The Abdus Salam International Centre for Theoretical Physics

United Nations Educational, Scientific and Cultural Organization
IAEA International Atomic Energy Agency

2350-3

Workshop on Quantum Simulations with Ultracold Atoms

16 - 20 July 2012

Repulsive polarons in a strongly interacting Fermi gas

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Repulsive polarons in a strongly interacting Fermi gas

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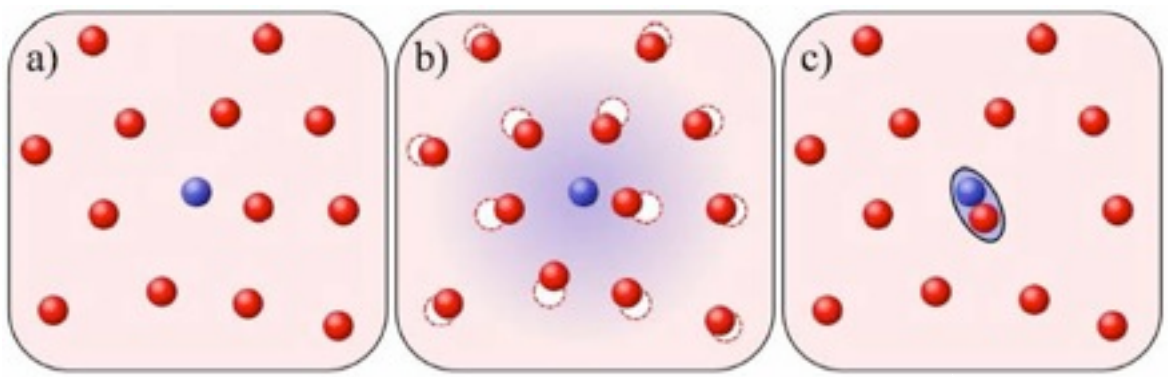
C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, P. Massignan, GMB, F. Schreck, R. Grimm, Nature **485**, 615 (2012)

Outline

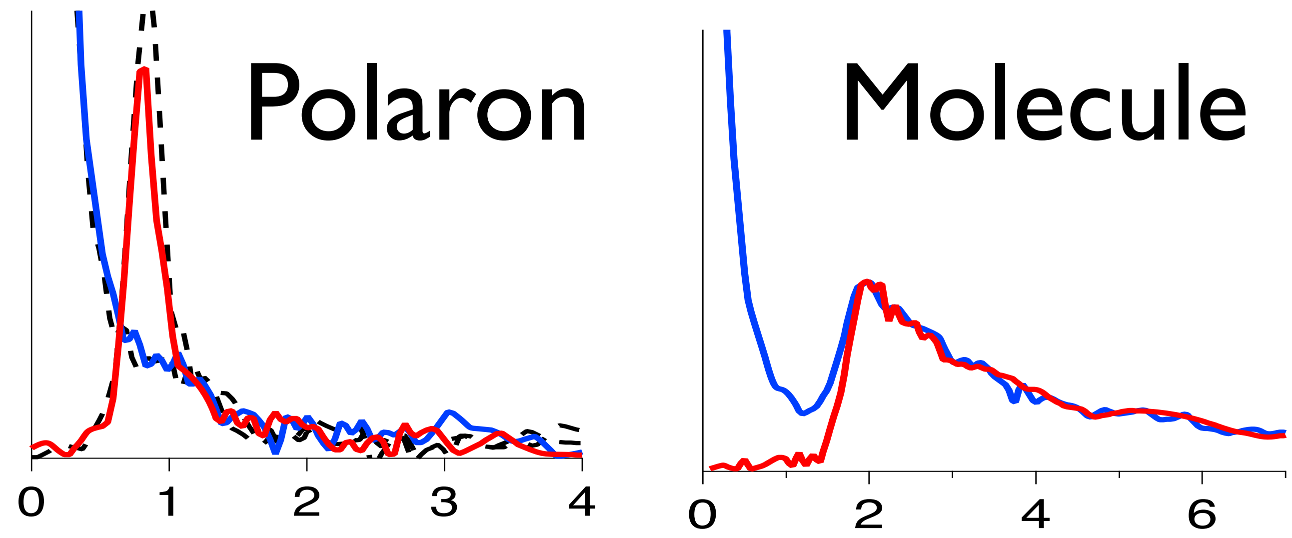
- Polarons & molecules: Main concepts & results
- 2-body physics: broad vs. narrow resonances
- Many-body theory & comparison with experiments
- Itinerant Ferromagnetism

Polarons and molecules

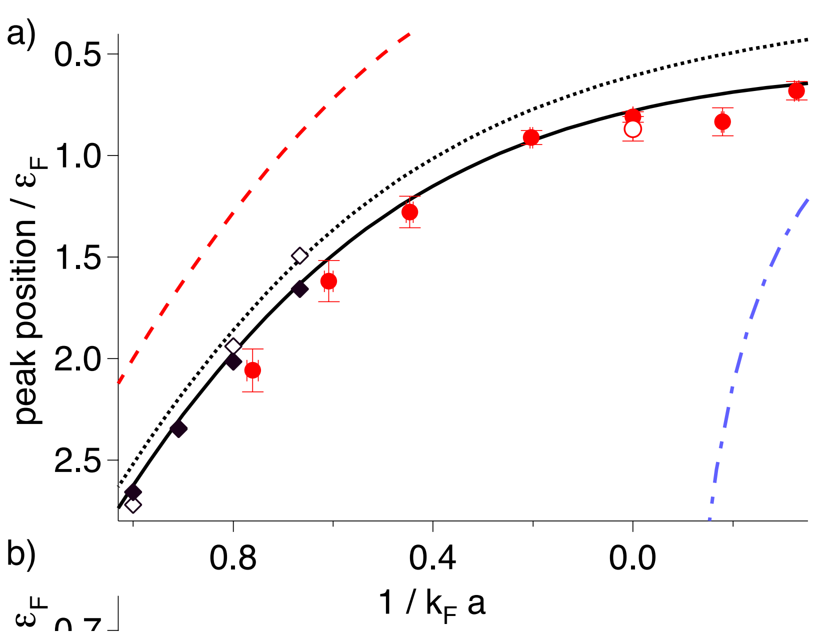
One ↓ in a Fermi sea of ↑'s

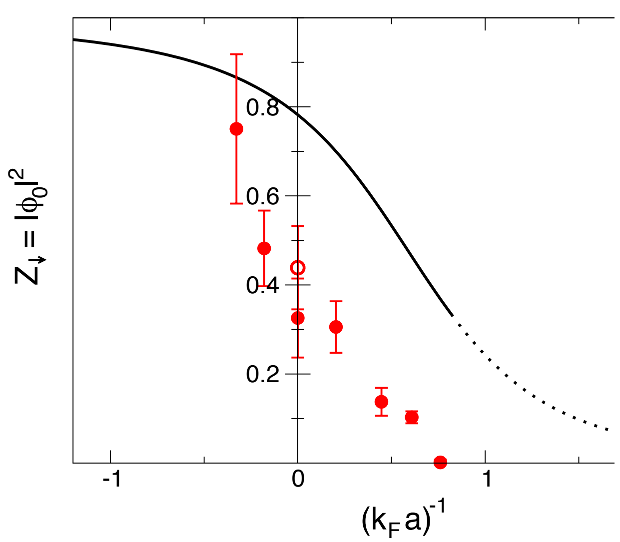


Increasing interaction

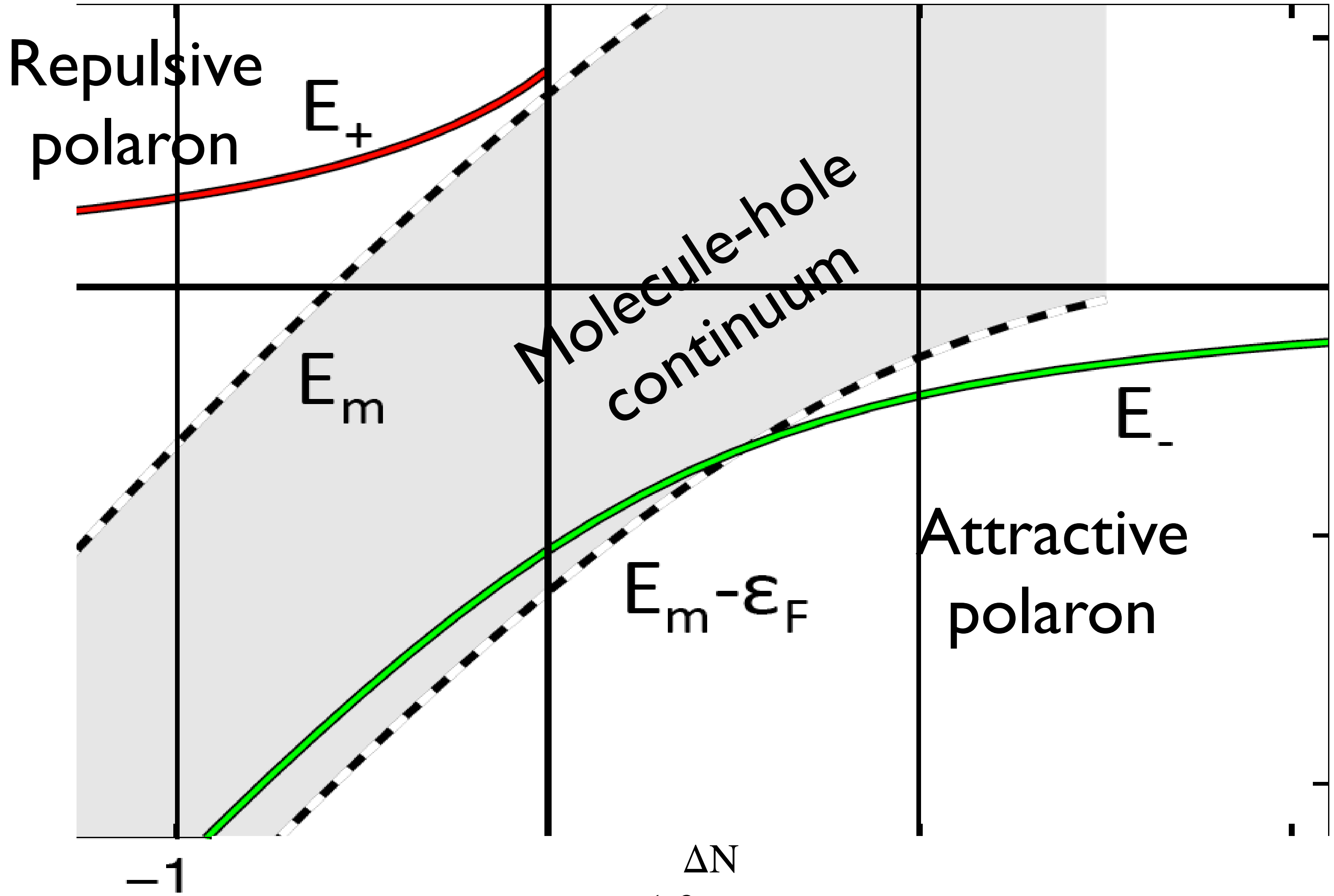


Zwierlein group





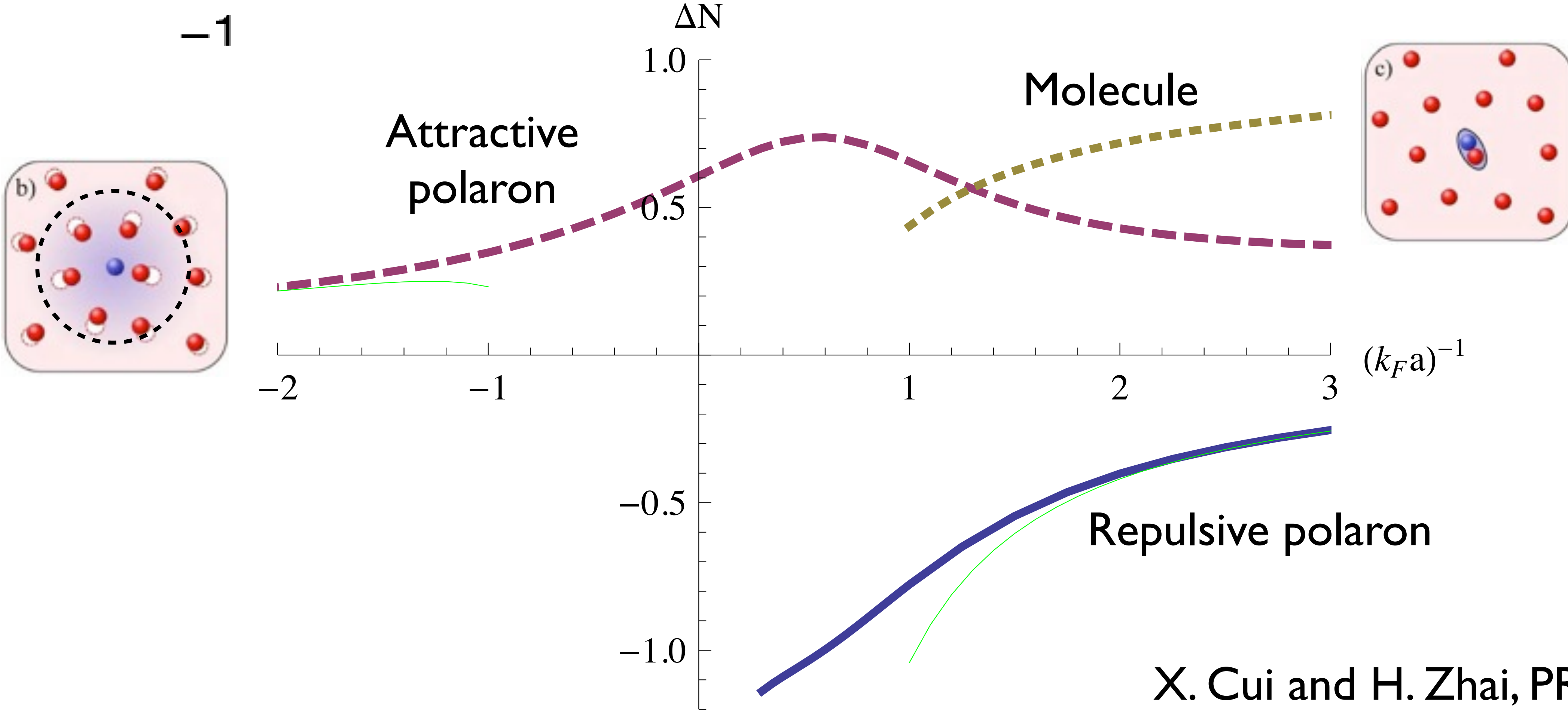
Chevy, Mora, Zwerger,
Punk, Combescot,
Leyronas, Recati, Lobo,
Prokof'ev, Svistunov, ...



