Quantum Integrability-to-Ergodicity Transition as a Hilbert-Schmidt Rotation of Observables

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Preview

Preview

- Proposition:

  The sum of the ensemble variance of the temporal means and the ensemble mean of the temporal variances remains approximately constant across the integrability-to-ergodicity transition

- Example of a Sinai-type billiard
- Example of long-range interacting hard-core bosons
- Ensemble variance of temporal means as a cos^2 of the Hilbert-Schmidt (HS) angle between the observable and integrals of motion

  HS geometry of density matrices for Quantum Information: 164 arXiv articles
  HS geometry of observables: 0 arXiv, hints in Suzuki's quantum extension of Mazur's theorem (Physica 51 (1971))

- IPR as an angle between the original and perturbed integrals of motion
- An application of the HS geometry: Optimal integrals of motion for GGE
- Empirical manifestations: fluctuations in the Newton Cradle system

Preview

We suggest

$$\tan^2[\Theta_A] \equiv \frac{Var_{MC}[Mean_t[A]]}{Mean_{MC}[Var_t[A]]}$$

as a measure of the position of an observable A on the (Integral of Motion)-(Thermalizable Observable) continuum.

Sinai billiard

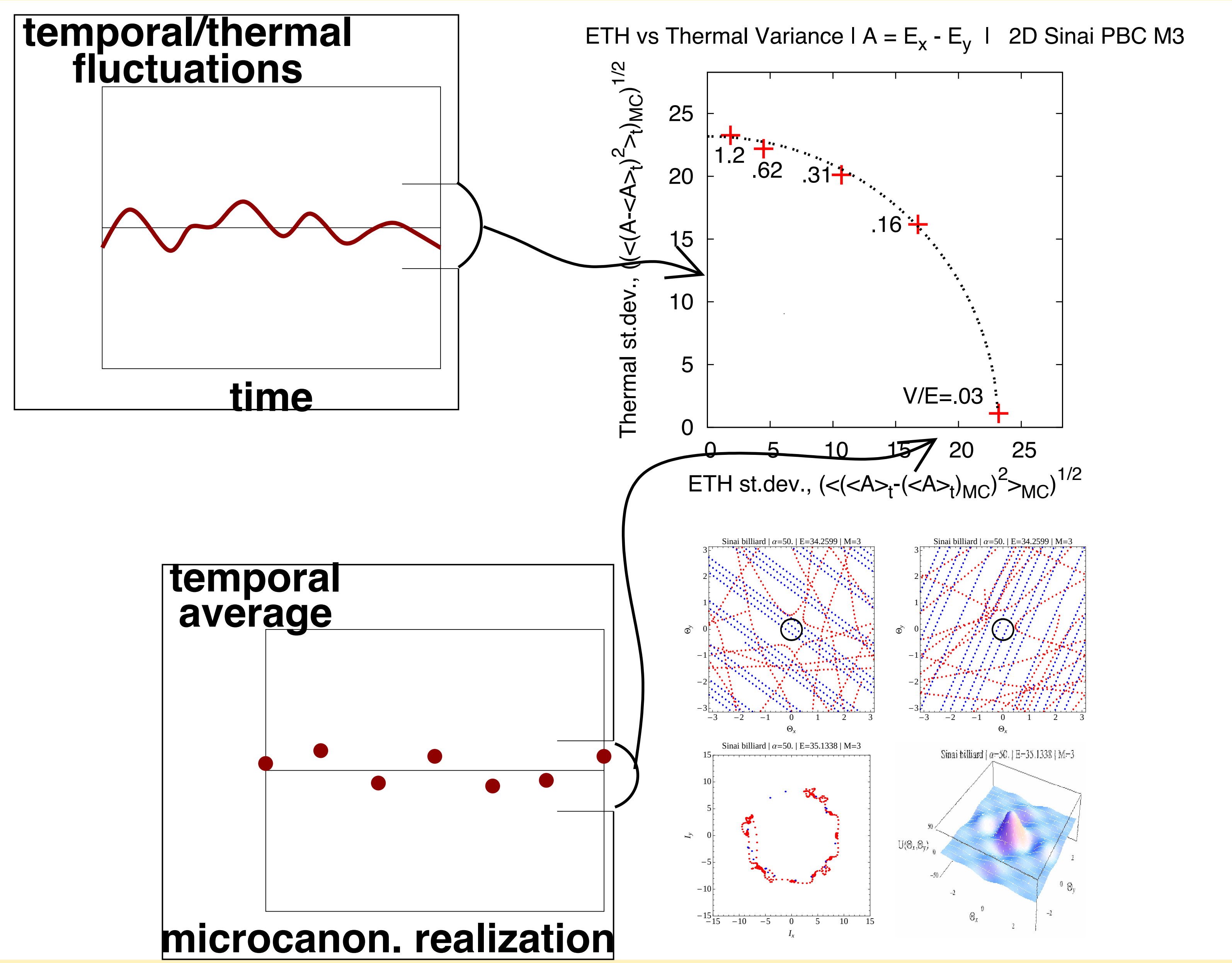
A Sinai-type billiard

Movies:

_R__trajectory01.gif

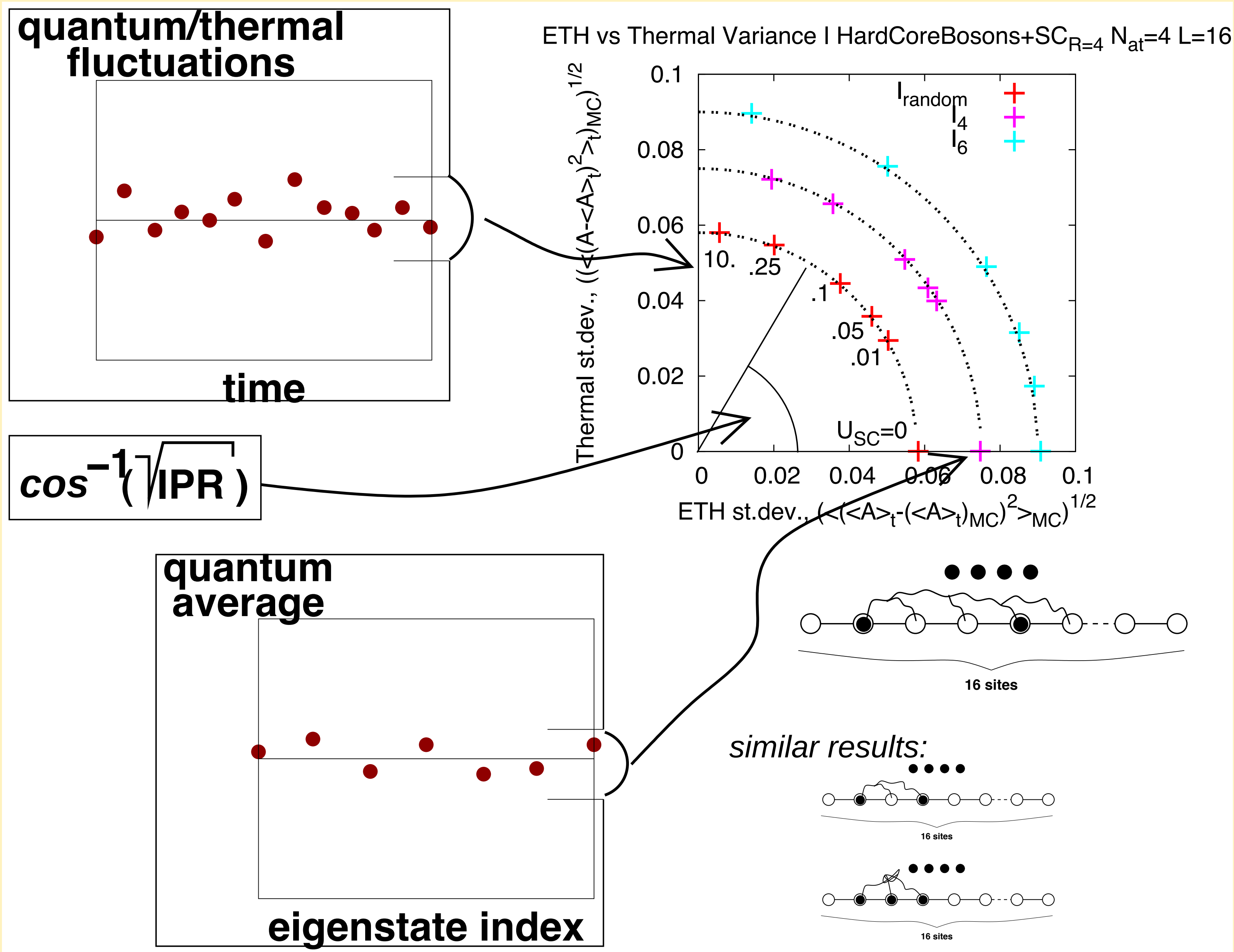
_R__trajectory02.gif

A Sinai-type billiard: $Var_{MC}[Mean_t[A]]$ vs. $Mean_{MC}[Var_t[A]]$



Hard-Core Bosons

HCB+NNNNN: $Var_{MC}[Mean_t[A]]$ vs. $Mean_{MC}[Var_t[A]]$



"[T]he angel of geometry and the devil of algebra share the stage, illustrating the difficulties of both."

Hermann Well

The Hilbert-Schmidt (HS) inner product between two matrices:

$$(\hat{A}|\hat{B}) \equiv Tr[\hat{A}^\dagger \hat{B}] \quad .$$

For Hermitian matrices, HS product is invariant under unitary transformations:

$$(\hat{A}^\dagger = \hat{A})\&(\hat{B}^\dagger = \hat{B}) \Rightarrow (\hat{U}\hat{A}\hat{U}^{-1}|\hat{U}\hat{B}\hat{U}^{-1}) = (\hat{A}|\hat{B}) \quad .$$

In this case, the unitary transformations become a (small) subgroup of the group of HS rotations: the latter preserve the HS norm, defined as

$$||\hat{A}|| \equiv \sqrt{Tr[\hat{A}^2]} \quad .$$

Some definitions

Introduce:

(a) a microcanonical window: $\mathcal{W}_{MC} \equiv \{|\alpha\rangle \quad | \quad E_\alpha \in [E_{min},\, E_{max}]\}$, where $|\alpha\rangle$ and E_α are the eigenstates and eigenenergies of a Hamiltonian $\hat{H}$;

(b) a HS normalized identity operator: $\hat{\mathcal{I}} = (N_{MC})^{-1/2} I$;

(c) a space of the diagonal (w.r.t. $\hat{H}$) observables: $\mathcal{L}_{d,\,\hat{H}} \equiv Span[\{|\alpha\rangle\langle\alpha| \quad | \quad |\alpha\rangle \in \mathcal{W}_{MC}\}]$;

(d) a space of the off-diagonal (w.r.t. $\hat{H}$) observables: $\mathcal{L}_{o-d,\,\hat{H}} \equiv Span[\{2^{-1/2}(|\alpha\rangle\langle\beta| + h.c.)\} \cup i2^{-1/2}(|\alpha\rangle\langle\beta| - h.c.)\} \quad | \quad |\alpha\rangle \in \mathcal{W}_{MC}; |\beta\rangle \in \mathcal{W}_{MC}; \quad \beta > \alpha \quad \}]$.

