Lecture 3 ICTP

Scale invariance, universality, and quantum criticality of 2D quantum Gas

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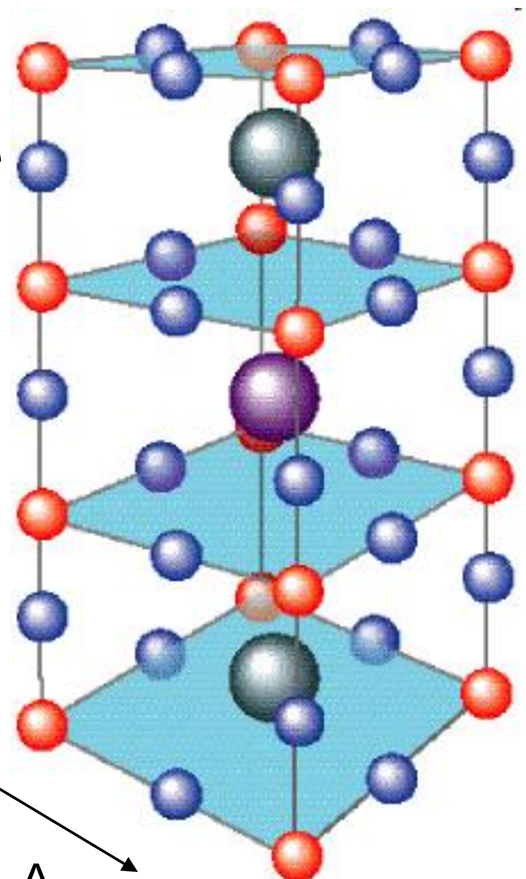
Quantum simulation and universality

Time scale
1ps



T=100 K

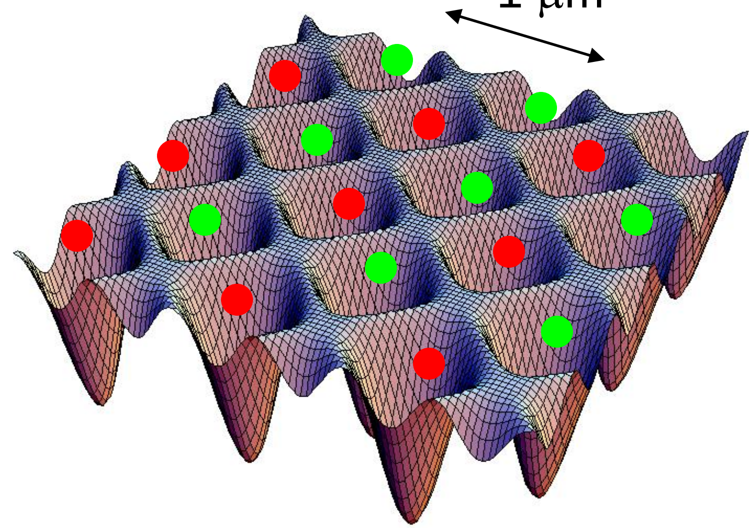
1 A

Examples: Condensed matter, nuclear physics, **HEP**, QIP, **cosmology**…

Time scale
1ms

T=100 nK

1 μm



Ultracold atoms in optical lattices

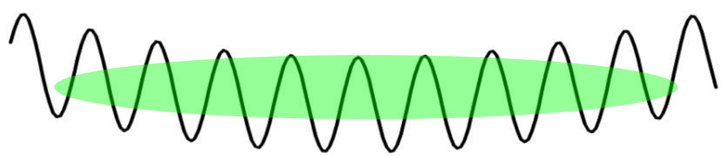
Bose-Hubbard model *(Fisher et al., PRB 1989, Greiner et al., Nature 2002)*

$$\hat{H} = -t\sum_{<i,j>}(\hat{b}_i^+\hat{b}_j + \hat{b}_j^+\hat{b}_i) + U\sum_{i}\frac{\hat{n}_i(\hat{n}_i - 1)}{2} - \sum_i \mu_i \hat{n}_i$$

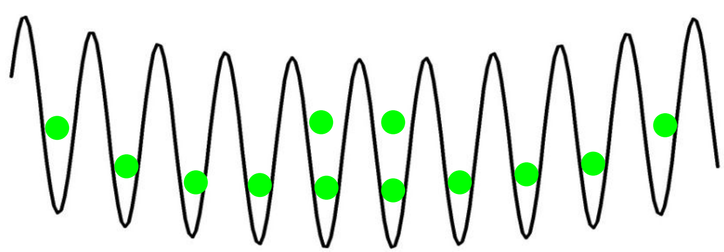
Tunneling Interaction Trap potential

Density distribution

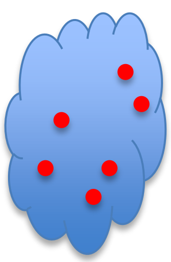
t >> U
Superfluid

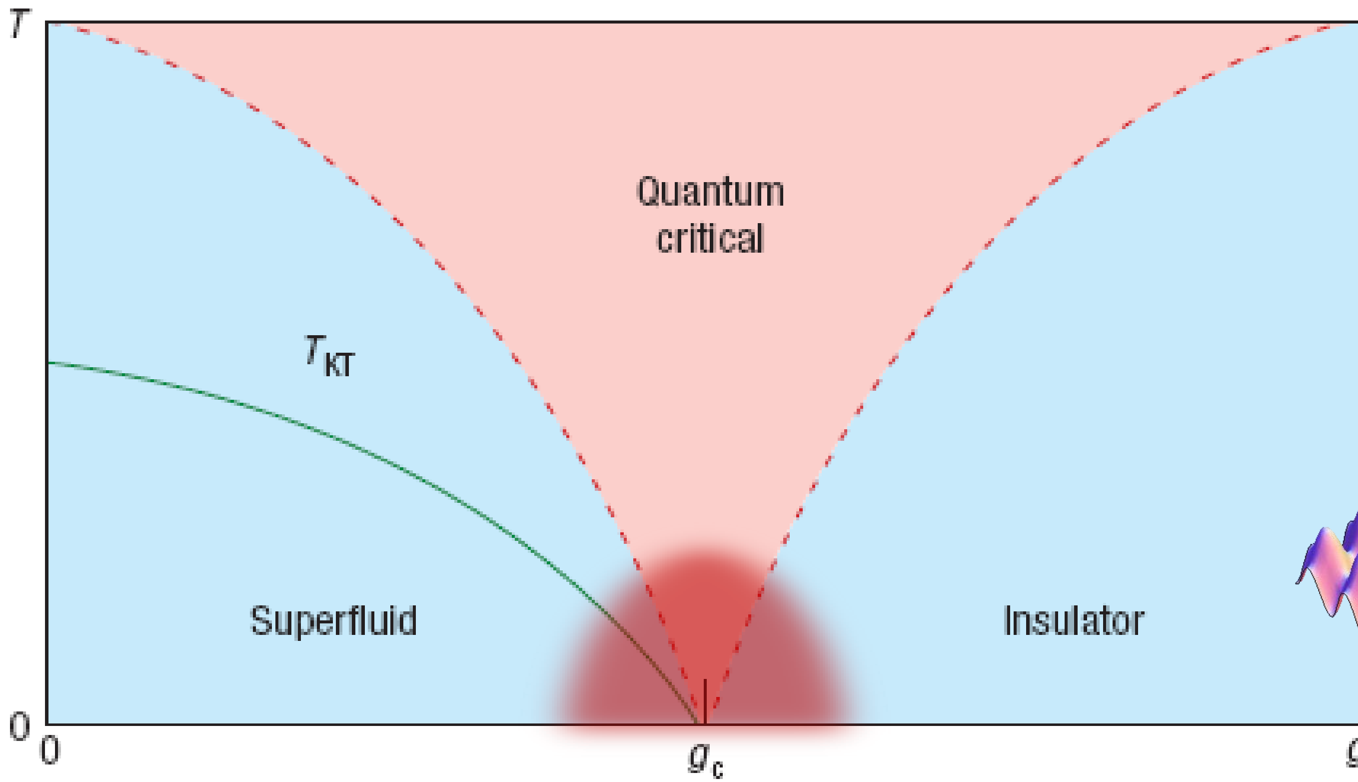


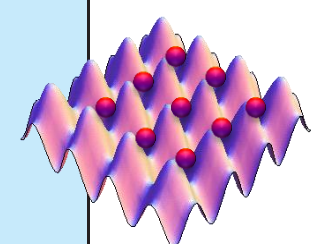
t << U
Mott insulator



Quantum phase transition and scale invariance







Ads-CFT duality: Sachdev, Nature physics (2007)

$$\frac{n-n_c}{T^{d/z+1-1/\nu z}} = h\left(\frac{\mu-\mu_c}{T^{1/\nu z}}\right)$$

Critical thermodynamics: Zhou and Ho, PRL (2010)

Hazzard and Mueller , PRA (2011)

AdS-CFT (gauge-gravity) duality

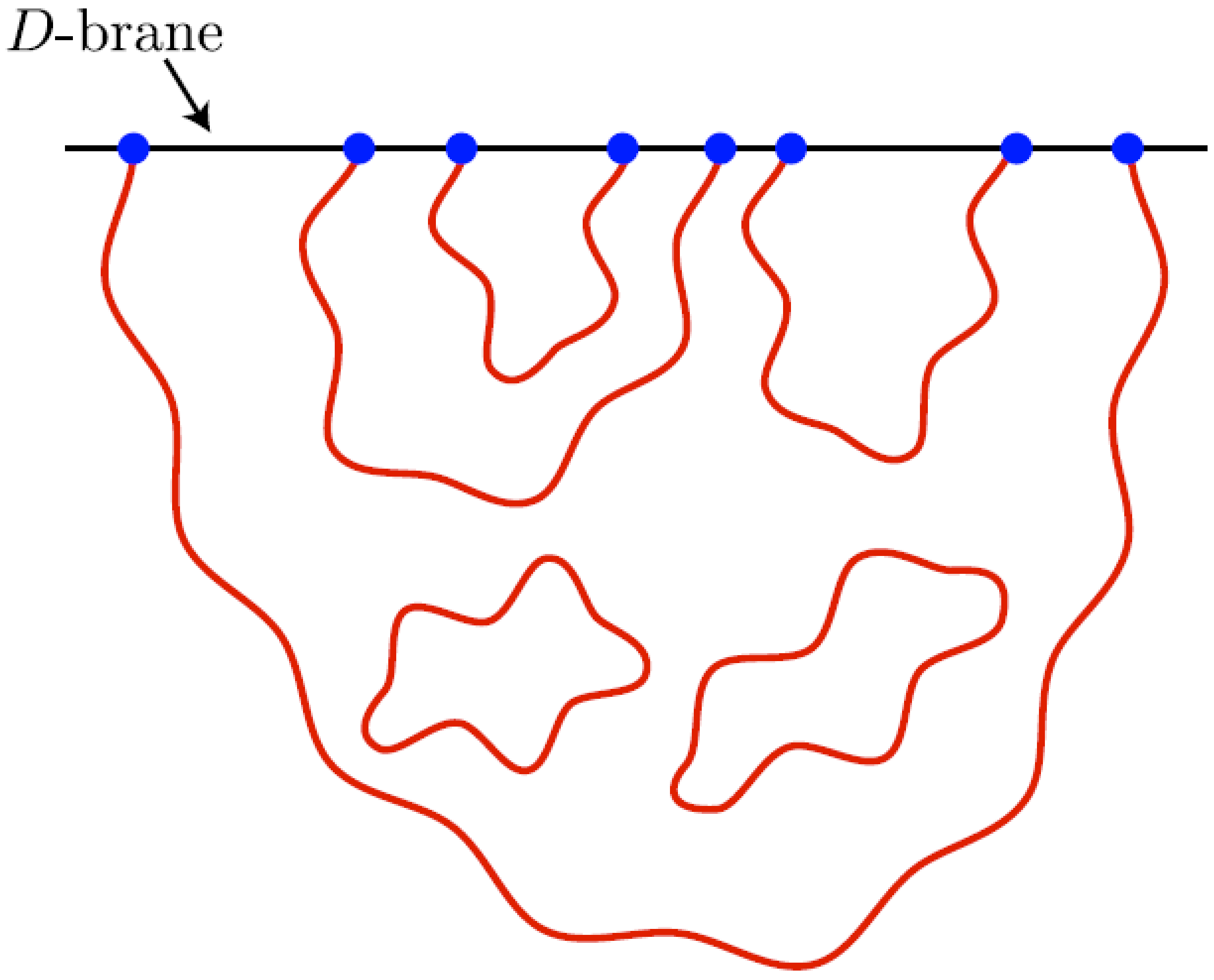


Figure: Subir Sachdev (2011)

Juan Maldacena (1997)

Synopsis

Equilibrium Physics

- **Mott domains in optical lattices**
- **2D gas: Scale invariance and universality**
- **2D lattice: Quantum criticality**

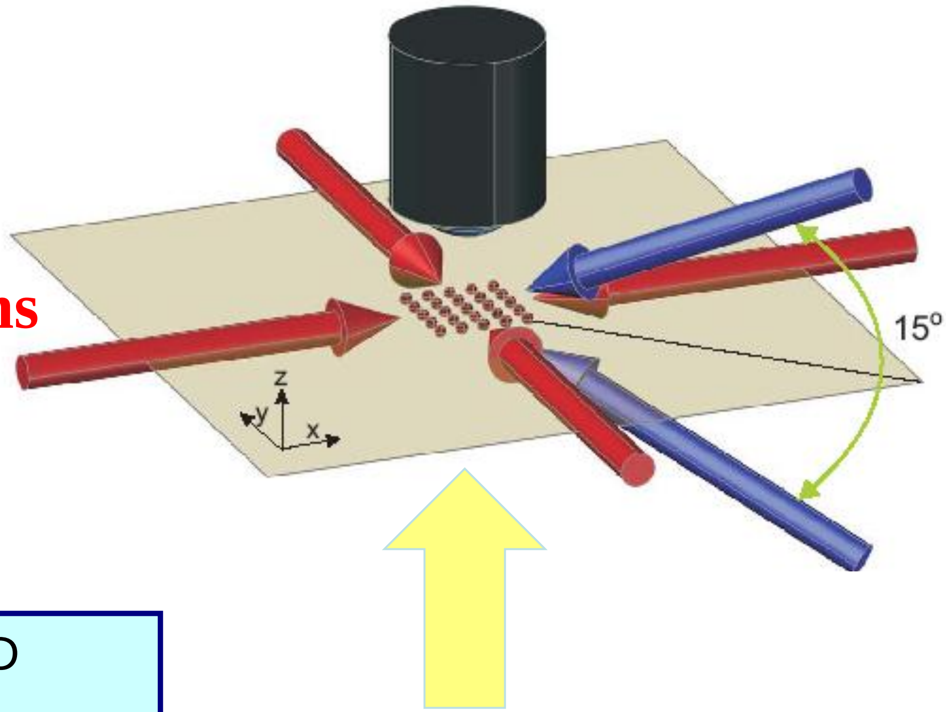
Many-body dynamics

- **Quantum quench and Sakharov oscillations**
- **Gauge-Gravity duality**

In situ Imaging a single layer of 2D gas

Microscope objective

4 lattice beams



2 Vertical compress. beams

Imaging beam

Cesium atoms in 2D

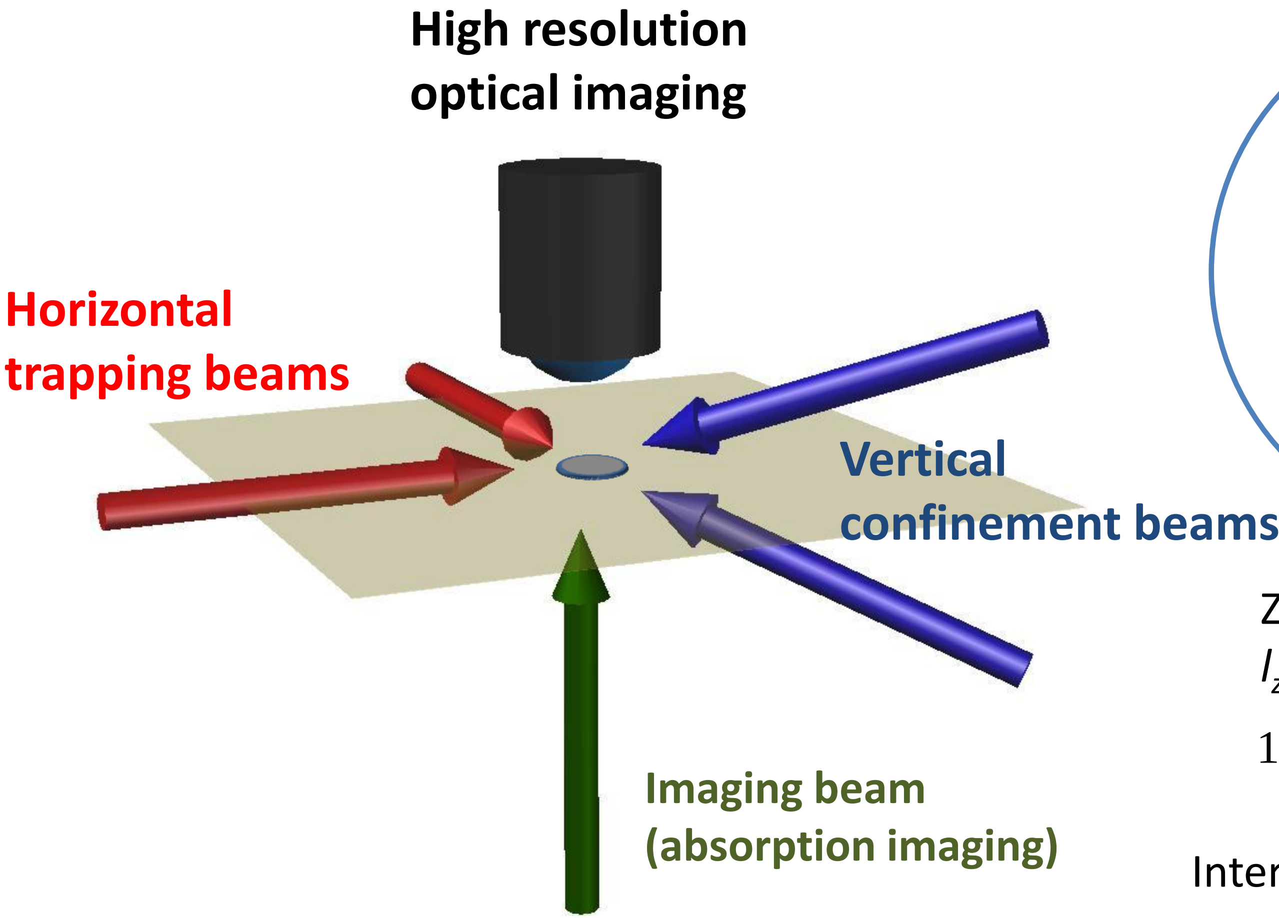
$$15\,\hbar\omega_r \sim k_B T \text{ and } \mu \sim \frac{\hbar\omega_z}{10}$$