Number of atoms $\Delta N_\uparrow$ in dressing cloud:

$$\delta\mu_\uparrow = \frac{\partial^2\epsilon}{\partial n_\uparrow \partial n_\downarrow} + \frac{\partial^2\epsilon}{\partial n_\uparrow \partial n_\uparrow}\Delta N_\uparrow = 0$$

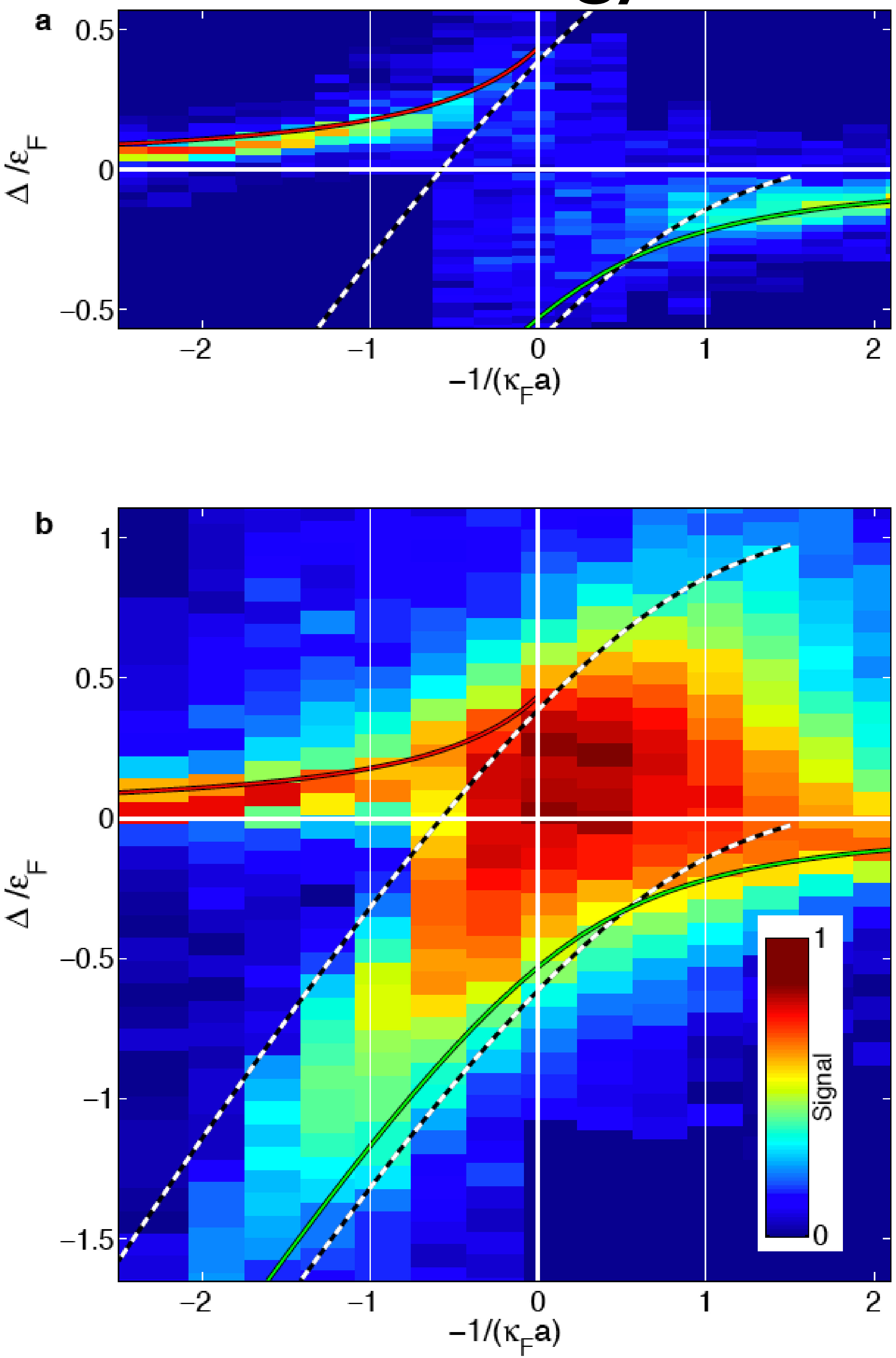
$$\Delta N_\uparrow = -\left(\frac{\partial \mu_\downarrow}{\partial \epsilon_F}\right)_{n_\downarrow}$$



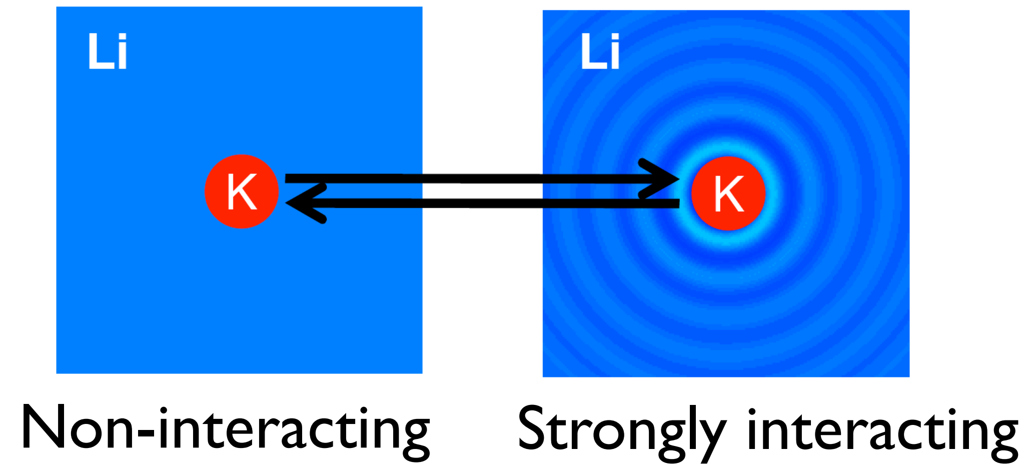
X. Cui and H. Zhai, PRA **81**, 041602 (2010)

^{40}K-^{6}Li experiments by Grimm group

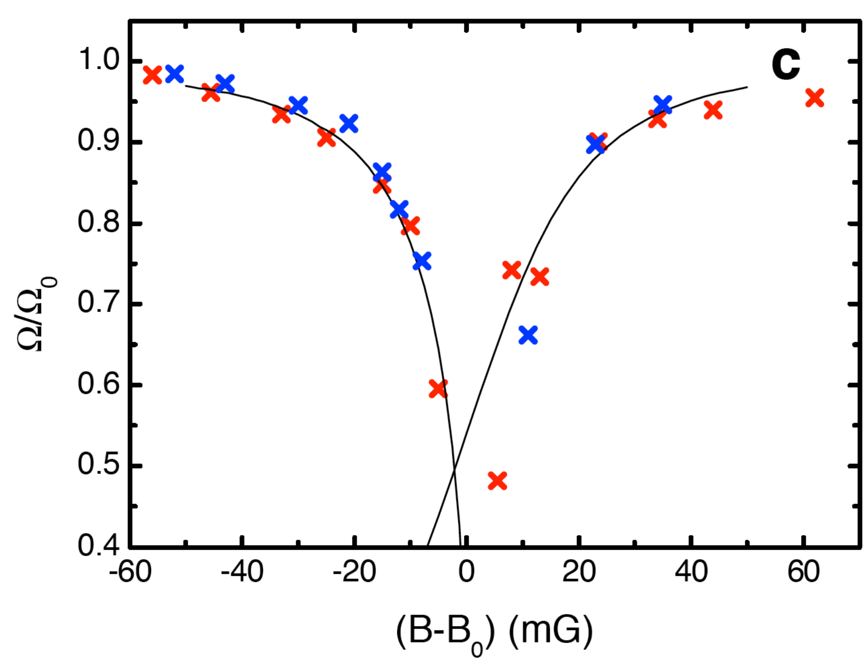
Energy



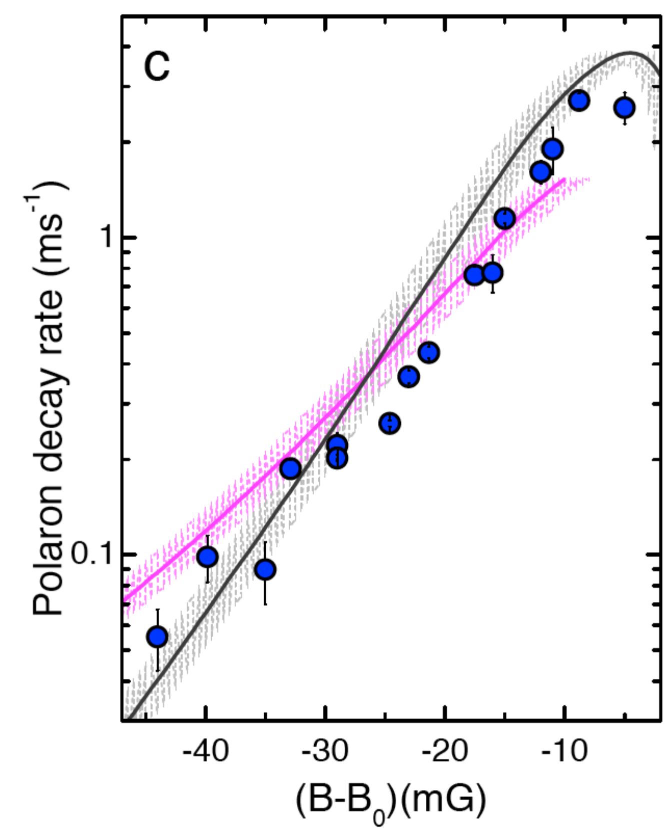
RF flip



QP residue

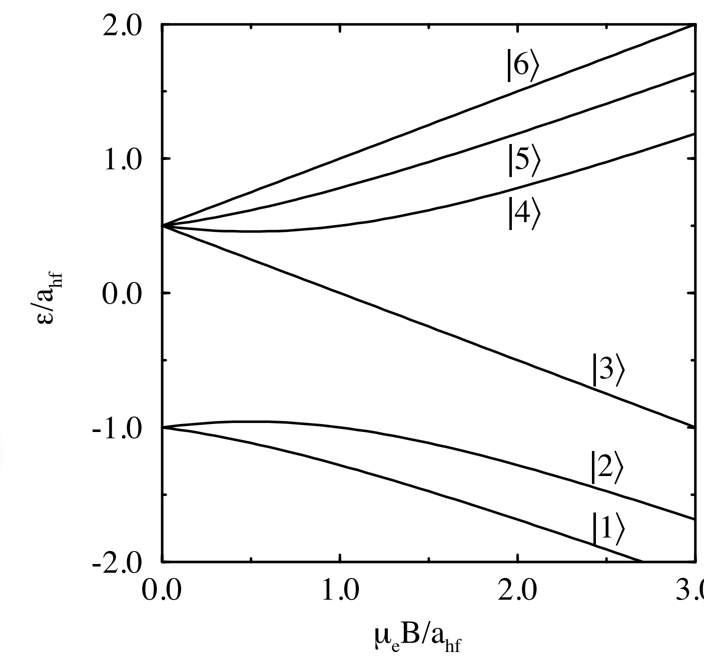


Decay rate

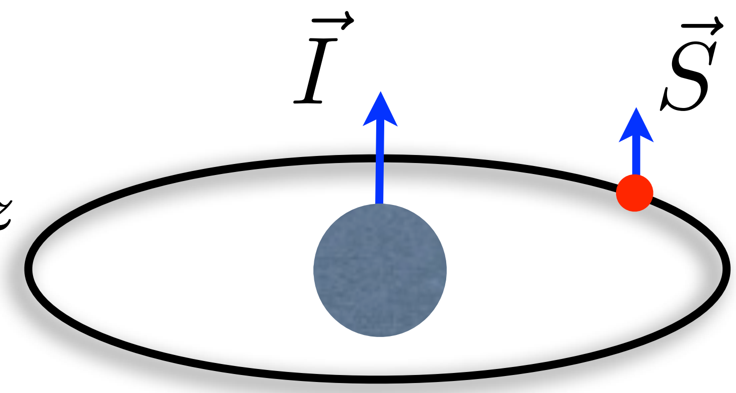


2-body physics

^{6}Li



Hyperfine Hamiltonian:

