Ultracold Atoms in Controlled Disorder II:

Quantum Transport and Anderson Localization

in Dimension d>1

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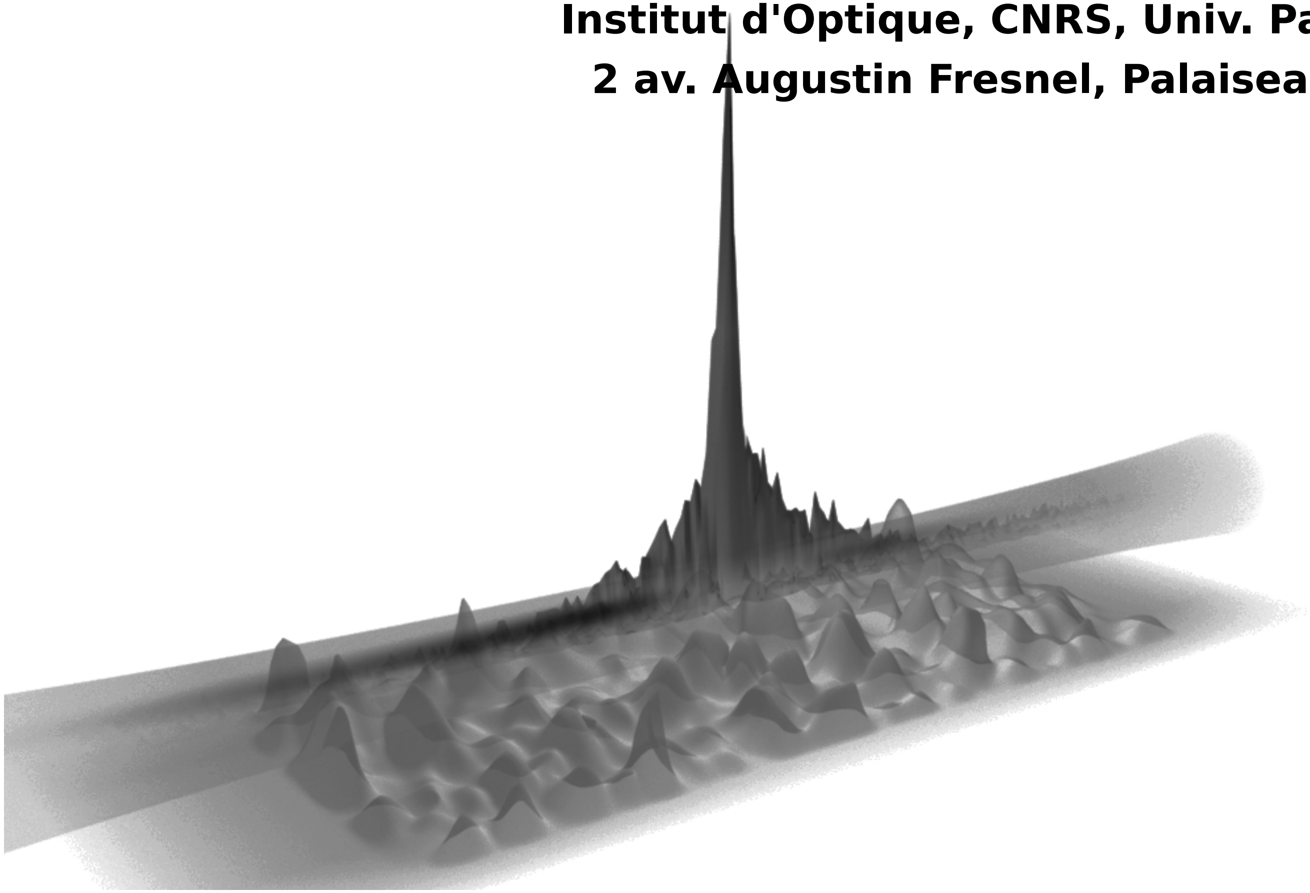
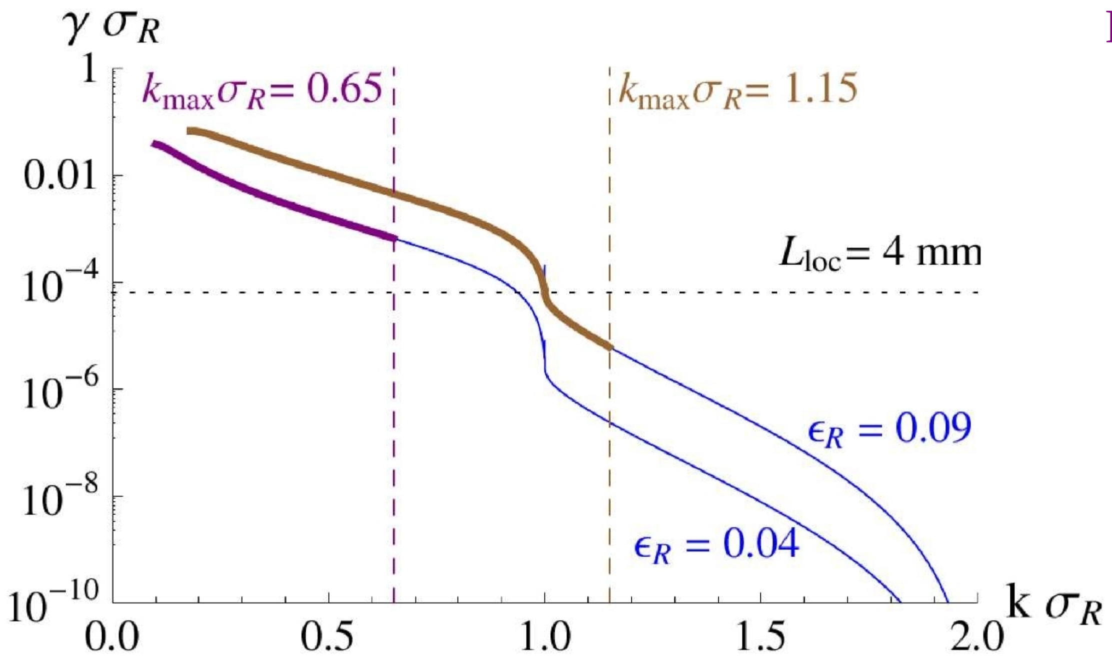
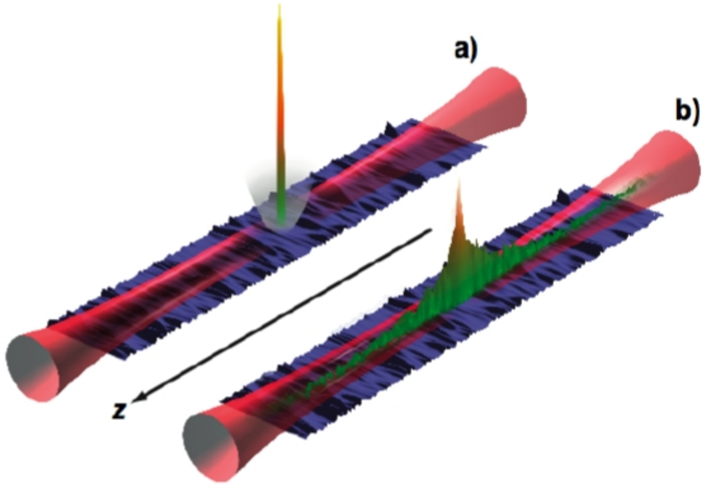


Anderson Localization in 1D Speckles : A Reminder

LSP *et al.*, Phys. Rev. Lett **98**, 210401 (2007)
P. Lugan *et al.*, Phys. Rev. A **80**, 023605 (2009)
J. Billy *et al.*, Nature **453**, 891 (2008)





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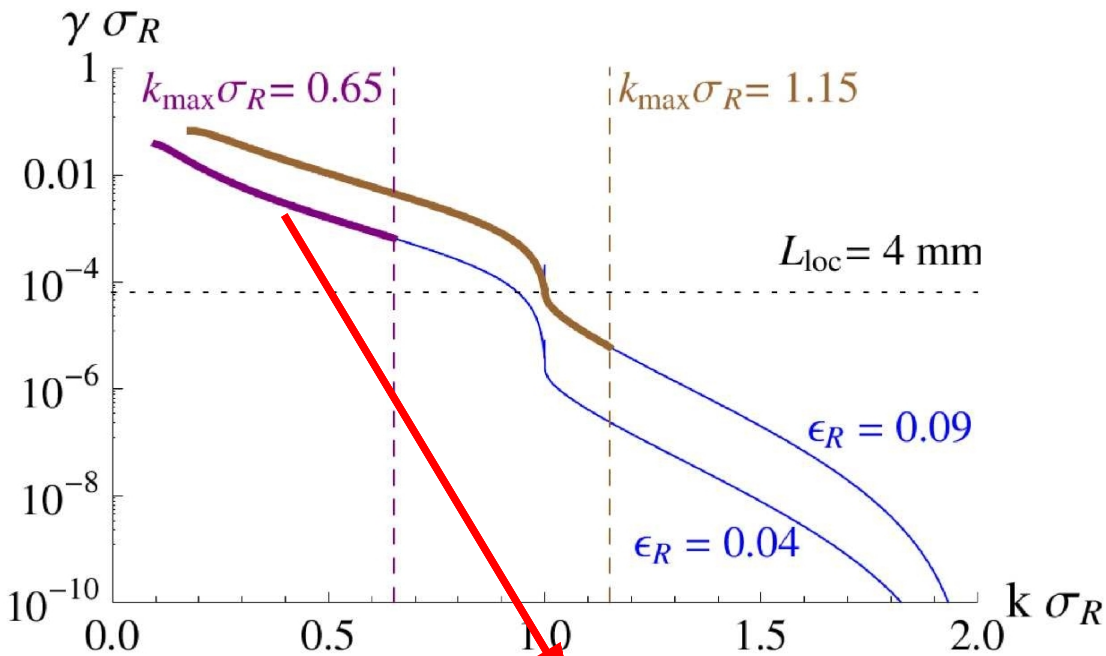
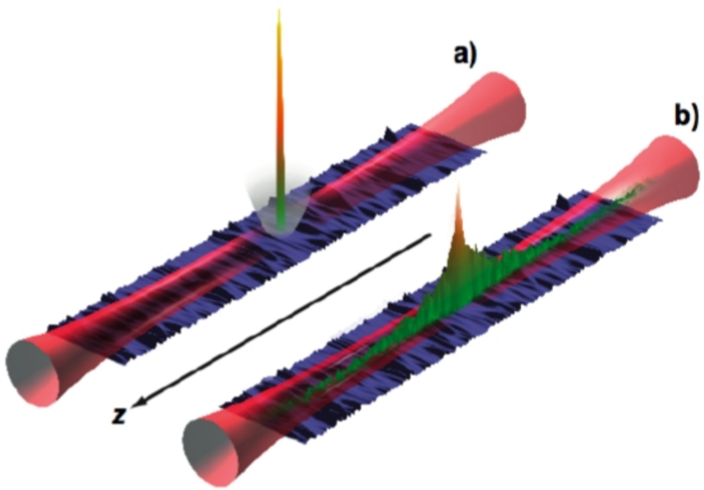
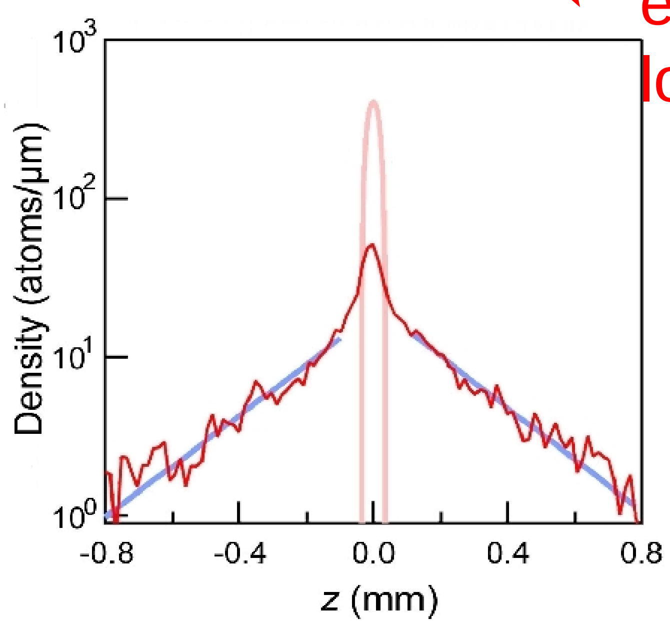




exponential localization



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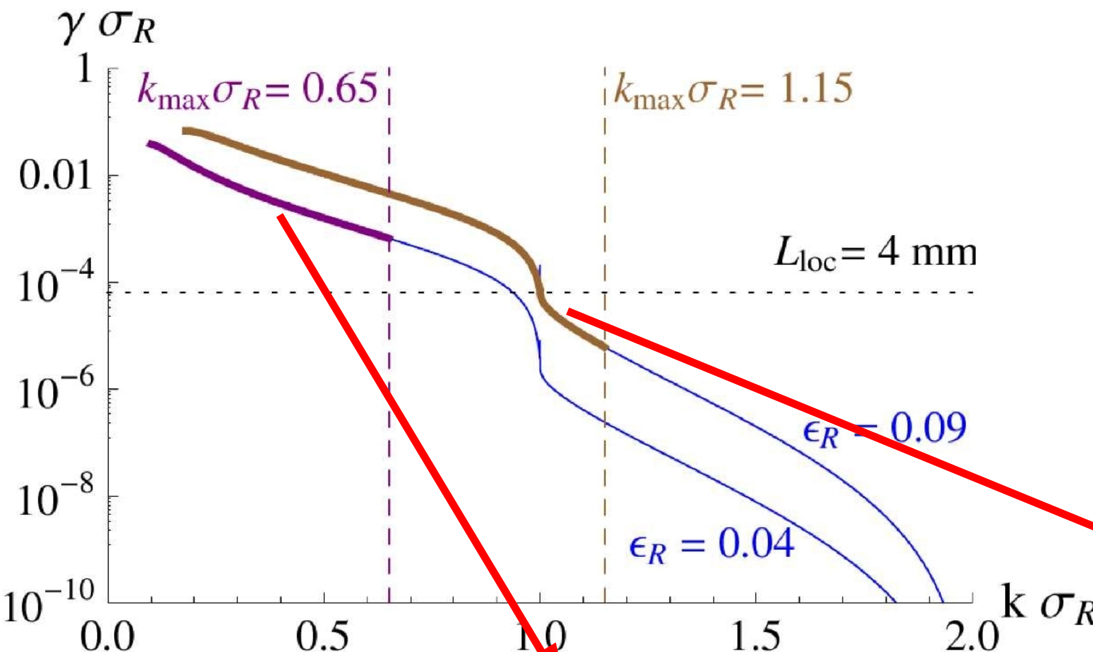
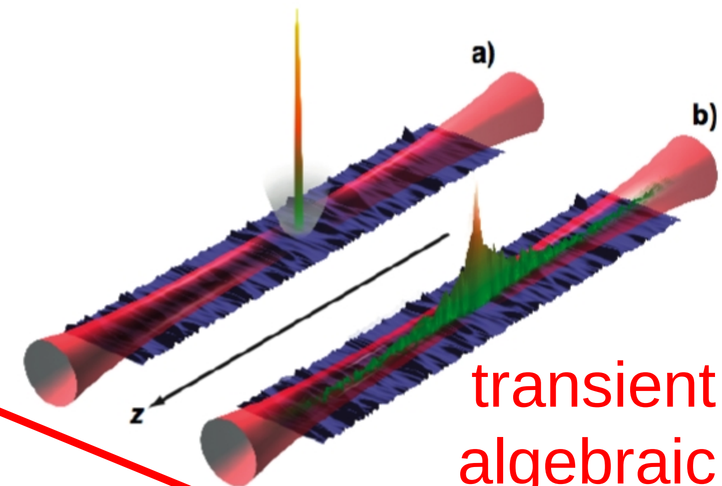
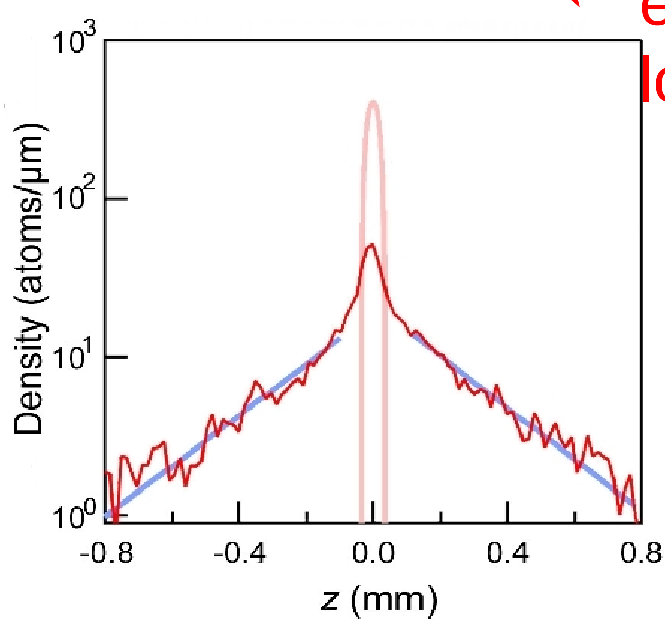
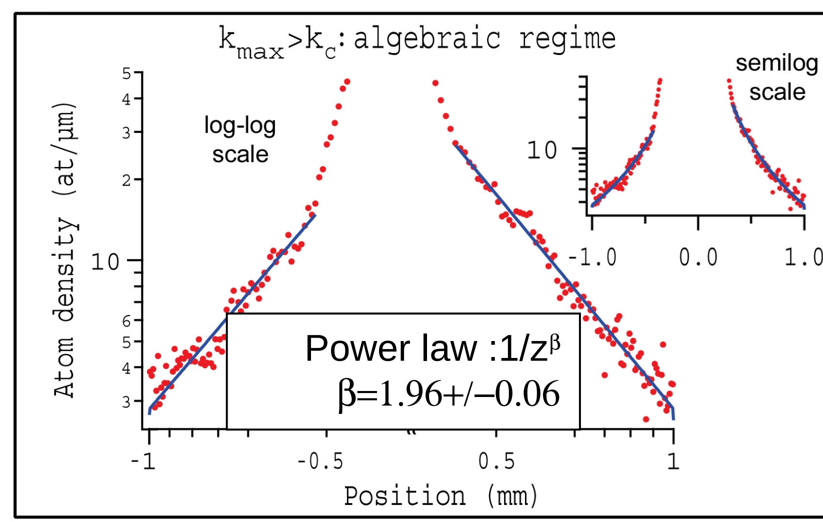




exponential localization

transient algebraic localization





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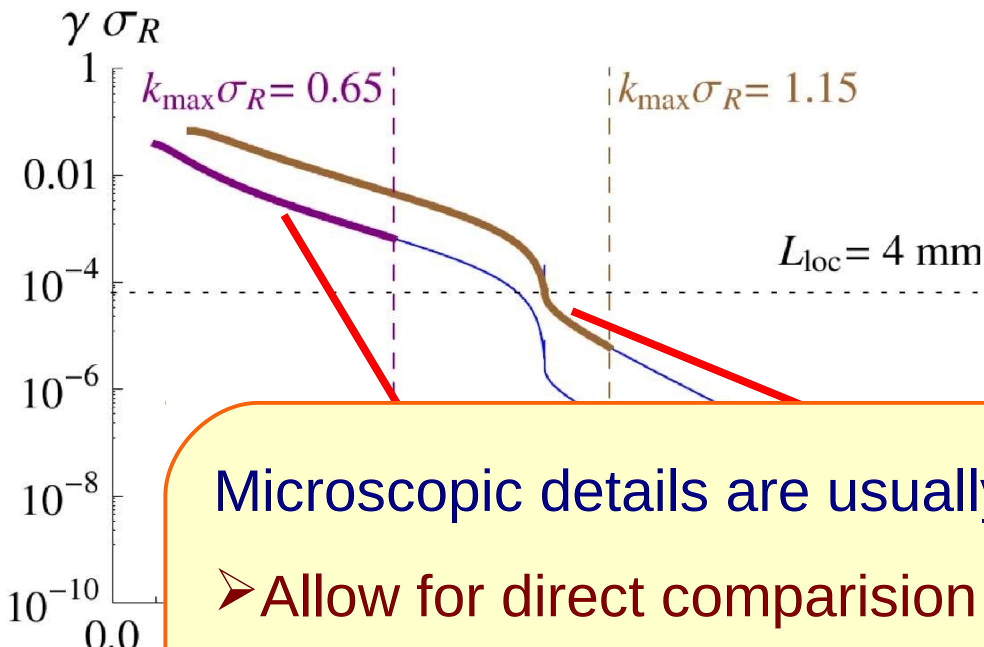