The integrability-ergodicity-to-HS dictionary

Then:

ensemble variance of the temporal means (ETH variance) $\equiv$

$$\left(N_{MC}^{-1}\sum_{\alpha}^{\mathcal{W}_{MC}} A_{\alpha,\,\alpha}^{2} - \langle\hat{A}\rangle_{MC}\right) / \left(\langle\hat{A}^{2}\rangle_{MC} - \langle\hat{A}\rangle_{MC}^{2}\right) = \cos^{2}(\hat{A}\widehat{\ }\mathcal{L}_{d,\,\hat{H}}) - \cos^{2}(\hat{A}\widehat{\ }\hat{\mathcal{I}})$$

ensemble mean of the temporal (quantum) variance $\equiv$

$$N_{MC}^{-1}\left(\sum_{\alpha,\,\beta\neq\alpha}^{\mathcal{W}_{MC}} A_{\alpha,\,\beta}^{2}\right) / \left(\langle\hat{A}^{2}\rangle_{MC} - \langle\hat{A}\rangle_{MC}^{2}\right) = \cos^{2}(\hat{A}\widehat{\ }\mathcal{L}_{o-d,\,\hat{H}})$$

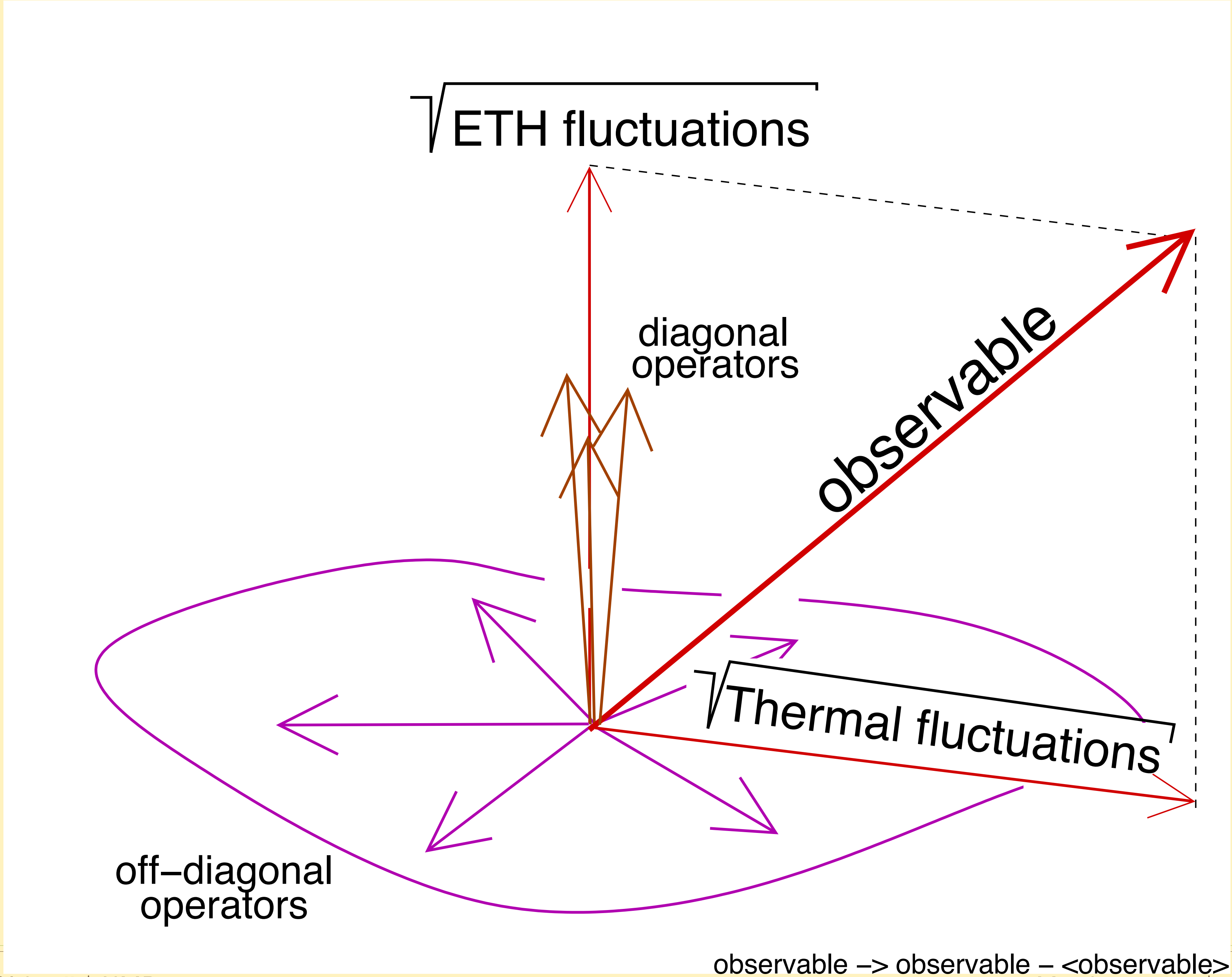
ensemble variance $\equiv$

$$\langle\hat{A}^{2}\rangle_{MC} - \langle\hat{A}\rangle_{MC}^{2} = N_{MC}^{-1}\left(||\hat{A}^{2}|| - N_{MC}^{-1}||\hat{A}||^{2}\right)$$

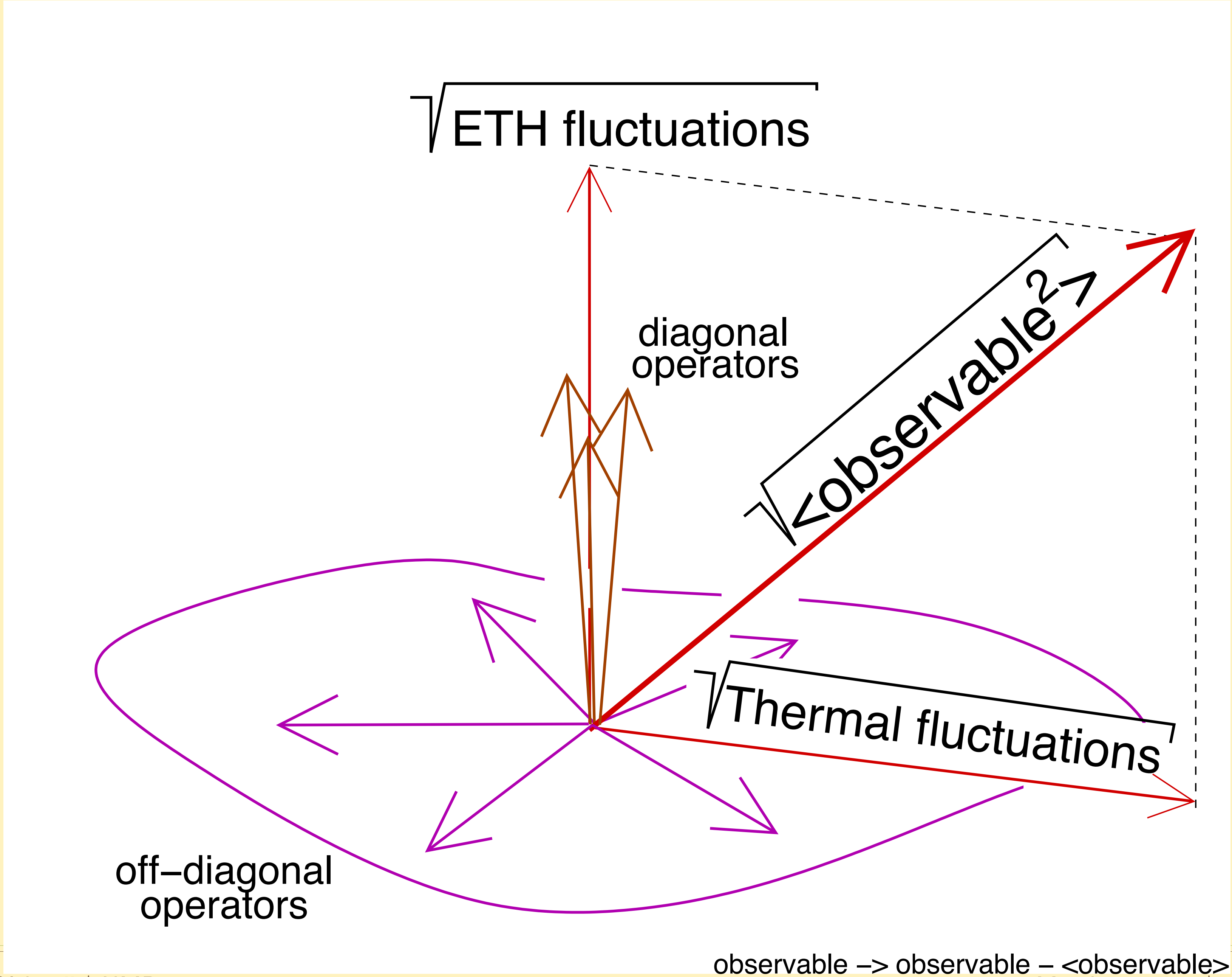
inverse participation ratio (ITR) between the eigenstates of an integrable ($\hat{H}_0$)

$$N_{MC}^{-1}\sum_{\alpha,\,\alpha_o}^{\mathcal{W}_{MC}} |\langle\alpha_0|\alpha\rangle|^{4} = \cos^{2}(\mathcal{L}_{d,\,\hat{H}_0}\widehat{\ }\mathcal{L}_{d,\,\hat{H}})$$

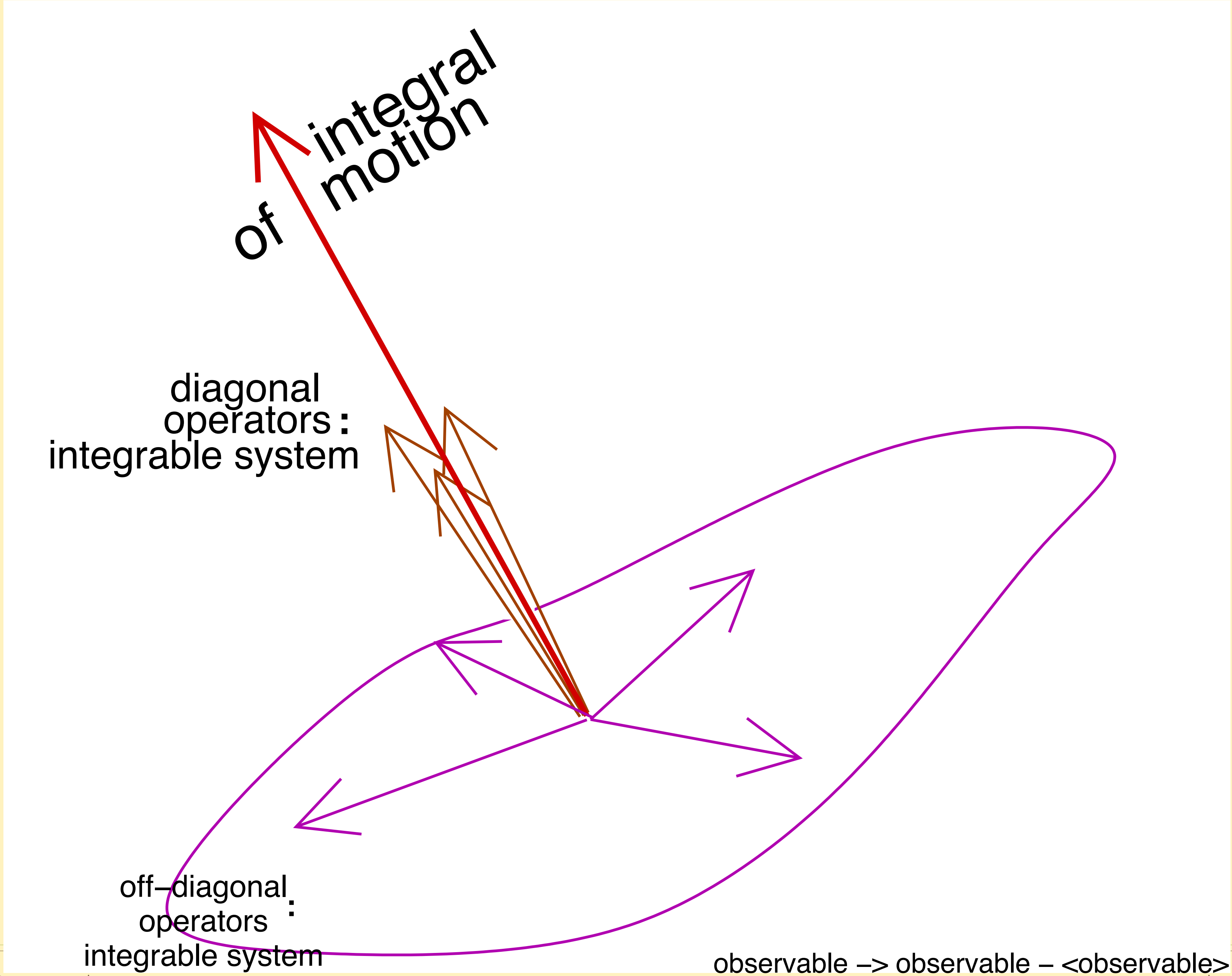
HS Geometry of the Conservation-Thermalization transition