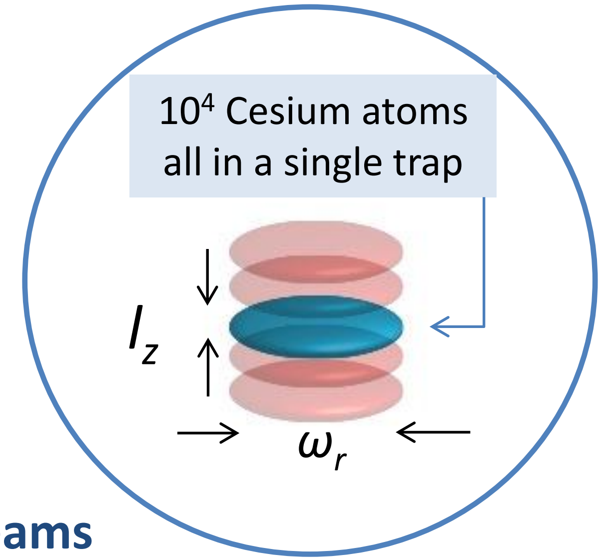
Gemelke et al., Nature 460 (2009)

See also: (Harvard) Bakr et al., Nature 462 (2009)

(MPQ) Sherson et al., Nature 467 (2010)

In situ probing a monolayer of 2D quantum gases





Zero-point oscillator length

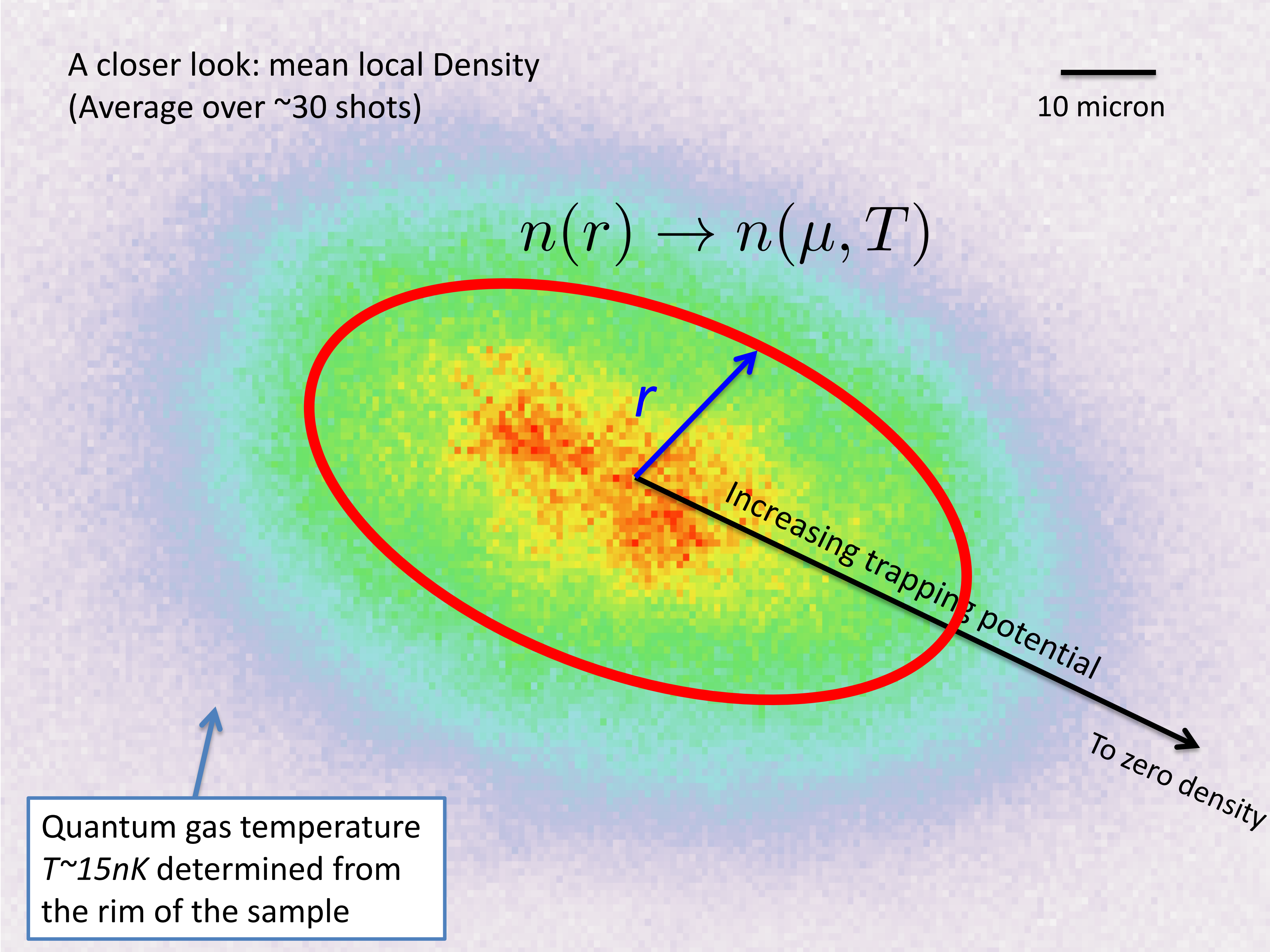
l_z: 200nm

$15\hbar w_r = k_B T$ and $m = \frac{\hbar w_z}{10}$

Interaction strength $g = \frac{\sqrt{8\pi}a}{l_z}$

a: scattering length

A closer look: mean local Density
(Average over ~30 shots)



Scaling symmetry in 2D Bose gases

2D gas with <u>constant</u> interaction *g*

Pitaevskii and Rosch, PRA (1997)

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{ij} g\delta^{(2)}(r_{ij})$$

~ L^{-2} ~ L^{0} ~ L^{-2}

$$H\psi(\lambda x) = \lambda^{-2} H\psi(x)$$

Equation of state (EoS) $n(\mu, T) = \lambda_{dB}^{-2} F(\tilde{\mu})$ $\tilde{\mu} = \dfrac{\mu}{k_B T}$

Universal function

Scale invariance on density observables in 2D

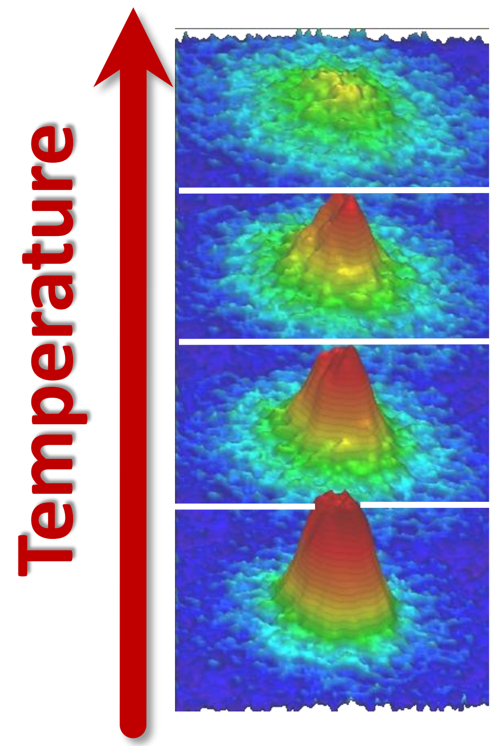
- Equation of state

$$n\lambda_{dB}^2 = F\left(\frac{\mu}{k_B T}\right)$$

- Local density fluctuation

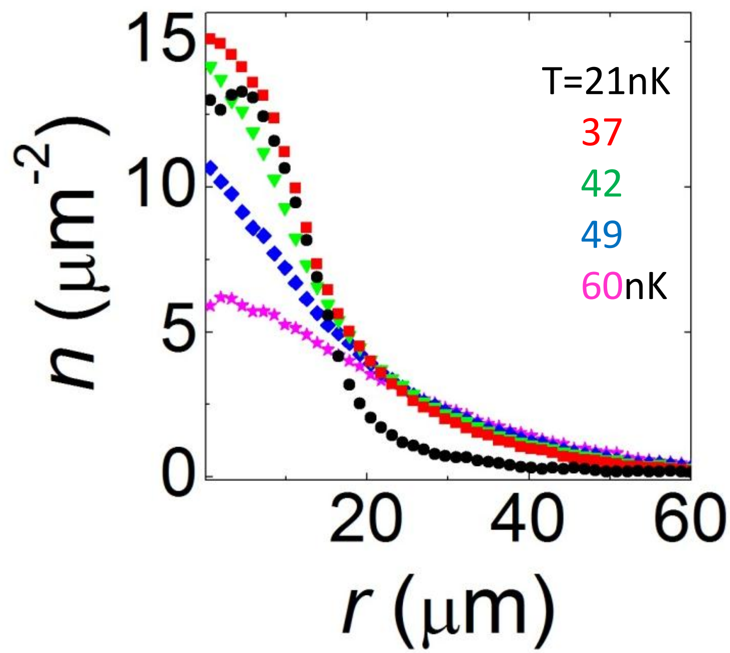
$$\delta n^2 \lambda_{dB}^4 = G\left(\frac{\mu}{k_B T}\right)$$

F(*x*), *G*(*x*): universal functions

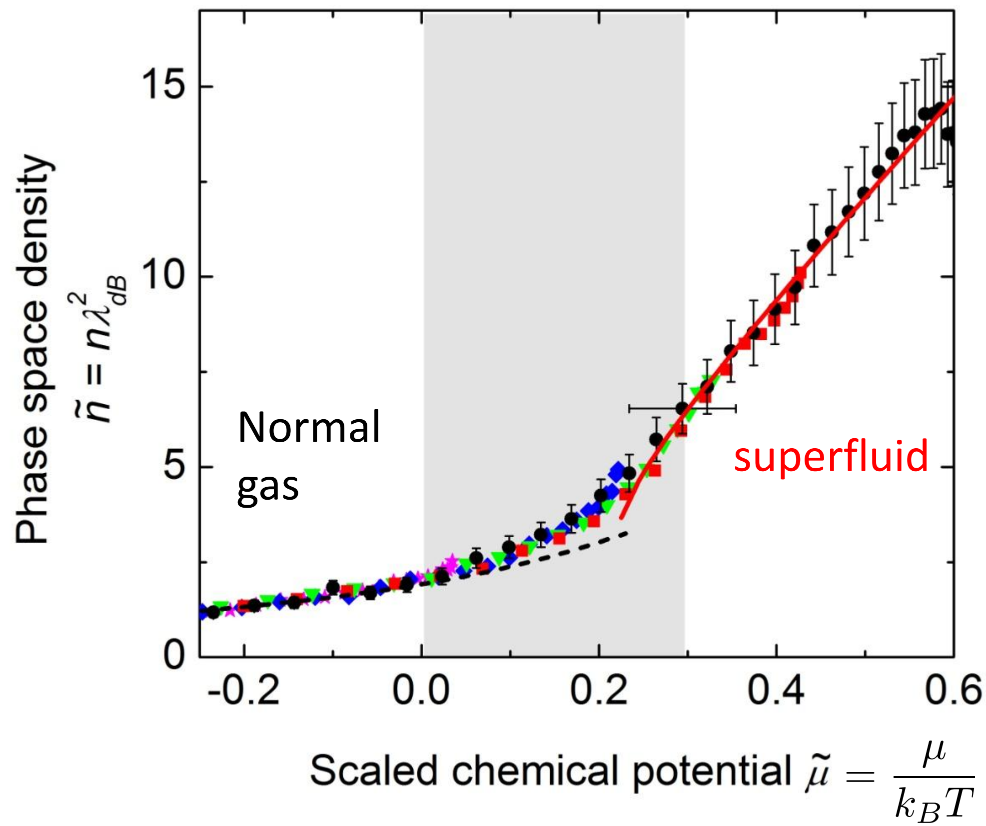


Observation of scale Invariance based on density observables

g=0.26 (a=10nm)



$$\tilde{n} = F(\tilde{\mu})$$



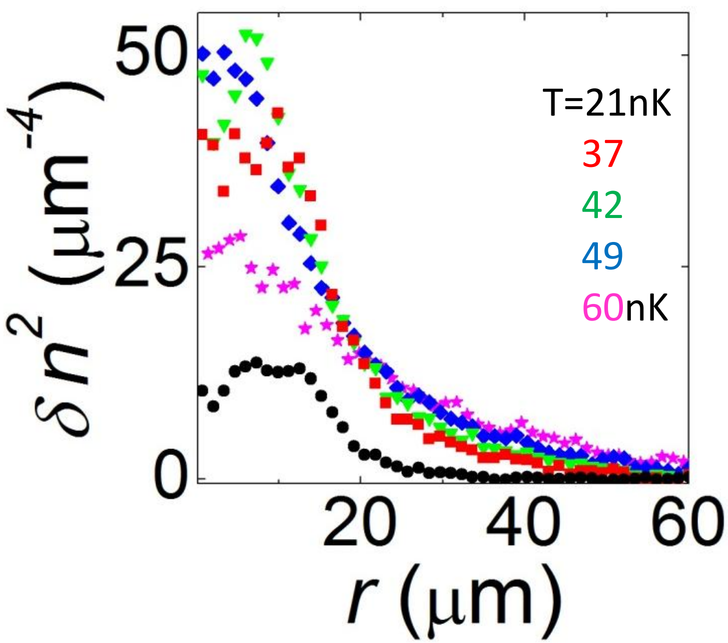
Mean-field (superfluid)

Mean-field (normal gas)