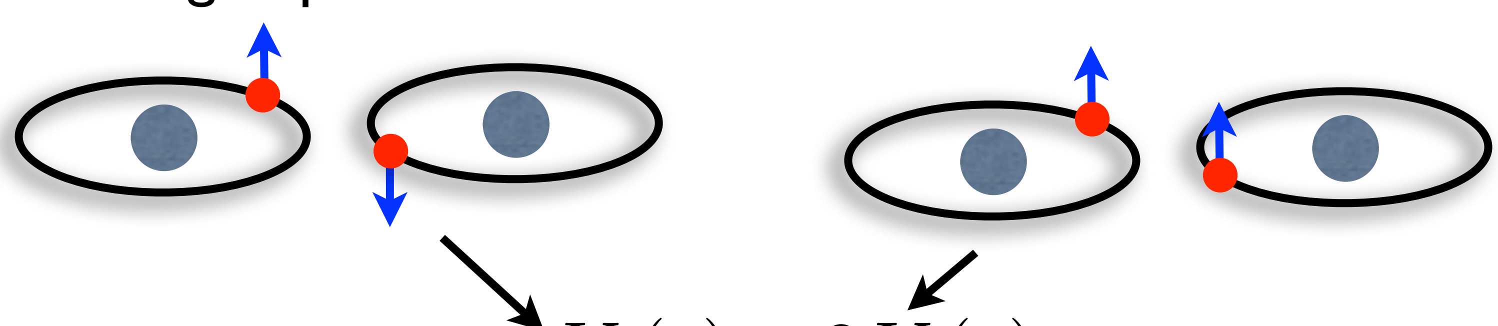


$$\hat{H}_{\text{spin}} = A\,\vec{I}\cdot\vec{S} + C\,S_z + D\,I_z$$

$$\hat{H}_{\text{spin}}|\alpha\rangle = \epsilon_\alpha|\alpha\rangle$$

$$|\alpha\rangle \equiv |F, m_F\rangle$$

$$\vec{F} = \vec{S} + \vec{I}$$



Atom-atom interaction: $$V(r) = \frac{V_s(r) + 3\,V_t(r)}{4} + [V_t(r) - V_s(r)]\,\vec{S}_1\cdot\vec{S}_2$$

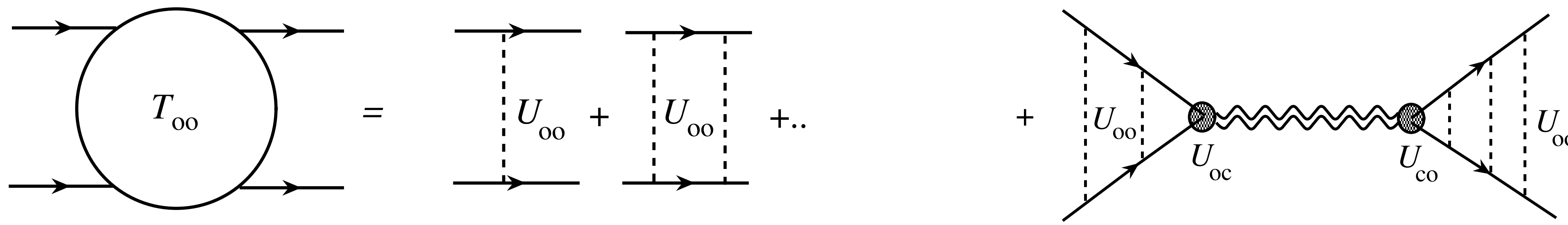
$[\hat{H}_{\text{spin}}, \hat{V}] \neq 0$ Mixes hyperfine states $\Rightarrow$ **Scattering channels**

Effective low-energy interaction:

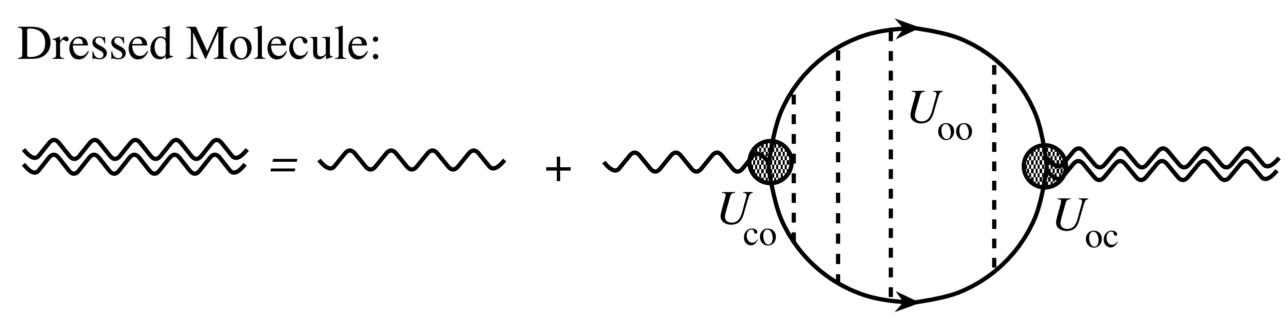
$$U = \frac{2\pi}{m_r}\left[\frac{a_s + 3a_t}{4} + (a_t - a_s)\vec{S}_1\cdot\vec{S}_2\right]$$

Scattering matrix:

$$T = \frac{T_{\rm bg}}{1 - T_{\rm bg}\Pi} + \frac{g^2}{\omega - K^2/2M - \Delta\mu(B - B_0) + g^2\Pi}$$

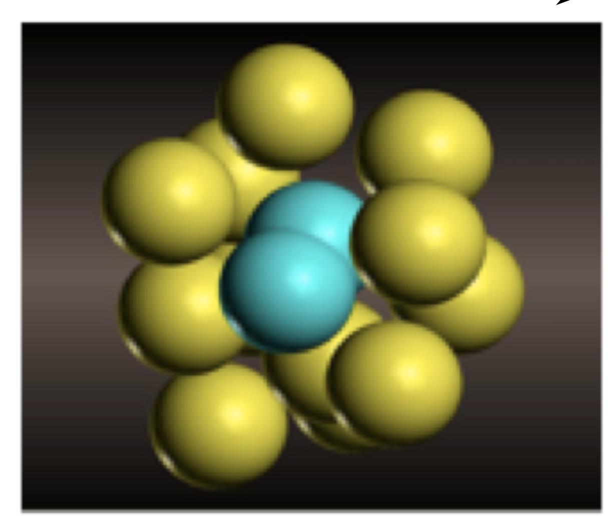


“Dressed” molecule



“<u>Landau Theory</u>”

Interaction expressed in terms of observable 2-body parameters



$$r_{\rm eff}\, a_{\rm bg} = -\frac{1}{\Delta\mu\Delta B m_r} \propto \frac{1}{g^2}$$

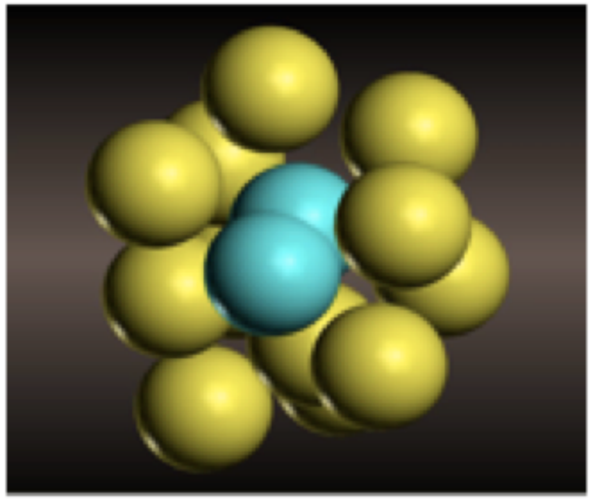
$$g^2 = T_{\rm bg}\Delta\mu\Delta B$$

GMB, E. Kolomeitsev, and A. D. Jackson, PRA **71** 052713 (2005)

"Broad" resonance

$$k_F r_{\text{eff}} \ll 1 \qquad \frac{g^2}{\epsilon_F} \gg \frac{1}{m_r k_F}$$

Single channel

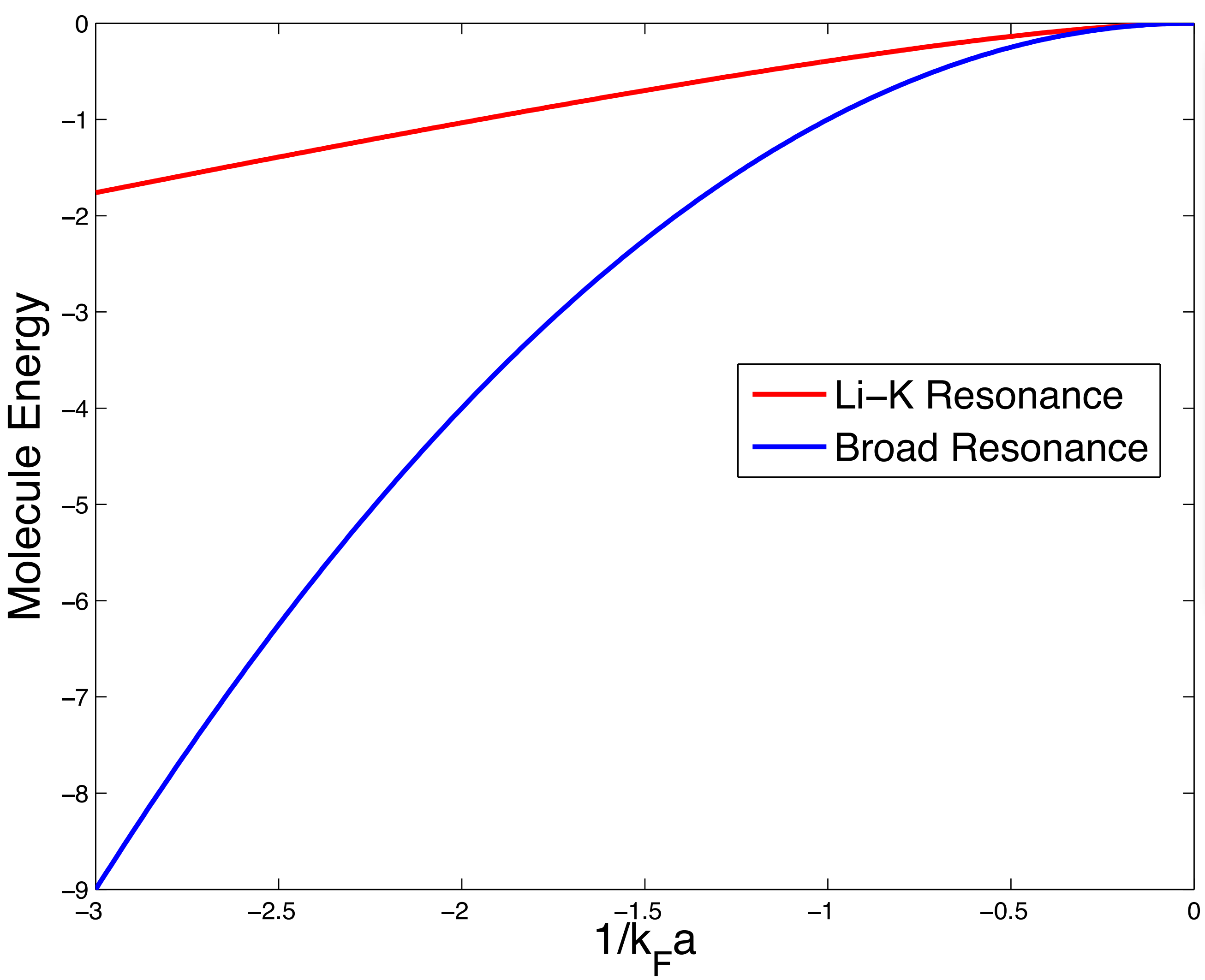


"Narrow" resonance

$$k_F r_{\text{eff}} \gtrsim 1 \qquad \frac{g^2}{\epsilon_F} \ll \frac{1}{m_r k_F}$$

Multi-channel

Molecule energy



^{40}K - ^{6}Li resonance:

$$B_0 = 154,72G \quad \Delta B = 880mG$$

$$a_{bg} = 63,0a_0 \quad \Delta\mu = 1,64\mu_B$$

$$|k_F r_{\text{eff}}| \simeq 1,9$$

$$E_B = \frac{\hbar^2}{2m_r a^{*2}} \qquad a^* = \frac{r_{\text{eff}}}{1-\sqrt{1-2r_{\text{eff}}/a}}$$

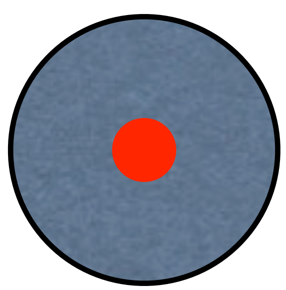
$$\rightarrow \frac{\hbar^2}{2m_r a^2}$$

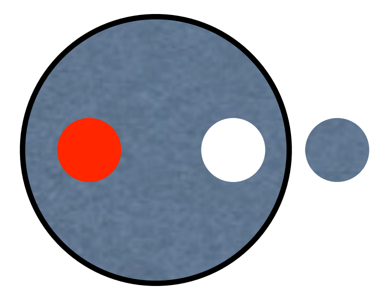
for broad resonance

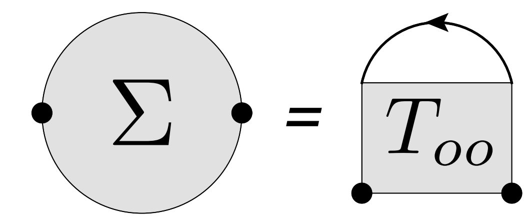
Many-body theory

Polaron:

$$|\psi_P\rangle = \sqrt{Z} a^\dagger_{0\downarrow}|\mathrm{FS}\rangle + \sum_{q<k_F<k} \phi_{\mathbf{k},\mathbf{q}} a^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} a^\dagger_{\mathbf{k}\uparrow} a_{\mathbf{q}\uparrow}|\mathrm{FS}\rangle + \ldots$$

Zero holes: 

One hole: 