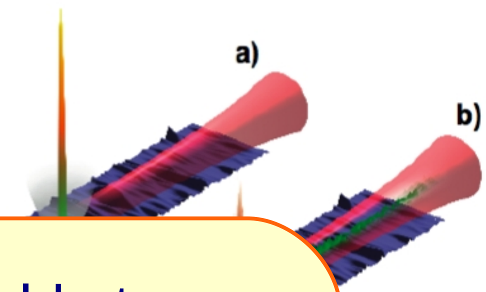

Microscopic details are usually important in ultracold atoms

- Allow for direct comparision of experiments and theory
- Combination of (i) finite-range correlations in 1D speckle potentials and (ii) broad energy distribution can lead to algebraic localization

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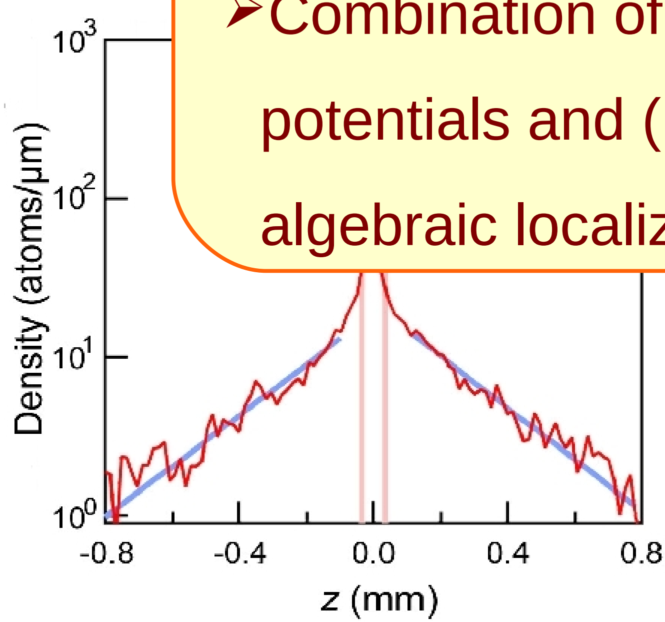
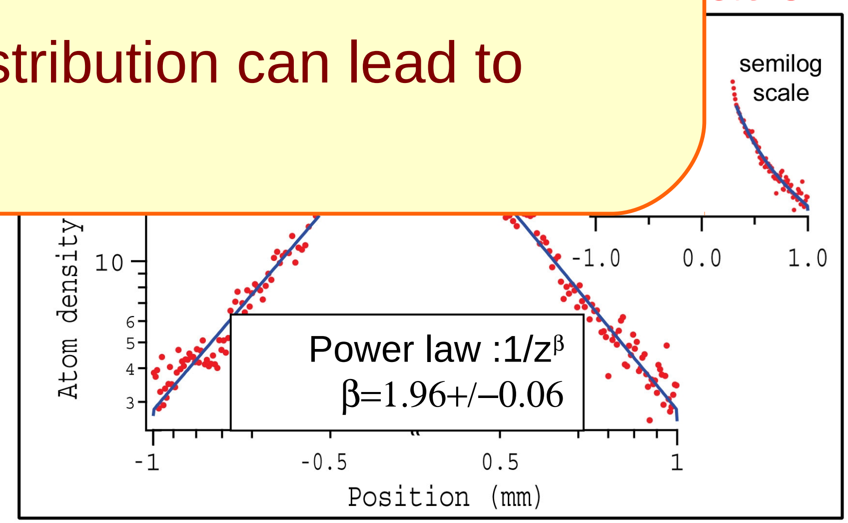
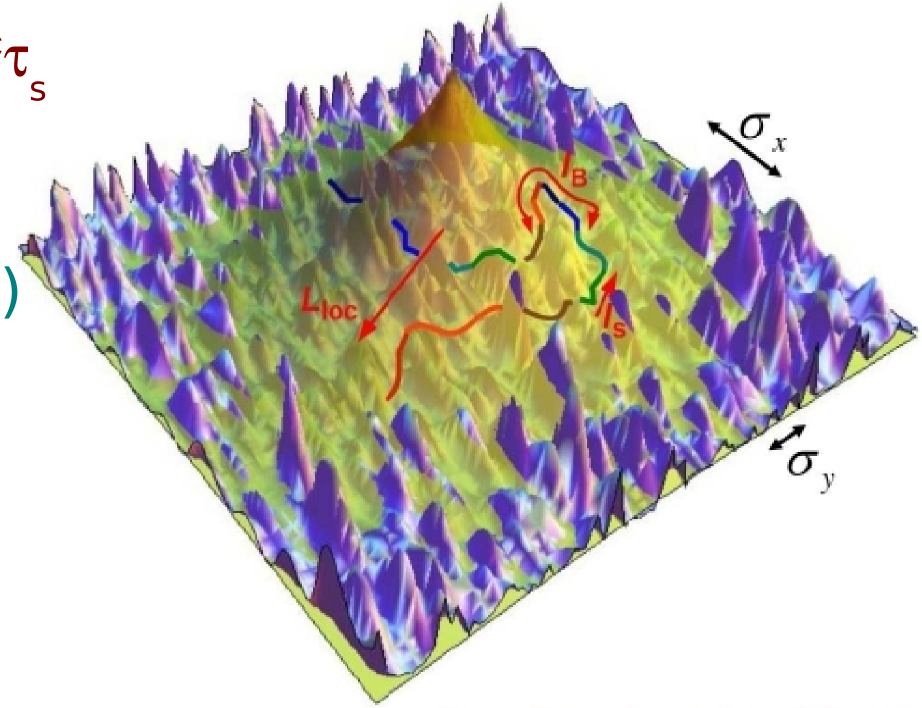




Basics of Transport Theory in Disordered Media

- Single scattering
  - phase loss ➔ finite life time τ_s of $|k\rangle$ states
  - τ_s given by the Fermi golden rule
  - scattering mean free path $l_s(k) = v^*\tau_s$
- Multiple scattering
  - diffusion (for vanishing interference)
  - transport mean free path

    $l_B(E) \geq l_s(k_E)$ ➔ $D_B(E) = (1/d)\, v^* l_B$
- Interference effects
  - weak localization $D_*(E) < D_B(E)$
  - strong localization $D_*(E) = 0$; exponential localization



➔ stronger for smaller k^*l_B (coherence) and smaller d (return prob.)

Outlook

Lecture #2

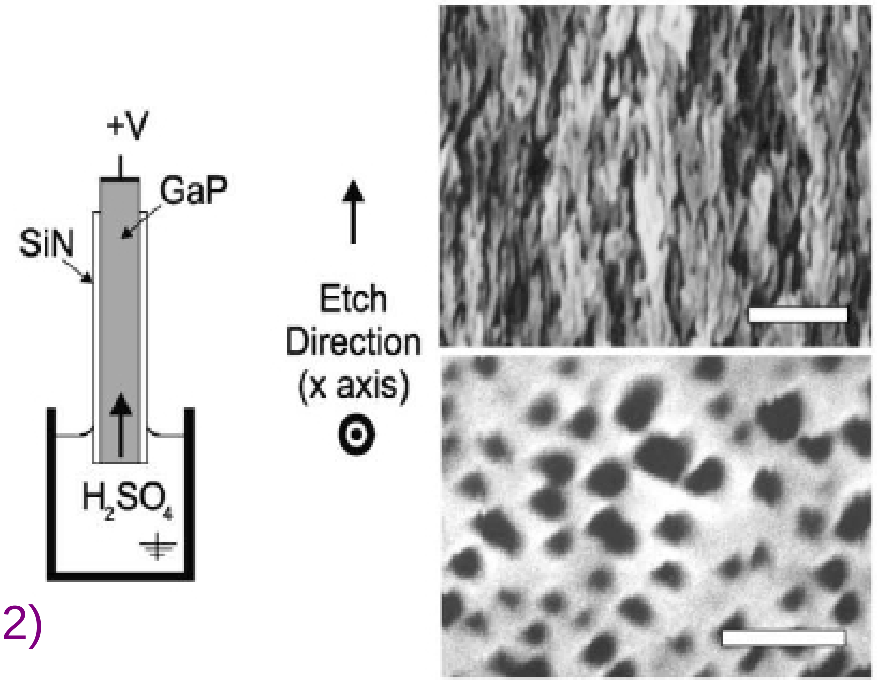
1. The (not always) nice features of 3D speckle potentials

2. Quantum transport theory in anisotropic 3D disorder
   - 2.1 Incoherent transport
   - 2.2 Quantum corrections
   - 2.3 Self-consistent approach
   - 2.4 Discussion of recent experiments

3. About the Mobility edge
   - 3.1 Shifted on-shell approach
   - 3.2 Discussion of recent experiments

Wave Propagation in Anisotropic Disorder

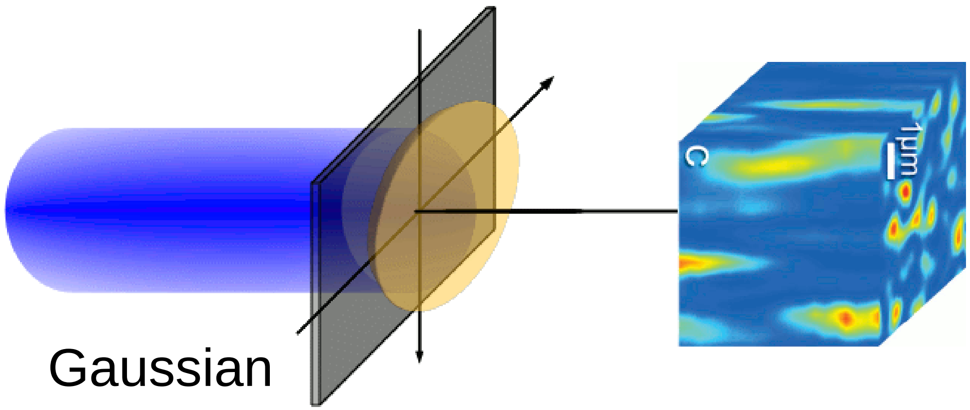
- Relevant to many materials
  - Electrons in MOFSETs
    Bishop *et al.*, Phys. Rev. B **30**, 3539 (1984)
  - Biomedical imaging
    Nickell *et al.*, Phys. Med. Biol. **45**, 2873 (2000)
  - Light in anisotropic materials
    Kao *et al.*, Phys. Rev. Lett. **77**, 2233 (1996)
    Wiersma *et al.*, Phys. Rev. Lett. **83**, 4321 (1999)
    Johnson *et al.*, Phys. Rev. Lett. **89**, 243901 (2002)
    Gurioli *et al.*, Phys. Rev. Lett. **94**, 183901 (2005)



from Johnson *et al.* (2002)

- Theory and models
  - Isotropic (eg point-like) scatterers in anisotropic materials
  - Etched scatterers in isotropic materials
    Wölfle & Bhatt, Phys. Rev. B **30**, 3542 (1984) ; van Tiggelen *et al.*, Phys. Rev. Lett. **77**, 639 (1996) ; Stark & Lubensky, Phys. Rev. E **55**, 514 (1997) ; Kaas *et al.*, Phys. Rev. Lett. **100**, 123902 (2008)
  - Complex correlation function for ultracold atoms

Anisotropic 3D Speckle Potentials



Gaussian intensity profile

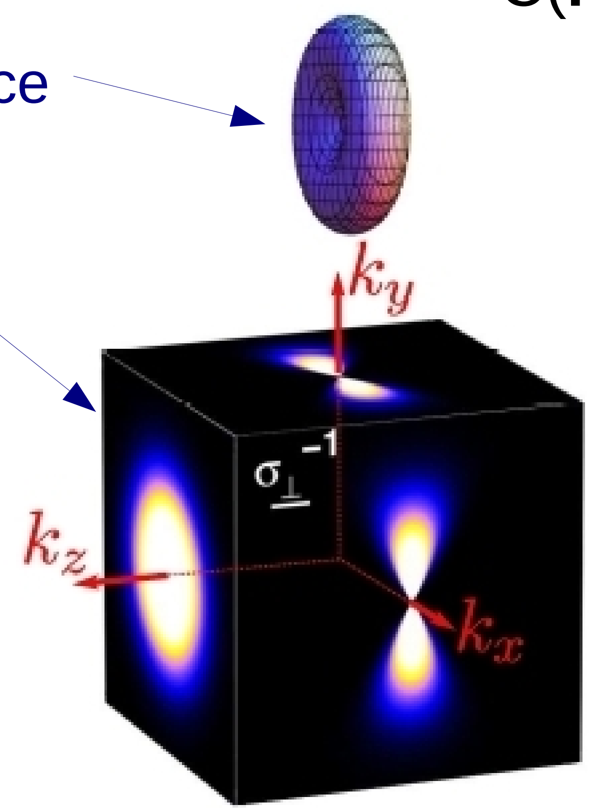
$$C(\mathbf{r}) = V_{\mathrm{R}}^2 c_{1\mathrm{sp}}^2(x,y,z)$$

$$c_{1\mathrm{sp}}(x,y,z) = \frac{1}{\sqrt{1+4z^2/\sigma_\parallel^2}} \exp\left[-\frac{(x^2+y^2)/2\sigma_\perp^2}{1+4z^2/\sigma_\parallel^2}\right]$$

C(**k**)

iso-value surface

in-plane plots



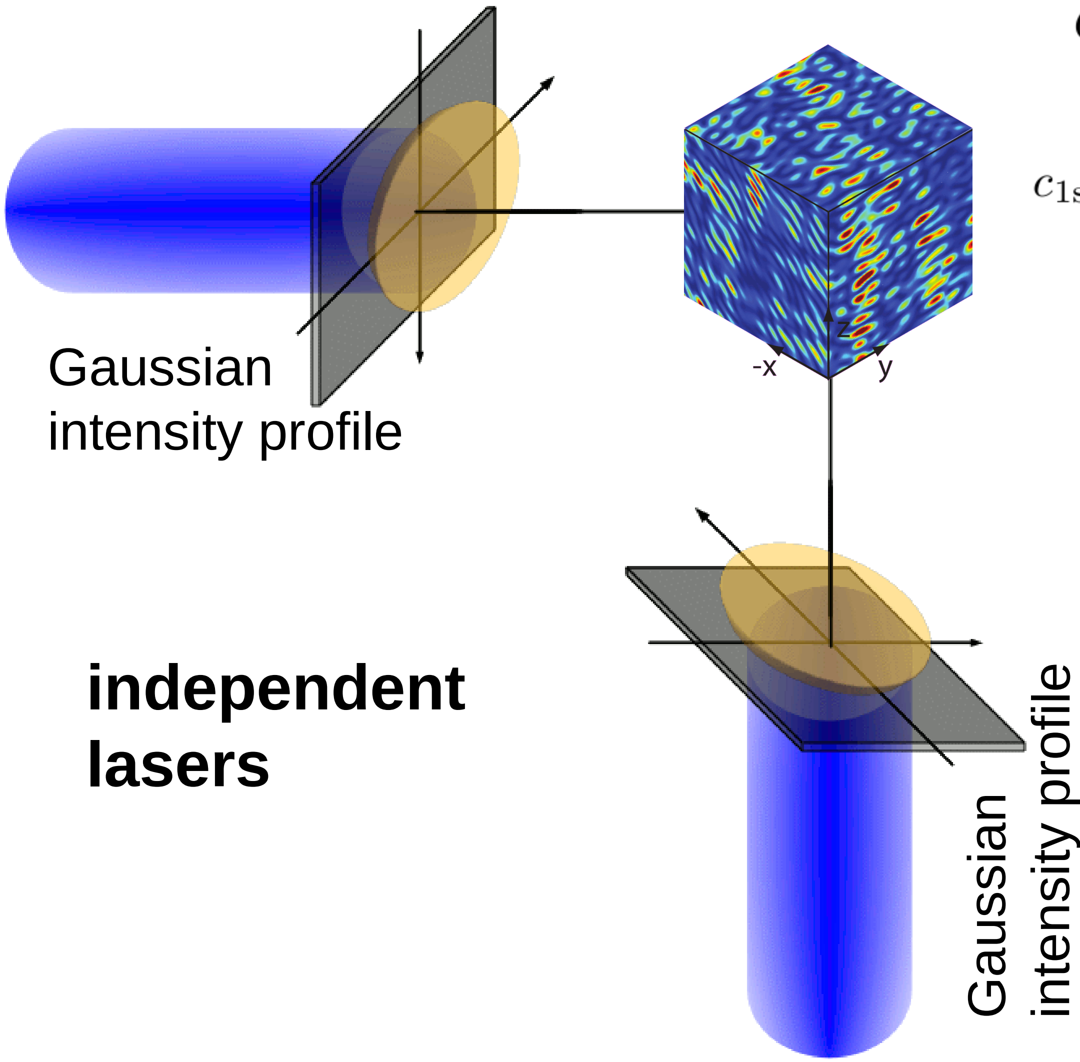
Configuration used in DeMarco's group (Urbana Champain, USA)

Kondov *et al.*, Science **334**, 66 (2011)

