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The unitary Fermi gas: a benchmark case for many-body physics

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THE UNITARY FERMI GAS: A BENCHMARK CASE FOR MANY-BODY PHYSICS

Haussmann/Rantner/Cerrito/Zw. PR **A75**, 023610 '07

Enss/Haussmann/Zw. Ann. Phys. **326**, 777 '11

Enss/Haussmann arXiv:1207.3103

Schmidt/Rath/Zw. arXiv:1201.4310 **Efimov physics**

textbook model for fermionic superfluids

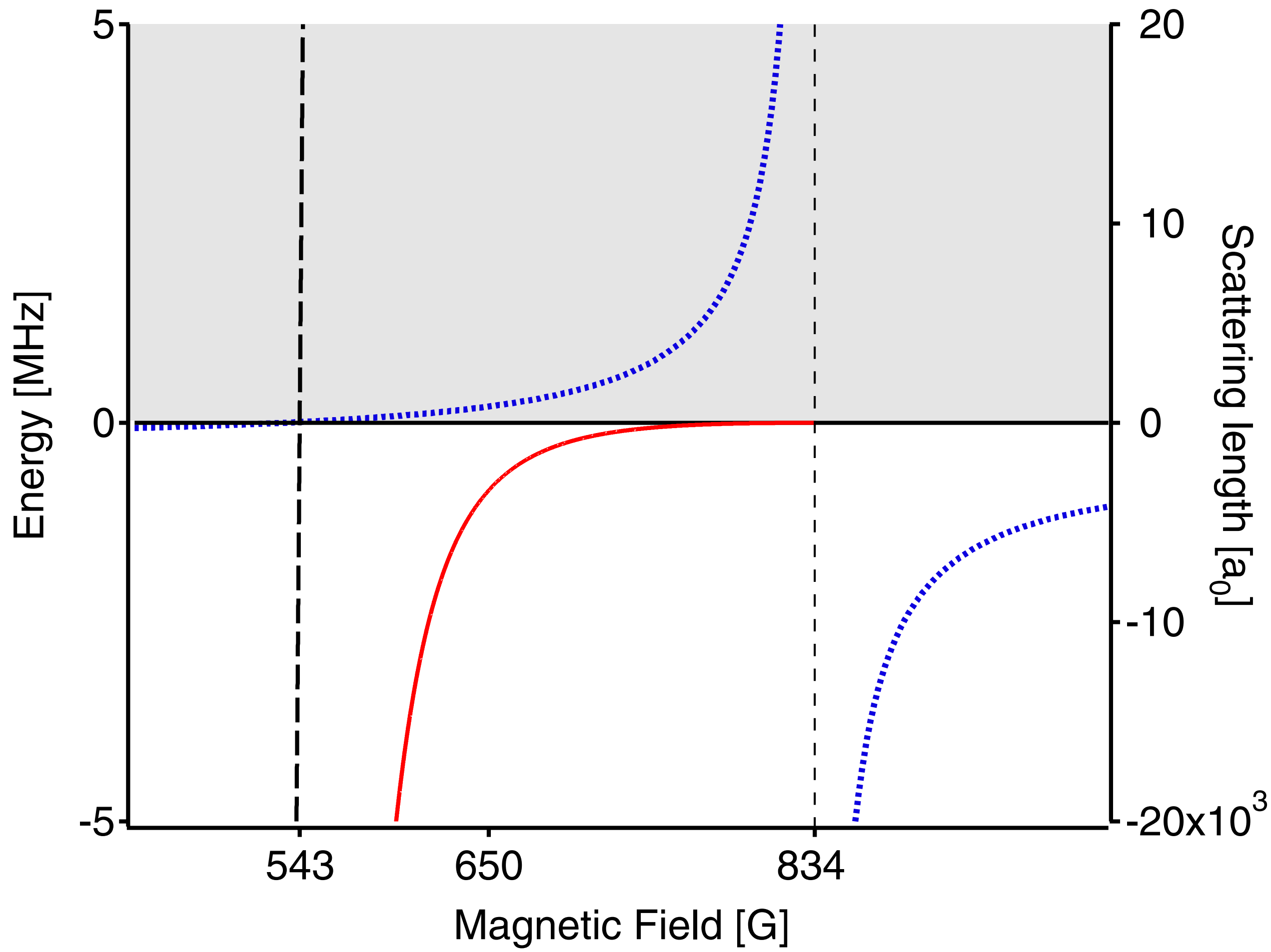
BCS '57 Fermions $\uparrow\downarrow$ with density $n = k_F^3/3\pi^2$ and

attractive two-particle interaction $V_{\uparrow\downarrow}(\boldsymbol{x}) = \bar{g} \cdot \delta(\boldsymbol{x})$

pairs form and condense at $T_c \sim \exp -\frac{1}{|\bar{g}|N(0)} \ll T_F$

what happens at infinite coupling $g = \infty$?

