Summer School on Quantum Many-Body Physics of Ultra-Cold Atoms and Molecules

Lecture 1: Overview of tools in cold atom experiments
(11:30 am, July 12, 2012)

Lecture 2: In situ imaging: density, fluctuations, correlations and equation of state
(3:30 pm, July 12, 2012)

Lecture 3: Scale invariance, universality and quantum criticality of 2D quantum gases
(9 am, July 13, 2012)

WORKSHOP ON QUANTUM SIMULATIONS WITH ULTRACOLD ATOMS

Talk: Exploring Universal Quantum Physics in few- and many-body atomic systems
(9:40am, July 17, 2012)

Cheng Chin, James Franck institute and Department of Physics, University of Chicago

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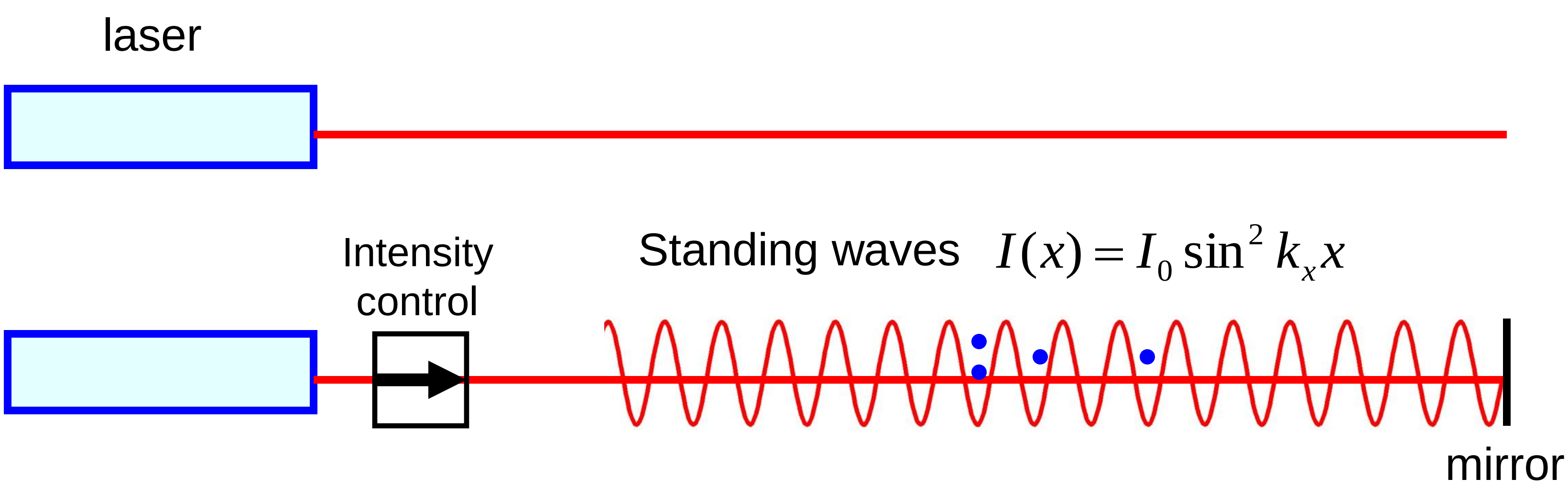
What is an optical lattice?



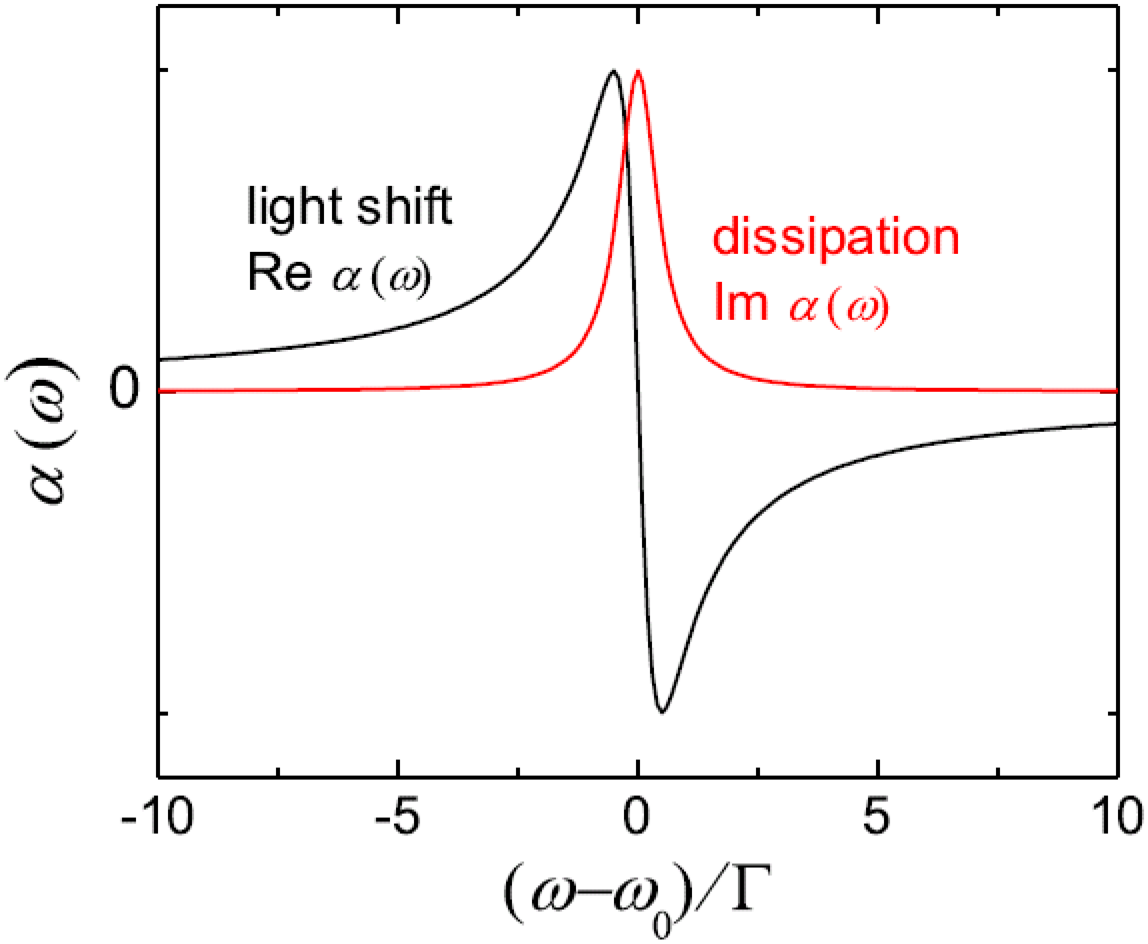
Atomic polarization $\quad P = \varepsilon E$

Optical potential $\quad V = -\int P \cdot dE = -\frac{1}{2}\varepsilon E^2 = -\frac{1}{2}\alpha_{AC} I$

α_{AC} : AC polarizability

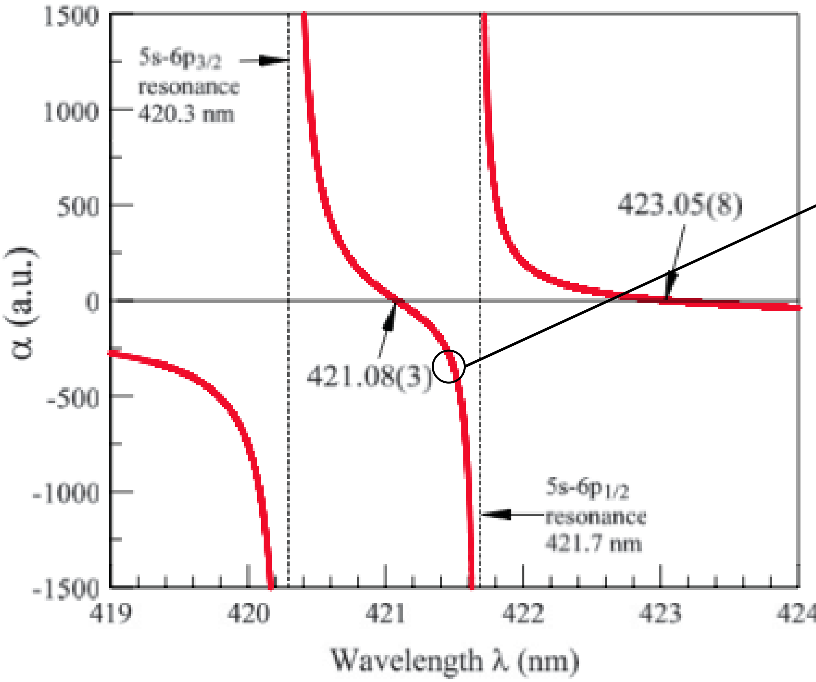
Optical lattice potential $\quad V(x) = -\frac{1}{2}\alpha_{AC} I_0 \sin^2 k_x x$

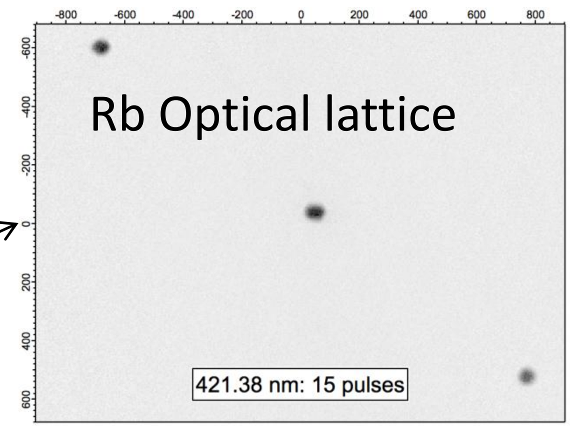
AC polarizability



Real atom: multiple transitions

Theoretical Light Shifts





Variations:

Magic wavelengths:

Precision measurement

Independent addressing

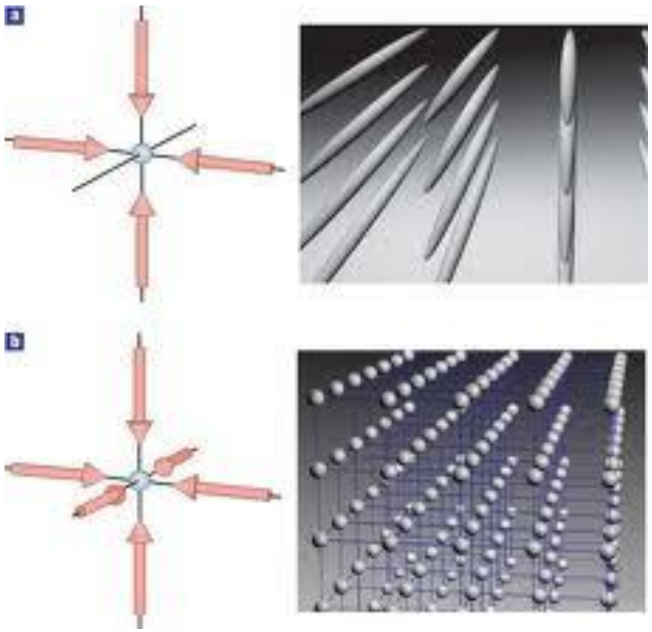
Spin-dependent lattice:

Lattice in higher dimensions

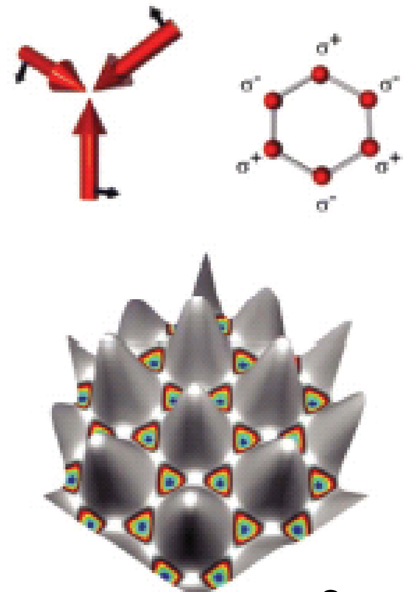
Arora et al. PRA 84 043401 (2011)

Proto group, NIST & JQI

Optical lattice in higher dimensions



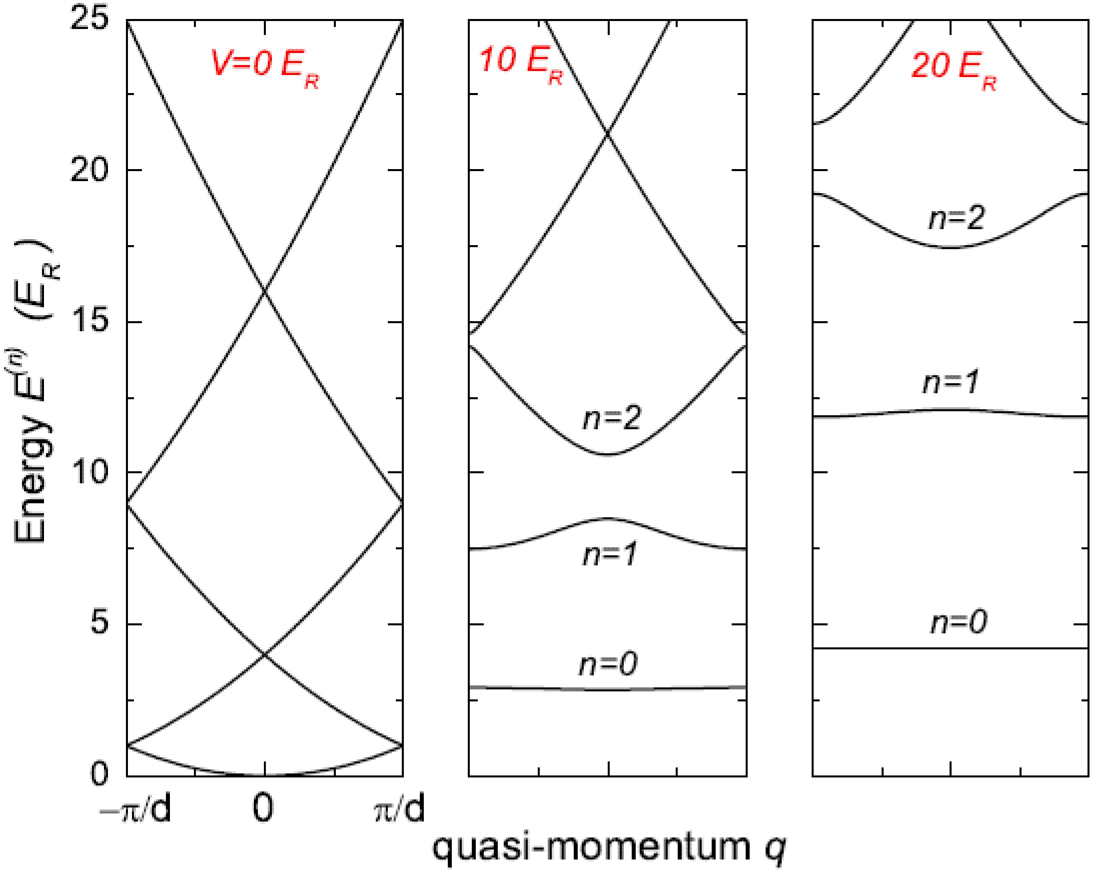
I. Bloch, Munich and MPQ



Sengstock, Hamburg

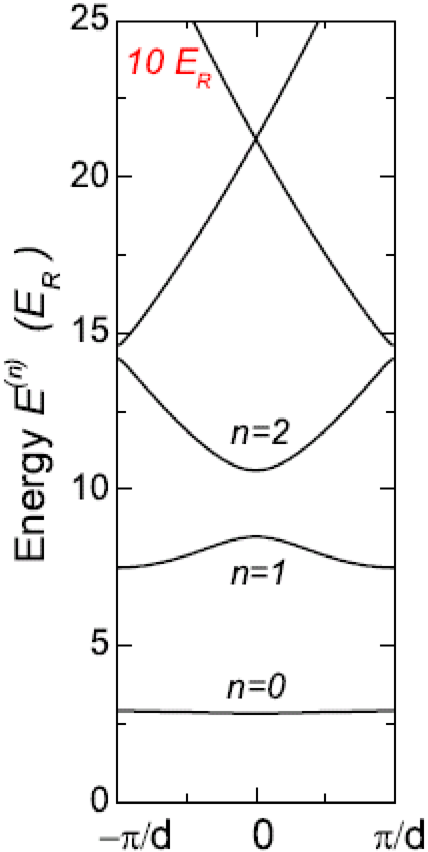
$$V(\vec{x}) = -\frac{1}{2}\alpha(\omega) < |Re[\sum_i \vec{E}_i e^{i(\vec{k}_i\cdot\vec{x}-\omega_i t+\phi_i)}]|^2 >$$

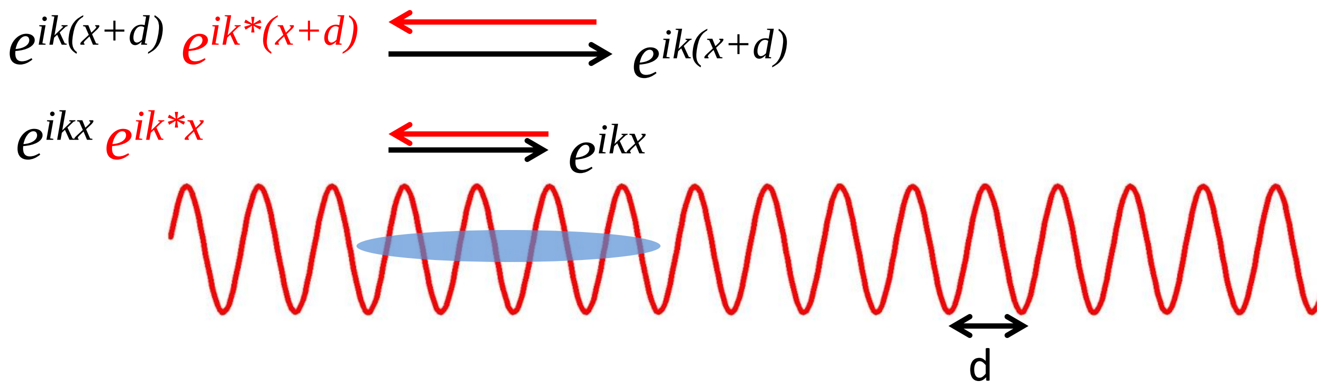
Band structure in optical lattice



$$-\frac{\hbar^2}{2m}\partial_x^2\psi_i(x) + V\sin^2(kx)\psi_i(x) = E_i\psi_i(x)$$