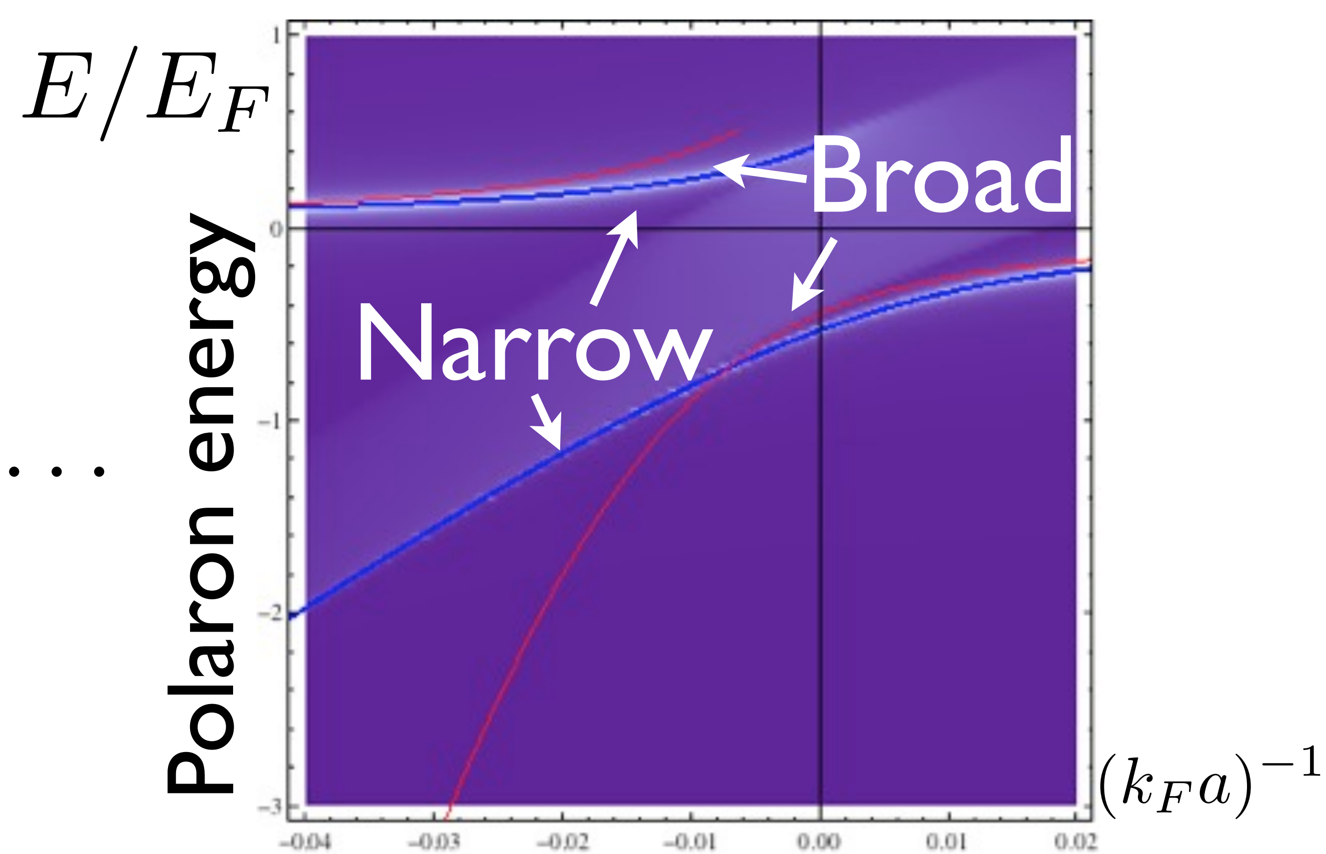


F. Chevy PRA **74** 063628 (2006)

Molecule:

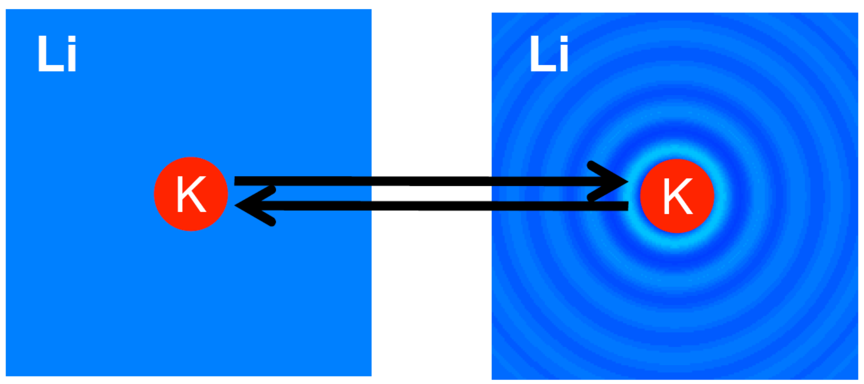
$$|\psi_M\rangle = \sum_{\mathbf{k}} \psi_k a^\dagger_{-\mathbf{k}\downarrow} a^\dagger_{\mathbf{k}\uparrow}|\mathrm{FS}\rangle + \ldots$$

$$\psi_{\mathbf{k}} \propto \frac{1}{1+k^2a^2} \Leftrightarrow \psi_M(r) \propto \frac{e^{-r/a}}{r}$$



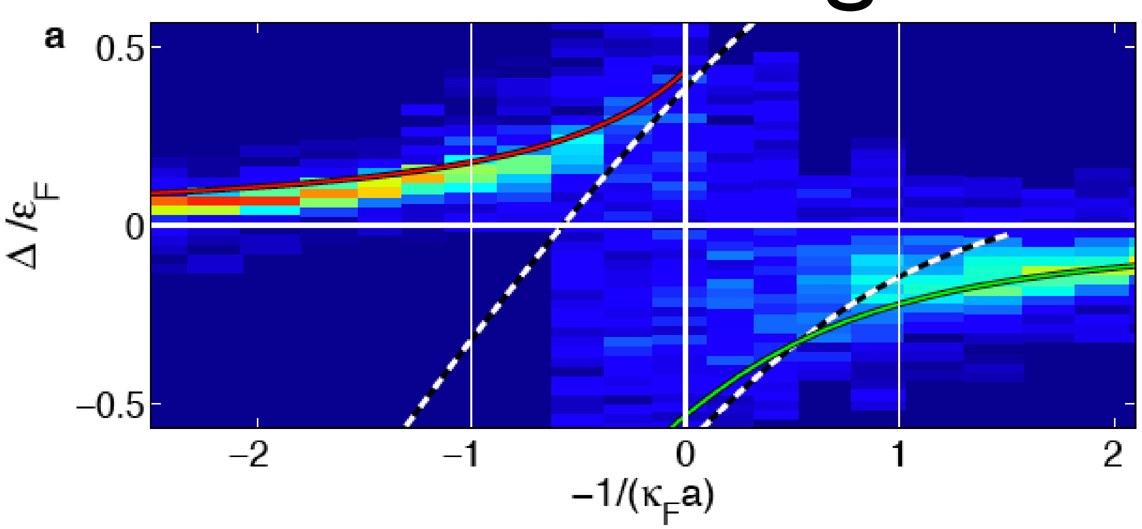
Results & experiments

RF flip

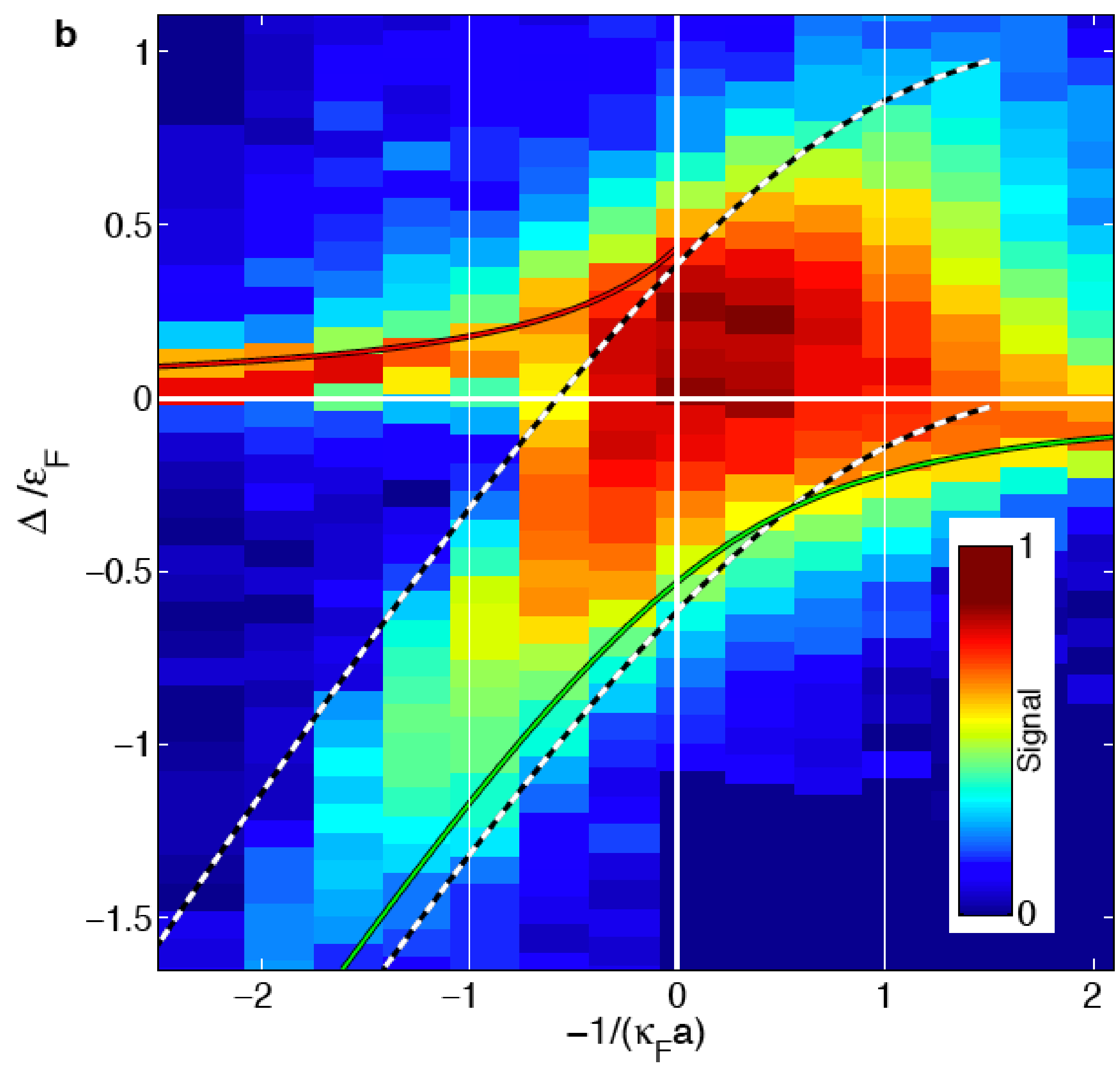


Non-interacting   Strongly interacting

Polaron energies



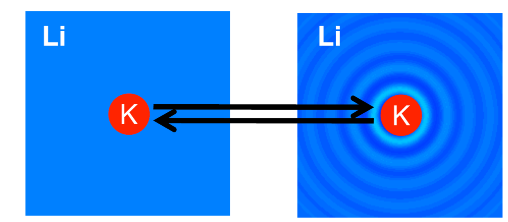
Molecule-hole continuum



Polaron quasiparticle residue

$$|\psi_P\rangle = \sqrt{Z} a^\dagger_{0\downarrow}|\mathrm{FS}\rangle + \sum_{q<k_F<k} \phi_{\mathbf{k},\mathbf{q}} a^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} a^\dagger_{\mathbf{k}\uparrow} a_{\mathbf{q}\uparrow}|\mathrm{FS}\rangle + \ldots$$

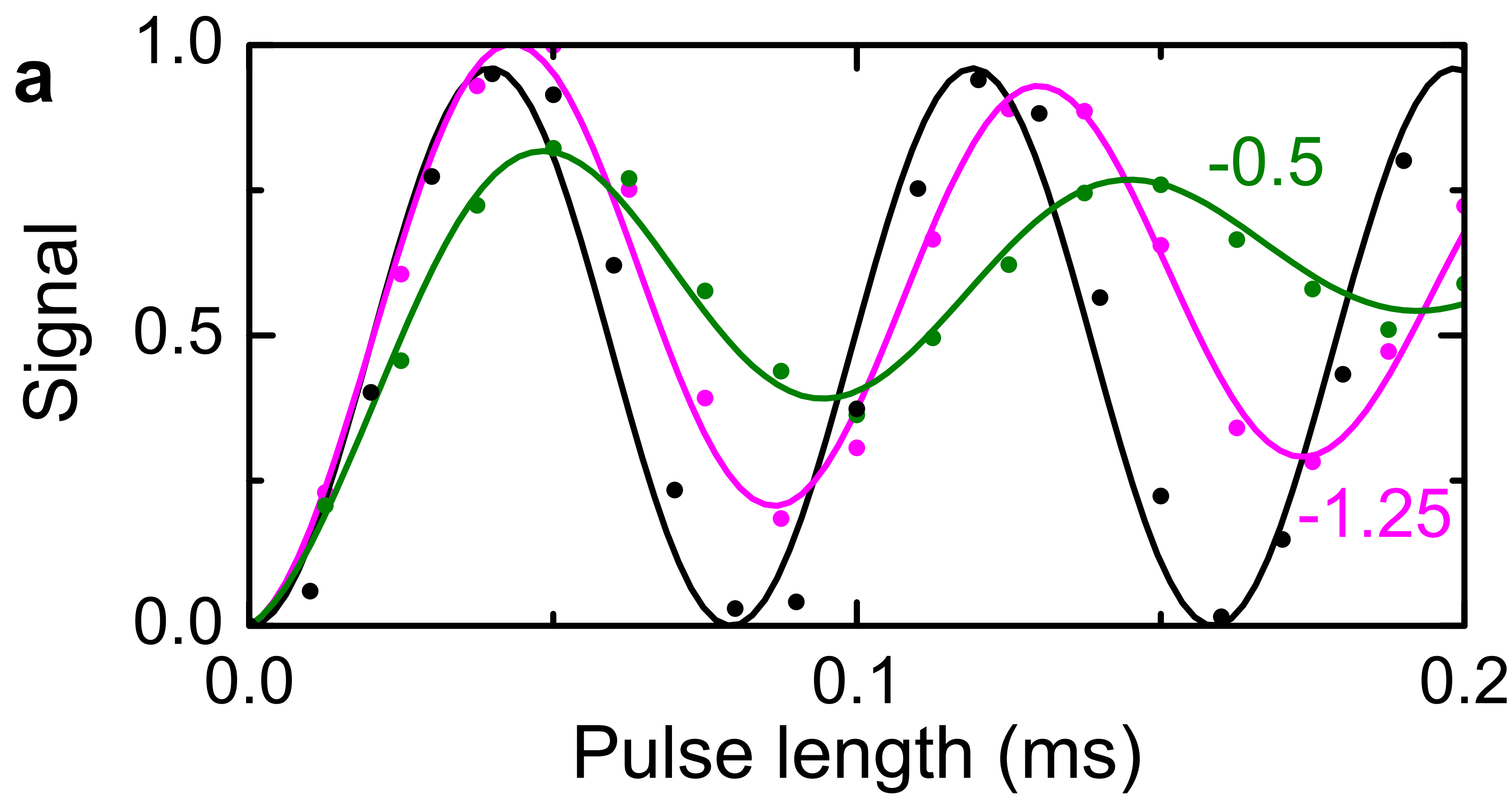
RF-probe momentum conserving $R \propto \Omega_0 \sum_{\mathbf{k}} (b^\dagger_{\downarrow\mathbf{k}} a_{\downarrow\mathbf{k}} + h.c.)$

