Two-dimensional Fermi gases

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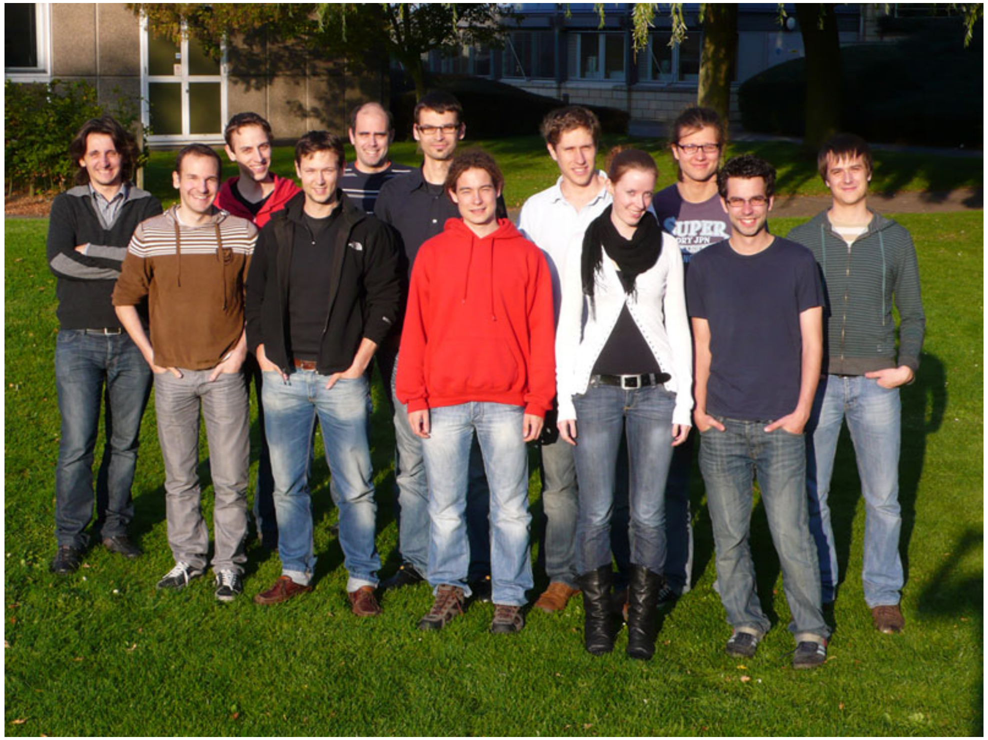
Our group in Cambridge



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Fermi gases	E. Vogt, M. Koschorreck, D. Pertot, L. Miller, **E. Cocchi**, **J. Bohn**
	[Special thanks: B. Fröhlich, M. Feld]
Ion trap QIP	H.-M. Meyer, M. Steiner

Open postdoc position

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Overview

Lecture 1: Atomic Fermi gases - experimental and theoretical background	Lecture 2: Fermion pairing in 3D and 2D	Lecture 3: 2D Fermi gases
Introduction	BEC-BCS crossover	Momentum-resolved rf spectroscopy
How to make a Fermi gas	Preparing 2D Fermi gases	Fermi liquid
Seeing the Fermi surface	Scattering in 2D	Pseudogap pairing
Interactions	Collective modes	Polarons

Literature

Books (general background)

C. Foot: Atomic Physics, OUP

C. Pethick, H. Smith: Bose-Einstein condensation in dilute gases, CUP

L. Pitaevski, S. Stringari: Bose-Einstein condensation, OUP

K. Levin, A. L. Fetter, D. M. Stamper-Kurn (eds): Ultracold Bosonic and Fermionic Gases, Elsevier

Review articles

I. Bloch, J. Dalibard, W. Zwerger: Many-body physics with ultracold gases, Rev. Mod. Phys. 80, 885 (2008)

V. M. Loktev, R. M. Quick & S. Sharapov: Phase fluctuations and pseudogap phenomena, Phys. Rep. 349, 1–123 (2001).

D.S. Petrov, D.M. Gangardt, G.V. Shlyapnikov: Low-dimensional trapped gases, J. Phys. IV France 116, 3-44 (2004).

Strongly interacting quantum systems

high densities

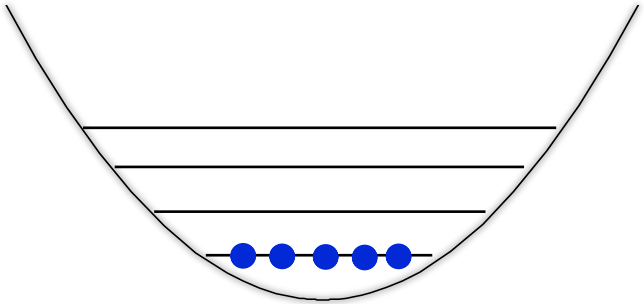
low temperatures

Strongly interacting: interaction energy dominates over kinetic energy

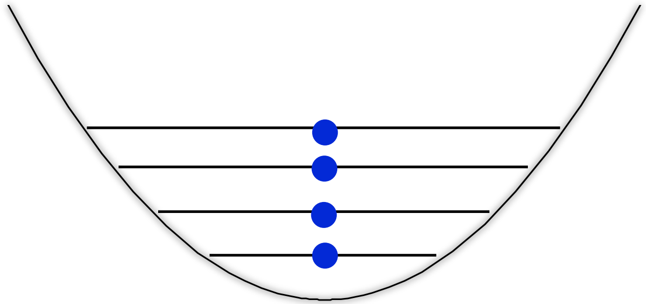
(It's all about the scale.)

Quantum degeneracy

Bosons



Fermions



$$T_D \approx \frac{2\pi\hbar^2}{mk_B} n^{2/3}$$

	m	n	T_c
Nuclear matter	n	10^{38} cm^{-3}	10^{12} K / 30 MeV
Electron gas	m_e	10^{23} cm^{-3}	50000 K / 10 eV
Superfluid Helium	^{4}He	10^{22} cm^{-3}	5 K / 1 meV
Atomic quantum gas	^{40}K	10^{13} cm^{-3}	100 nK / 10 peV

Why cold atoms?

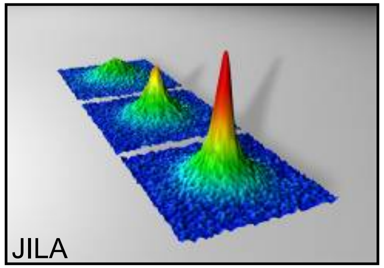
GREAT EXPERIMENTAL CONTROL...

- extremely pure systems: no dirt, impurities,...
- all parameters are tuneable:
  kinetic energy, interaction strength, spin composition, dimensionality, ...
- energy scales/time scales are so slow that parameters can be tuned in real time

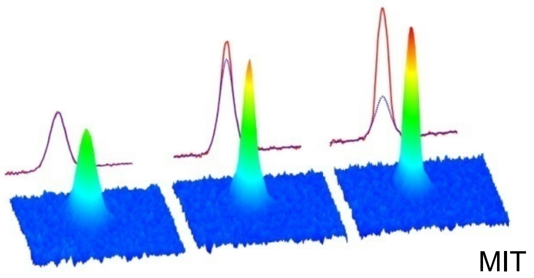
... and a precise microscopic understanding from first principles.

A few highlights

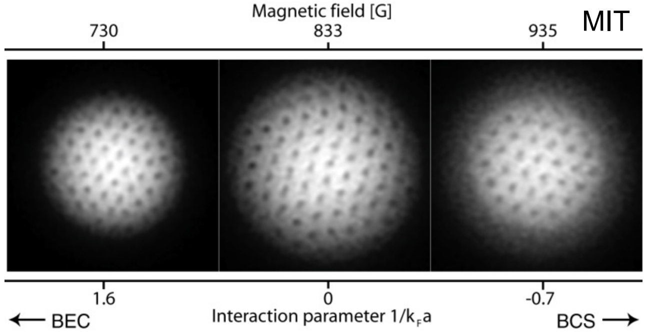
BEC-BCS crossover



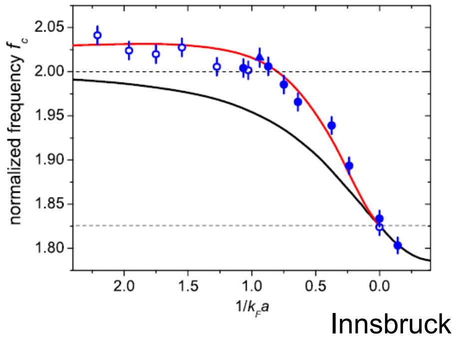
Bose-Einstein condensation of pairs of fermions



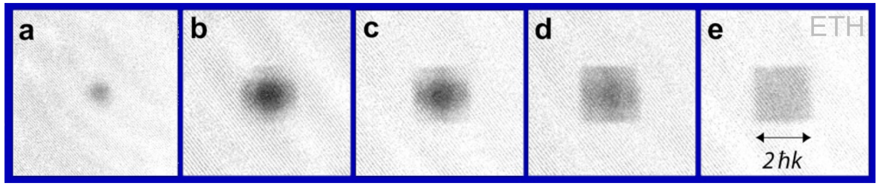
Quantized vortices



Collective (shape) oscillations



Fermionic atoms in optical lattices



How to make a quantum gas

Problems when reaching low temperatures

- Everything (except Helium) solidifies below 1K
  → work at low densities (timescale for solidification $\propto n^3$)

- Cooling becomes increasingly difficult since the heat capacity of the coolant vanishes ($\propto T^3$)
  → try new cooling method!

The road to Nanokelvin temperatures

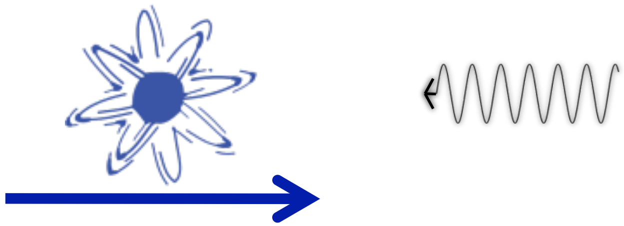
1. Laser cooling

2. Magnetic or optical trapping

3. Evaporative cooling

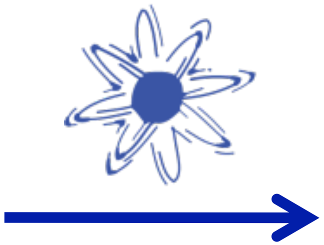
4. Imaging and temperature measurement

Cooling with lasers: the mechanism

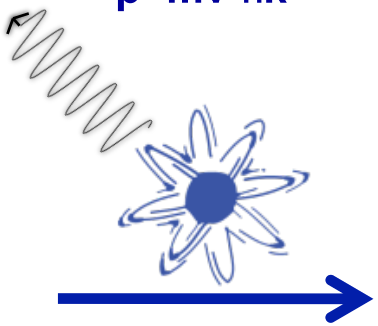


p=mv

$$\nu_{abs} = \nu_{atom} - \frac{v}{\lambda}$$



p=mv-ħk



p=mv-ħk+ħk' but <ħk'>=0 !

$$\nu_{emis} = \nu_{atom}$$

$$\Delta E = -h\frac{v}{\lambda}$$

Proposed by Hänsch, Schawlow, Wineland, Demehlt in 1975

Can this work at all?

Doppler shift

excited state decay rate ≈ scattering rate

Let's calculate the deceleration:

$$a = \frac{1}{m}\frac{dE}{dx} = \frac{1}{m}\frac{\Delta E}{\Delta t}\frac{dt}{dx} = -\frac{1}{m}\frac{h\nu}{\lambda}\Gamma\frac{1}{\mathrm{v}} \approx 10^4 g$$

(that is huge!)

It works: atoms can be stopped from 300 m/s to rest within 0.01 s !

What's the limit? → "Doppler limit"

laser detuning

Cooling rate:

$$\dot{E} \approx -\frac{1}{\hbar}\frac{\hbar^2 k^2}{2m} R_{scatt}(\Delta, I)\frac{2\Delta}{\Delta^2 + \Gamma^2/4} E_{kin}$$

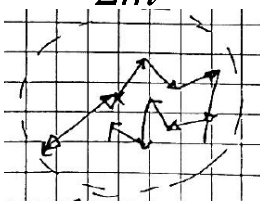
recoil energy

photon scattering rate

Lorentzian lineshape

Heating rate from photon recoil

$$\dot{E} \approx \frac{\hbar^2 k^2}{2m} R_{scatt}(\Delta, I)$$



random walk from photon recoil

limiting temperature

$$k_B T = \frac{\hbar\Gamma}{2} \approx 100\,\mu K$$

Laser cooling Nobel prize

The Nobel Prize in Physics 1997

"for development of methods to cool and trap atoms with laser light"

Steven Chu

1/3 of the prize

USA

Stanford University
Stanford, CA, USA

Claude Cohen-Tannoudji

1/3 of the prize

France

Collège de France; École Normale Supérieure
Paris, France

William D. Phillips

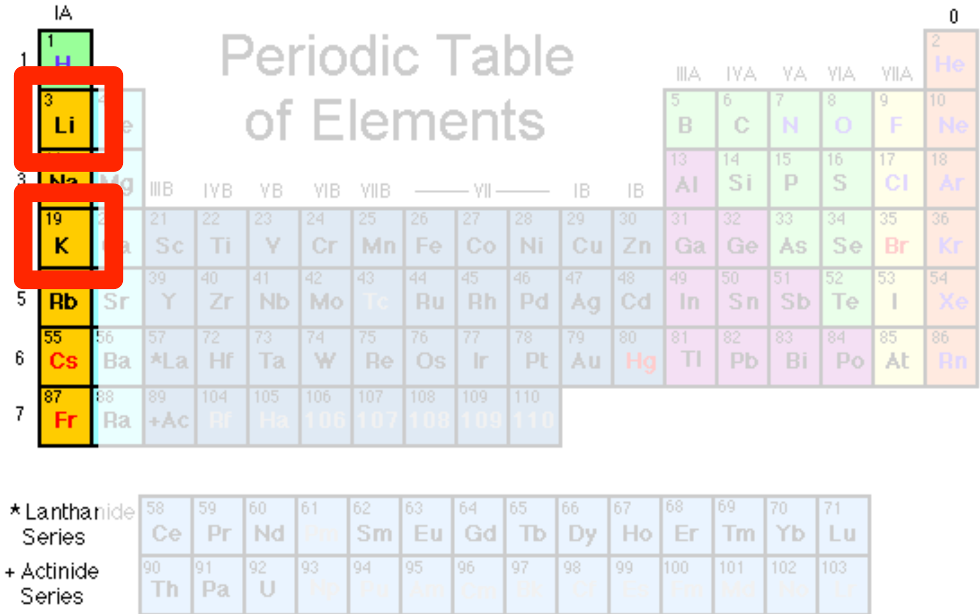
1/3 of the prize

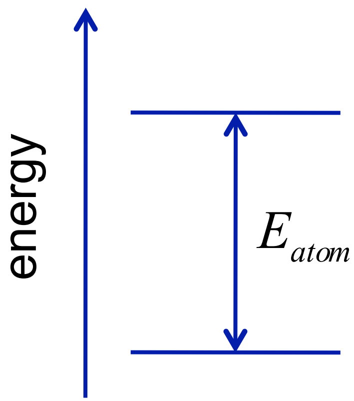
USA

National Institute of Standards and Technology
Gaithersburg, MD, USA

Cooling with lasers: atomic structure

Fermionic Isotopes: ^{6}Li, ^{40}K



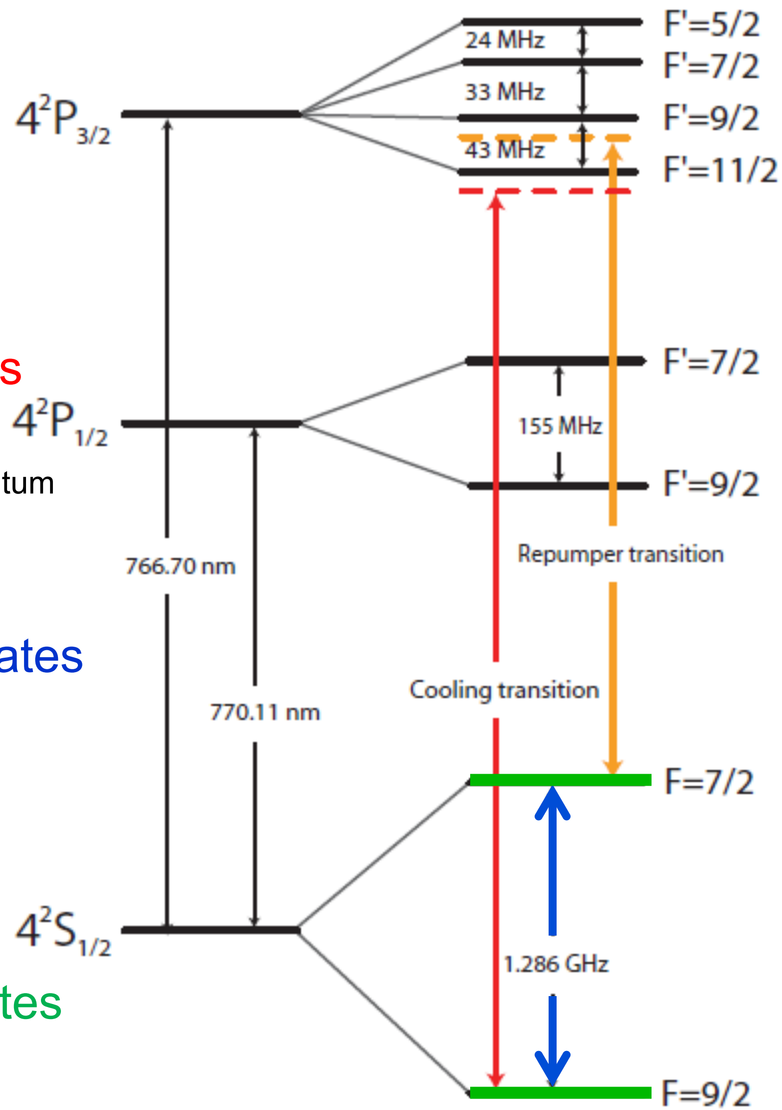


$$E_{atom} = h\nu_{atom}$$

element	wavelength λ_{atom}
H	121 nm (UV)
Li	671 nm (red)
Na	589 nm (yellow)
K	767 nm (red)
Rb	780 nm (infrared)
Cs	850 nm (infrared)

Level scheme of ^{40}K

(similar for all other alkali atoms)



Optical transitions
electric dipole transition between principal states of quantum numbers n, L, J = L+S

L = orbital angular momentum
S = spin

Microwave transitions
Magnetic dipole transition between hyperfine states with different quantum numbers F = I+J

I= nuclear spin

Radiofrequency transitions
magnetic dipole transition between Zeeman states of different quantum number m_F
(will be discussed in detail later)

The laser setup

Laser cooling in action

Movie from University of Munich/MPQ

Magnetic trapping

Interaction of an atom with a magnetic field

$$H = H_{gross} + \beta \vec{L}\cdot\vec{S} + A\vec{I}\cdot\vec{J} + g_J\mu_B\vec{J}\cdot\vec{B} - g_I\mu_N\vec{I}\cdot\vec{B}$$

Hyperfine Hamiltonian
|F,m_F> are eigenstates

Zeeman Hamiltonian
|m_I,m_J> are eigenstates

At low fields ($g_J\mu_B B < A$):

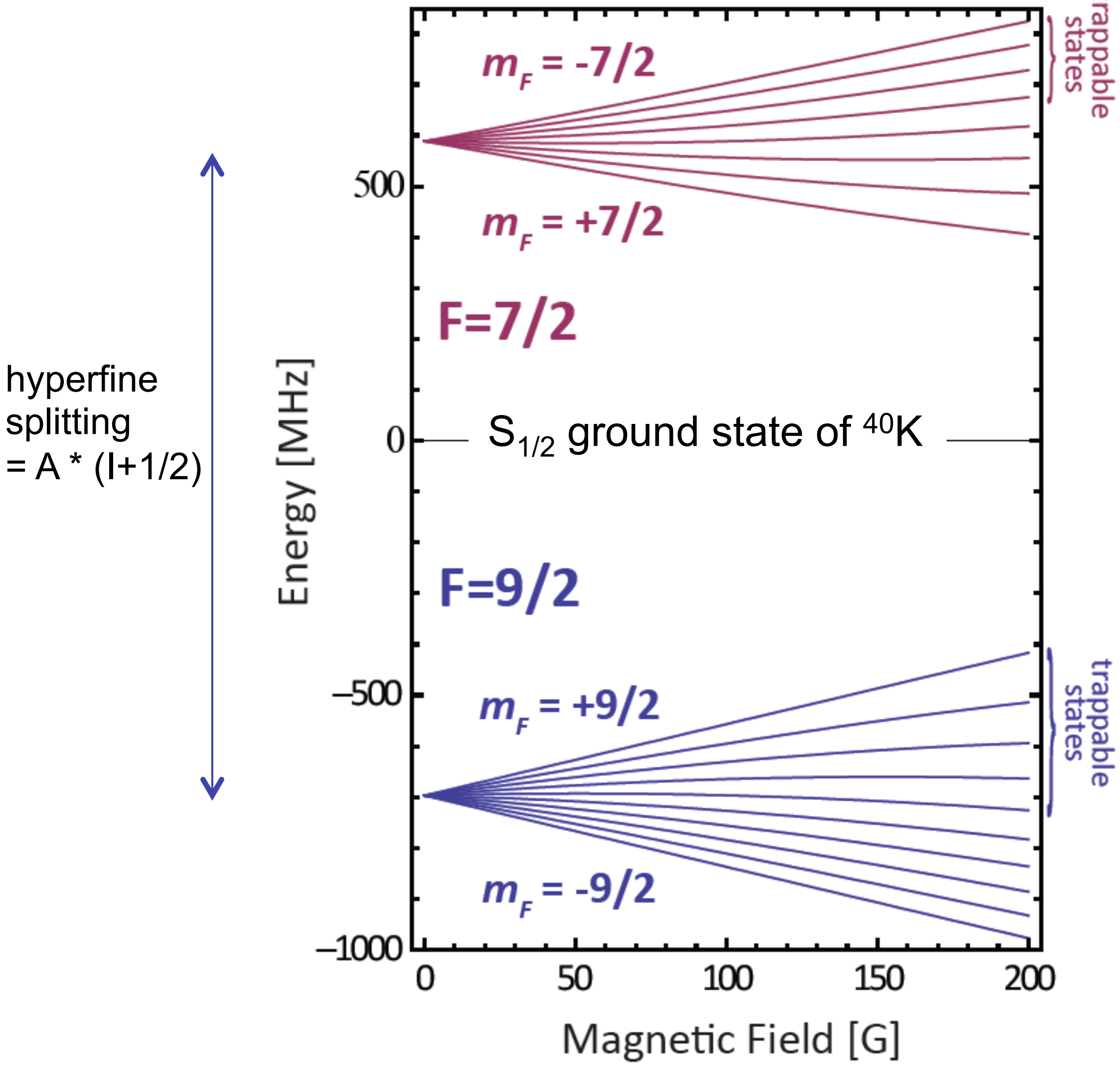
Magnetic field is perturbation
-> |F, m_F> remain good quantum numbers
Linear Zeeman effect: $E(B) \approx \text{const.} + g_F m_F \mu_B B$

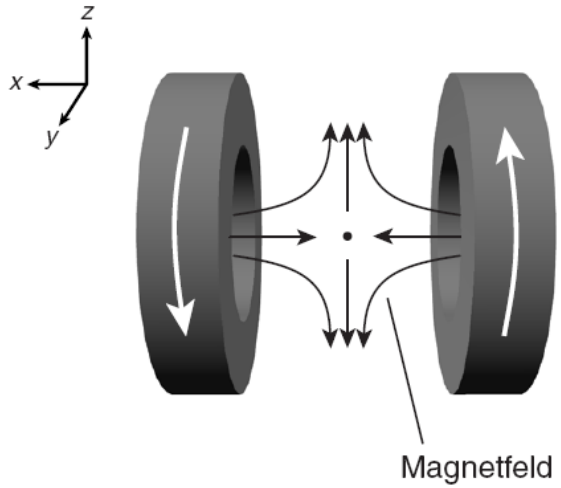
At high fields ($g_J\mu_B B > A$):

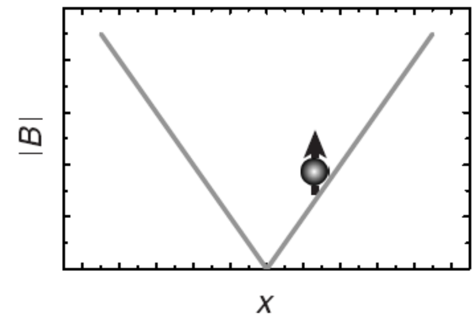
Hyperfine interaction is perturbation
-> |m_I,m_J> are good quantum numbers
$E(B) \approx \text{const.} + g_J m_J \mu_B B$

Interaction of atoms with a magnetic field

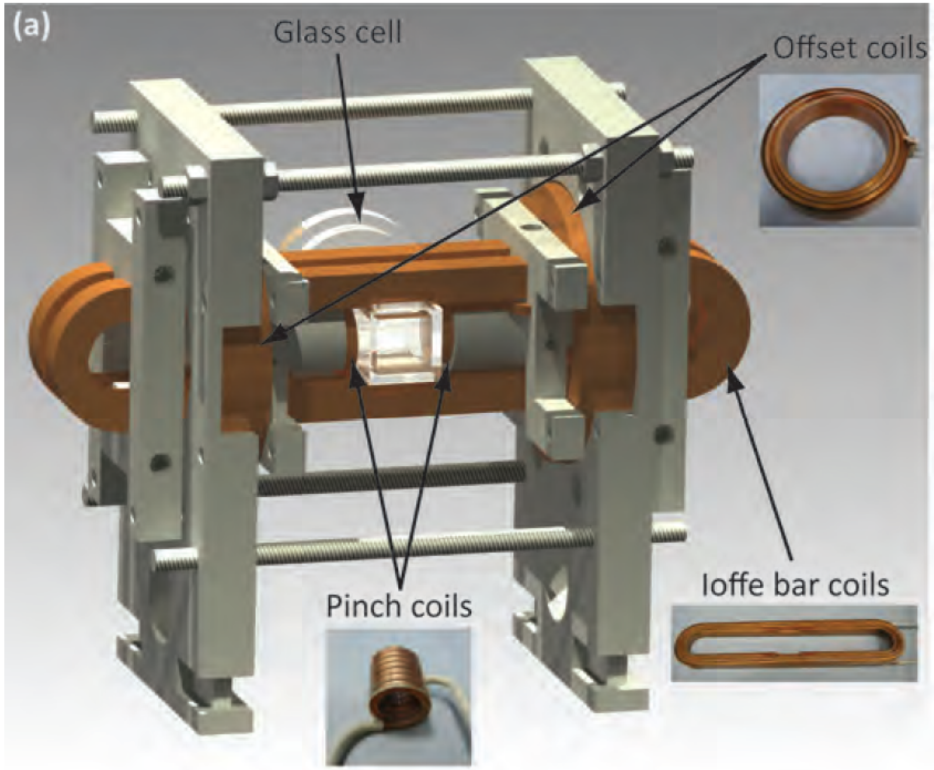
$$H = H_{gross} + \beta \vec{L}\cdot\vec{S} + A\vec{I}\cdot\vec{J} + g_J \mu_B \vec{J}\cdot\vec{B} - g_I \mu_N \vec{I}\cdot\vec{B}$$







How does this look like?



(the wires carry 500 Amps)

Magnetic field

$$\mathbf{B} = B_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + B' \begin{pmatrix} x \\ -y \\ 0 \end{pmatrix} + \frac{B''}{2} \begin{pmatrix} -xz \\ -yz \\ z^2 - \frac{1}{2}(x^2 + y^2) \end{pmatrix}$$

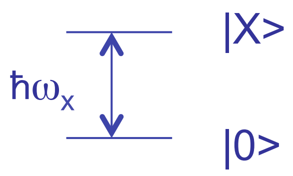
Potential

$$V(x,y,z) = \frac{m}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$$

three-dimensional harmonic oscillator

Optical dipole traps

Two-level atom



&

Laser field: $E(t) = E_o(t) cos(\omega t)$

$$\hat{H} = \hat{H}_o + \hat{H}_I$$

atomic two-level Hamiltonian $\hat{H}_0 = 0\,|0\rangle\langle 0| + \hbar\omega_x\,|X\rangle\langle X|$

electric-dipole interaction $\hat{H}_I = -\mu_{ox} E(t)\,|0\rangle\langle X|$ + c.c.

Rabi frequency: $\Omega_1 = -\mu E_o(t)/\hbar$

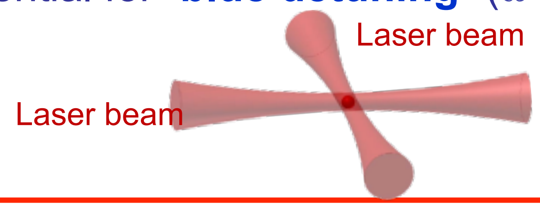
Detuning: $\Delta = \omega(t) - \omega_x$

Eigenenergies of the coupled system: $E_\pm = \frac{\hbar}{2}\left(\Delta(t) \pm \sqrt{\Delta(t)^2 + \Omega(t)_1^2}\right)$

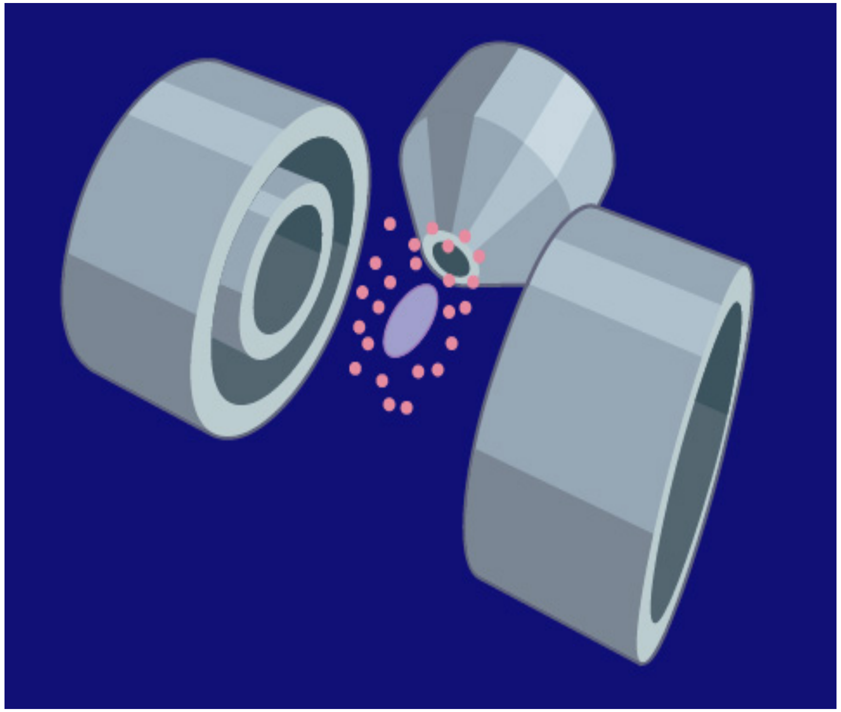
energy shift $\Delta E \approx \hbar \frac{\Omega_1^{\,2}}{4\Delta}$ for $\Delta >> \Omega$

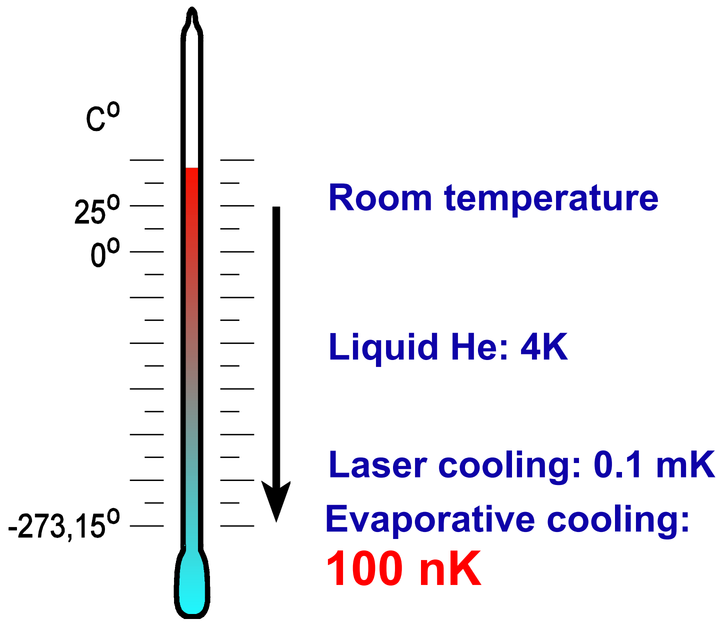
Optical dipole potential: $U_{\rm dip} = \frac{3\pi c^2}{2\omega_x^3}\frac{\Gamma}{\Delta} I(r).$ attractive potential for "**red detuning**" $(\omega<\omega_x)$
repulsive potential for "**blue detuning**" $(\omega<\omega_x)$

- potential depth proportional to laser intensity
- simply realized by (crossed) laser beams
- usually independent of internal states (for large detuning)



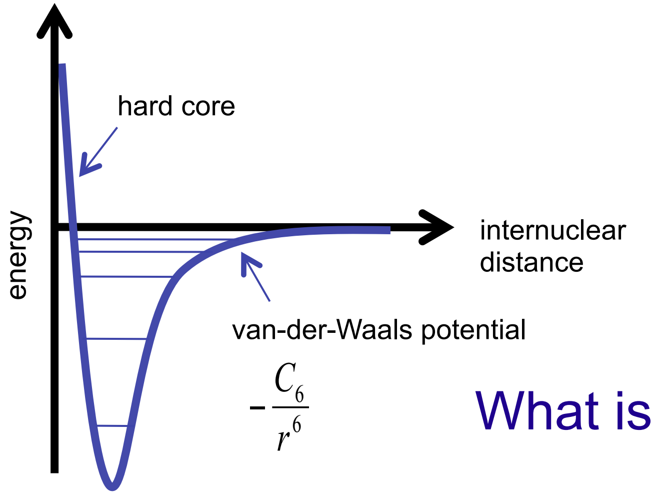
Getting cold: Evaporative cooling





Collisions are required!