Origin of the band structure: Bragg diffraction

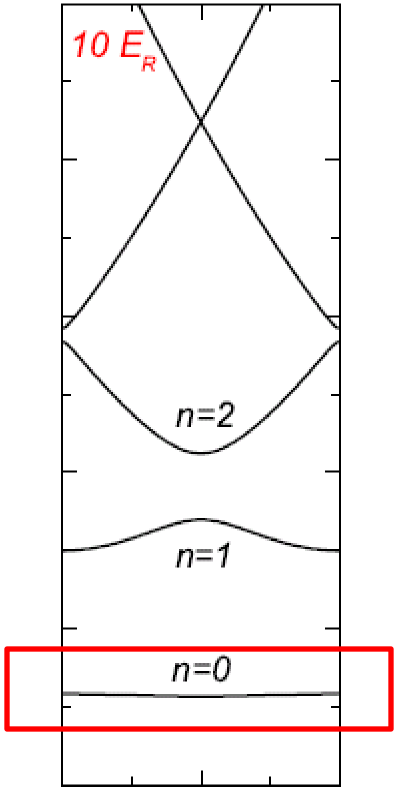




Condition for constructive interference:

$$(k+k^*)d=\pm 2\pi,\ \pm 4\pi,\ \pm 6\pi\ldots$$

Basic properties of particles in optical lattices



Eigen-states are delocalized Bloch waves $\phi_q(x)=e^{iqx}f(x)$

q is quasi-momentum

Eigen-energy E(q) is the dispersion

Localized wave packets, Wannier states, are defined as

$$w(x-x_i)=\sqrt{\frac{d}{2\pi}}\int_{-\pi/d}^{\pi/d} e^{iqx_i}\phi_q(x)dq$$

Hamiltonian in the Wannier state basis is

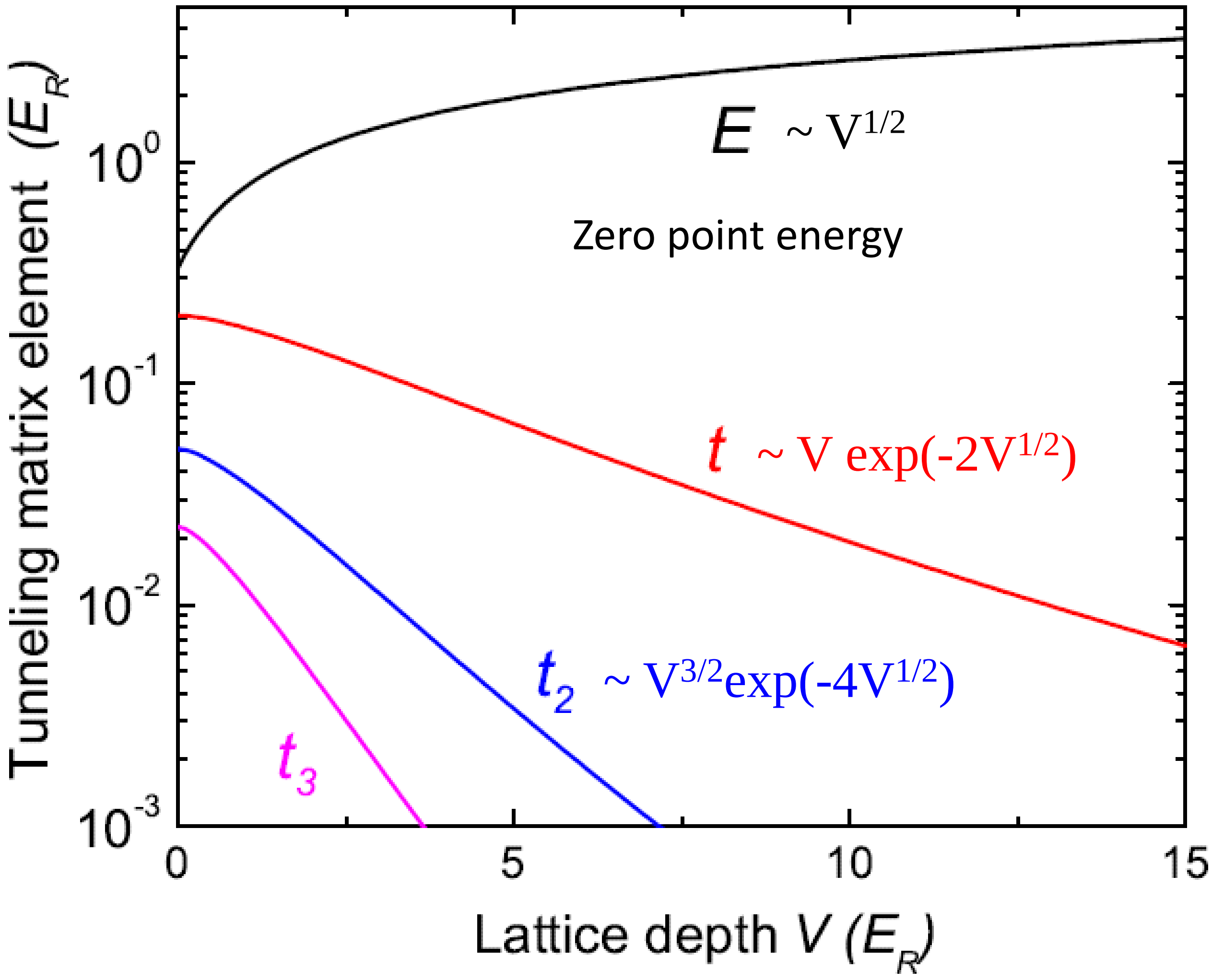
$$<w_i|H|w_j>=\begin{pmatrix} E & -t & t_2 & -t_3 & \dots \\ -t & E & -t & t_2 & \dots \\ t_2 & -t & E & -t & \dots \\ -t_3 & t_2 & -t & E & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

t: nearest neighbor tunneling

$t_{2:}$ next nearest neighbor tunneling

… are the Fourier expansion of E(q)

Energy scales in optical lattices



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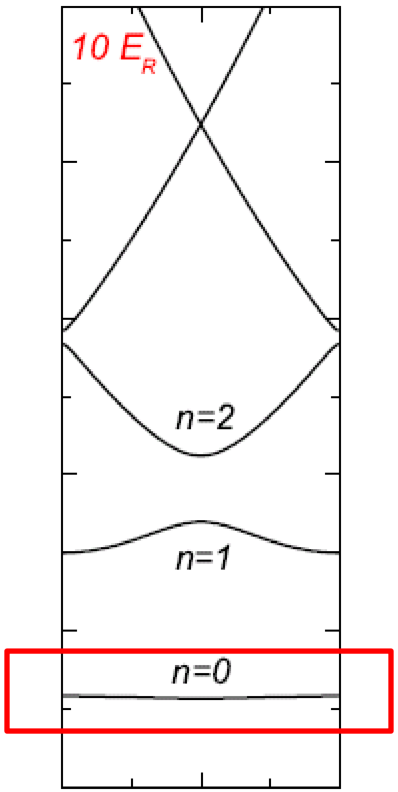

Quasi-momentum

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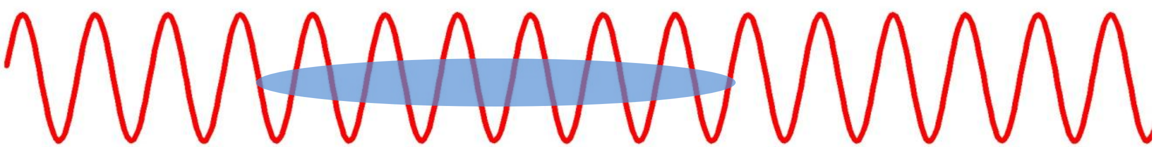
t: nearest neighbor tunneling

$t_{2:}$ next nearest neighbor tunneling

… are the Fourier expansion of E(q)

Interaction of atoms in lattices

General wave function $\hat{\psi} = \sum_i \hat{a}_i w(x - x_i)$



$$V = \frac{1}{2}\int \hat{\psi}^+(x_1)\hat{\psi}^+(x_2)V(x_1 - x_2)\hat{\psi}(x_1)\hat{\psi}(x_2)dx_1 dx_2$$

$$\approx \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$$

$$U = \frac{\hbar\omega}{\sqrt{2\pi}}\frac{a}{\ell}$$

V(x): (contact) interaction
$n_{i:}$ particle number at site *I*
a: scattering length
ω *: mean trap frequency*
l : mean oscillator length

Bose-Hubbard model *(Fisher et al., PRB 1989, Greiner et al., Nature 2002)*

$$\hat{H} = -t \sum_{<i,j>} (\hat{b}_i^+ \hat{b}_j + \hat{b}_j^+ \hat{b}_i) + U \sum_i \frac{\hat{n}_i(\hat{n}_i - 1)}{2} - \sum_i \mu_i \hat{n}_i$$