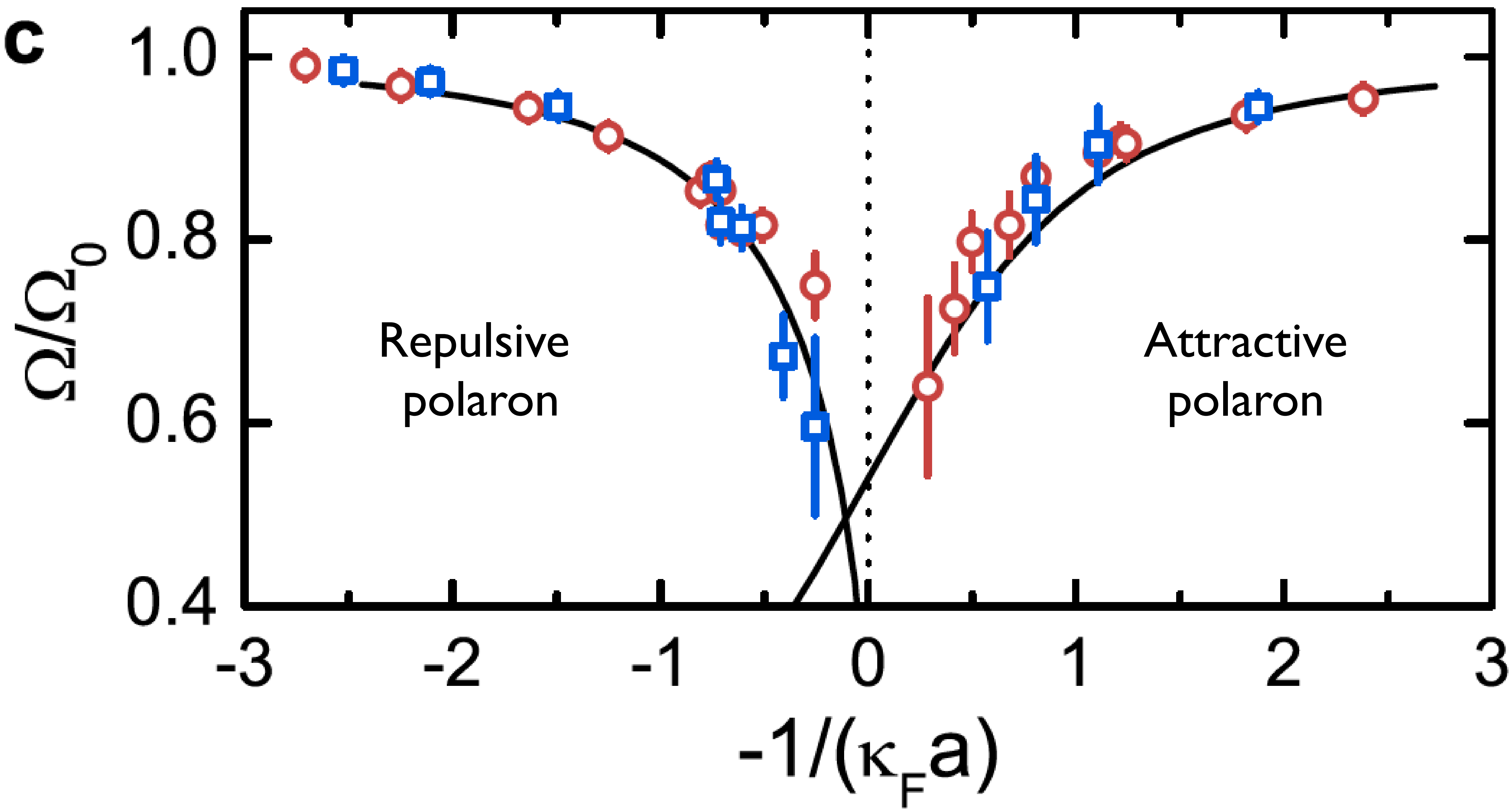


Initial state: $|I\rangle = b^\dagger_{\downarrow 0}|\mathrm{FS}\rangle$

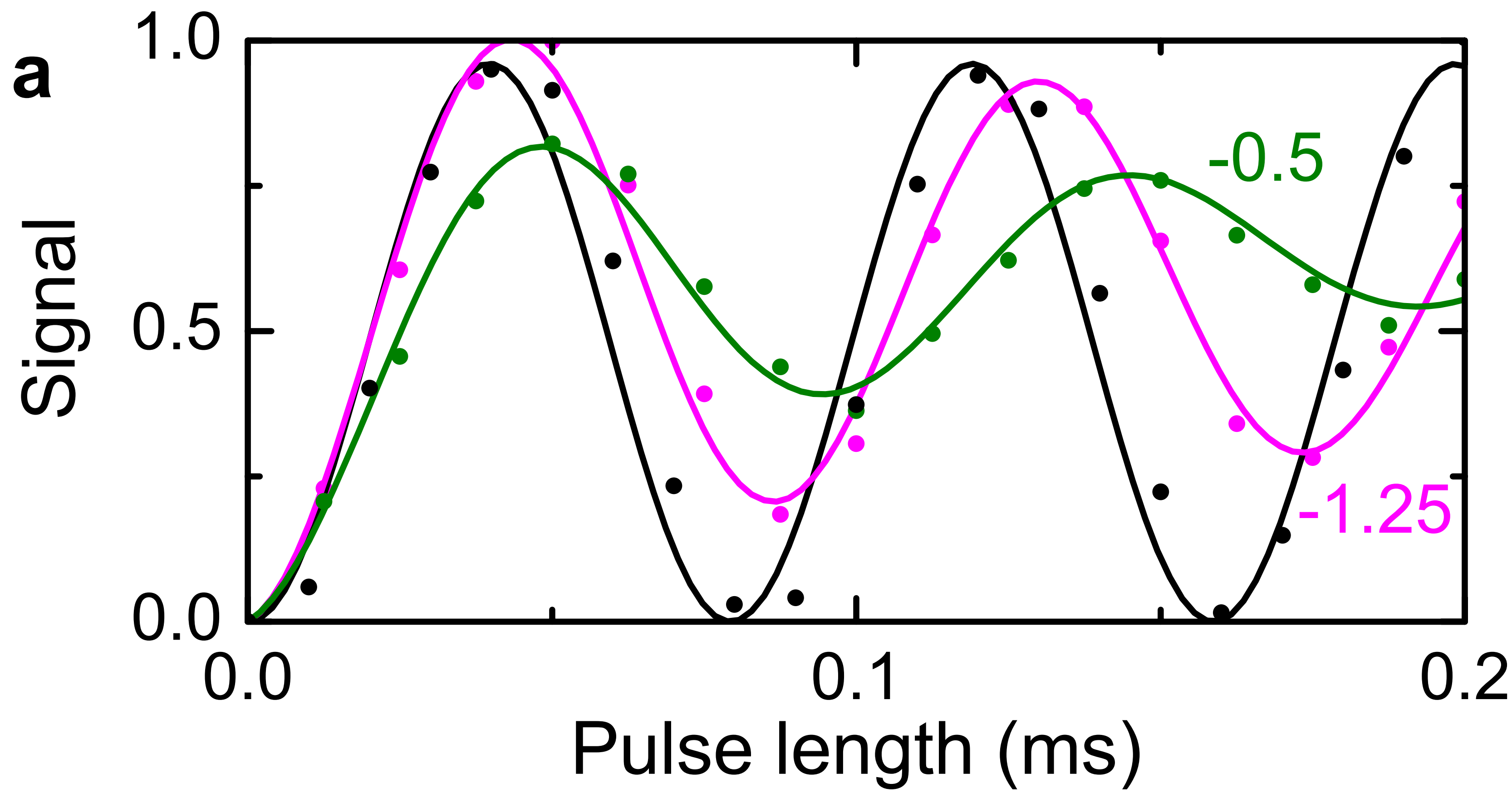
Rabi flipping frequency:

$$\Omega = \langle\psi_P|R|I\rangle = \sqrt{Z}\Omega_0$$

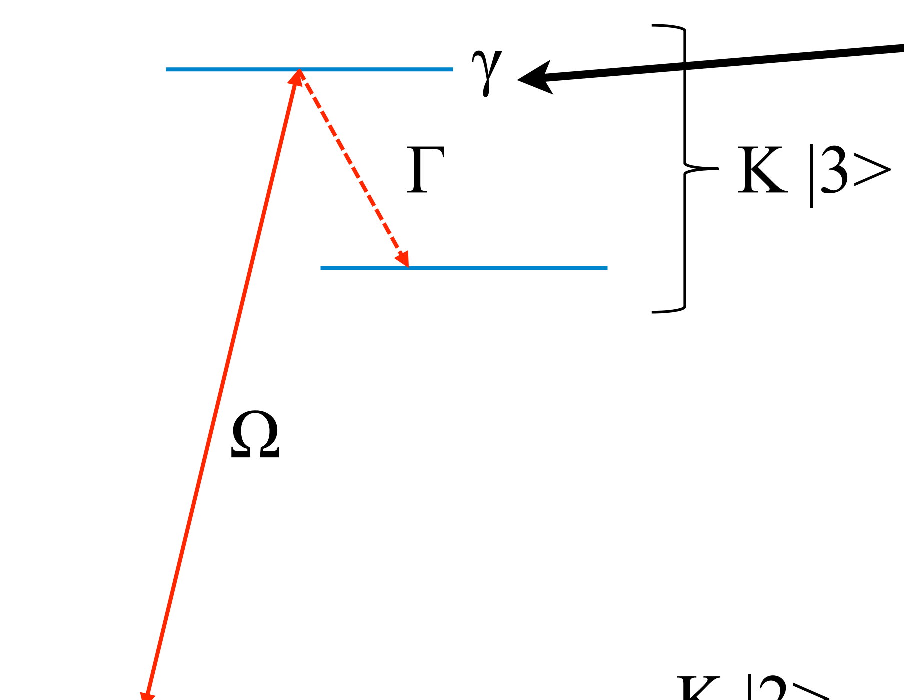



c
1.0
0.8
0.6
0.4
Ω/Ω0
-3
-2
-1
0
1
2
3
-1/(κFa)
Repulsive
polaron
Attractive
polaron

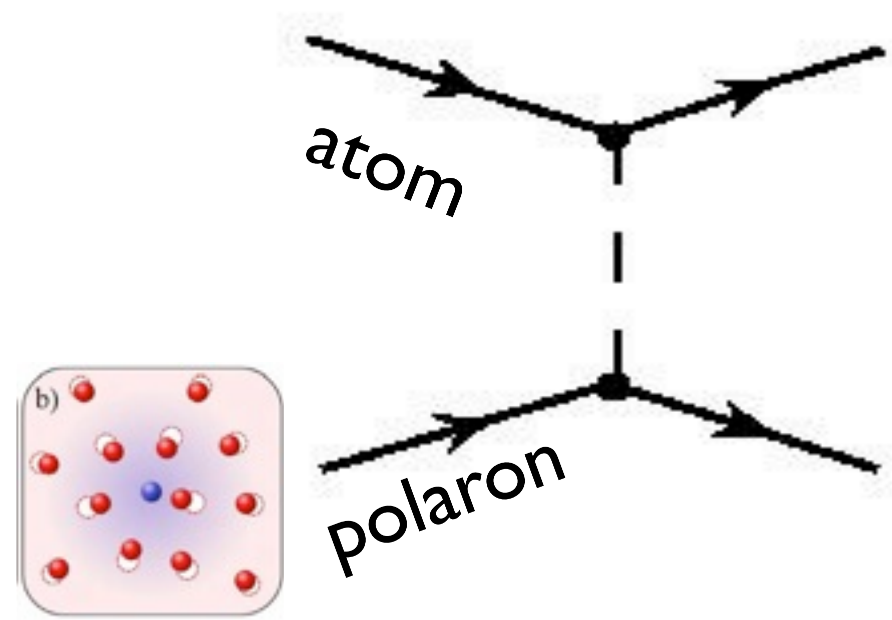
Damping of oscillations:



3-state model

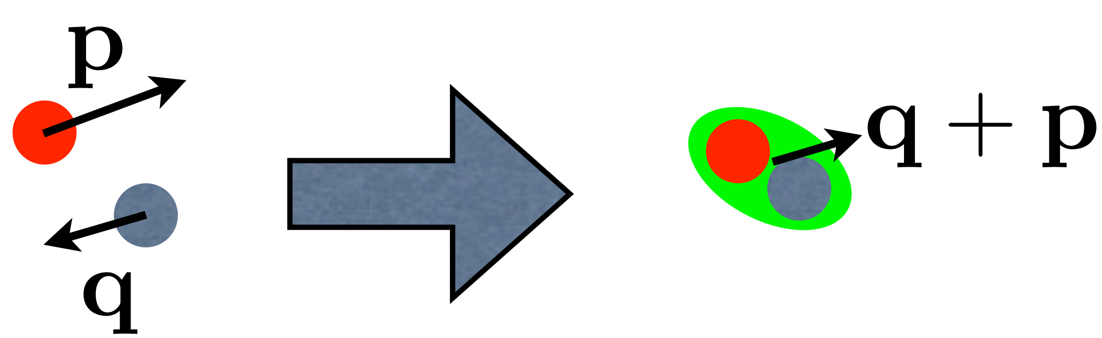


Collisional broadening:



Molecule wave function

RF-signal to molecule-hole continuum (BEC limit):



$$\Gamma_{2B}(\omega_{rf}) \propto \iint \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3} f(\xi_{p\downarrow}) f(\xi_{q\uparrow})$$

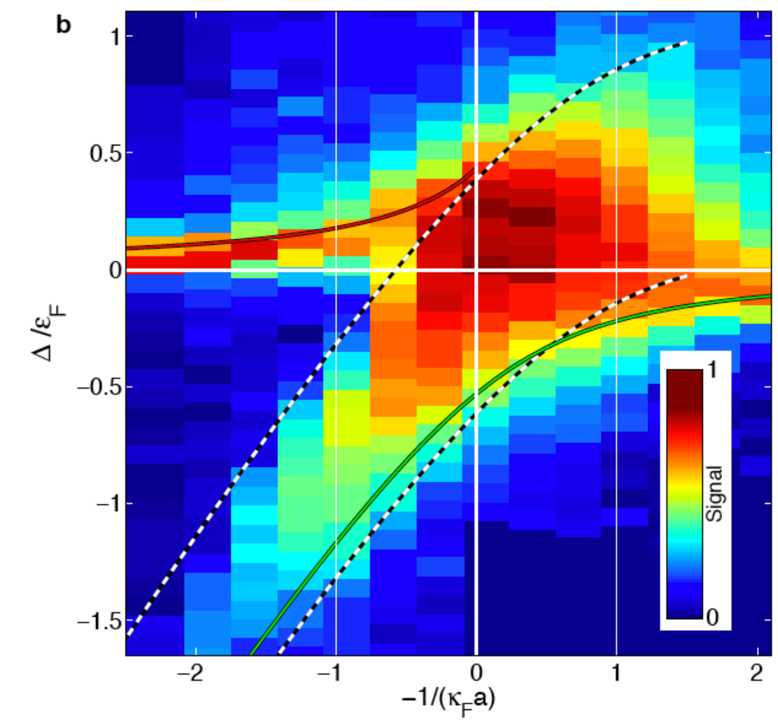
Overlap between molecule and plane wave

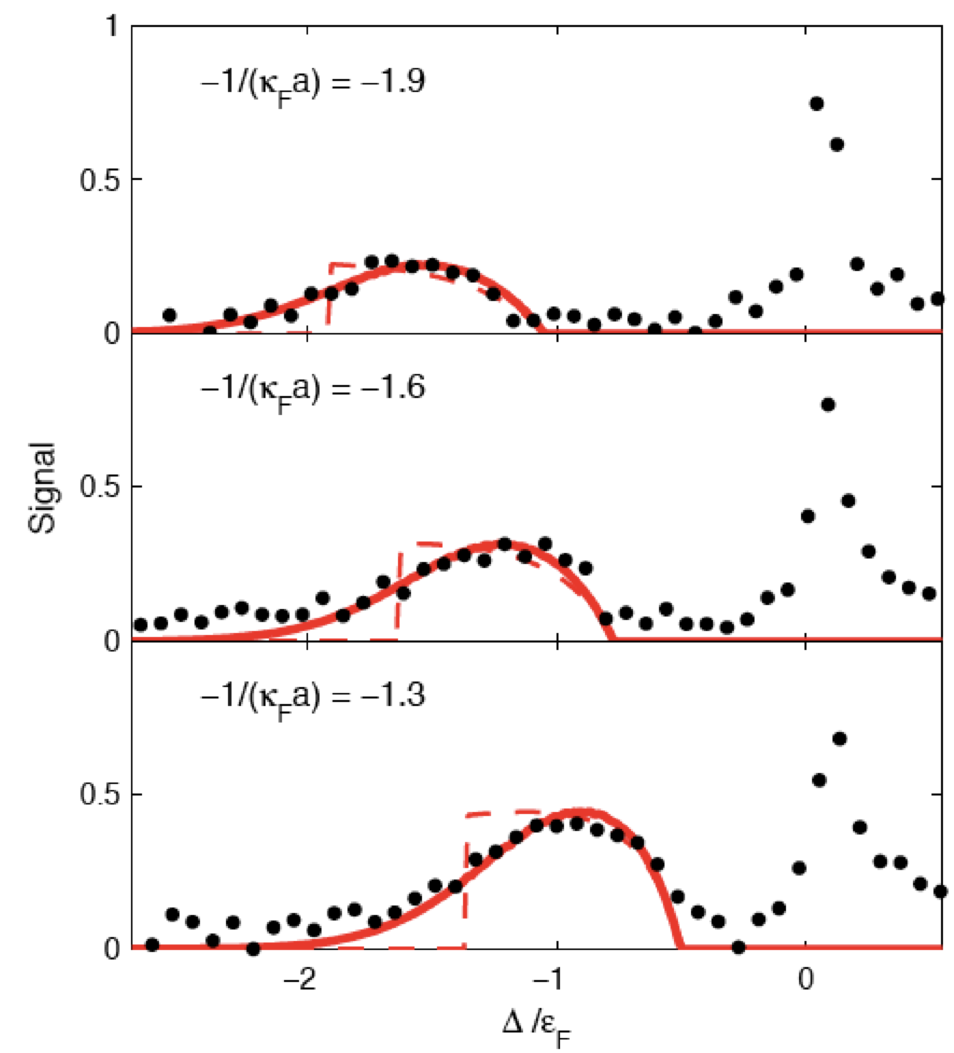
$$\frac{1}{\sqrt{1+4R^*/a}}\,\frac{8\pi {a^*}^3}{(1+k'^2 {a^*}^2)^2}$$

$$\delta(\omega_{rf} + |\omega_M| + \frac{k'^2}{2m_r})$$

$$\rightarrow \frac{8\pi a^3}{(1+k'^2a^2)^2}$$

for $|a/r_{eff}| \gg 1$

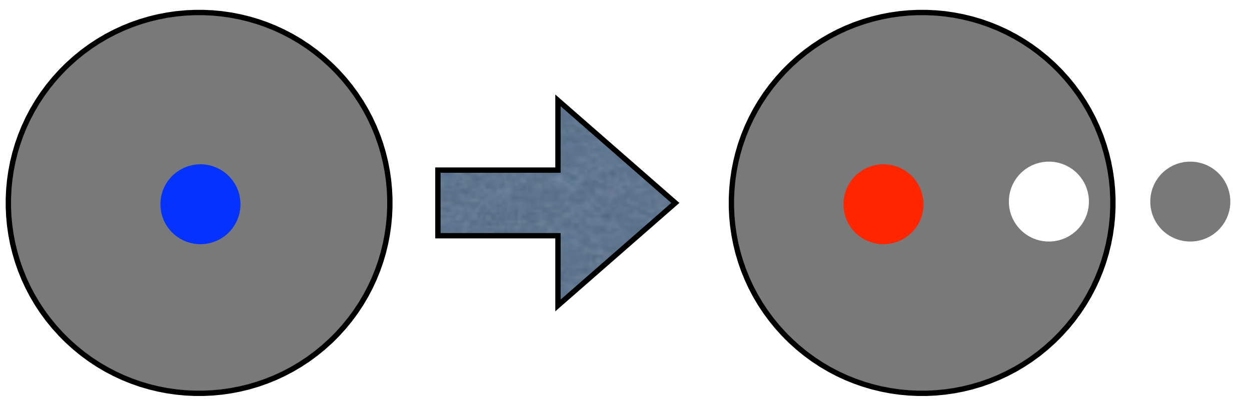




Repulsive Polaron Decay

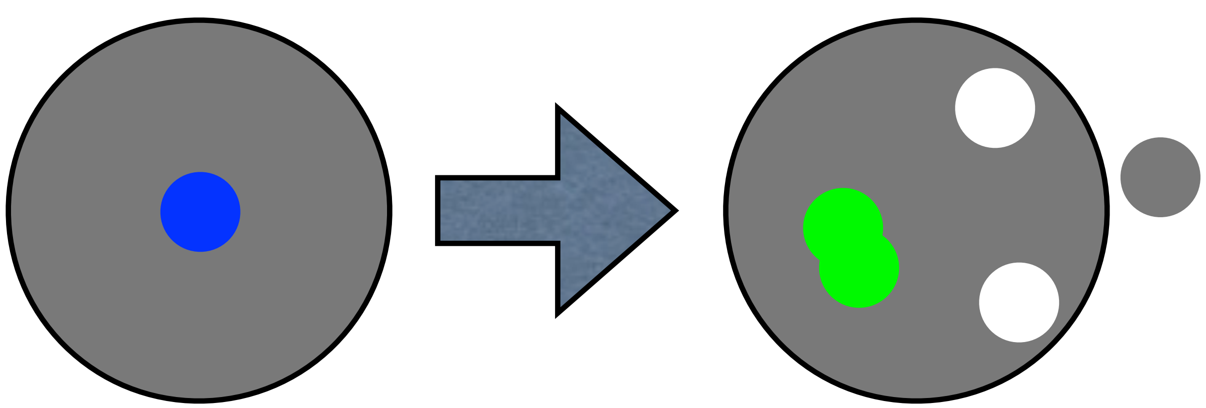
❶ Decay to attractive polaron:

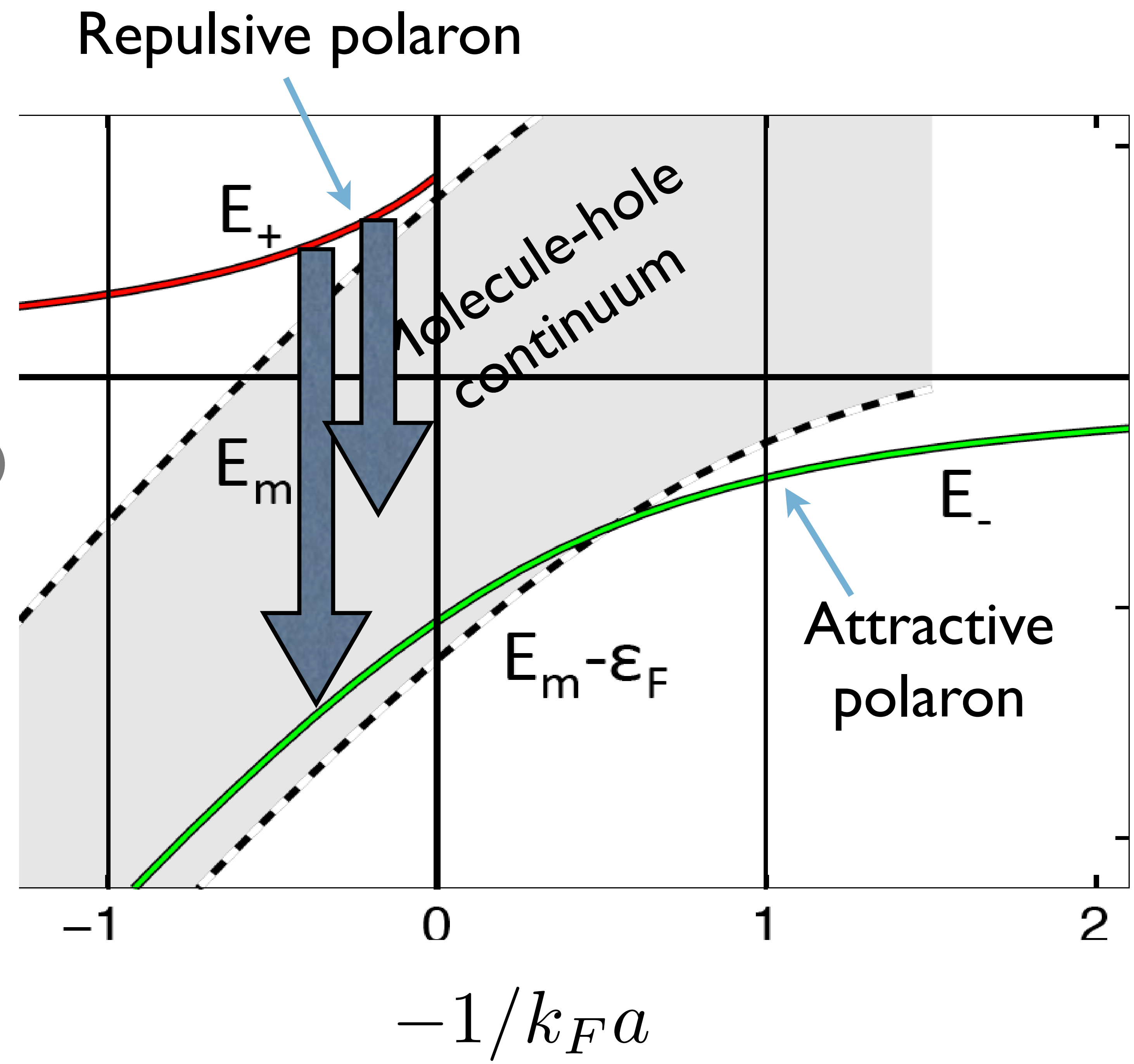
2-body process



❷ Decay to molecule:

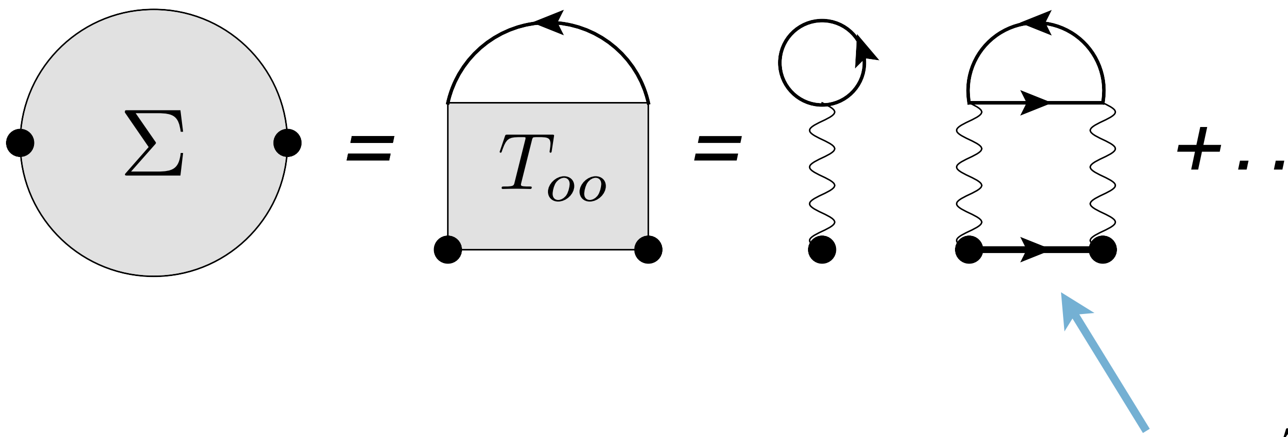
3-body process

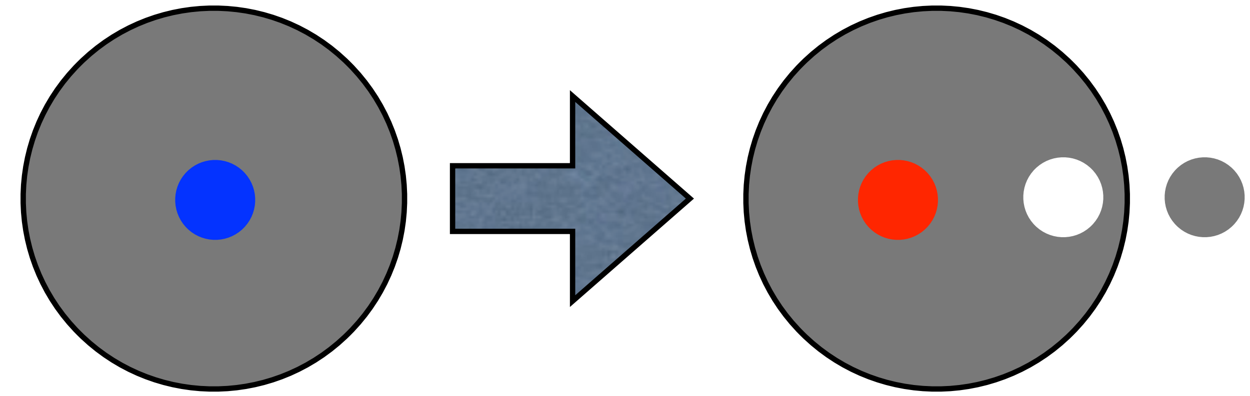




R. Schmidt and T. Enns, PRA **83** 063620 (2011)

2-body decay to attractive polaron:





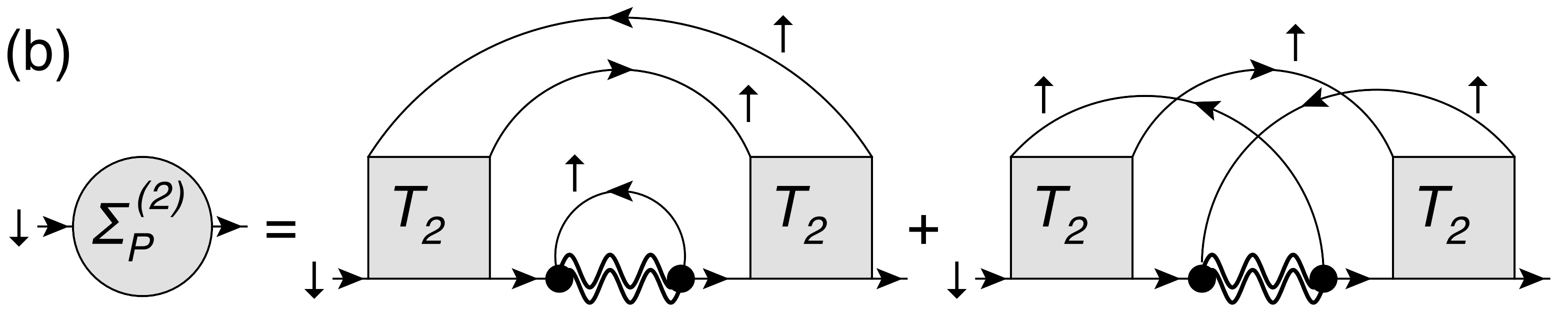
$$G_\downarrow = \frac{Z_+}{\hbar\omega - E_+} + \frac{Z_-}{\hbar\omega - E_-}$$

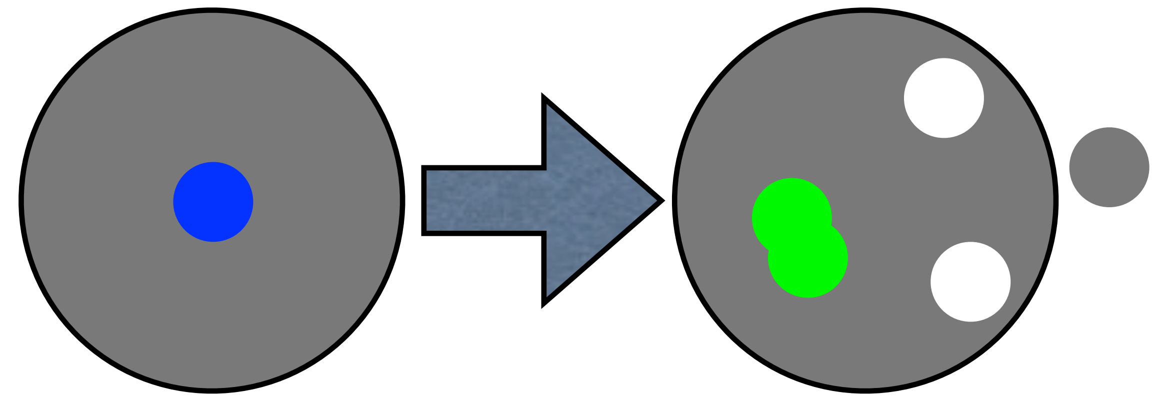
BEC-limit

$$\begin{aligned}\Gamma_{PP} &= \pi T_0^2 Z_- \int_{q<k_F<k} d^3\check{q}\, d^3\check{k}\, \delta(\Delta E + \epsilon_{\uparrow q} - \epsilon_{\uparrow k} - \epsilon^*_{\downarrow \mathbf{q}-\mathbf{k}}) \\ &= Z_- \frac{2}{3\pi}\sqrt{\frac{m_\uparrow (m_r^*)^3}{m_r^4}}\sqrt{\frac{\Delta E_{PP}}{\epsilon_F}}(k_F a)^2 \epsilon_F \propto k_F a\end{aligned}$$

P. Massignan and GMB, EPJD **65**, 83 (2011)

3-body decay to molecule + hole:





$$D(\mathbf{p},\omega) \simeq \frac{Z_M}{\omega - \omega_M - p^2/2m_M^*}.$$

$$F(\mathbf{q},\mathbf{k},\omega) = T_2(\mathbf{q},\omega+\xi_{q\uparrow})G^0_\downarrow(\mathbf{q}-\mathbf{k},\omega+\xi_{q\uparrow}-\xi_{k\uparrow})$$

$$\Gamma_P = \frac{g^2 Z_M}{2}\int d^3\check{q}\,d^3\check{k}\,d^3\check{q}'\left[F(\mathbf{q},\mathbf{k},\omega_P) - F(\mathbf{q}',\mathbf{k},\omega_P)\right]^2 \times\delta\left(\Delta\omega + \xi_{q\uparrow} + \xi_{q'\uparrow} - \xi_{k\uparrow} - (\mathbf{q}+\mathbf{q}'-\mathbf{k})^2/2m_M^*\right)$$

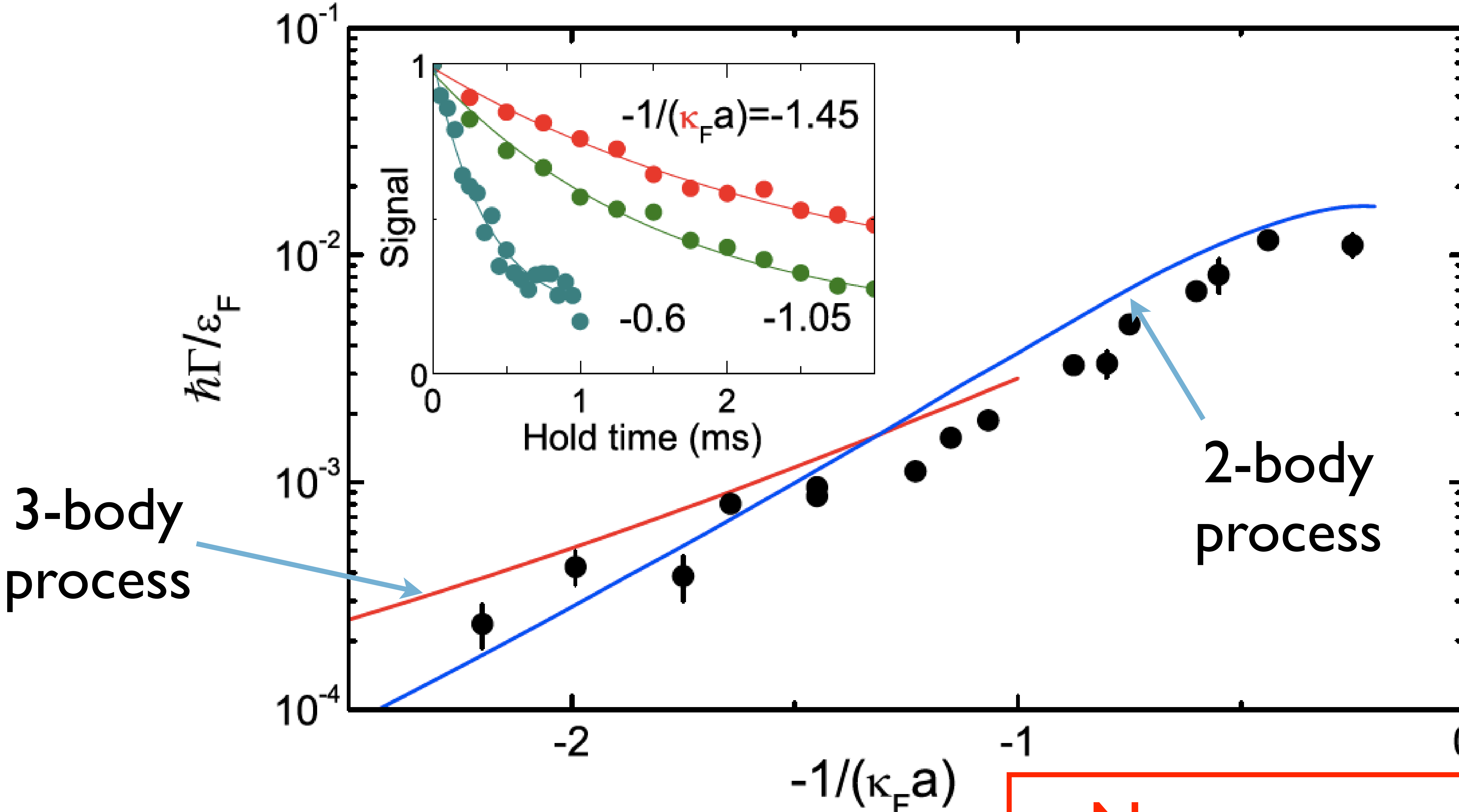
Broad resonance $\Gamma_P \propto (k_F a)^6 \epsilon_F \propto n_\uparrow^2 \epsilon_F$

Due to Fermi exclusion principle

GMB and P. Massignan, PRL **105** 020401 (2010)

D. S. Petrov, PRA **67** 010703 (2003)

Experiment



$(k_F a)^{-1}=-0.25$:

$$\hbar\Gamma/\epsilon_F = 0.01$$

$$\hbar\Gamma/E_+ = 0.03$$

Non-zero range gives ≈ 10 times longer life time. 1/e life time ≈400μs

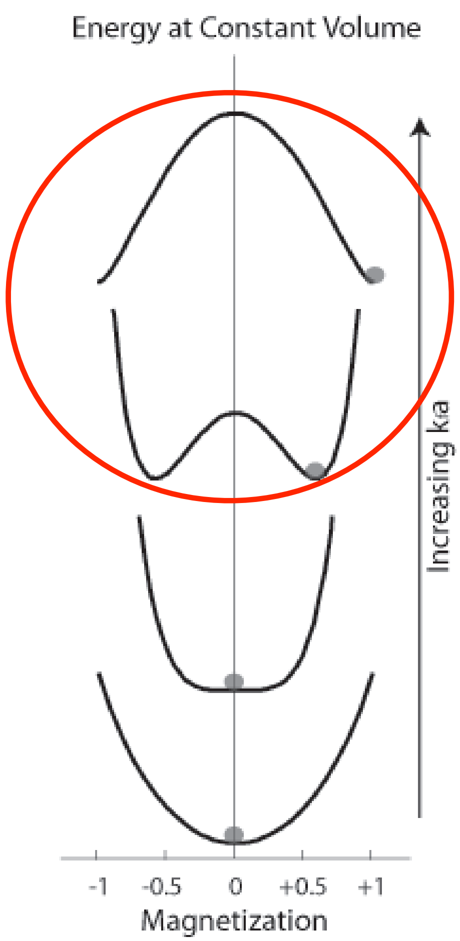
Itinerant ferromagnetism

Fermi gas with short range repulsive interactions

$$\hat{H} = -\int d^3r \hat{\psi}^\dagger_\sigma(\mathbf{r}) \frac{\nabla^2}{2m} \hat{\psi}_\sigma(\mathbf{r}) + g \int d^3r \hat{\psi}^\dagger_\uparrow(\mathbf{r}) \hat{\psi}^\dagger_\downarrow(\mathbf{r}) \hat{\psi}_\downarrow(\mathbf{r}) \psi_\downarrow(\mathbf{r})$$