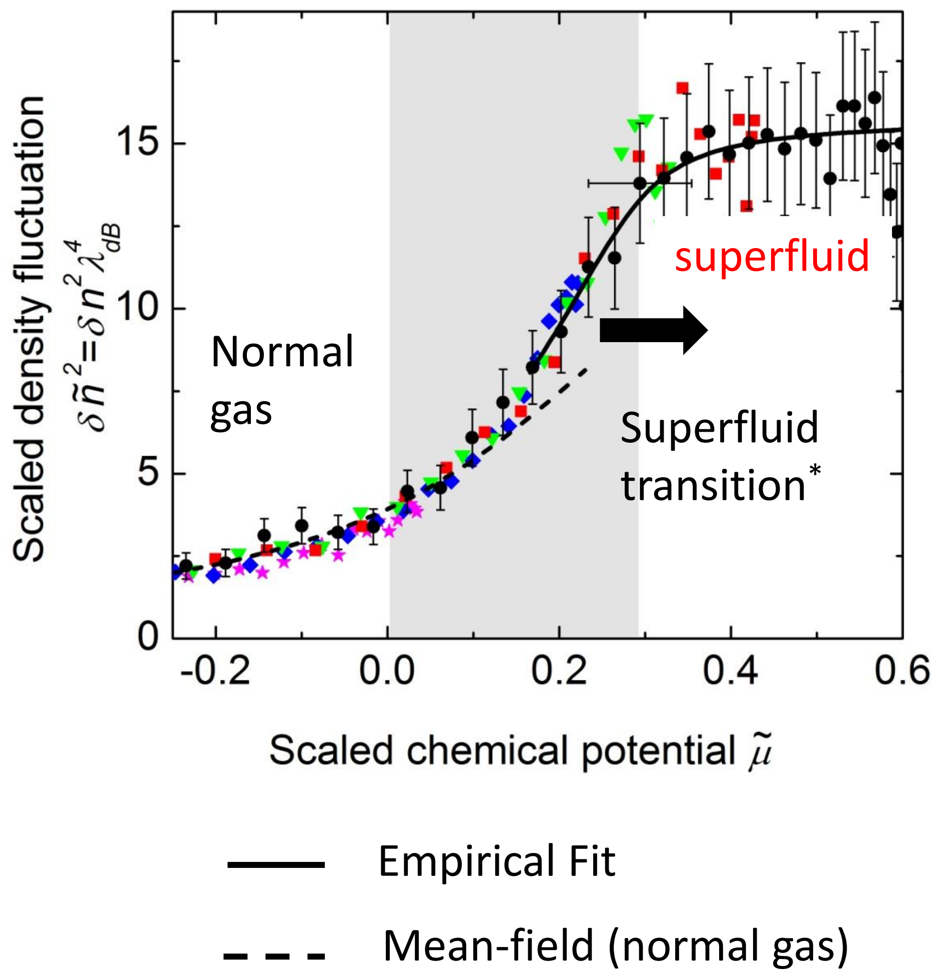
Mean-field: Prokof'ev et al., PRA 66, 043608 (2002)

C.-L. Hung et al., Nature 470, 236 (2011)

Observation of scale Invariance based on density observables



$$\delta\tilde{n}^2 = G(\tilde{\mu})$$



C.-L. Hung et al., Nature 470, 236 (2011)

For other 2D gas experiments, see also:
Dalibard(ENS), Cornell (JILA), Phillips(Gaithersburg)

*Superfluid transition: Berezinsky–Kosterlitz–Thouless (BKT) transition

Universality on density observables

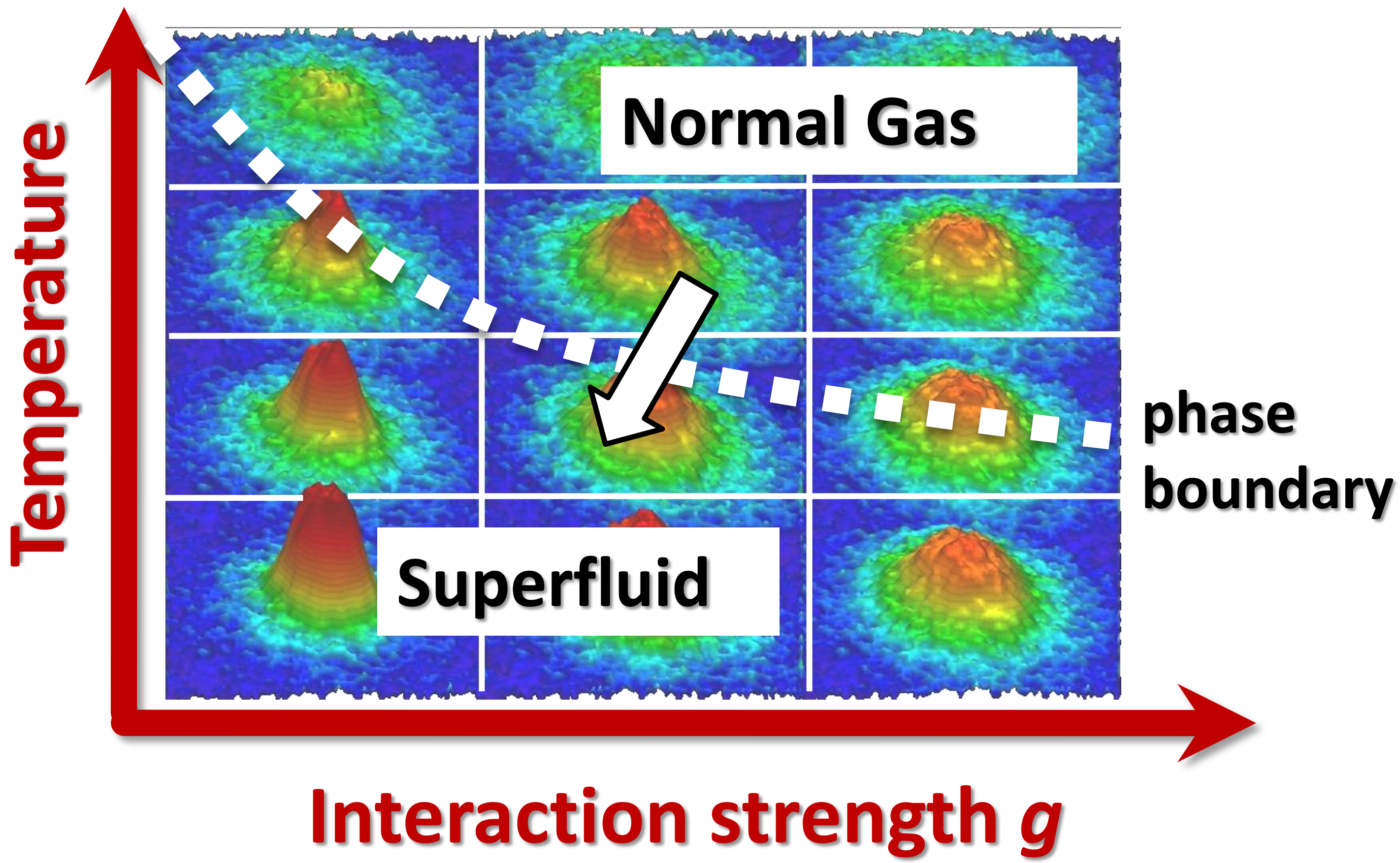
Near the BKT transition region:

- Equation of state

$$(n - n_c)\lambda_{dB}^2 = H\left(\frac{\tilde{\mu} - \tilde{\mu}_c}{g}\right)$$

H(*x*): universal function

Theory:
Prokof’ev et. al., PRA 66, 043608 (2002)

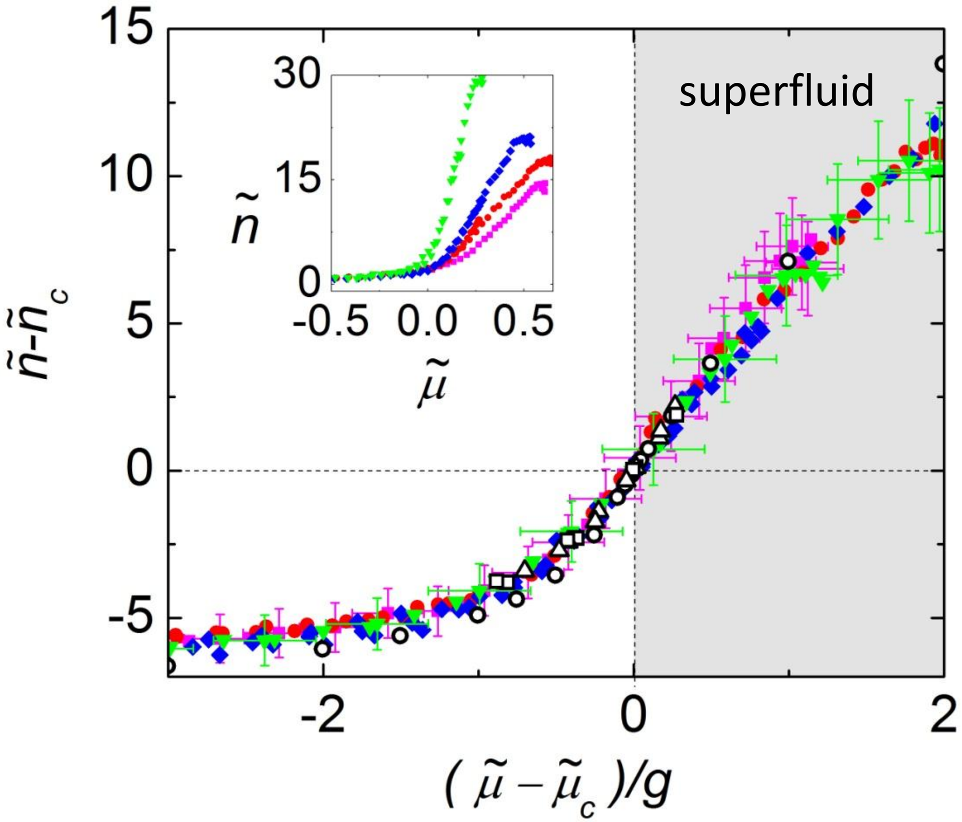


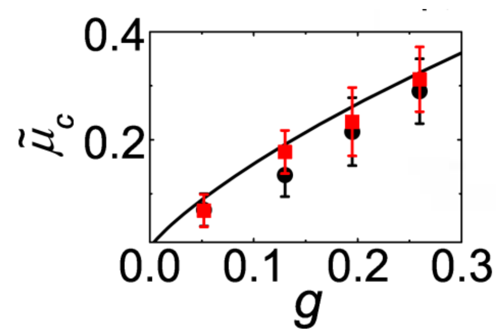
Controlling *g* by cesium Feshbach resonances
C. Chin et al. ,RMP 82, 1225 (2010)

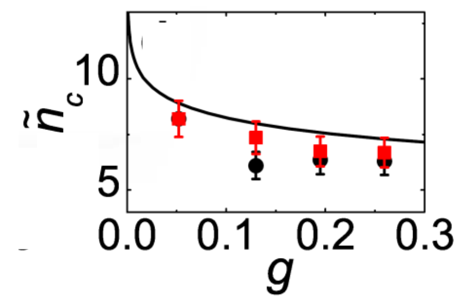
Observation of universality based on density observables

$$\tilde{n} - \tilde{n}_c = H\left(\frac{\tilde{\mu} - \tilde{\mu}_c}{g}\right)$$

g=0.26, 0.19, 0.13, 0.05







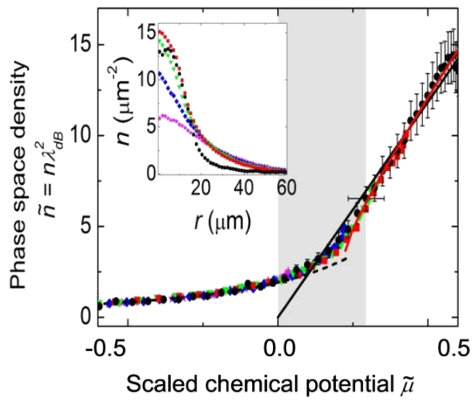
△ □ QMC, Holzmann et. al., PRA (2010)

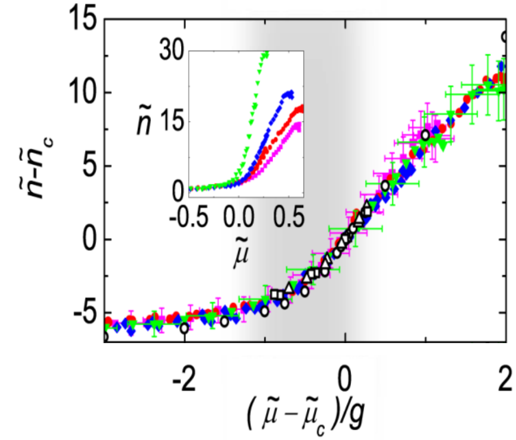
○ Classical field MC, Prokof'ev et. al., PRA (2002)

Classical field theory:
Prokof'ev et. al., PRL 87, 270402 (2001)
Sachdev, Demler, PRB 69,144504 (2004)

Experiment: C.-L. Hung et al., Nature 470, 236 (2011)

Scale invariance and universality in 2D gases





Scale-invariance

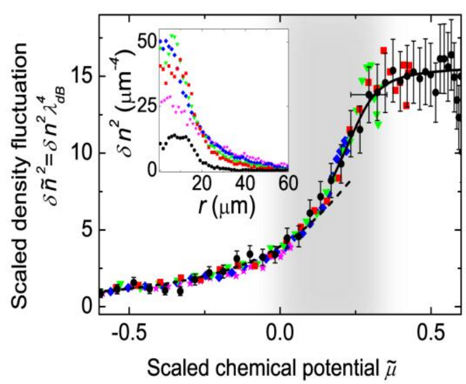
$$\tilde{n} = n\lambda_{dB}^2 = F\left(\frac{\mu}{kT}\right)$$

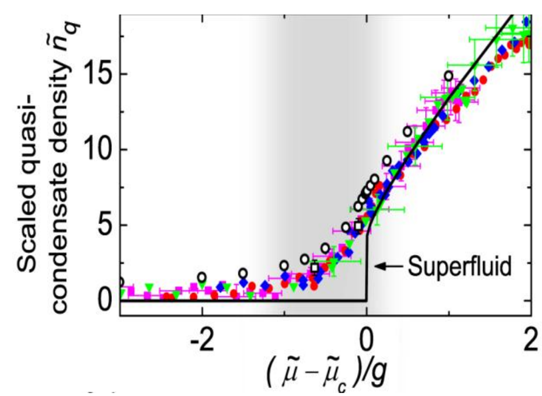
$$\delta\tilde{n}^2 = G\left(\frac{\mu}{kT}\right)$$

$$\tilde{n} - \tilde{n}_c = H\left(\frac{\mu-\mu_c}{kTg}\right)$$

$$\sqrt{\tilde{n}^2 - \delta\tilde{n}^2} = Q\left(\frac{\mu-\mu_c}{kTg}\right)$$

Universality

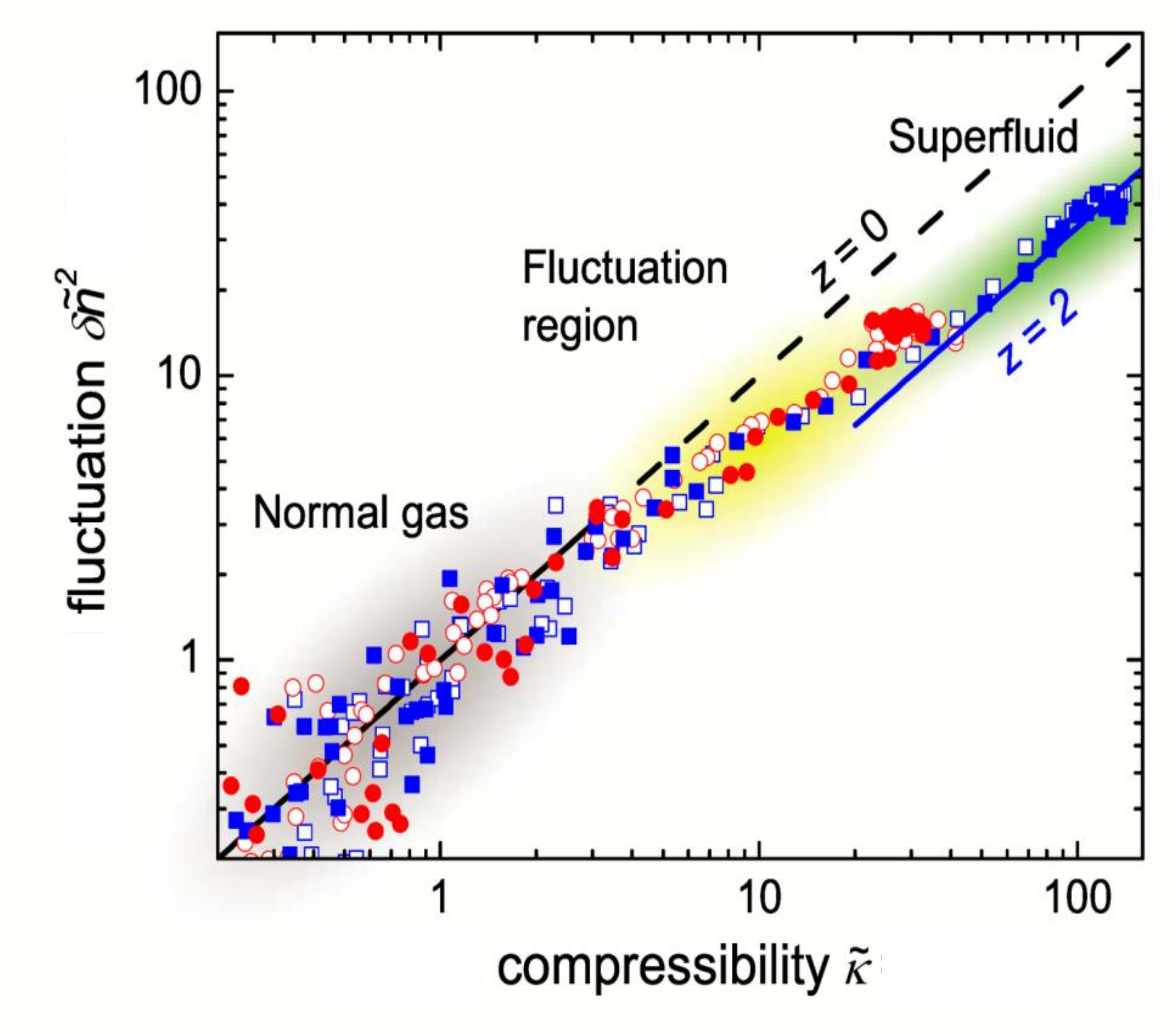




Exp: Hung et al., Nature(2011)