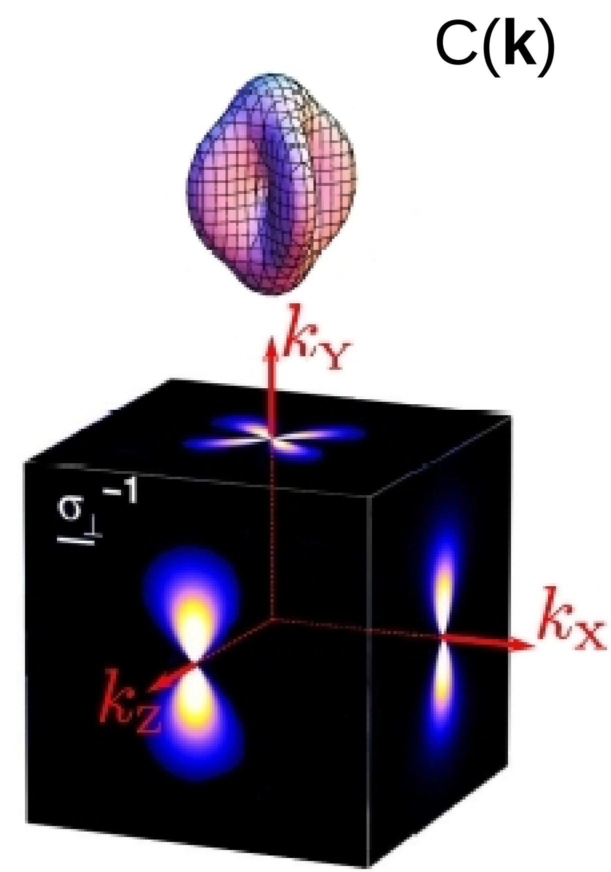
Anisotropic 3D Speckle Potentials

$$C(\mathbf{r}) = (V_\mathrm{R}/2)^2 \times \{c_{1\mathrm{sp}}^2(x,y,z) + c_{1\mathrm{sp}}^2(z,y,x)\}$$

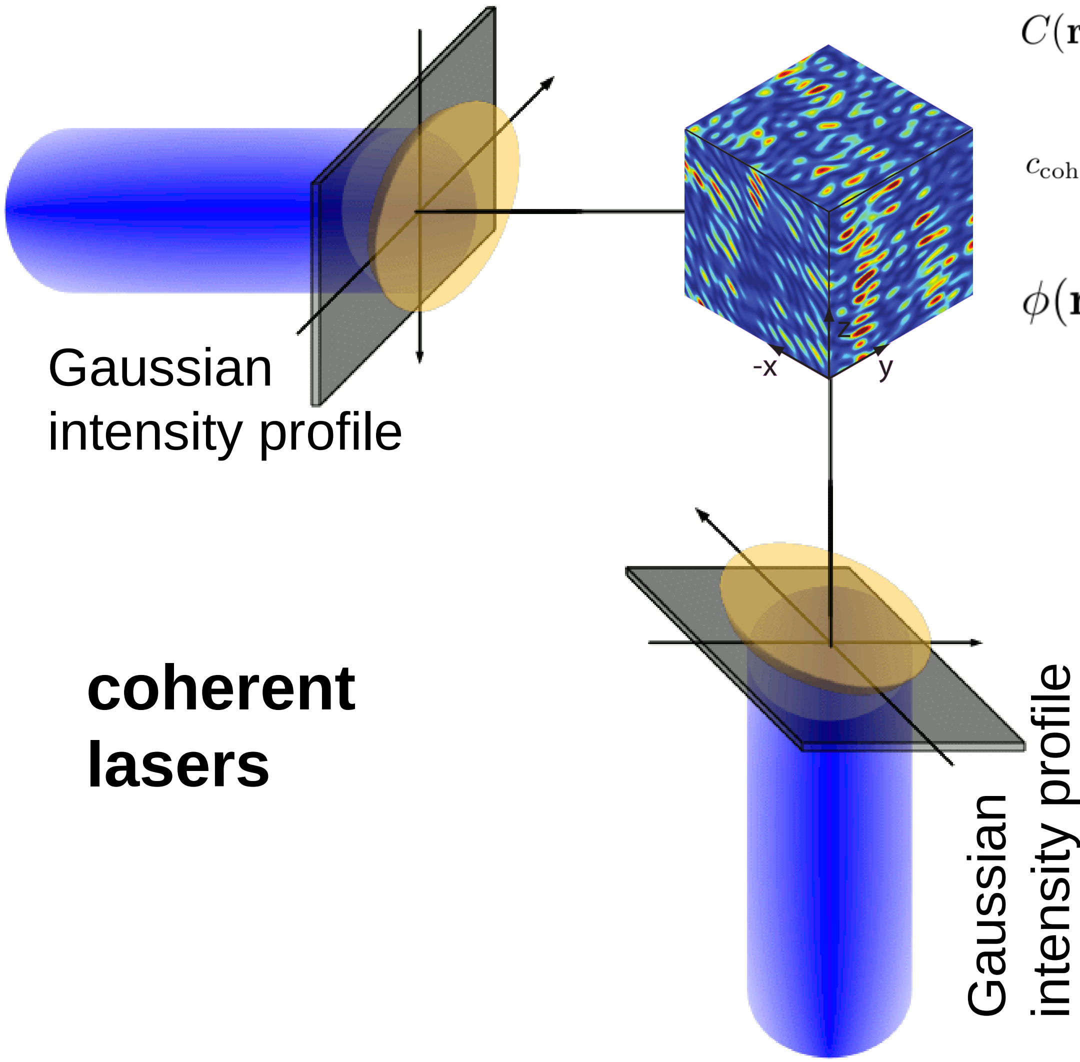
$$c_{1\mathrm{sp}}(x,y,z) = \frac{1}{\sqrt{1+4z^2/\sigma_\parallel^2}} \exp\left[-\frac{(x^2+y^2)/2\sigma_\perp^2}{1+4z^2/\sigma_\parallel^2}\right]$$



independent lasers



Anisotropic 3D Speckle Potentials



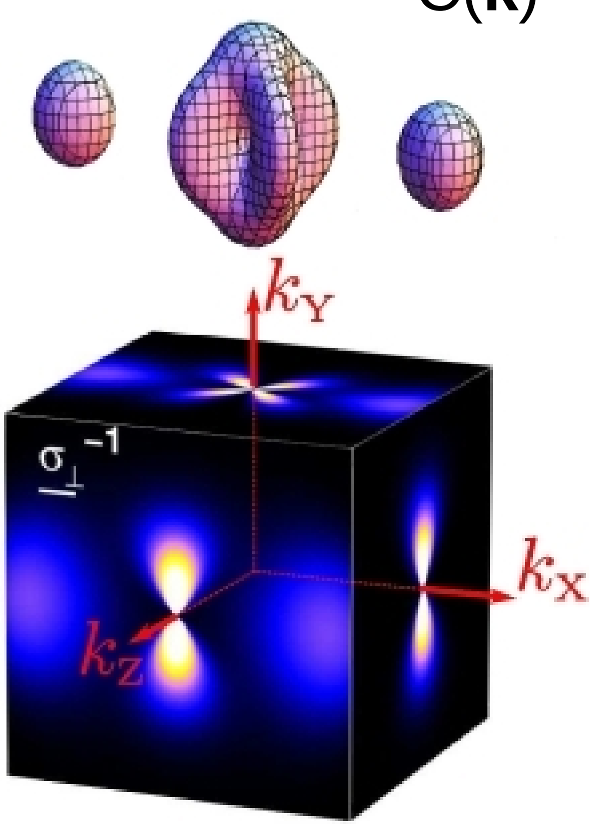
coherent lasers

$$C(\mathbf{r}) = (V_\mathrm{R}/2)^2 \times \{c_{1\mathrm{sp}}^2(x,y,z) + c_{1\mathrm{sp}}^2(z,y,x) + 2c_{\mathrm{coh}}(x,y,z)\}$$

$$c_{\mathrm{coh}}(\mathbf{r}) = c_{1\mathrm{sp}}(x,y,z) \times c_{1\mathrm{sp}}(z,y,x) \times \frac{(1+4\frac{xz}{\sigma_\parallel^2})\cos[\phi(\mathbf{r})] + 2\frac{x-z}{\sigma_\parallel}\sin[\phi(\mathbf{r})]}{\sqrt{1+4z^2/\sigma_\parallel^2}\sqrt{1+4x^2/\sigma_\parallel^2}}$$

$$\phi(\mathbf{r}) = \frac{2\pi}{\lambda_\mathrm{L}}(x-z) - \frac{z}{\sigma_\perp^2\sigma_\parallel}\frac{x^2+y^2}{1+4z^2/\sigma_\parallel^2} - \frac{x}{\sigma_\perp^2\sigma_\parallel}\frac{z^2+y^2}{1+4x^2/\sigma_\parallel^2}$$

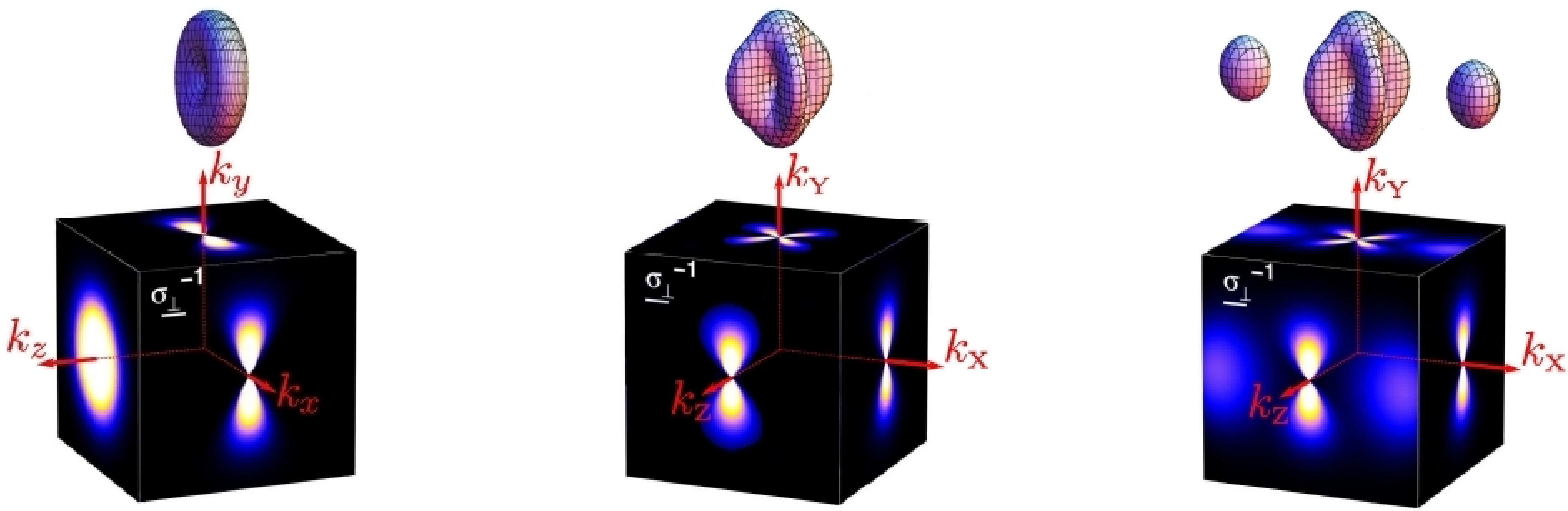
C(**k**)



Configuration used in Aspect's group (Institut d'Optique, France)

Jendrzejewski *et al.*, Nature Phys. **8**, 398 (2012)

Anisotropic 3D Speckle Potentials



« single-speckle » (Urbana Champain) | « incoherent-speckles » | « coherent-speckles » (Institut d'Optique)

- Three configurations with complex structured C(**k**)
- No rotation invariance (even upon anisotropic rescaling)

➢Predictions of quantum transport theory ?

➢Anisotropies of transport and localization ?

➢Mobility edge ?

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Quantum Transport

Rammer, *Quantum Transport Theory* (1998)

- Propagation of the density matrix (*ie* of the Wigner function)
  - Observable quantities

$$W(\mathbf{r},\mathbf{k},t) \equiv \int \frac{\mathrm{d}\mathbf{q}}{(2\pi)^d}\, e^{i\mathbf{q}\cdot\mathbf{r}} \left\langle \mathbf{k}+\frac{\mathbf{q}}{2}\right| \rho(t) \left|\mathbf{k}-\frac{\mathbf{q}}{2}\right\rangle$$

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$$\overline{W}(\mathbf{r},\mathbf{k},t-t_0) = \int \mathrm{d}\mathbf{r}' \int \frac{\mathrm{d}\mathbf{k}'}{(2\pi)^d}\, W_0(\mathbf{r}',\mathbf{k}')\, F(\mathbf{r}-\mathbf{r}',\mathbf{k},\mathbf{k}';t-t_0)$$

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propagation kernel of W

$$F(\mathbf{R},\mathbf{k},\mathbf{k}';t) \equiv \int \frac{dE}{2\pi} \int \frac{d\mathbf{q}}{(2\pi)^d} \int \frac{d\hbar\omega}{2\pi}\, e^{i\mathbf{q}\cdot\mathbf{R}}\, e^{-i\omega t}\, \Phi_{\mathbf{k},\mathbf{k}'}(\mathbf{q},\omega,E)$$

and

$$\overline{\langle \mathbf{k}_+|G(E_+)|\mathbf{k}'_+\rangle\langle \mathbf{k}'_-|G^\dagger(E_-)|\mathbf{k}_-\rangle} \equiv (2\pi)^d \delta(\mathbf{q}-\mathbf{q}')\Phi_{\mathbf{k},\mathbf{k}'}(\mathbf{q},\omega,E)$$

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→ propagation kernel of W

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and

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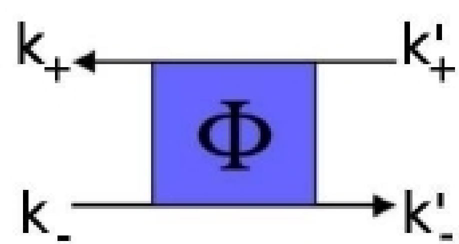
$$\Phi \equiv \overline{G \otimes G^\dagger} \equiv$$



Rammer, *Quantum Transport Theory* (1998)

- Bethe-Salpeter equation (exact)

$$\Phi = \overline{G} \otimes \overline{G^\dagger} + \overline{G} \otimes \overline{G^\dagger}\, \mathrm{U}\, \Phi$$

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all correlations in the propagation

$$\Phi = \boxed{\overline{G} \otimes \overline{G}^{\dagger}} + \boxed{\overline{G} \otimes \overline{G}^{\dagger}\, U\, \Phi}$$

uncorrelated propagation of ψ and ψ*

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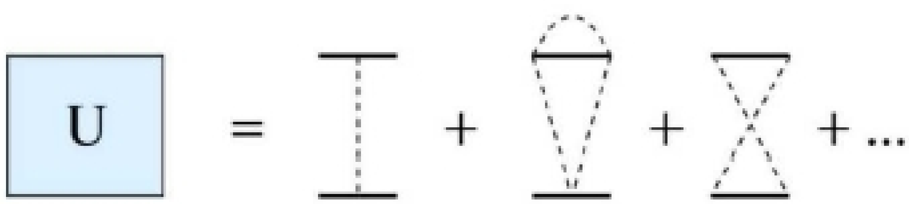
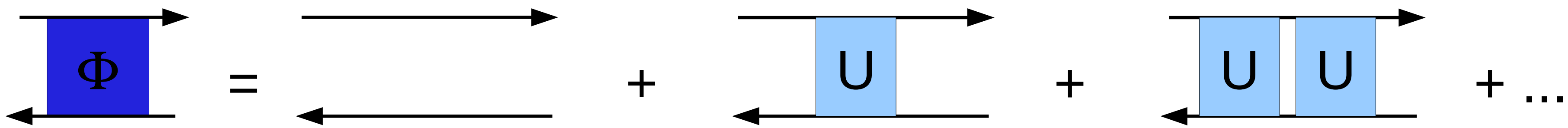
- Perturbative diagrammatic expansion





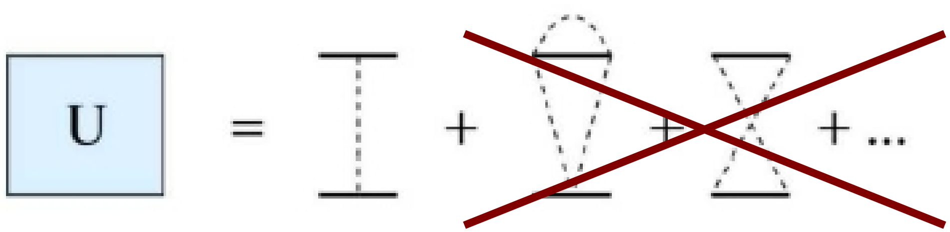
Incoherent (Boltzmann) Transport

Rammer, *Quantum Transport Theory* (1998)

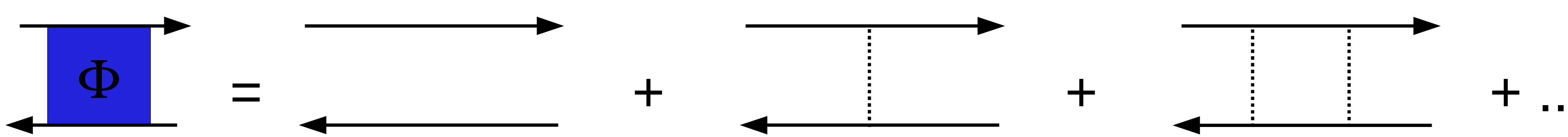
- 1^{st} step : neglect interference terms
  - independent scattering (Boltzmann approximation)
  - weak disorder (Born approximation)

$$\Phi = \overline{G} \otimes \overline{G^\dagger} + \overline{G} \otimes \overline{G^\dagger}\, U\, \Phi$$

- Perturbative diagrammatic expansion



$U_{k,k'}(q,\omega,E) = C(\mathbf{k}-\mathbf{k}')$



(ladder diagrams)

Incoherent (Boltzmann) Transport

Wölfle & Bhatt, Phys. Rev. B **30**, 3542 (1984)

- Long time ($\omega \to 0$) and large distance ($\mathbf{q} \to 0$) limit

$$\Phi_{\mathbf{k},\mathbf{k}'}(\mathbf{q},\omega,E) = \frac{2\pi}{\hbar N_0(E)} \frac{\delta(E-\epsilon(\mathbf{k}))\,\delta(E-\epsilon(\mathbf{k}'))}{-i\omega + \mathbf{q}\cdot\mathbf{D}_{\mathrm{B}}(E)\cdot\mathbf{q}}$$

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➔ On-shell approximation
(*ie* E ≈ $\hbar^2 k^2/2m$)

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➔ Energy-dependent « diffusion » pole

Incoherent (Boltzmann) Transport

Wölfle & Bhatt, Phys. Rev. B **30**, 3542 (1984)