(Very) basic properties of ultracold collisions



What is the range of a power-law potential $-\frac{C_n}{r^n}$ (n>2)?

particle prepared at a distance r has a kinetic energy $E_{kin} \geq \frac{\hbar^2}{m\,r^2}$
(Heisenberg limit due to the hard core)

potential energy of the particle $E_{pot} = \frac{C_n}{r^n}$

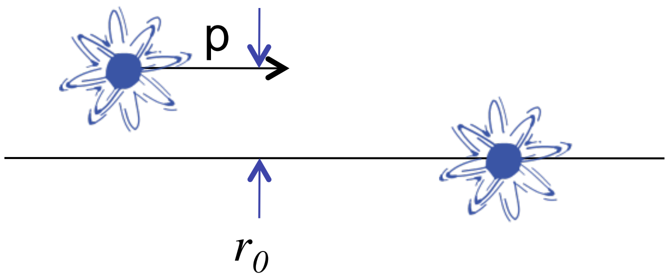
$$E_{kin} = E_{pot} \quad @ \quad r_0 = \left(\frac{mC_n}{\hbar^2}\right)^{\frac{1}{n-2}}$$

typical values for alkali atoms: r_0 = 100 a_{Bohr}

<u>Note:</u> this has nothing to do with the scattering length or the collisional cross section!

Collisions of ultracold atoms

Classically



$$p \approx \sqrt{mkT}$$

$$\boxed{p \cdot r_0 < \hbar \quad \rightarrow \quad T < 100\,\mu K}$$

Only s-wave collisions!

Quantum mechanical collisions

Schrödinger equation in relative coordinates

$$\left[-\frac{\hbar^2}{2m_\mathrm{r}}\Delta + V(\boldsymbol{r})\right]\psi_{\boldsymbol{k}}(\boldsymbol{r}) = E_k\psi_{\boldsymbol{k}}(\boldsymbol{r}).$$

Asymptotic wave function

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) \propto e^{ikr} + f(k, \boldsymbol{n}, \boldsymbol{n'})\frac{e^{ikr}}{r}.$$

incoming plane wave

outgoing spherical wave

Using partial wave decomposition
for outgoing wave

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = \sum_{l=0}^{\infty} P_l(\cos\theta)\frac{u_{k,l}(r)}{r}$$

Legendre polynomials

$$-\frac{\hbar^2}{2m_\mathrm{r}}\frac{d^2}{dr^2}u_{k,l}(r) + \left(V(r) + \frac{l(l+1)\hbar^2}{2m_\mathrm{r}r^2} - E_k\right)u_{k,l}(r) = 0.$$

l=0

$$f(k,\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\left(e^{2i\delta_l(k)} - 1\right)P_l(\cos\theta) \quad = \frac{1}{k\cot\delta_0(k) - ik}.$$

l=0

s-wave scattering length a
(measures phase shift in collision)

$$k\cot\delta_0(k) \approx -\frac{1}{a} + R_\mathrm{eff}\frac{k^2}{2}$$

<u>single parameter!</u>

Scattering length vs. cross section

Scattering length can be positive or negative.

Total scattering cross section follows from optical theorem:

$$\sigma(k) = \frac{4\pi}{k}\mathrm{Im}(f).$$

$$\begin{array}{ll} ka \ll 1 & \sigma \simeq 4\pi a^2 \\ ka \gg 1. & \sigma \simeq 4\pi/k^2 \end{array}$$

naively expected

unitary cross-section independent of interaction potential

Fermions at low temperatures

total wave function is antisymmetric for fermions:

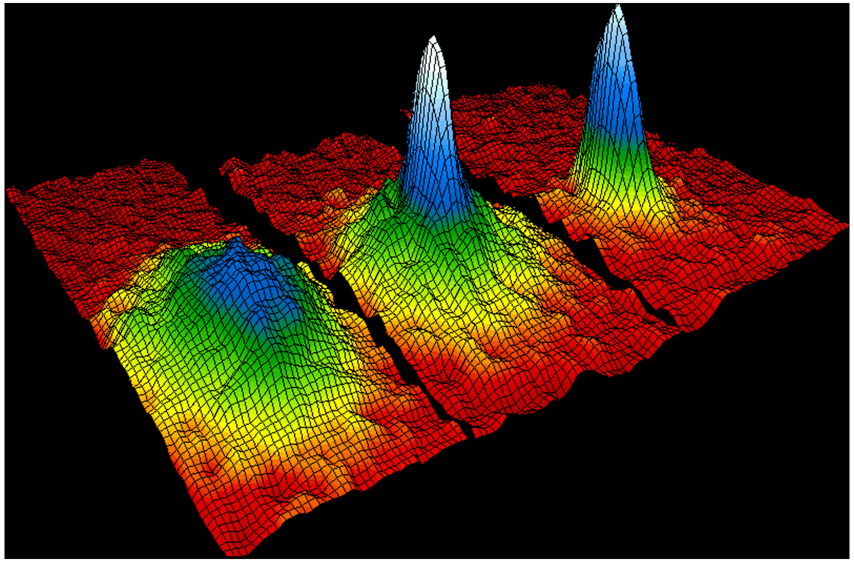
$$|\psi\rangle = \quad |\psi_{spatial}\rangle \otimes |\psi_{spin}\rangle$$

no s-wave collisions possible
(pair correlation length: λ_{dB})

anti-symmetric	$|\uparrow\uparrow\rangle$	spin-polarized
anti-symmetric	$|\downarrow\downarrow\rangle$	
anti-symmetric	$|\uparrow\downarrow + \downarrow\uparrow\rangle$	spin mixture
symmetric	$|\uparrow\downarrow - \downarrow\uparrow\rangle$	

Cooling fermions is MUCH more difficult than bosons
- use mixture of different spin states
- use mixture with a different species, e.g. bosons

Nobel prize 2001



The Nobel Prize in Physics 2001

"for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"

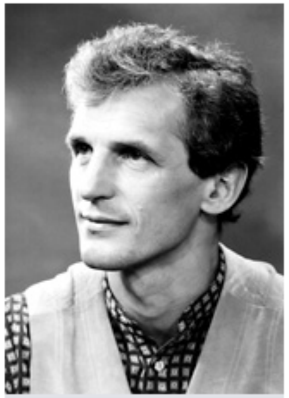
Eric A. Cornell

1/3 of the prize

USA

University of Colorado, JILA
Boulder, CO, USA

b. 1961



Wolfgang Ketterle

1/3 of the prize

Federal Republic of Germany

Massachusetts Institute of Technology (MIT)
Cambridge, MA, USA

b. 1957

Carl E. Wieman

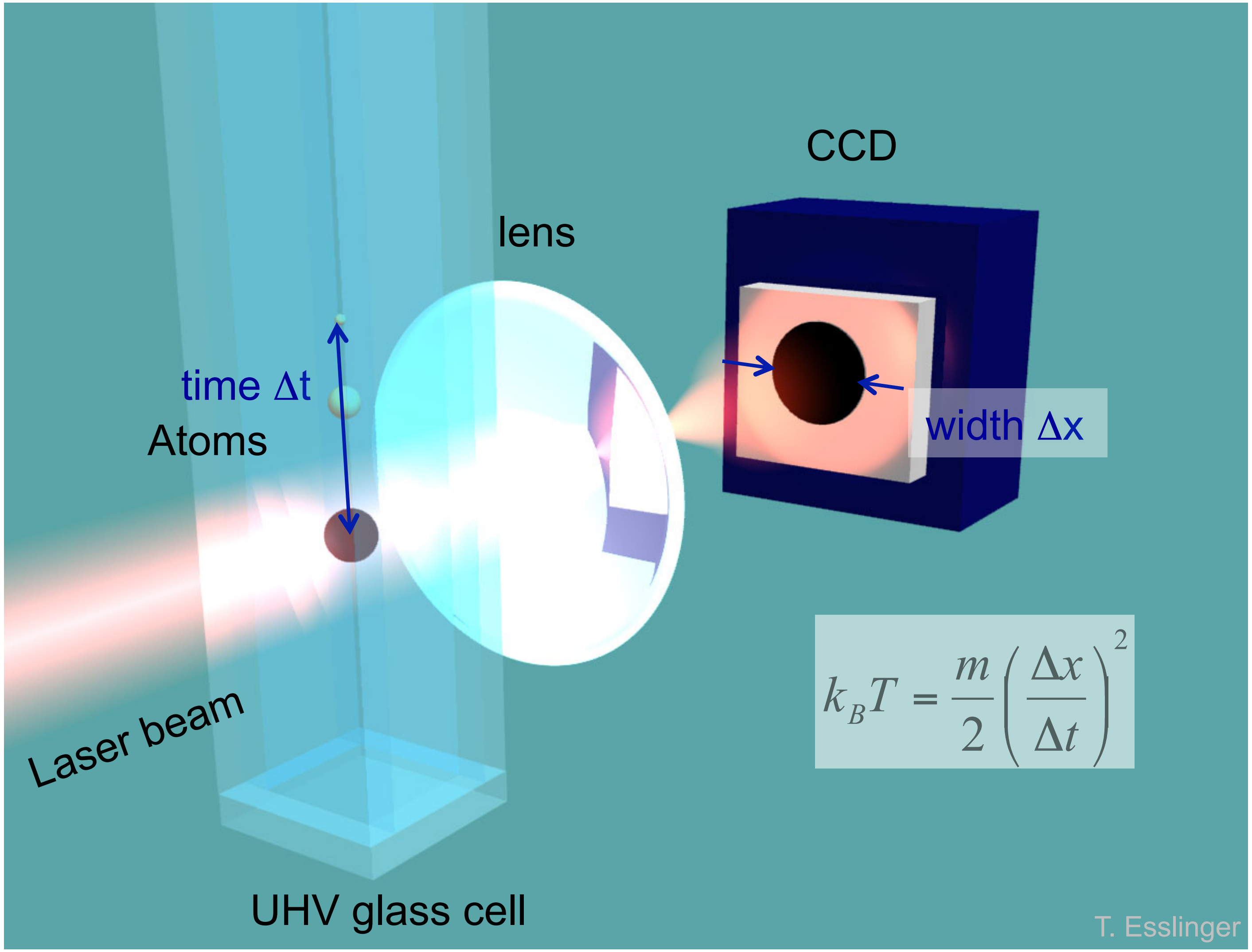
1/3 of the prize

USA

University of Colorado, JILA
Boulder, CO, USA

b. 1951

How to measure temperature

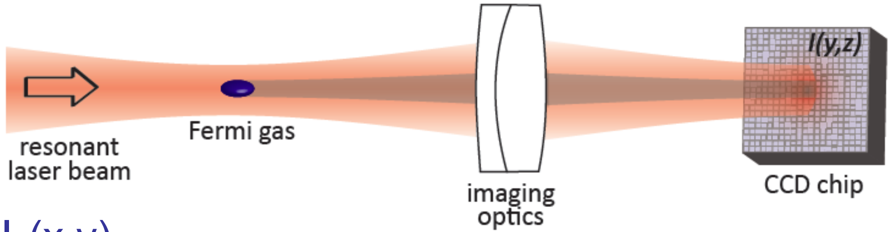


$$k_B T = \frac{m}{2}\left(\frac{\Delta x}{\Delta t}\right)^2$$

Absorption imaging

Light attenuation:

$$I = I_0 \cdot e^{-D(x,y)}.$$



$I_0(x,y)$

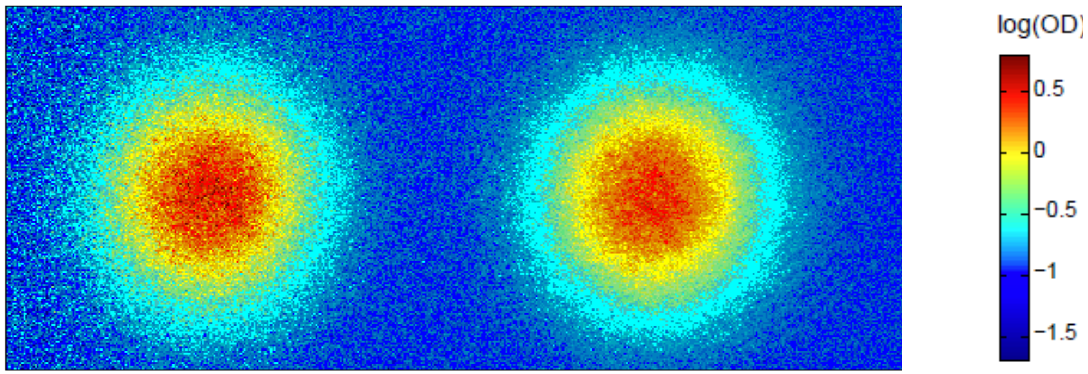
$$D(x,y) = \frac{\sigma_0}{1 + \frac{I}{I_{\rm Sat}} + \frac{4\Delta^2}{\Gamma^2}} \int n(x,y,z)dz.$$

Absorption cross section: $\sigma_0 = \frac{3\lambda^2}{2\pi}$

λ: Resonance wavelength; $\lambda=2\pi c/\omega_0$
Γ: Scattering rate
$\Delta= \omega-\omega_0$: Detuning:
I_{sat}: Saturation intensity

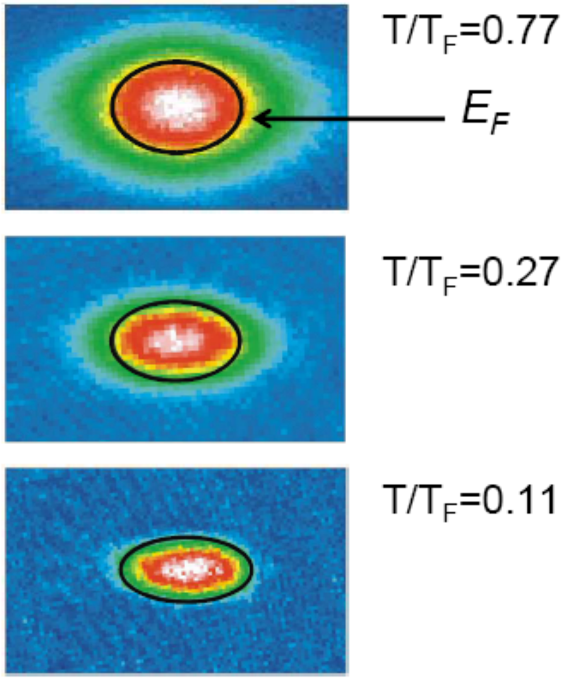
|-9/2> |-7/2>

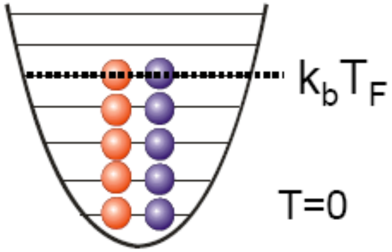
Image of D(x,y)

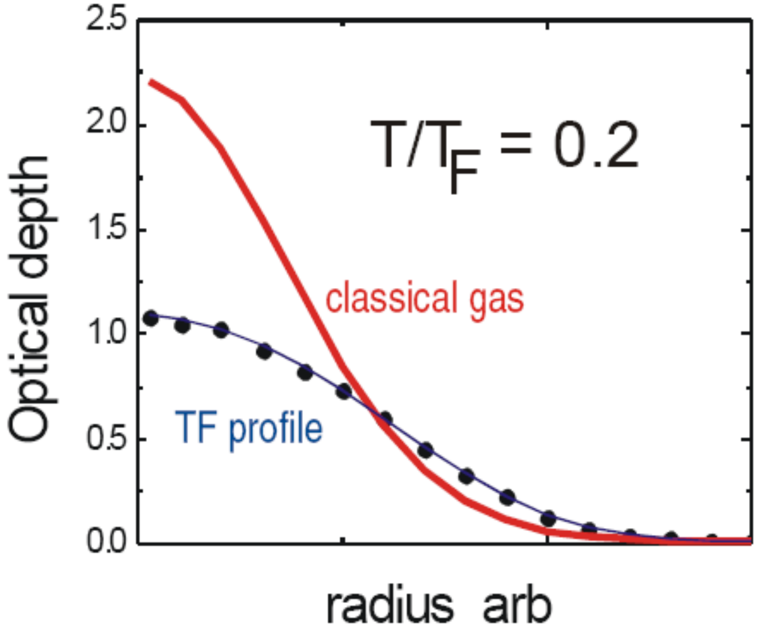


Fermi degeneracy

momentum distribution







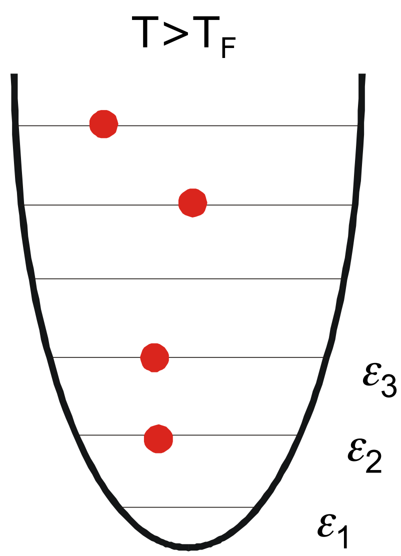
Jin group, JILA

Basic properties of Fermions at low temperature

Ultracold fermions in a trap

Fermi-Dirac distribution determines population of energy level ε_i :

$$\langle n_i \rangle = \frac{1}{e^{(\epsilon_i - \mu)/k_B T} + 1}.$$



Chemical potential μ results from normalization of particle number

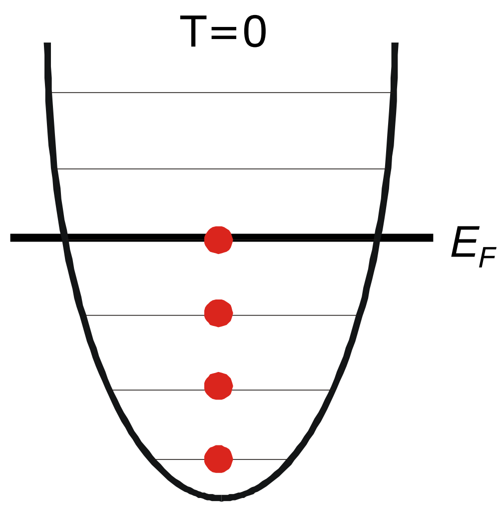
$$N = \int_0^\infty d\epsilon \frac{g(\epsilon)}{\mathcal{Z}^{-1} e^{\epsilon/k_B T} + 1}$$

$\mathcal{Z} = e^{\mu/k_B T}$
(fugacity)

Density of states of 3D-harmonic oscillator:

$$g(\epsilon) = \epsilon^2 / 2(\hbar\overline{\omega})^3$$

$\rightarrow$ $\mathrm{Li}_n(-\mathrm{x}) = \sum_{k=1}^{\infty} (-x)^k / k^n$ Poly-logarithmic function of order n



At T=0 this simplifies to:

$$N = \int_0^{E_F} d\epsilon g(\epsilon),$$

Fermi energy: $E_F = (6N)^{1/3}\, \hbar\omega$

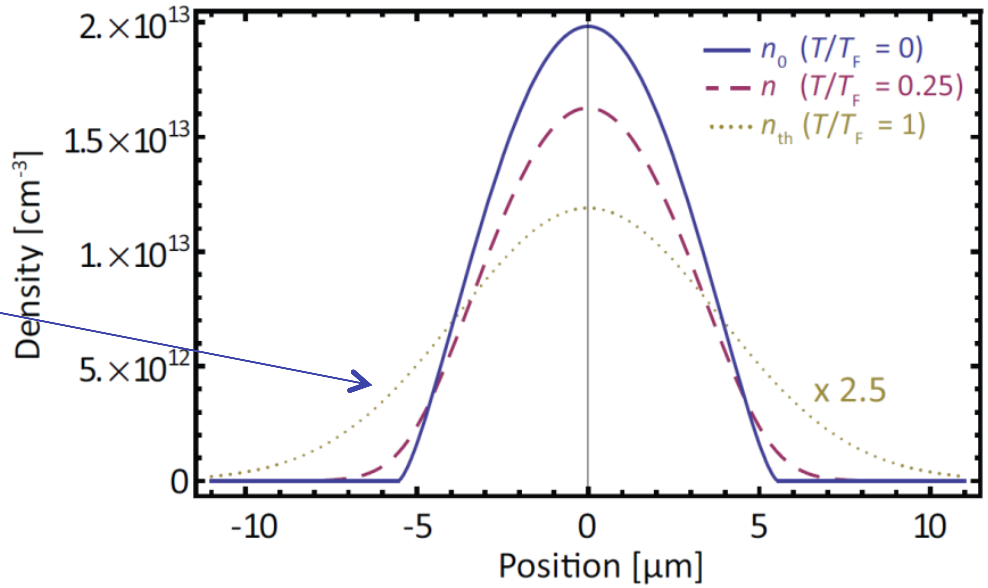
Density profile

Density profile is calculated from phase-space distribution function

$$n(\boldsymbol{r}) = \frac{1}{(2\pi\hbar)^3}\int d^3p\,\wp(\boldsymbol{r},\boldsymbol{p}) \quad \text{with} \quad \wp(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{\mathcal{Z}^{-1}e^{\left(\frac{p^2}{2m}+\frac{1}{2}m\omega_x^2x^2+\frac{1}{2}m\omega_y^2y^2+\frac{1}{2}m\omega_z^2z^2\right)/k_BT}+1},$$

$$n(\boldsymbol{r}) = -\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\mathrm{Li}_{3/2}\left(-\mathcal{Z}e^{-\frac{m\omega_x^2}{2k_BT}x^2}e^{-\frac{m\omega_y^2}{2k_BT}y^2}e^{-\frac{m\omega_z^2}{2k_BT}z^2}\right).$$

Gaussian for $T \gg T_F$

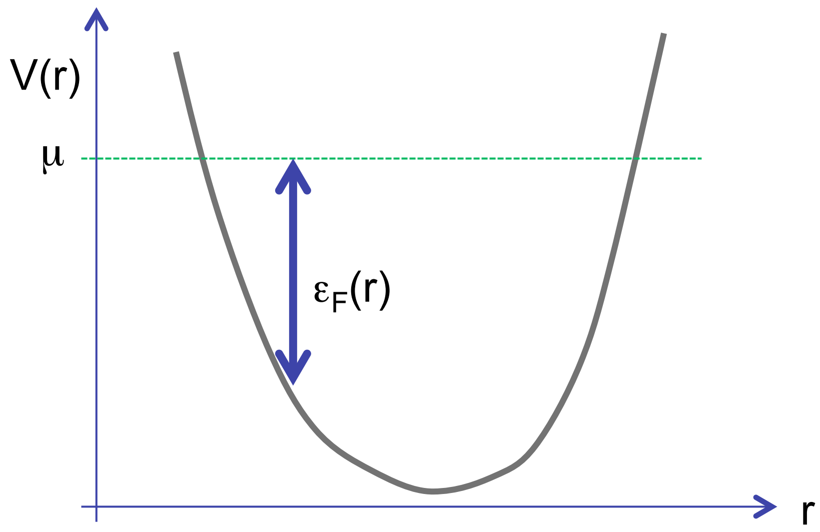


Density profile – local density approximation

At T=0 for many particles:

Assume locally homogeneous system with slowly varying local Fermi energy

$$\epsilon_F(\boldsymbol{r}) = \mu(T=0) - \underbrace{\frac{1}{2}m\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)}_{V(\boldsymbol{r})},$$

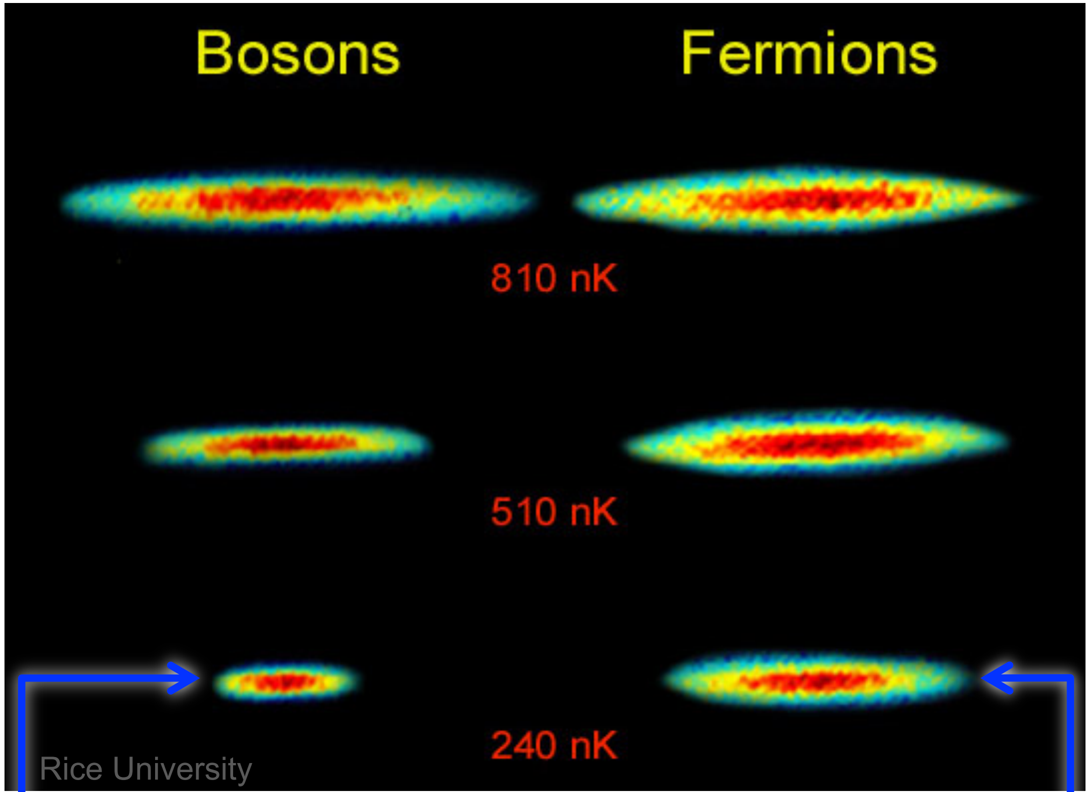


Local density is then

$$\begin{aligned} n(\boldsymbol{r}) &= k_F(\boldsymbol{r})^3/(6\pi^2) \\ &= \frac{4}{3\sqrt{\pi}}\left(\frac{m}{2\pi\hbar^2}\right)^{3/2}(\mu - V(\boldsymbol{r}))^{3/2} \end{aligned}$$

Requires: $E_F >> \hbar\omega$, i.e. $N>>1$

Bosons vs. Fermions



Bosons shrink to ground state

Pauli's exclusion principle prevents fermions from shrinking

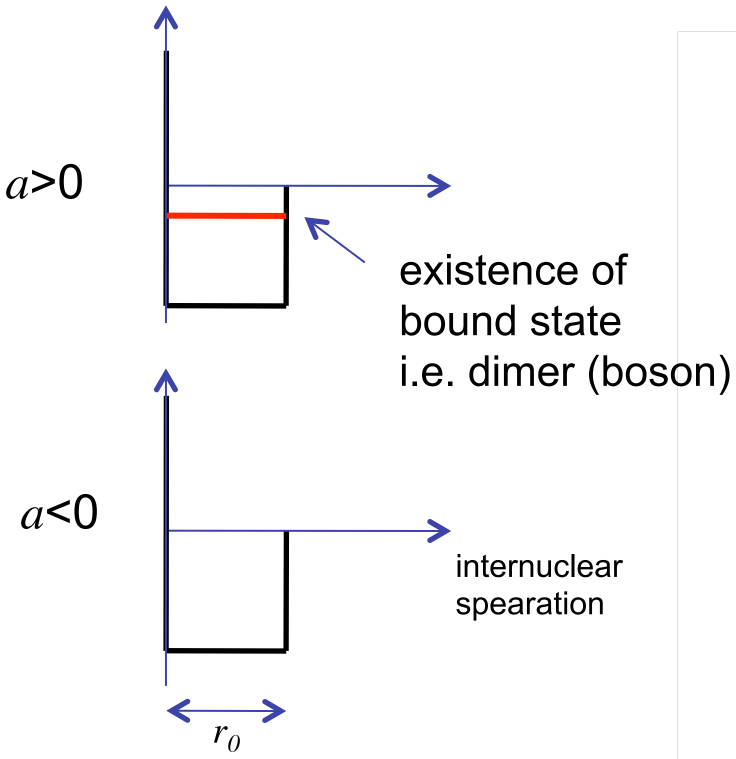
Overview

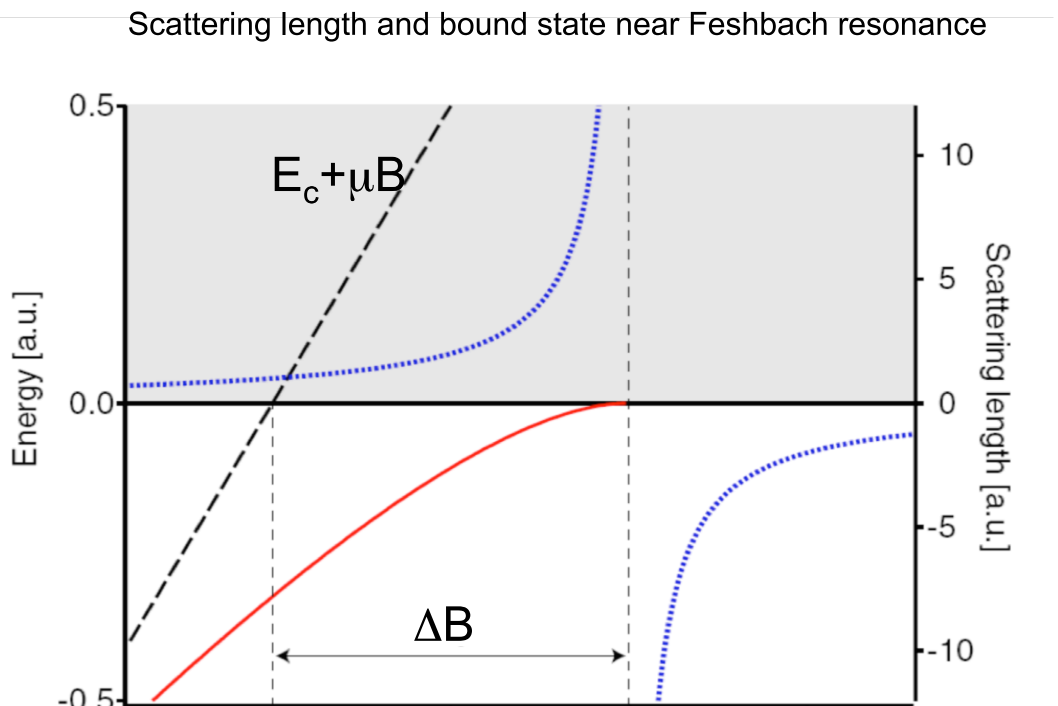
Lecture 1: Atomic Fermi gases - experimental and theoretical background	Lecture 2: Fermion pairing in 3D and 2D	Lecture 3: 2D Fermi gases
Introduction How to make a Fermi gas Seeing the Fermi surface Interactions	BEC-BCS crossover Preparing 2D Fermi gases Interactions in 2D Collective modes	Momentum-resolved rf spectroscopy Fermi liquid Pseudogap pairing Polarons