Theory: Prokof'ev and Svistunov, PRA (2002)

Fluctuation-Dissipation Theorem (FDT)



FDT: $$\sum_j \delta n_i \delta n_j + \delta n_i^2 = k_B T \frac{\partial n_i}{\partial \mu}$$

Interacting 2D Bose gases in a 2D optical lattice

(breaking the scale invariance symmetry)

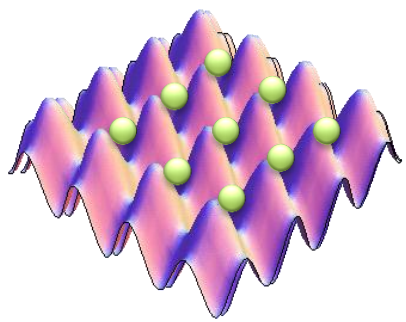
Bose-Hubbard Model

$$H = -t\sum_{\langle i,j\rangle}\hat{b}_i^\dagger\hat{b}_j + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1) - \mu_i \hat{n}_i$$

$$H = \sum_i \frac{p_i^2}{2m} + V(\sin^2 kx_i + \sin^2 ky_i) + g\sum_{ij}\delta(x_{ij})\delta(y_{ij})$$

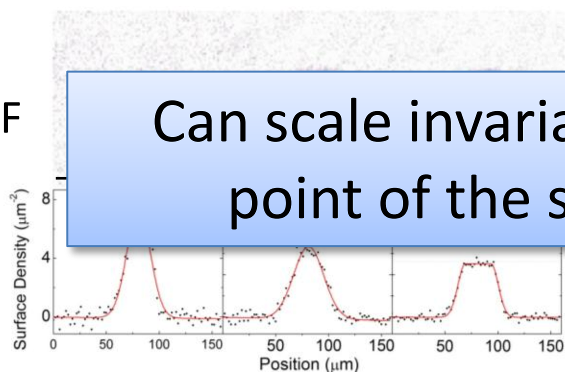
Fischer, Phys. Rev. B, 40, 546 (1989) Jaksch, PRL 81, 3108 (1998)

t: tunneling energy
U: On-site interaction
μ_i: local chemical potential



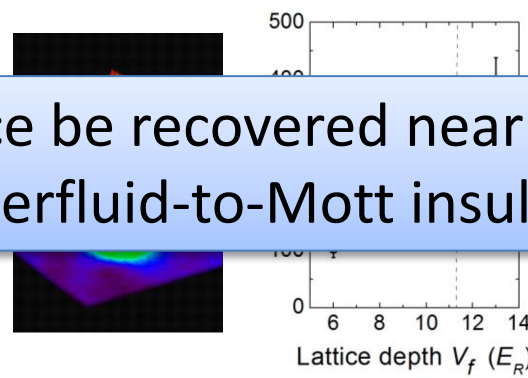
Competition between U, t, or μ induces superfluid-to-Mott insulator transition with site occupation number N_{MI}=0, 1, 2,…

In situ signature



Gemelke et al., Nature (2009)

Mass + entropy transport



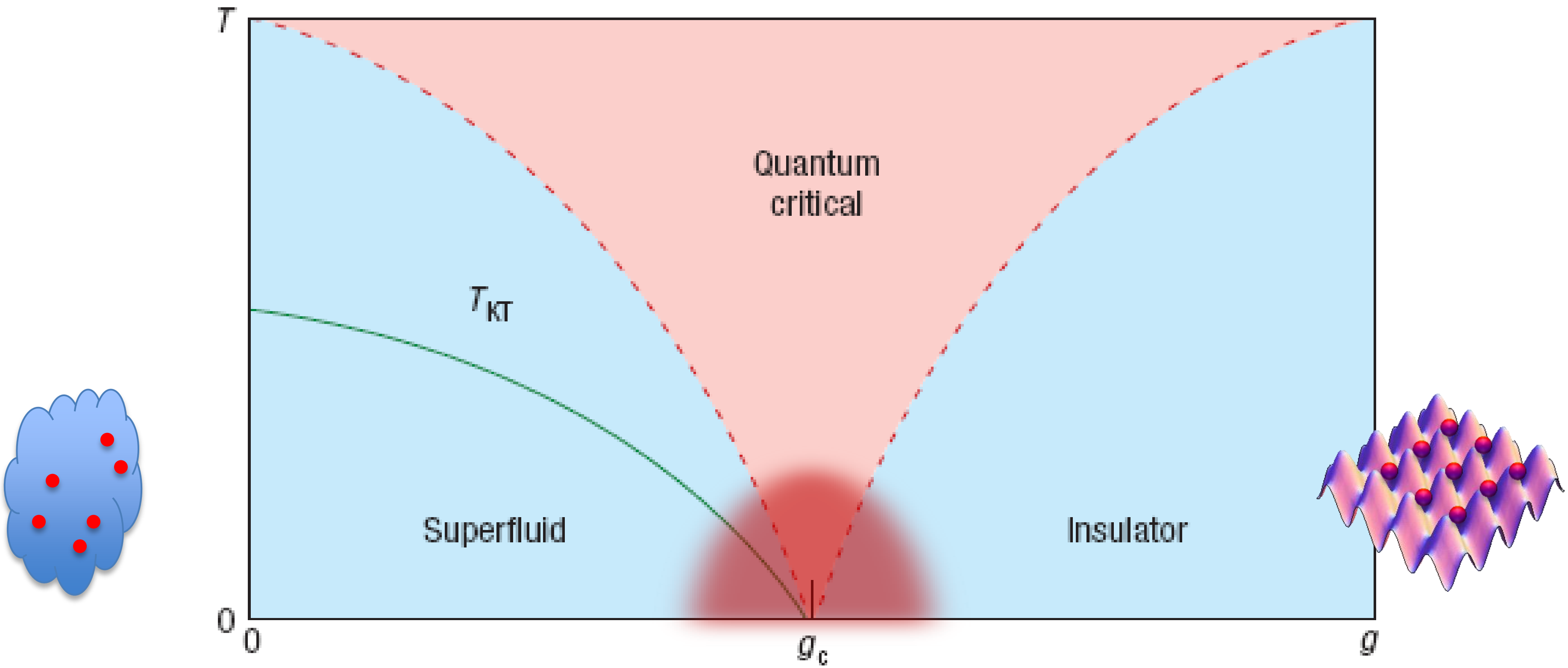
C.-L. Hung et al., PRL (2010)

Sherson et al., Nature (2010)

Can scale invariance be recovered near a quantum critical point of the superfluid-to-Mott insulator transition?

SF: superfluid MI: Mott insulator

Quantum phase transition of bosons in 2D lattices



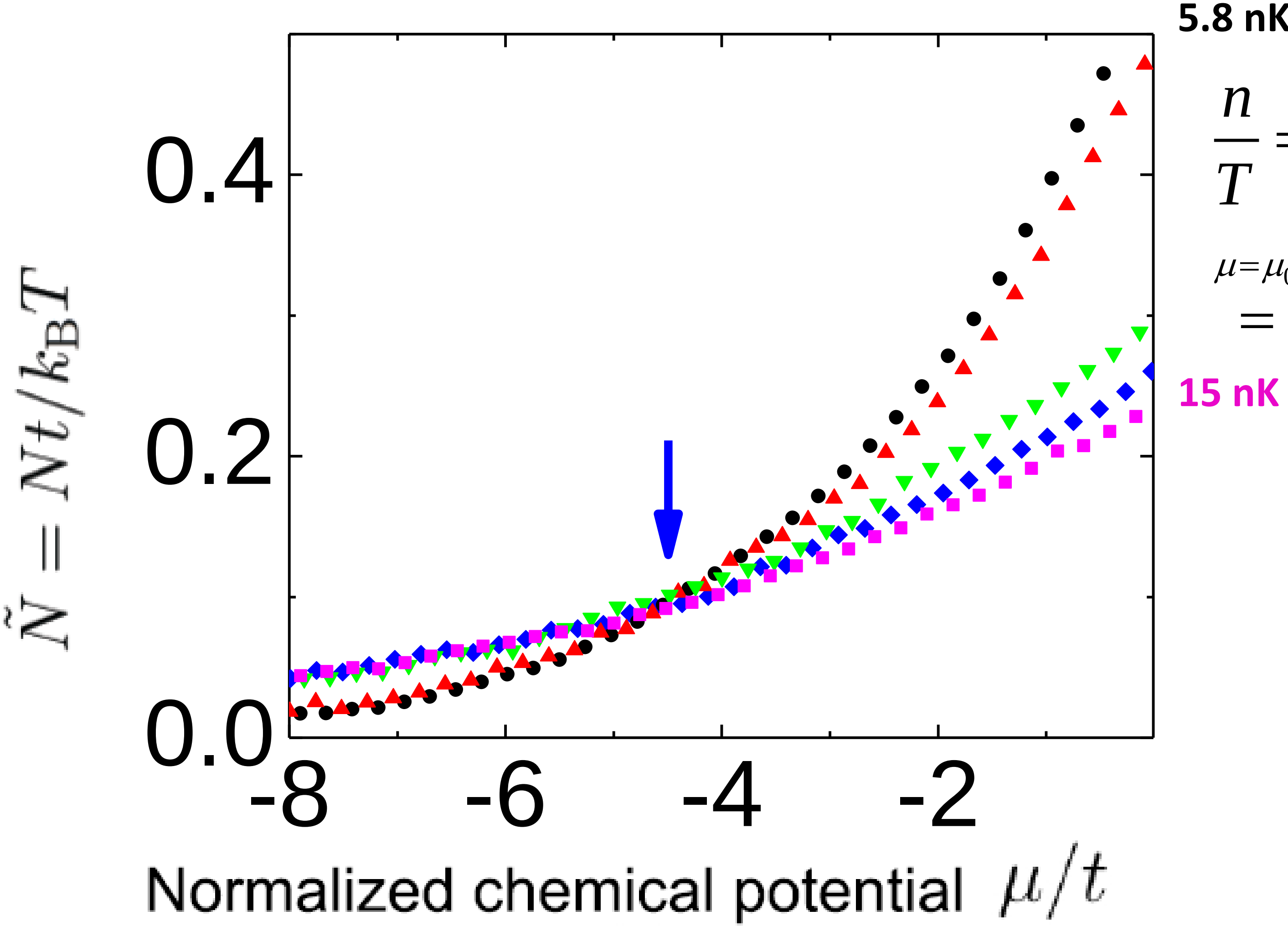
Ads-CFT duality: Sachdev, Nature physics (2007)

$$\frac{n-n_c}{T^{d/z+1-1/\nu z}} = h\left(\frac{\mu-\mu_c}{T^{1/\nu z}}\right)$$

Critical thermodynamics: Zhou and Ho, PRL (2010)

Hazzard and Mueller , PRA (2011)

1/5: identify the quantum critical point.

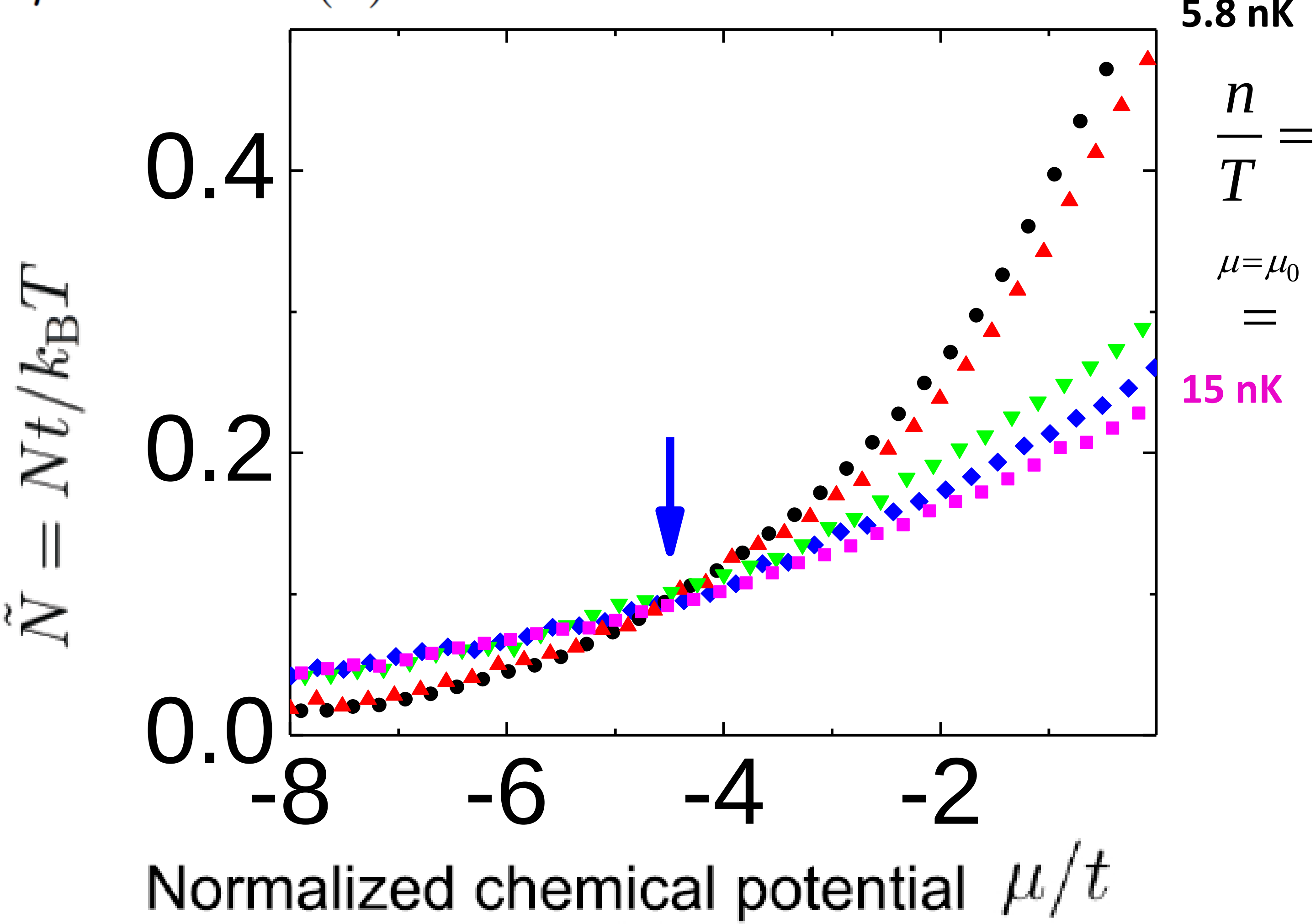


$$\frac{n}{T} = f\left(\frac{\mu - \mu_0}{T}\right)$$

$$\overset{\mu=\mu_0}{=} f(0)$$

1/5: identify the quantum critical point.