Tunneling

Interaction

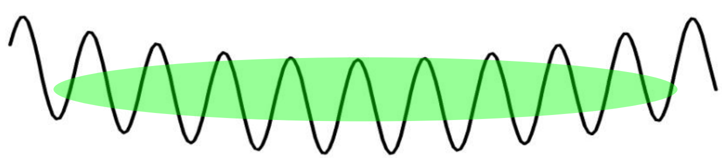
Trap potential

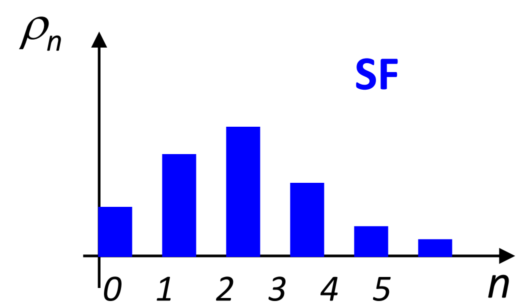
Density distribution

Number statistics

t >> U

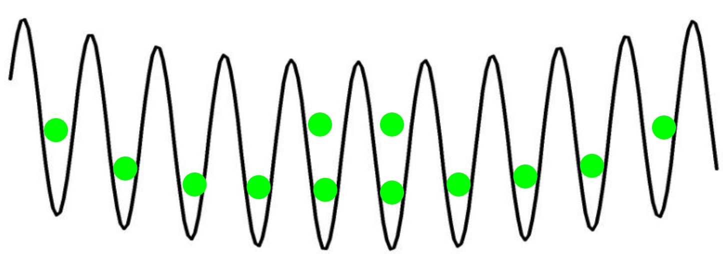
Superfluid

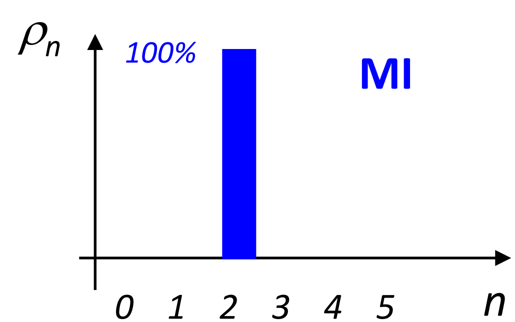




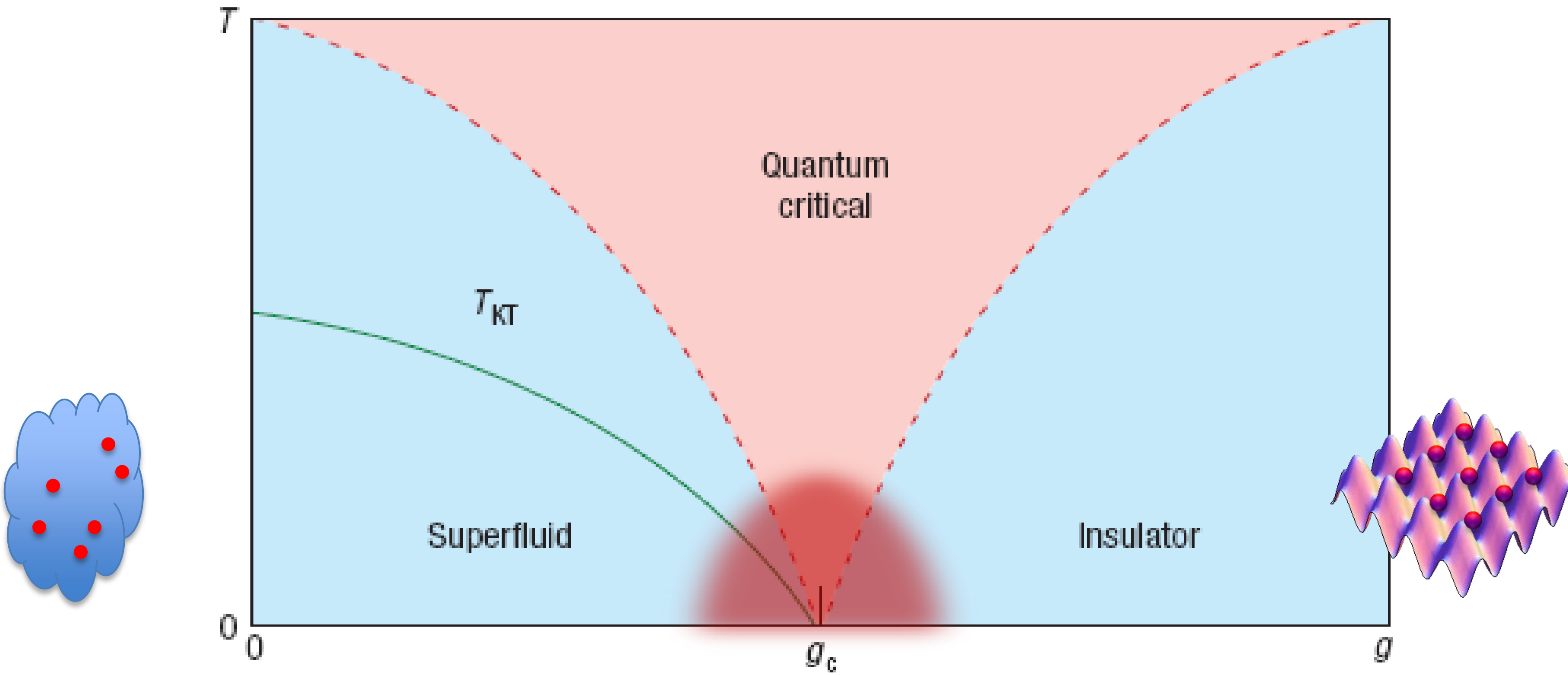
t << U

Mott insulator





Quantum phase transition of bosons in 2D lattices



Ads-CFT duality: Sachdev, Nature physics (2007)

$$\frac{n-n_c}{T^{d/z+1-1/\nu z}} = h\left(\frac{\mu-\mu_c}{T^{1/\nu z}}\right)$$

Critical thermodynamics: Zhou and Ho, PRL (2010)

Hazzard and Mueller , PRA (2011)

AdS-CFT duality

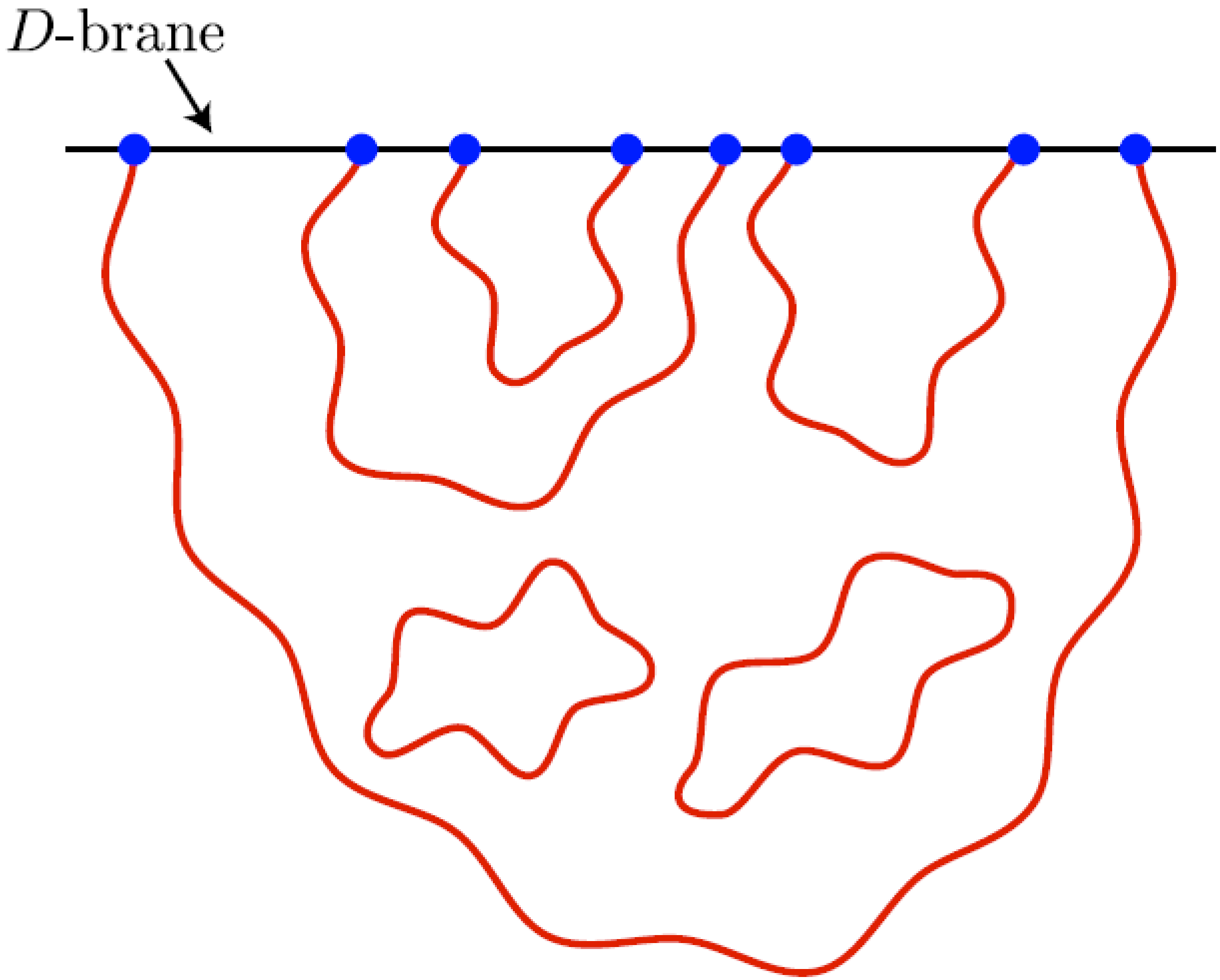
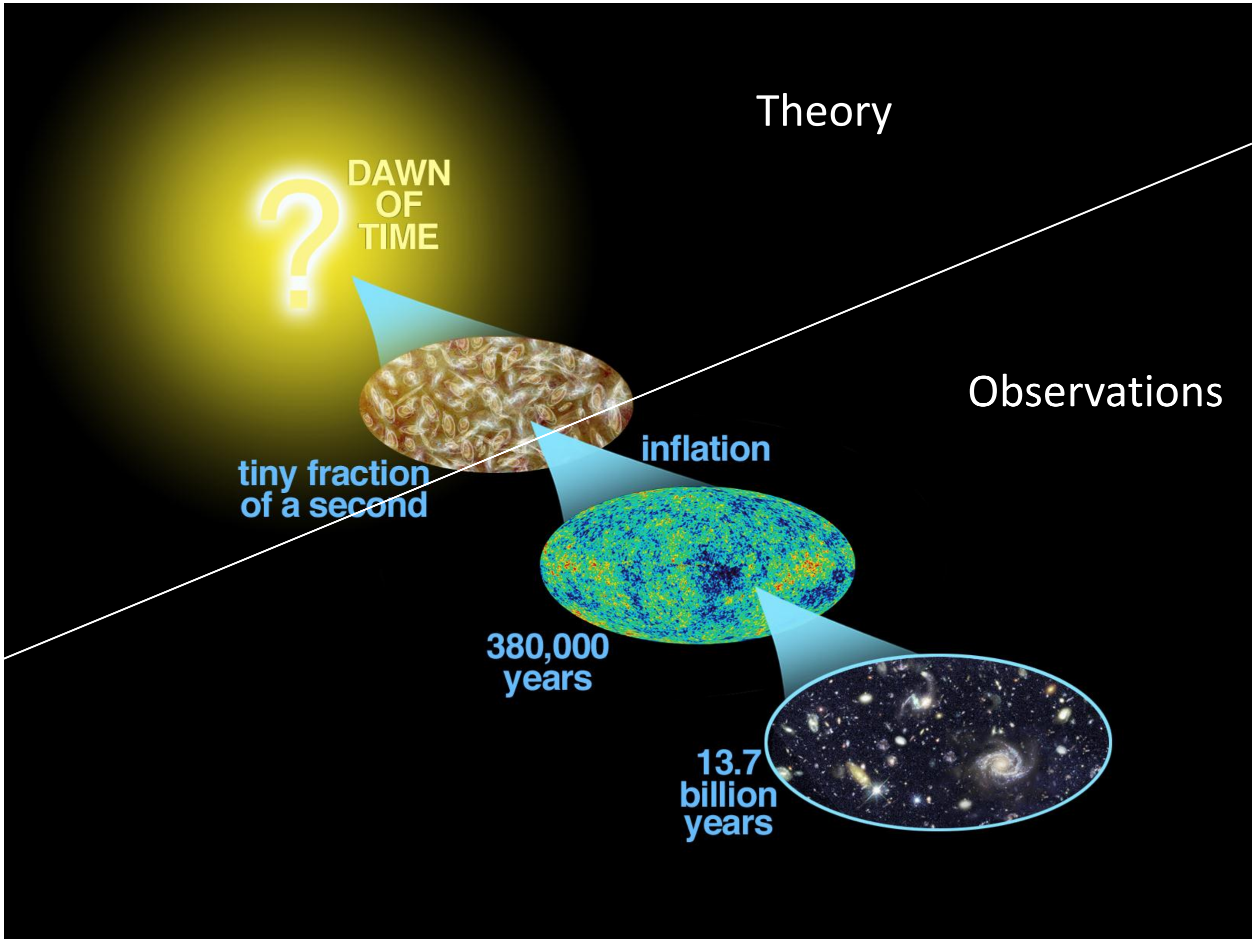