Converting between fermions and bosons

Elementary massive particles are fermions

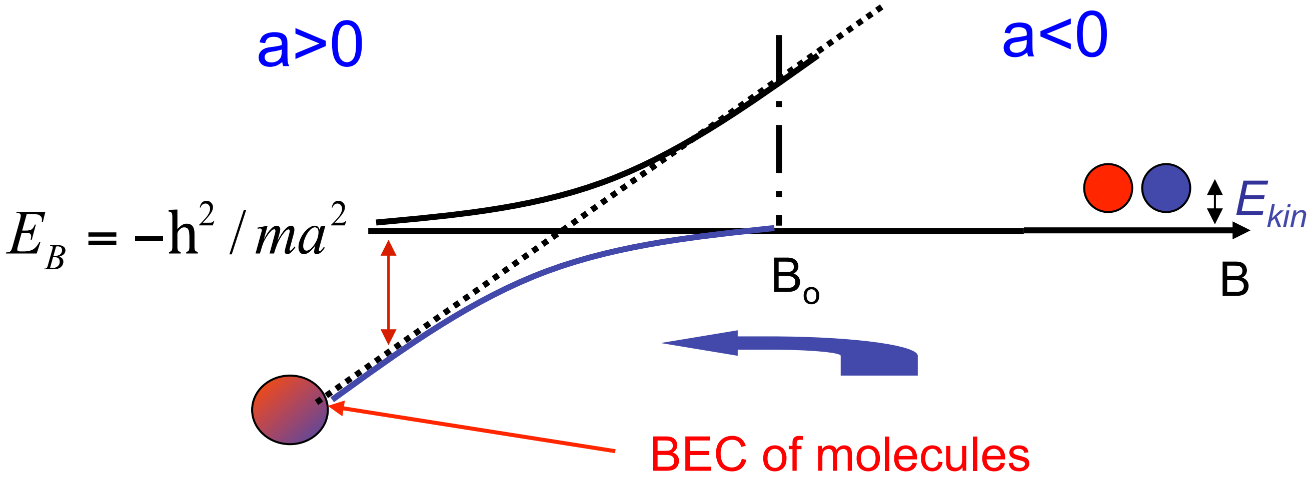
=> bosons are composite particles





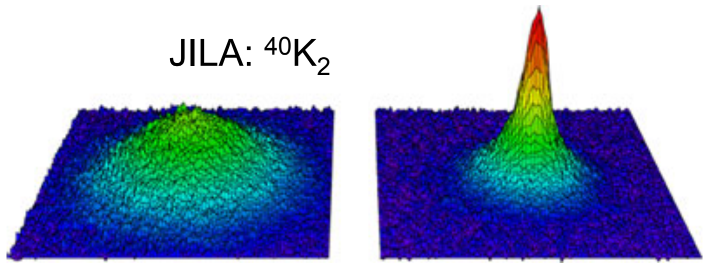
$$E_m = \frac{\hbar^2}{m\,(a - r_0)^2}$$

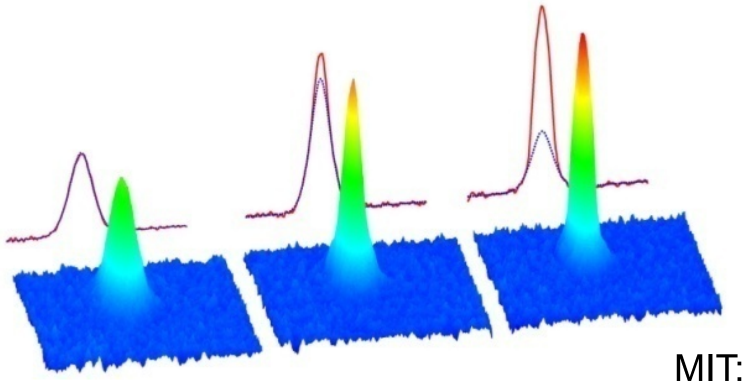
Converting fermions to bosons



Bose-Einstein condensation of pairs of fermions

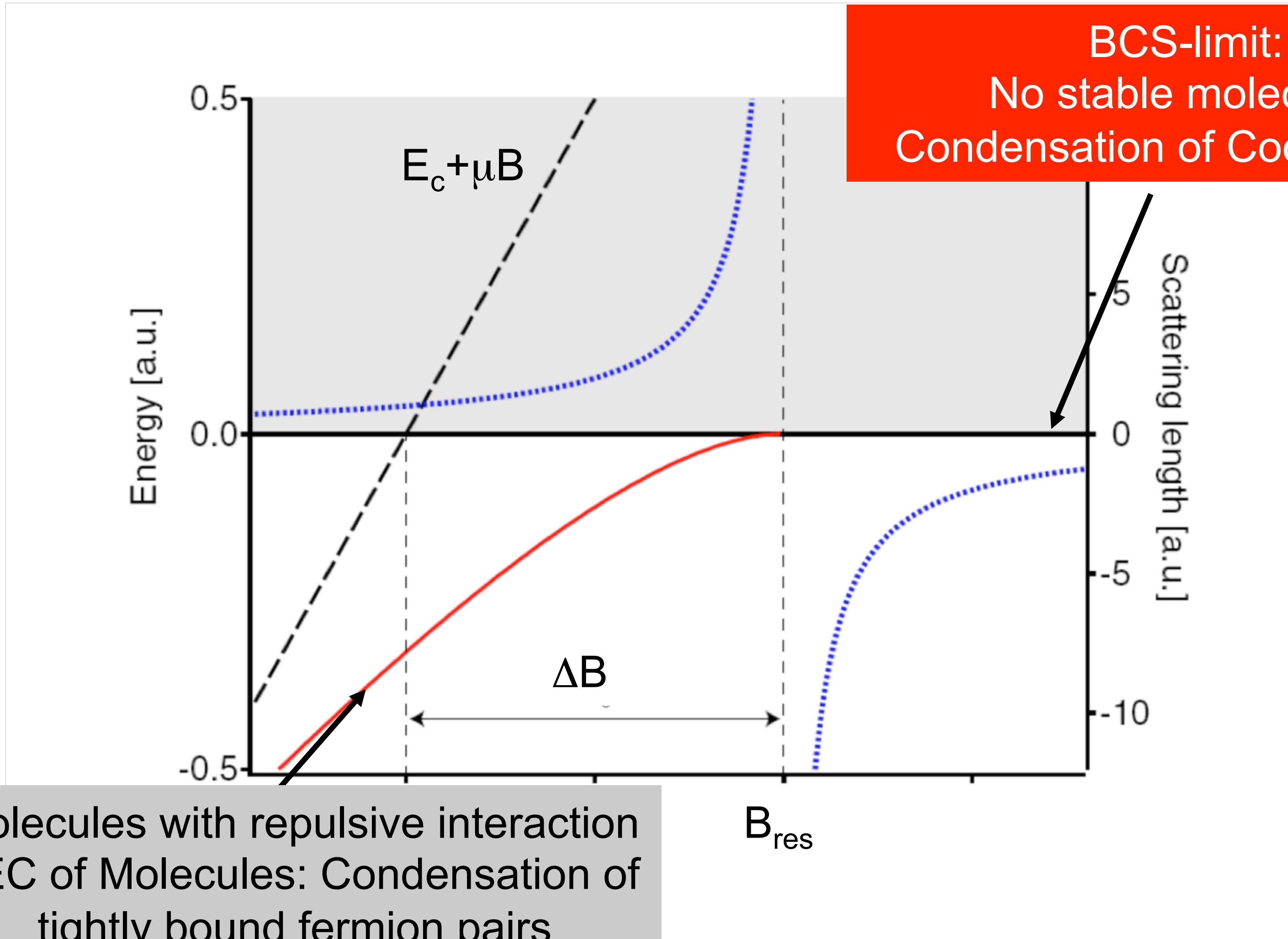
JILA: $^{40}K_2$





MIT: $^{6}Li_2$

Feshbach resonance



BCS-limit:
No stable molecules
Condensation of Cooper pairs

Molecules with repulsive interaction
BEC of Molecules: Condensation of
tightly bound fermion pairs
(last lecture)

The “BCS” regime

<u>Bardeen, Cooper, Schrieffer, 1957:</u>

- Superconductivity of metals at low temperature
- Weak attractive interactions
- Pairs in momentum space
- Quasi-particle excitation spectrum has a gap

Bound states and dimensionality

Search for bound state in the Schrödinger equation $\left[-\frac{\hbar^2}{2m_r}\Delta + V(\boldsymbol{r})\right]\psi_{\boldsymbol{k}}(\boldsymbol{r}) = E_k\psi_{\boldsymbol{k}}(\boldsymbol{r}).$

with binding energy $E_B=-\hbar^2k^2/m$

short-range interaction
$V(\mathbf{q}) \approx V_0$ for $q<1/r_0$
$V(q) = 0$ for $q>1/r_0$

=> solve $\frac{\hbar^2}{m}\left(\Delta - k^2\right)\psi = V\psi.$

in momentum space: $\psi_k(\mathbf{q}) = -\frac{m}{\hbar^2}\frac{1}{q^2+k^2}\int\frac{d^nq'}{(2\pi)^n}V(\mathbf{q}-\mathbf{q}')\psi_k(\mathbf{q}').$

[integrate both sides over q and divide by $\int\frac{d^nq}{(2\pi)^n}\psi_k(\mathbf{q})$]

density of states

$$-\frac{1}{V_0} = \frac{1}{\Omega}\int_{\epsilon<E_{R_0}} d\epsilon\frac{\rho_n(\epsilon)}{2\epsilon+|E|},$$

$E_{R0}=\hbar^2/mr_0^2$

Is there a weakly bound state ($|E|<<E_{R0}$) for infinitesimal attraction? => depends on density of states!

3D: $\rho(\varepsilon) \sim \varepsilon^{1/2}$

$$E_{3D} = -\frac{8}{\pi^2}E_{R_0}\frac{(|V_0|-V_{0c})^2}{|V_0|^2}.$$

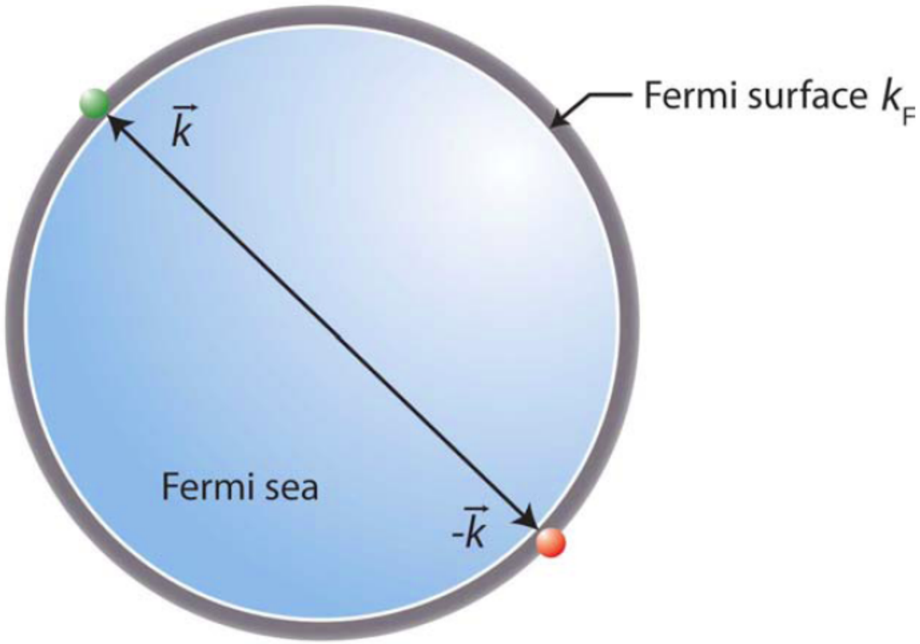
minimum V_{0c} required for pairing

2D: $\rho(\varepsilon)$ = const.

$$E_{2D} = -2E_{R_0}e^{-\frac{2\Omega}{\rho_{2D}|V_0|}}$$

always a bound state, even for V_0->0!

Cooper pairing



Cooper's idea:
Pairing is only on the surface of a non-interacting Fermi sea.

Binding energy below $2E_F$

$$E_B = \frac{8}{e^2} E_F e^{-\pi/k_F|a|}.$$

Many-body Hamiltonian: pairing between fermions of opposite k and spin only.

$$\hat{H} = \sum_{k,\sigma} \epsilon_k n_{k\sigma} + V_0 \sum_{k,k'} c^\dagger_{k'\uparrow} c^\dagger_{-k'\downarrow} c_{-k\downarrow} c_{k\uparrow}.$$

$c_{k\sigma}$: fermionic annihilation operator for momentum k and spin σ
$n_{k\sigma}$: density operator

Introduce fermionic pair operators: $b_k = c_{-k\downarrow} c_{k\uparrow}$

$$\hat{H} = \sum_k 2\epsilon_k b^\dagger_k b_k + V_0 \sum_{k,k'} b^\dagger_{k'} b_k.$$

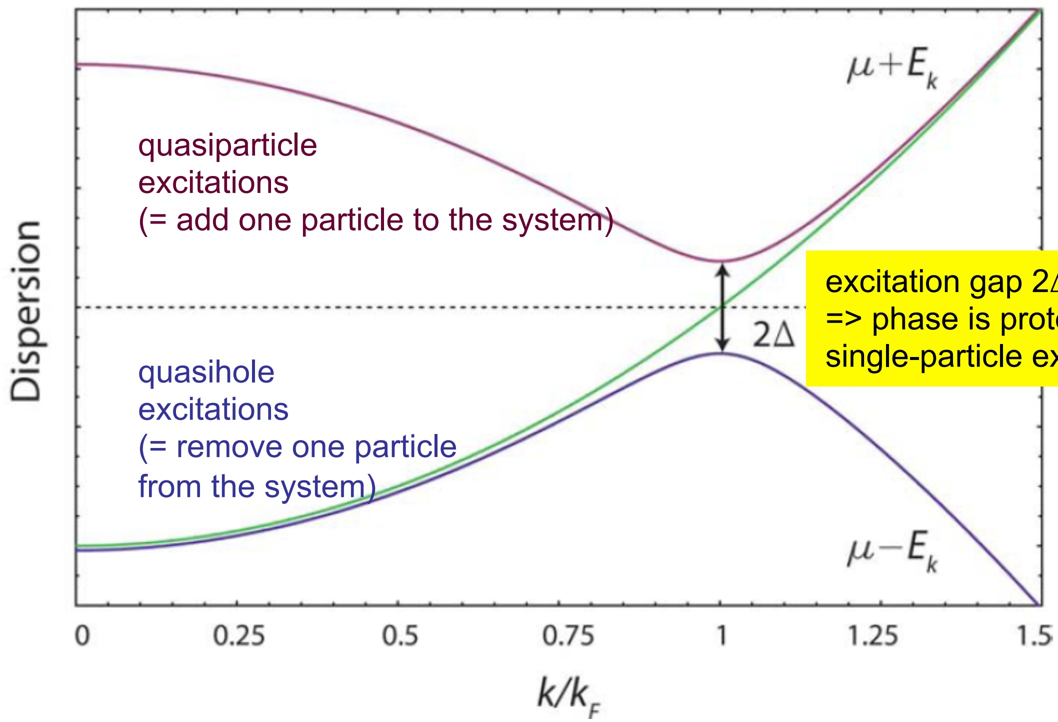
BCS superfluidity

Bardeen, Cooper, Schrieffer: variational ansatz (use trial wave function to minimize energy)

$$|\psi_0\rangle = \prod_k \frac{1 + g_k b_k^\dagger}{(1+g_k^2)^{1/2}} |0\rangle.$$

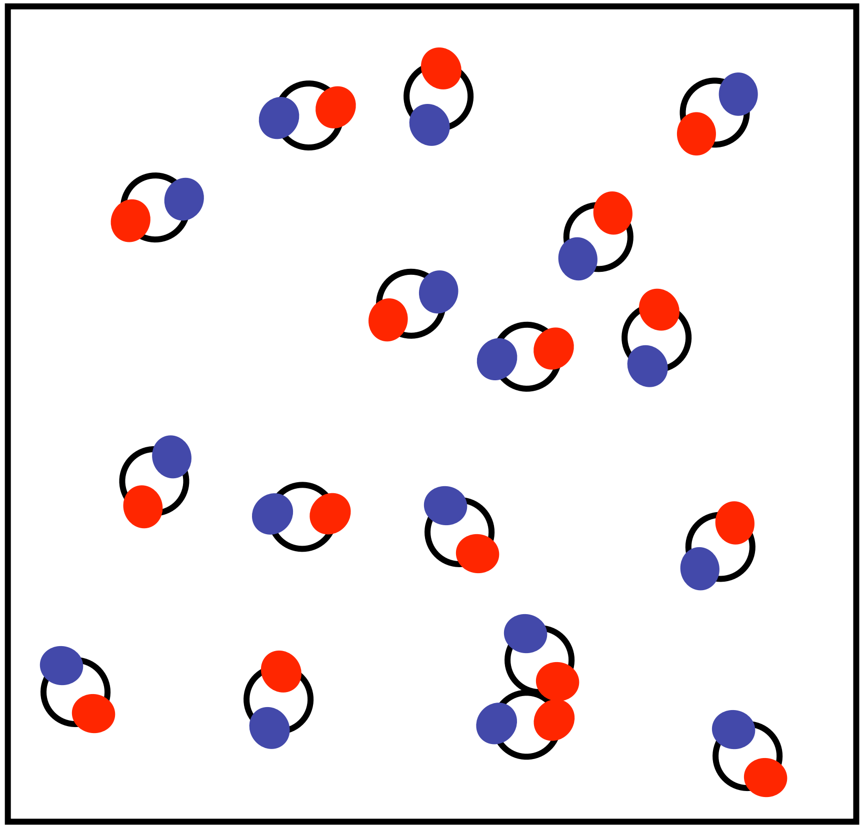
BCS dispersion relation

$$E_k = \left[(\epsilon_k - \mu)^2 + \Delta^2\right]^{1/2}$$

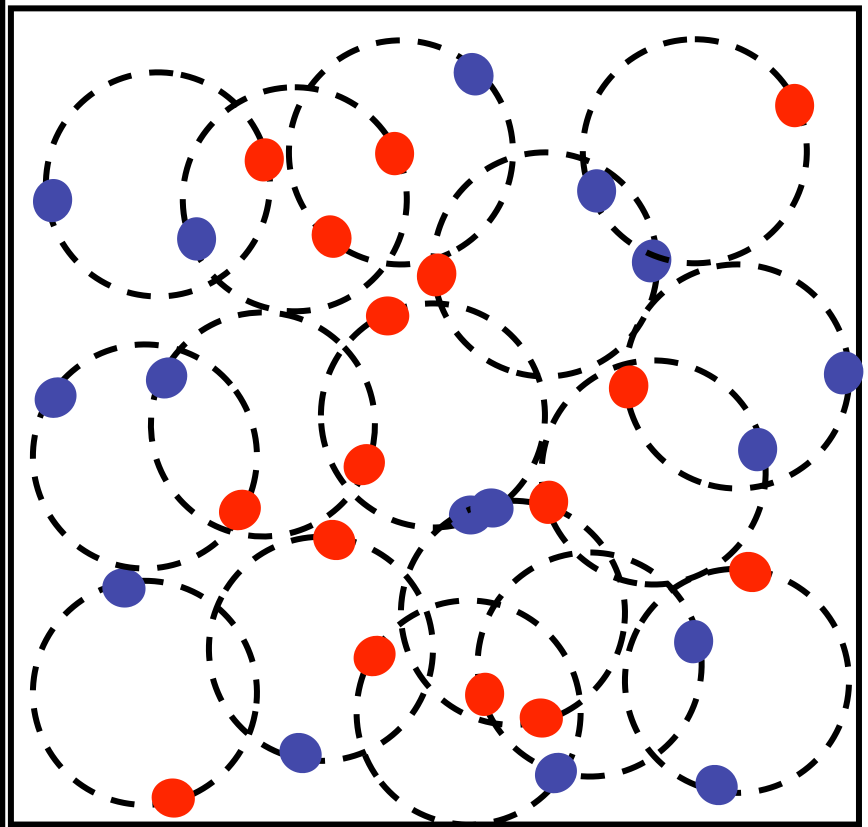


excitation gap 2Δ
=> phase is protected from
single-particle excitation

$$\mu \approx E_\mathrm{F} \qquad \text{and} \qquad \Delta \approx \frac{8}{e^2} e^{-\pi/2k_\mathrm{F}|a|} E_\mathrm{F}.$$

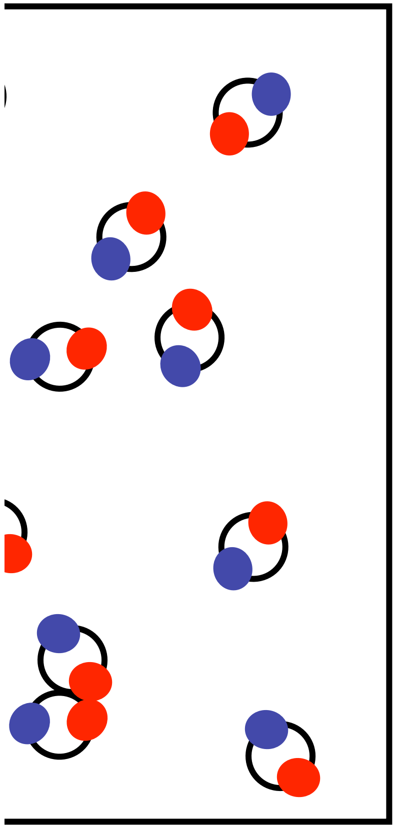


Bose Einstein condensate
of molecules



BCS superfluid

W. Ketterle



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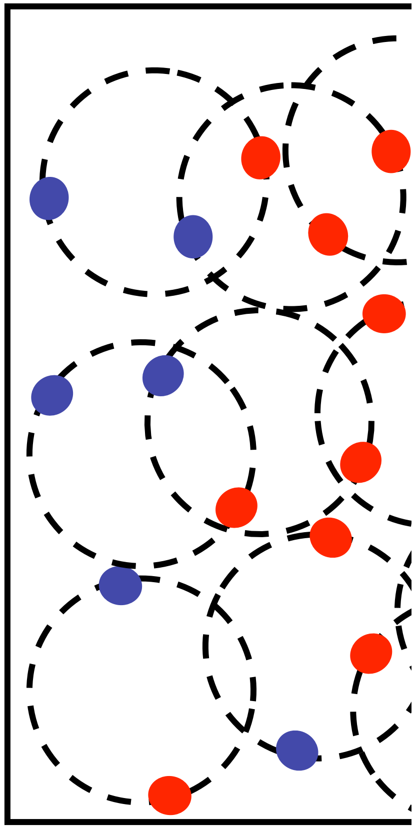
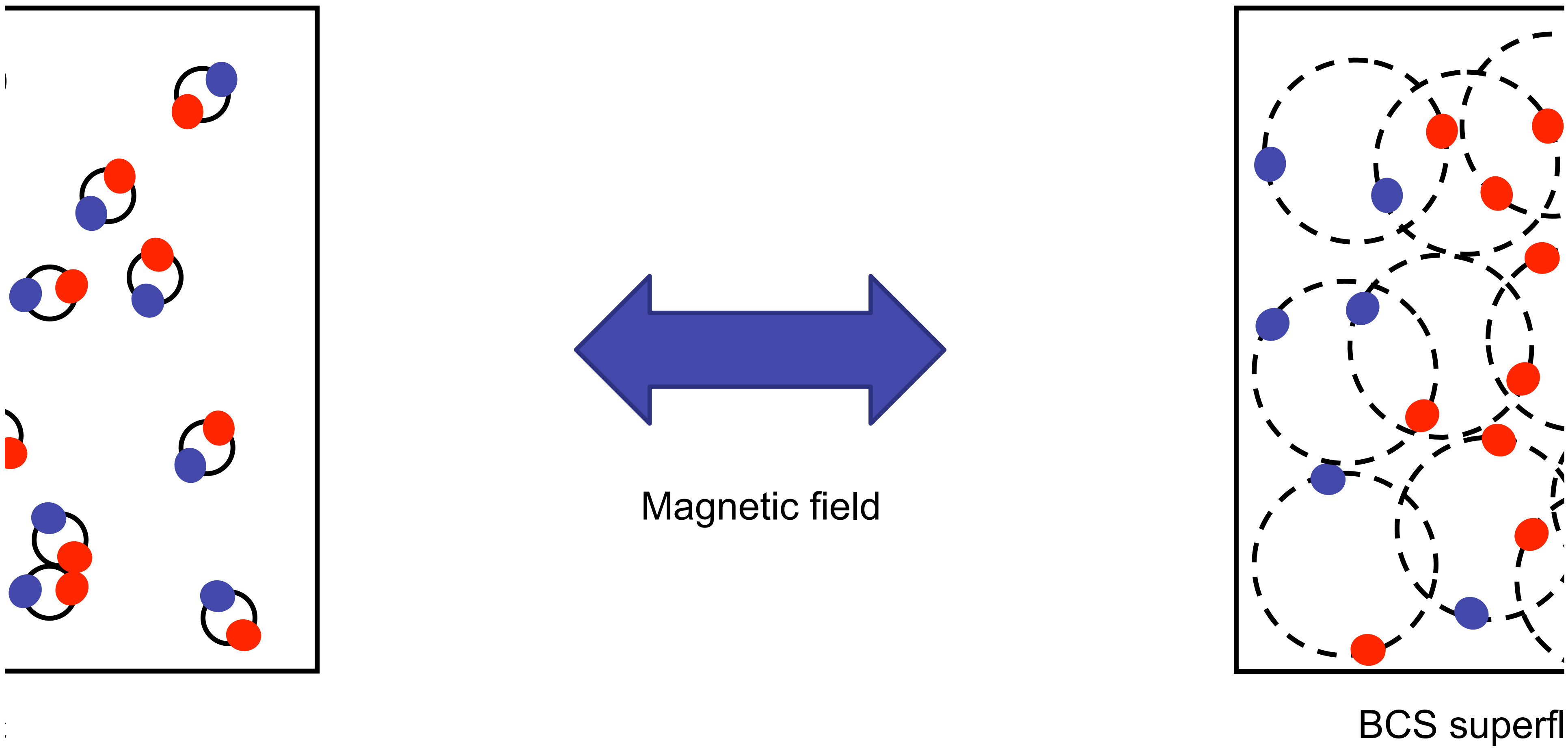
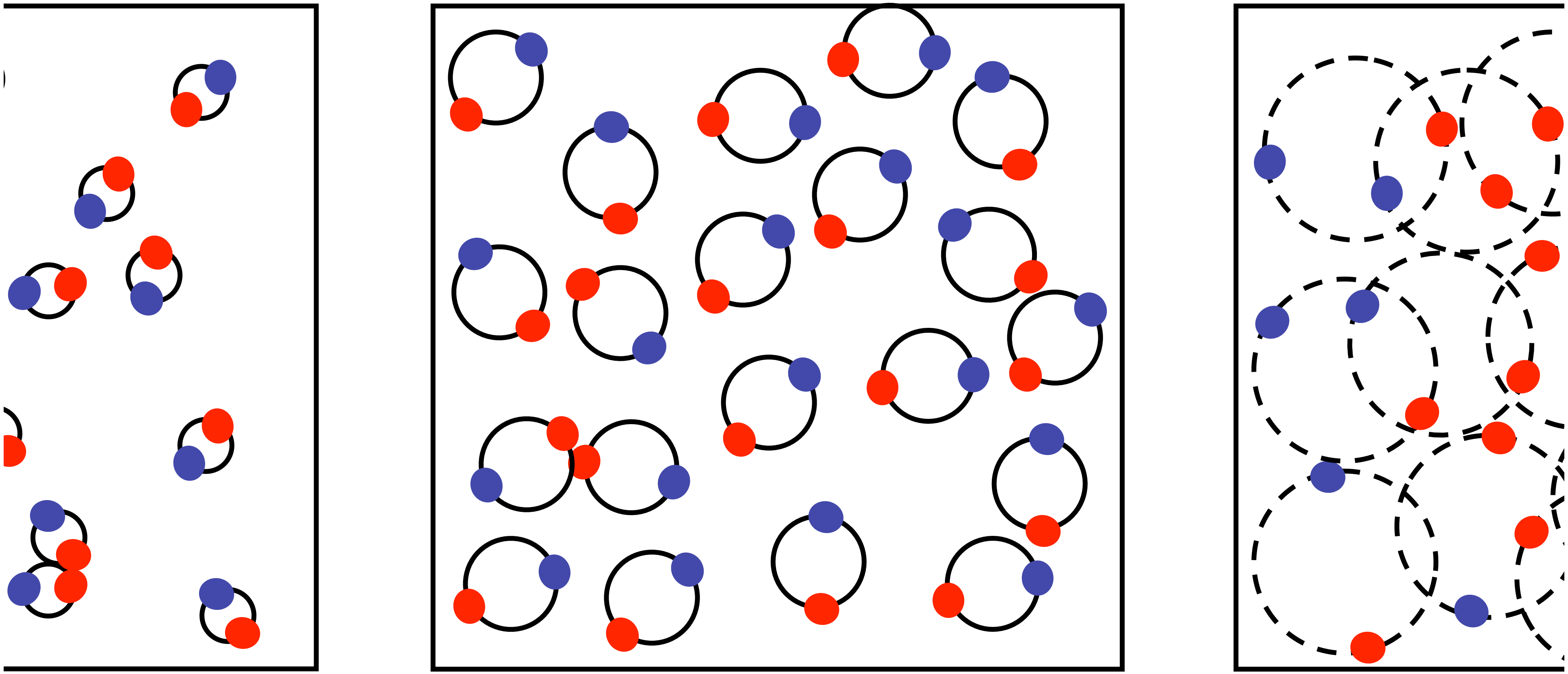