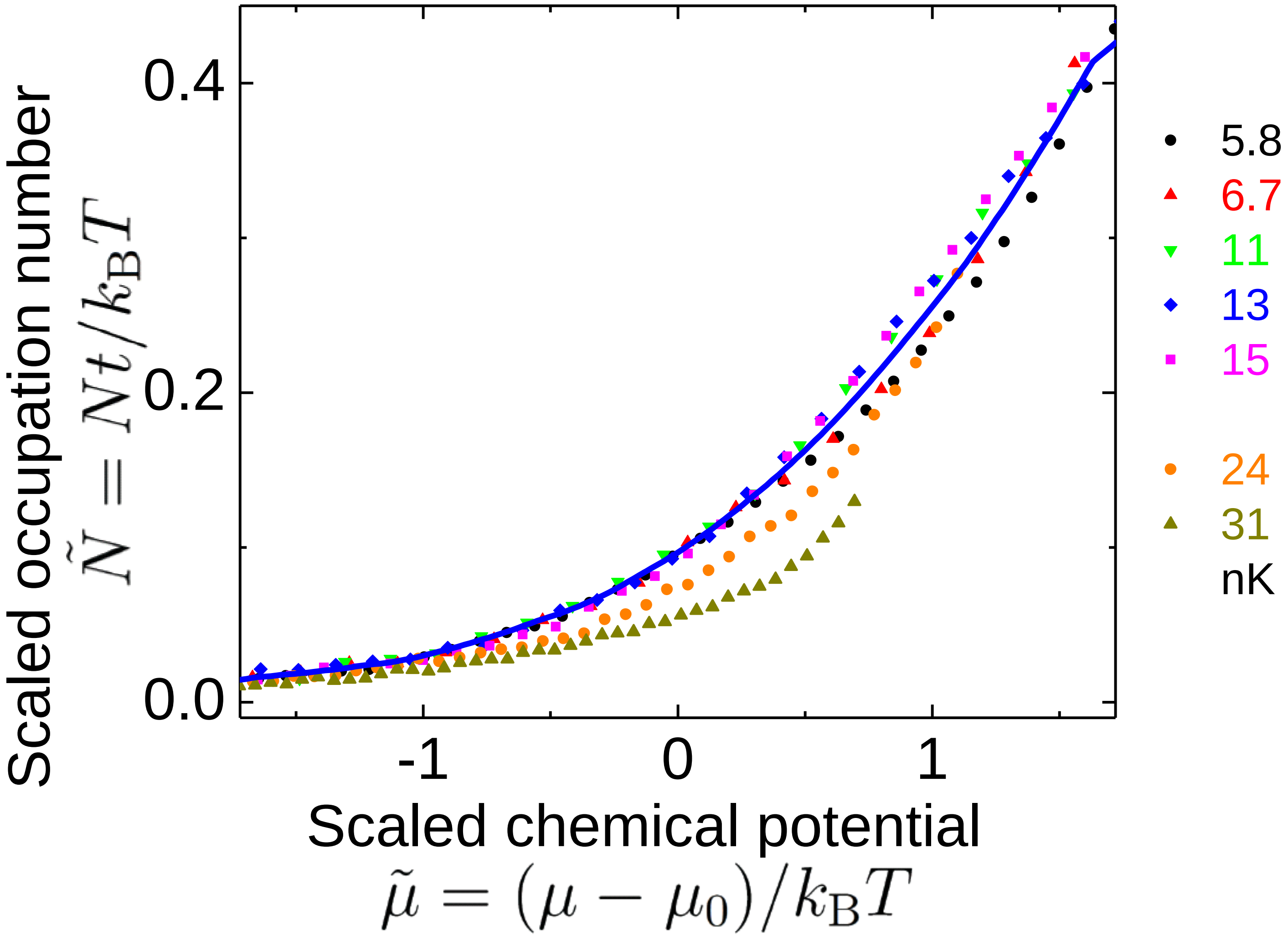
$\mu_0 = -4.5(6)t$ (theory: -4t) t: tunneling



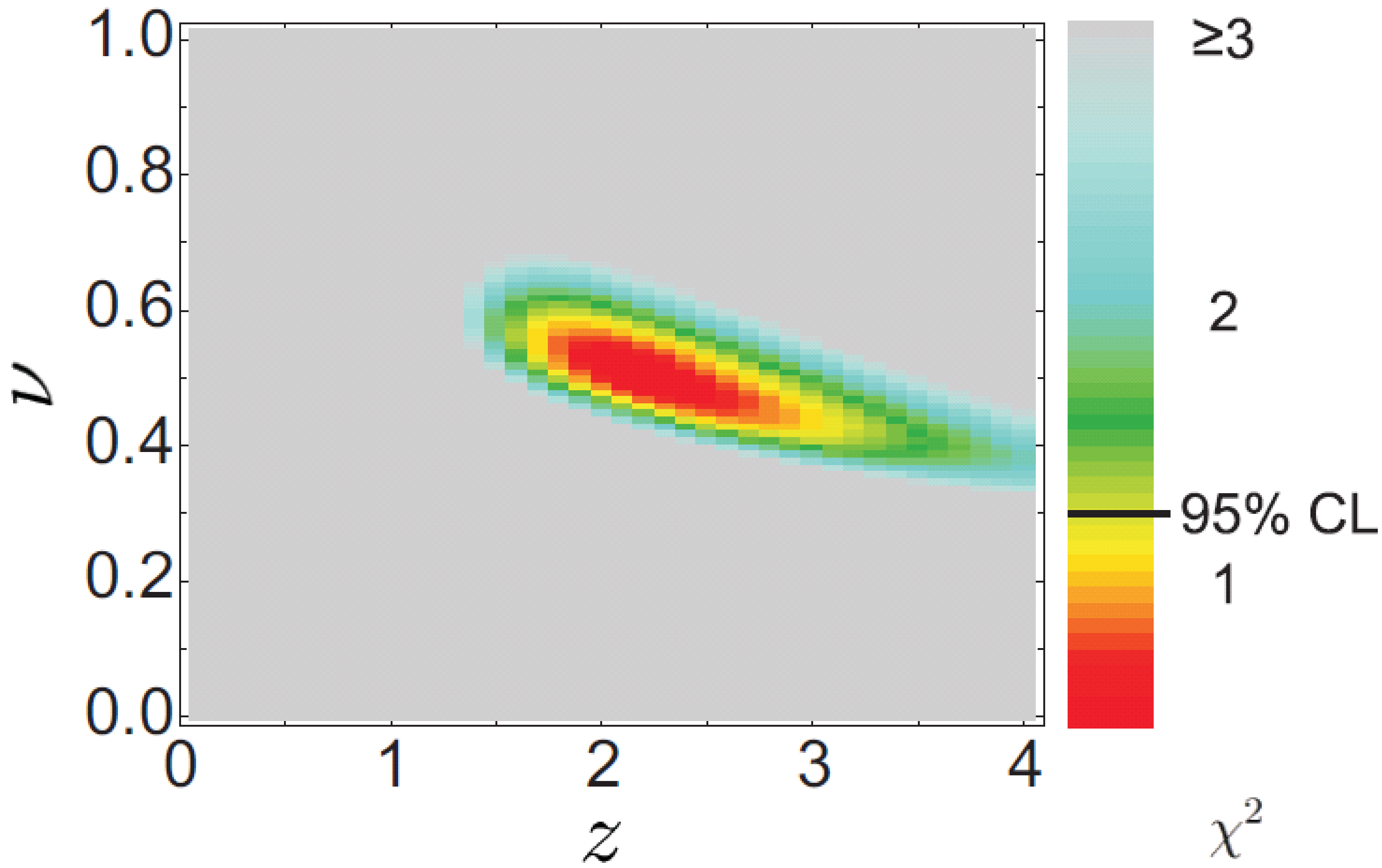
$$\frac{n}{T} = f\left(\frac{\mu - \mu_0}{T}\right)$$

$$\overset{\mu=\mu_0}{=} f(0)$$

2/5: show the universal scaling.



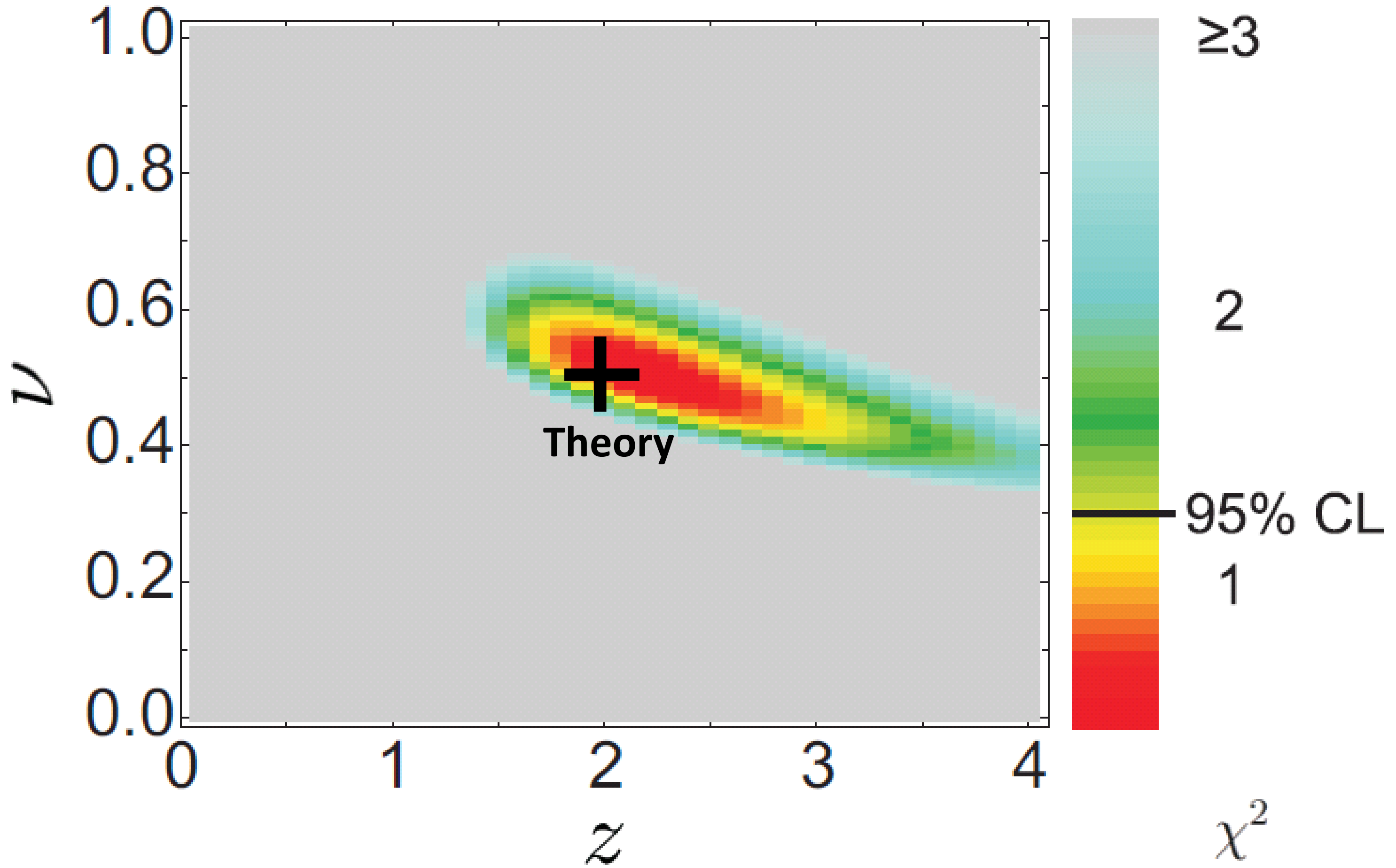
3/5: constrain the critical exponents.



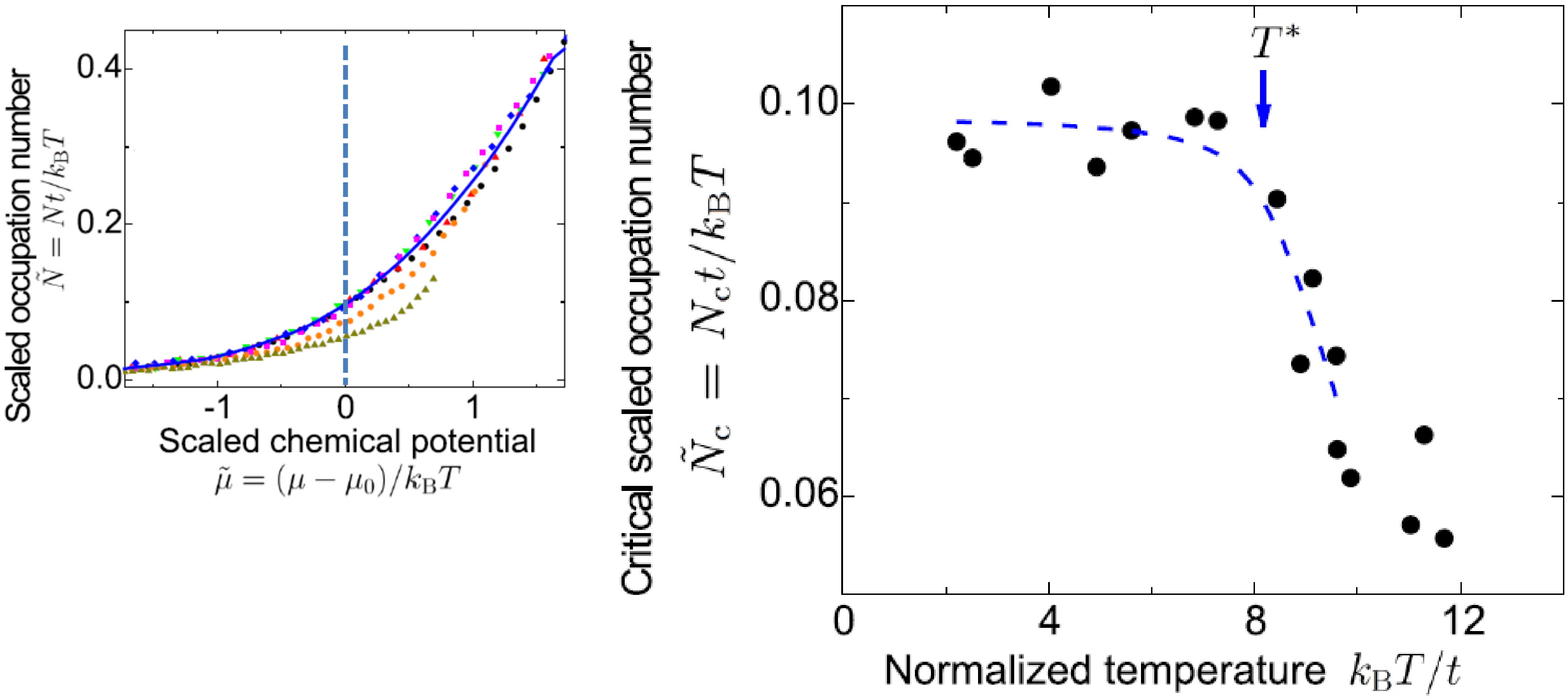
3/5: constrain the critical exponents.

Best fit: $z = 2.2^{+1.0}_{-0.5}$ and $\nu = 0.52^{+0.09}_{-0.10}$, based on $\mu_0 = -4.5t$

theory: $z = 2, \nu = 1/2$ t: tunneling



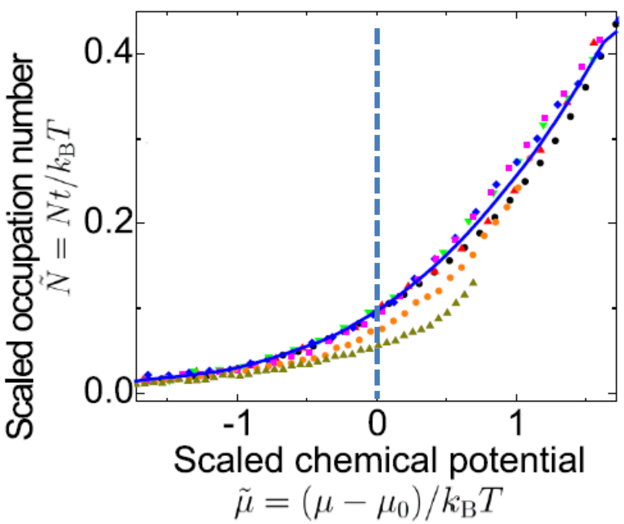
4/5: To how high temperature does quantum criticality persist?

