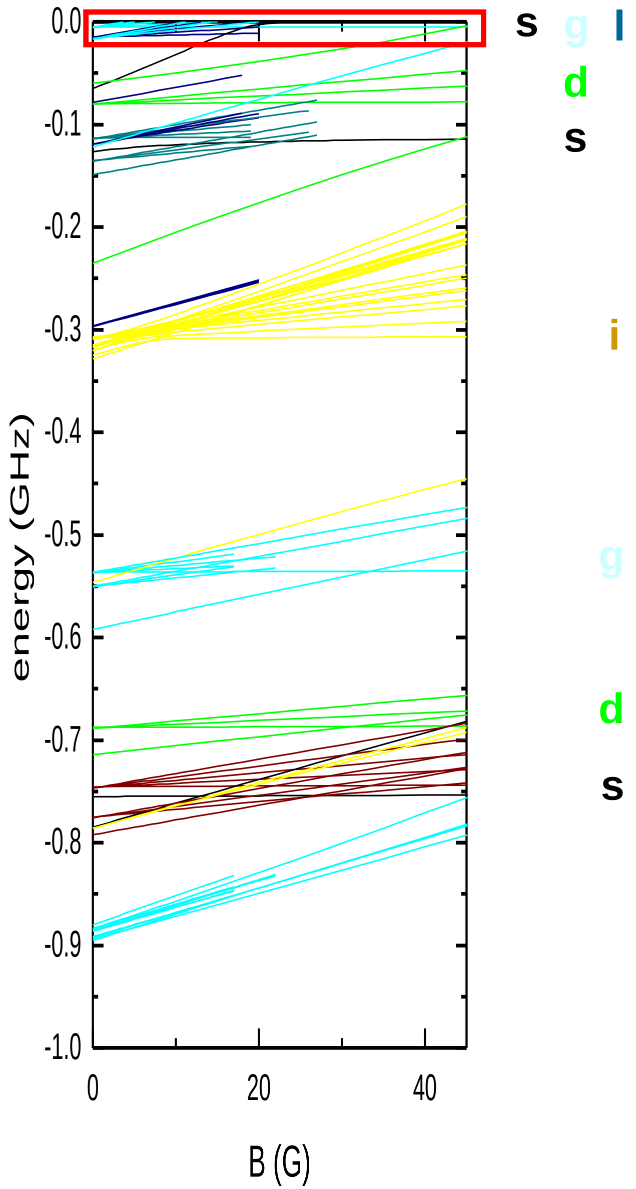
$$T_{fi} = T_{fi}^{0} + \frac{\langle \chi_f^- | V | \phi \rangle \langle \phi | V | \chi_i^+ \rangle}{E - E_\phi + i\Gamma/2}$$