- Energy-dependent Boltzmann diffusion tensor

$$
\begin{aligned}
D_{\mathrm{B}}^{i,j}(E) \quad &= \frac{1}{N_0(E)}\Bigg\{\Big\langle \tau_{E,\hat{\mathbf{k}}}\, v_i\, v_j \Big\rangle_{\hat{\mathbf{k}}|E} \\
&+\frac{2\pi}{\hbar}\sum_{\lambda_E^n \neq 1} \frac{\lambda_E^n}{1-\lambda_E^n}\Big\langle \tau_{E,\hat{\mathbf{k}}}\, v_i\, \phi^n_{E,\hat{\mathbf{k}}} \Big\rangle_{\hat{\mathbf{k}}|E}\Big\langle \tau_{E,\hat{\mathbf{k}}}\, v_j\, \phi^n_{E,\hat{\mathbf{k}}} \Big\rangle_{\hat{\mathbf{k}}|E}\Bigg\}
\end{aligned}
$$

Incoherent (Boltzmann) Transport

Wölfle & Bhatt, Phys. Rev. B **30**, 3542 (1984)

- Energy-dependent Boltzmann diffusion tensor

scattering time

$$\tau_{E,\hat{\mathbf{k}}} = \hbar/2\pi \langle \tilde{C}[k_E(\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \rangle_{\hat{\mathbf{k}}'|E}$$

velocity v ∝ k

$$v_i = \hat{\mathbf{u}}_i \cdot \nabla_{\mathbf{k}} \epsilon(\mathbf{k})/\hbar$$

$$D_{\mathrm{B}}^{i,j}(E) = \frac{1}{N_0(E)} \left\{ \left\langle \tau_{E,\hat{\mathbf{k}}} v_i v_j \right\rangle_{\hat{\mathbf{k}}|E} + \frac{2\pi}{\hbar} \sum_{\lambda_E^n \neq 1} \frac{\lambda_E^n}{1-\lambda_E^n} \left\langle \tau_{E,\hat{\mathbf{k}}} v_i \phi_{E,\hat{\mathbf{k}}}^n \right\rangle_{\hat{\mathbf{k}}|E} \left\langle \tau_{E,\hat{\mathbf{k}}} v_j \phi_{E,\hat{\mathbf{k}}}^n \right\rangle_{\hat{\mathbf{k}}|E} \right\}$$

orbitals

$$\frac{2\pi}{\hbar} \left\langle \tau_{E,\hat{\mathbf{k}}} \tilde{C}[k_E(\hat{\mathbf{k}} - \hat{\mathbf{k}}')] \phi_{E,\hat{\mathbf{k}}'}^n \right\rangle_{\hat{\mathbf{k}}'|E} = \lambda_E^n \phi_{E,\hat{\mathbf{k}}}^n$$

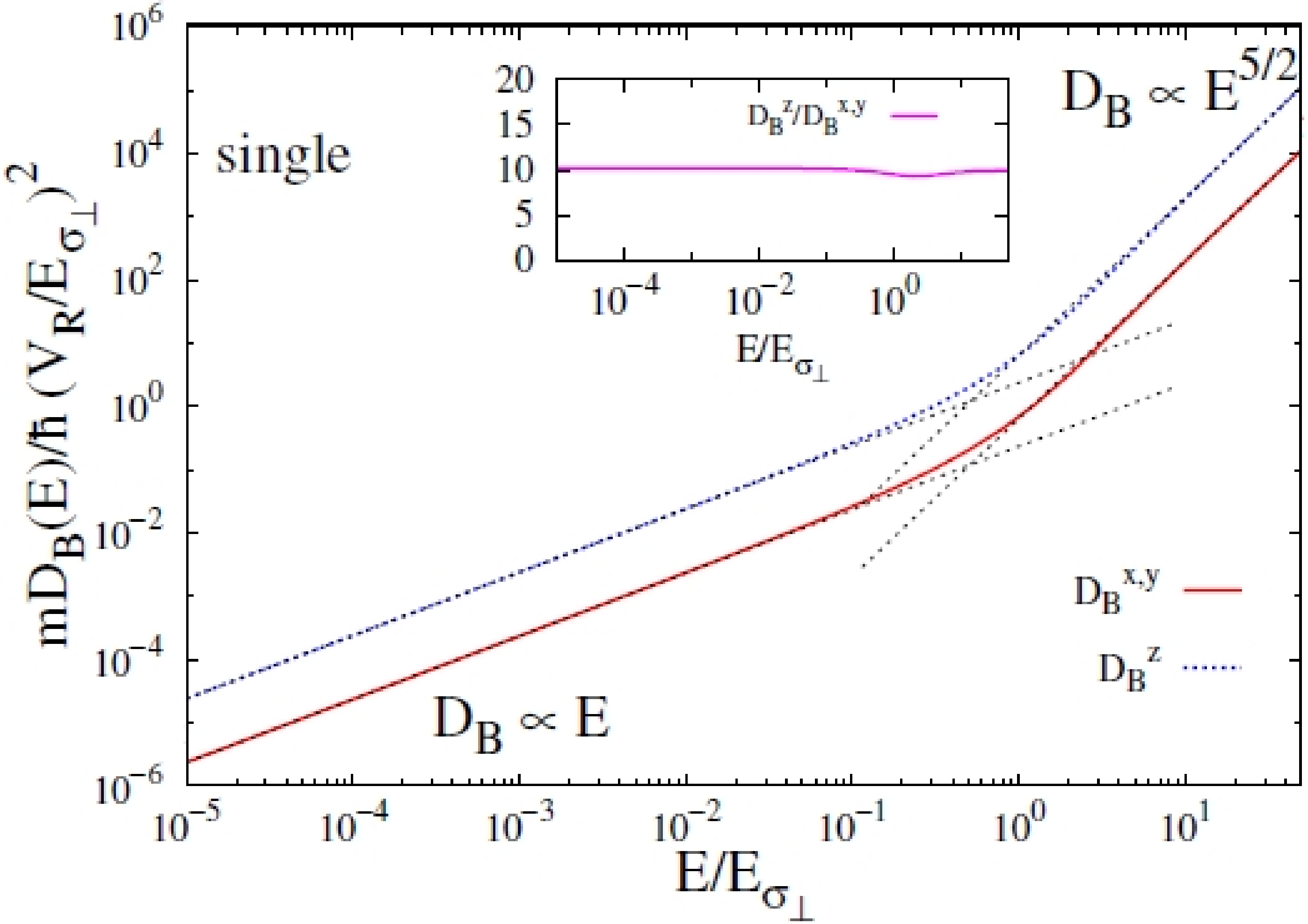
angular integration

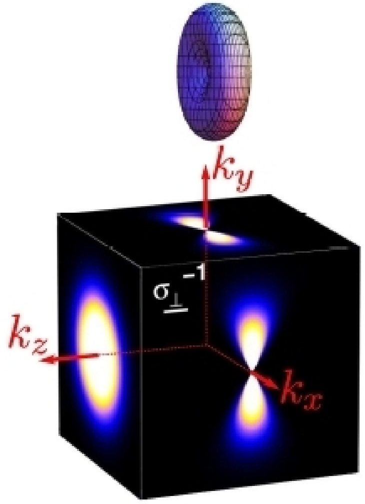
$$\langle \dots \rangle_{\hat{\mathbf{k}}|E} \equiv \int \frac{\mathrm{d}^d \mathbf{k}}{(2\pi)^d} \dots \delta[E - \epsilon(\mathbf{k})]$$

Incoherent (Boltzmann) Transport

Piraud *et al.*, arXiv:1112.2859

- Results : single-speckle configuration





For E→0

*) D(E) ~ E

*) no white-noise limit

*) persisting anisotropy

For E→∞

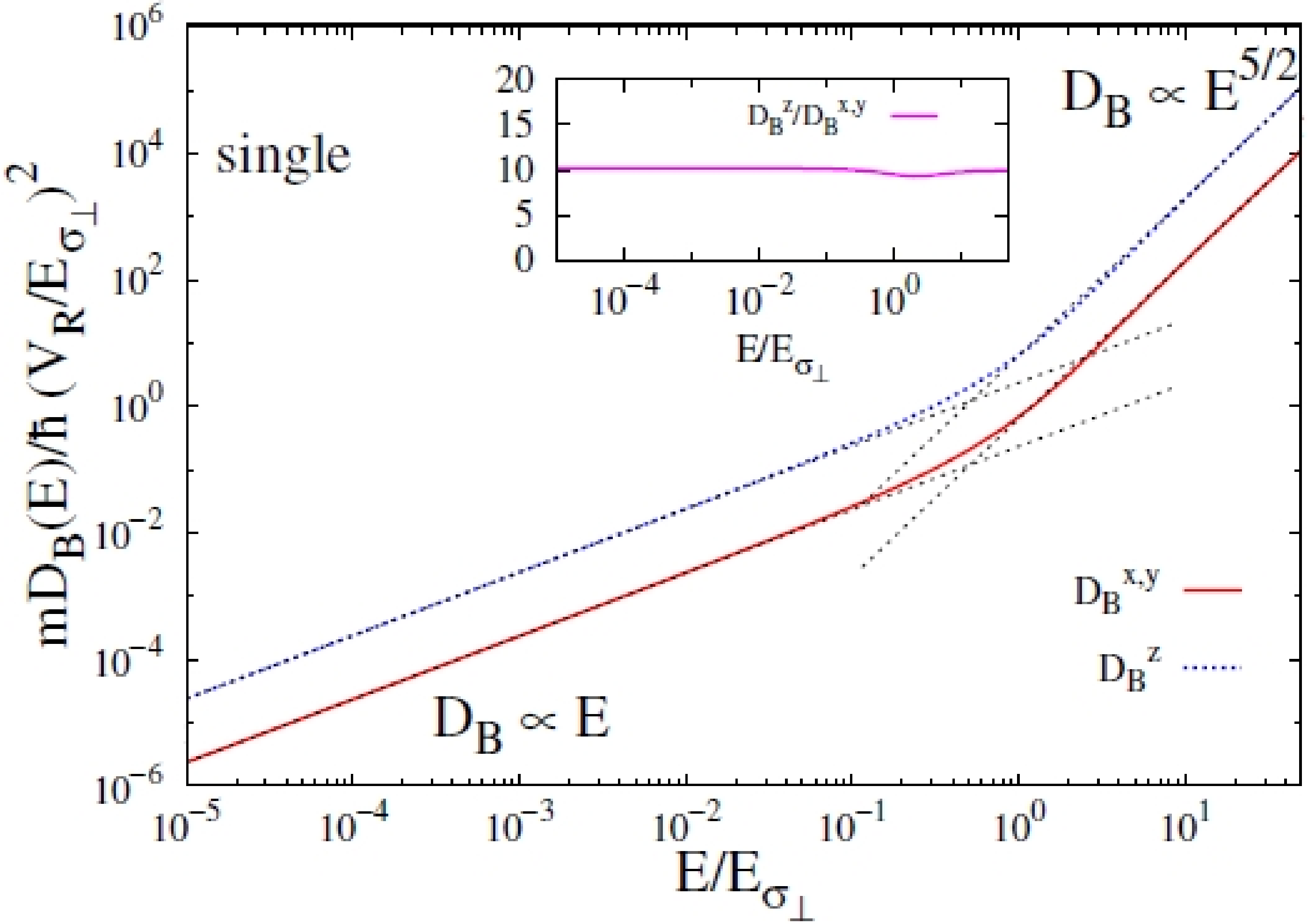
*) D(E) ~ $E^{5/2}$

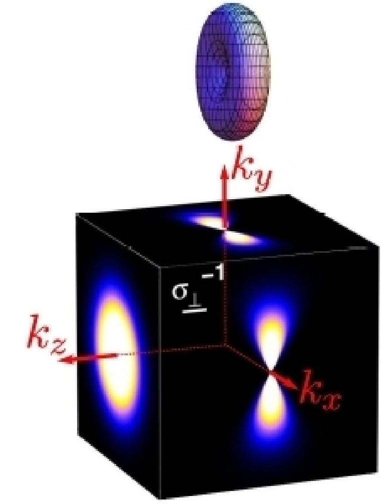
similar scalings with E as for other models (isotropic speckles) ; Kuhn *et al.*, NJP (2007)

Incoherent (Boltzmann) Transport

Piraud *et al.*, arXiv:1112.2859

- Results : single-speckle configuration





For E→0

*) D(E) ~ E

*) no white-noise limit

*) persisting anisotropy

For E→∞

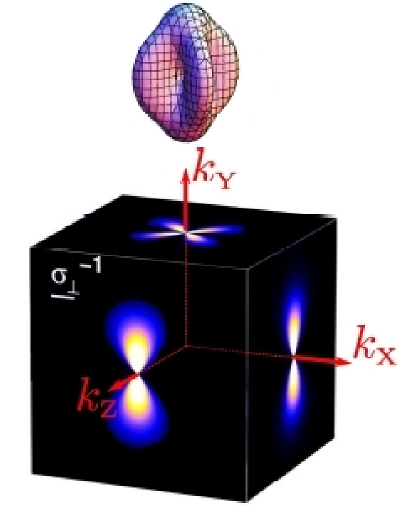
*) D(E) ~ $E^{5/2}$

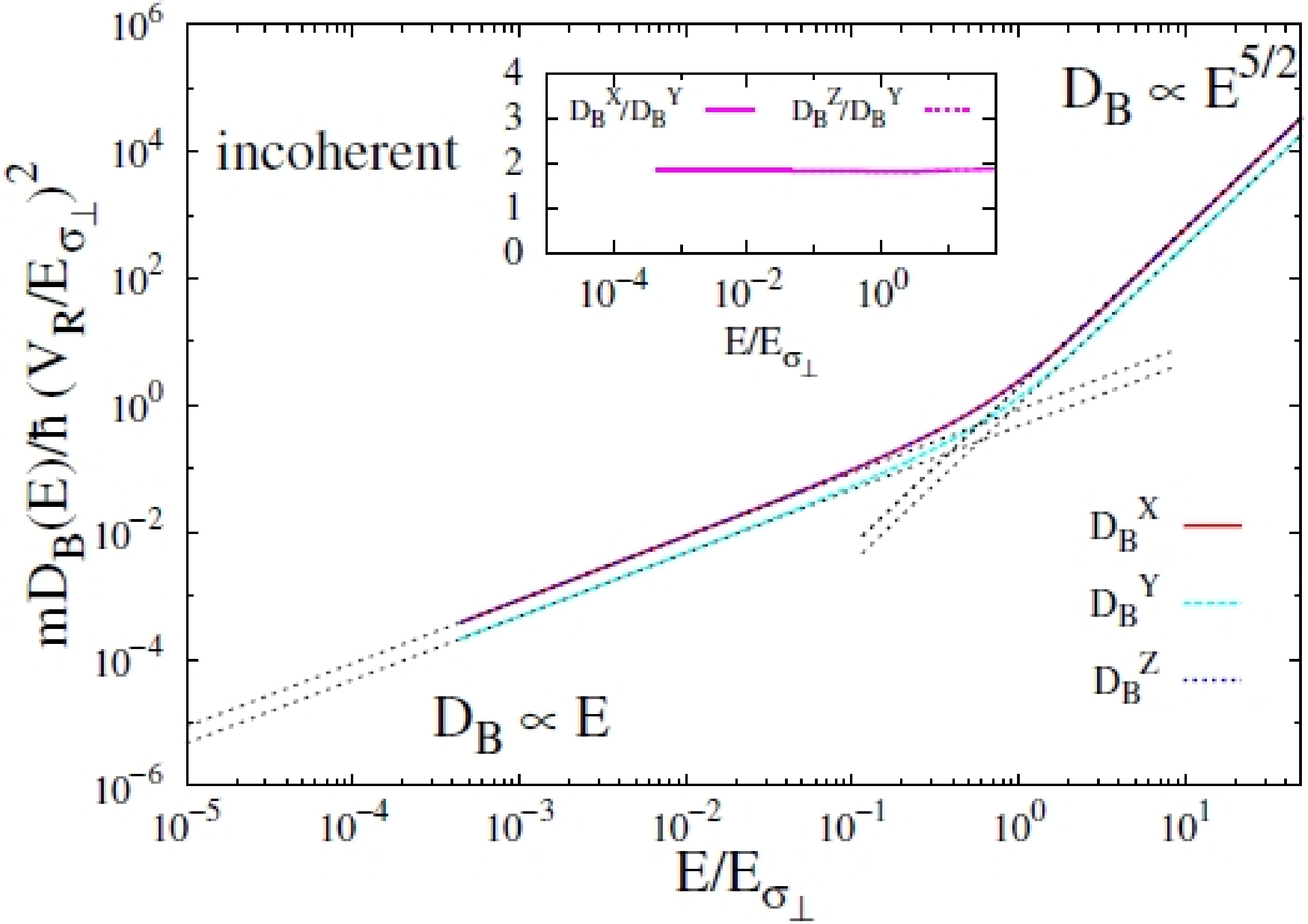
~constant transport anisotropy $D_B^Z/D_B^{X,Y} \approx 10$