Figure: Subir Sachdev (2011)

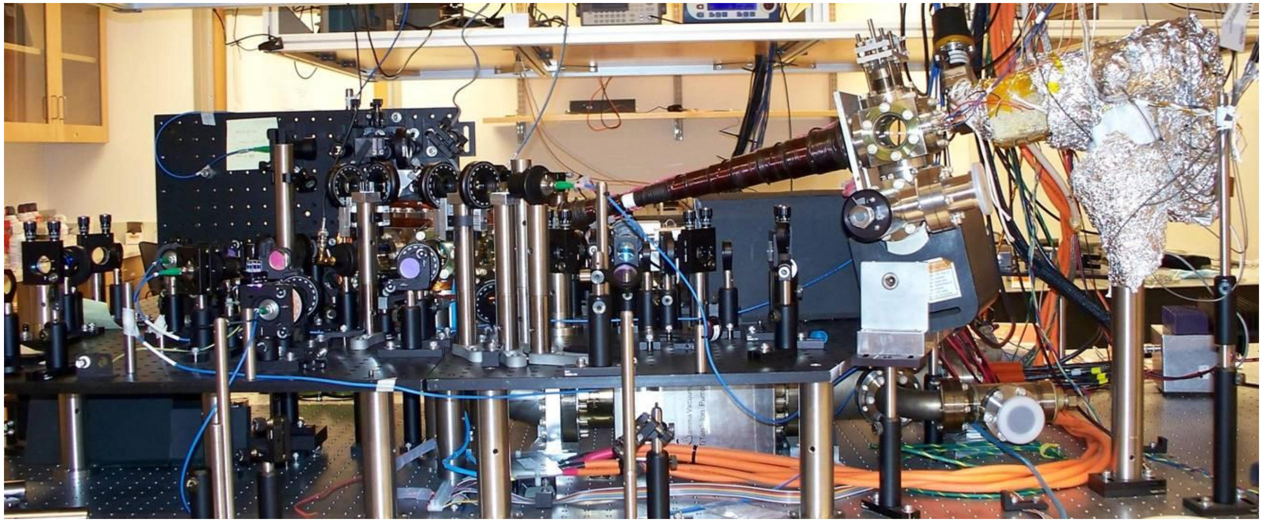
Juan Maldacena (1997)

Evolution of early universe

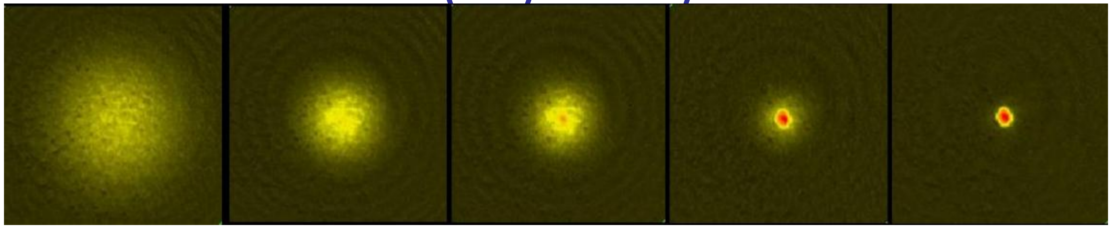


Source: NASA/WMAP Science Team

Cesium-133 Bose-Einstein condensate apparatus



First cold atom apparatus at Univ. of Chicago (10/2006)



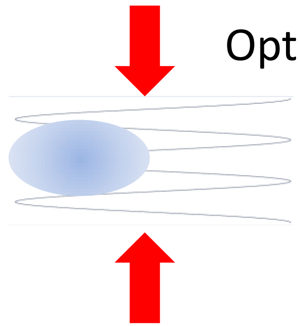
Fast evaporative cooling and Bose condensation (11/07/2007)

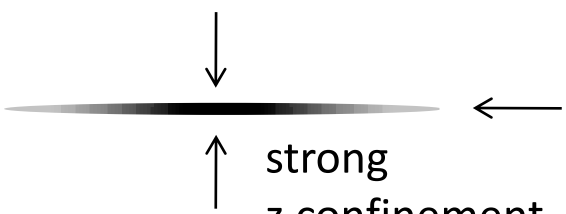
Hung, Zhang, Gemelke, and Chin, PRA (2008)