→

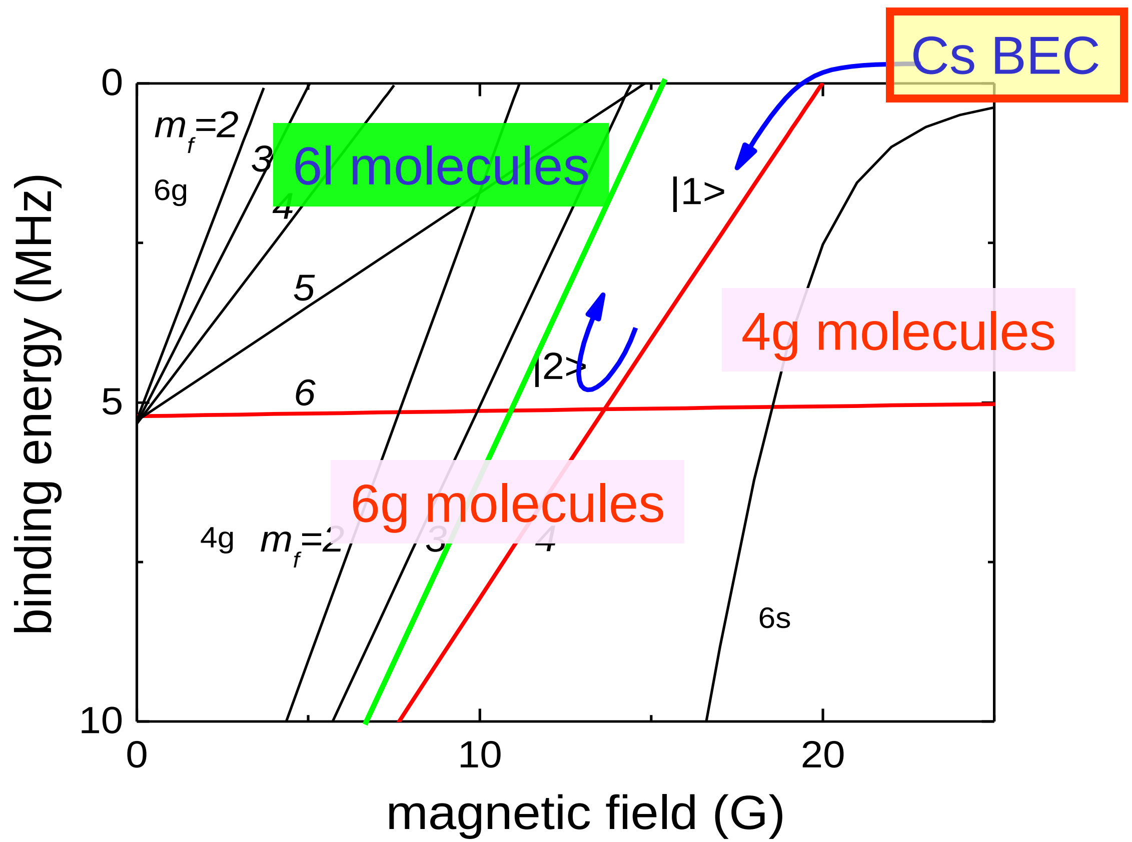
Scattering length:

$$a = a_{bg}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

Cs_2 molecular structure near the continuum



G-wave molecules: orbital angular momentum L = 4 hbar
G-wave resonance: coupling which changes L by 4hbar
due to high order spin-orbit interaction



E. Tiesinga and P. Julienne, NIST

Feshbach resonances in cold atom collisions

TABLE IV. Properties of selected Feshbach resonances. The first column describes the atomic species and isotope. The next three columns characterize the scattering and resonance states, which include the incoming scattering channel (ch.), partial wave ℓ, and the angular momentum of the resonance state ℓ_c. This is followed by the resonance location B_0, the width Δ, the background scattering length a_{bg}, the differential magnetic moment $\delta\mu$, the dimensionless resonance strength s_{res}, the background scattering length in van der Waals units $r_{bg}=a_{bg}/\bar{a}$, and the bound state parameter ζ from Eq. (52). Here a_0 is the Bohr radius and μ_B is the Bohr magneton. Definitions are given in Sec. II. The last column gives the source. A string "na" indicates that the corresponding property is not defined. For example a_{bg} is not defined for p-wave scattering.

Atom	ch.	ℓ	ℓ_c	B_0 (G)	Δ (G)	a_{bg}/a_0	$\delta\mu/\mu_B$	s_{res}	r_{bg}	ζ	Reference
^{6}Li	*ab*	*s*	*s*	834.1	−300	−1405	2.0	59	−47	1400	Bartenstein *et al.*, 2005
	ac	*s*	*s*	690.4	−122.3	−1727	2.0	29	−58	850	Bartenstein *et al.*, 2005
	bc	*s*	*s*	811.2	−222.3	−1490	2.0	46	−50	1200	Bartenstein *et al.*, 2005
	ab	*s*	*s*	543.25	0.1	60	2.0	0.001	2.0	0.001	Strecker *et al.*, 2003
	aa	*p*	*p*	159.14	na	na	2.0	na	na	na	Zhang *et al.*, 2004; Schunck *et al.*, 2005
	ab	*p*	*p*	185.09	na	na	2.0	na	na	na	Zhang *et al.*, 2004; Schunck *et al.*, 2005
	bb	*p*	*p*	214.94	na	na	2.0	na	na	na	Zhang *et al.*, 2004; Schunck *et al.*, 2005
^{7}Li	*aa*	*s*	*s*	736.8	−192.3	−25	1.93	0.80	−0.79	0.31	Strecker *et al.*, 2002; Pollack *et al.*, 2009[a]
^{23}Na	*cc*	*s*	*s*	1195	−1.4	62	−0.15	0.0050	1.4	0.004	Inouye *et al.*, 1998; Stenger *et al.*, 1999[a]
	aa	*s*	*s*	907	1	63	3.8	0.09	1.5	0.07	Inouye *et al.*, 1998; Stenger *et al.*, 1999[a]
	aa	*s*	*s*	853	0.0025	63	3.8	0.0002	1.5	0.0002	Inouye *et al.*, 1998; Stenger *et al.*, 1999[a]