Incoherent (Boltzmann) Transport

Piraud *et al.*, arXiv:1112.2859

- Results : incoherent-speckles configuration





For E→0

*) D(E) ~ E

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*) persisting anisotropy

For E→∞

*) D(E) ~ $E^{5/2}$

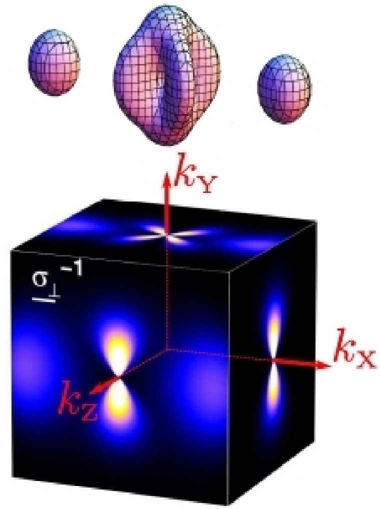
~constant transport anisotropy

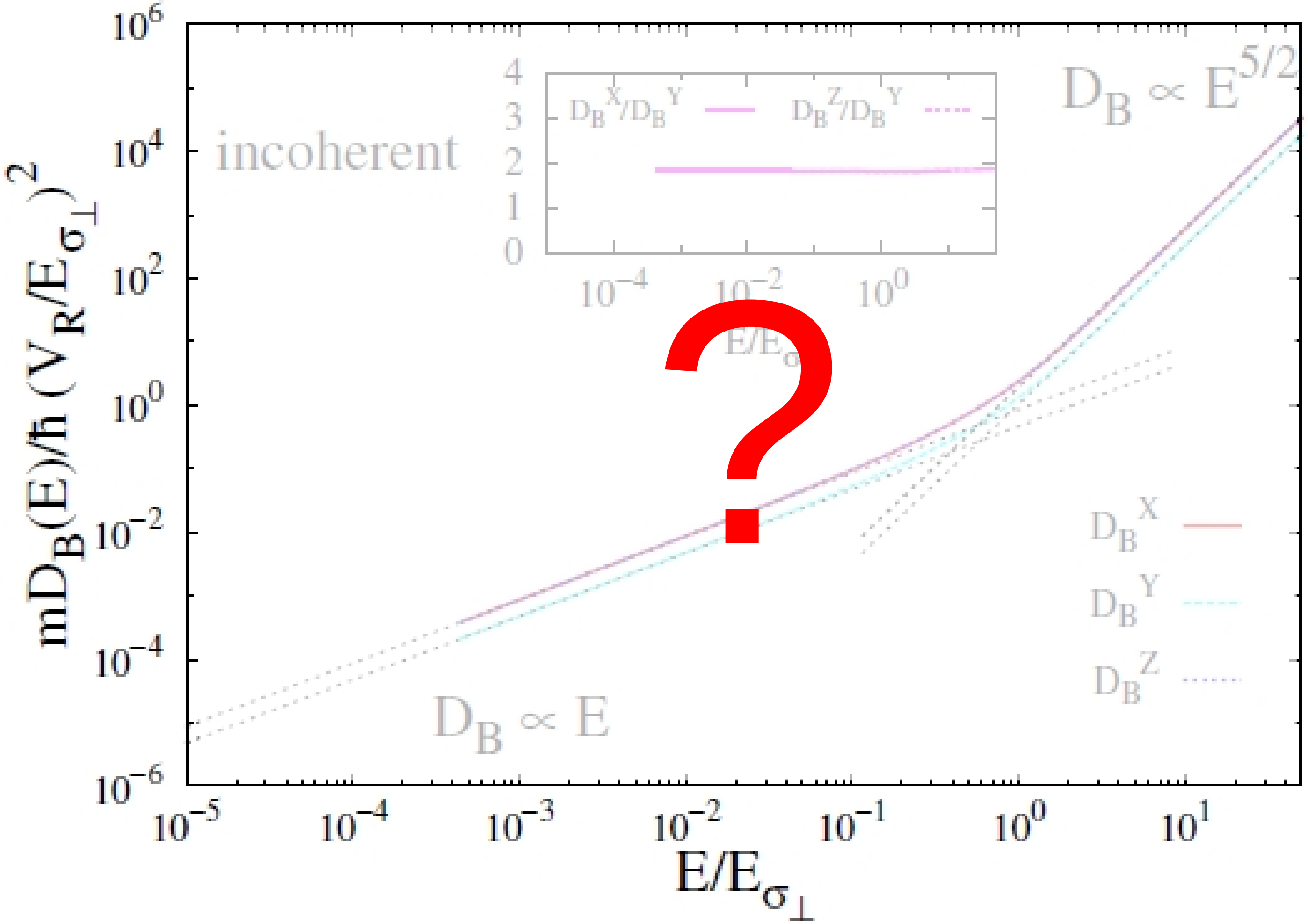
$D_B^Z/D_B^{X,Y} \approx 2$

Incoherent (Boltzmann) Transport

Piraud *et al.*, arXiv:1112.2859

- Results : coherent-speckles configuration

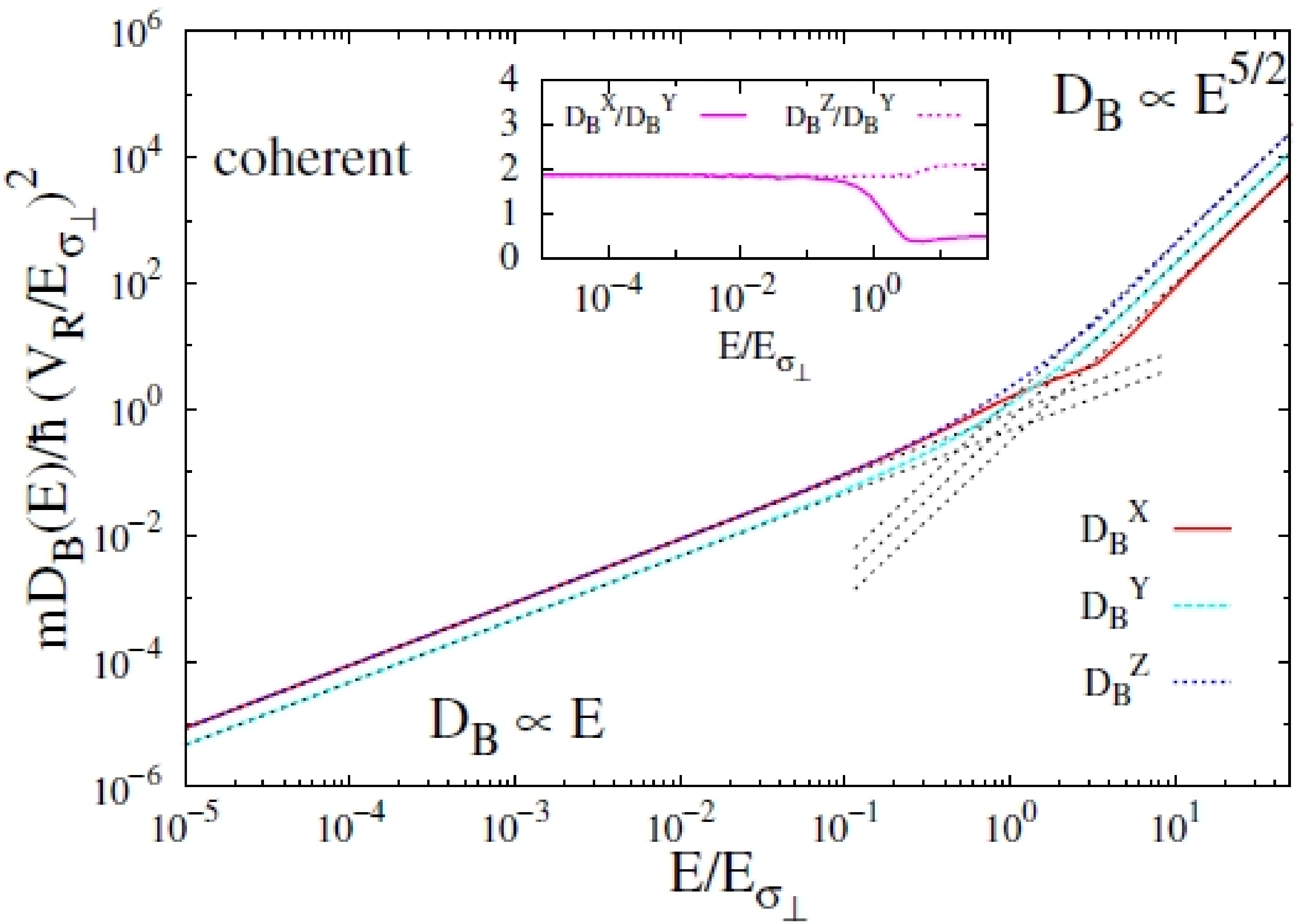


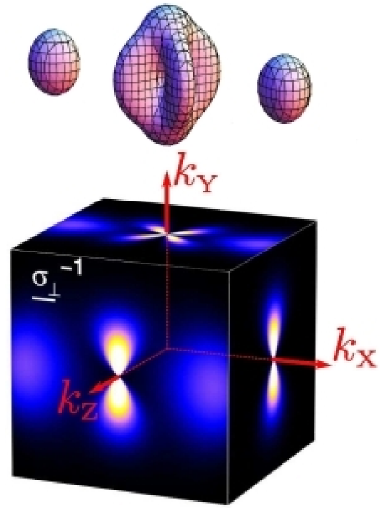


Incoherent (Boltzmann) Transport

Piraud *et al.*, arXiv:1112.2859

- Results : coherent-speckles configuration





For $E \ll E_{\sigma\perp}$

$D_B^X/D_B^Y \approx D_B^Z/D_B^Y \approx 2$

For $E \gg E_{\sigma\perp}$

$D_B^Z/D_B^Y \approx 2$

$D_B^X/D_B^Y \approx 0.5$

Inversion of transport anisotropy

Outlook

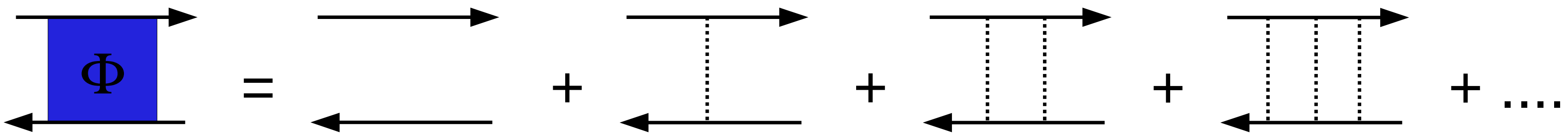
Lecture #2

1. The (not always) nice features of 3D speckle potentials

2. Quantum transport theory in anisotropic 3D disorder
    - 2.1 Incoherent transport
    - 2.2 Quantum corrections
    - 2.3 Self-consistent approach
    - 2.4 Discussion of recent experiments

3. About the Mobility edge
    - 3.1 Shifted on-shell approach
    - 3.2 Discussion of recent experiments

- Boltzmann approximation

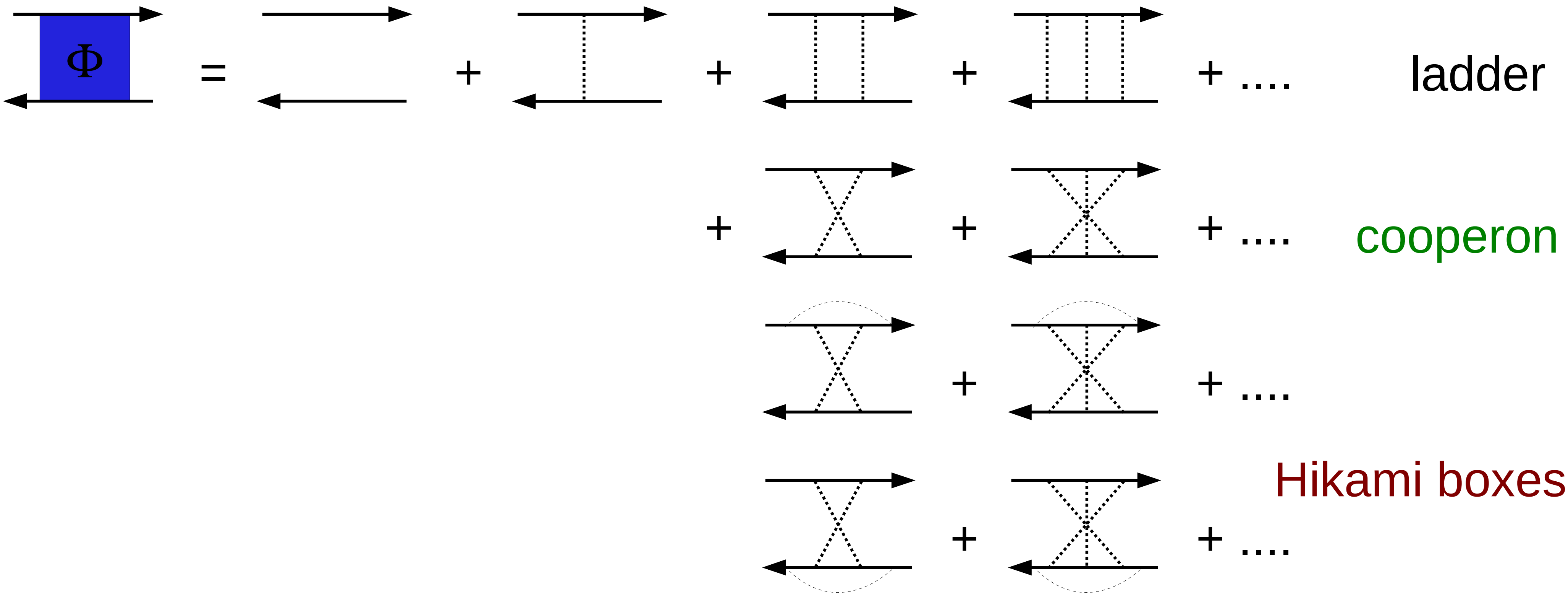


ladder

Quantum Corrections (Interference Terms)

Vollhardt & Wölfle, in *Electronic Phase Transitions* (1992)

- Take into account the maximally-crossed diagrams



$$\frac{\mathbf{D}_*(E,\omega)}{\mathbf{D}_\mathrm{B}(E)} = 1 - \frac{1}{\pi\hbar N_0(E)} \int \frac{\mathrm{d}\mathbf{q}}{(2\pi)^d} \frac{1}{-i\omega + \mathbf{q}\cdot\mathbf{D}_\mathrm{B}(E)\cdot\mathbf{q}}$$

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- Self-consistent theory

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↓

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Self-Consistent Solution

Vollhardt & Wölfle, in *Electronic Phase Transitions* (1992)

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- Self-consistent solution for $\omega \to 0$
  - Energy-threshold E_c (« mobility edge », see later)

$$D_\mathrm{B}^\mathrm{av}(E_\mathrm{c}) \equiv \det\{\mathbf{D}_\mathrm{B}(E_\mathrm{c})\}^{1/3} = \hbar/\sqrt{3\pi}m$$

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  - For $E<E_c$, $D_*(E,\omega) \to -i\omega L_{loc}(E)^2$, with $L_{loc}(E)$ a real definite positive tensor

Localization regime

Self-Consistent Solution

Vollhardt & Wölfle, in *Electronic Phase Transitions* (1992)

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  - For $E<E_c$, $D_*(E,\omega)\to -i\omega L_{loc}(E)^2$, with $L_{loc}(E)$ a real definite positive tensor

Localization regime

  - For $E>E_c$, $D_*(E,\omega)\to D_*(E)$, a real definite positive tensor

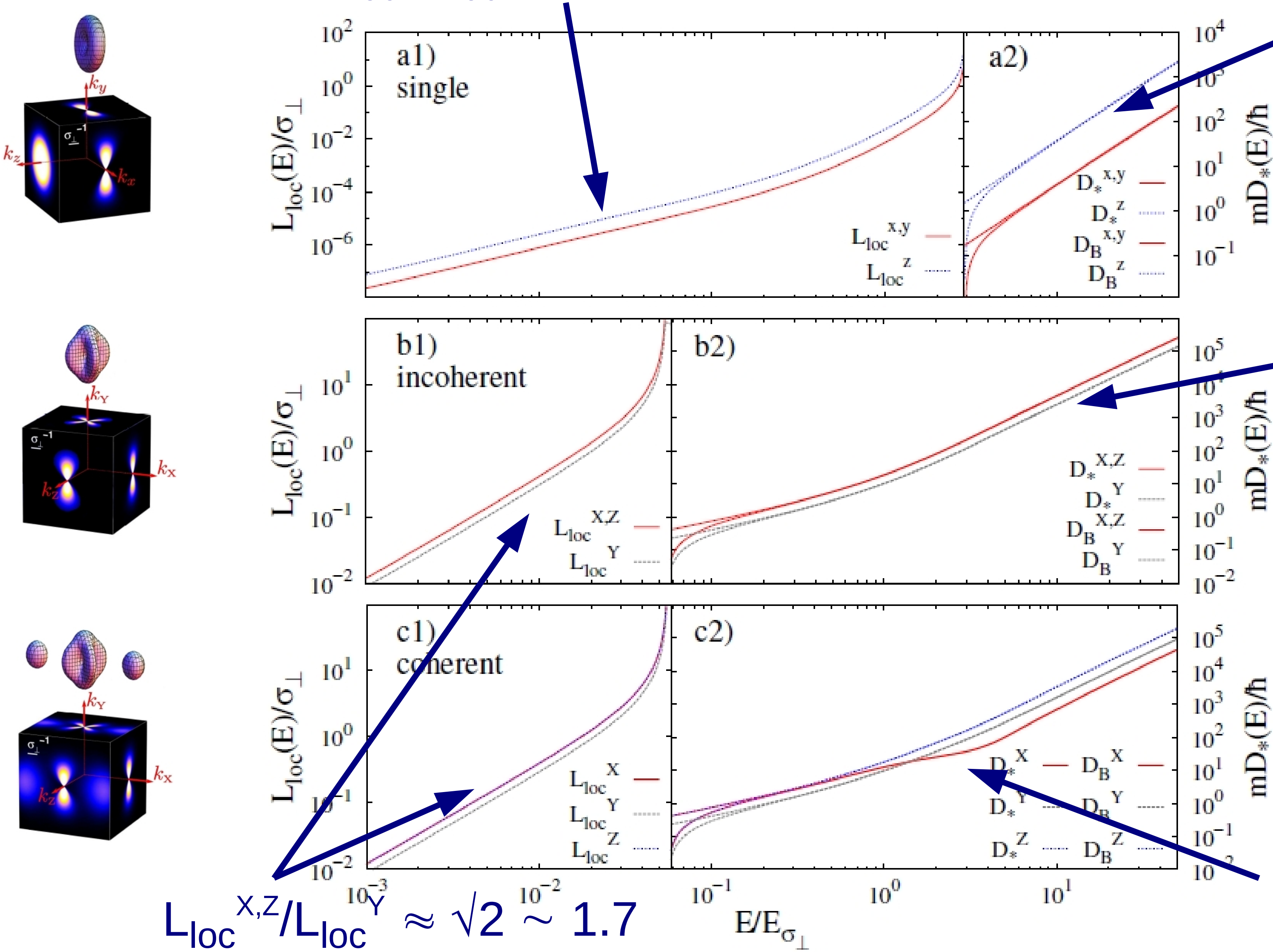
Diffusive regime

Self-Consistent Solution

Piraud *et al.*, arXiv:1112.2859

$L_{loc}^{z}/L_{loc}^{x,y} \approx \sqrt{10} \sim 3$

$D_*^{z}/D_*^{x,y} \approx 10$



$D_*^{z}/D_*^{x,y} \approx 2$

inversion of anisotropy

$L_{loc}^{X,Z}/L_{loc}^{Y} \approx \sqrt{2} \sim 1.7$

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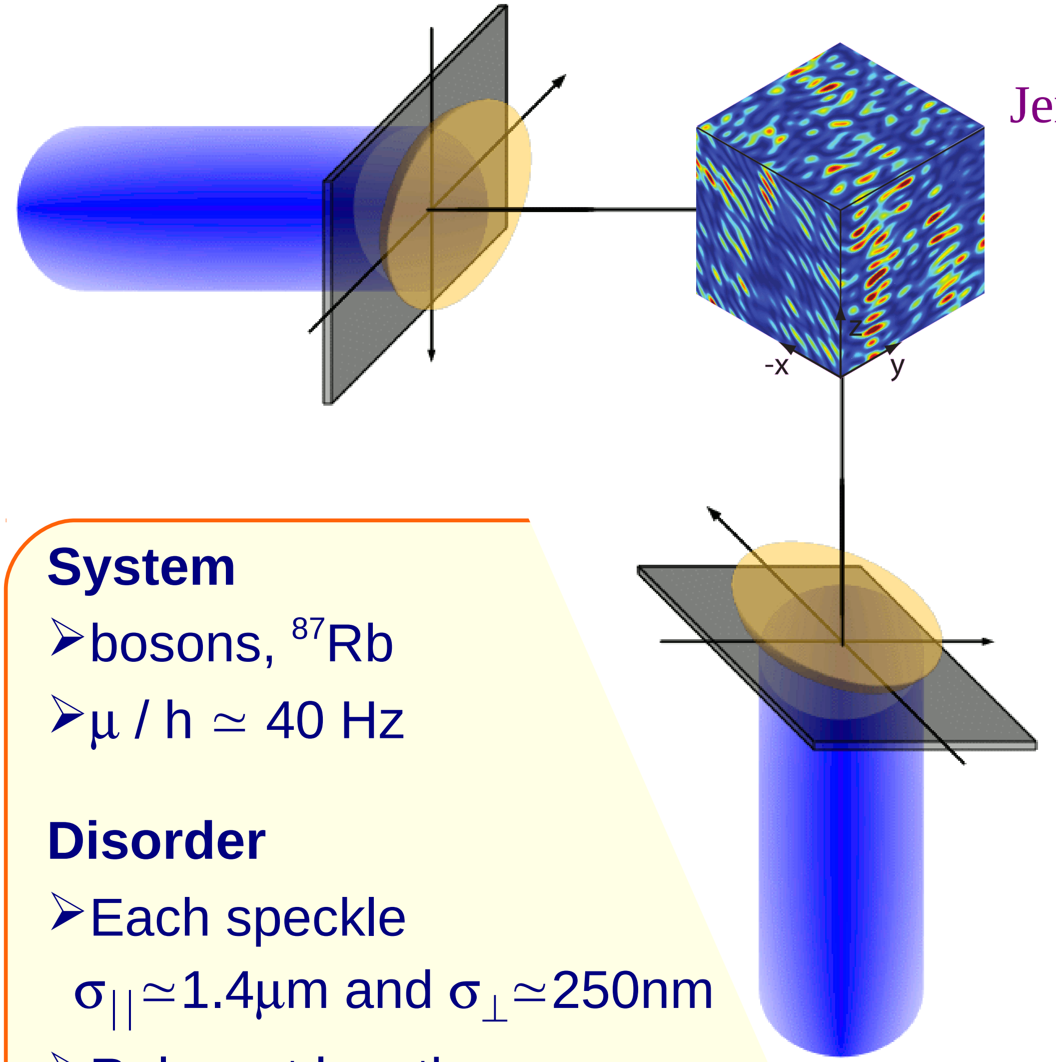
Comparison to Experiments (1/2)

experiment @ Aspect's group
(Institut d'Optique, France)
Jendrzejewski *et al.*, Nature Phys. **8**, 398 (2012)