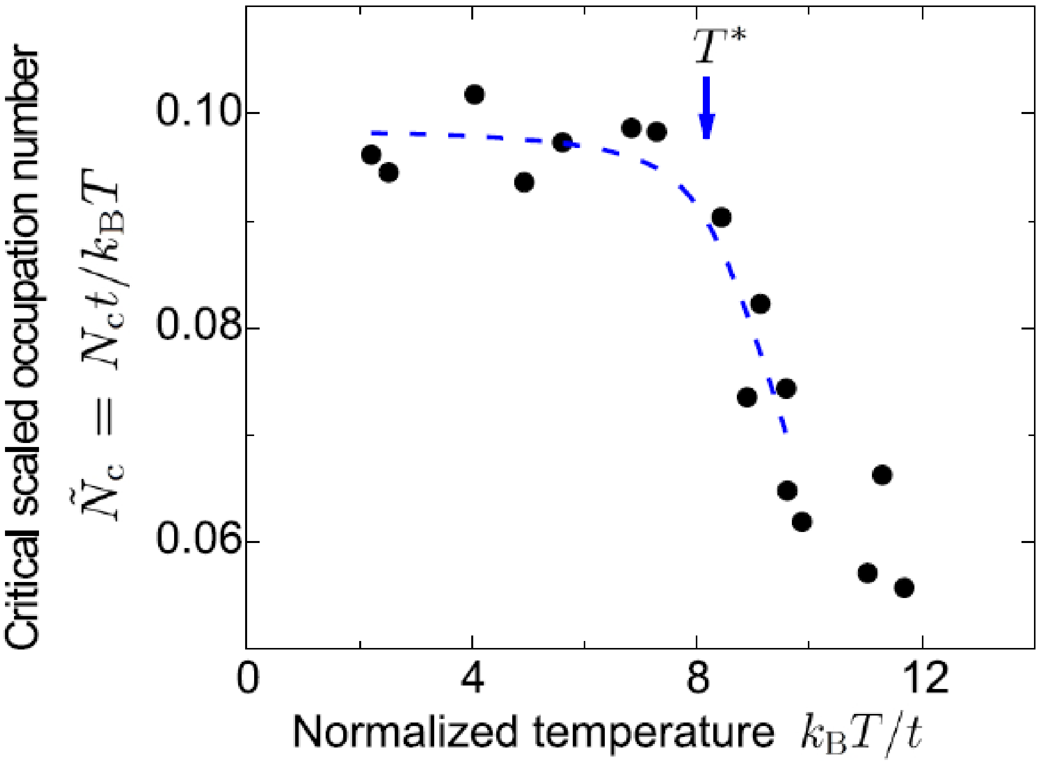


4/5: To how high temperature does quantum criticality persist?

$k_B T^* \approx 8t$ (theory : $6t$; bandwidth: $8t$)

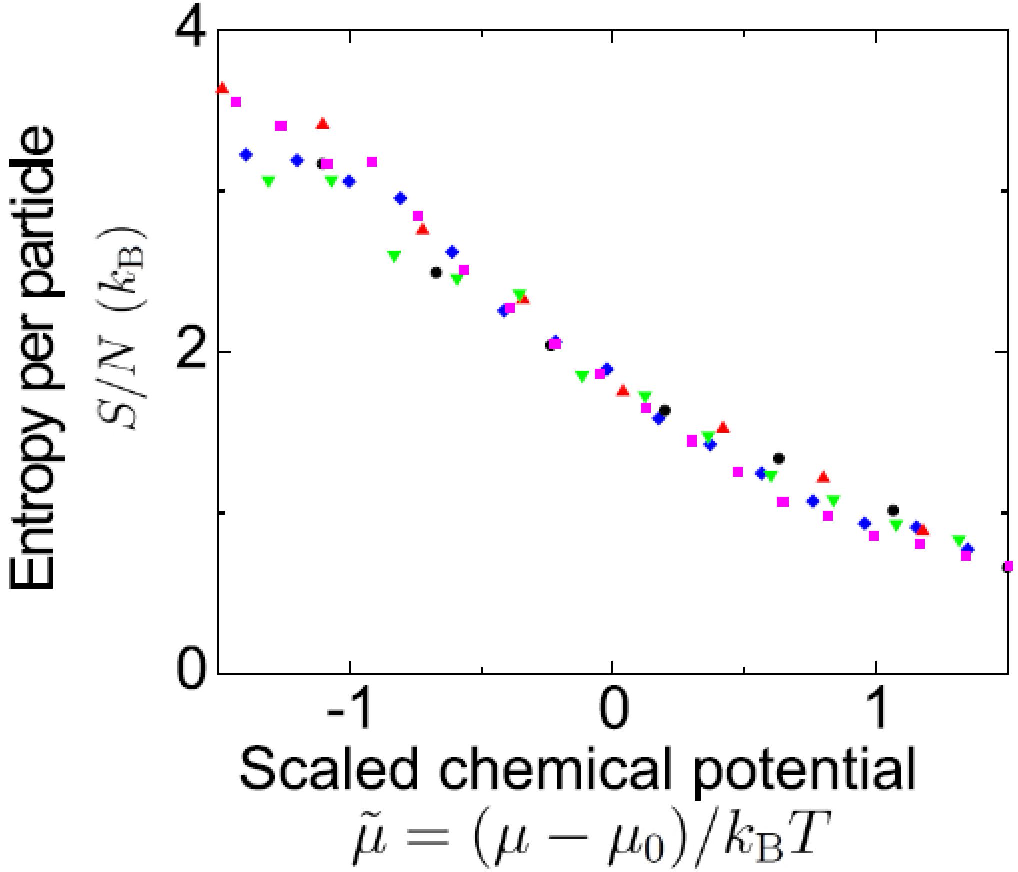
Here t is the tunneling.



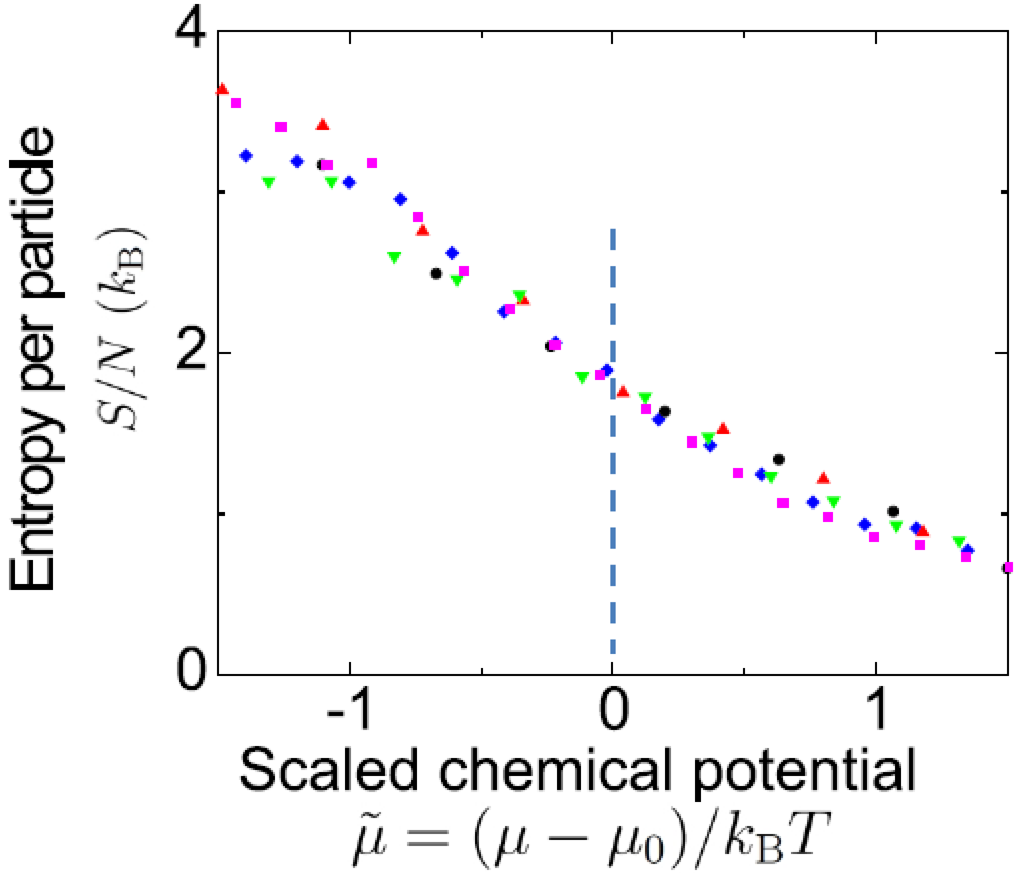


Theory: PRA 83, 031605(R) (2011), D.-W. Wang group

5/5: Entropy per particle in critical regime



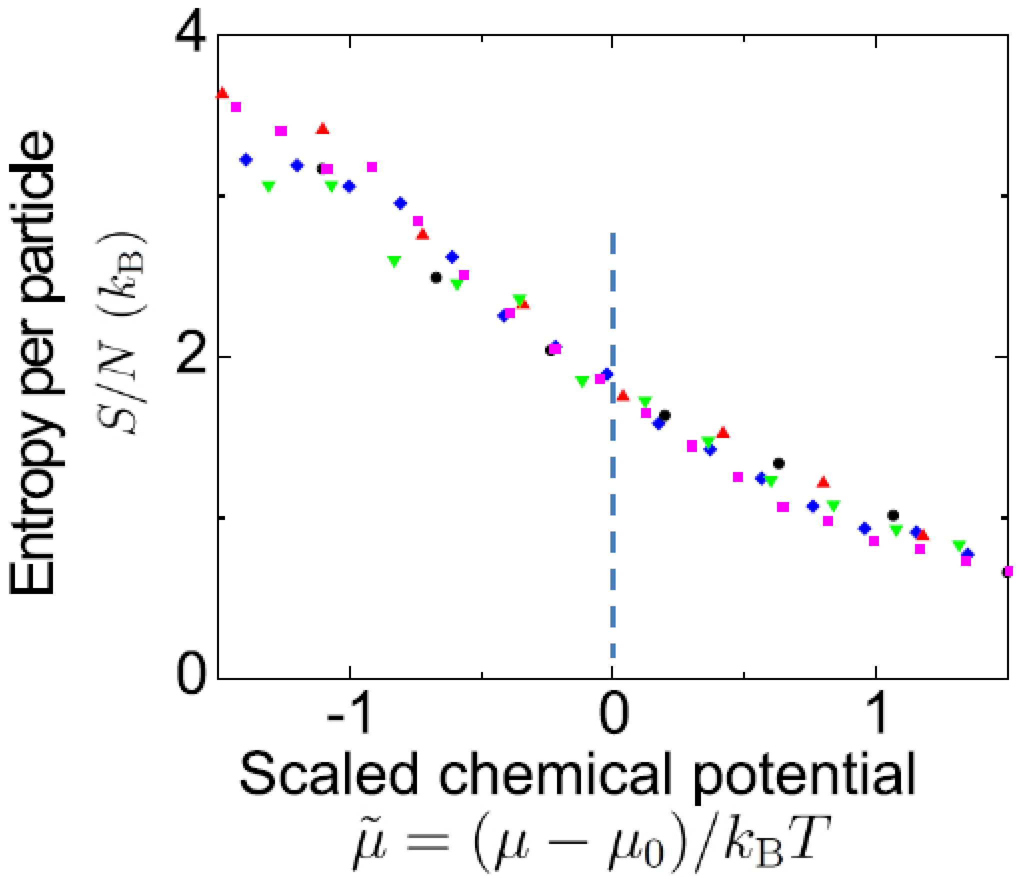
5/5: Entropy per particle in critical regime



$$S/Nk_B = a - b\tilde{\mu}$$

a=1.8(1), b = 1.1(1)

5/5: Entropy per particle in critical regime



$$S/Nk_B = a - b\tilde{\mu}$$

a=1.8(1), b = 1.1(1)

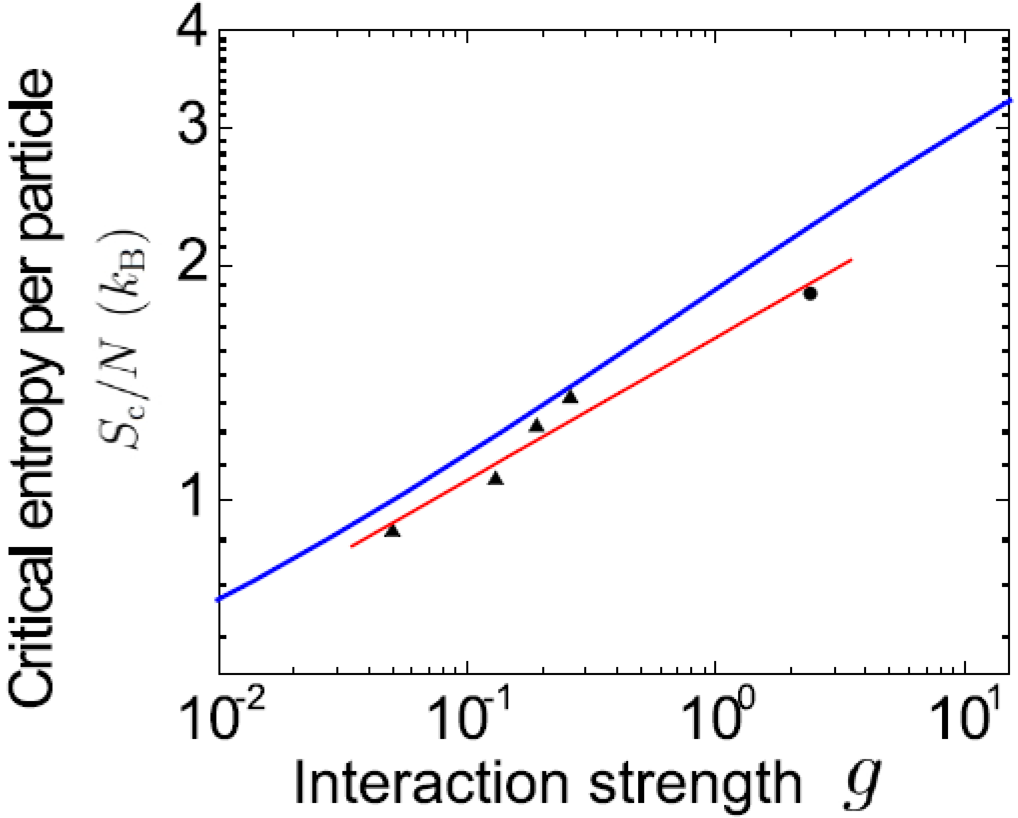
$$P = Cn^x (k_B T)^y$$

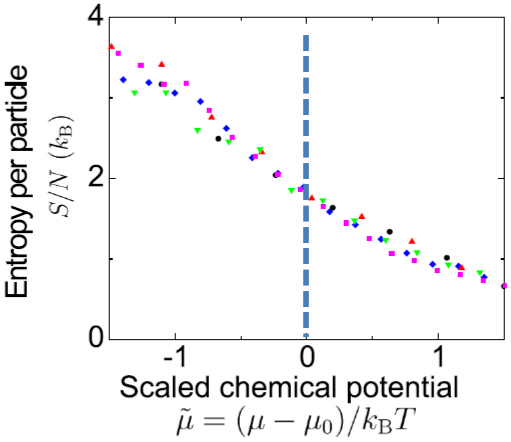
$$x = \frac{2}{1+b} \approx 0.95(5)$$

$$y = \frac{2b}{1+b} \approx 1.05(5)$$

$$C = 0.8(2)(td^2)^{-0.05(5)}$$

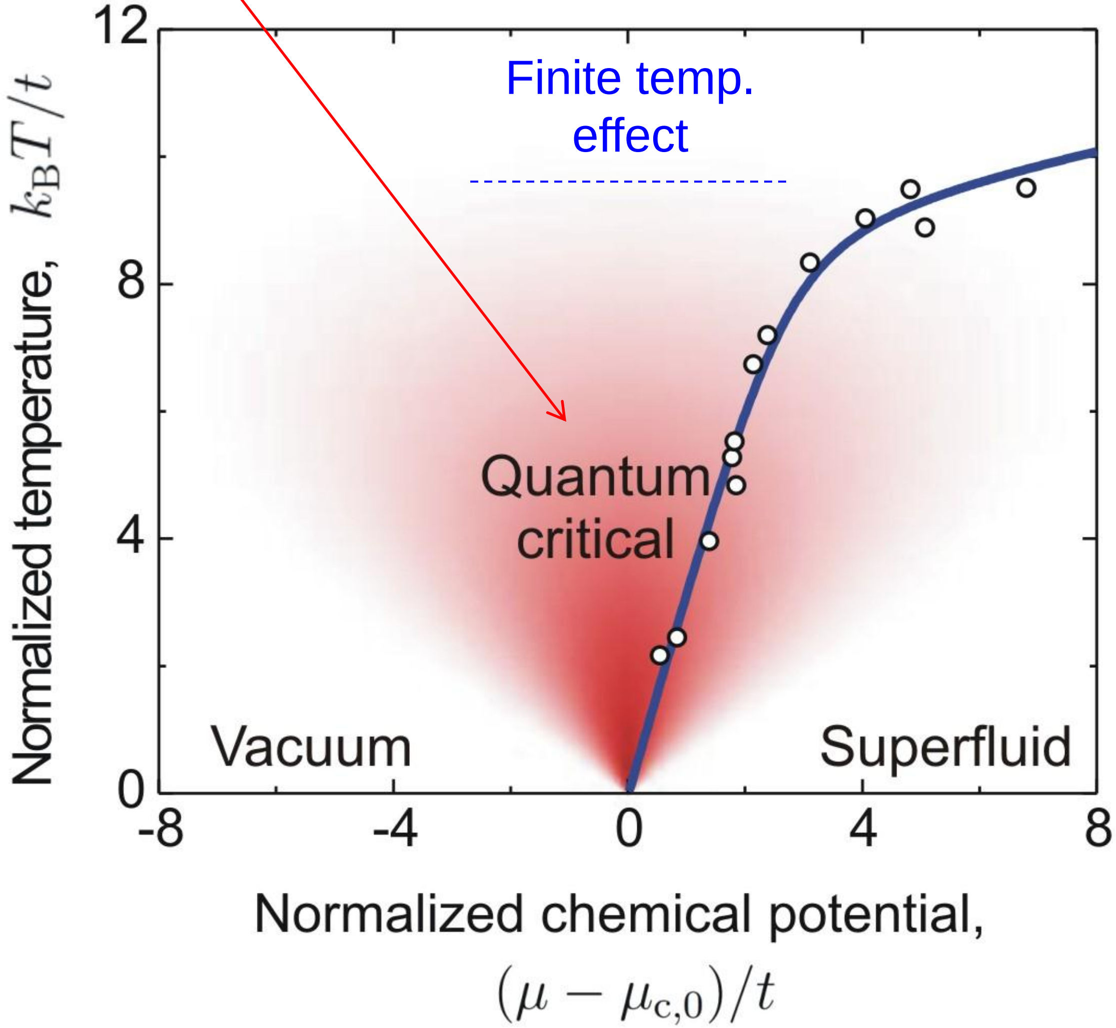
5B/5: Entropy per particle in critical regime