Reference: CC, Rudolf Grimm, Paul Julienne, Eite Tiesinga, RMP (2012)

Summary of Feshbach resonances

Tools to control atomic interactions

B field tunability: simulating condensed matter, nuclear physics,

Many-body applications: BEC-BCS crossover, Hubbard model…

Pairing atoms into molecules: Feshbach molecules, Efimov trimers

Toward quantum manipulation: Coherent control of entangelment

New ideas

Test of fundamental constant variation

Dipolar quantum gas

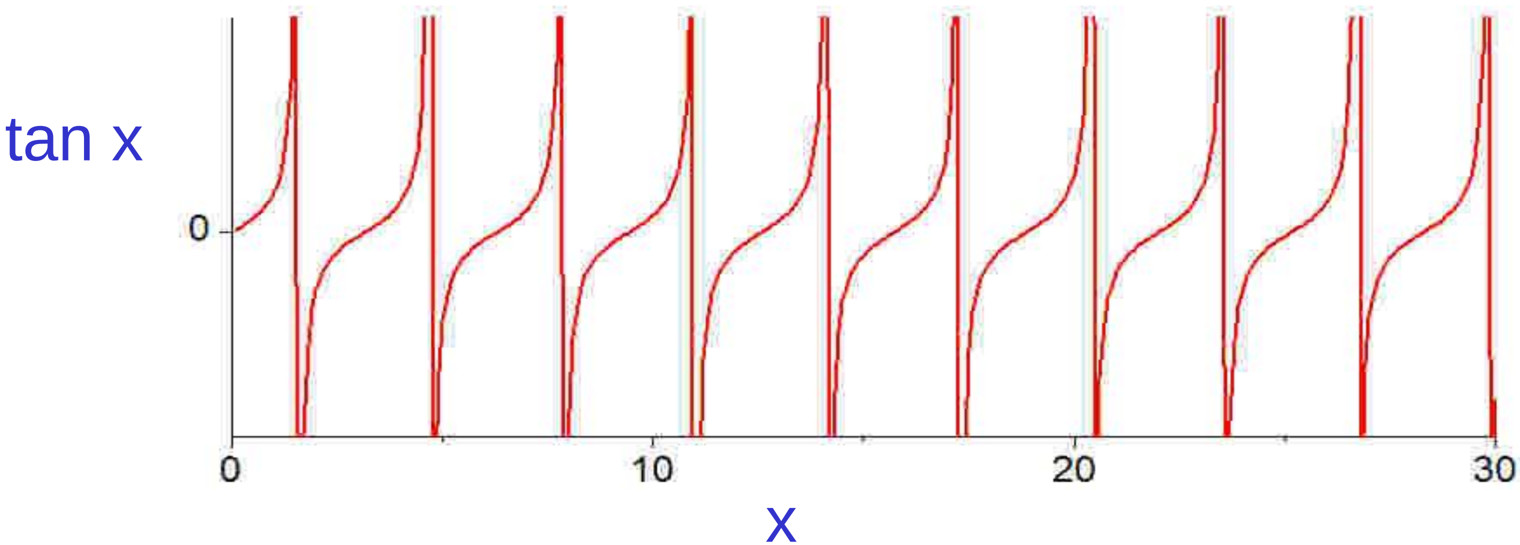
Strongly interacting systems

Anecdote of Richard Feynman

"I can calculate anything to 10% in 1 minute, if the question can be described in 10s."