BCS superfl

W. Ketterle



W. Ketterle

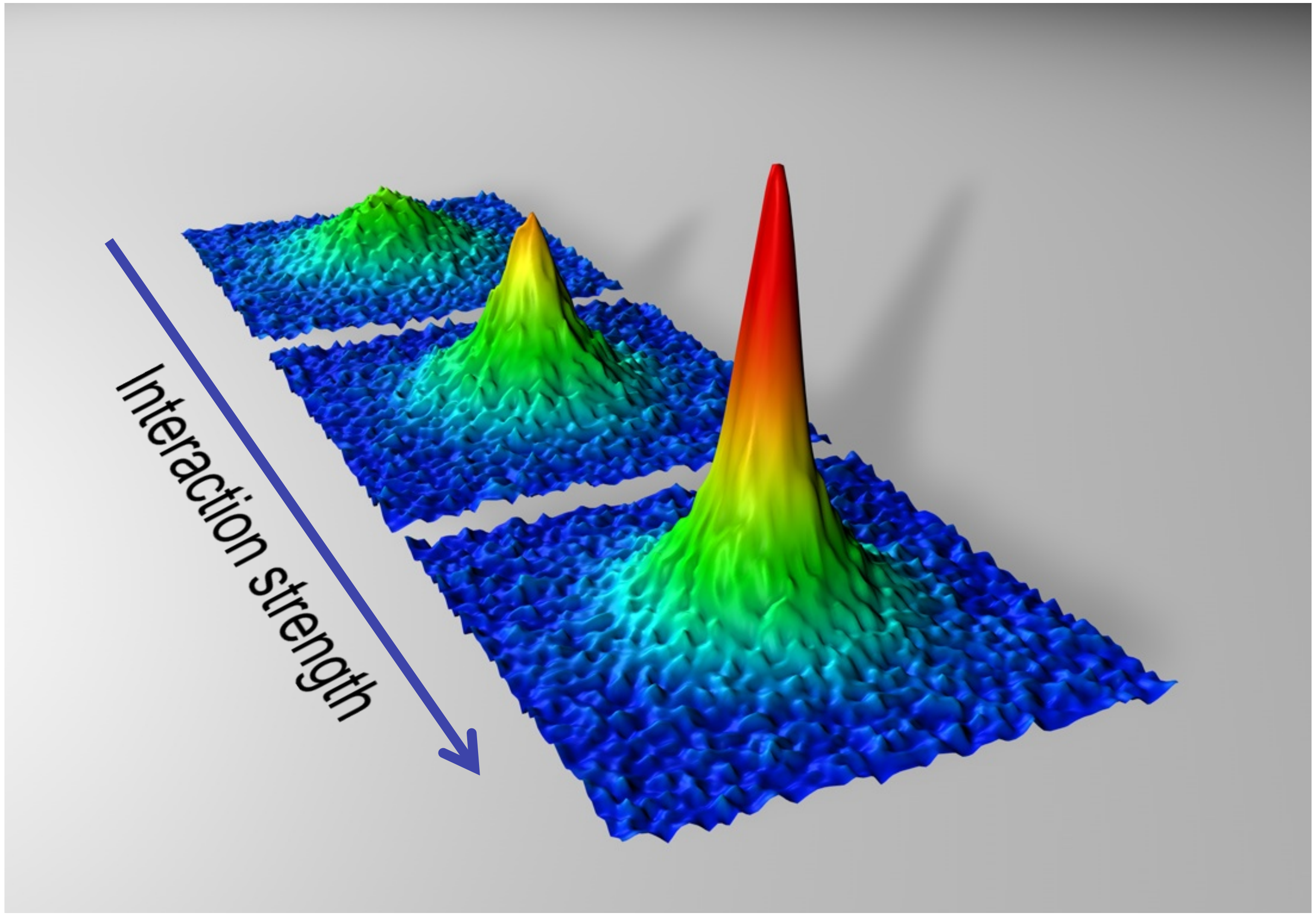


Crossover superfluid

BCS superfl

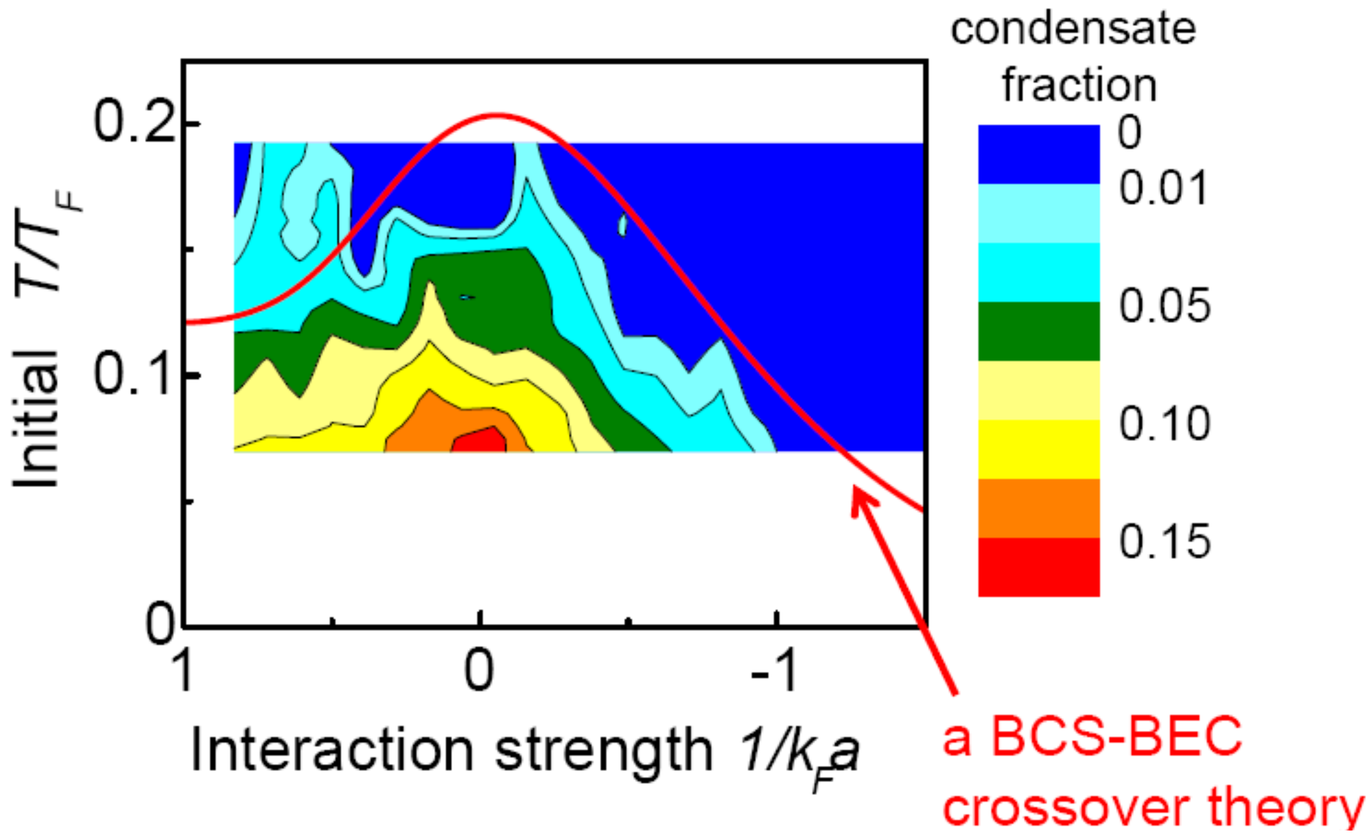
W. Ketterle

Fermi condensates



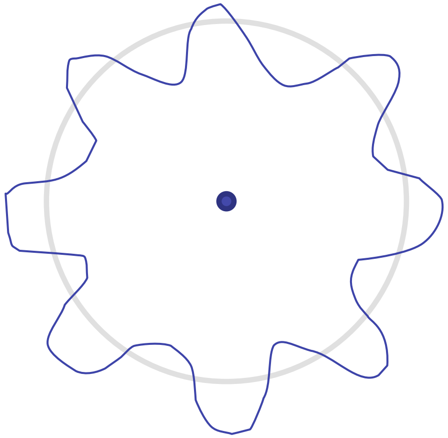
C. A. Regal, M. Greiner, and D. S. Jin, PRL 92, 040403 (2004)

The BEC-BCS crossover



Jin goup, JILA

Superfluidity: Quantized circulation



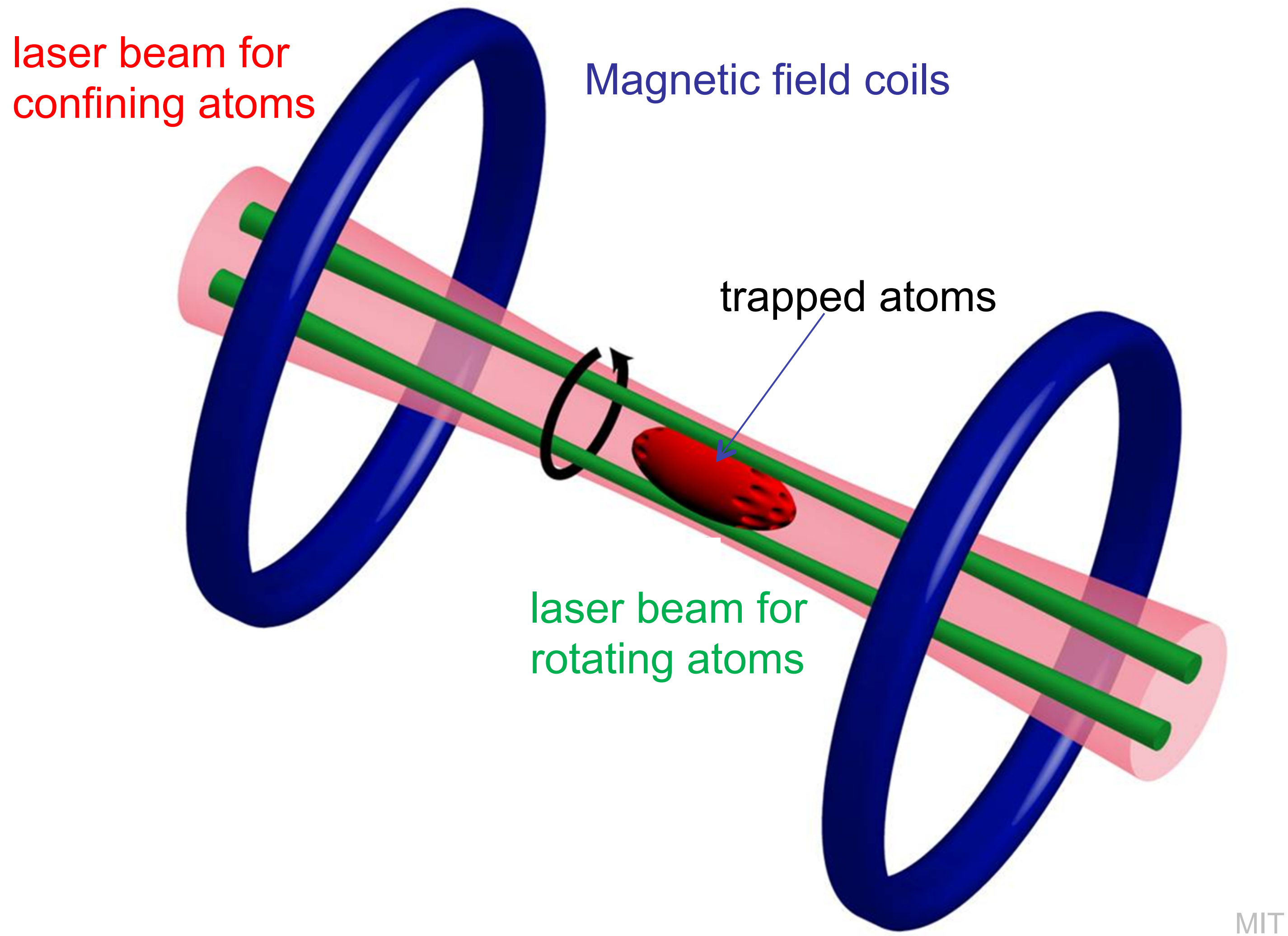
$$\lambda = \frac{h}{mv} \quad \text{(de Broglie)}$$

$$2\pi r = n\lambda = \frac{nh}{mv}$$

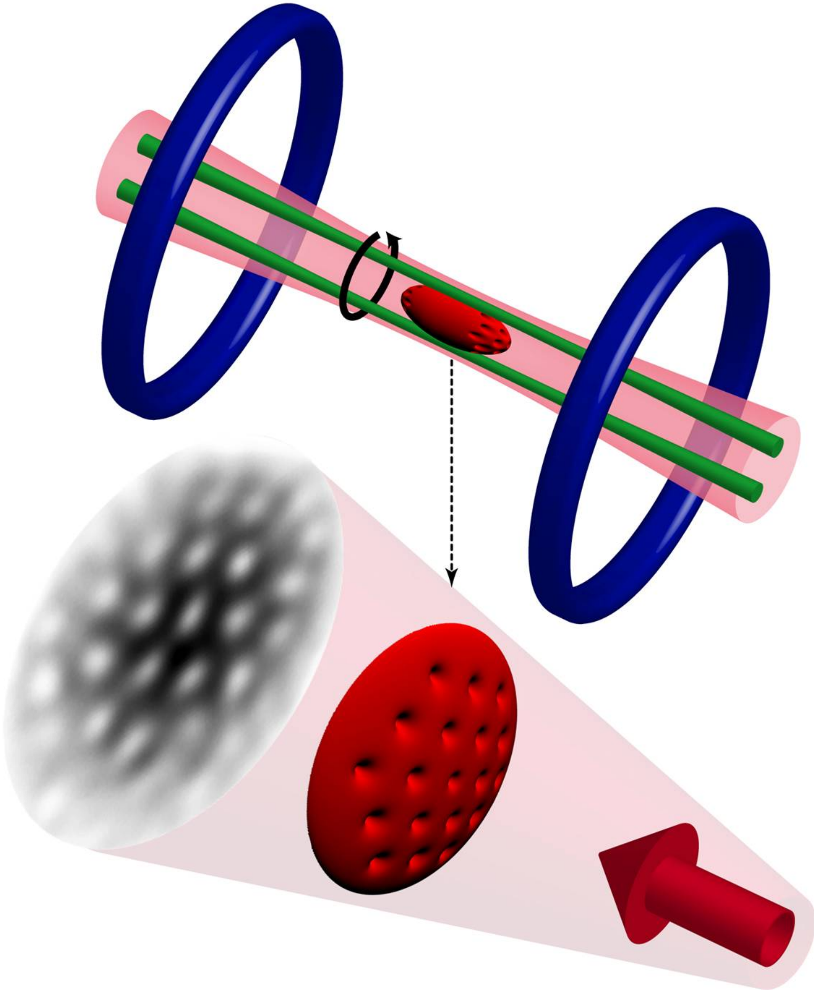
$$\Rightarrow v \cdot 2\pi r = \oint \vec{v} \cdot \vec{ds} = n\frac{h}{m}$$

Quantization: Integer number of matter waves on a circle

Spinning a strongly interacting Fermi gas

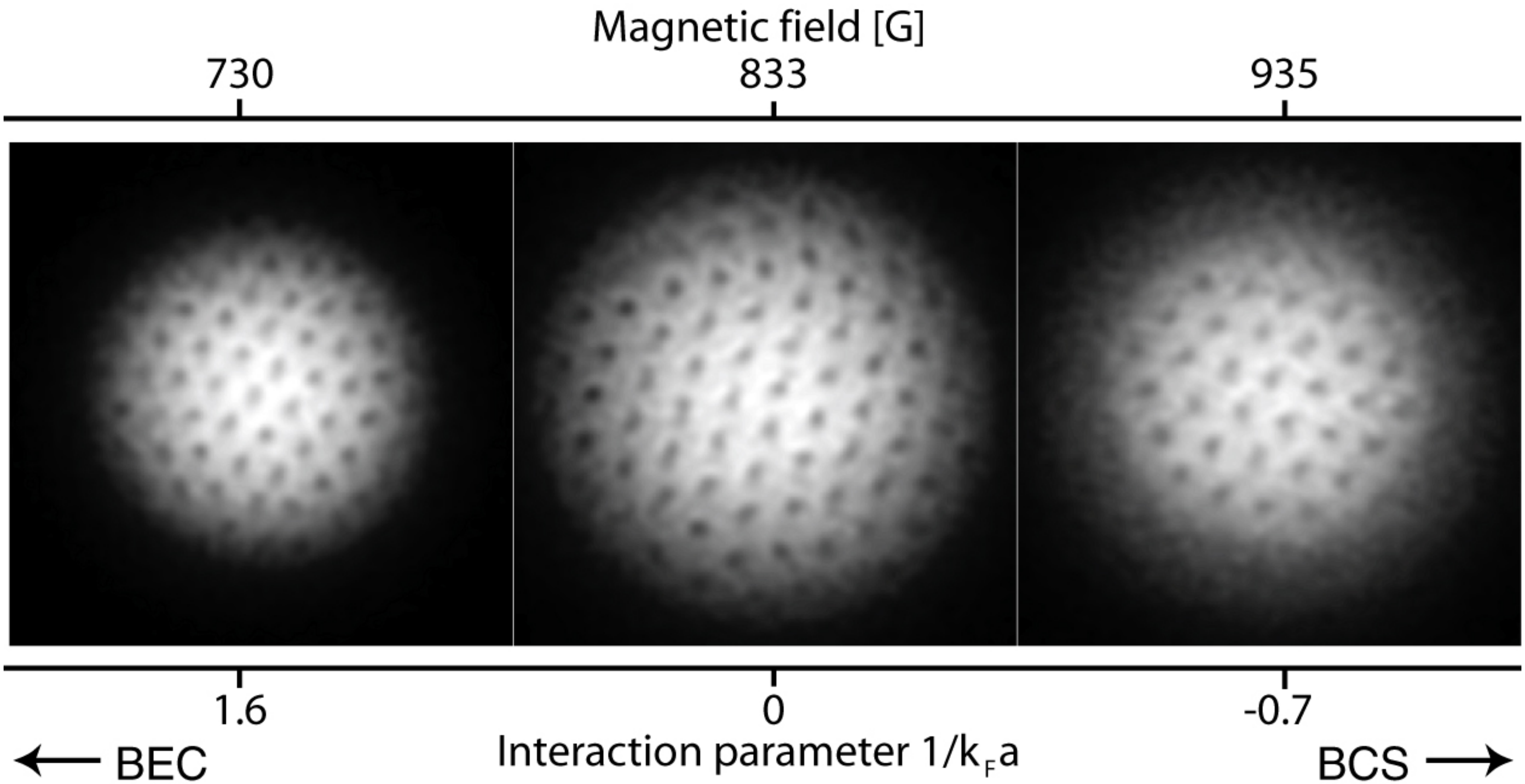


Spinning a strongly interacting Fermi gas



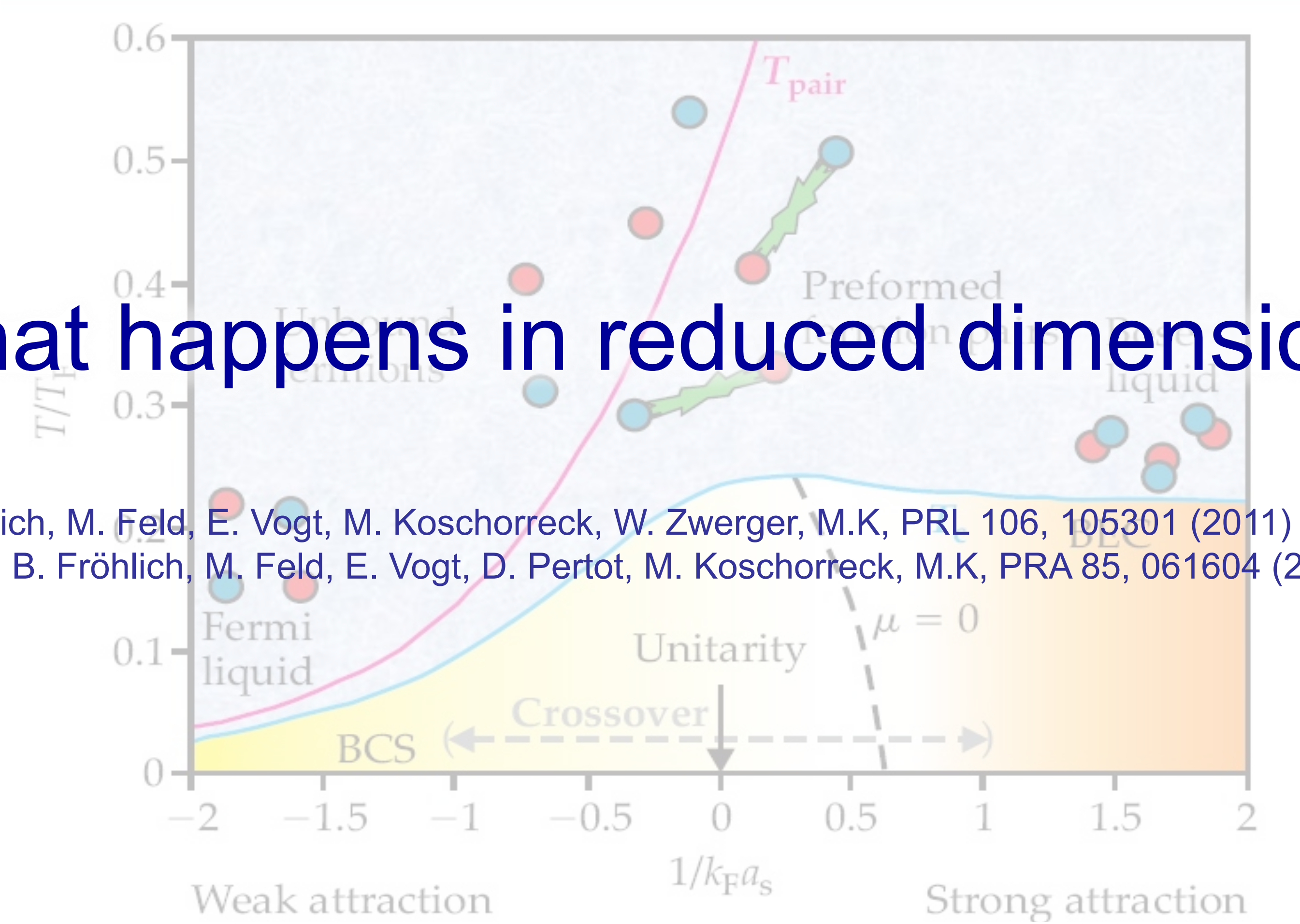
MIT

Vortex lattices in the crossover



M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle,
Nature 435, 1047-1051 (2005)

BEC-BCS crossover



What happens in reduced dimensions?

B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, W. Zwerger, M.K, PRL 106, 105301 (2011)
S. Baur, B. Fröhlich, M. Feld, E. Vogt, D. Pertot, M. Koschorreck, M.K, PRA 85, 061604 (2012)

Sa de Melo, Physics Today (2008)

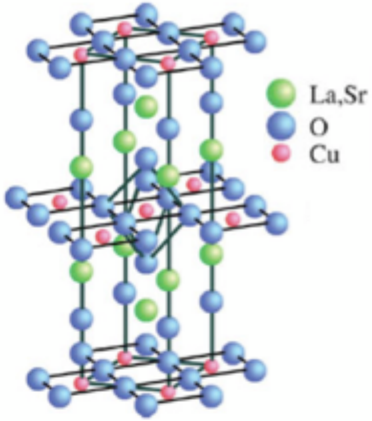
Two-dimensional Fermi gases

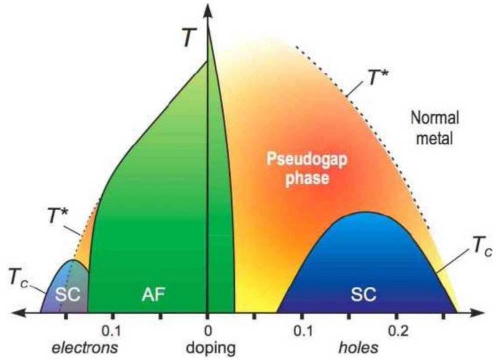
Two-dimensional gases:

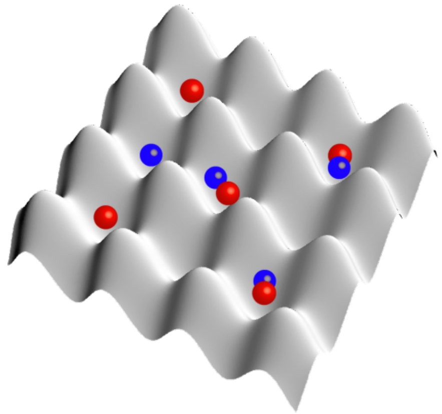
"the grand challenge" of condensed matter physics

High-T_c superconductors:

- After 25 years of research still no breakthrough in understanding (nor to solve the energy crisis)
- Material is to complicated to understand even the basic mechanism







Cold atomic gases provide tuneable model system

- fermionic atoms take the role of electrons
- lattice created by standing wave laser fields

→ build quantum simulator (Feynman)

Two-dimensional Fermi gases

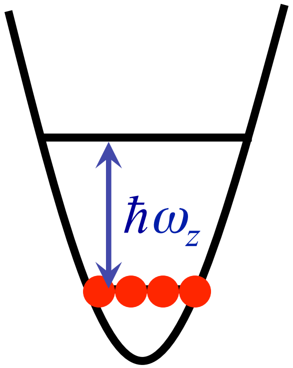
1. Two-body scattering in 2D
2. Basic properties of 2D Fermi gases
3. Collective modes (sound)

} today

4. Momentum-resolved rf spectroscopy
5. Fermi liquid
6. Pseudogap pairing
7. Fermi polaron in 2D

} tomorrow

Quasi-2D geometry



<u>Conditions for 2D</u>

$$E_F\,,\; k_B T \ll \hbar\omega_z$$

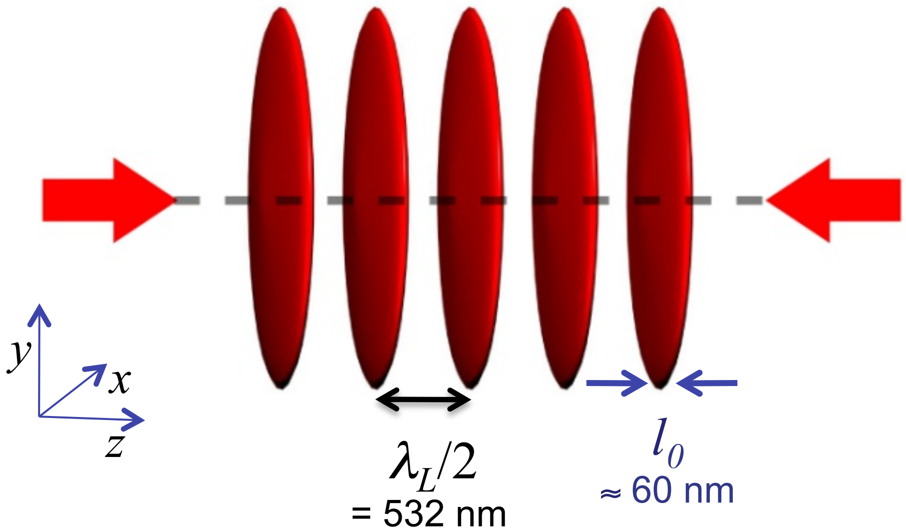
Strong axial confinement required

$E_F \approx h \times 8\,\text{kHz}$

$\omega_z \approx 2\pi \times 80\,\text{kHz}$

$\omega_\perp \approx 2\pi \times 130\,\text{Hz}$

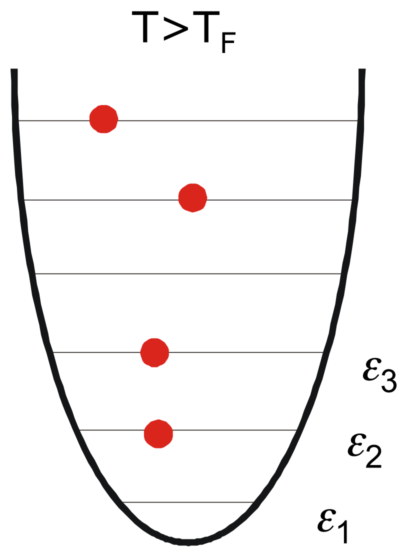
Optical lattice: array of 2D quantum gases



- lattice depth 83 E_{rec}
- hopping rate 0.002 Hz
- ~ 2000 Fermions per spin state
- ~ 30 "pancakes" / layers

B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, W. Zwerger, MK, PRL 106, 105301 (2011)
other 2D Fermi gases: Inguscio, Grimm, Esslinger, Turlapov, Vale, Zwierlein

Ultracold fermions in a 2D trap



Fermi-Dirac distribution determines population of energy level ε_i :

$$\langle n_i \rangle = \frac{1}{e^{(\epsilon_i - \mu)/k_B T} + 1}.$$

Chemical potential μ results from normalization of particle number

$$N = \int_0^\infty d\epsilon \frac{g(\epsilon)}{\mathcal{Z}^{-1} e^{\epsilon/k_B T} + 1} = -\left(\frac{k_B T}{\hbar \overline{\omega}}\right)^2 \mathrm{Li}_2(-\mathcal{Z})$$

Density of states of 2D-harmonic oscillator:

$$g(\epsilon) = \epsilon / \hbar^2 \overline{\omega}^2$$

At T=0 this simplifies to:

$$N = \int_0^{E_F} d\epsilon g(\epsilon),$$

Fermi energy: $E_{F,2D}=(2N)^{1/2}\,\hbar\omega$

T=0 density distribution: $n_F^0(\mathbf{r}) = \frac{m}{2\pi\hbar^2}\left(\mu - V(\mathbf{r})\right)$

Scattering in two dimensions

(Formalism analogous to three dimensions)

Schrödinger equation in relative coordinates $\left[-\frac{\hbar^2}{2m_r}\Delta + V(\boldsymbol{r})\right]\psi_{\boldsymbol{k}}(\boldsymbol{r}) = E_k\psi_{\boldsymbol{k}}(\boldsymbol{r}).$

Asymptotic wave function

$$\psi_{\boldsymbol{k}}(\boldsymbol{\rho}) \propto e^{ik\rho} - f_{2\mathrm{D}}(k,\phi)\sqrt{\frac{i}{8\pi k\rho}}e^{ik\rho}.$$

incoming plane wave

(k: wave vector of relative motion)

outgoing cylindrical wave

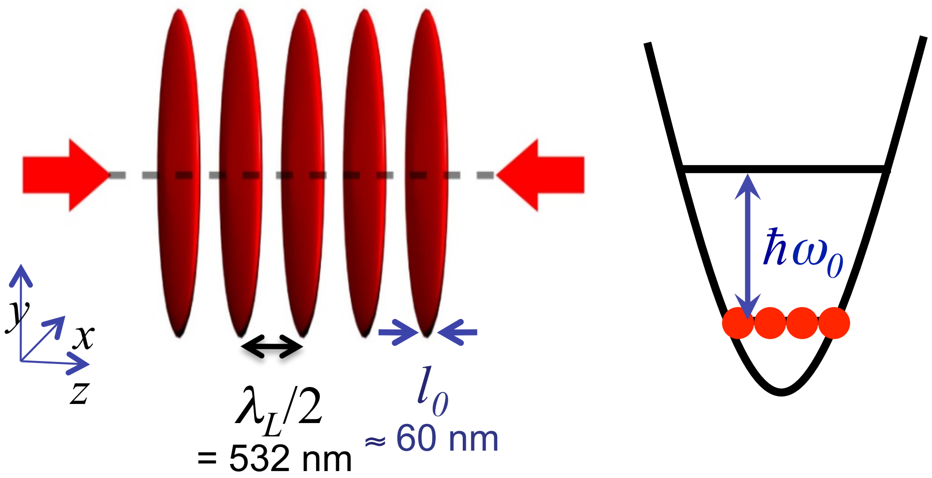
For s-wave collisions and short-range interactions:

$$f_{2\mathrm{D}}(k) = \frac{2}{i}\left(1 - e^{2i\delta_0}\right) \quad \text{with} \quad \pi\cot\delta_0(E) = \ln\frac{E}{E_b} + O(2mR_0^2E/\hbar^2),$$

s-wave scattering amplitude in 2D: $f_{2\mathrm{D}}(k) = \frac{4\pi}{\ln\left(\frac{E_b}{E_k}\right) + i\pi}$

- one parameter E_b - the 2D bound state
- define 2D scattering length: $E_B = \hbar^2/ma_{2D}^2$
- energy dependent

Quasi-two dimensional confinement



Quasi-2D

$$E_F\,,\, k_B T \ll \hbar\omega_0$$

$$E_F \approx h \times 8\ \mathrm{kHz}$$
$$\omega_0 \approx 2\pi \times 80\ \mathrm{kHz}$$
$$\omega_\perp \approx 2\pi \times 130\ \mathrm{Hz}$$

- Kinematics of the gas is 2D, since $E_F \ll \hbar\omega_z$
- Interaction potential is 3D: $l_0 > r_{eff}$
  => Three-dimensional scattering length a_{3D} parameterizes the interaction
  (good for Feshbach resonance)

=> in experiments we realize quasi-2D confinement!