Quantum phase transition in 2D lattice

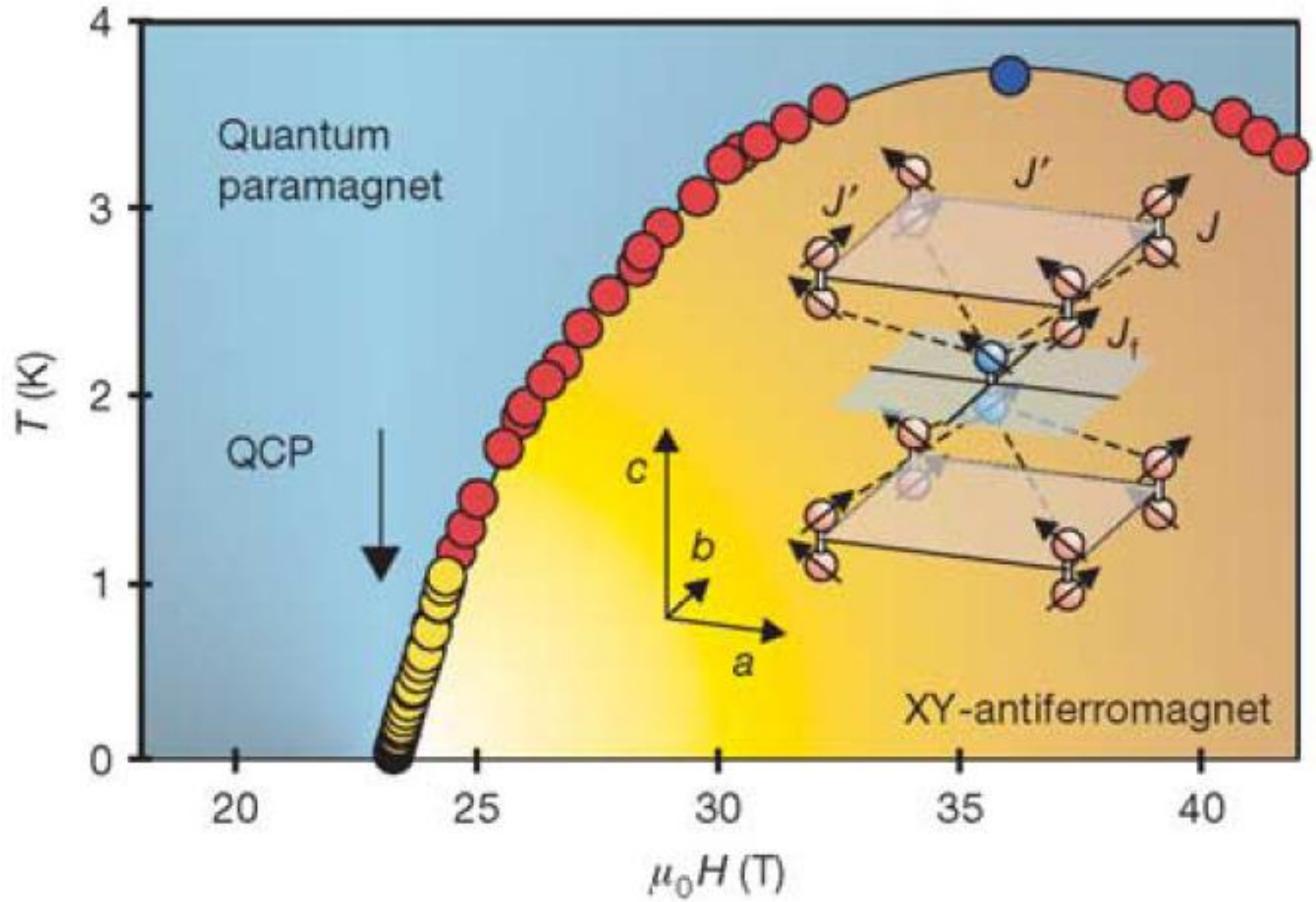
critical scaling law $$\frac{n}{T^{d/z+1-1/\nu z}} = G\left(\frac{\mu-\mu_{c,0}}{T^{1/\nu z}}\right)$$



$$\frac{n-n_c}{T} = F\left(\frac{\mu-\mu_c}{T}\right)$$

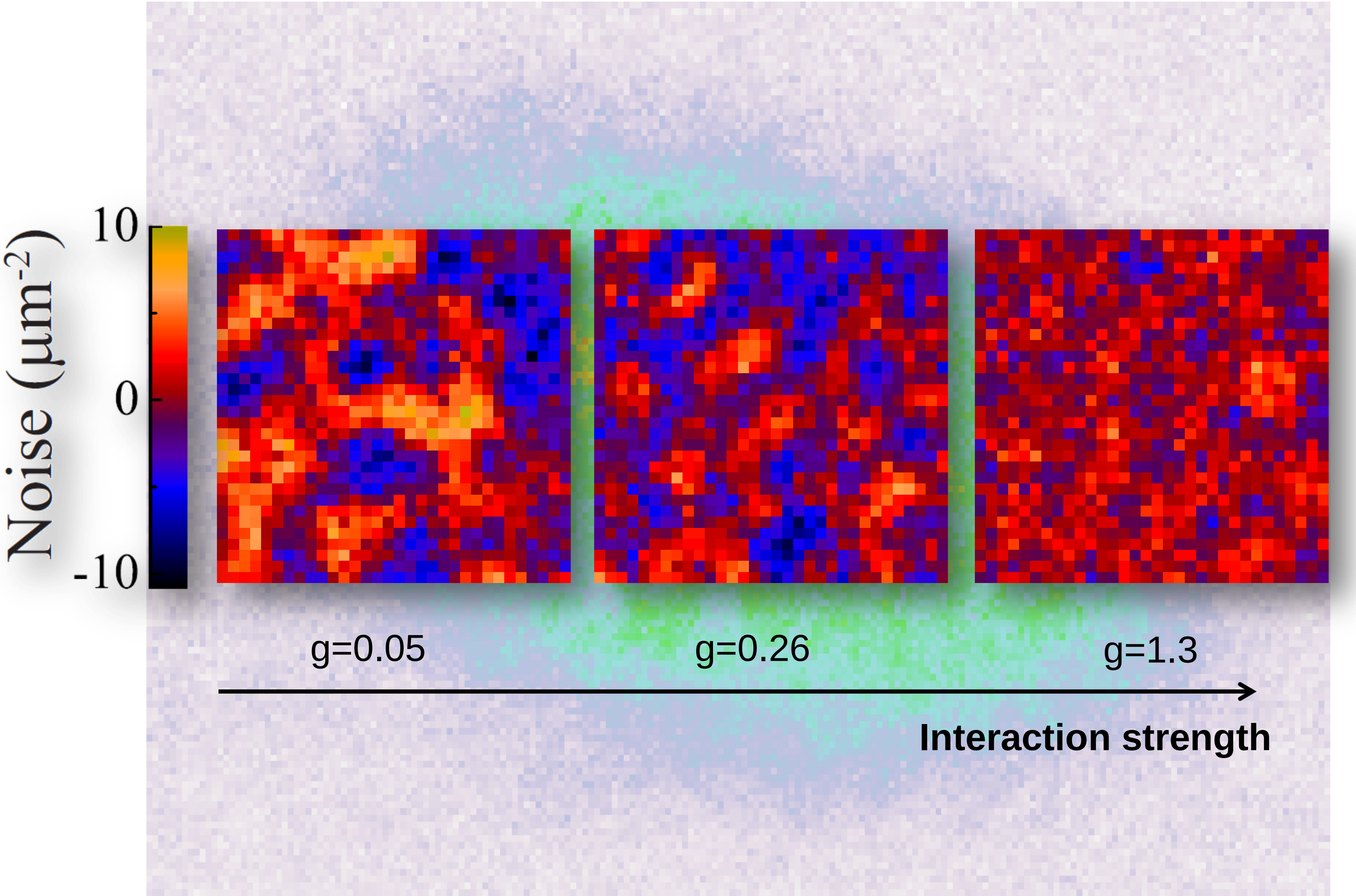
Our paper: X. Zhang, C. Hung, S. Tung and CC, Science (2012)

Quantum Criticality near a 2D BEC of spin-triplets



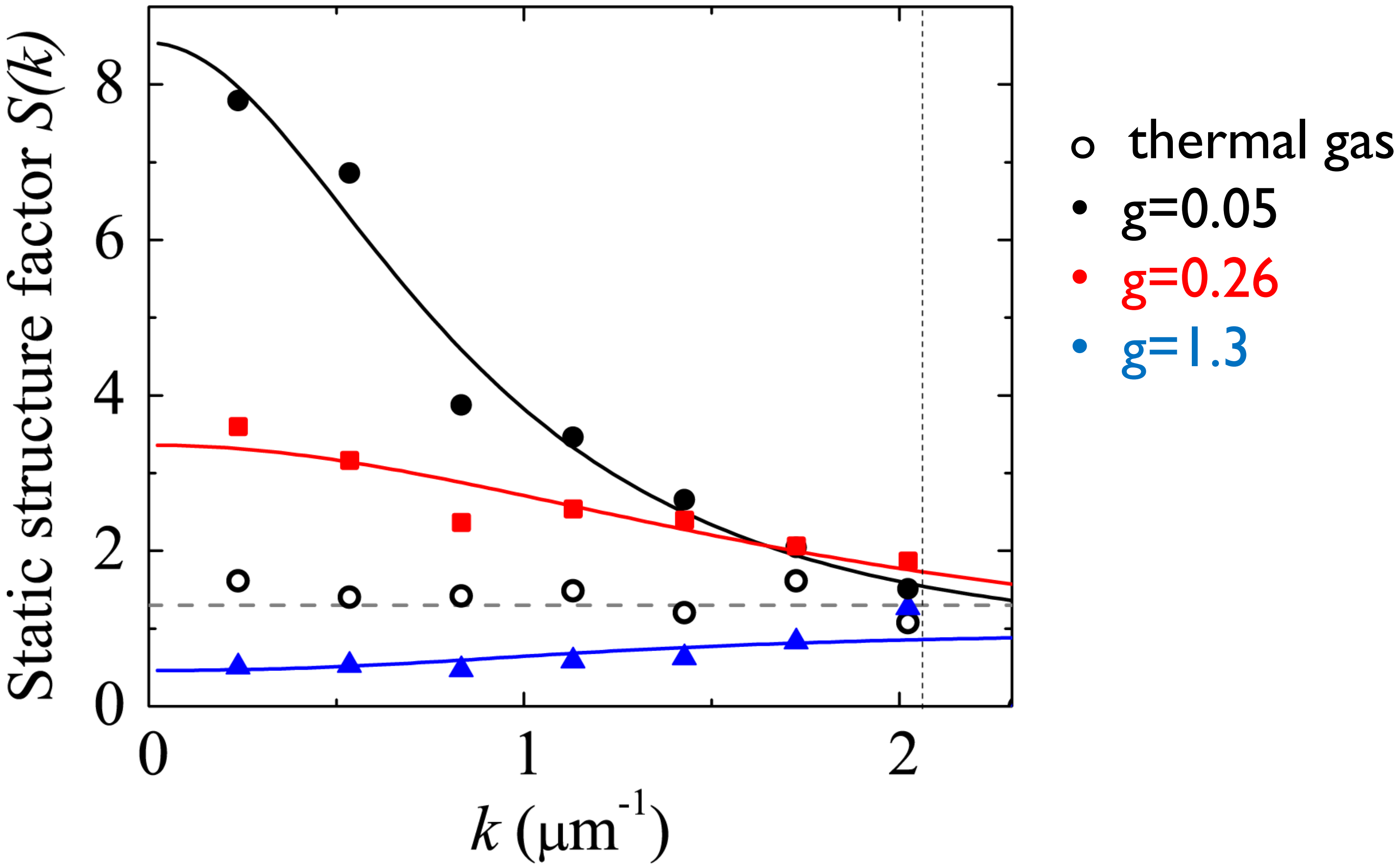
I.R. Fisher group: S.E. Sebastian et al., Nature 441 617 (2006)

$$\delta n = n - \langle n \rangle$$



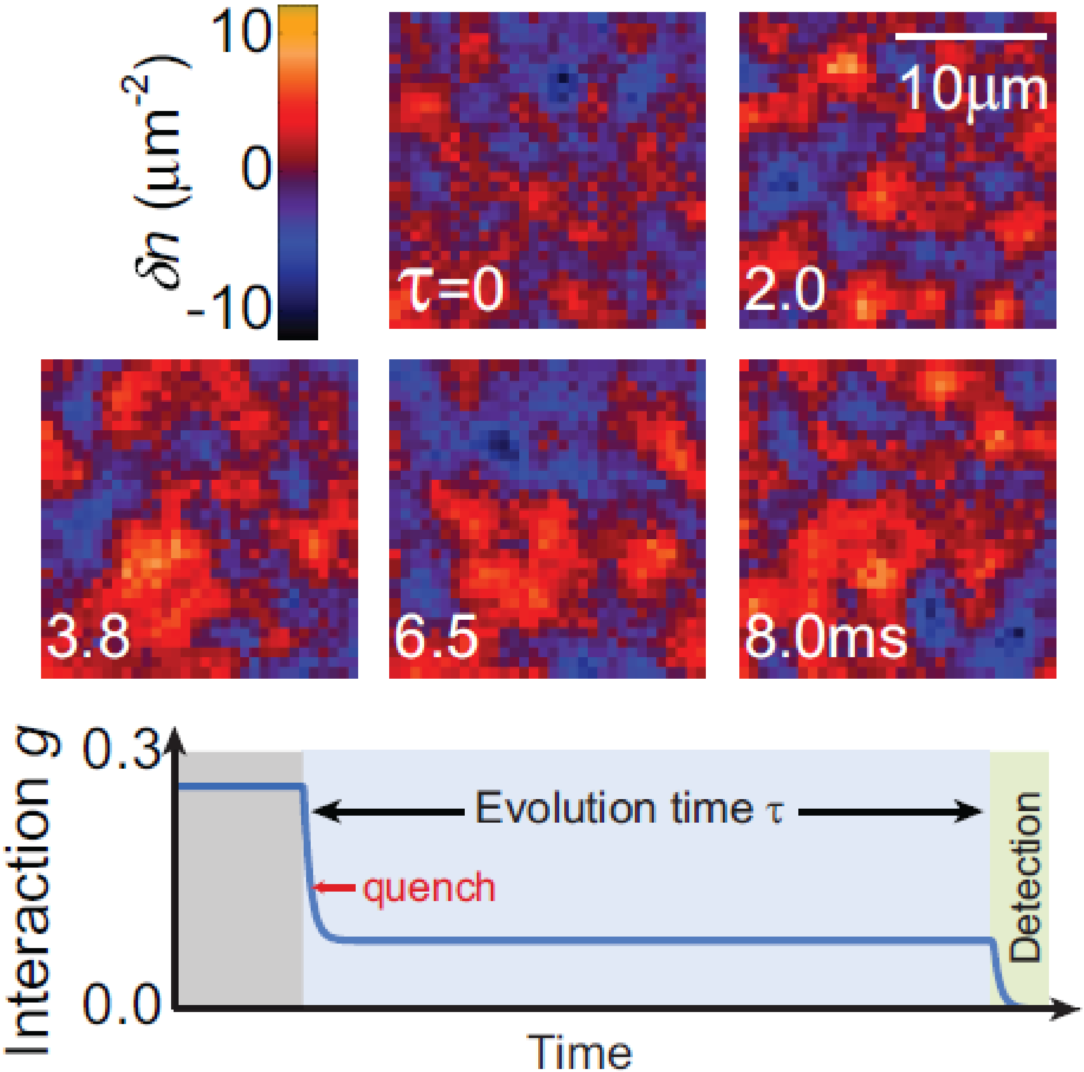
Static structure factor $S(k)=n^{-1}<\delta n(-k)\delta n(k)>$