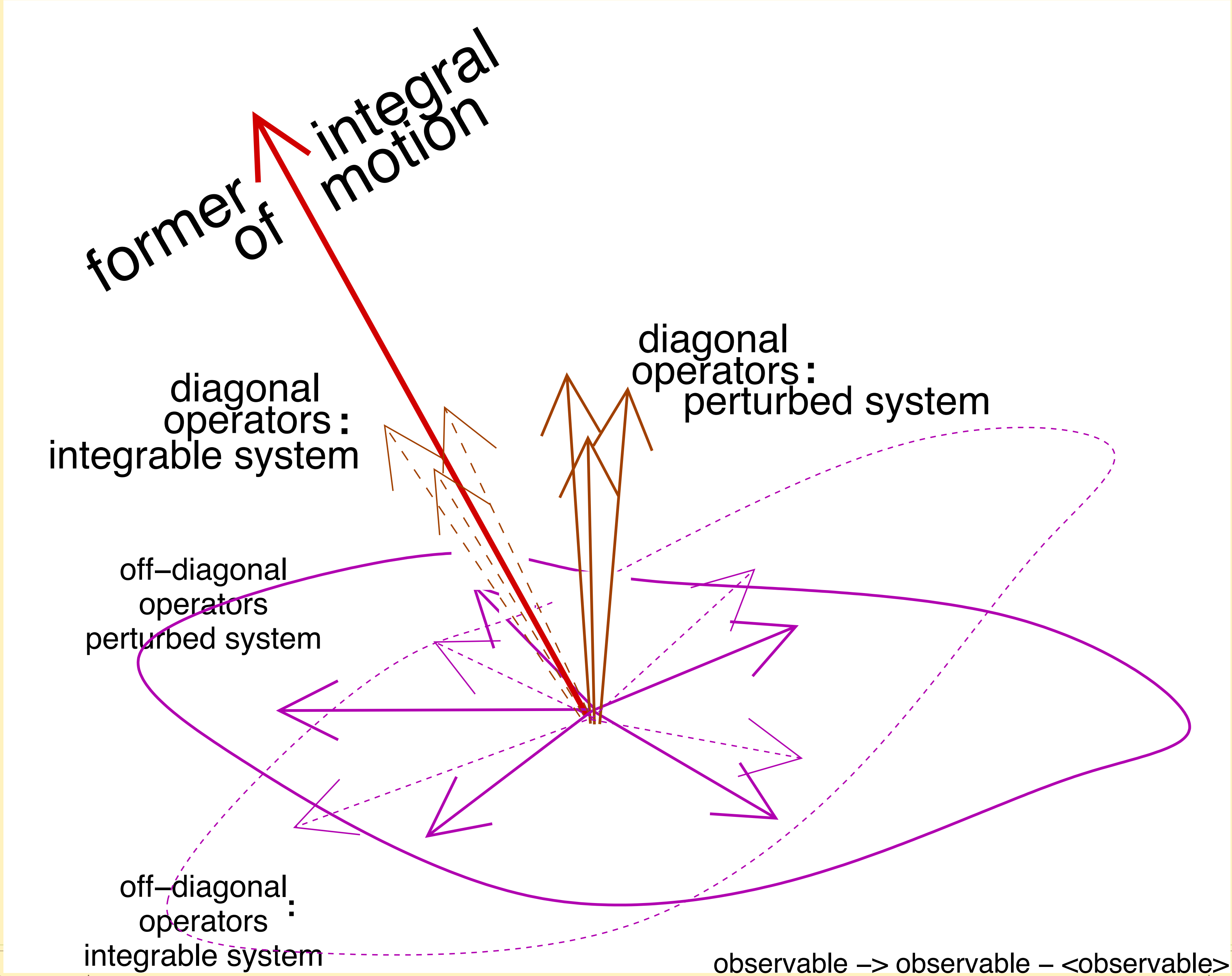
HS Geometry of the Conservation-Thermalization transition



HS Geometry of the Integrability-Ergodicity Transition



HS Geometry of the Integrability-Ergodicity Transition

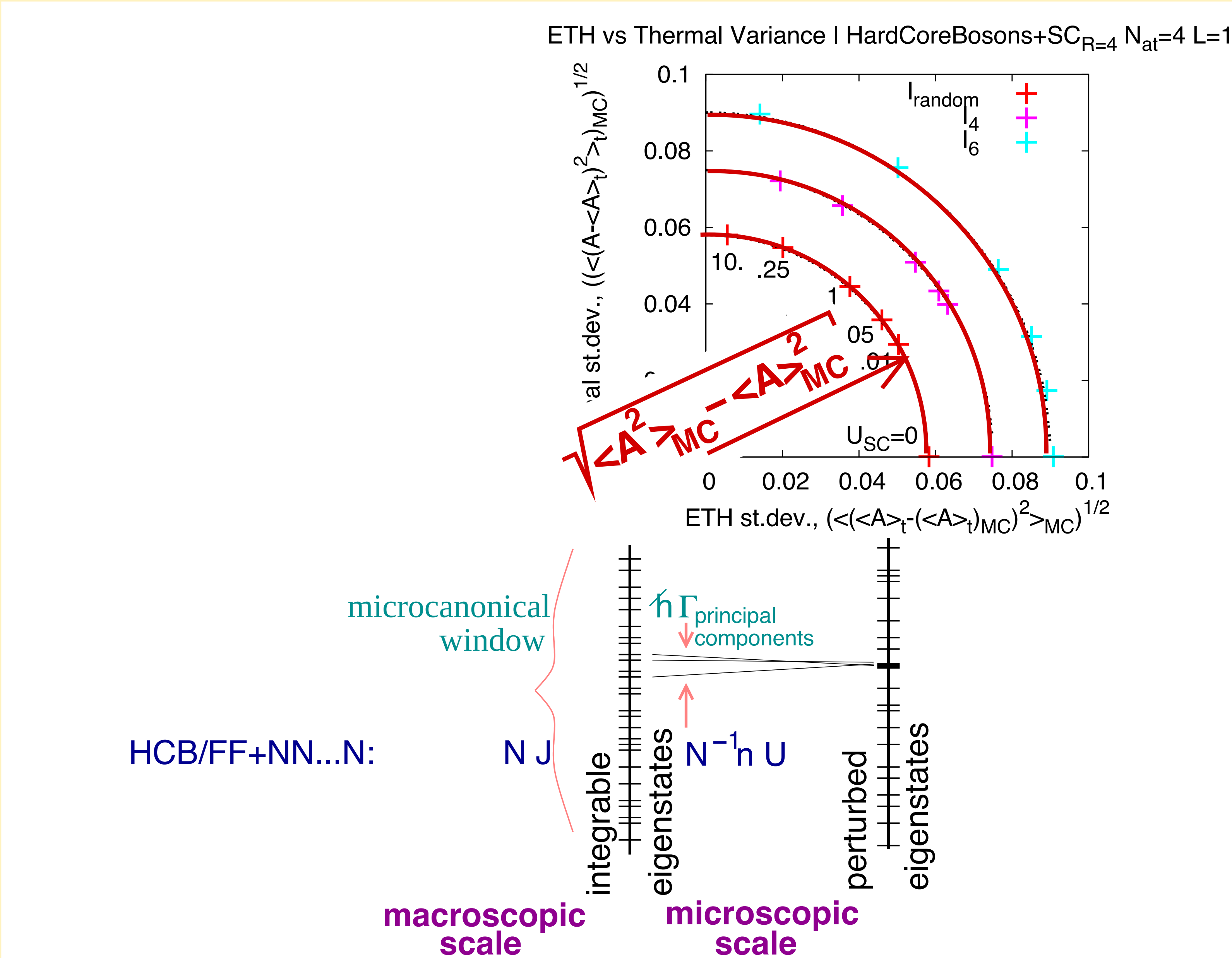


Justification for the HS rotation

An implicit assumption:

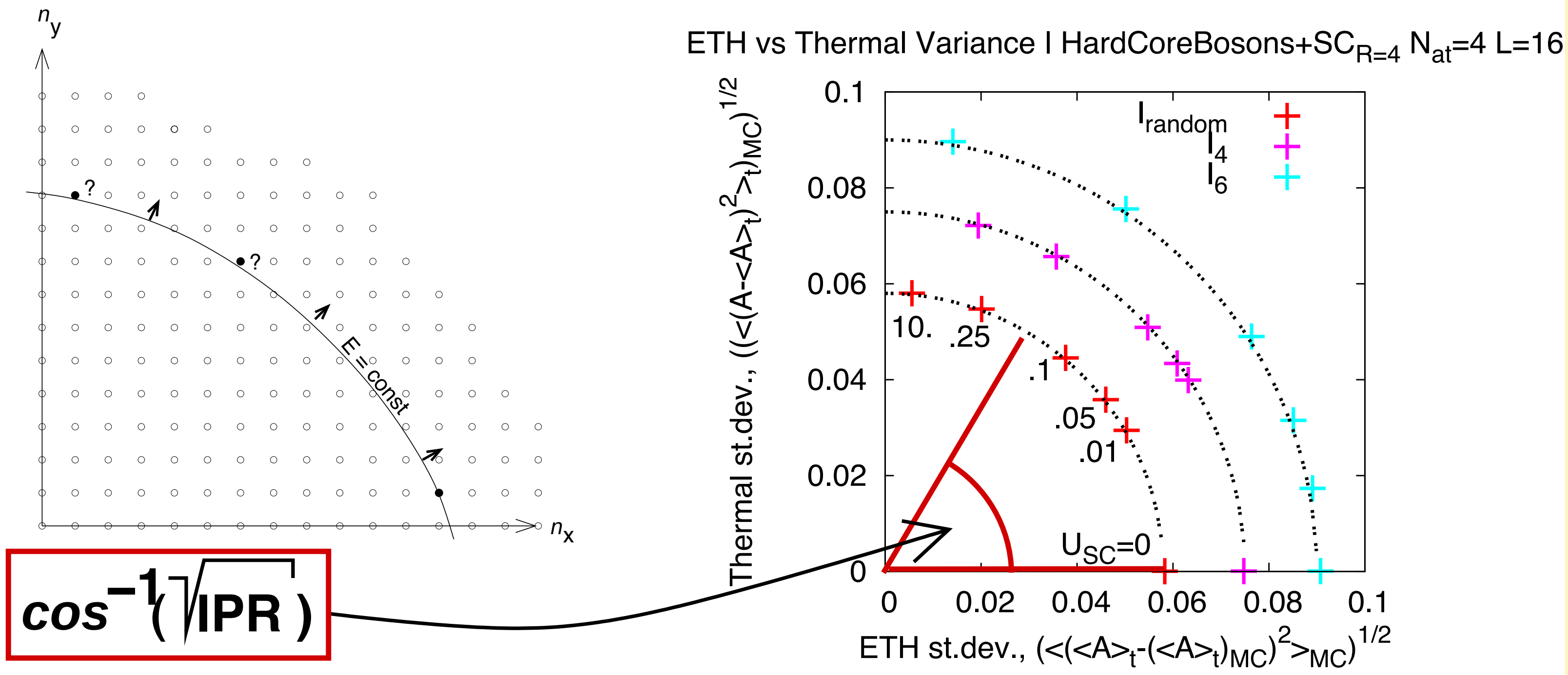
> under a perturbation, the states inside the microcanonical window couple mainly between themselves, not to the states outside the window