However, for an harmonic oscillator we can still separate COM and relative motion:

Schrödinger equation in relative coordinates

$$\left[-\frac{\hbar^2}{2m_r}\Delta + V(r) + \frac{V_{ho}}{2} - \frac{\hbar\omega_0}{2}\right]\psi_{\boldsymbol{k},\nu}(\boldsymbol{r}) = E_{\boldsymbol{k},\nu}\psi_{\boldsymbol{k},\nu}(\boldsymbol{r}),$$

Scatt. amplitude and bound state in quasi-2D

Solving the scattering problem in quasi-2D results in the scattering amplitude
(Petrov, Shlyapnikov, PRA 2001)

$$f_{00}(k) = \frac{4\pi}{\sqrt{2\pi}\frac{l_0}{a} + \ln\left(\frac{0.905}{\pi l_0^2 k^2}\right) + i\pi}$$

scattering only within
the lowest state of the HO

the trap plays a crucial role!

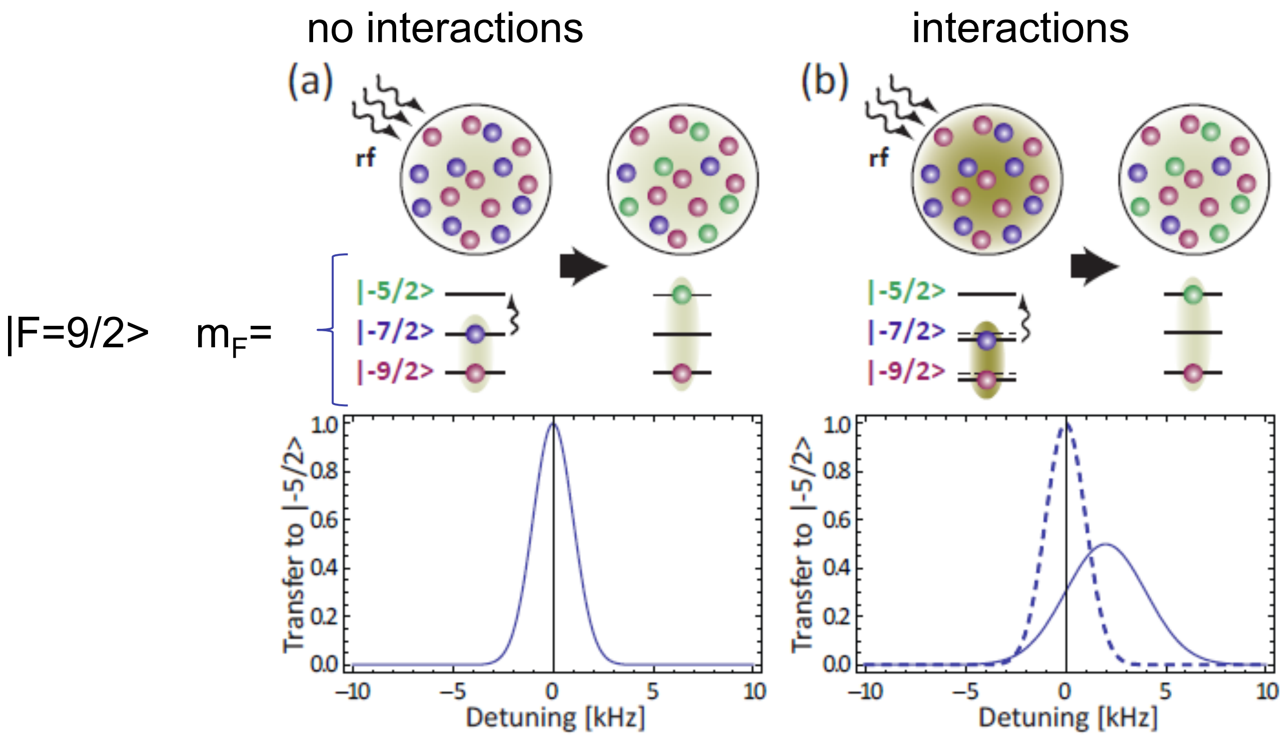
Solving for the bound state in quasi-2D

$$E_b = 0.905 \frac{\hbar\omega_0}{\pi} e^{-\sqrt{2\pi} l_0/|a|}$$ (for $-a<l_0$)

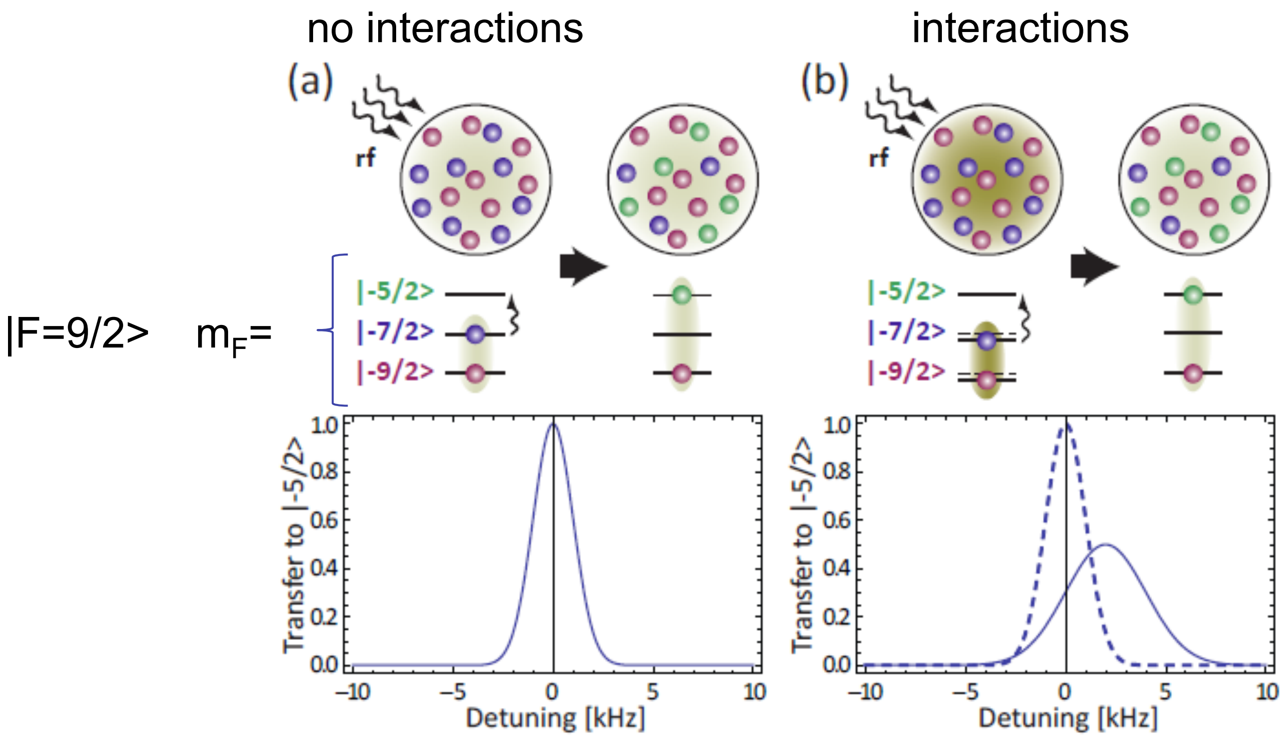
s-wave scattering amplitude in quasi-2D: $f_{2D}(k) = \frac{4\pi}{\ln\left(\frac{E_b}{E_k}\right) + i\pi}$

- one parameter E_b
- energy dependent
- qualitatively the same as in true 2D

Radio-frequency spectroscopy



Radio-frequency spectroscopy



Transition rate by Fermi's golden rule: $\Gamma_0(\omega) = \frac{2\pi}{\hbar} \sum_f |\langle f|\hat{V}|i\rangle|^2 \delta(\hbar\omega + E_i - E_f),$

Rabi frequency

$$\Gamma_0(\omega) = \frac{\sqrt{2\pi}\hbar\Omega_R^2}{m\omega^2 l_0^2 g'\left(E_B^{12}/\hbar\omega_0\right)} \times \sum_{j=0}^{\infty} \frac{(2j-1)!!}{(2j)!!} \theta\left(\hbar\omega - E_B - 2\hbar\omega_0 j\right).$$

$g(x) = \int_0^\infty \frac{du}{\sqrt{4\pi u^3}} \{1 - e^{-xu}[(1 - e^{-2u})/(2u)]^{-1/2}\}$

axial mode index

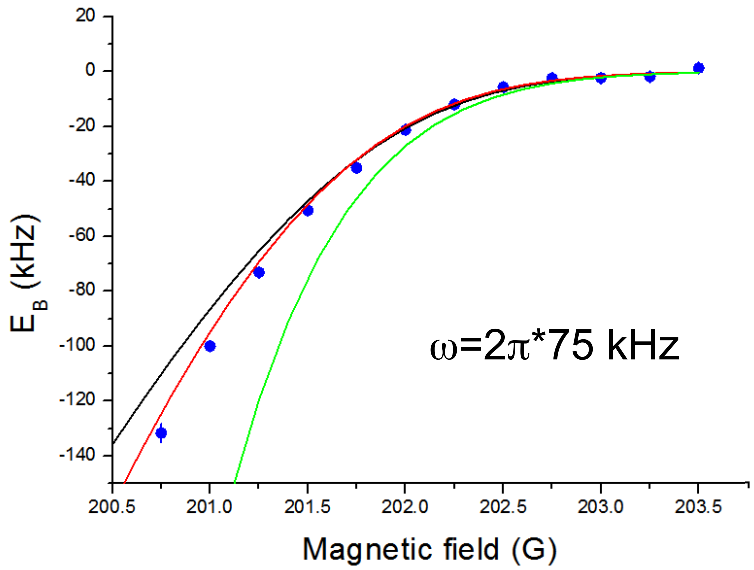
S. Baur, B. Fröhlich, M. Feld, E. Vogt, D. Pertot, M. Koschorreck, M.K, PRA 85, 061604 (2012)

Confinement-induced bound state

Bound state in quasi-2D:
(Petrov, Shlyapnikov, PRA 2001)

$$E_b = 0.905 \frac{\hbar\omega_0}{\pi} e^{-\sqrt{2\pi} l_0/|a|}$$ (for $-a<l_0$)

(green curve)



Black curve: correction for large and positive a
(Bloch, Dalibard, Zwerger, Rev. Mod. Phys. 2008)

Red curve: correction for finite range of the potential
(S. Baur, B. Fröhlich, M. Feld, E. Vogt, D. Pertot, M. Koschorreck, M.K, PRA 85, 061604 (2012))

$$-\frac{1}{a_s(\epsilon)} = -\frac{1}{a_s} + \frac{k^2 r_{\text{eff}}}{2} + \cdots.$$ (see last lecture)

in quasi-2D this means: $$\frac{l_z}{a_s} \rightarrow \frac{l_z}{a_s} + \frac{r_{\text{eff}}}{2l_z}\left(\frac{E_B}{\hbar\omega_z} - 1/2\right)$$

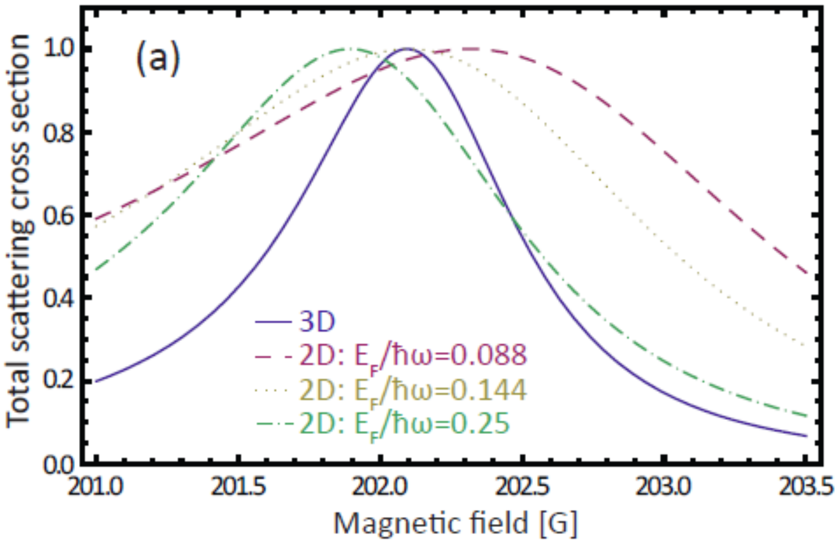
Confinement-induced Feshbach resonance

$$f_{00}(k) = \frac{4\pi}{\sqrt{2\pi}\frac{l_0}{a} + \ln\left(\frac{0.905}{\pi l_0^2 k^2}\right) + i\pi}$$

- Scattering amplitude is tuneable via 3D scattering length
- resonance structure is modified:

Im[f(k)] has maximum at $-\sqrt{2\pi}\frac{l_0}{a} = \ln\left(\frac{0.905}{\pi l_0^2 k^2}\right)$ (or $\ln\left(\frac{E_b}{E_k}\right)$=0)

Resonance position is defined by a, l_0, and k (or E_B and k)



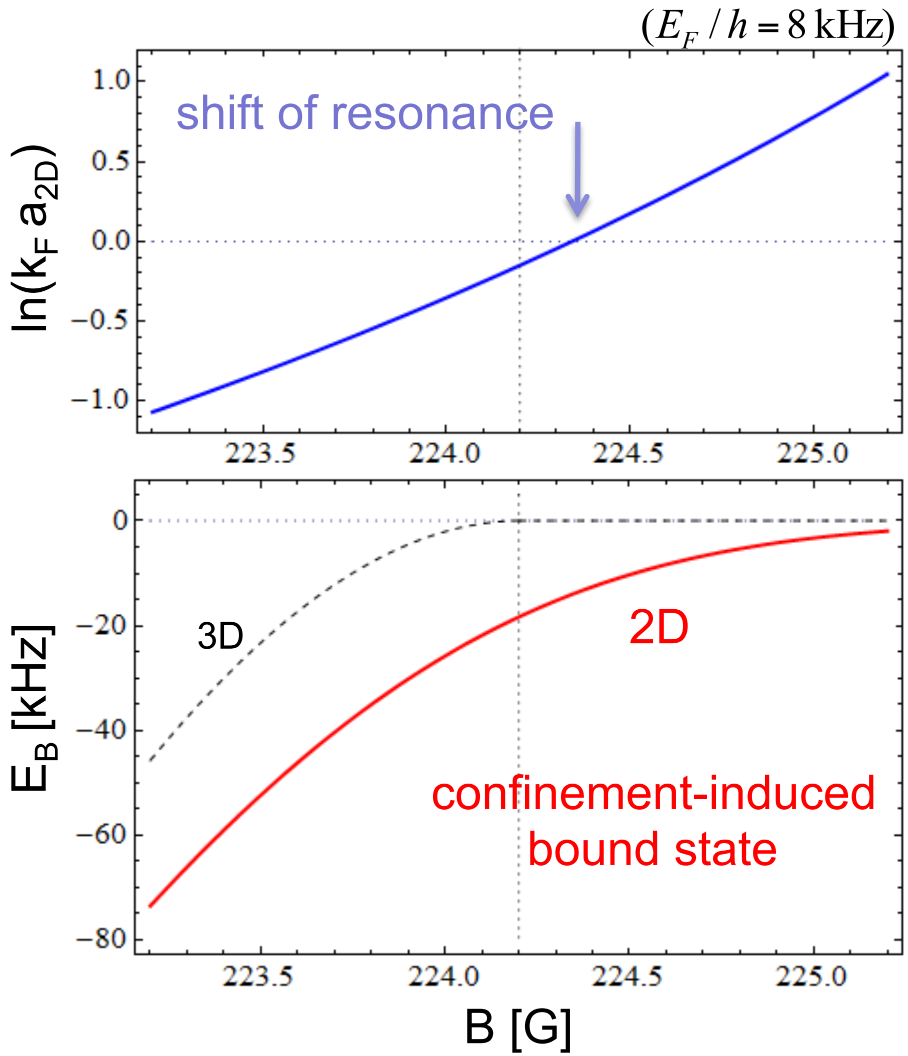
Mean-field coupling in a 2D Fermi gas

Many-body Hamiltonian $$H = \sum_i \frac{p_i^2}{2m} + V_{\text{ext}} + g \sum_{i\uparrow, j\downarrow} \delta(\mathbf{r}_i - \mathbf{r}_j),$$

coupling constant:

$$g_{2D} = -\frac{2\pi\hbar^2}{m} \frac{1}{\ln(k_F a_{2D})}$$

(Bloom, PRB 1975)

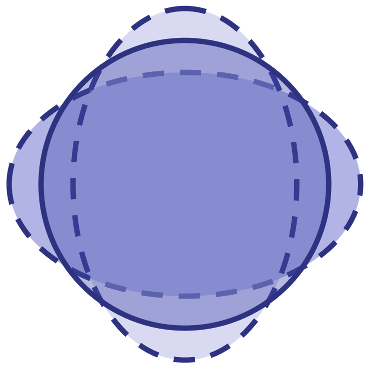


2D Fermi gases & collective modes

E. Vogt, M. Feld, B. Fröhlich, D. Pertot, M. Koschorreck, M.K., PRL 108, 070404 (2012)

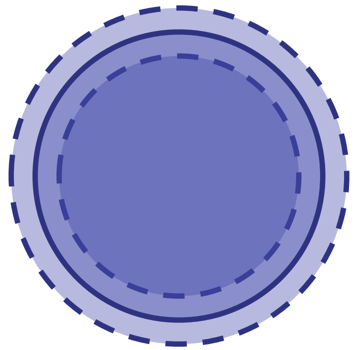
Collective modes of a 2D Fermi gas

Collective modes: Excitations with momentum $k \ll k_F$, e.g. sound propagation



Quadrupole mode

• insensitive to EoS
• measures shear viscosity



Breathing mode

• measures compressibility
• measures bulk viscosity

Scale invariance

General Hamiltonian with interaction $H_0 = \sum_i -\frac{1}{2m}\Delta_i + \sum_{i<j} V(\boldsymbol{r}_i - \boldsymbol{r}_j)$

dilute the system: $\boldsymbol{r} \to \lambda \boldsymbol{r}, \Psi(\boldsymbol{r}) \to \lambda^{d/2}\Psi(\lambda \boldsymbol{r}),$

$$H_0 \to \frac{1}{\lambda^2}\sum_i -\frac{1}{2m}\Delta_i + \sum_{i<j} V(\lambda(\boldsymbol{r}_i - \boldsymbol{r}_j)).$$

Scale invariance in a homogeneous system: $H(\lambda x)= H(x)/\lambda^2$ if $V(\lambda \boldsymbol{r}) = V(\boldsymbol{r})/\lambda^2$

This works for $V=g/r^2$ and delta-interaction in two dimensions $V(\boldsymbol{r}-\boldsymbol{r}') = \frac{1}{2} g\,\delta^2(\boldsymbol{r}-\boldsymbol{r}')$

Scale invariant systems have unique properties
- simple equation of state (like ideal gas: $P=2\varepsilon/D$)
- vanishing bulk viscosity

In cylindrically symmetric trap: scale invariance is replaced by SO(2,1) Lorentz symmetry
(Pitaevskii/Rosch, 1997)

<u>Two remarkable predictions:</u>
1. Breathing mode: $\omega_B=2\omega_\perp$ (independent of interaction strenght!)
2. bulk viscosity is zero

For bosons: Pitaevskii/Rosch, Perrin/Olshanii, Chin, Dalibard

Quantum anomaly

In two dimensions there are a few complications:

- delta-function interaction is not well-defined => regularization required
- Interaction strength depends on density:

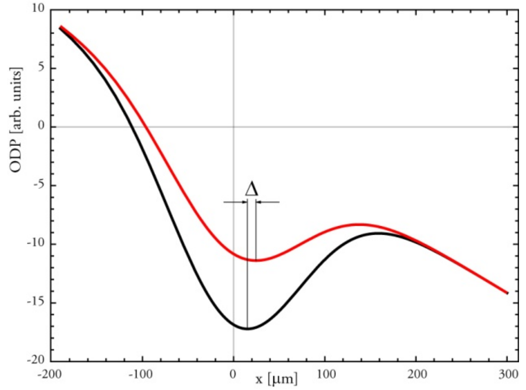
$$g = \frac{\hbar^2}{m} \frac{2\pi}{\ln(k_F a_{2D})}$$

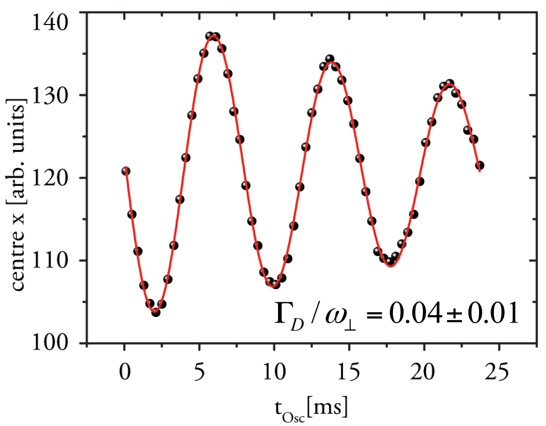
=> Quantum anomaly shifts mode by $\delta\omega/\omega \sim a/l_0$

(Olshanii et al., PRL 2010)

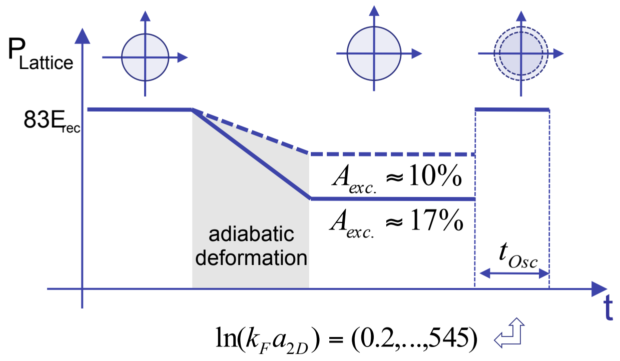
Collective excitations

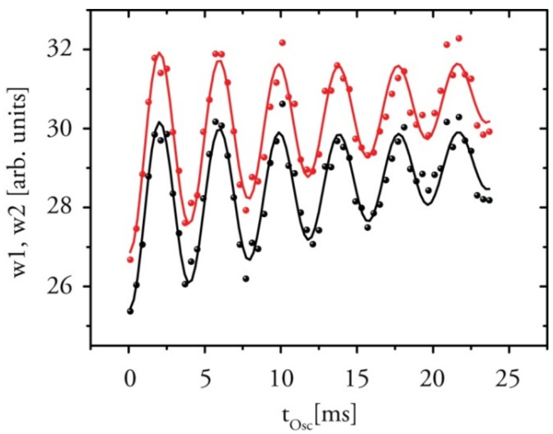
Dipole Mode
($\omega_\perp$ - calibration)



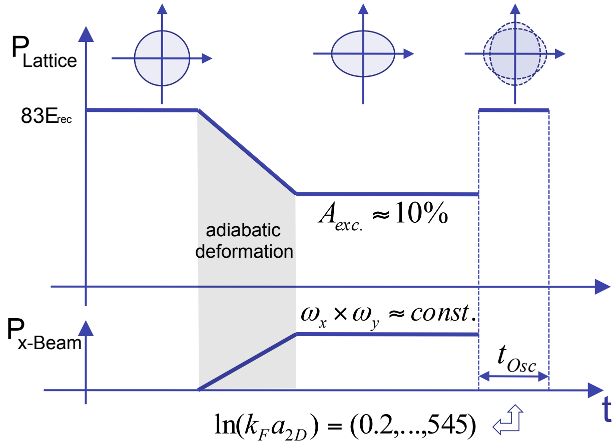


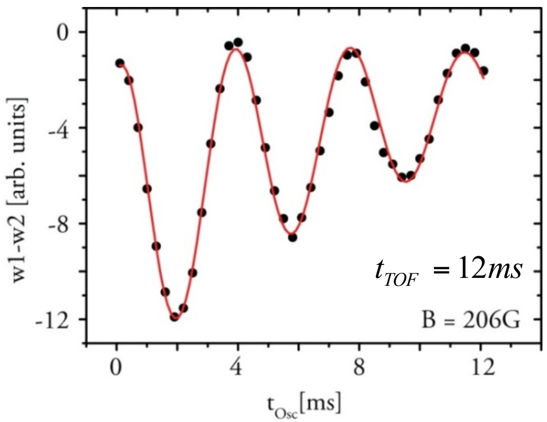
Breathing Mode





Quadrupole Mode





Damping & Viscosity

2D-Energy dissipation rate:

shear viscosity — bulk viscosity

$$\dot{E} = -\frac{1}{2}\int d^2r\,\boxed{\eta(\vec{r})}\left(\partial_k v_i + \partial_i v_k - \delta_{ik}\nabla\cdot\vec{v}\right)^2 - \int d^2r\,\boxed{\zeta(\vec{r})}\left(\nabla\cdot\vec{v}\right)^2$$

Quadrupole Mode

$$\mathrm{v}_{\mathbf{Q}}(r) = a\cdot[x\hat{e}_x - y\hat{e}_y]\cdot\cos(\omega_Q t)$$

$$\curvearrowright\quad \left\langle \dot{E} \right\rangle_t = -2a^2\int d^2r\cdot\eta(\vec{r})$$

Breathing Mode

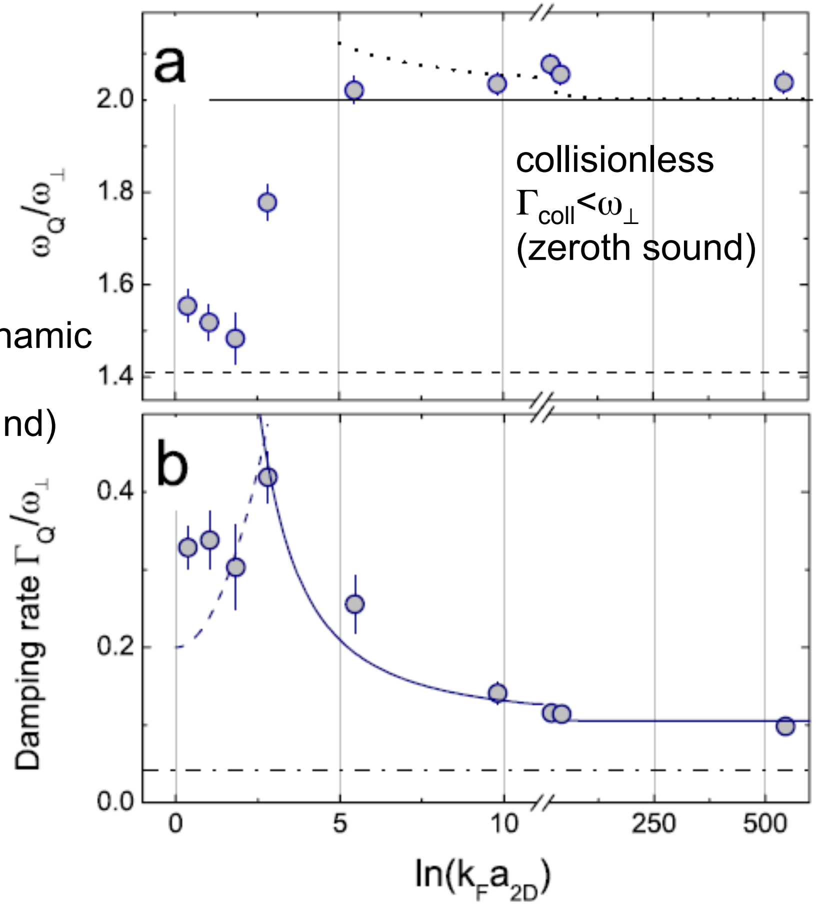
$$\mathrm{v}_{\mathbf{B}}(r) = b\cdot[x\hat{e}_x + y\hat{e}_y]\cdot\cos(\omega_B t)$$

$$\curvearrowright\quad \left\langle \dot{E} \right\rangle_t = -2b^2\int d^2r\cdot\zeta(\vec{r})$$

Damping rate: $\Gamma \propto \left\langle \dot{E} \right\rangle_t$

Collective modes

Quadrupole mode

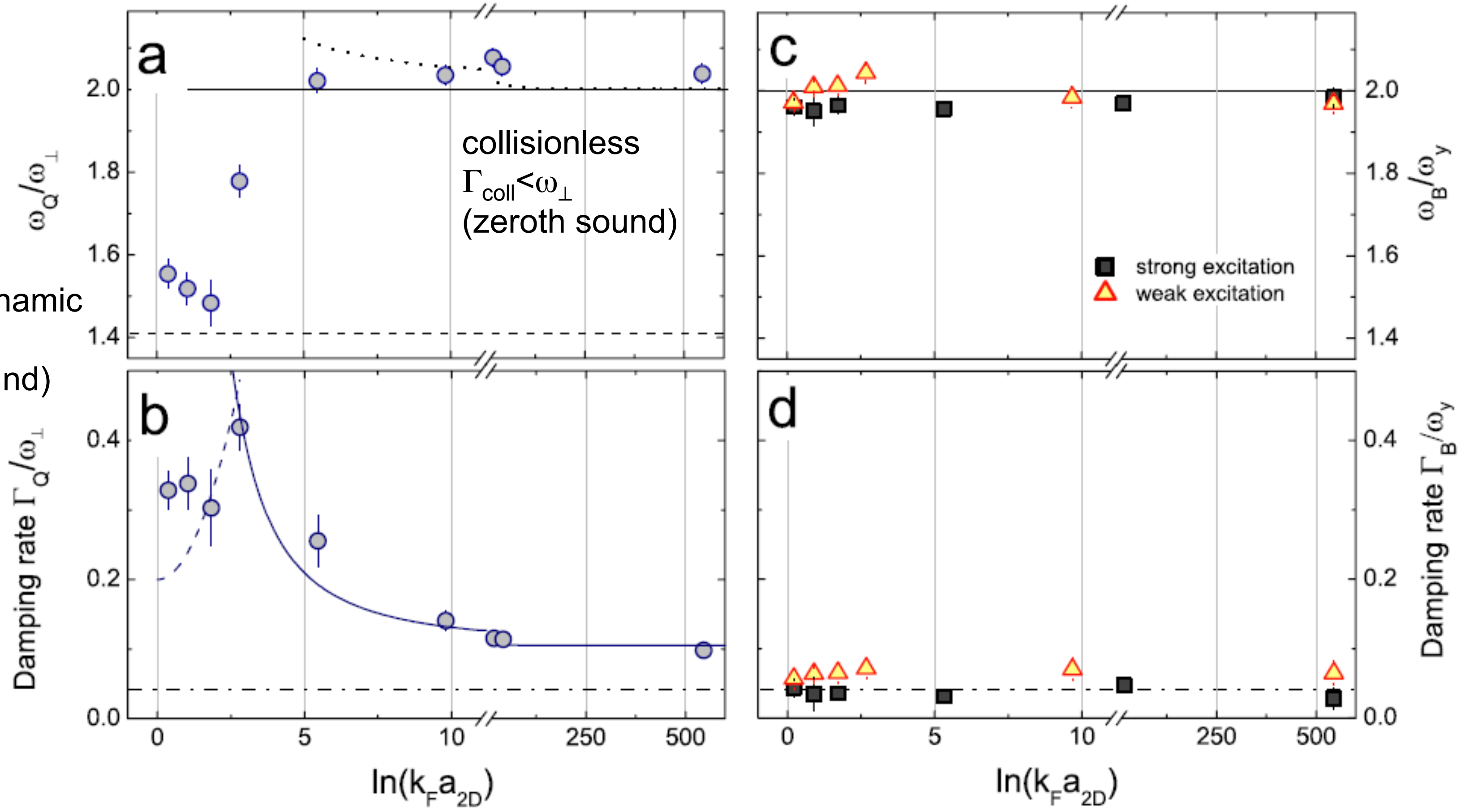


hydrodynamic
$\Gamma_{coll}>\omega_\perp$
(first sound)