- power spectrum of the density fluctuations

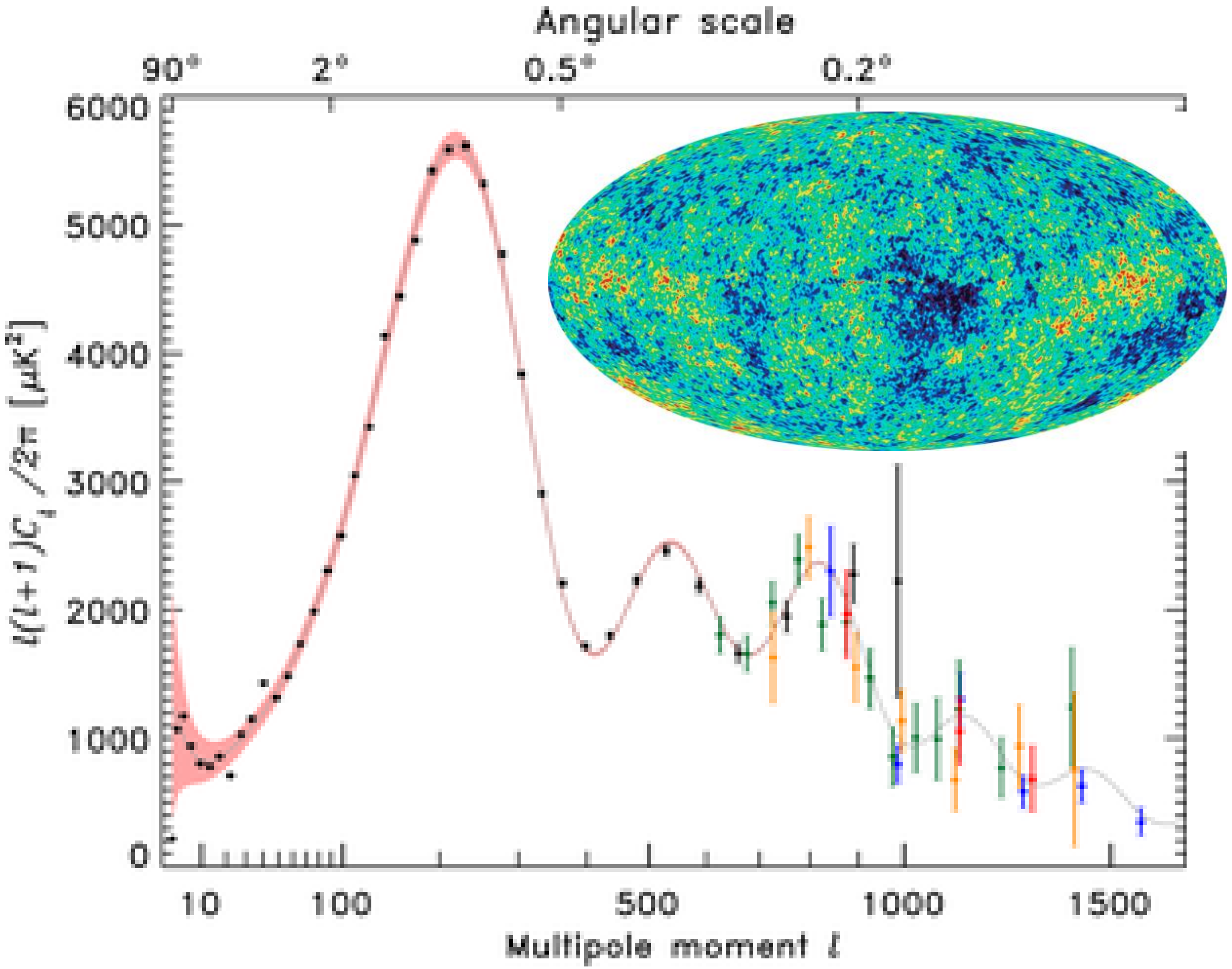


Hung, Zhang, Gemelke, Tung and CC, New Journal of Physics (2011)

Quantum Quench (from g=0.05 to 0.26 in 0.1 ms)



Sakharov Oscillations in CMB



Origin of Sakharov oscillations in cosmic microwave background

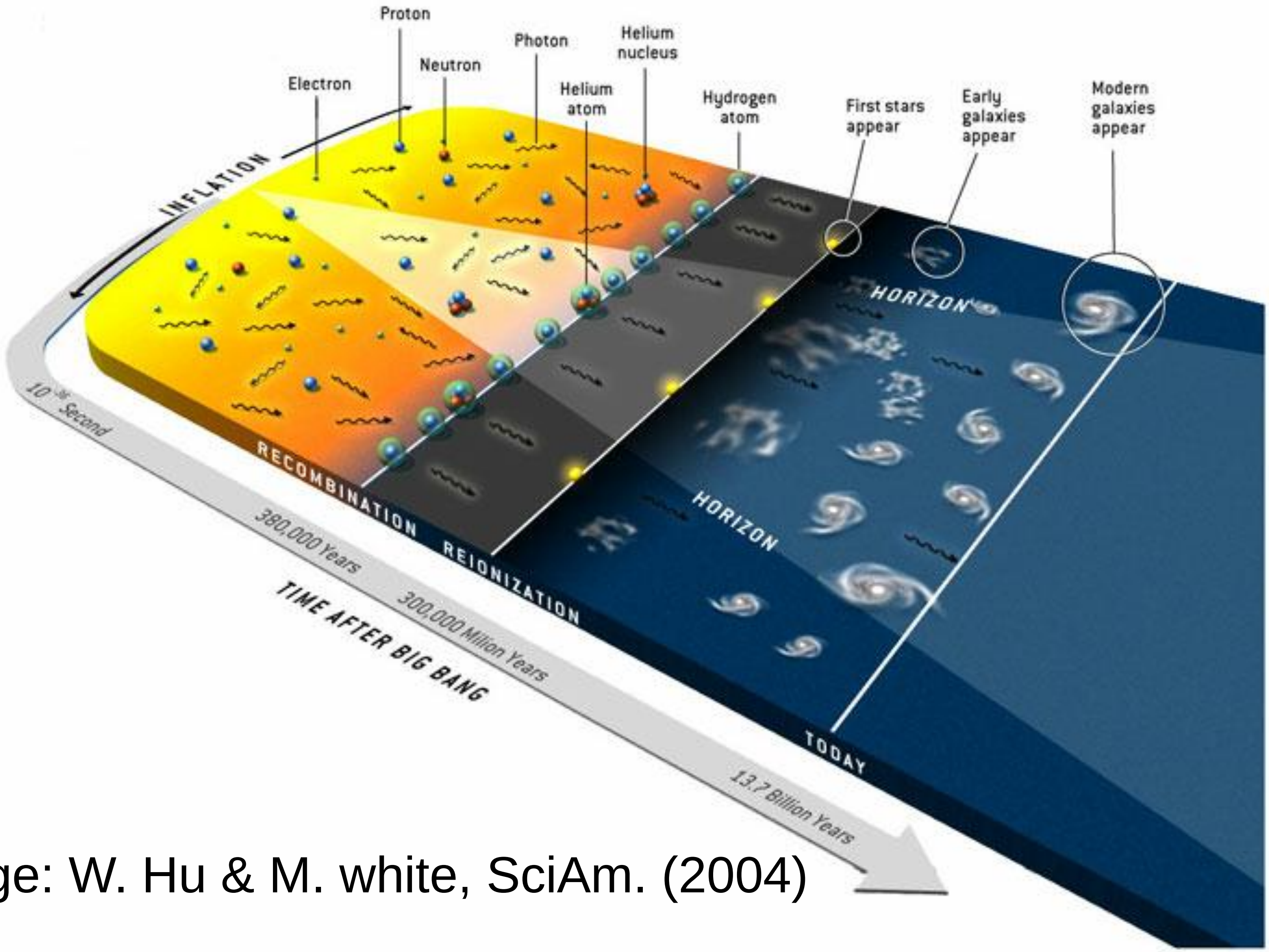


Image: W. Hu & M. white, SciAm. (2004)

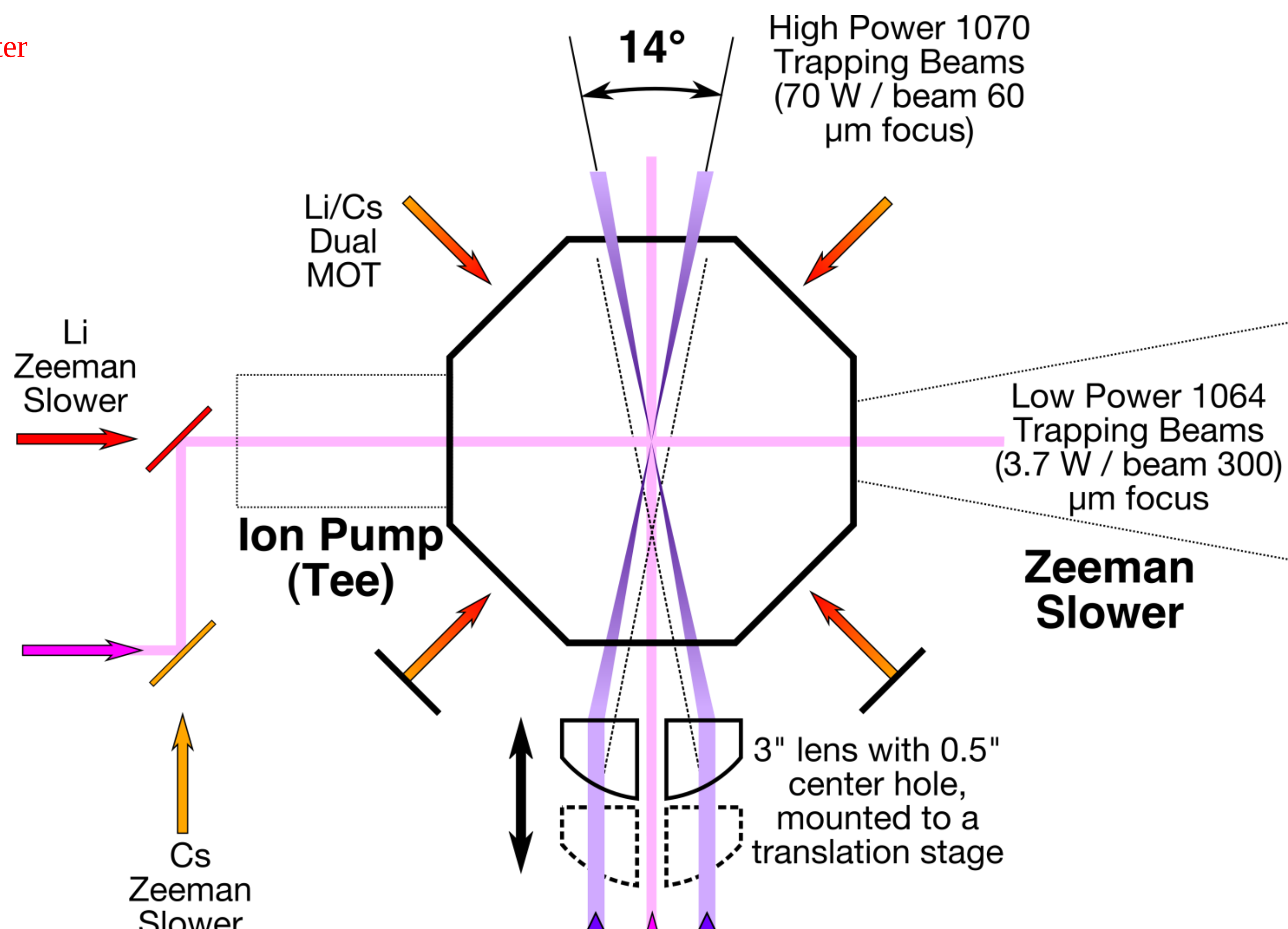
Conclusion and (near) future projects

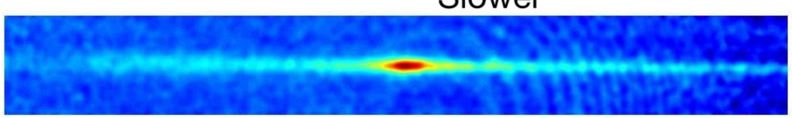
1. Quantum criticality
   – Conformal symmetry of 2D critical gas
   – Quantum critical transport and test of gauge-gravity duality
   – Quantum critical quench?

2. Extension beyond bosons
   – In situ imaging of Fermi gas
   – Heavy Boson (Cs) and light fermion (Li) mixture
   – Scalable quantum information processing based on 2-color lattices

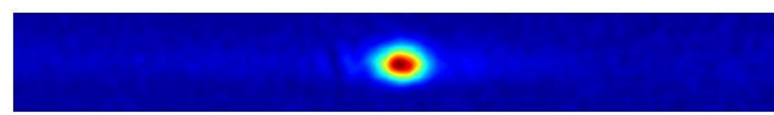
Li - Cs mixture experiment Progress

please see our poster



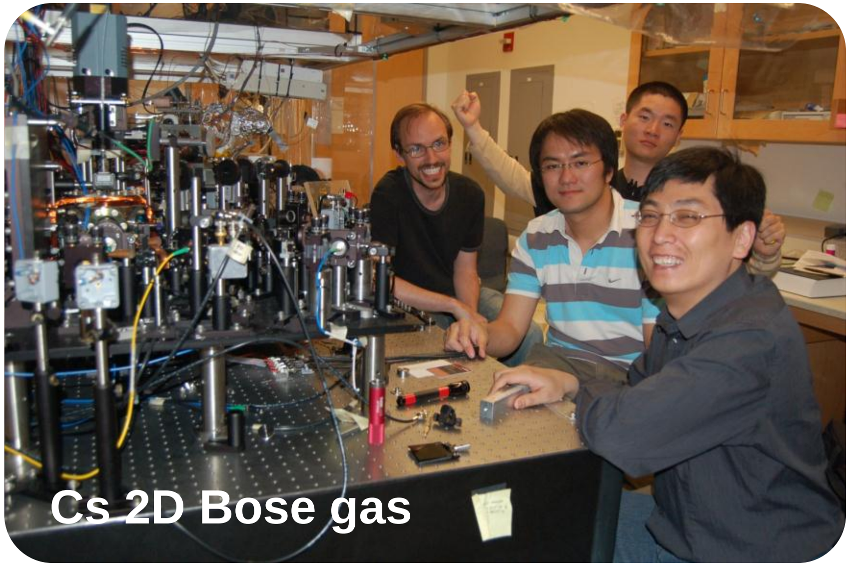


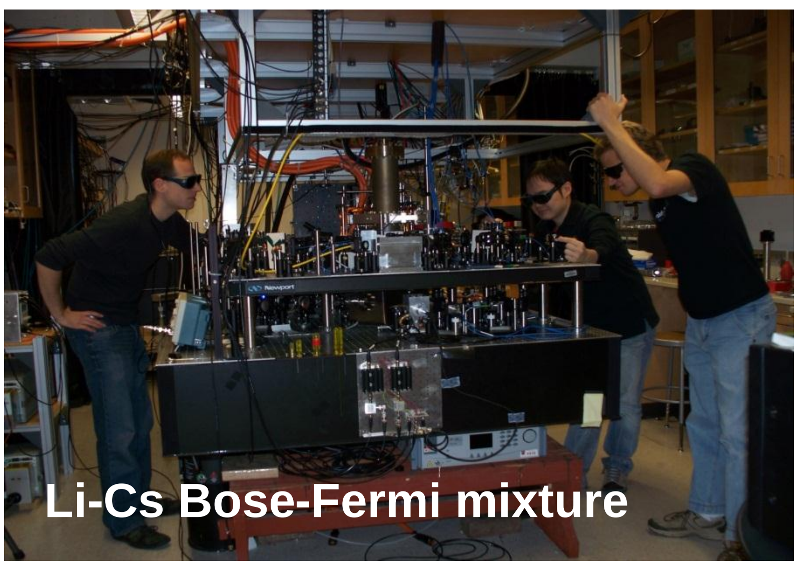
N = 106 6Li atoms in the high-power crossed dipole trap, T=500 µK



N = 106 133Cs atoms in the low-power crossed dipole trap, T=4 µK

Experiments





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