System

- bosons, ^{87}Rb
- μ / h ≃ 40 Hz

Disorder

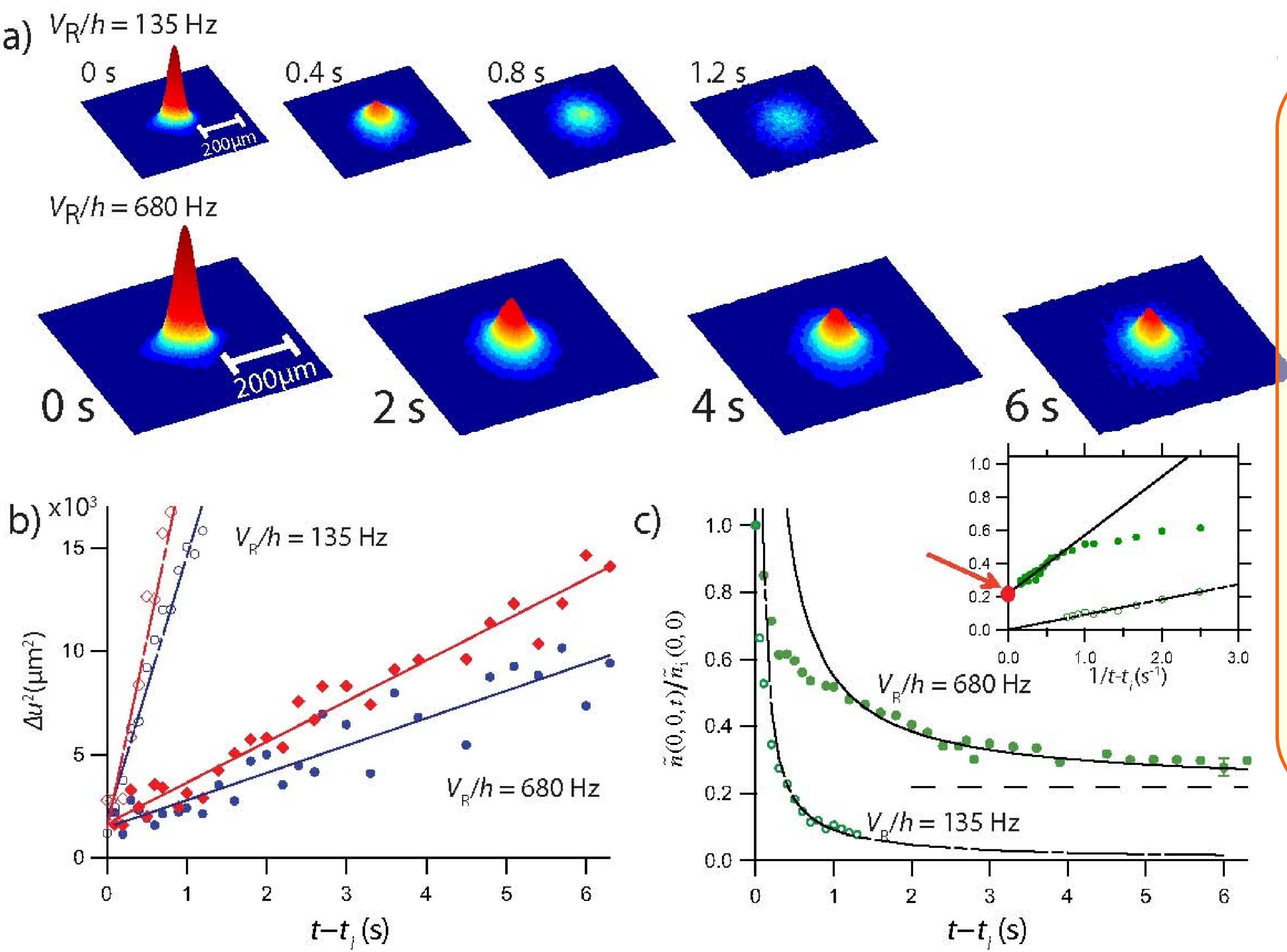
- Each speckle
  $\sigma_{||} \simeq 1.4\mu$m and $\sigma_{\perp} \simeq 250$nm
- Relevant lengths
  $\sigma_x \simeq$ 110nm ; $\sigma_y \simeq$ 270nm; $\sigma_x \simeq$ 80nm
- $V_R/k_B \simeq$ 0 – 1100 Hz (blue-detuned)

Experimental sequence

- Prepare a trapped BEC (evaporation ; no disorder)
- Trap suddenly shut off (t=0)
- Free expansion (0<t<50ms) : interaction energy decreases
- Disorder suddenly switched on (t=50ms)
- Expansion in the disorder (variable duration)
- Image (density column profiles)

Comparison to Experiments (1/2)

experiment @ Aspect's group
(Institut d'Optique, France)
Jendrzejewski *et al.*, Nature Phys. **8**, 398 (2012)



Main results

- ➢Twofold behavior
  - overall diffusion
  - localized part
- ➢Moderate but clear transport anisotropy

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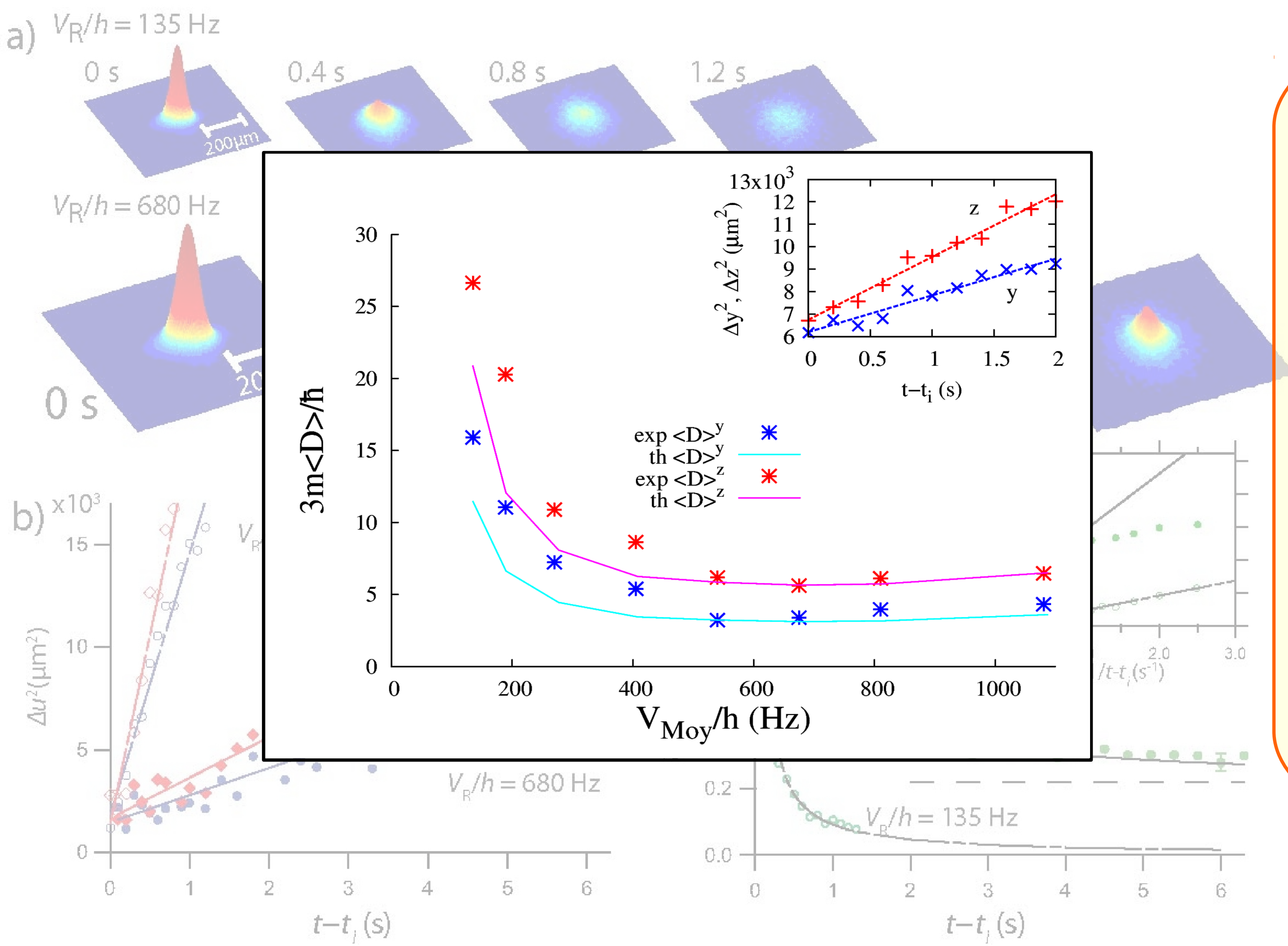
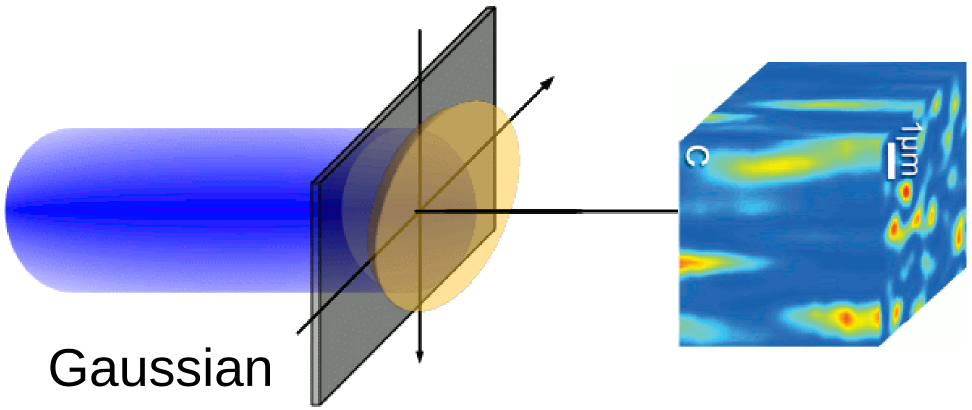


Main results

- ➢Twofold behavior
  - – overall diffusion
  - – localized part
- ➢Moderate but clear transport anisotropy
- ➢Quantitative agreement for diffusion tensor

Comparison to Experiments (1/2)

experiment @ DeMarco's group
(Urbana Champain, USA)
Kondov *et al.*, Science **334**, 66 (2011)



Gaussian
intensity profile

System

- Polarized fermions, ^{40}K (no interactions)
- $T \simeq$ 200-1500 nK

Disorder

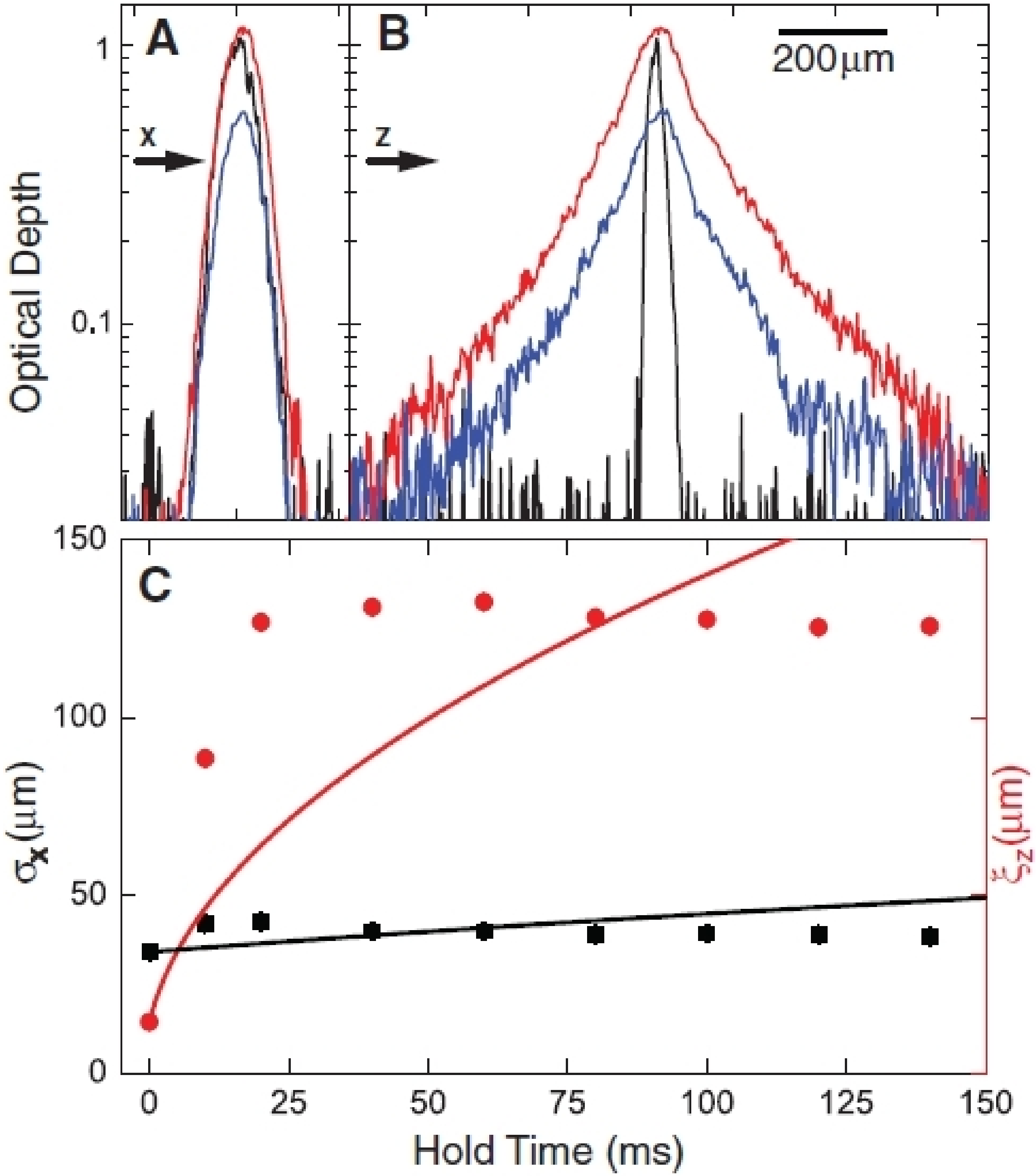
- $\sigma_{||} \simeq 1.6\ \mu$m and $\sigma_{\perp} \simeq 270$ nm
- disorder anisotropy $\simeq 5.9$
- $V_R/k_B \simeq 0 - 1000$ nK (blue-detuned)

Experimental sequence

- Prepare ultracold trapped fermions (evaporation; $|\uparrow\rangle$ & $|\downarrow\rangle$)
- Spin polarization : select $|\uparrow\rangle$
- Disorder ramped up within 200ms
- Trap suddenly shut off (t=0)
- Expansion in the disorder (variable duration)
- Image (density column profiles)

Comparison to Experiments (1/2)

experiment @ DeMarco's group
(Urbana Champain, USA)
Kondov *et al.*, Science **334**, 66 (2011)



Main analysis

- Twofold structure
  - mobile (red)
  - localized (blue)
- Very strong anisotropy ($\gg$3)

Comparison to Experiments (1/2)

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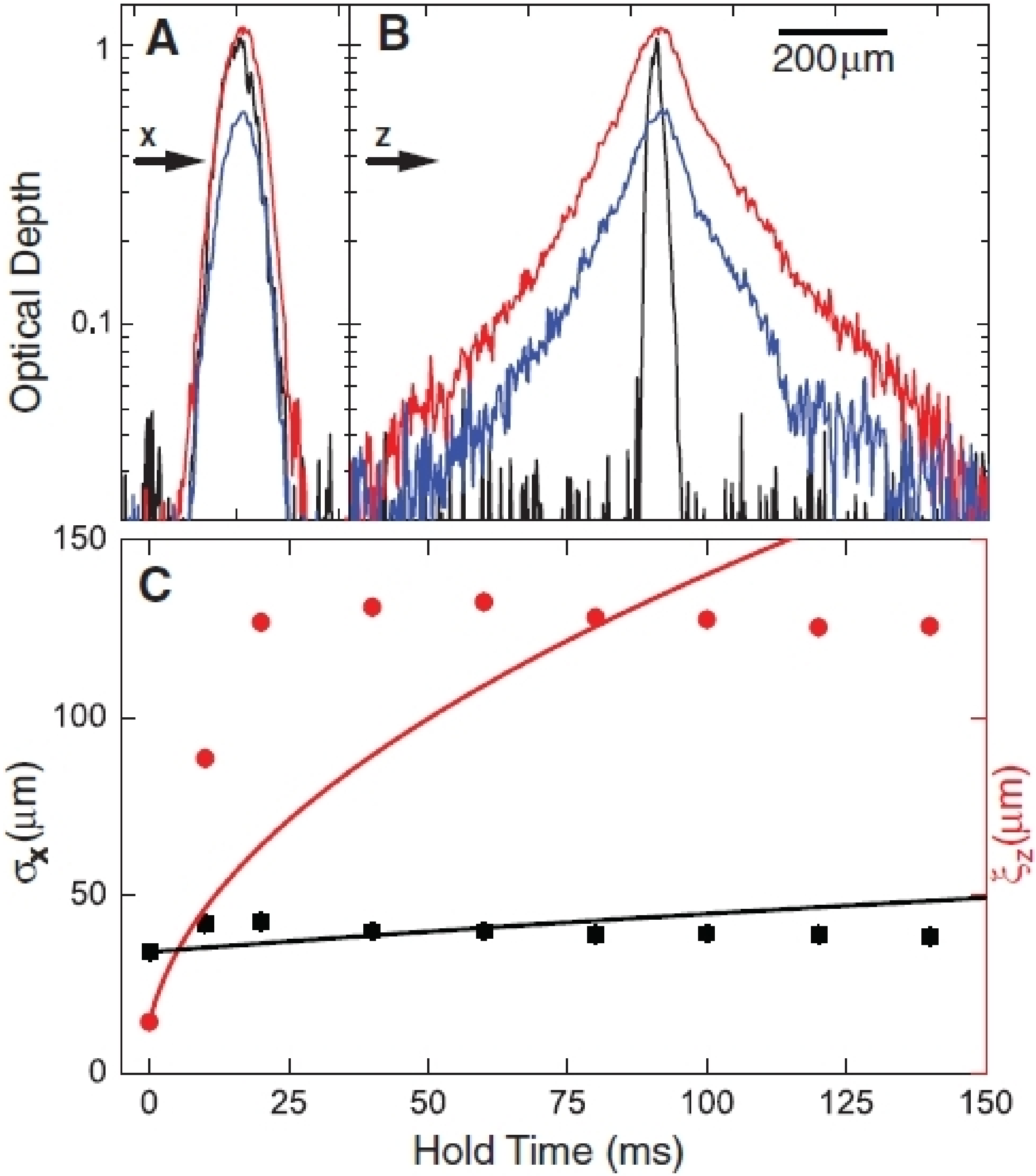

Main analysis

- Twofold structure
  - mobile (red)
  - localized (blue)
- Very strong anisotropy ($\gg$3)

Strong discrepancy with theory

Need :

- more experimental insight (diffusive part, dynamics in the transverse direction, ...) ?
- refined theory ?