E. Vogt, M. Feld, B. Fröhlich, D. Pertot, M. Koschorreck, M.K., PRL 108, 070404 (2012)

Collective modes

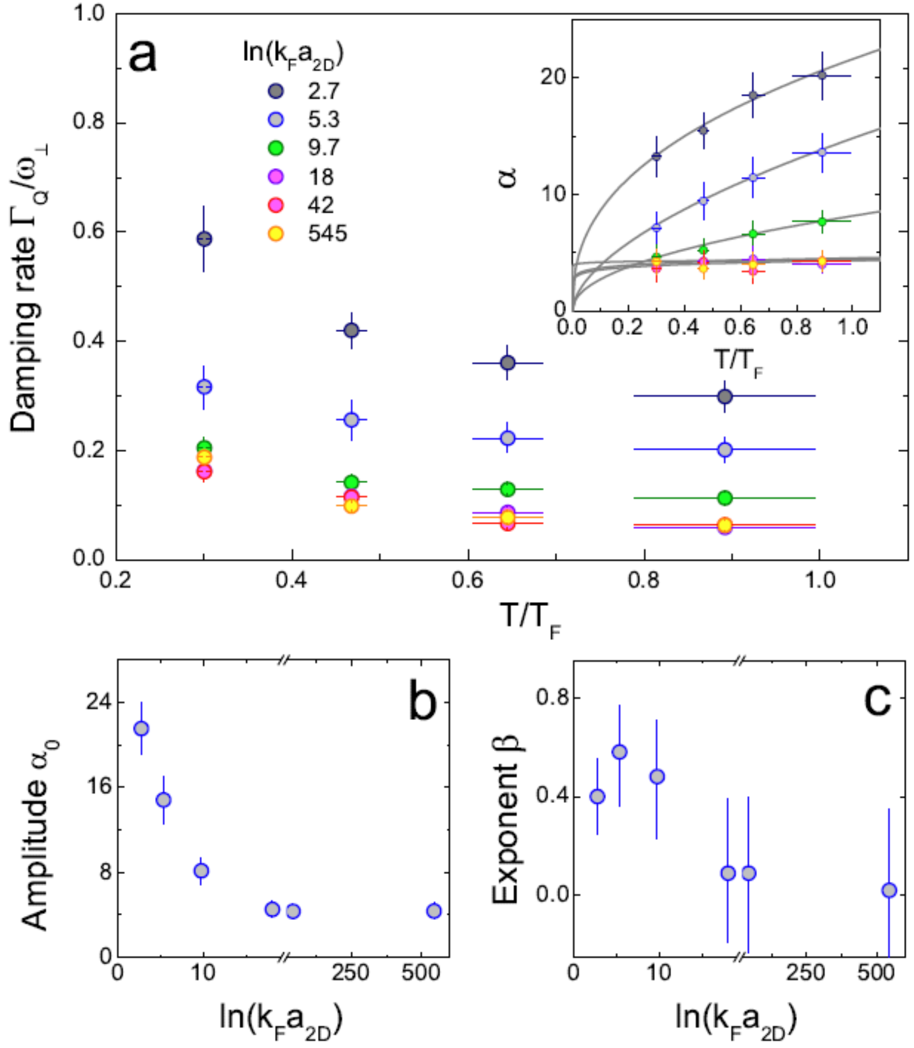
Quadrupole mode | Breathing mode



hydrodynamic
$\Gamma_{coll}>\omega_\perp$
(first sound)

E. Vogt, M. Feld, B. Fröhlich, D. Pertot, M. Koschorreck, M.K., PRL 108, 070404 (2012)

Temperature dependent damping



Shear viscosity

$$\eta = \hbar\, n\, \alpha\left(T/T_F\right)$$

dimensionless function

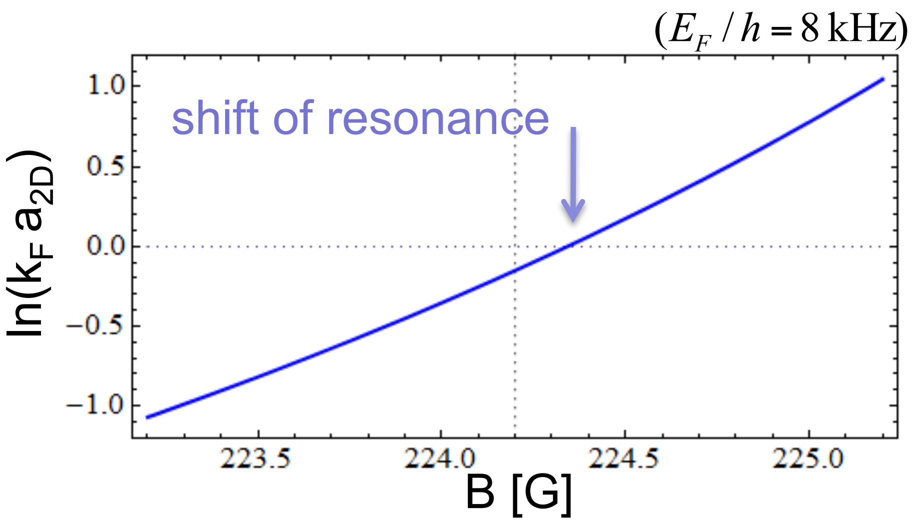
$$\alpha\left(T/T_F\right) = \alpha_0 \times \left(T/T_F\right)^{\beta}$$

E. Vogt, M. Feld, B. Fröhlich, D. Pertot, M. Koschorreck, M.K., PRL 108, 070404 (2012)

Overview

Lecture 1: Atomic Fermi gases - experimental and theoretical background	Lecture 2: Fermion pairing in 3D and 2D	Lecture 3: 2D Fermi gases
Introduction	BEC-BCS crossover	Momentum-resolved rf spectroscopy
How to make a Fermi gas	Preparing 2D Fermi gases	Fermi liquid
Seeing the Fermi surface	Scattering in 2D	Pseudogap pairing
Interactions	Collective modes	Polarons

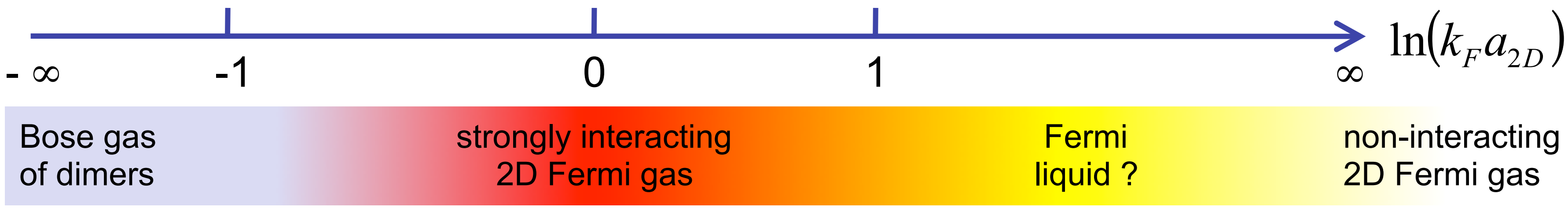
Reminder of basic 2D (last lecture)



Mean-field coupling in 2D
(Bloom 1975)

$$g_{2D} = -\frac{2\pi\hbar^2}{m}\frac{1}{\ln(k_F a_{2D})}$$

$$\left(g_{3D} = \frac{4\pi\hbar^2}{m} a_{3D}\right)$$



- BKT transition at $T_{BKT} \approx 0.1\ T_F$ in the strongly interacting regime
- T_{BKT} decays exponentially towards weak attractive interactions (as in 3D)

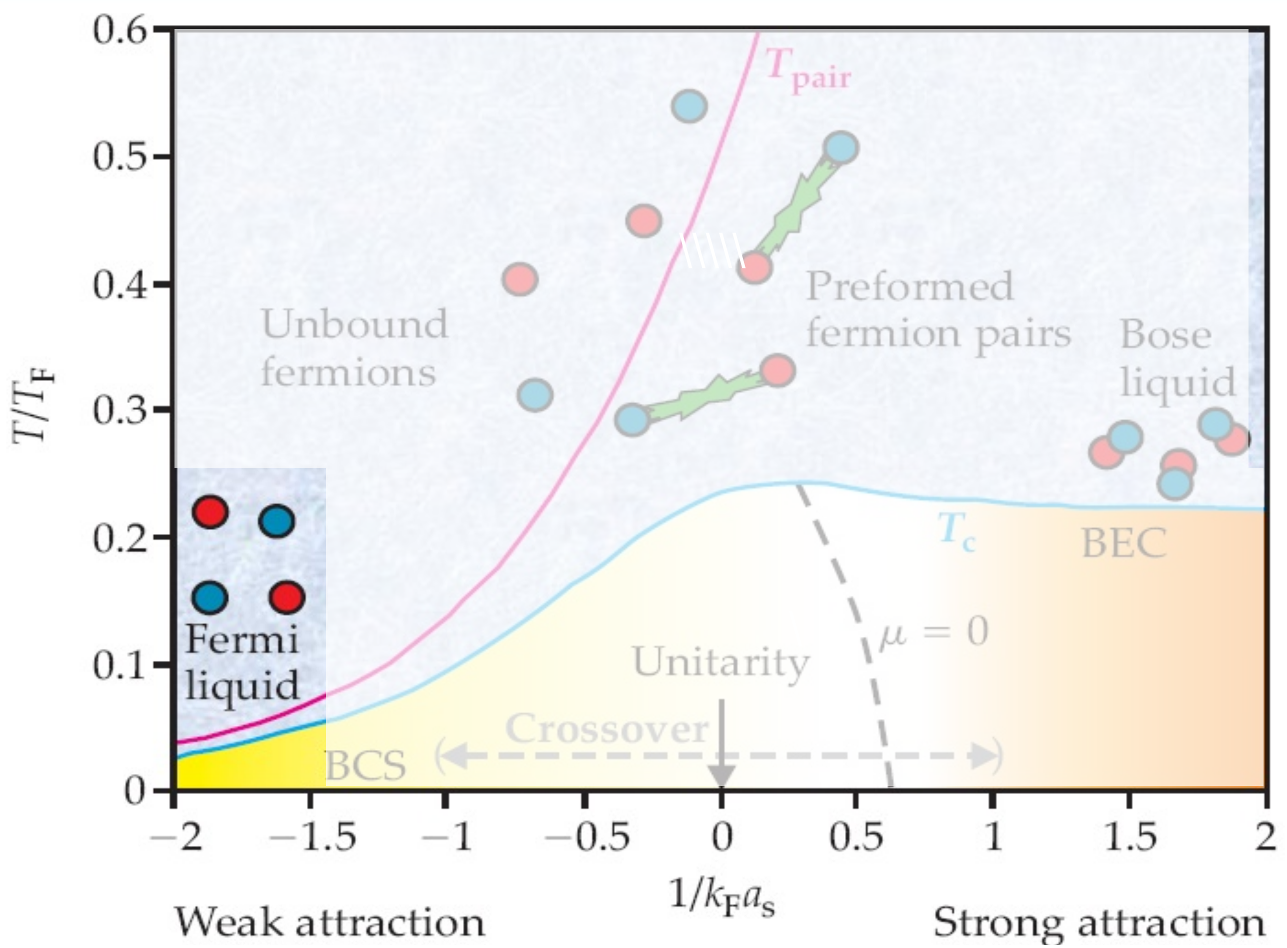
Theory: Bloom, P.W. Anderson, Randeria, Shlyapnikov, Petrov, Devreese, Julienne, Duan, Zwerger, Giorgini, Sa de Melo, **...**

Momentum-resolved rf spectroscopy &

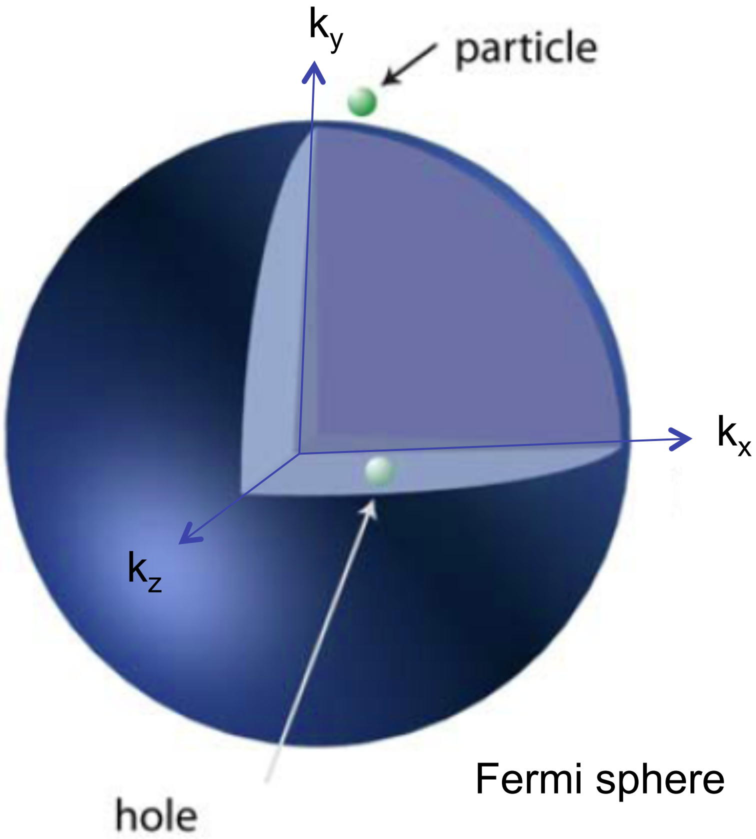
Fermi liquid in two dimensions

B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, W. Zwerger, M.K, PRL 106, 105301 (2011)
B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, M.K., C. Berthod, T. Giamarchi, arXiv:1206.5380 (2012)

Fermi liquid regime



Concept of the Fermi liquid



Single particle excitations in a non-interacting Fermi gas

"Particle excitation": $k > k_F$
"Hole excitation": $k < k_F$

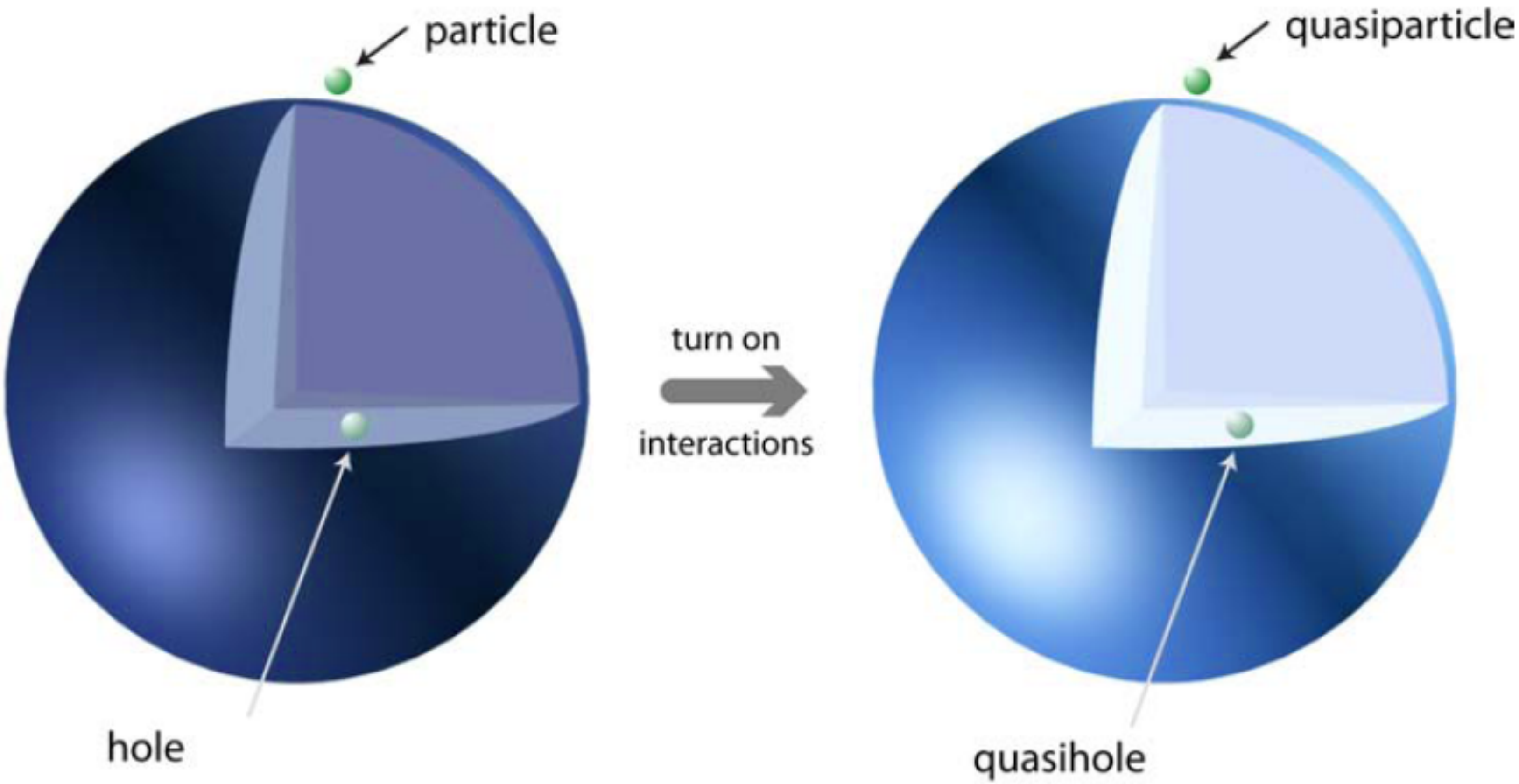
Excitation energy:

$$E - E_0 = \sum_{\boldsymbol{k}} \frac{\hbar^2 k^2}{2m} \delta n_{\boldsymbol{k}},$$

$$\delta n_{\boldsymbol{k}} = n_{\boldsymbol{k}} - n^0_{\boldsymbol{k}}$$

equilibrium occupation

Concept of the Fermi liquid



Particle and hole excitation get dressed by interactions with the Fermi sea
→ “quasiparticle” excitations (Landau, 1957)

Quasiparticle excitations

- Landau-Fermi liquid particles are fermionic
- finite lifetime $1/t \sim (k-k_F)^2$ (long-lived near the Fermi surface)
- effective mass: $m^*/m > 1$, depending on interaction strength
- Conceptual simplification of interacting Fermi systems: quasiparticles form non-interacting Fermi gas with renormalized properties

Microscopic theory

Hamiltonian

$$\hat{H} = \underbrace{\sum_{\mathbf{k},\sigma} \epsilon_k n_{\mathbf{k}\sigma}}_{H_0} + \underbrace{\sum_{\substack{\mathbf{k},\mathbf{k}',\mathbf{q}\\ \sigma,\sigma'}} V(q) c^\dagger_{\mathbf{k}+\mathbf{q},\sigma} c^\dagger_{\mathbf{k}'-\mathbf{q},\sigma'} c_{\mathbf{k}'\sigma'} c_{\mathbf{k}\sigma}}_{V}$$

free Fermi gas — Interactions

Spectral function: probability to make a single particle/single hole excitation with well-defined energy ω and momentum k

$$A(k,\omega) = \sum_m |\langle m|c_k^\dagger|\psi_0\rangle|^2 \delta(\omega - E_m) + \sum_m |\langle m|c_k|\psi_0\rangle|^2 \delta(\omega - E_m).$$

Free Fermi gas: $A^0(k,\omega) = \delta(\omega - \epsilon_k),$

Microscopic theory

With weak interactions

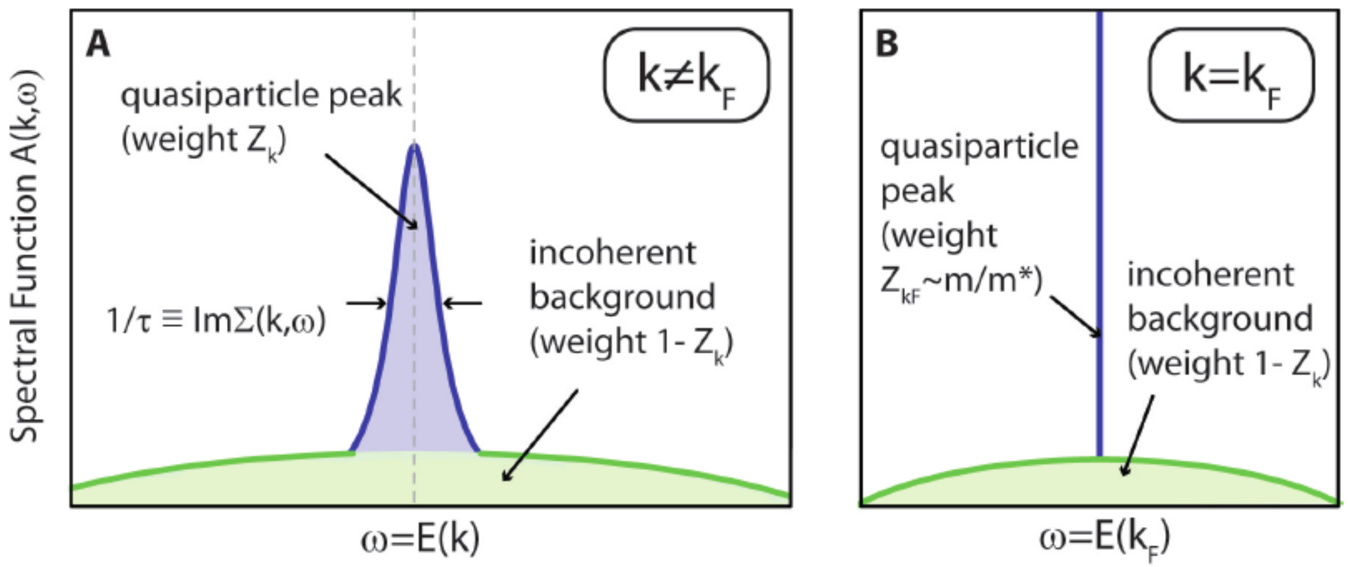
$$A(k,\omega) = -\frac{1}{\pi}\frac{\mathrm{Im}\,\Sigma(k,\omega)}{(\omega - \epsilon_k - \mathrm{Re}\,\Sigma(k,\omega))^2 + (\mathrm{Im}\,\Sigma(k,\omega))^2}$$

(Lorentzian)

shift = dispersion

width = decay rate

$$E(k) = E_\mathrm{F} + \frac{k_\mathrm{F}}{m^*}(k - k_\mathrm{F}) + O(k^2)$$



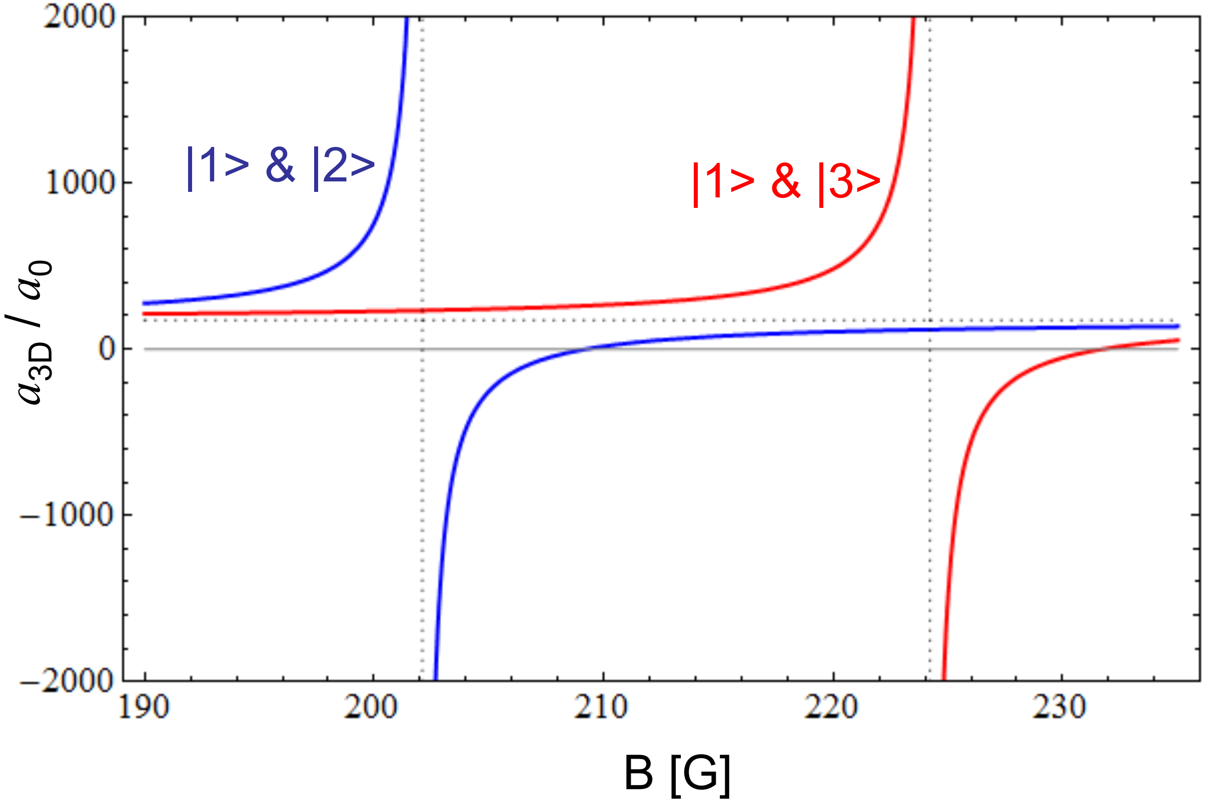
Fermi liquid parameters in 2D

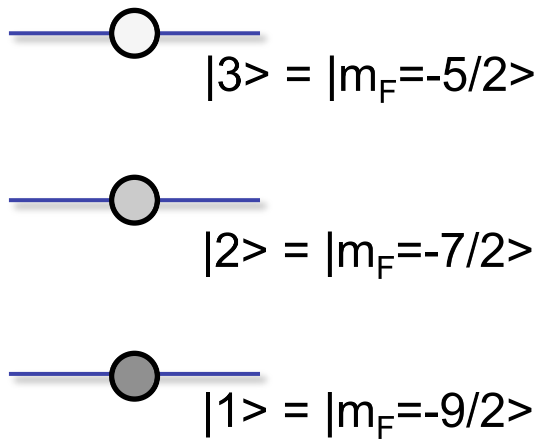
$$\frac{m^*}{m} = 1 + \frac{c}{\ln\left(k_{\mathrm{F}} a_{2\mathrm{D}}\right)^2},$$

Bloom, PRB 1975

Preparing interacting 2D systems

Feshbach resonances in ^{40}K



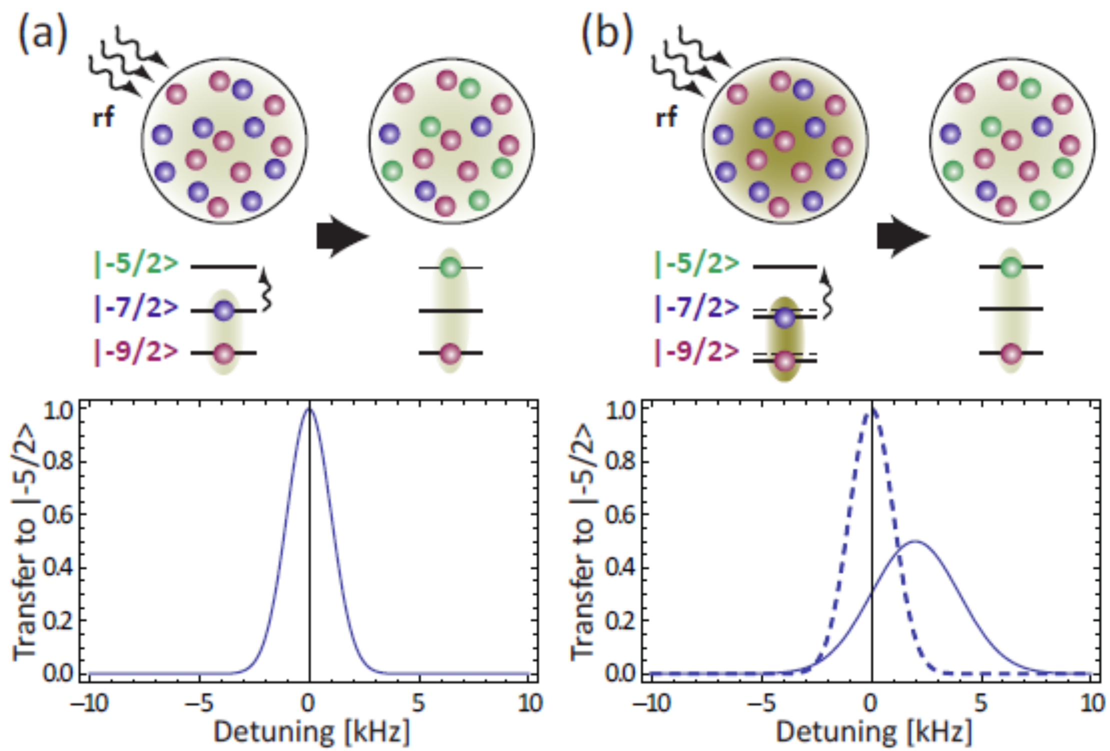


|2> & |3>

$$a_{BG} = 174 a_0$$

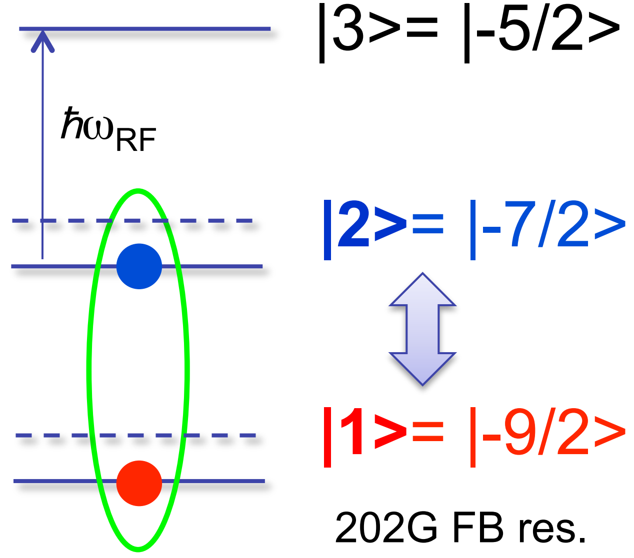
Radio-frequency spectroscopy

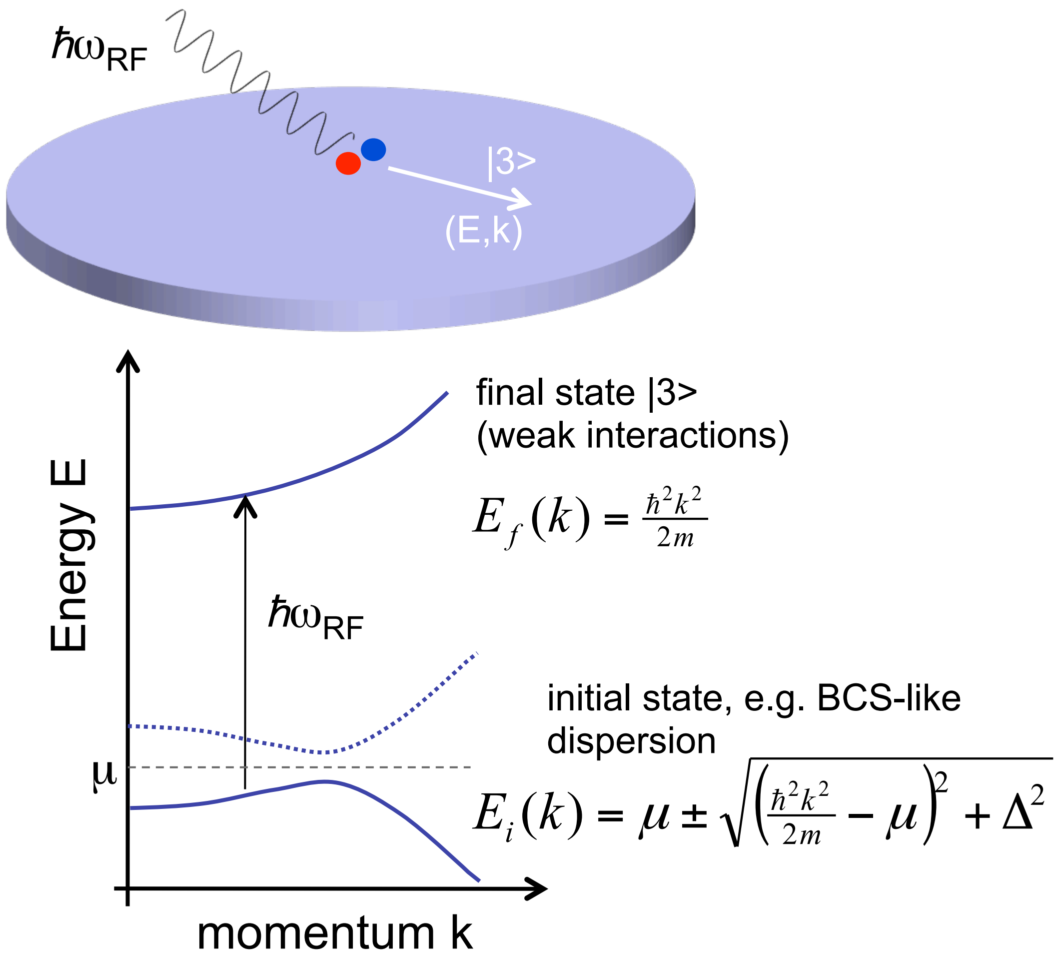
no interactions interactions



Does not tell us the dispersion!