Preparation of a 2D quantum gas

Optical lattice

Bose condensate

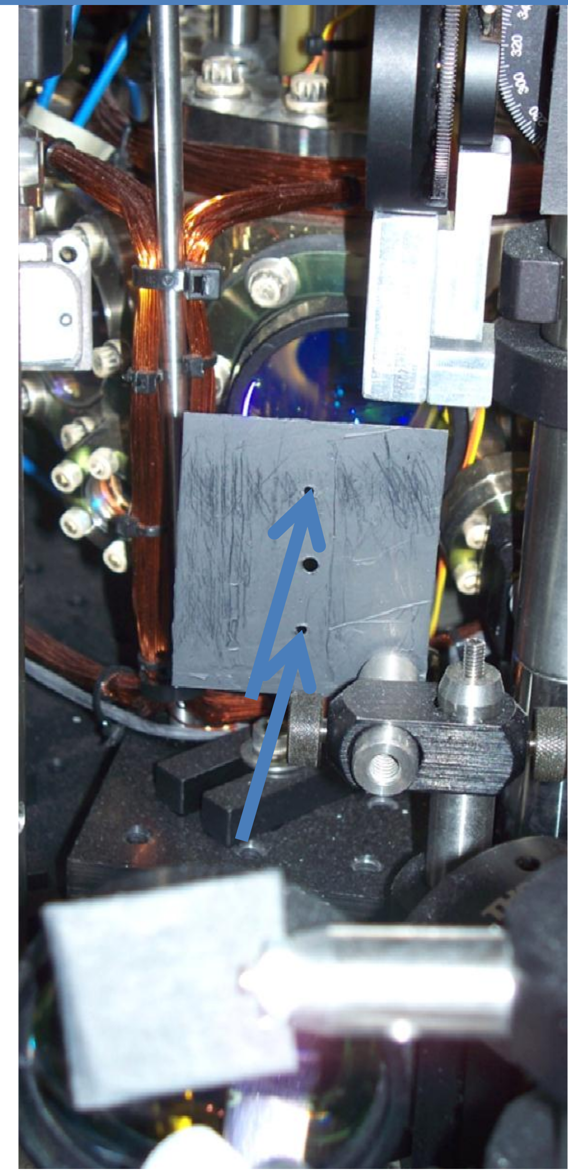




weak
radial confinement

strong
z confinement

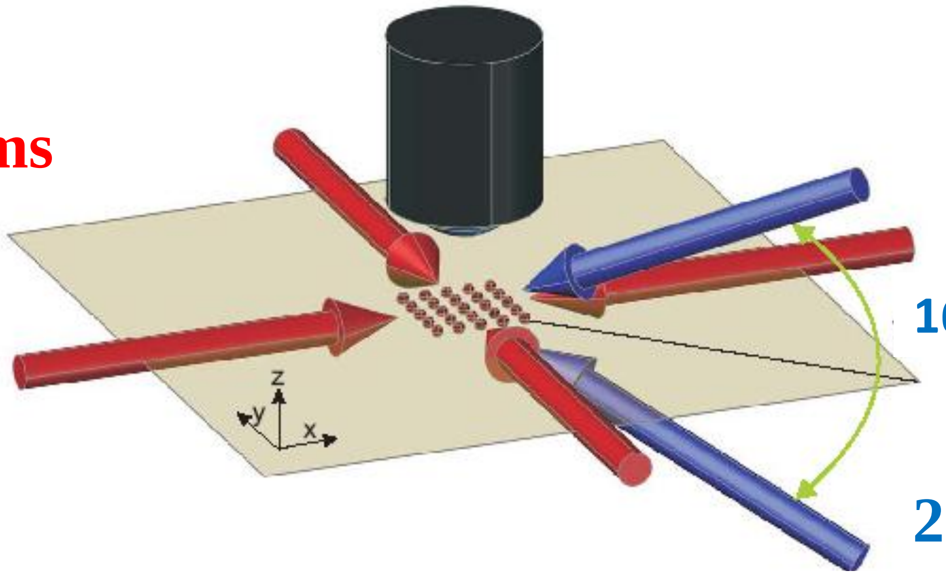
$$\hbar\omega_r << k_B T, \mu << \hbar\omega_z$$



Atoms in a 2D optical lattice

Microscope objective

4 lattice beams

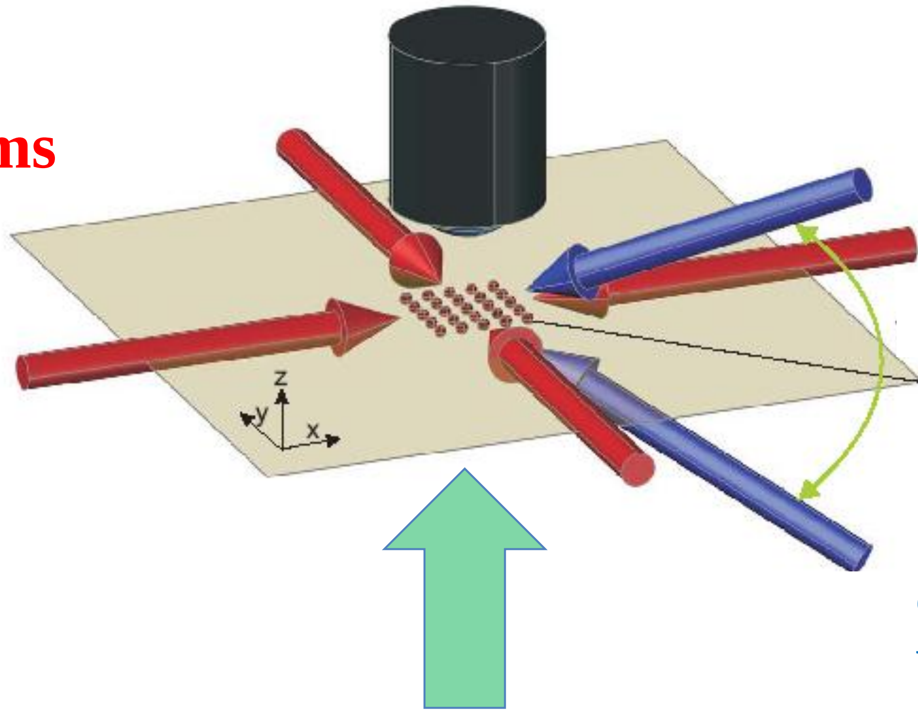


16 degrees

2 Vertical compress. beams

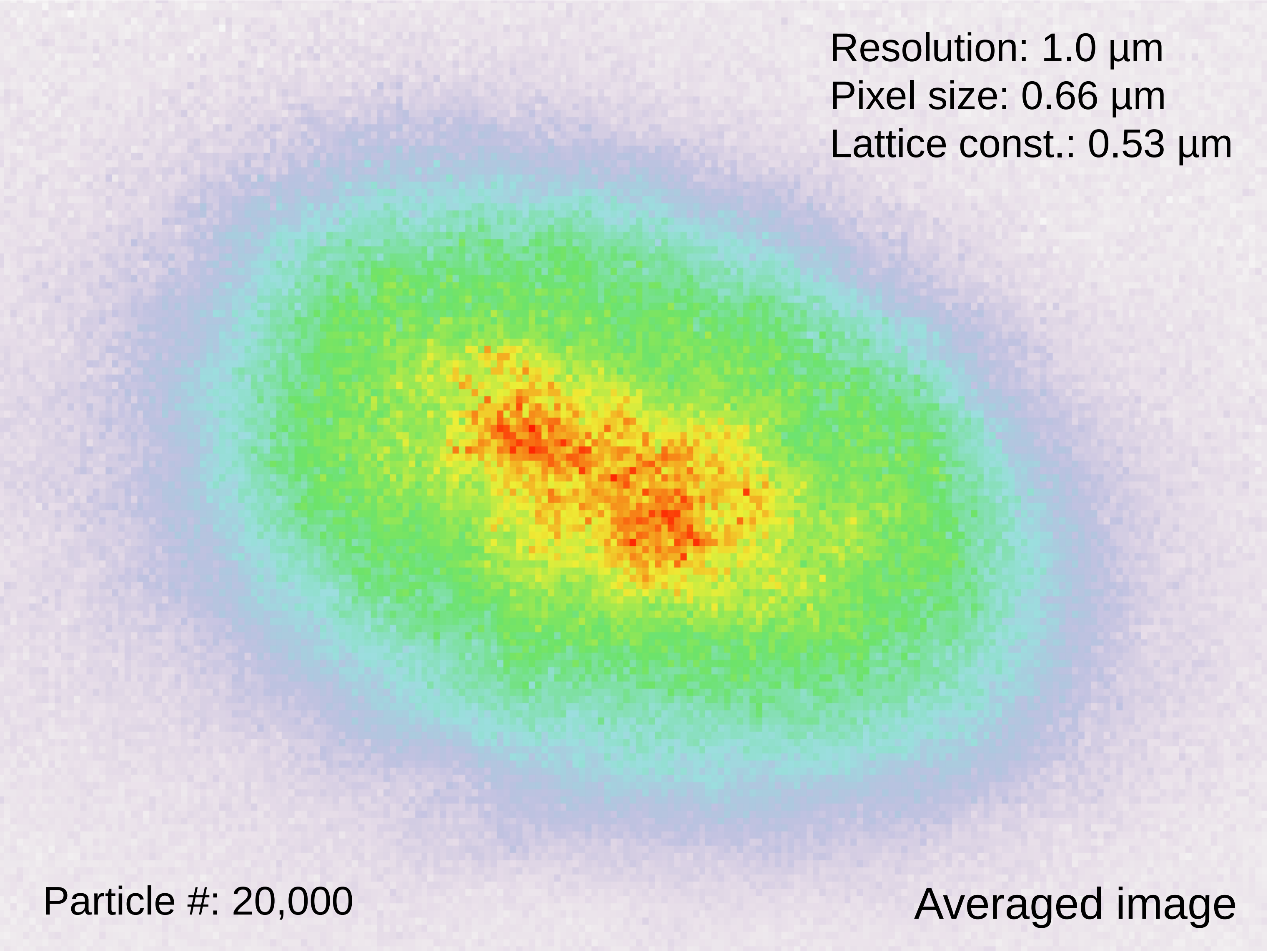
*Gemelke, **Zhang**, Hung, Chin, Nature 460, 995 (2009)*