Justification for the HS rotation

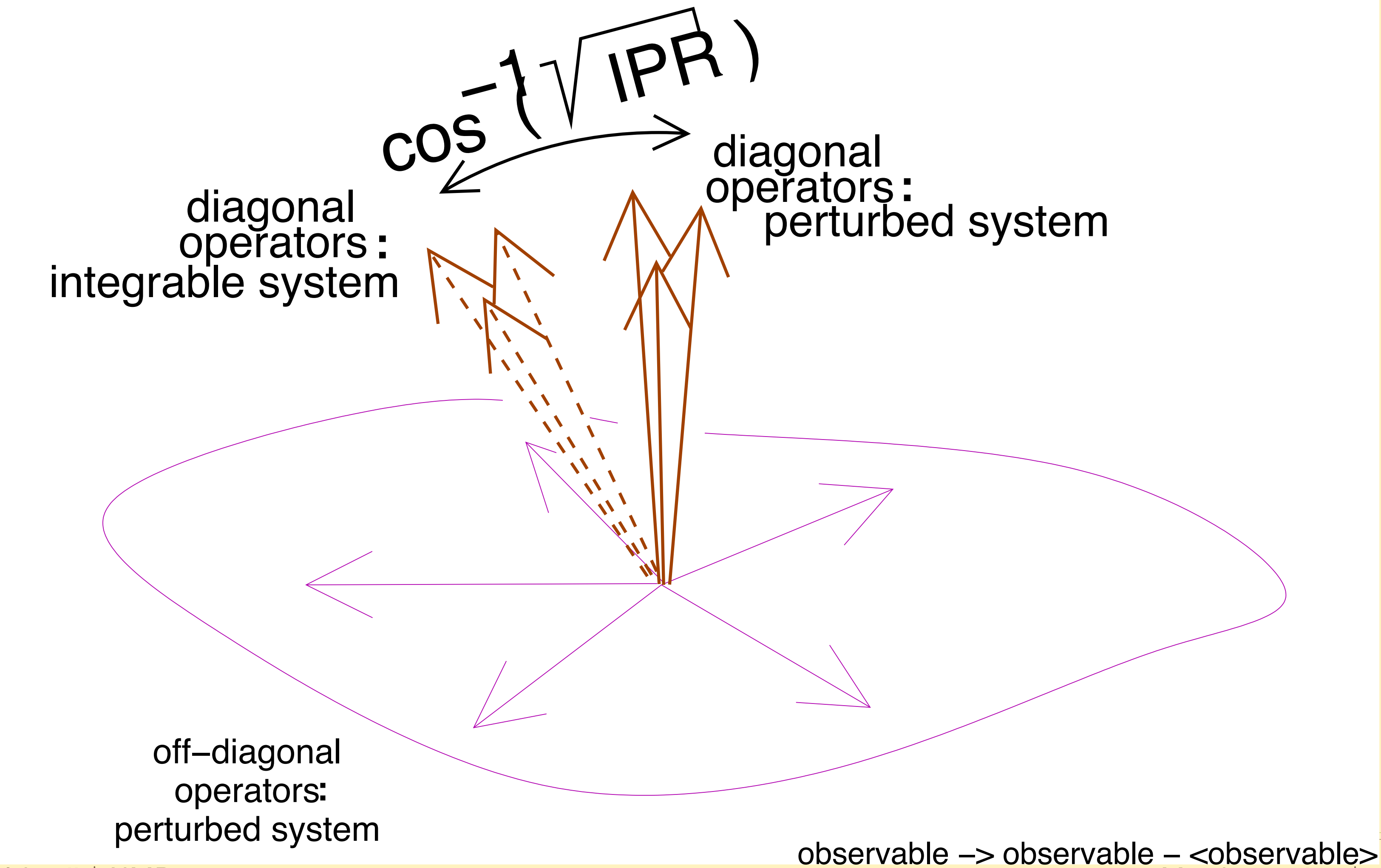


IPR

Universality in Disappearance of Integrals of Motion



Universality in Disappearance of Integrals of Motion



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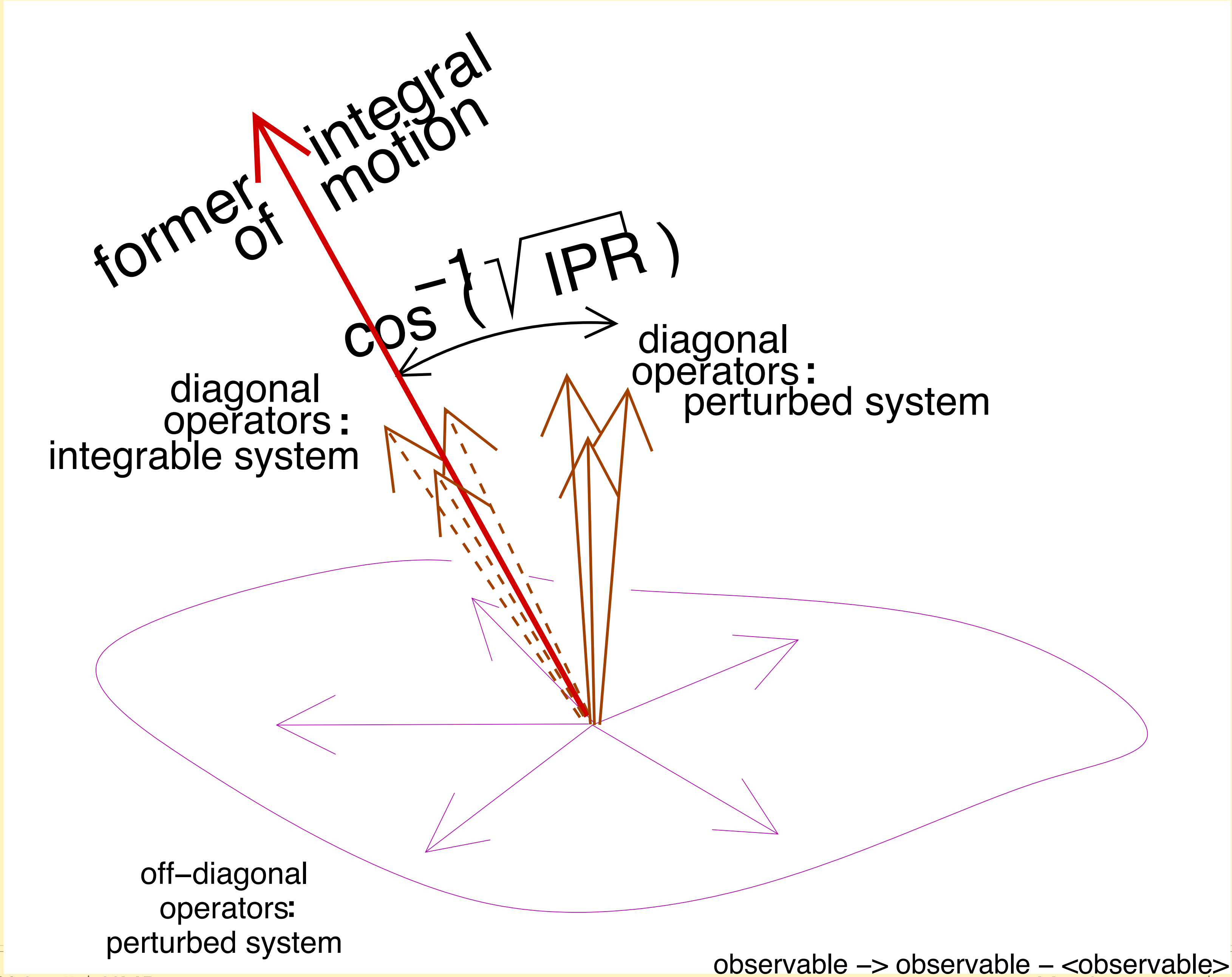

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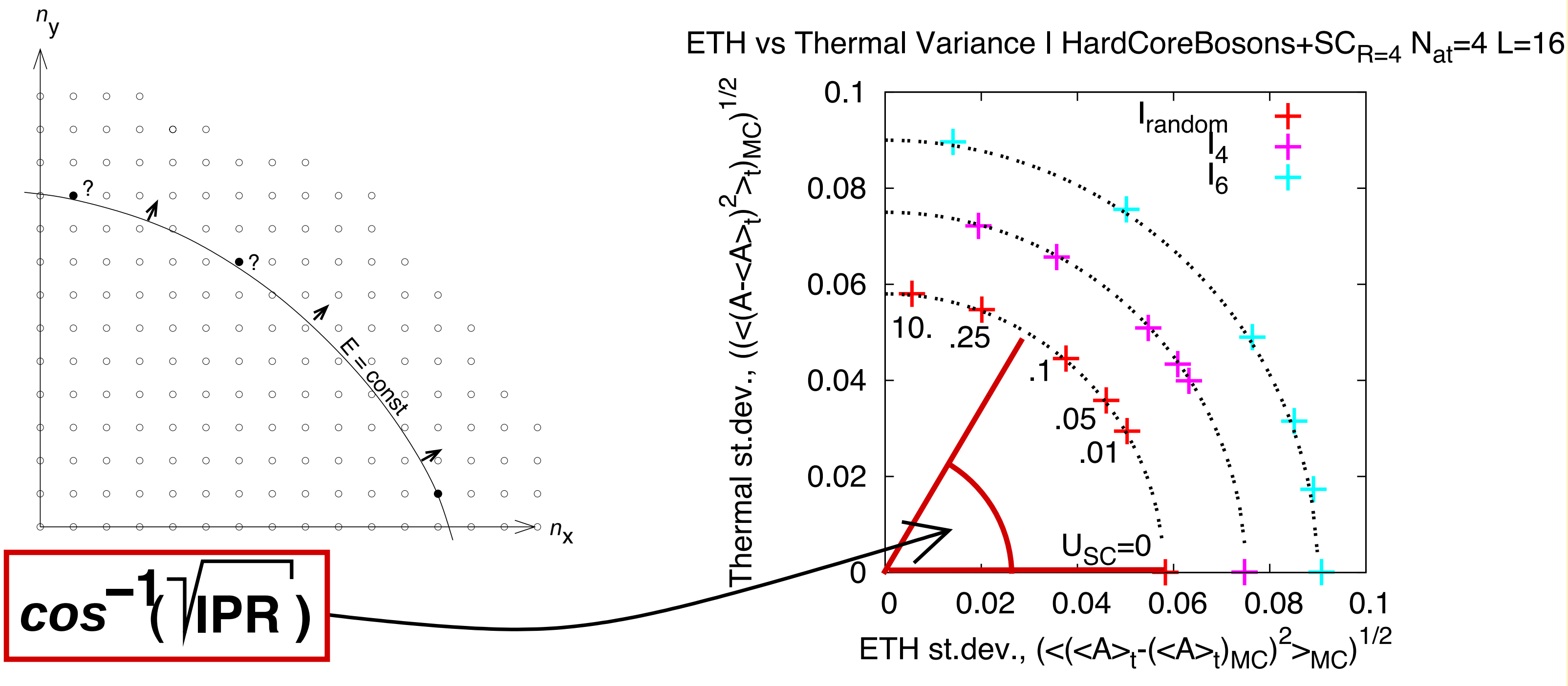

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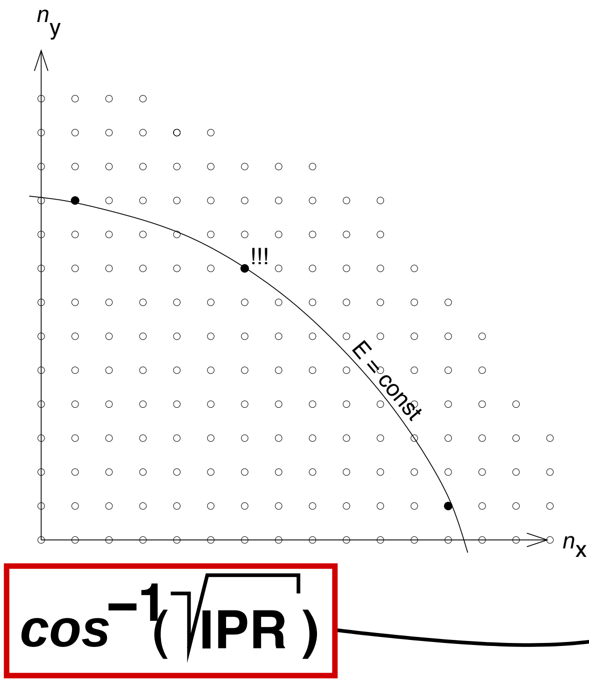