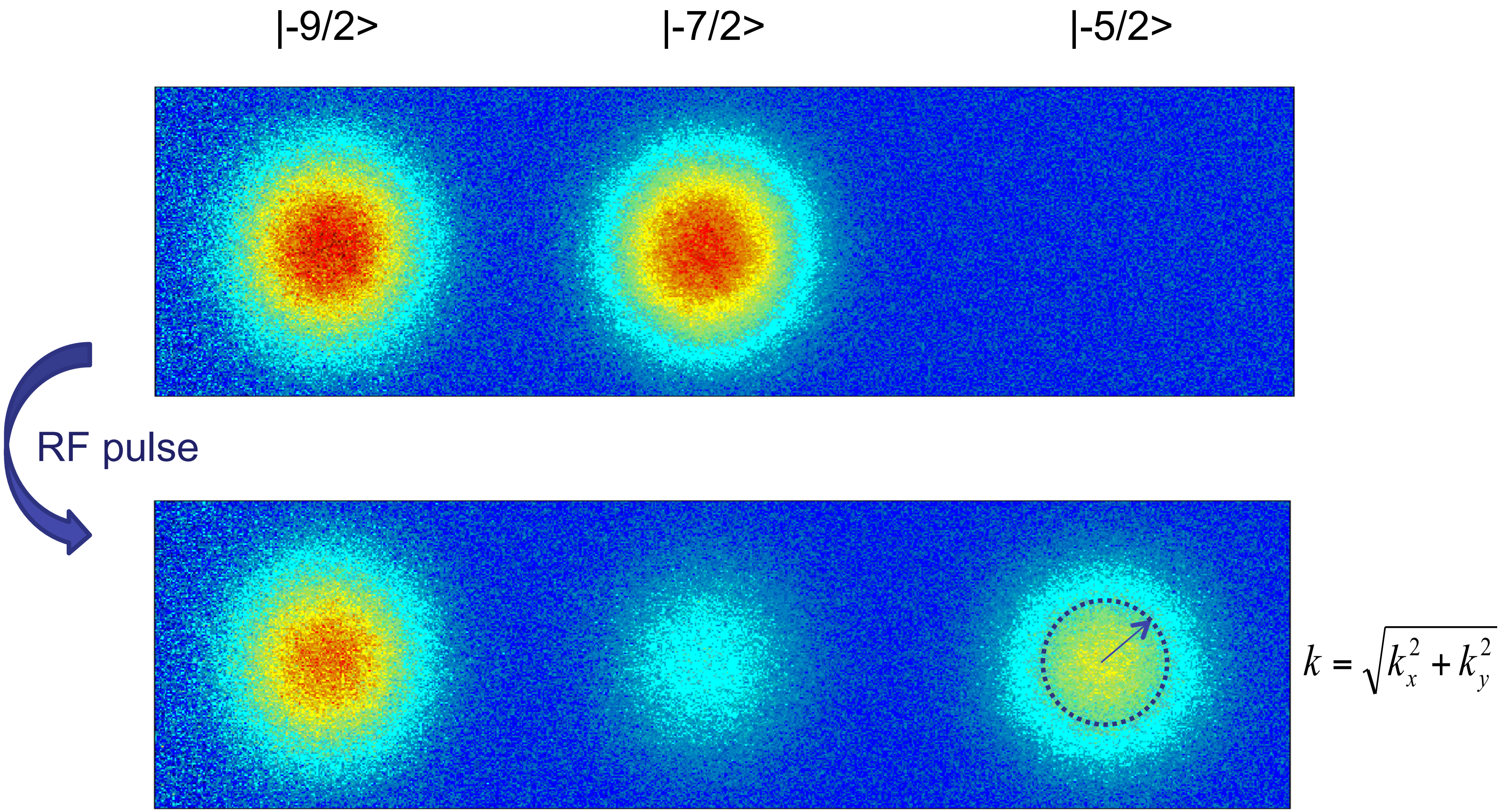
Momentum-resolved RF spectroscopy *("ARPES")*





ARPES in 3D Experiment: Jin *Theory:* Georges, Strinati, Levin, Ohashi, Zwerger, Drummond, ...

Population of the spin states



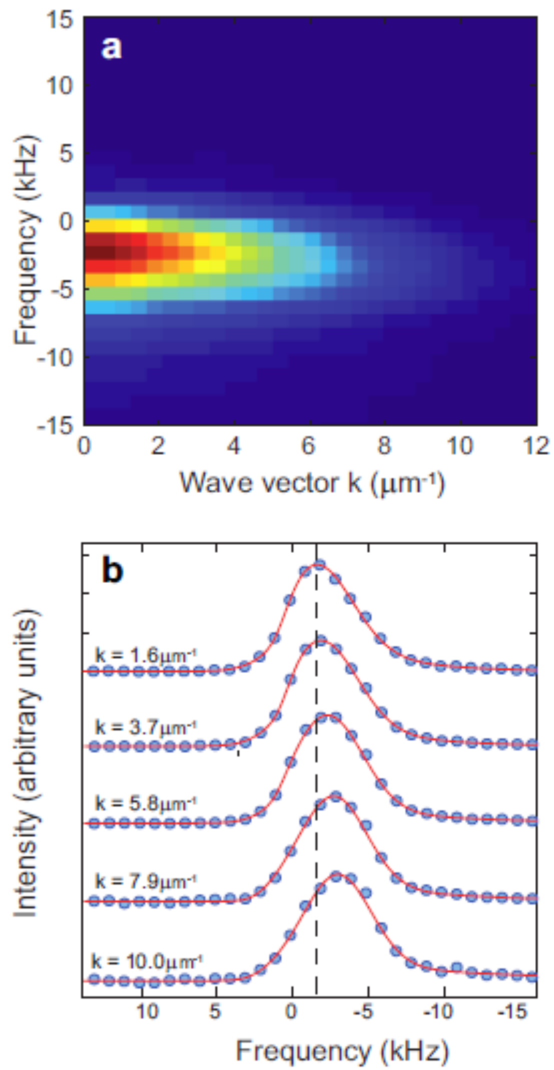
Spectroscopy of a 2D Fermi liquid

<u>Fermi liquid:</u>

E_F, $k_BT < \hbar\omega$ (two-dimensional)

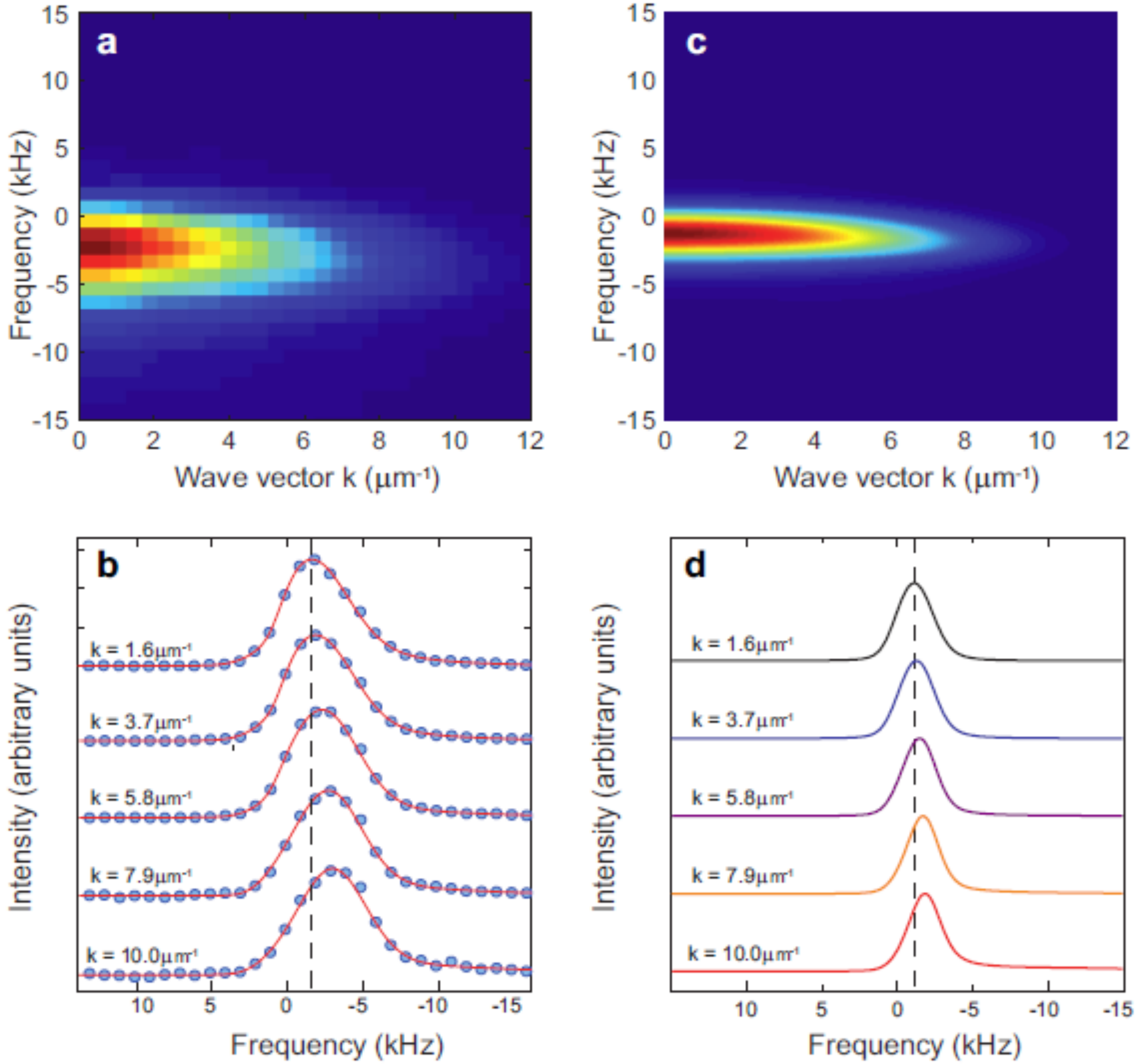
$E_B < k_BT$ (no pairing)

$g=1/\ln(k_Fa_{2D}) < 1$ (weak interactions)



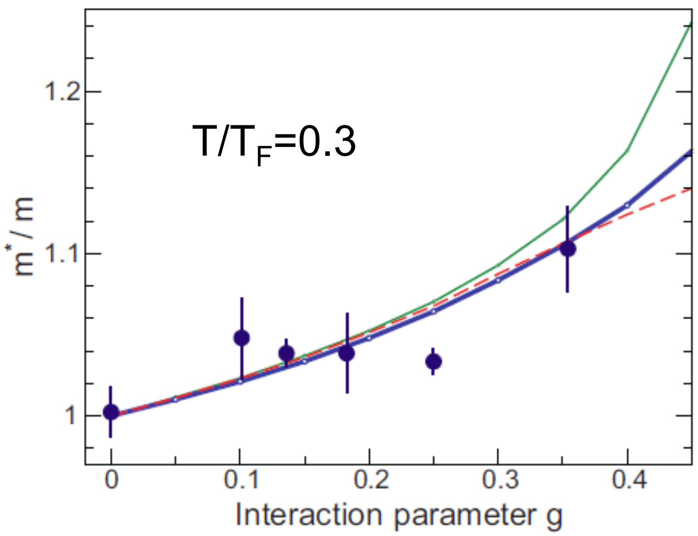
B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, M.K., C. Berthod, T. Giamarchi, arXiv:1206.5380 (2012)

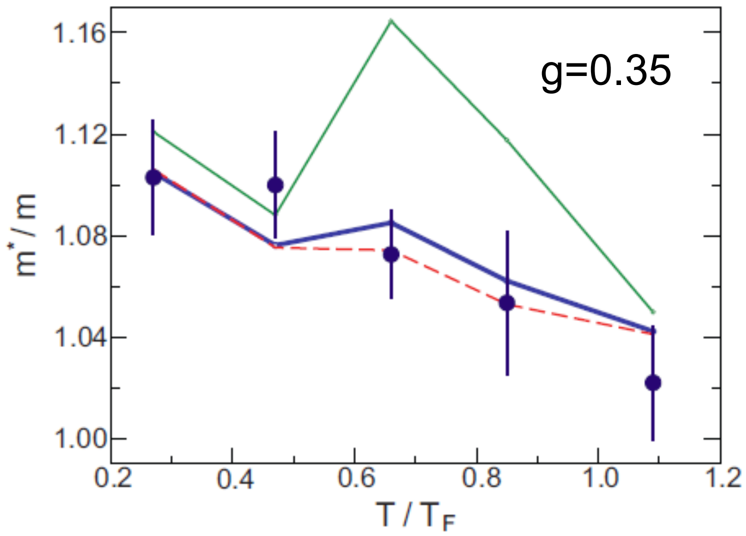
Comparison with theory



B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, M.K., C. Berthod, T. Giamarchi, arXiv:1206.5380 (2012)

Effective mass parameter



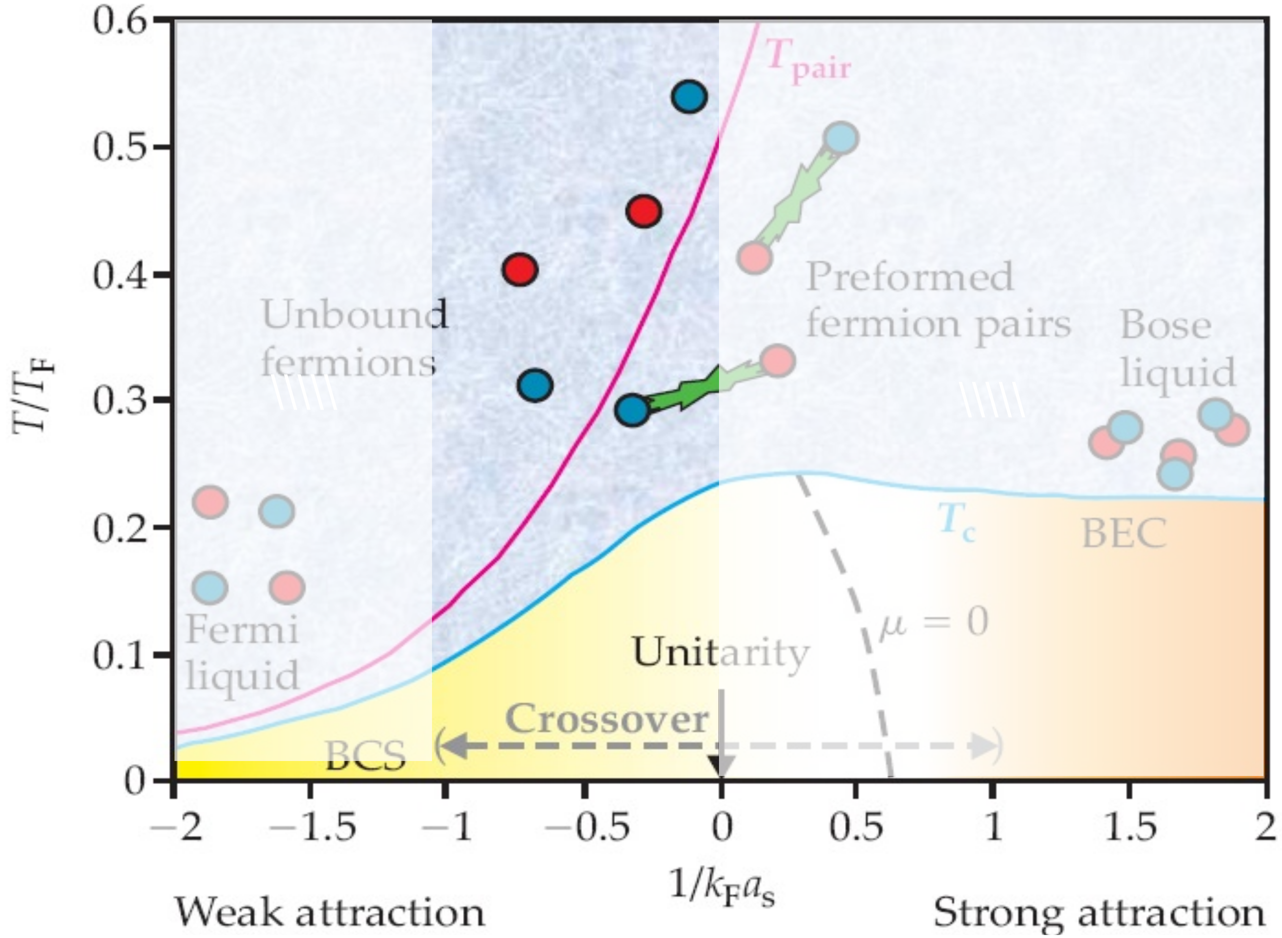


B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, M.K., C. Berthod, T. Giamarchi, arXiv:1206.5380 (2012)

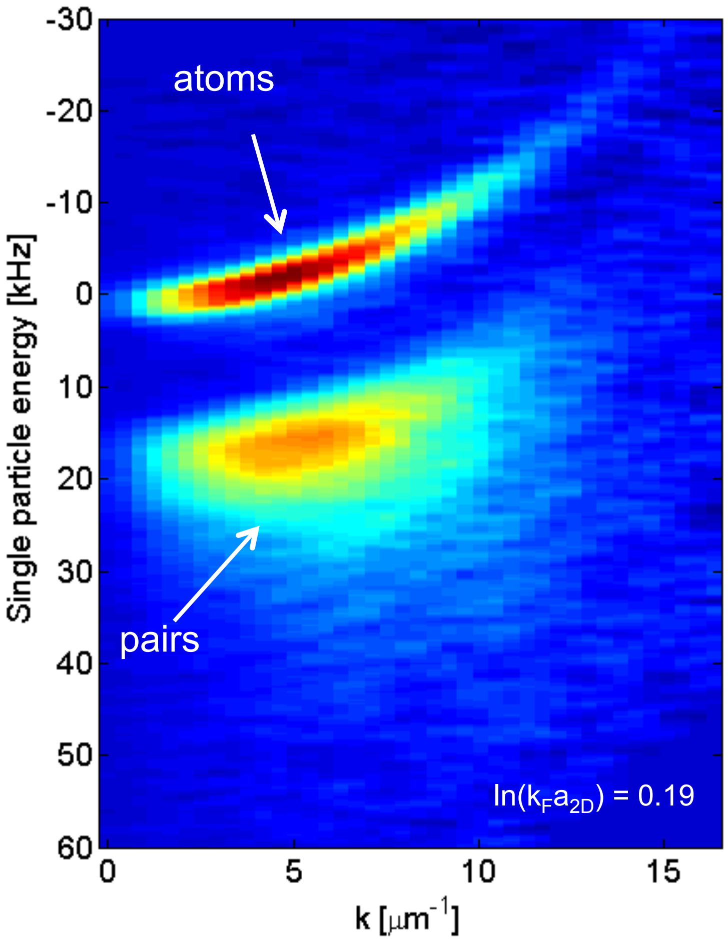
Pseudogap pairing

M. Feld, B. Fröhlich, E. Vogt, M. Koschorreck, M.K., Nature 480, 75 (2011)

Pairing regime



ARPES spectra at strong interactions



"BCS" side

$\ln(k_F a_{2D}) > 0$, $E_B < E_F$

no isolated dimers, attractive interactions, pairs are huge compared to inter-particle spacing

"Condensation energy" of Cooper pairs
(MF theory, T=0, Randeria 1989)

$$E_{th}(k=0) = \frac{\Delta^2}{2E_F} = E_B$$

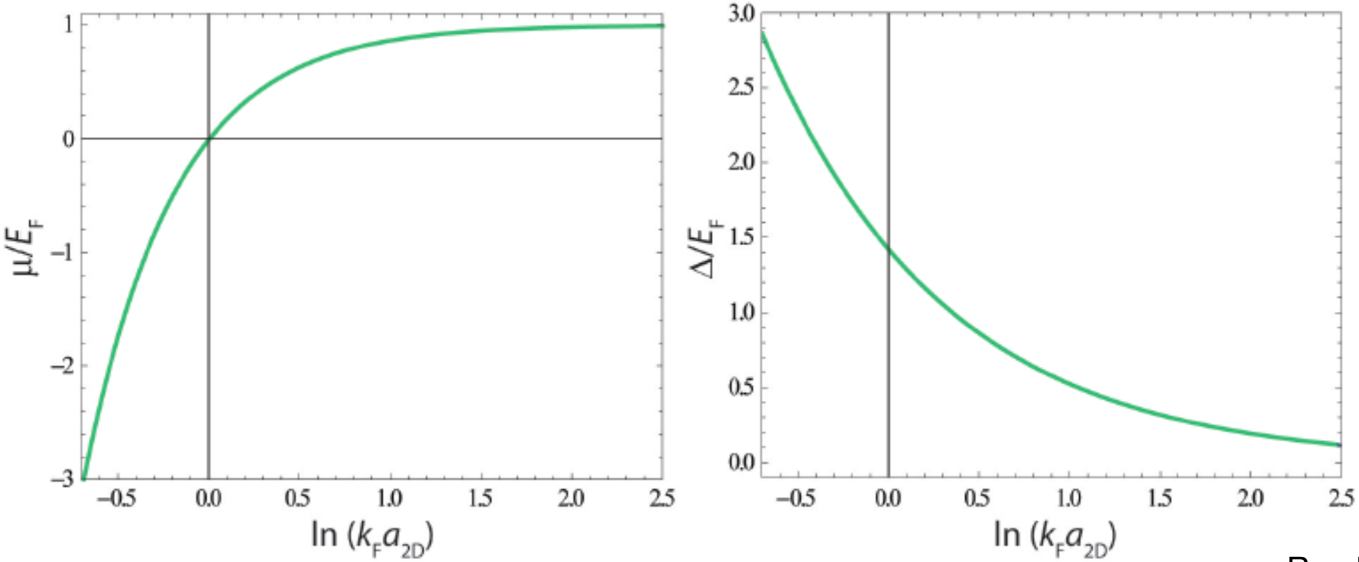
Only the case in 2D
(in 3D: $E_B = 0$ on the BCS side)

BCS pairing in two dimensions at T=0

Number equation $\int_0^\infty d\epsilon_k \left(1 - \frac{\epsilon_k - \mu}{[(\epsilon_k - \mu)^2 + \Delta^2]^{1/2}}\right) = 2E_\mathrm{F} \longrightarrow \left(\mu^2 + \Delta^2\right)^{1/2} + \mu = 2E_\mathrm{F}.$

Gap equation $\left(\mu^2 + \Delta^2\right)^{1/2} - \mu = E_b,$

$$\Delta = (2E_\mathrm{F}E_b)^{1/2} \qquad \text{and} \qquad \mu = E_\mathrm{F} - \frac{E_b}{2}.$$



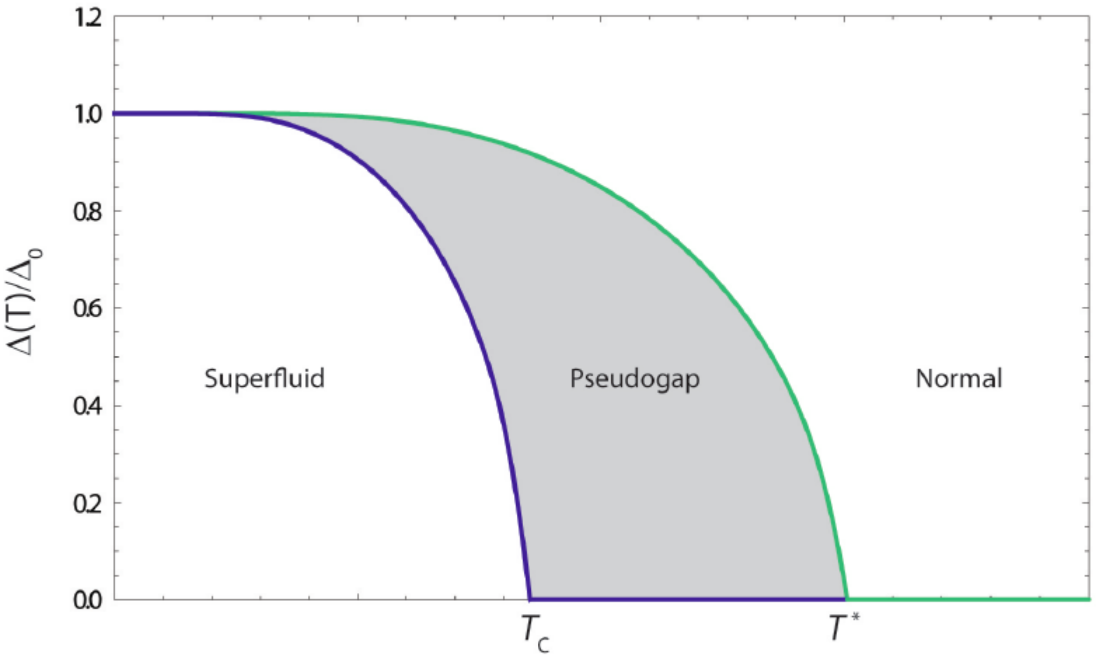
Randeria et al. 1989

Mean-field theory at T>0

Gap equation
$$-\frac{1}{V_0} = \sum_{\boldsymbol{k}} \frac{1}{2E_{\boldsymbol{k}}}\tanh\frac{E_{\boldsymbol{k}}}{2k_{\mathrm{B}}T}$$

Number equation
$$N_0 = \sum_{\boldsymbol{k}} \left(1 - \frac{\epsilon_{\boldsymbol{k}} - \mu}{E_{\boldsymbol{k}}}\tanh\frac{E_{\boldsymbol{k}}}{2k_{\mathrm{B}}T}\right).$$

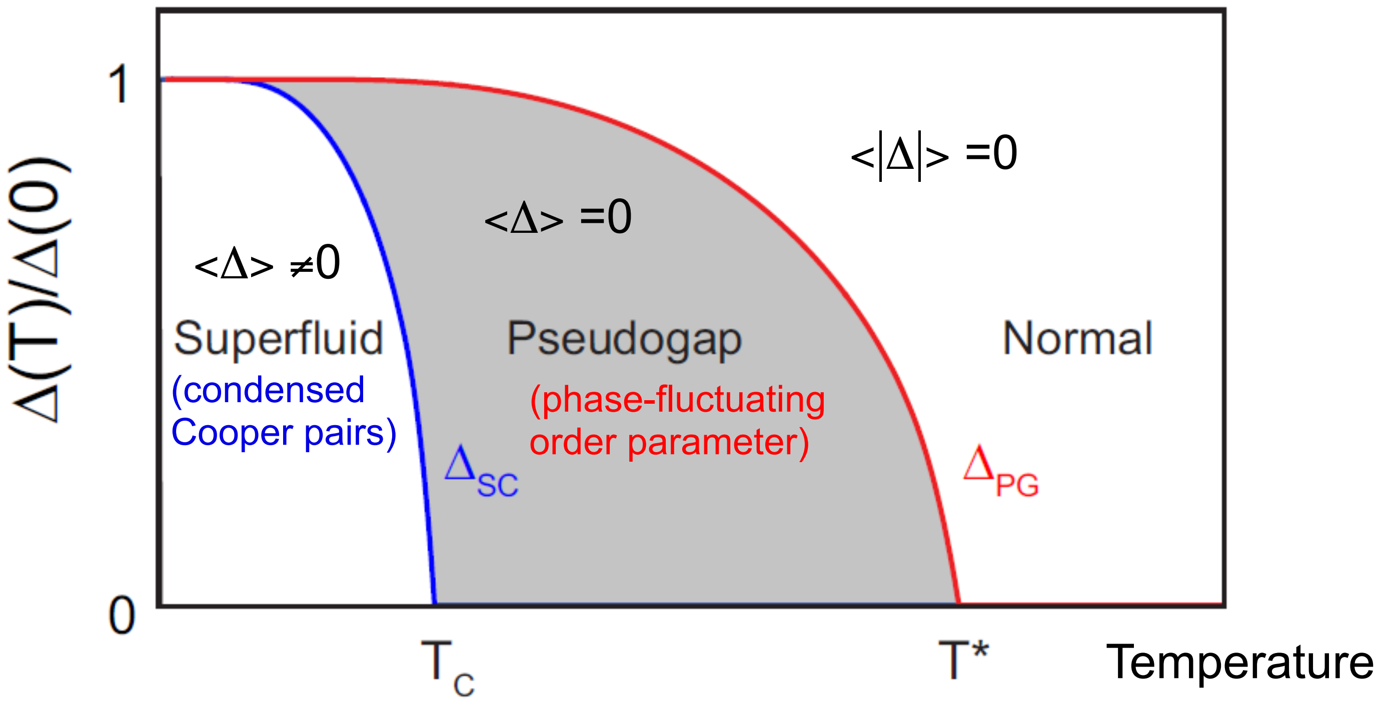
Numerical solution



In 3D weak coupling BCS: $T_c \approx T^*$ (pairs condense as they form)