Stoner theory: $E = \frac{3}{5} n \epsilon_F [(1+\eta)^{5/3} + (1-\eta)^{5/3} + A(1+\eta)(1-\eta)]$

$$\eta = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow} \qquad A \propto g \propto k_F a$$



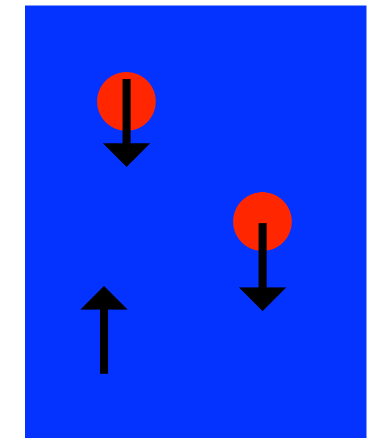
Not observable due to pairing instability

C. Sanner *et al.*, PRL **108**, 240404 (2012)

D. Pekker *et al.*, PRL **106**, 050402 (2011)

We have accurate theory in the limit N↓«N↑

Mixed phase energy:

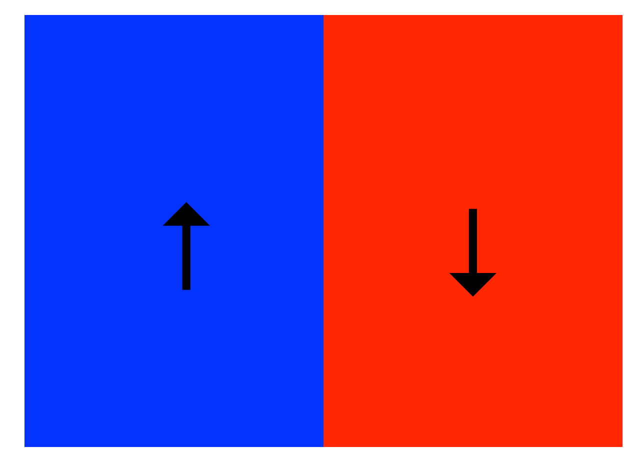


$$E(N_\uparrow, N_\downarrow, T) = N_\uparrow \varepsilon_1^0(N_\uparrow/V, T) + N_\downarrow \varepsilon_2^0(N_\downarrow/V, T) + N_\downarrow \varepsilon_1^0(N_\uparrow/V, T) A(T)$$

S. Pilati *et al.*, PRL **105**, 030405 (2010)

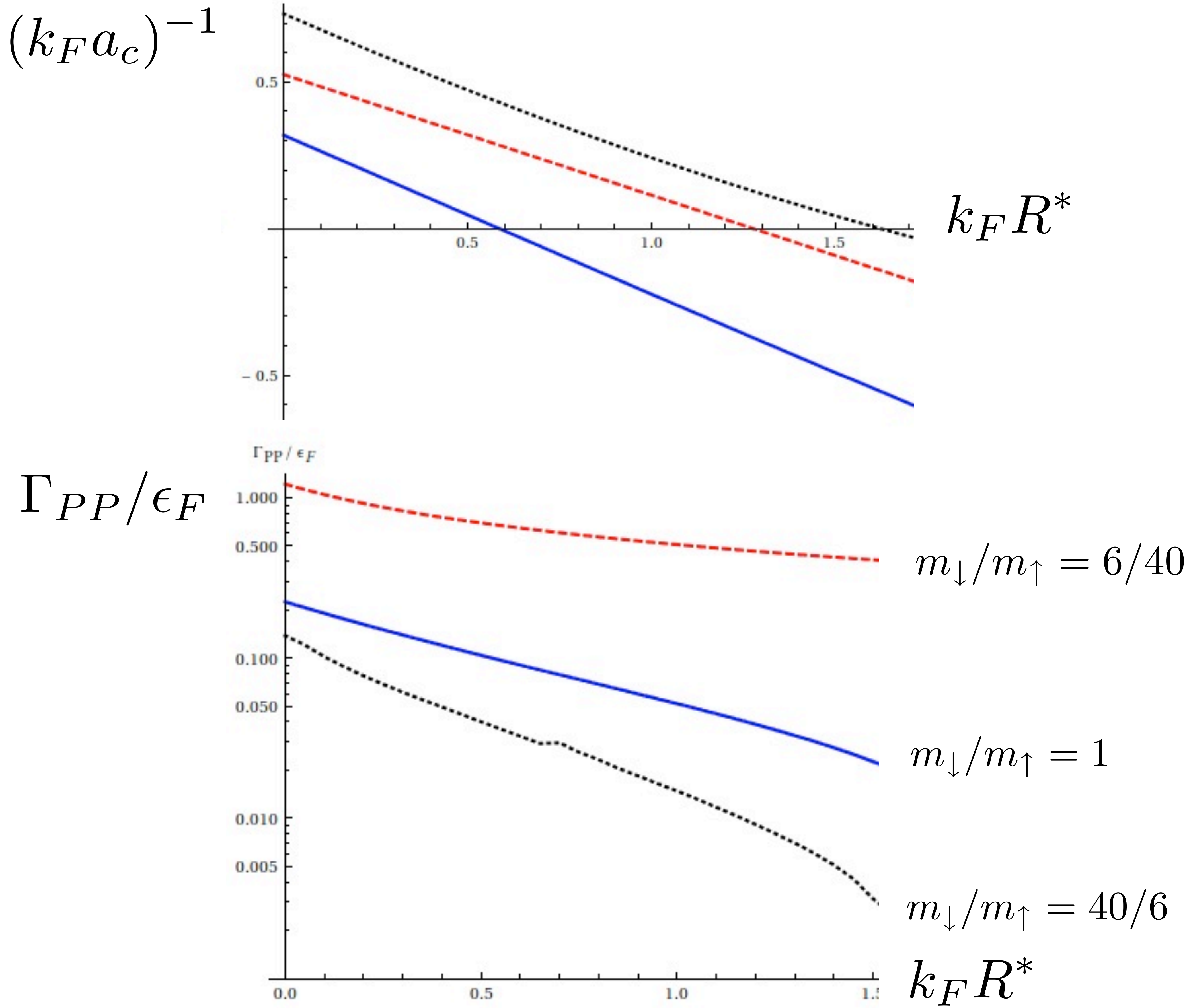
Phase separated energy:

$$E(N_\uparrow, N_\downarrow, T) = N_\uparrow \varepsilon_1^0(N_\uparrow/V_\uparrow, T) + N_\downarrow \varepsilon_2^0(N_\downarrow/V_\downarrow, T)$$



Condition for phase separation at T=0

$$A \geq \frac{5}{3}\left(\frac{m_1}{m_2}\right)^{3/5} \quad \Leftrightarrow \quad E_+ \geq \epsilon_F$$



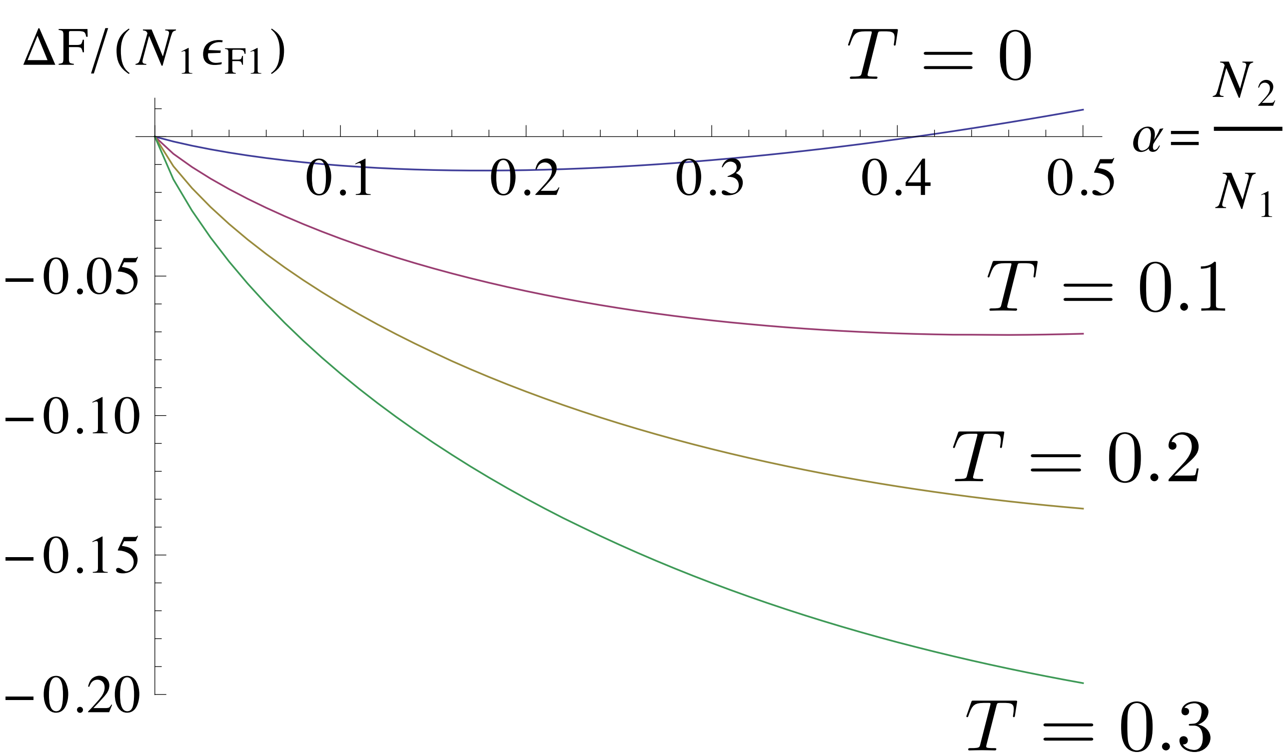
Ferromagnetism with narrow Feschbach resonance?

Minimize free energy for T>0 $F = E - TS$

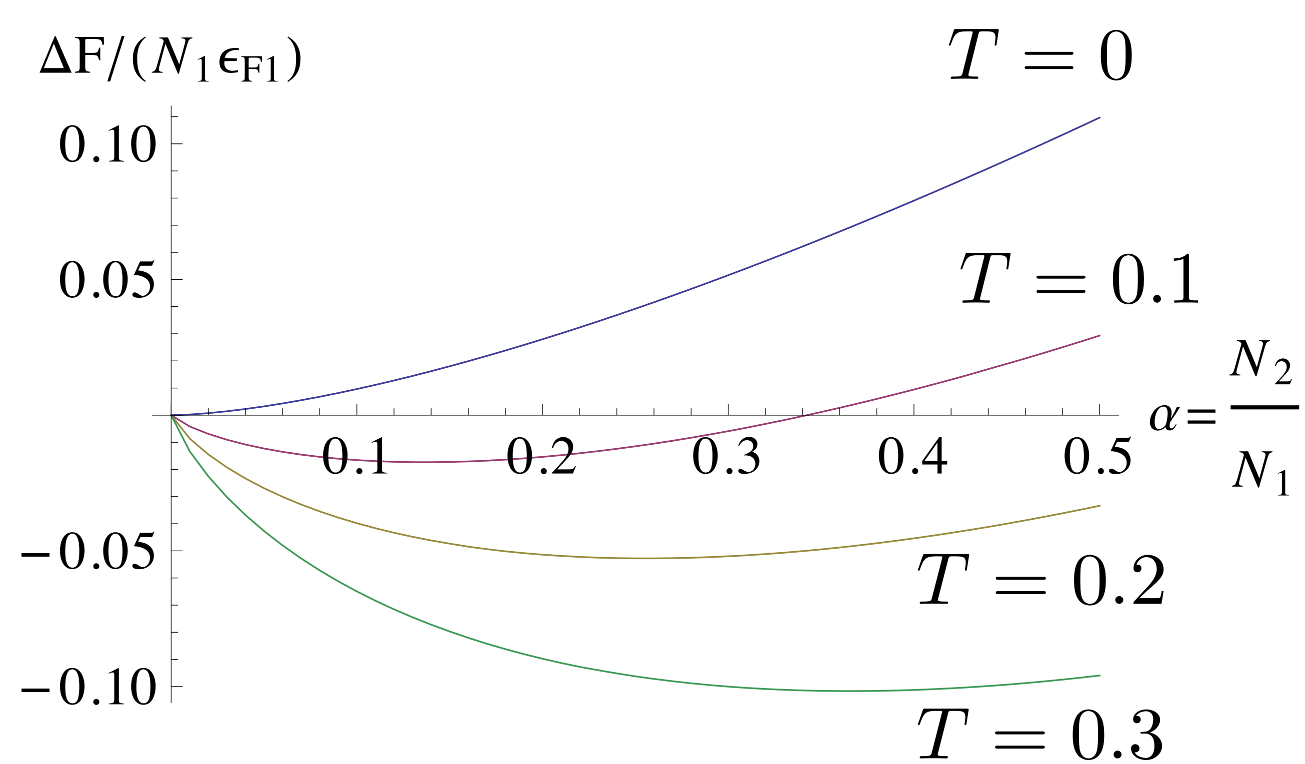
Entropy of mixing (ideal mixture):

$$\Delta S_{\mathrm{mix}} = -Nk_B[y \log y + (1-y)\log(1-y)] \qquad y = \frac{N_\downarrow}{N}$$

$E_+/\epsilon_F = 0.8$



$E_+/\epsilon_F = 1$



Phase diagram

Homogeneous

Phase separated

Conclusions

- Long lived repulsive polaron
- Excellent agreement between theory & experiment
- Narrow resonance increases stability of repulsive polaron
- Ferromagnetism for narrow resonance?