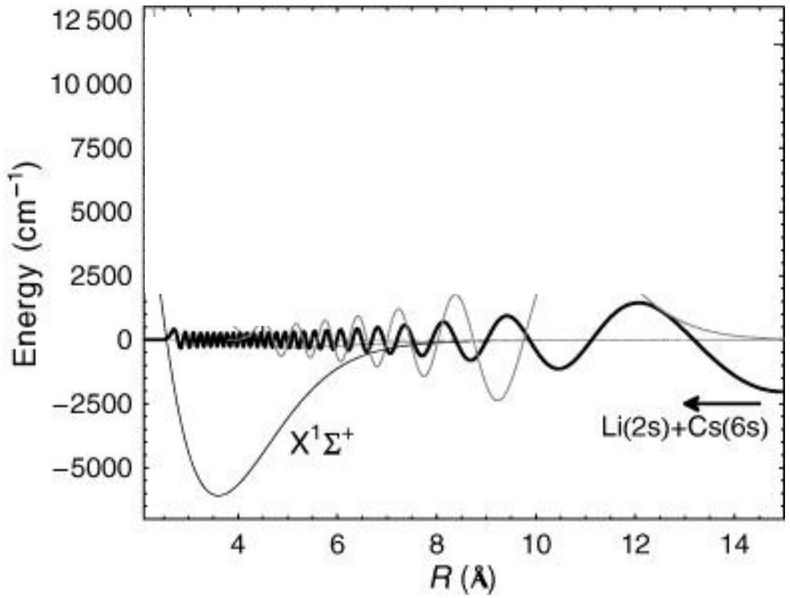
- "Surely you are joking, Mr. Feynman!"

$\tan 10^{10} = ?$



Periodicity and divergence…

Scattering length and scattering phase shifts



$$a = \bar{a}\left(1 - \tan(\phi - \pi/8)\right)$$

$$\phi = \int_{r_i}^{\infty} \sqrt{\frac{m}{\hbar^2} |V(r)|}\, dr$$

- **Molecular potential: electromagnetic**
- **Molecular mass: baryonic**

⇒Collision phase shift $\phi \propto N\,(m_e/m_p)^{1/2}$, N=# of mol. states

$$\frac{\delta a}{a} = \frac{\phi}{\pi}\,\frac{(a-\bar{a})^2 + \bar{a}^2}{a\bar{a}}\,\frac{\delta\beta}{\beta} = K\,\frac{\delta\beta}{\beta},$$

What is sensitive to the change of fundamental constants?

$$\frac{\delta X}{X} = K \frac{\delta m_e / m_p}{m_e / m_p}$$

$$X = 1000 \frac{m_e}{m_p}$$

$$\frac{\delta X}{X} = \frac{\delta m_e / m_p}{m_e / m_p}$$

$$X = (\frac{m_e}{m_p})^4 = \beta^4$$

$$\frac{\delta X}{X} = 4 \frac{\delta m_e / m_p}{m_e / m_p}$$

$$X = \sin 1000 (\frac{m_e}{m_p})$$

$$\frac{\delta X}{X} = 1000 \frac{\delta m_e / m_p}{m_e / m_p}$$

Feshbach resonances and variation of m_p/m_e

$$\frac{\delta a}{a} = \frac{M}{2}\frac{(a-a_{bg})^2}{aa_{bg}}\frac{1}{\rho(E)\Delta E}\frac{\delta\beta}{\beta}$$