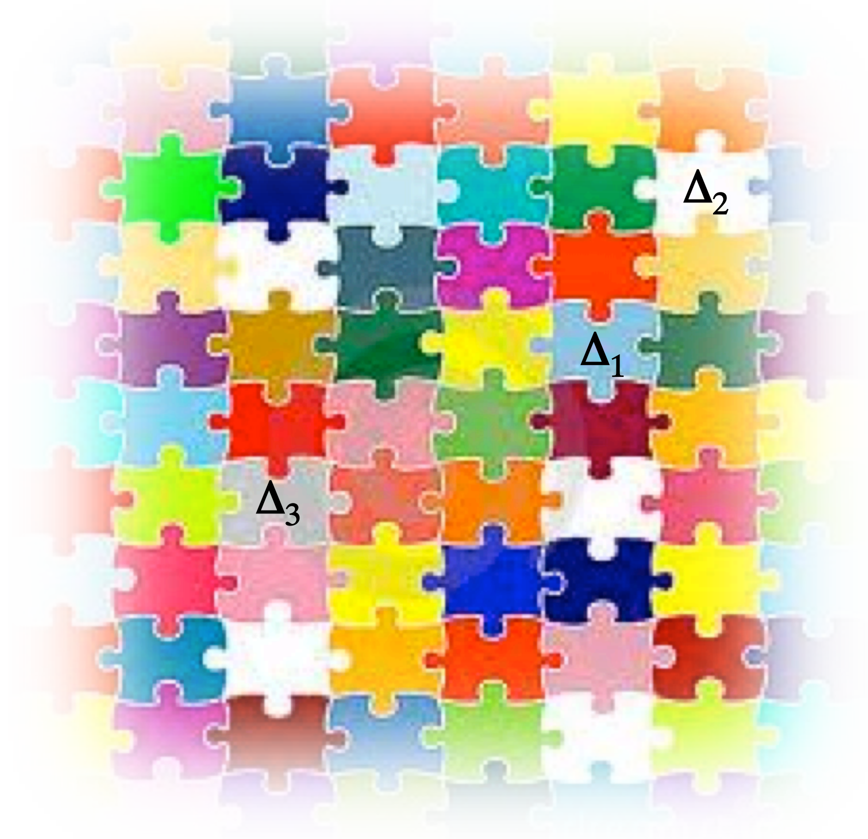
Pairing pseudogap phenomenon

complex order parameter $\Delta(T,x) = |\Delta(T,x)| e^{i\vartheta(T,x)}$



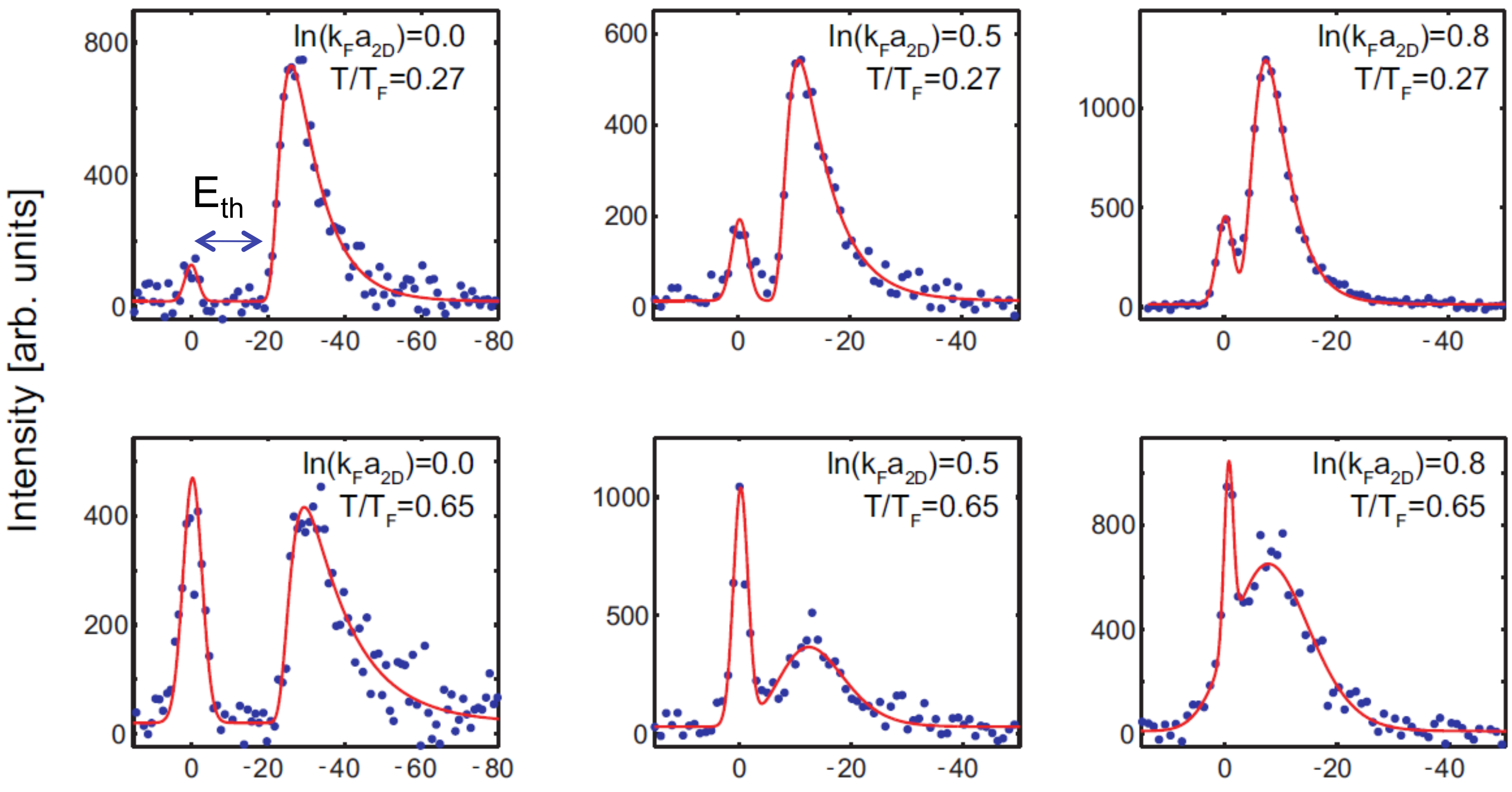
Pairing pseudogap phenomenon

complex order parameter $\Delta(T,x) = |\Delta(T,x)| e^{i\vartheta(T,x)}$



spatial average $\langle\Delta\rangle = 0$ but $\langle|\Delta|\rangle > 0$

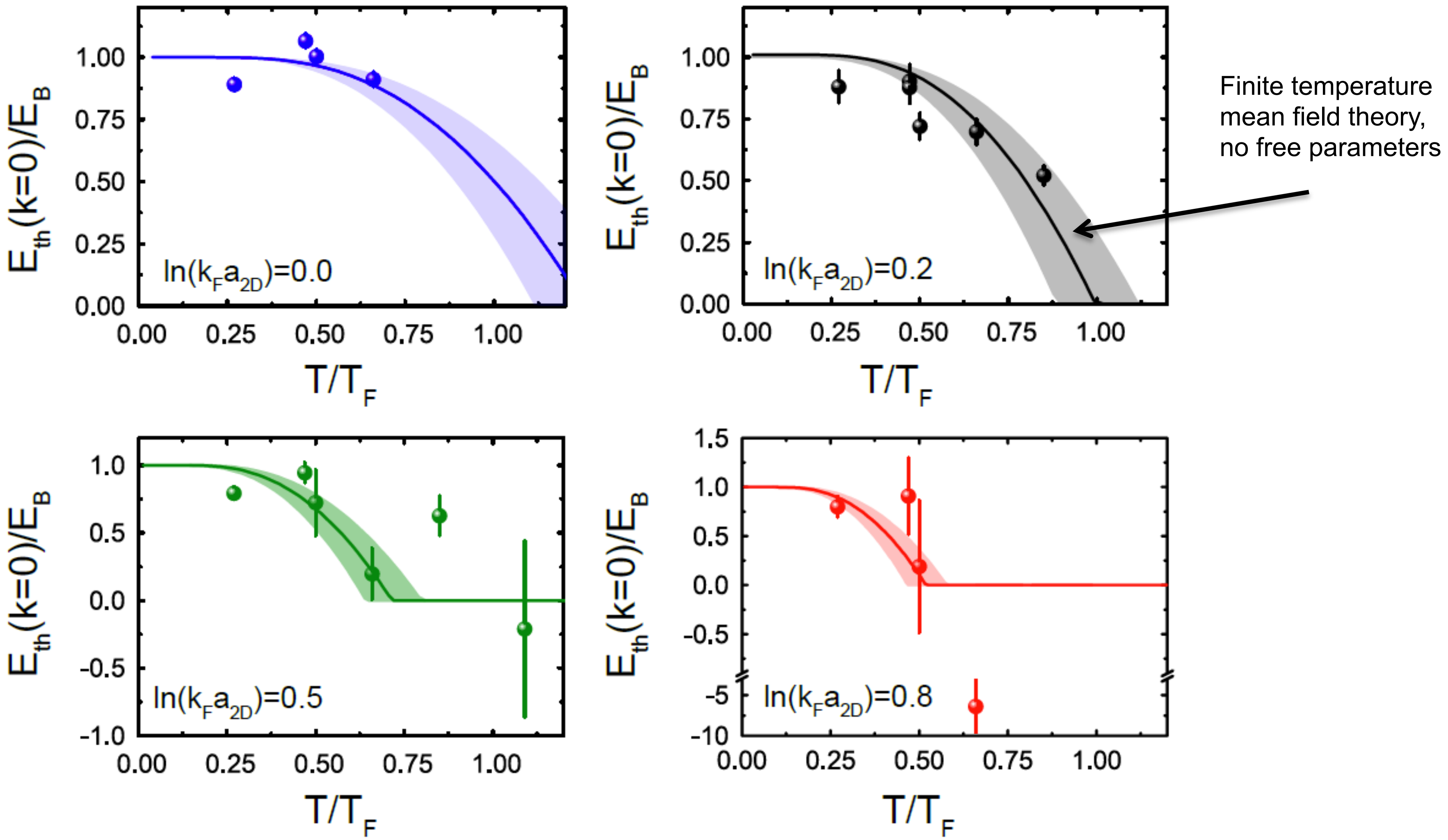
Spectra at k=0: Determining the pseudogap



$$E_{th}(k=0) = \frac{\Delta^2}{2E_F}$$

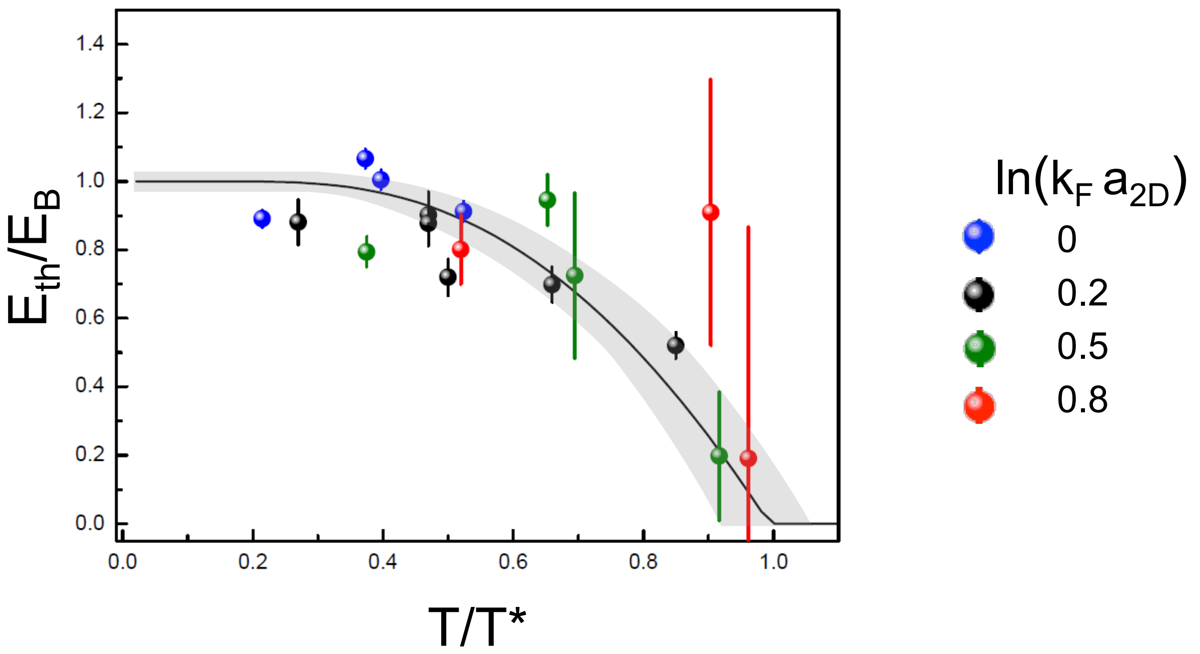
M. Feld, B. Fröhlich, E. Vogt, M. Koschorreck, M.K., Nature 480, 75 (2011)

Temperature dependence



M. Feld, B. Fröhlich, E. Vogt, M. Koschorreck, M.K., Nature 480, 75 (2011)

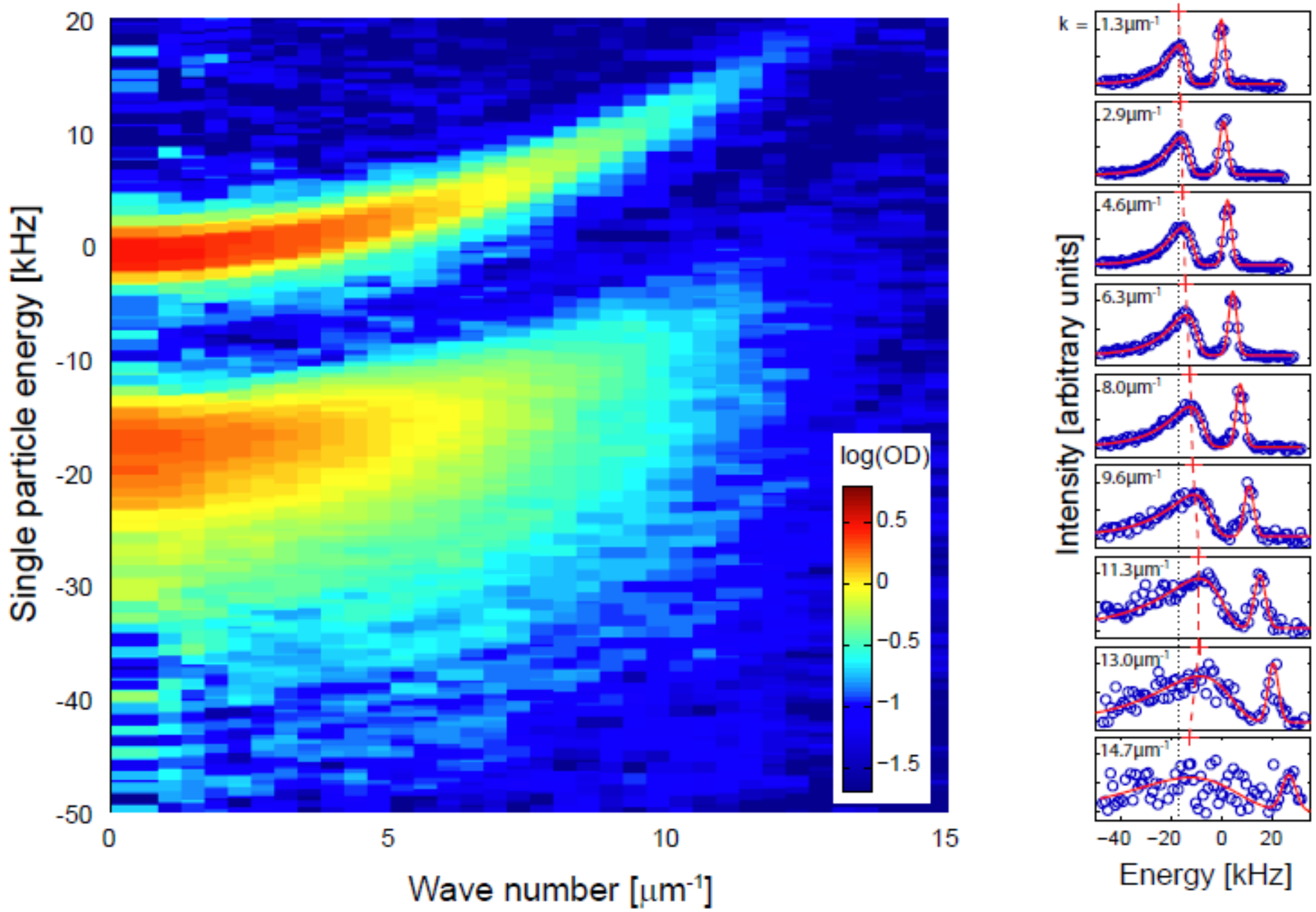
Pairing pseudogap



In 3D: Observation of Fermion condensates below $T/T_F \approx 0.15$ [by projection onto molecules]

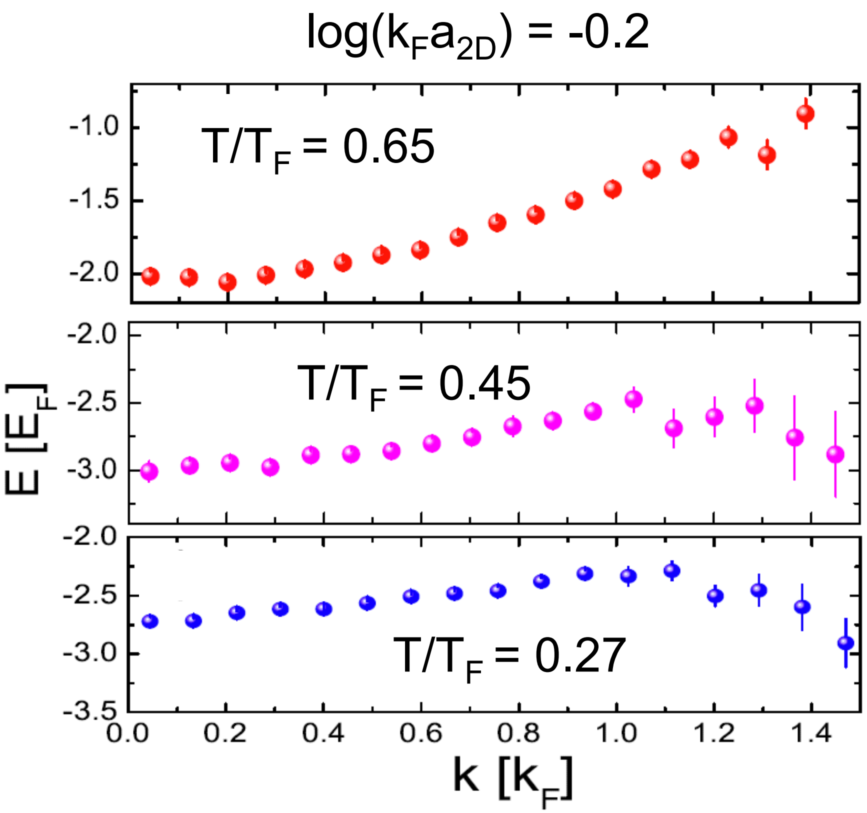
In 2D: No condensation observed.

Back-bending of dispersion relation



M. Feld, B. Fröhlich, E. Vogt, M. Koschorreck, M.K., Nature 480, 75 (2011)

Back-bending of dispersion relation



BCS-like dispersion
at low T

M. Feld, B. Fröhlich, E. Vogt, M. Koschorreck, M.K., Nature 480, 75 (2011)

Fermi polarons

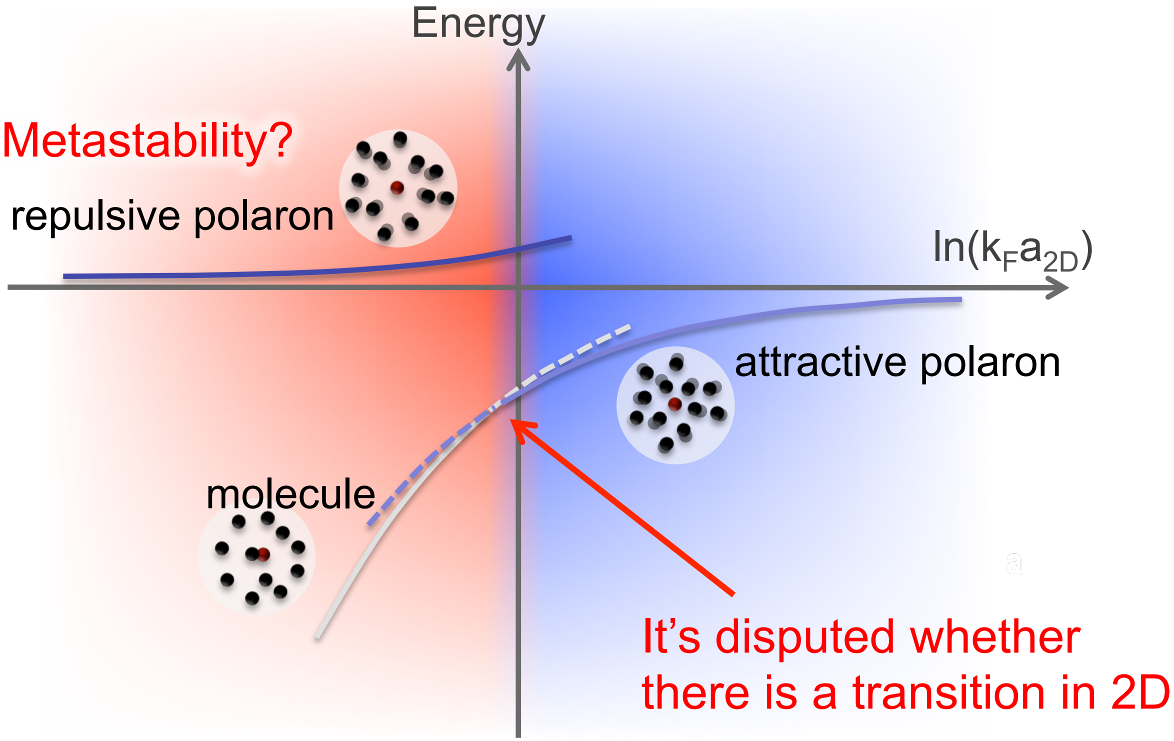
M. Koschorreck, D. Pertot, E. Vogt, M. Feld, B. Fröhlich, M.K., Nature 485, 619 (2012)

Strongly imbalanced Fermi gases in 2D

The "N+1" problem: one | ↓ > impurity in a large | ↑ > Fermi sea

$$|P\rangle = \alpha_0 c^\dagger_{0\downarrow}|N\rangle + \frac{1}{\Omega}\sum_{\mathbf{k},\mathbf{q}} \alpha_{\mathbf{kq}} c^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{q}\uparrow}|N\rangle,$$

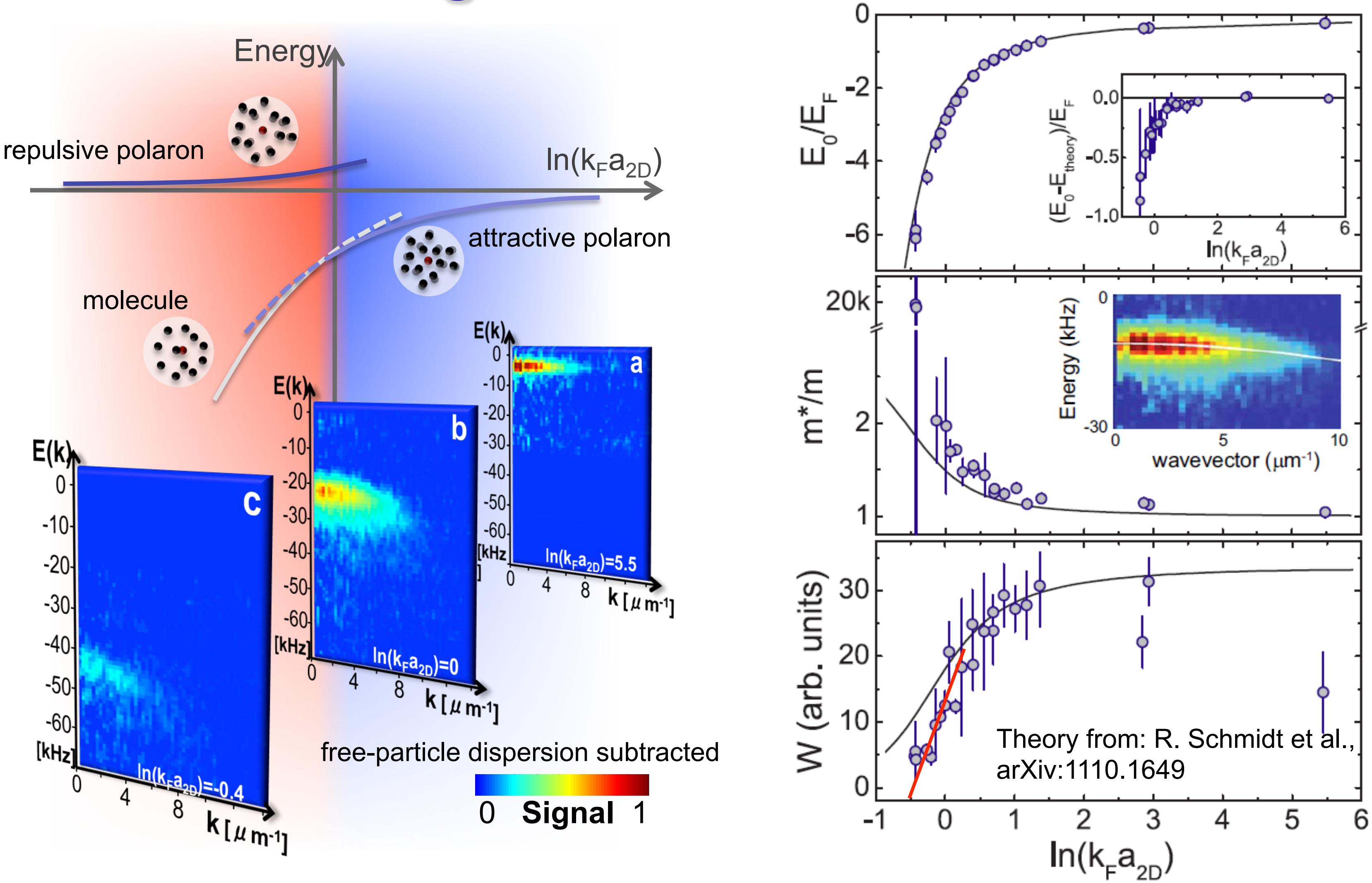
- Mobile impurity interacting with a Fermi sea of atoms
- Tunable interactions
- Polaron properties determine phase diagram of imbalanced Fermi mixtures



3D Theory: Bruun, Bulgac, Chevy, Giorgini, Lobo, Prokofiev, Stringari, Svistunov, ...
3D Expmt: Zwierlein, Salomon, Grimm

2D Theory: Bruun, Demler, Enss, Parish, Pethick, Recati, ...

Characterizing the attractive polaron



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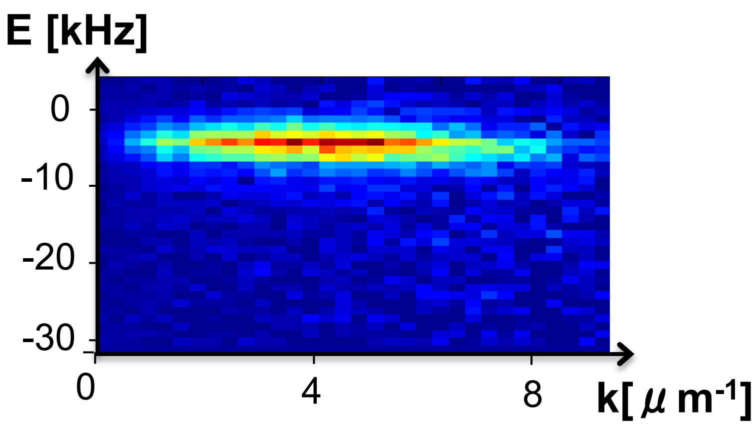
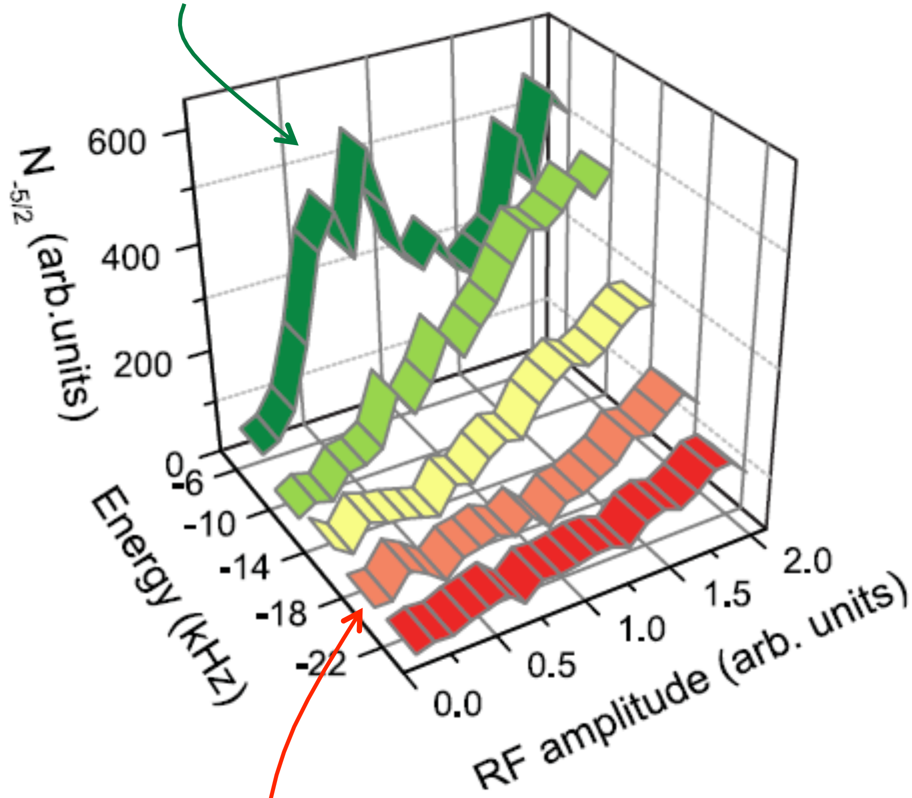
Coherence of the polaron



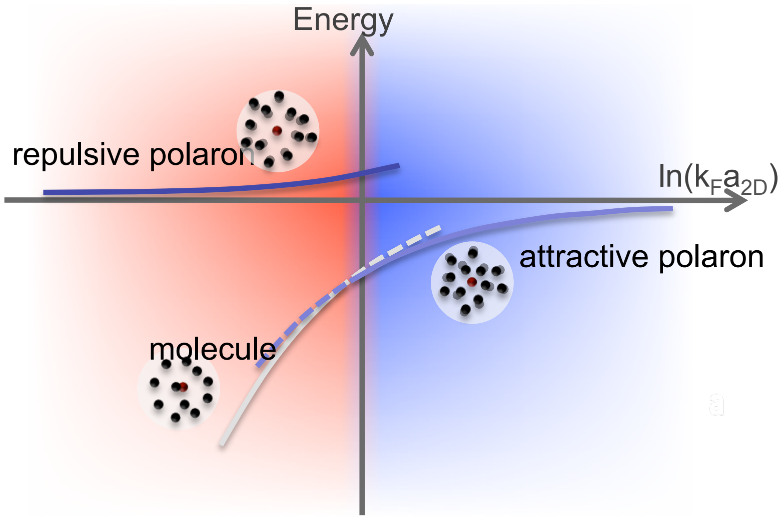
Rabi oscillations between polaron and free particle

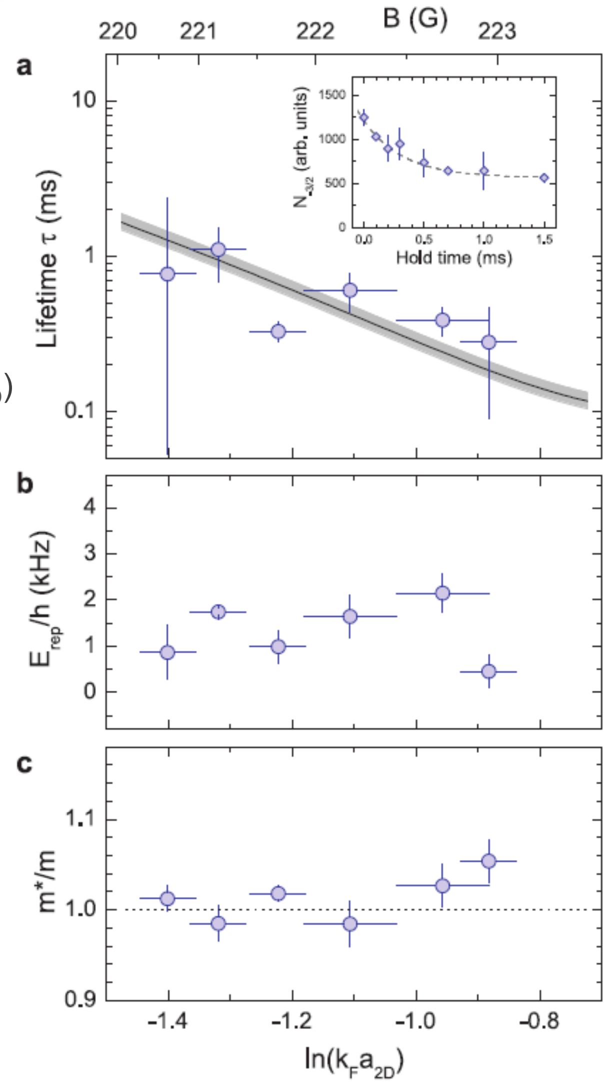


Incoherent transfer: rate ~ amplitude

M. Koschorreck, D. Pertot, E. Vogt, M. Feld, B. Fröhlich, M.K., Nature 485, 619 (2012)

Repulsive polaron





Theory:
V. Ngampruetikorn et al., arXiv:1110.6415

Energies comparable to
R. Schmidt, T. Enss, V. Pietilä, E. Demler, arXiv:1110.1649

Similar experiments in 3D:
Ketterle & Grimm groups

M. Koschorreck, D. Pertot, E. Vogt, M. Feld, B. Fröhlich, M.K., Nature 485, 619 (2012)

Summary

Lecture 1: Atomic Fermi gases - experimental and theoretical background	Lecture 2: Fermion pairing in 3D and 2D	Lecture 3: 2D Fermi gases
Introduction How to make a Fermi gas Seeing the Fermi surface Interactions	BEC-BCS crossover Preparing 2D Fermi gases Scattering in 2D Collective modes	Momentum-resolved rf spectroscopy Fermi liquid Pseudogap pairing Polarons