Perform in situ absorption imaging

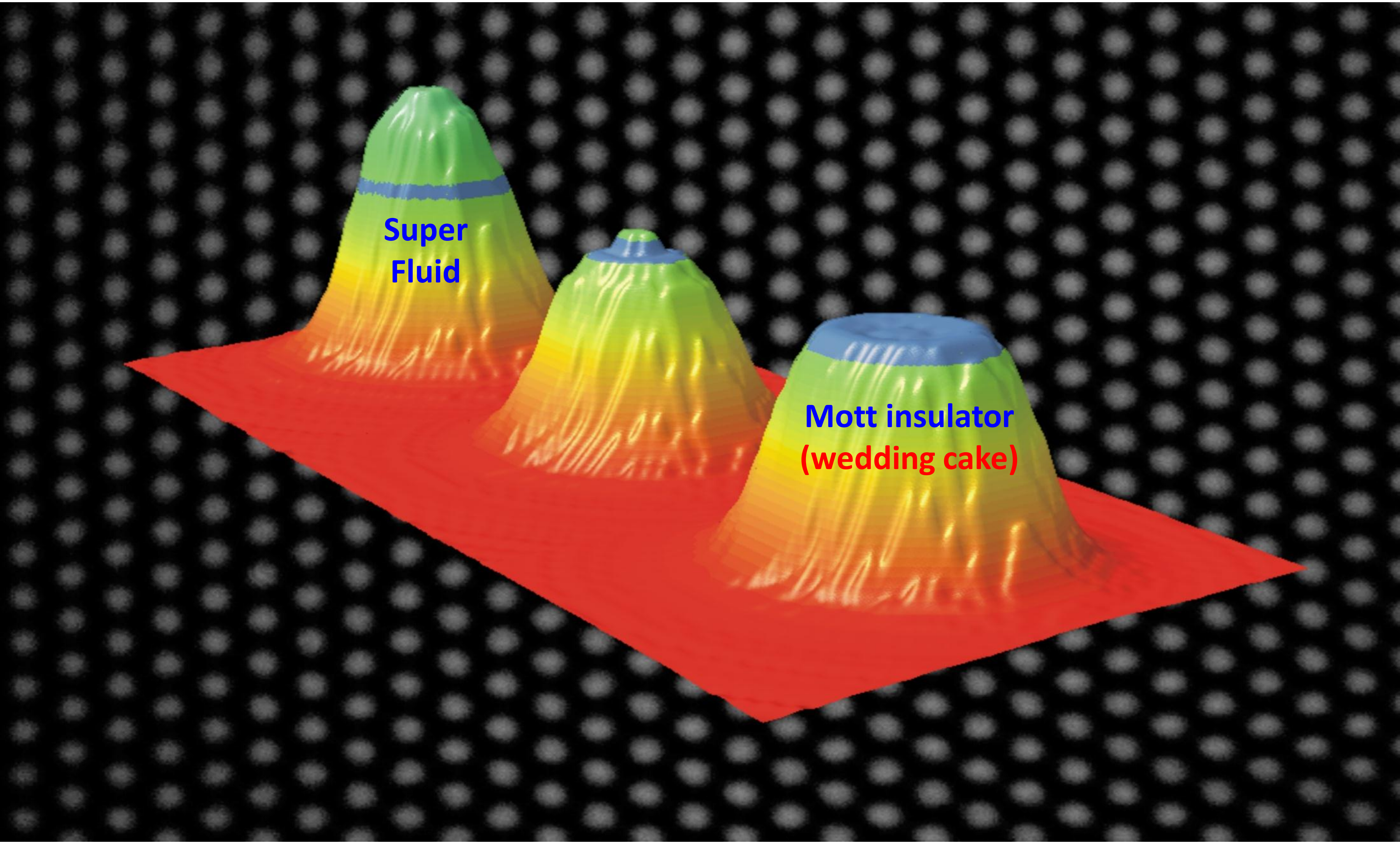


*Gemelke, **Zhang**, Hung, Chin, Nature 460, 995 (2009)*

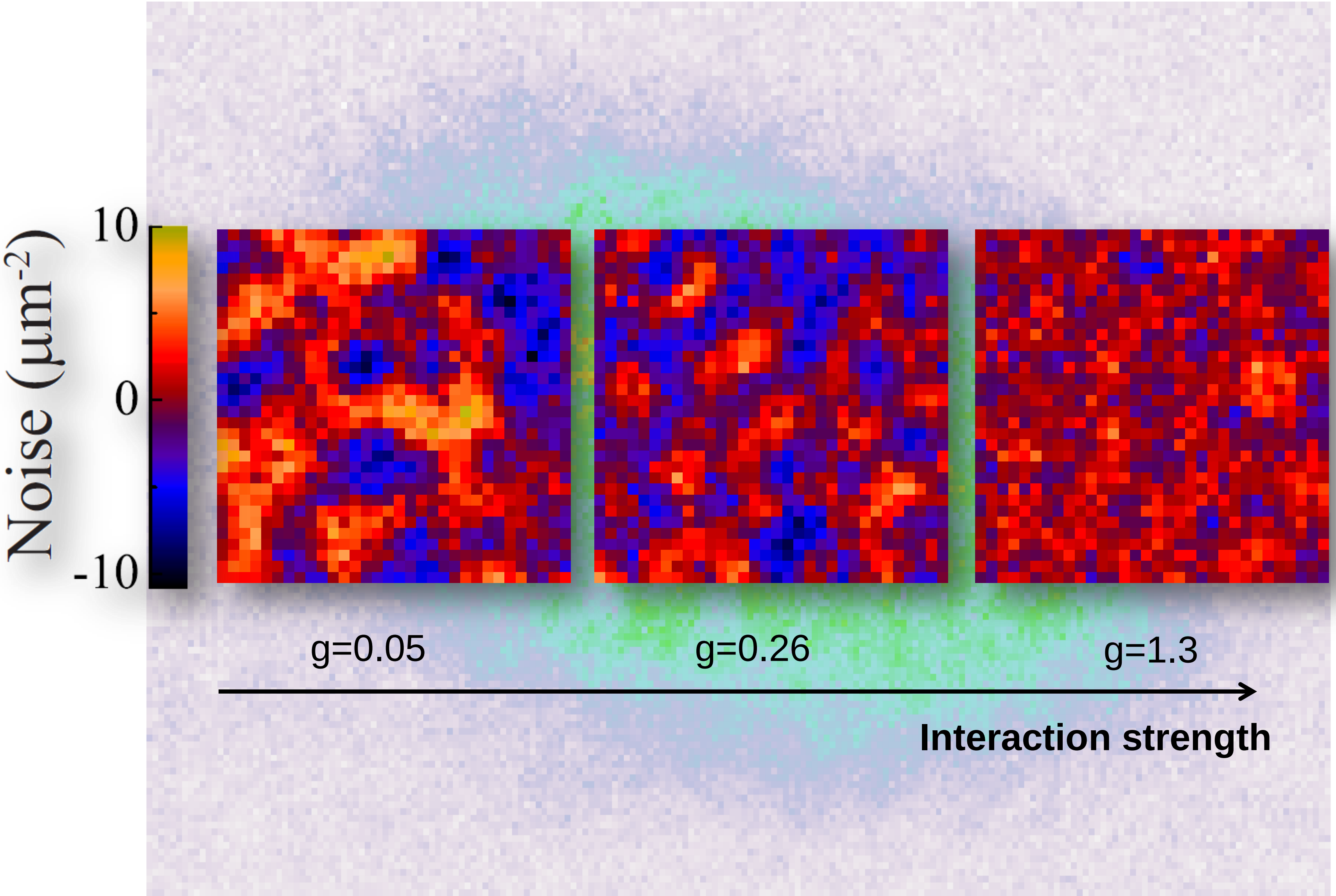
A closer look



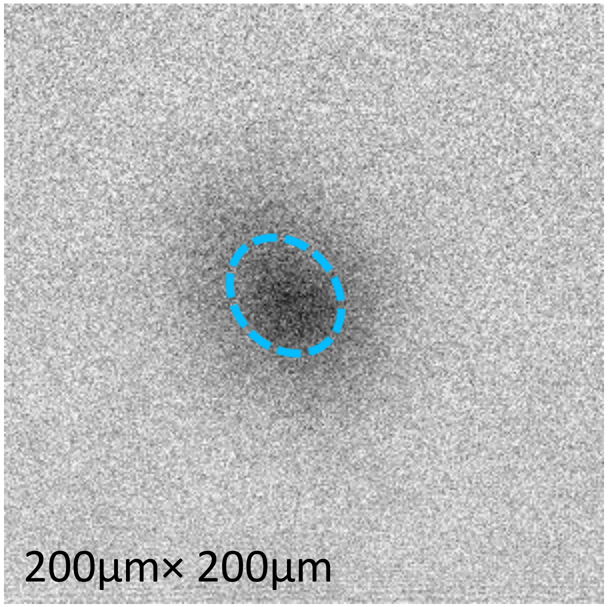
Density profiles of atoms in 2D optical lattices

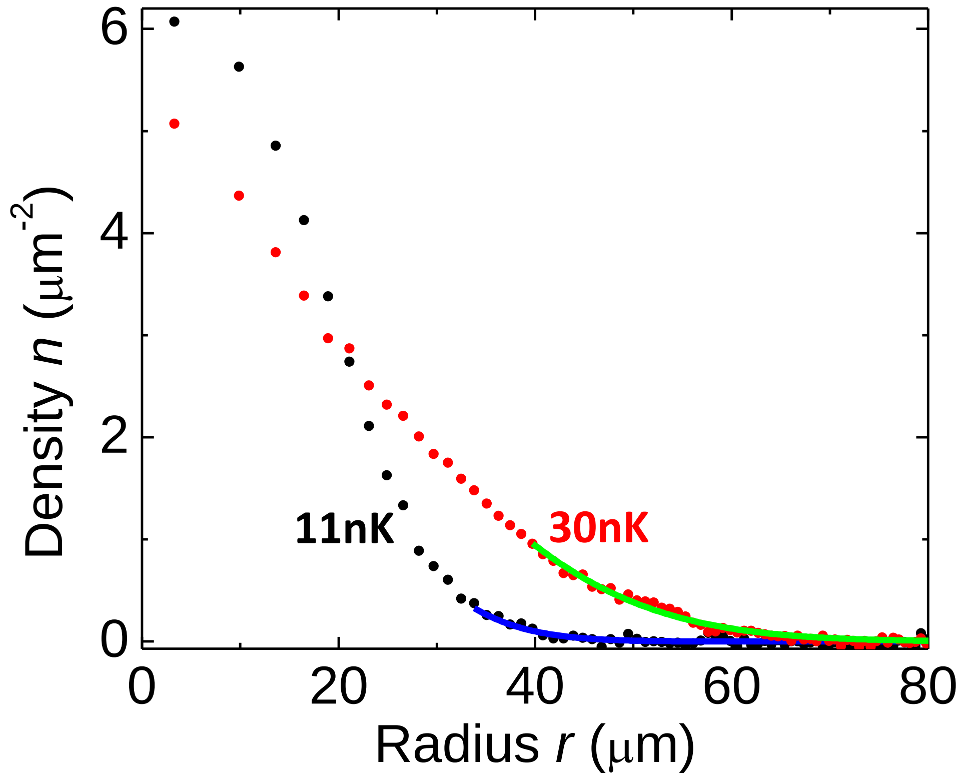


$$\delta n = n - \langle n \rangle$$



Extract temperature T and chemical potential μ_m





$$\mu = \mu_m - V(r)$$

Extracting T and μ_m by fitting the low-density thermal tail based on a mean-field formula