M: # of bound states
P(E): density of state
ΔE: coupling strength
β: m_e/m_p

1. Many molecular states
2. Large scattering length
3. Low density of state (DOS)
4. Weak coupling strength

Overall enhancement factor $K=10^9$

5. Optical Feshbach resonance
(“another 1000x, no magnetic field dependence”, Jun Ye)

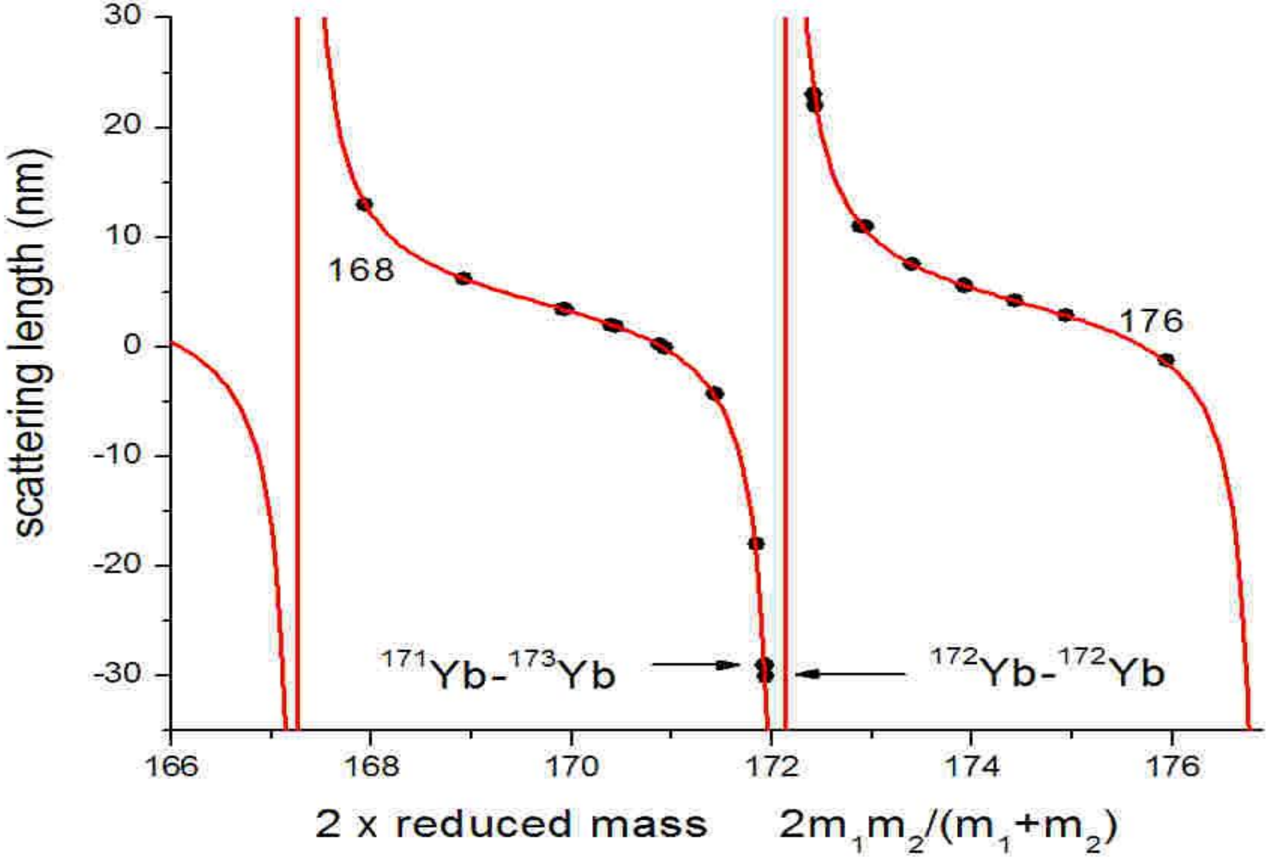
Refs: CC, M Koslov and V. Flambaum, NJP 2009 and CC, V. Flambaum PRL (2006)

Experiment progress:

Yb experiment (Takahashi, Kyoto)

Cs clock experiment (Gibble, Penn State)