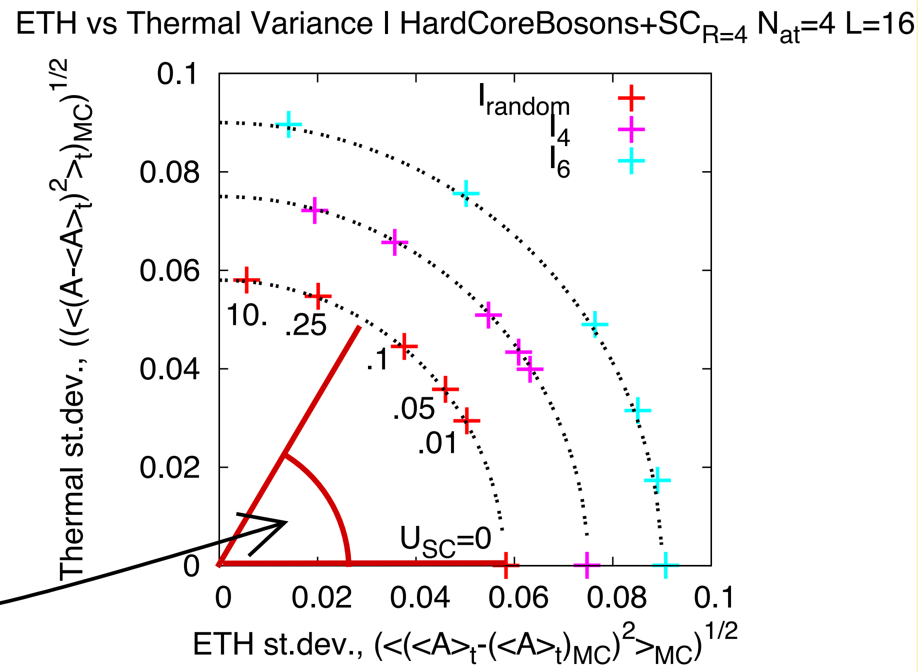

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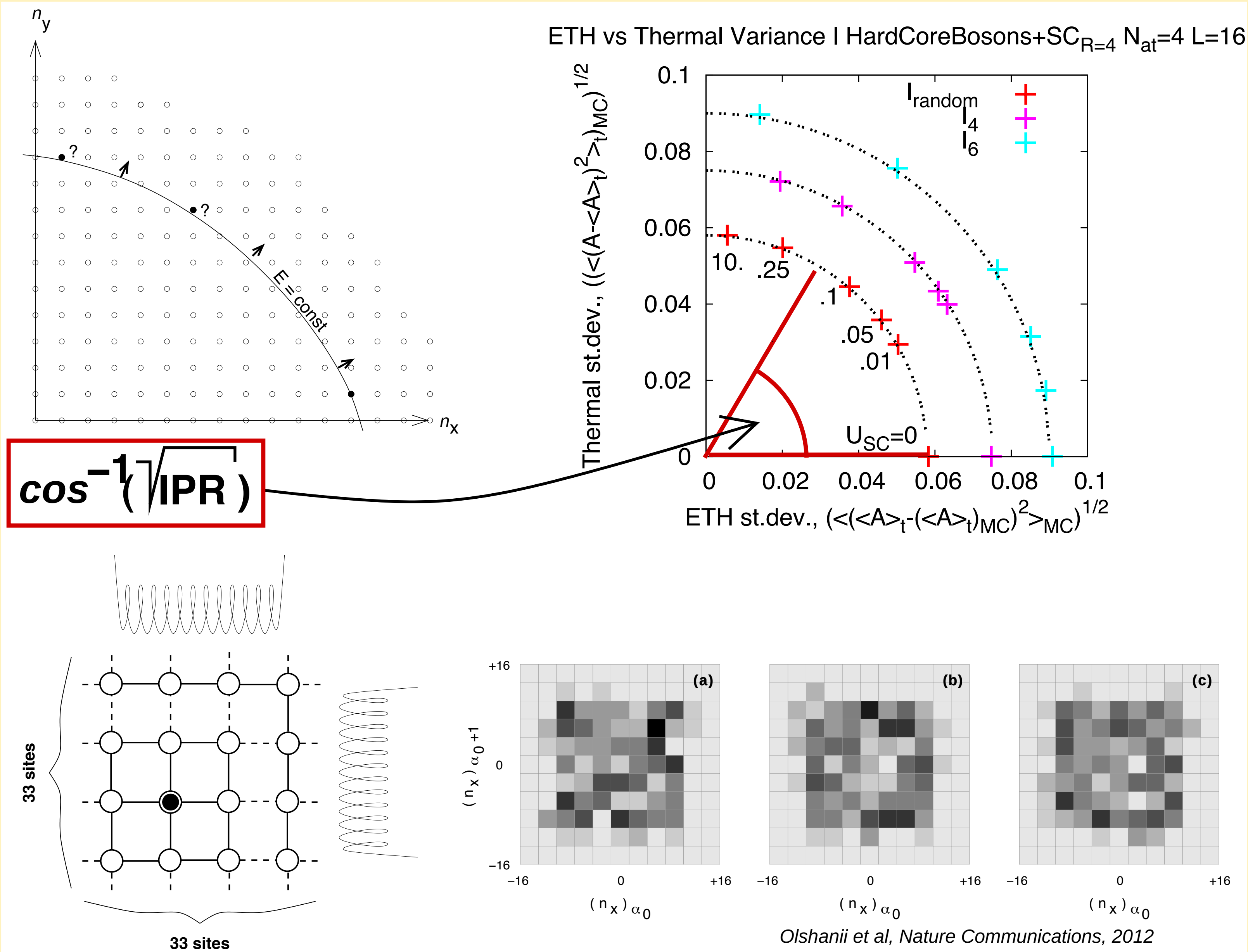
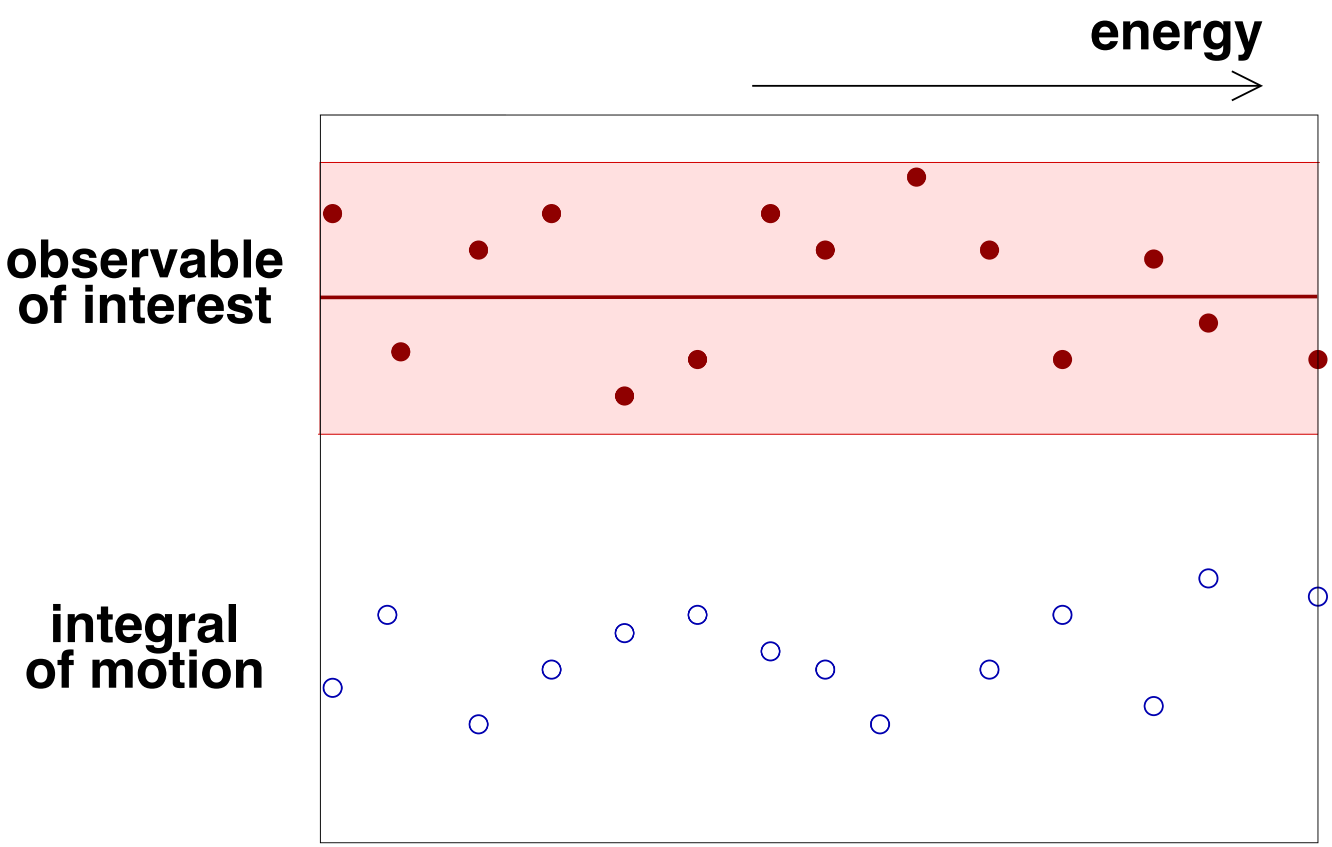