scattering length in ^{6}Li $\quad gN(0) \to 2k_F a/\pi$



Outline

I) **Unitary gas thermodynamics**

II) **Viscosity and spin-diffusion**

III) **Universality of the 3-body parameter in Efimov physics**

The unitary Fermi gas at $a = \infty$ $\boldsymbol{x} \to \lambda \boldsymbol{x}$ gives

$H \to H/\lambda^2$ **scale invariance** $\to$ **Tr T**$= 0$ $\to$

pressure $p = 2\epsilon/3$ Ho '04 bulk viscosity $\zeta = 0$ Son '07

ground state $p(\infty) = \xi \cdot p_F^{(0)}$ **Bertsch-parameter** ξ

determines cloud size in a trap $R_{TF} = R_{TF}^{(0)} \cdot \xi^{1/4}$

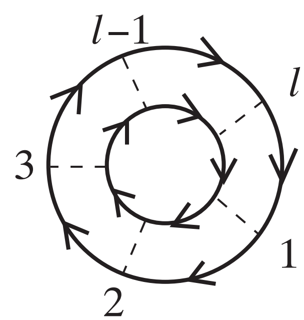
universal numbers $\xi \simeq 0.36$, $T_c \simeq 0.16\, T_F$, $\Delta_0 \simeq 0.46\, \epsilon_F$

transport: **shear viscosity** $\eta(T_c) \simeq 0.5\, \hbar n$ Shuryak '04

Many-body theory pseudopotential $V_{\uparrow\downarrow}(\boldsymbol{x}) = \bar{g}(\Lambda)\,\delta(\boldsymbol{x})$

Luttinger/Ward '60 $\Omega = -T \ln Z = \Omega[\widehat{G}]$

$$\Omega[\widehat{G}] = \beta^{-1}\Big(-\tfrac{1}{2}\mathrm{Tr}\{-\ln \widehat{G} + [\widehat{G}_0^{-1}\widehat{G} - 1]\} - \Phi[\widehat{G}]\Big)$$

Ladder-approximation $\Phi[G] = \sum_{l=1}^{\infty}$ 

$\delta\Omega[\widehat{G}]/\delta\widehat{G} = 0$ variational principle for $\mathcal{G}(\boldsymbol{k},\tau)$ and $\mathcal{F}(\boldsymbol{k},\tau)$

Haussmann/Rantner/Cerrito/Zw. '07, PR A75, 023610

why does Luttinger-Ward work well ?