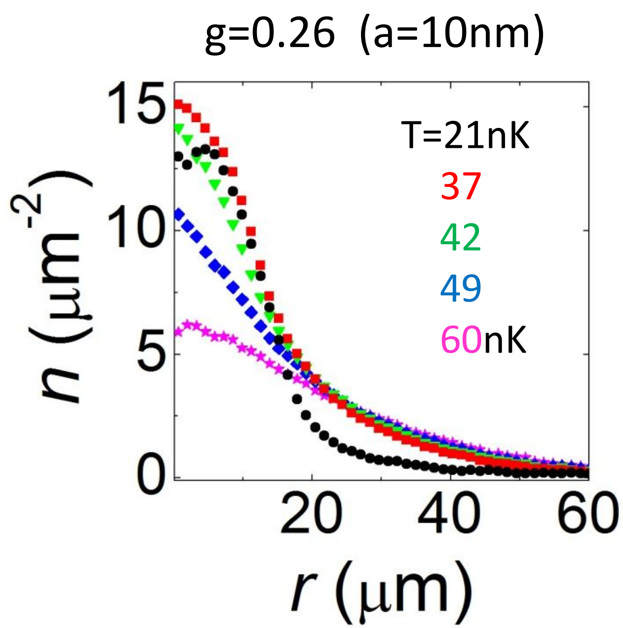
$$n(r) = d^{-2} \sum_{l=1}^{\infty} \left[I_0\left(\frac{2lt}{k_B T} \right) \right]^2 \exp\left[\frac{l\left(\mu_m - V(r) - 2Und^2 \right)}{k_B T} \right]$$

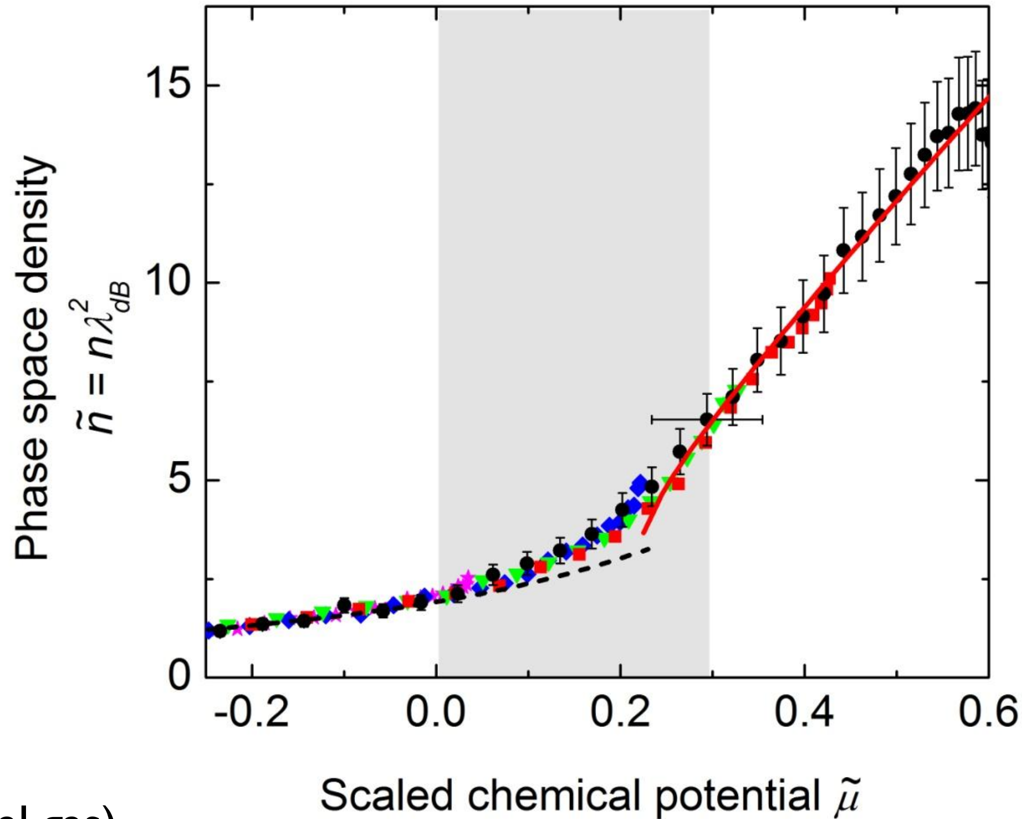
d = 0.532µm; **t**: tunneling; **U**: interaction; $\mathbf{I_0}$: 0^{th} Bessel

Theory:
 Ho & Zhou, Nature Physics 6, 131 ('10)
Exp:
 Dalibard ('10), Zwierlein ('12), Chin ('11), …

Equation of state of a 2D gas
And scale invariance

$$\tilde{n} = n\lambda_{dB}^2 = F\left(\frac{\mu}{kT}\right)$$





- - - - Mean-field (normal gas)

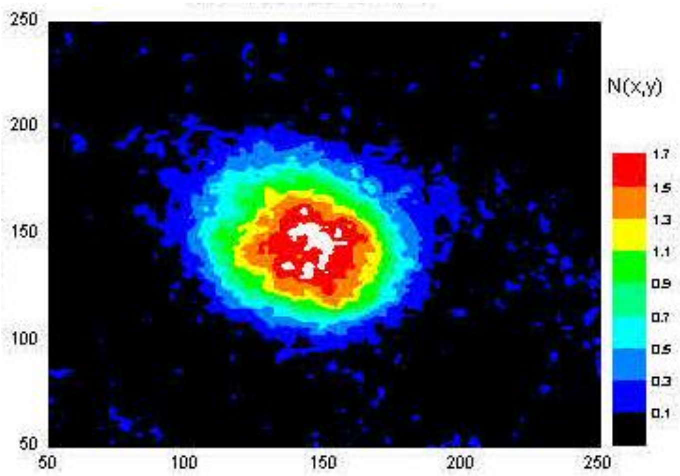
—— Mean-field (superfluid) Prokof'ev and Svistunov, PRA (2002)

Chicago Experiment: Hung, Zhang, Gemelke and CC, Nature 470, 236-239 (2011)

ENS experiment: Yefsah et. Al., arXiv:1106.0188 (2011)

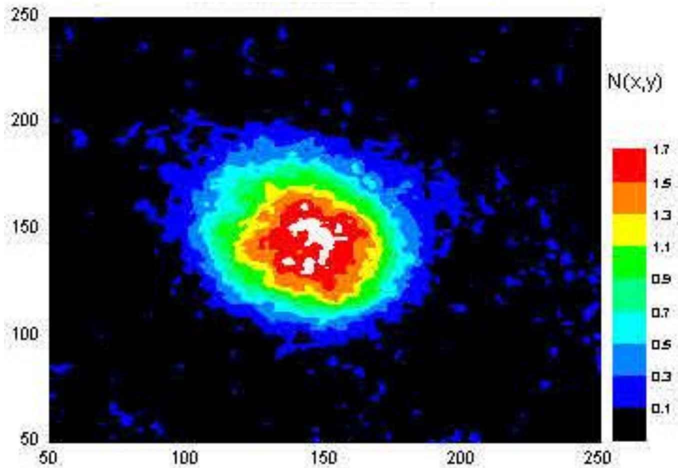
Thermodynamics tomography (equilibrium)

Density n(x,y)

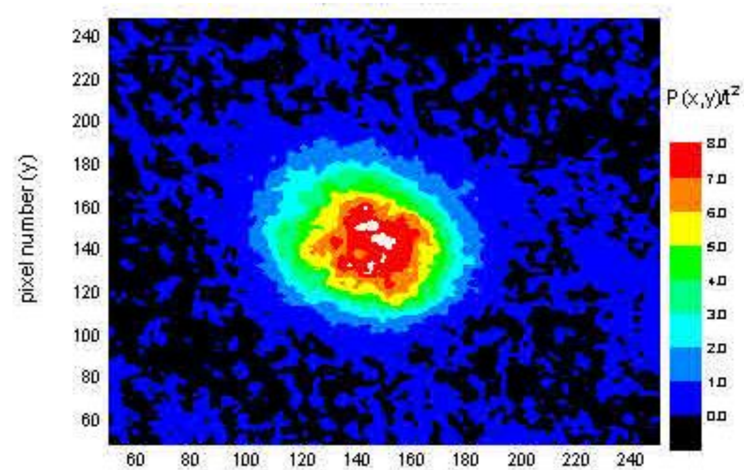


Thermodynamics tomography

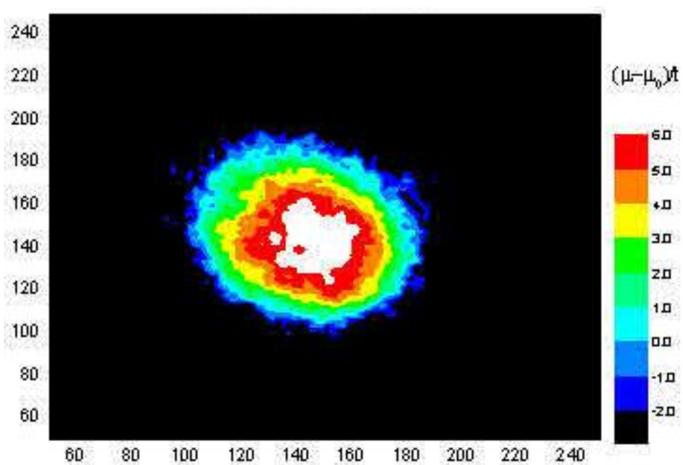
Density n(x,y)



Pressure n(x,y)



Chemical potential n(x,y)



Entropy per particle S/N(x,y)