Olshanii et al, Nature Communications, 2012

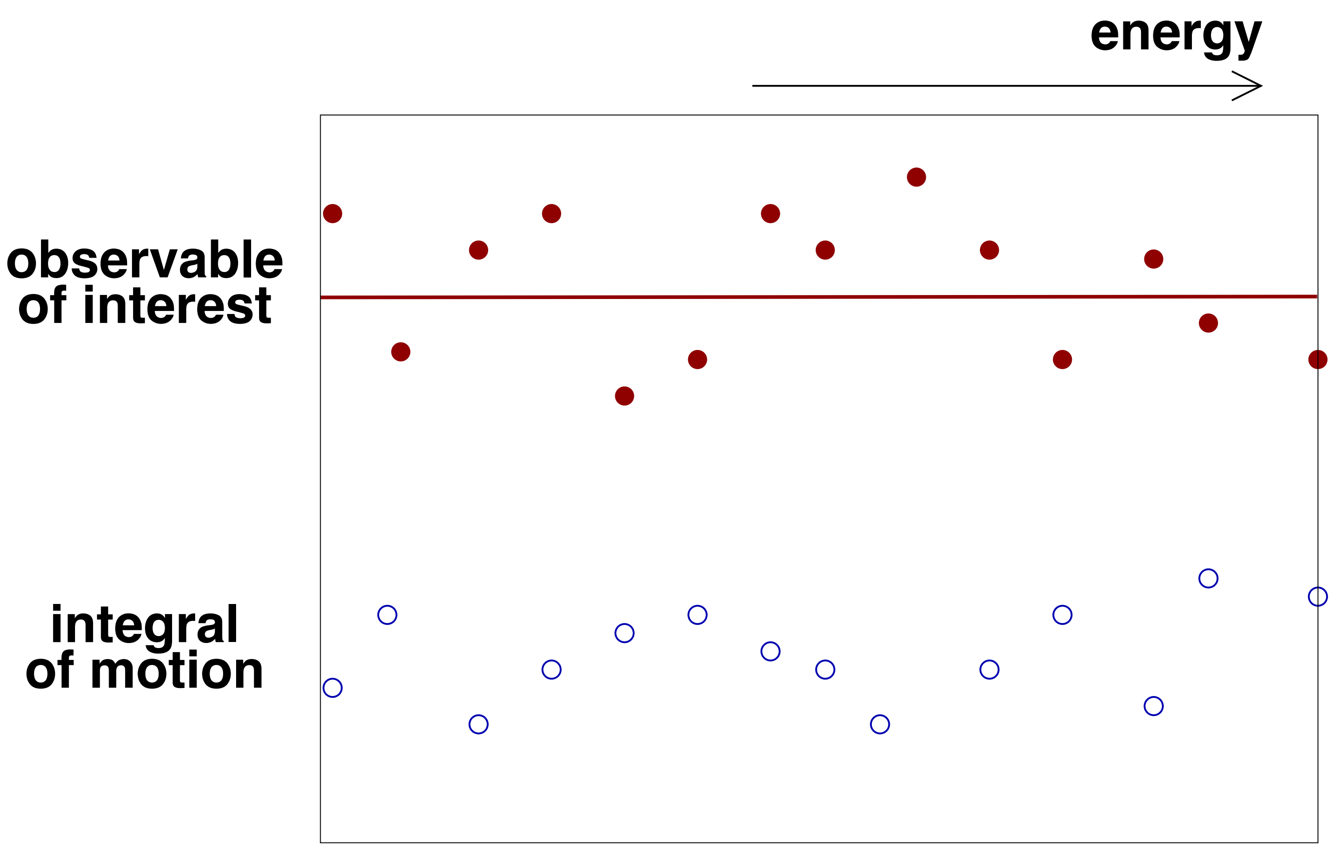
Optimizing the GGE

Optimizing the GGE

Conventional microcanonical vs generalized Gibbs microcanonical ensemble



Conventional microcanonical vs generalized Gibbs microcanonical ensemble



Optimizing the GGE

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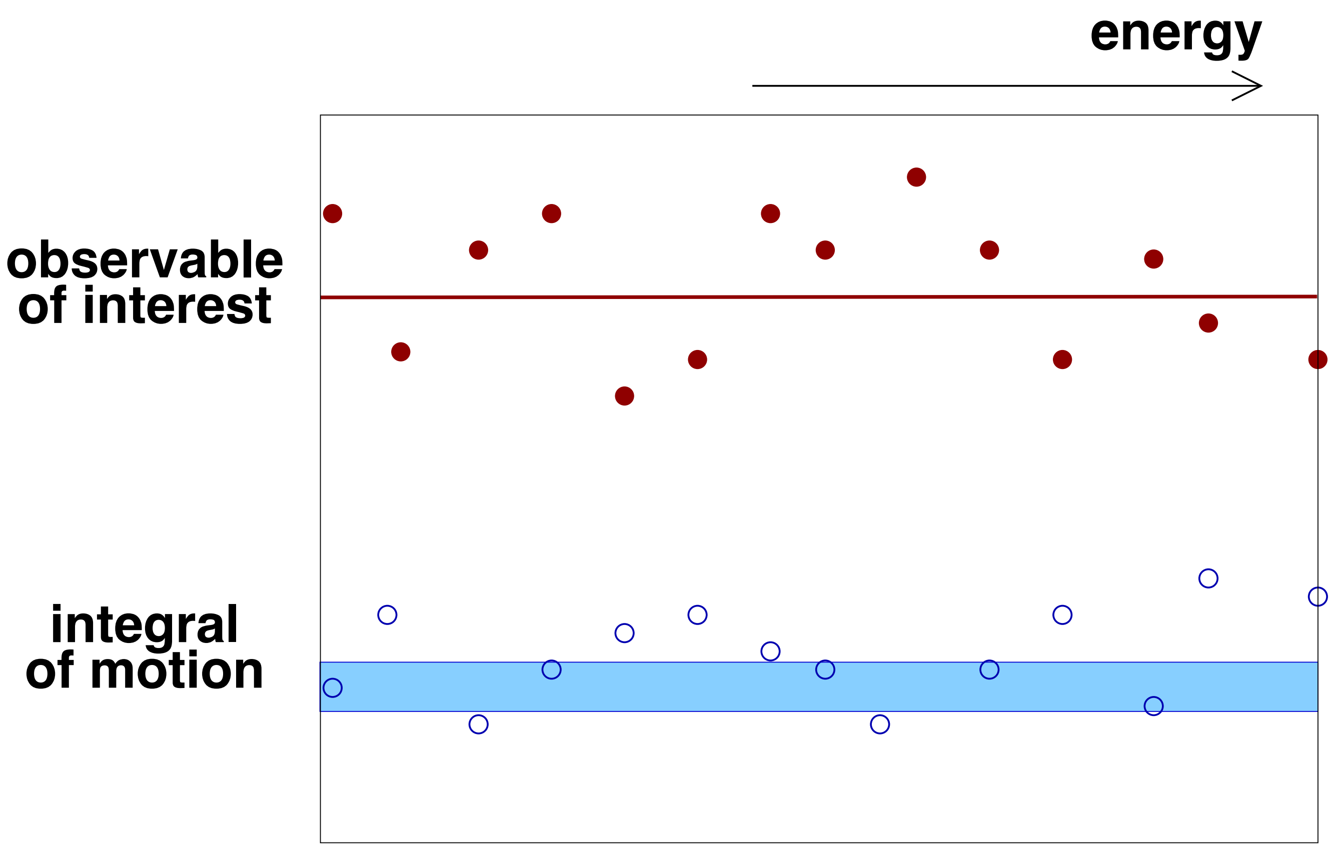

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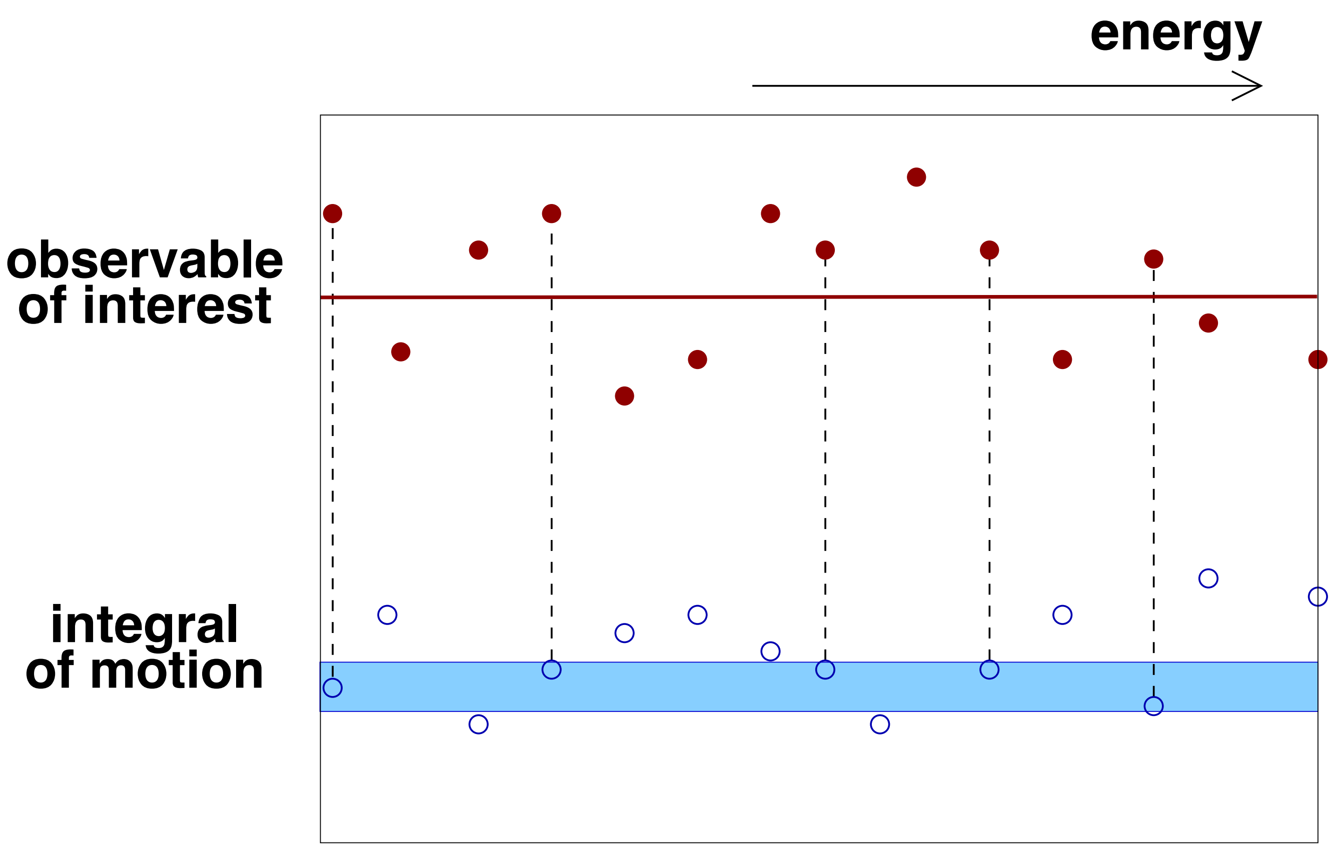