Outlook

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About the Mobility Edge

- On-shell approximation (used so far)
  - Roughly assumes that $A(k,E) \approx \delta(E-\hbar^2k^2/2m)$
  - It leads to E_c such that $D_{\mathrm{B}}^{\mathrm{av}}(E_{\mathrm{c}}) \equiv \det\{\mathbf{D}_{\mathrm{B}}(E_{\mathrm{c}})\}^{1/3} = \hbar/\sqrt{3\pi}m$
  - Agrees with Ioffe-Regel criterion (ie localization for $k^* l_B \sim 1$)

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  - It leads to E_c such ... $\{D_B(E_c)\}^{1/3} = \hbar/\sqrt{3\pi}m$
  - Agrees with ... Regel criterion (ie localization for k*l_B~1)

Not accuracte for determining E_c !

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  - Agrees with ... Regel criterion (ie localization for $k^* l_B \sim 1$)

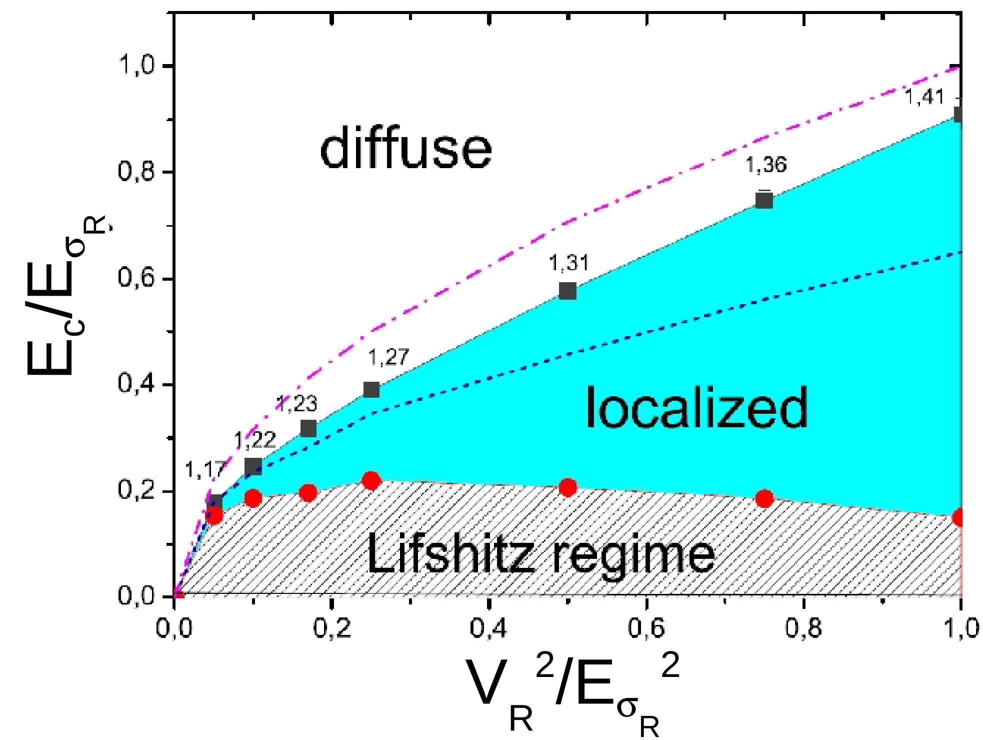
Not accuracte for determining E_c !

- Beyond the on-shell approximation : SCBA + Kubo

$C(\mathbf{r}) = V_R^{\,2}\,\mathrm{sinc}^2\left(|\mathbf{r}|/\sigma_R\right)$

Here, convention is $\overline{V} = V_R$

A. Yedjour & B.A. Van Tiggelen,
Eur. Phys. J. **58**, 249 (2010)



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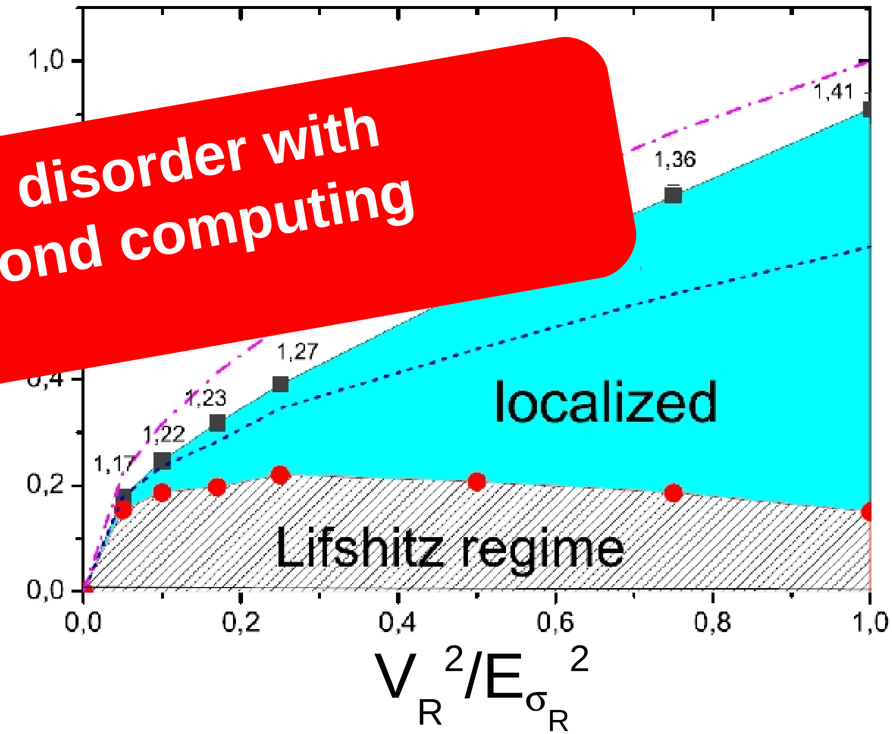
A. Yedj...
Eur. Phy... (2010)



Can hardly be applied for 3D disorder with structured correlations (beyond computing cluster possibilities) !

About the Mobility Edge

Piraud *et al.*, arXiv:1112.2859

- Beyond the on-shell approximation : shifted on-shell

spectral function

real part of the self-energy

imaginary part of the self-energy

$$A(\mathbf{k},E)=\frac{2\Sigma''(\mathbf{k},E)}{[E-\epsilon(\mathbf{k})-\Sigma'(\mathbf{k},E)]^2+[\Sigma''(\mathbf{k},E)]^2}$$

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finite lifetime to $|\mathbf{k}\rangle$

$\rightarrow \tau_s(\mathbf{k})$

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imaginary part of the self-energy

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energy shift (found self-consistently)

$E - \varepsilon(k_E) - \Delta(E) = 0$

$\Delta(E) = \Sigma''(k_E,E)$

finite lifetime to $|\mathbf{k}\rangle$

$\rightarrow \tau_s(\mathbf{k})$

About the Mobility Edge

Piraud *et al.*, arXiv:1112.2859

Shifted on-shell prescription: $\varepsilon(\mathbf{k}) = E' = E - \Delta(E)$

- all preceeding quantities D_B, D_*, L_{loc} regarded as function of E' (instead of E)
- mobility edge : $E_c - \Delta(E_c) = E'_c$

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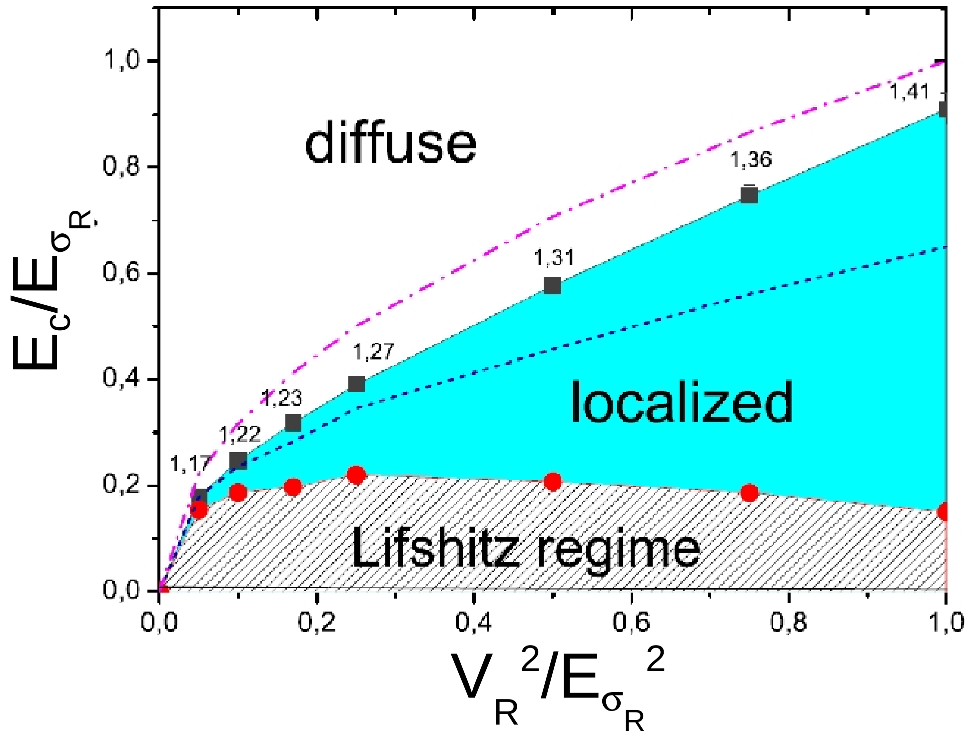

About the Mobility Edge

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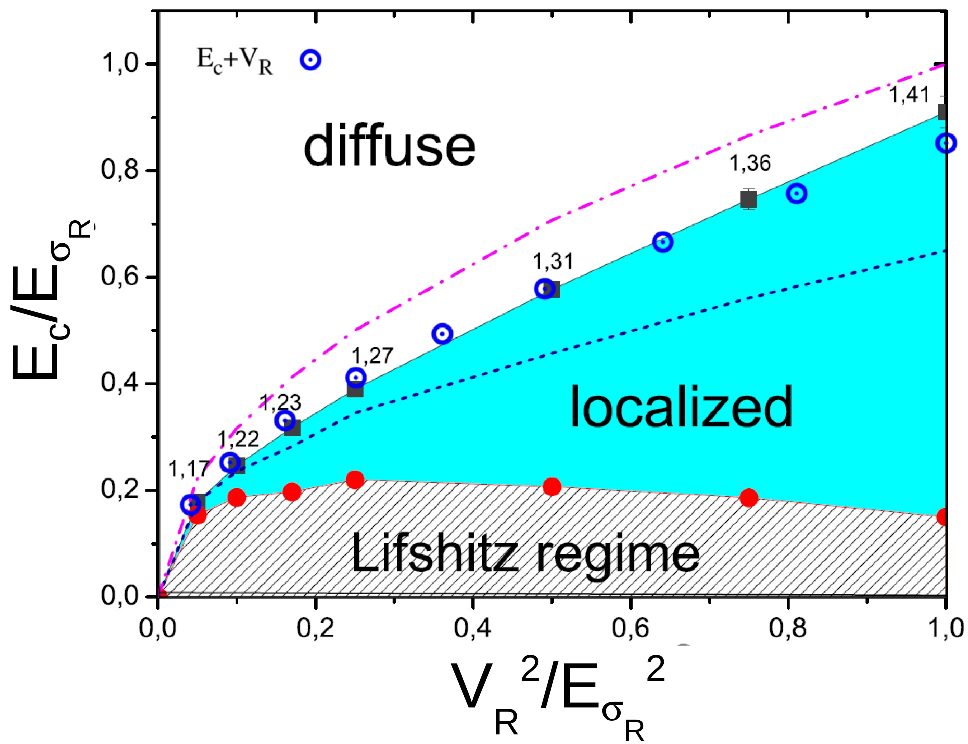


About the Mobility Edge

Piraud *et al.*, arXiv:1112.2859

- Self-consistent energy shift

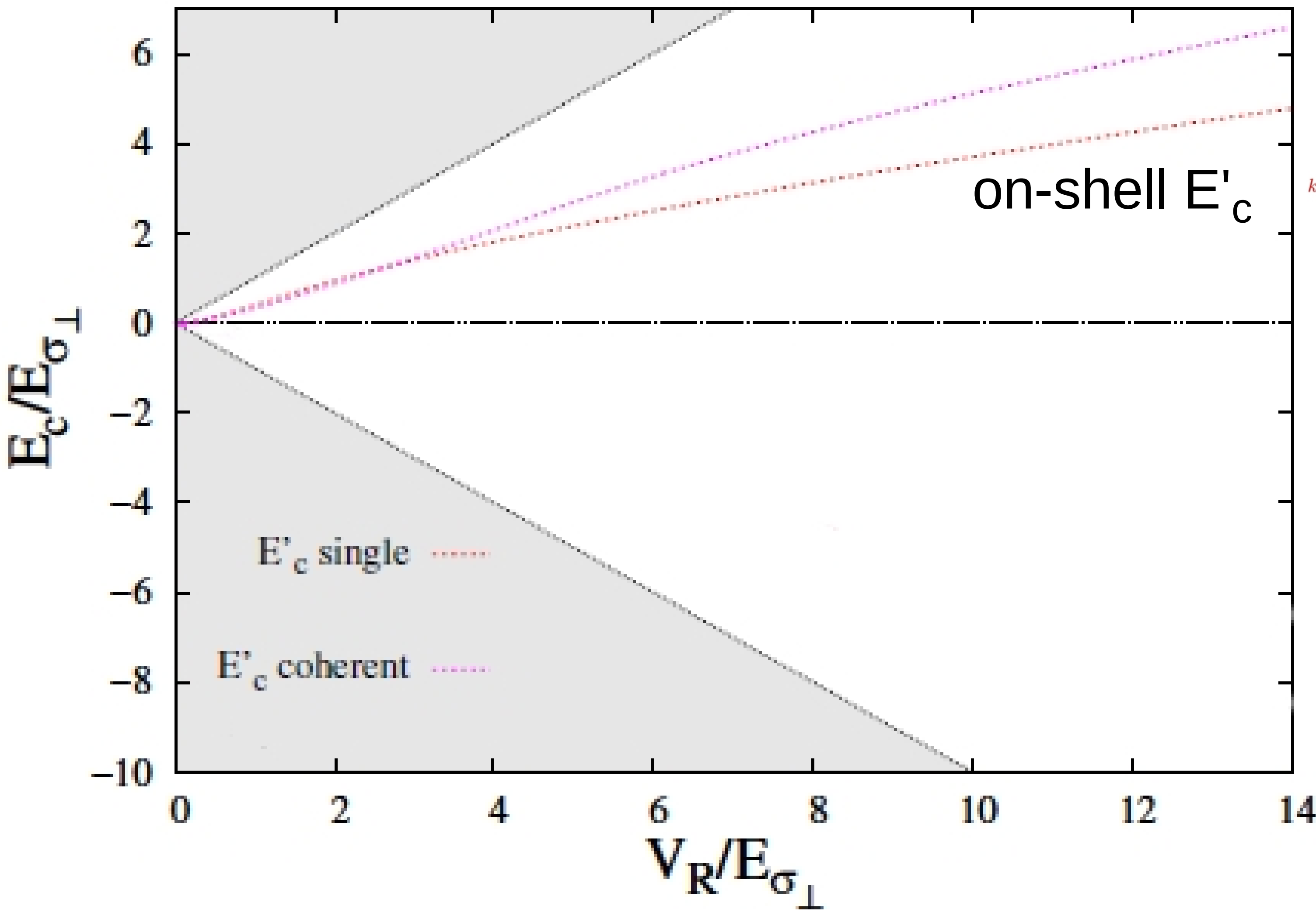
$$A(\mathbf{k},E) = \frac{2\Sigma''(\mathbf{k},E)}{[E-\epsilon(\mathbf{k})-\Sigma'(\mathbf{k},E)]^2+[\Sigma''(\mathbf{k},E)]^2}$$

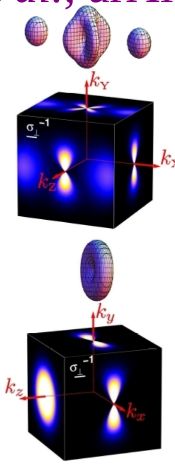
angle average (OK within 10-15%) → pure energy shift

$$\Delta(E) \equiv \frac{1}{4\pi}\int_{\epsilon(\mathbf{k})=E-\Delta(E)} \mathrm{d}\Omega_{\hat{\mathbf{k}}}\ \Sigma'(\mathbf{k},E)$$