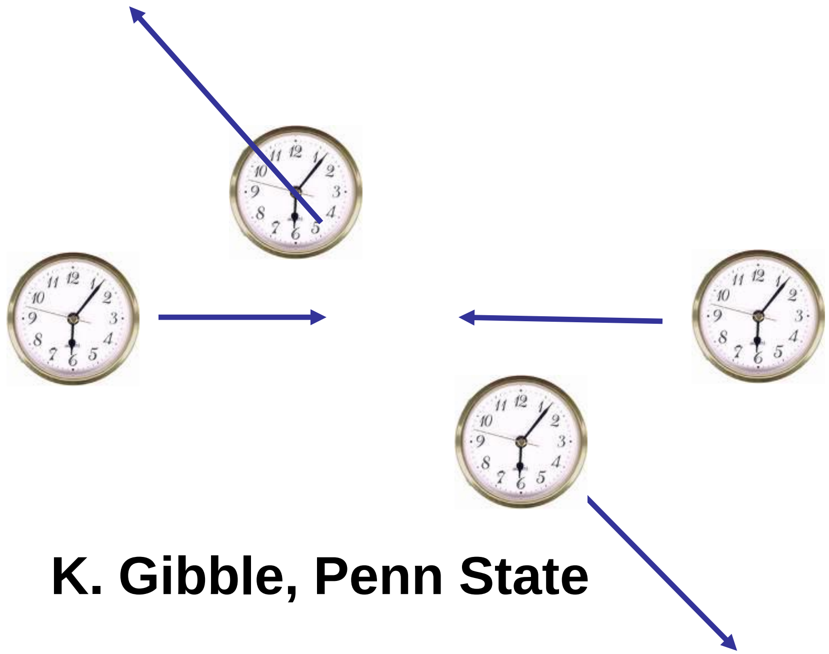
Scattering lengths of Yb isotopes



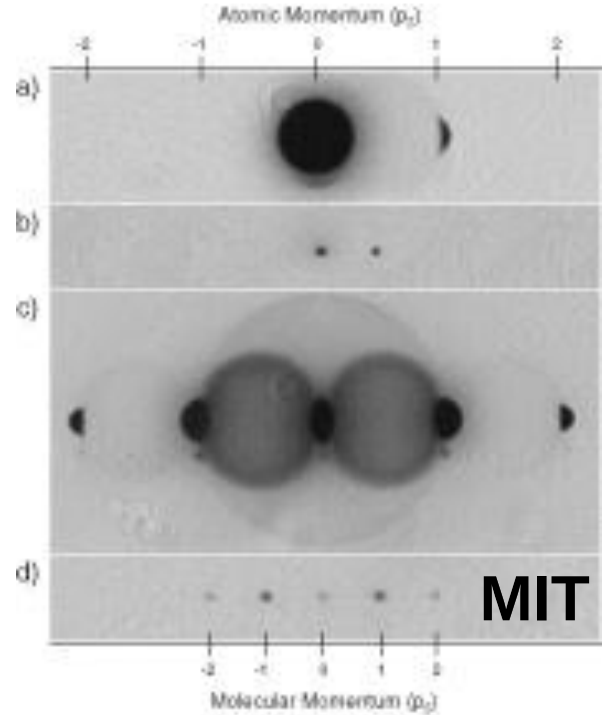
$$\frac{\delta a}{a} = 1230 \frac{\delta\, m_{atom} / m_e}{m_{atom} / m_e}$$

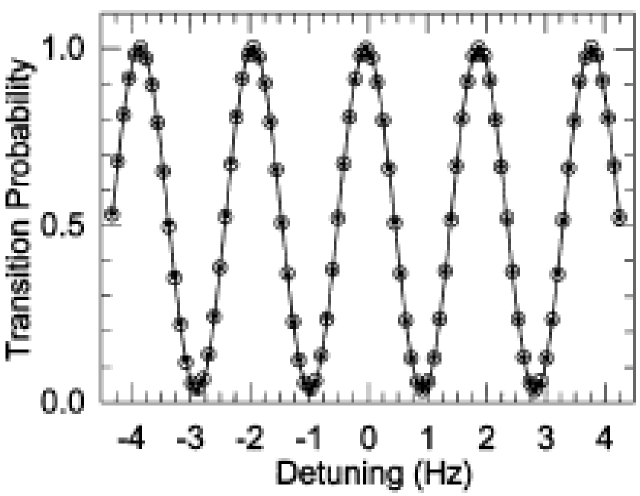
Data from Takahashi, Kyoto
Theory: Paul Julienne

Colliding Cs atomic Clocks



K. Gibble, Penn State





Expected precision: few μHz

$\delta a/a = 10^{-5} \sim 10^{-6}$

$\Rightarrow \; \delta\beta/\beta = 10^{-14} \sim 10^{-15}$

R. A. Hart et al., Nature **446**, 892-895 (2007).