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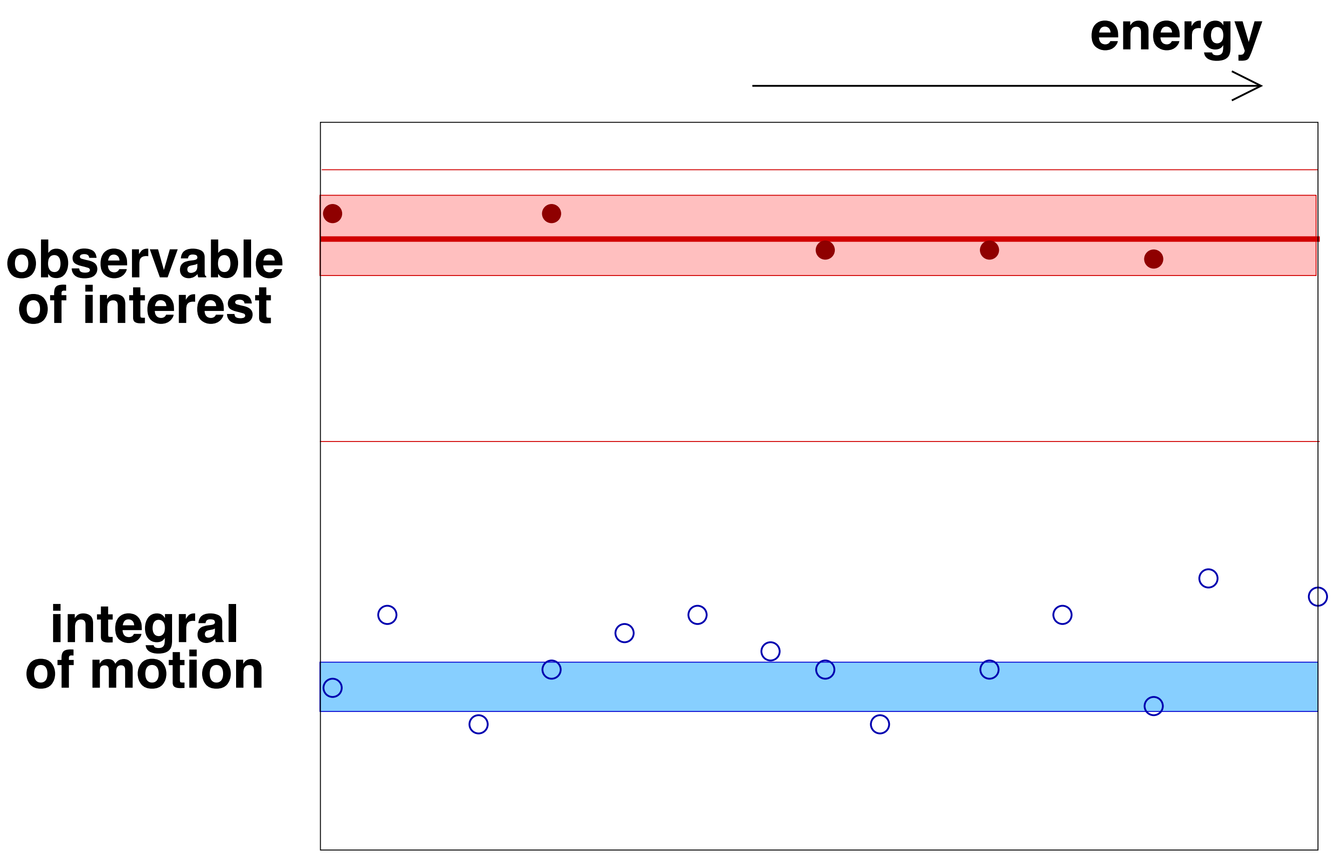
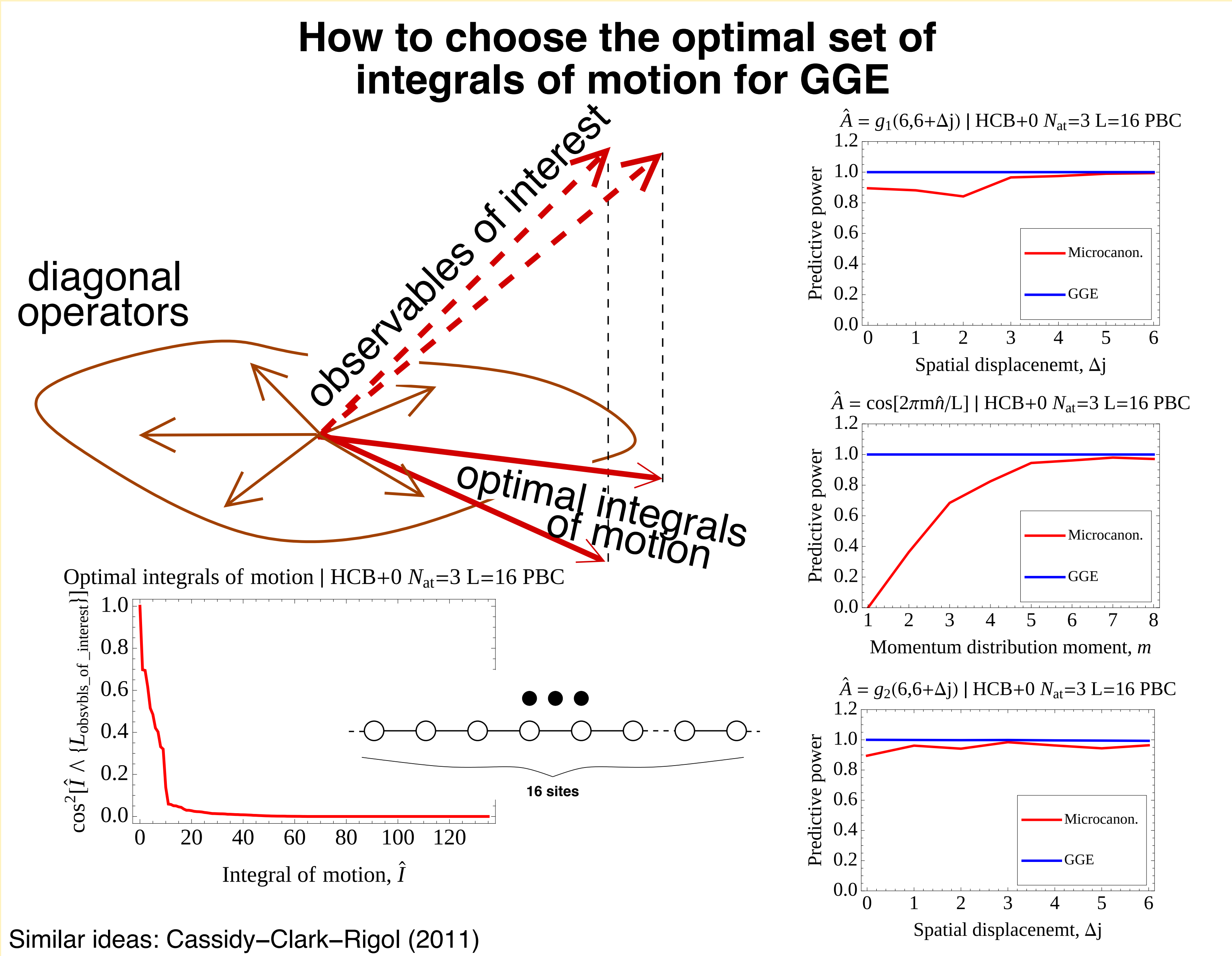
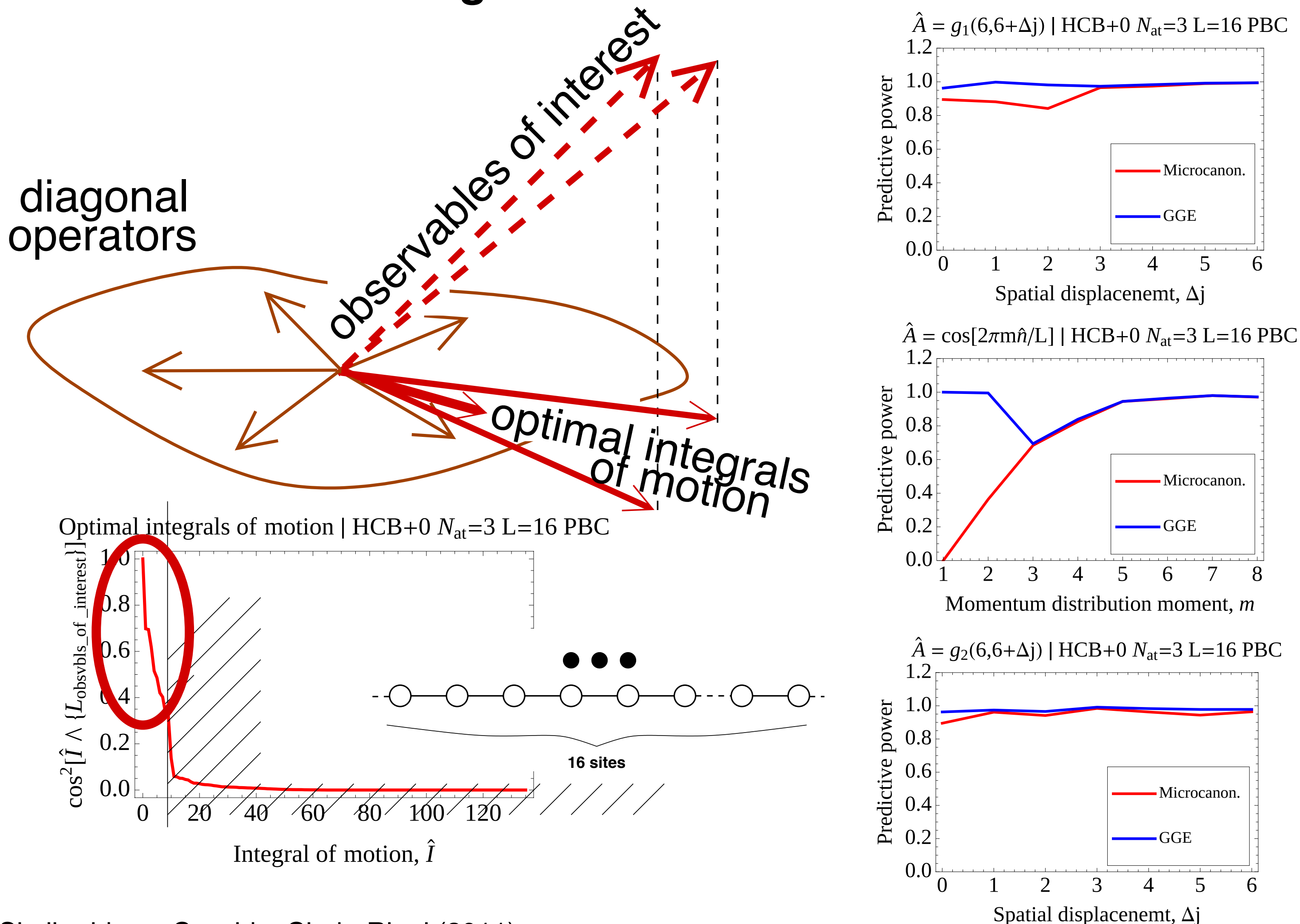


Optimizing the GGE

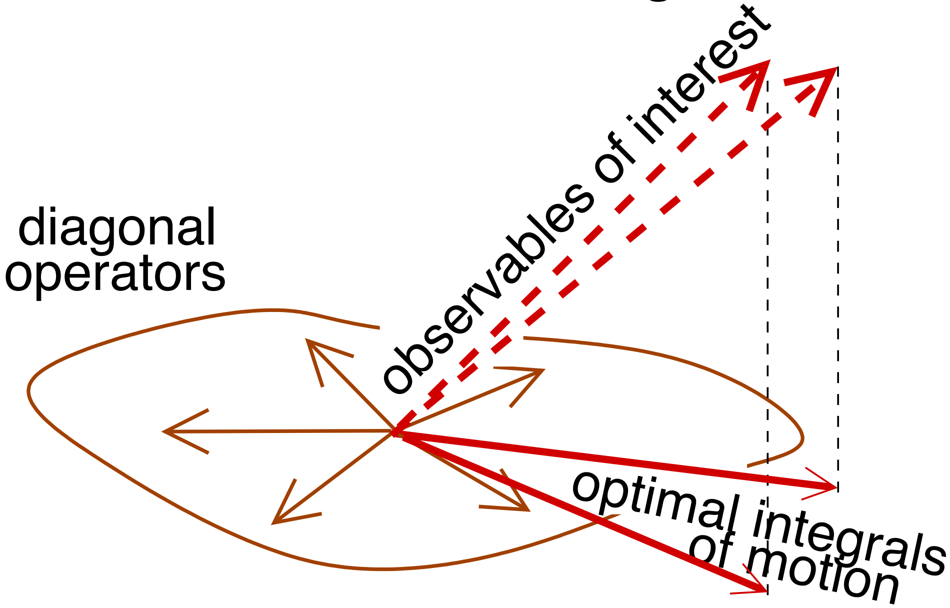


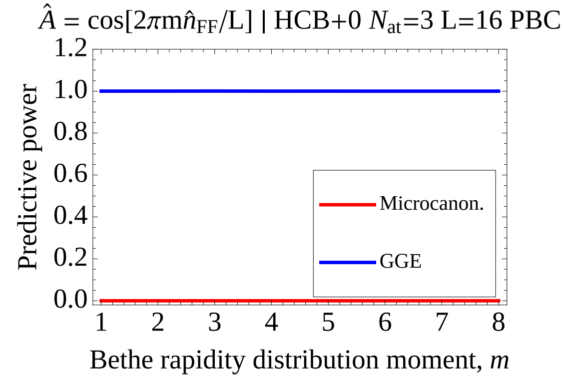
How to choose the optimal set of integrals of motion for GGE

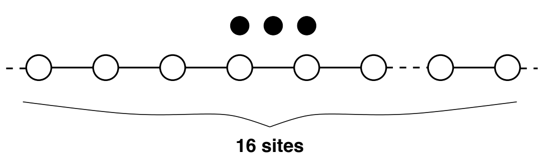


Similar ideas: Cassidy–Clark–Rigol (2011)