About the Mobility Edge

Piraud *et al.*, arXiv:1112.2859





About the Mobility Edge

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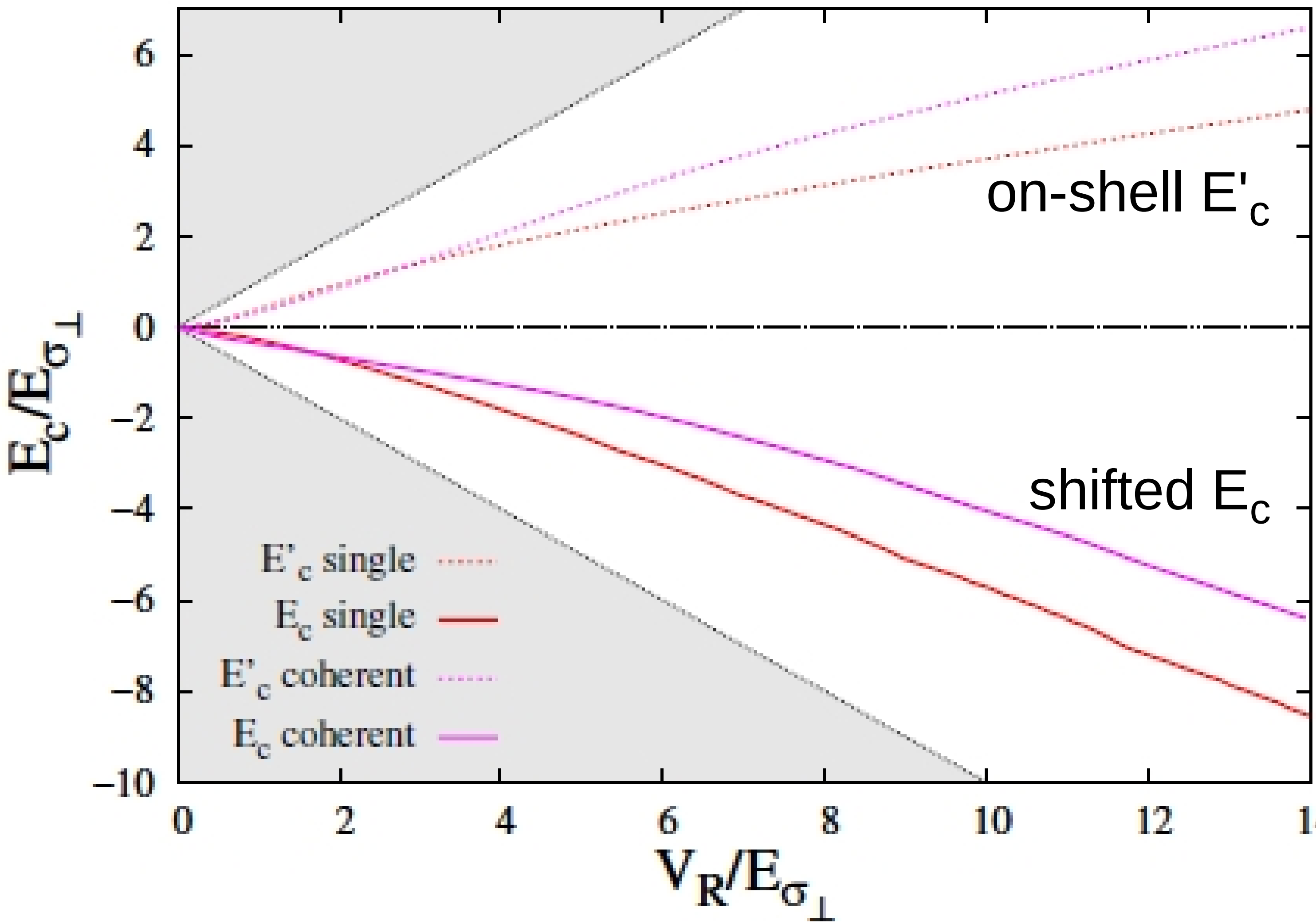

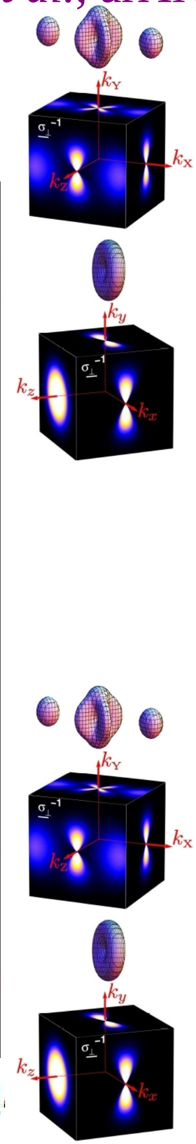

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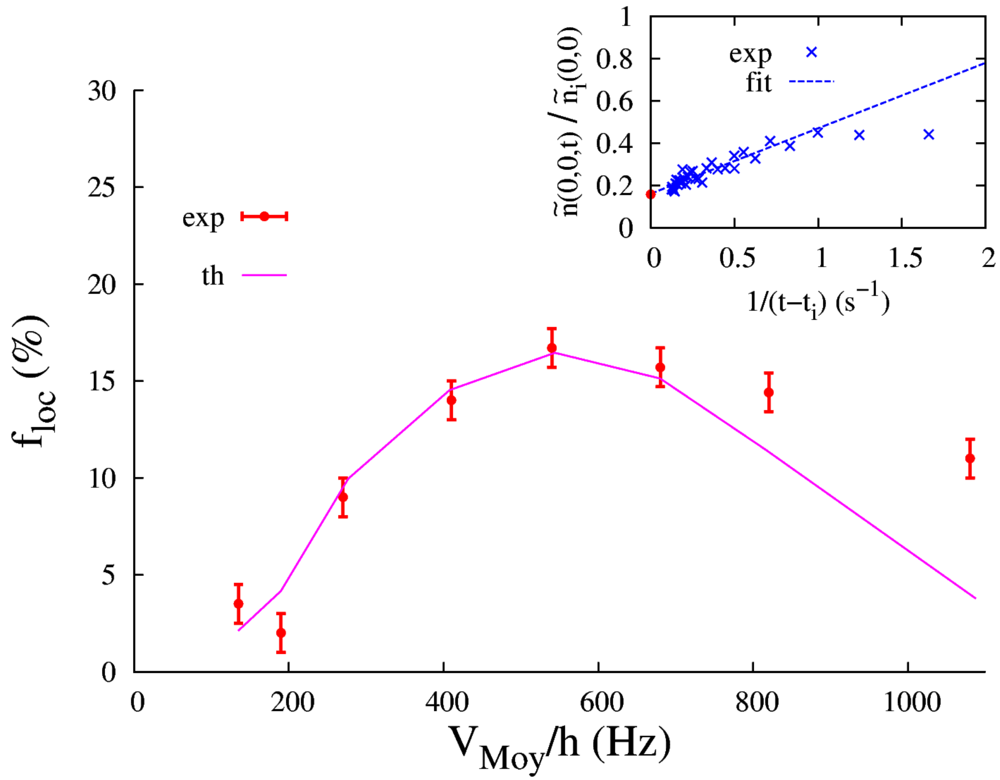

Main results

- From the twofold structure, extract the localized fraction
  - f_{loc} « ~↗ » when V_R ↗
- Numerical determination of the energy distribution $\mathcal{D}(E)$ + on-shell SC theory of AL
  - fair agreement, provided, we introduce a heuristic shift in $\mathcal{D}(E)$

Comparison to Experiments (2/2)

experiment @ Aspect's group
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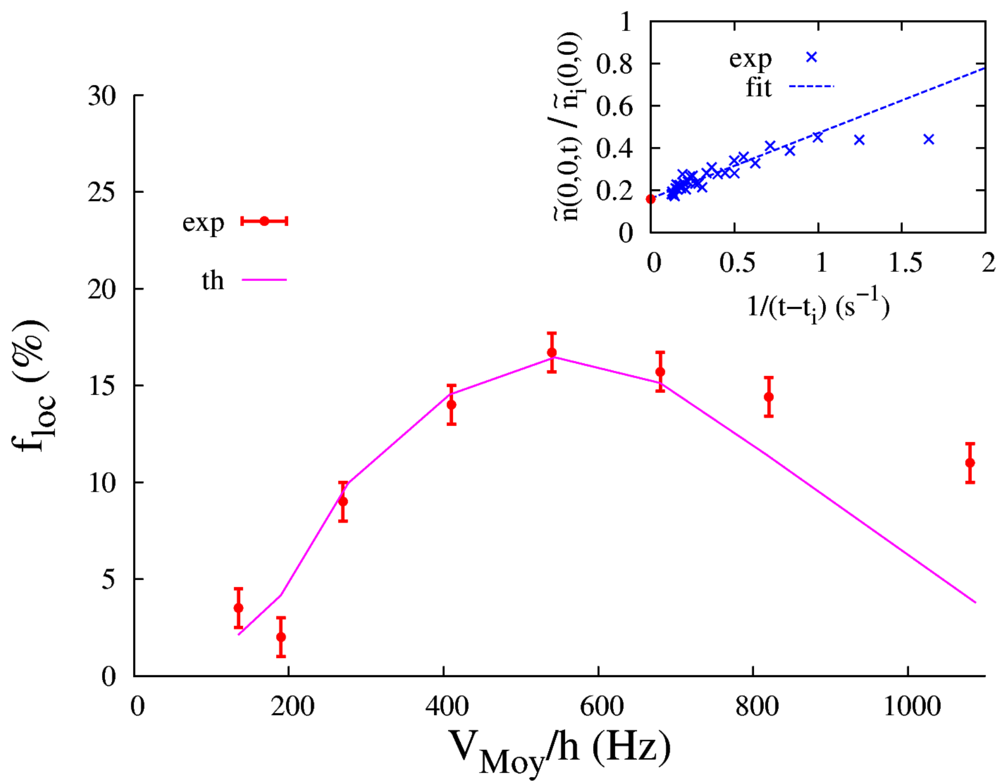
Jendrzejewski *et al.*, Nature Phys. **8**, 398 (2012)



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The heuristic shift may be explained by the real part of the self-energy $\Delta(E)$ as calculated here (agreement within a factor <1.7).

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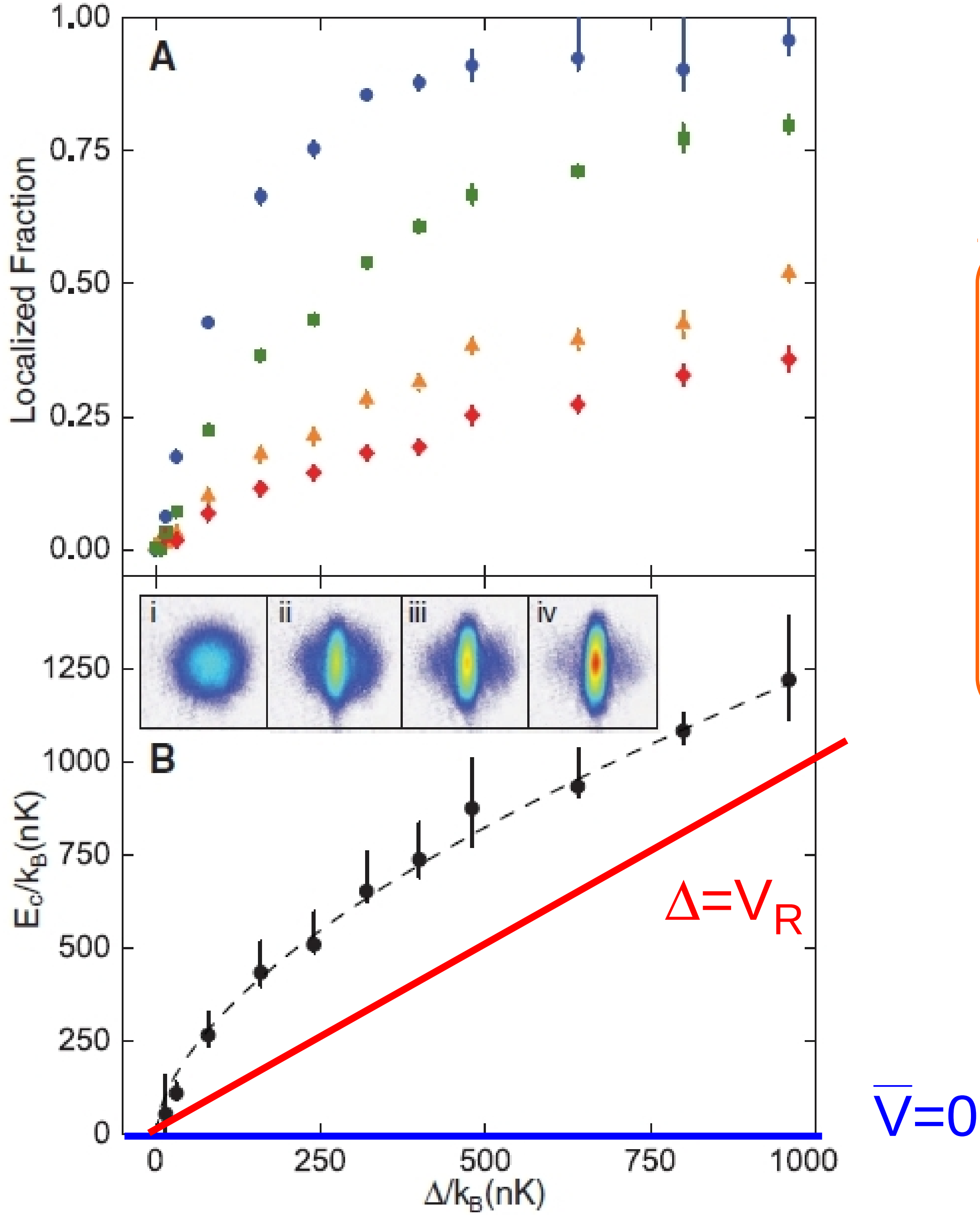
experiment @ DeMarco's group
(Urbana Champain, USA)
Kondov *et al.*, Science **334**, 66 (2011)



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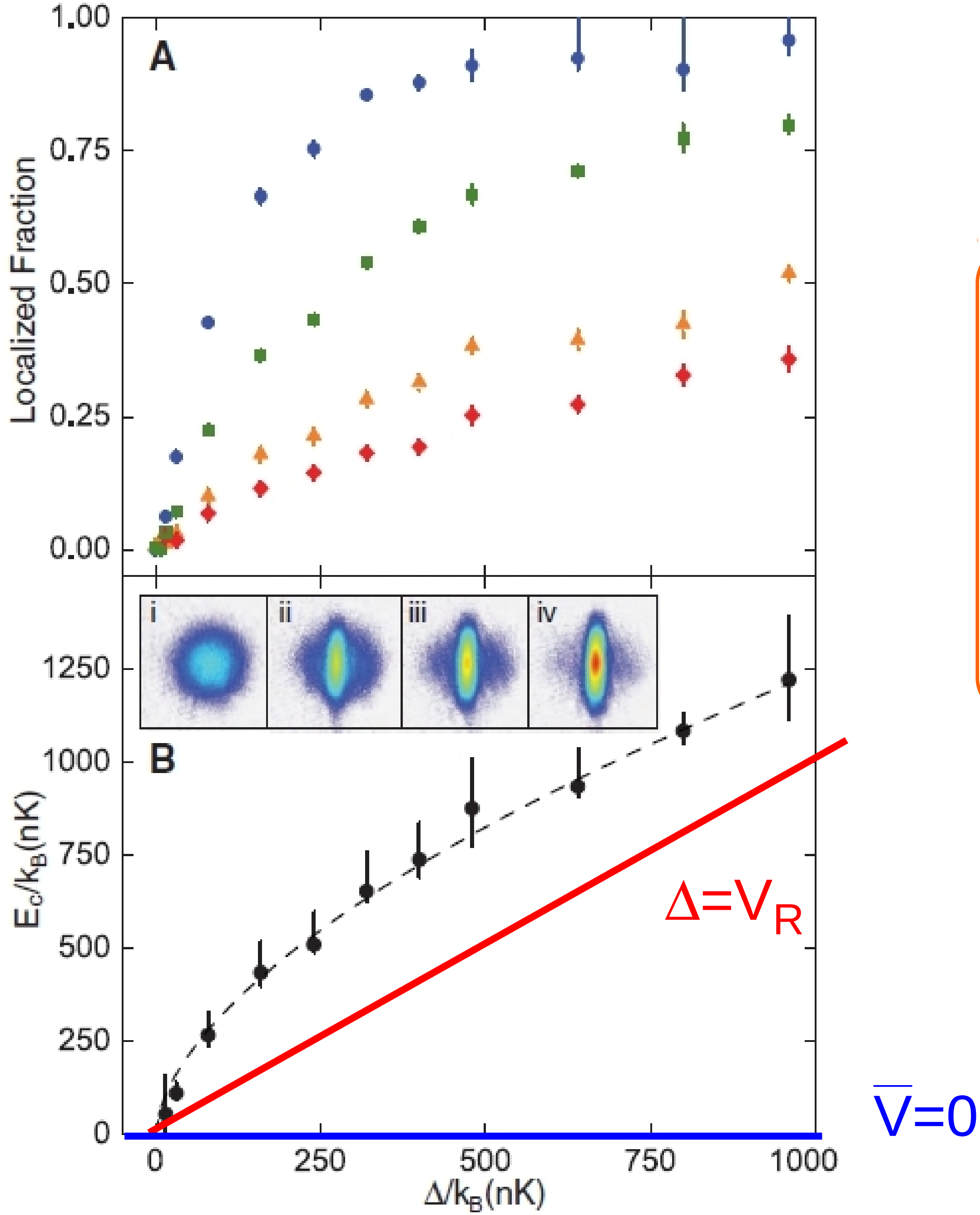

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In contrast, we predict $E_c<0$ (*ie* $E_c<\overline{V}$). However, experimental method does not allow for finding negative energies ....

Anderson Localization in a 3D Speckle Potential : Conclusions

- Self-consistent theory of AL for 3D anisotropic speckles
  - Coherent-speckles (Institut d'Optique)
    - quantitative agreement for the diffusion tensor
    - inversion of anisotropy to be observed (use suited imagery ...)
  - Single-speckle (Urbana Champain)
    - ~constant anisotropy : $D^z/D^{x,y} = 10$ (diffusion not observed)

      $L_{loc}^{z}/L_{loc}^{x,y} = 3.2$ ($\neq$ experimental images)

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  - Significant effect of $\Sigma'(\mathbf{k},E) \rightarrow$ shifted on-shell
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- Perspectives : beyond « shifted on-shell »
  - Calculate $A(\mathbf{k},E)$ self-consistently
  - Use Kubo formalism (collaboration with B.A. Van Tiggelen)

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