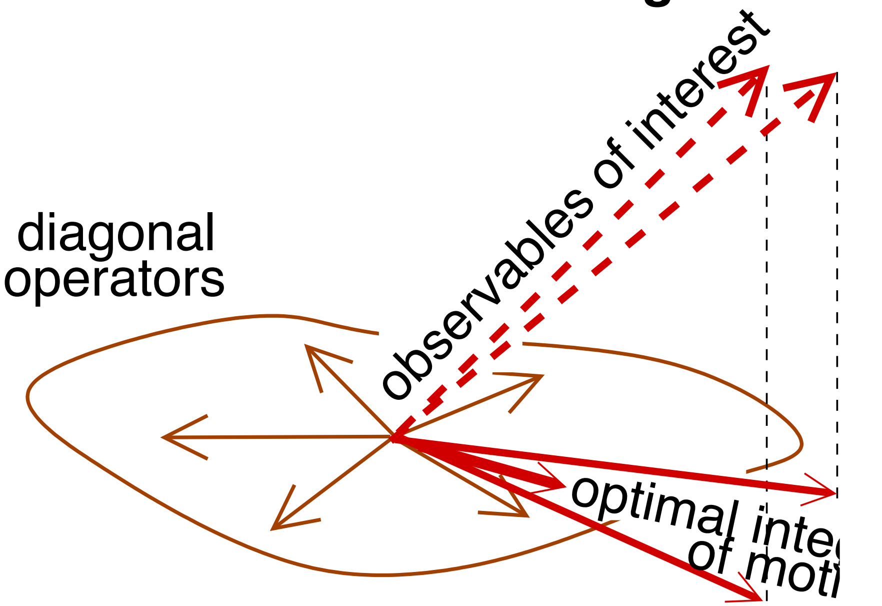
How to choose the optimal set of integrals of motion

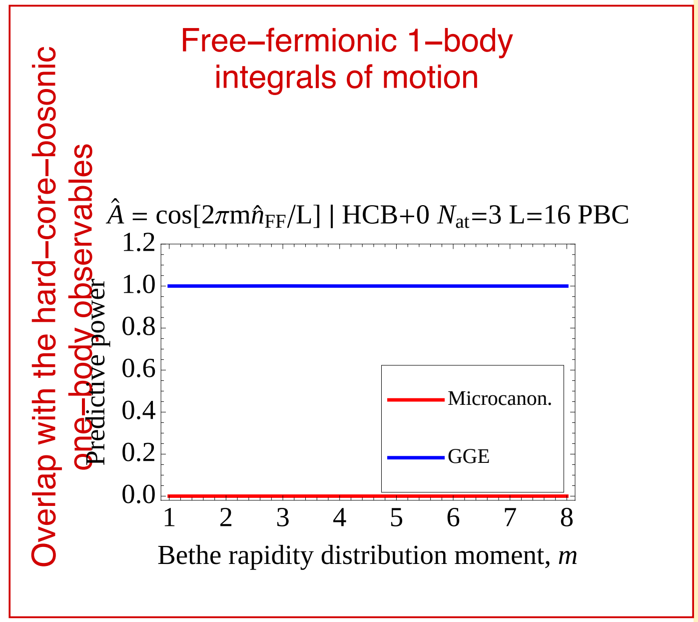


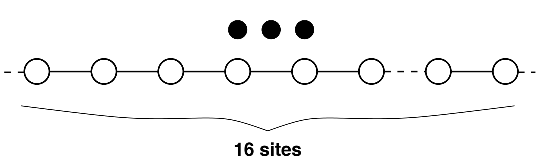




How to choose the optimal set of integrals of motion





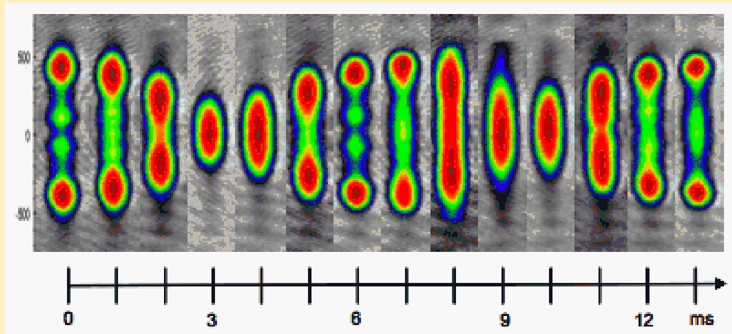


Take home:

in free–fermionic integrable systems, the Hilbert–Schmidt angle between the free–fermionic and "system proper" one–body observables is close to zero

Empirical Manifestations: Newton Cradle

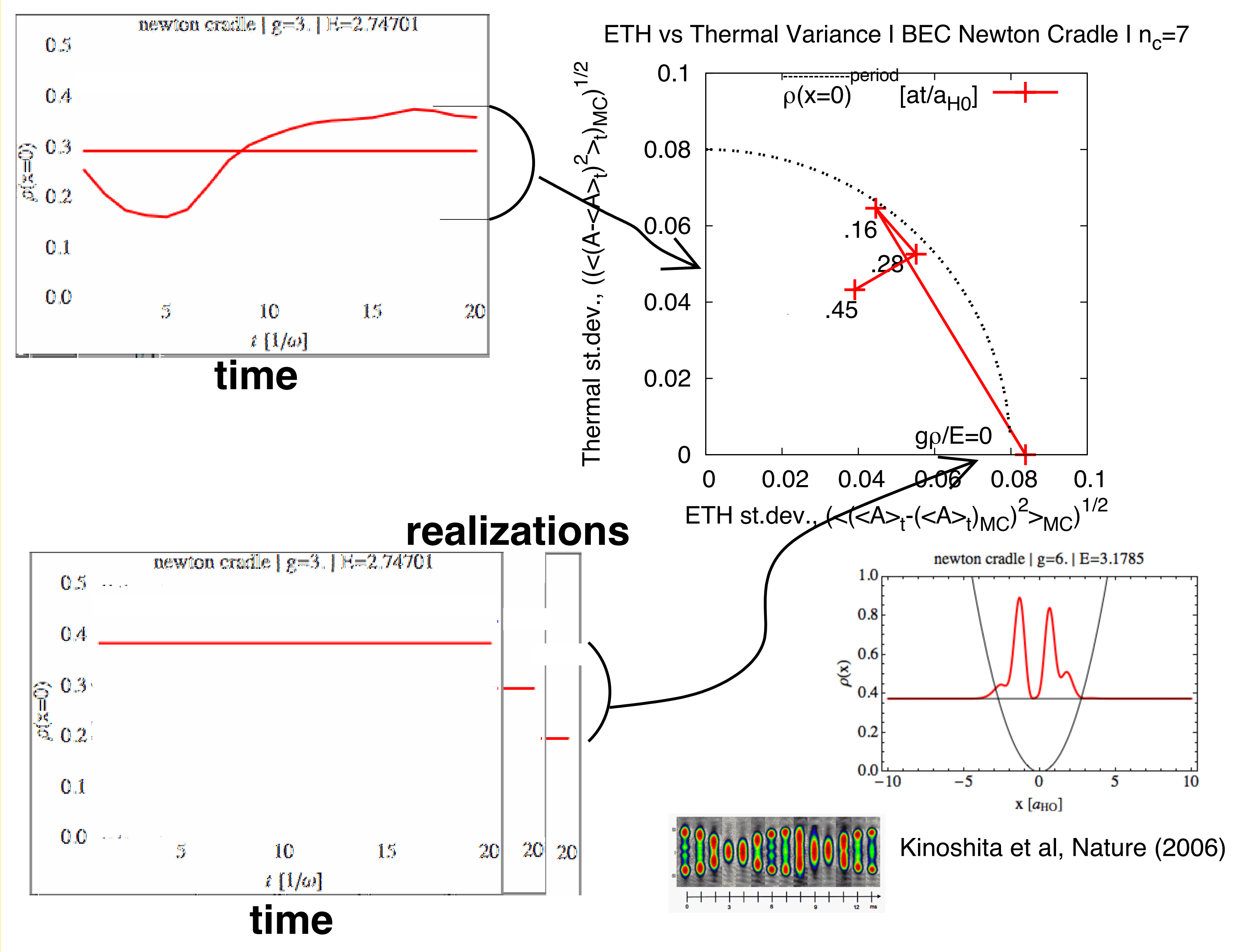
A hot Newton Cradle in mean-field



Kinoshita et al, Nature (2006)

Movies:
_R__newton_time_evolution.gif
_R__newton_A.gif
_R__newton_B.gif

Newton Cradle: $Var_{MC}[Mean_t[A]]$ vs. $Mean_{MC}[Var_t[A]]$

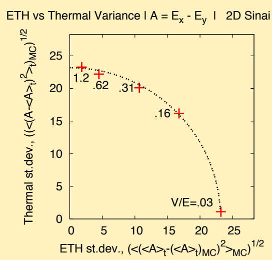


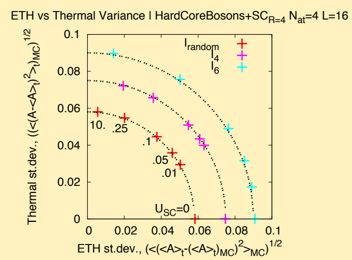
Concusions

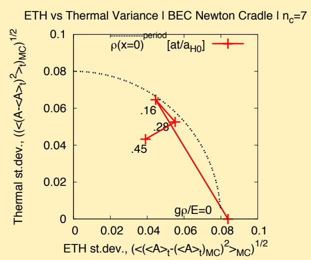
In this presentation we

- suggested the following proposition:

  The sum of the ensemble variance of the temporal means and the ensemble mean of the temporal variances remains approximately constant across the integrability-to-ergodicity transition

- 



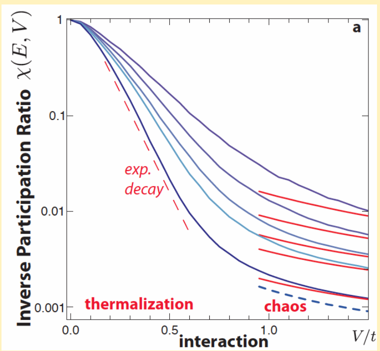


- linked the proposition to the Hilbert-Schmidt (HS) geometry of the observables: ETH variance $= cos^2$(HS angle between the observable and integrals of motion); IPR $= cos^2$(HS angle between the original and perturbed integrals of motion);
- found a way to identify the optimal integrals of motion for GGE;
- Suggested an experimental test for the proposition: a study of fluctuations in the Newton Cradle system

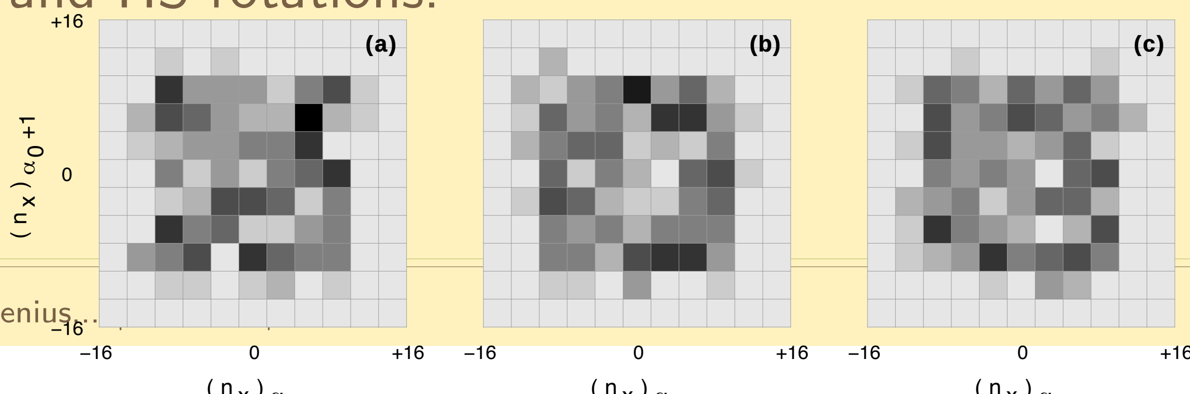
Outlook

system parameters $\underbrace{\longrightarrow}_{[1]}$ IPR $\underbrace{\longrightarrow}_{[2]}$ ergodicity of an observable

1. Silvestrov (1998); Neuenhahn-Marquart (2008)
source of universality: simplicity of the Fermi surface



2. Flambaum-Izrailev (1997); Rigol-Santos (2010)
source of universality: randomization of eigenvalues of integrals of motion in integrable systems, Olshanii et al. (2012). Study relationship between the U and HS rotations.



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