- it is **conserving** $\rightarrow$ all th. dyn. relations are obeyed

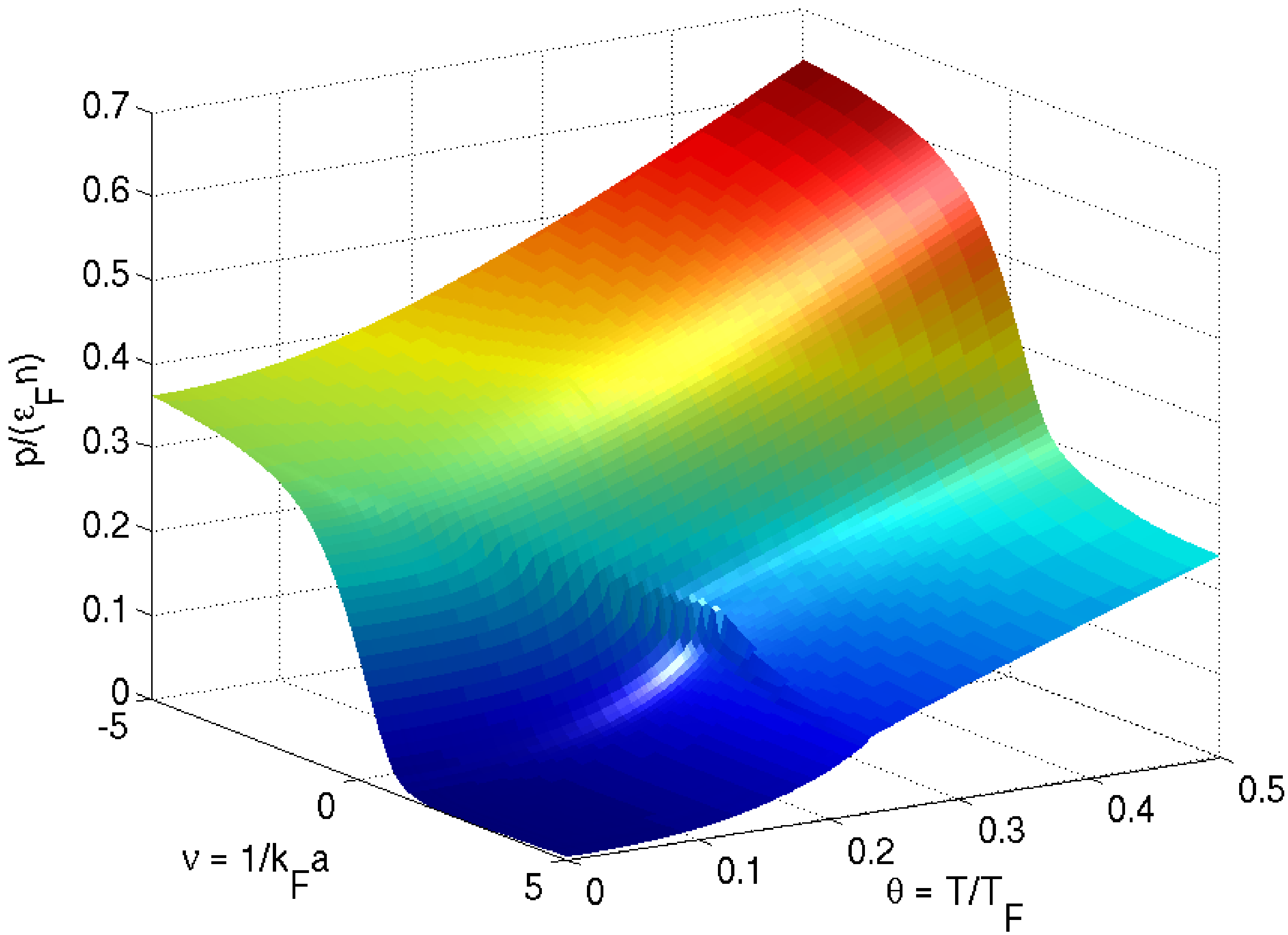
- it obeys the **Tan relations**

$$\mathcal{L}_E = \mathcal{L}[\psi_\sigma] + \mathcal{L}[\Phi] + \tilde{g}\Big(\bar{\Phi}_B \psi_\uparrow \psi_\downarrow + \text{h.c.}\Big)$$

change of Ω with scattering length $\dfrac{\partial \Omega}{\partial(-1/a)} =$

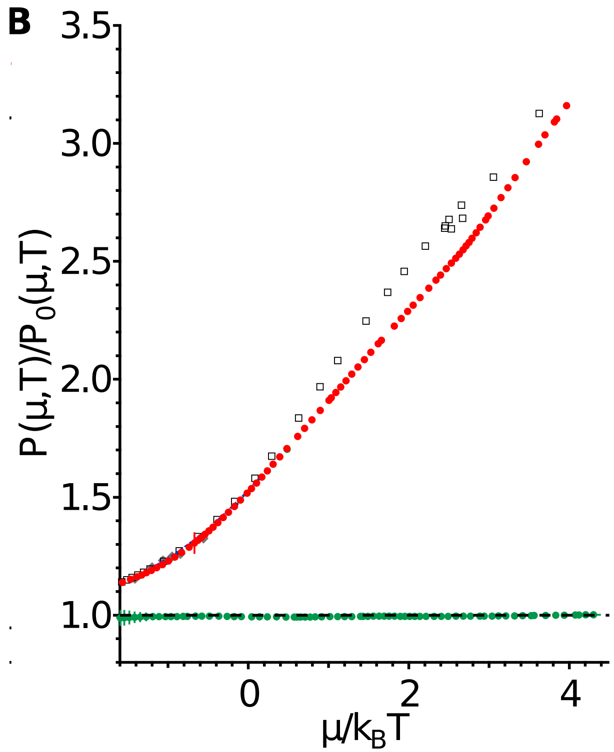
$$= \text{Tr}\left[G_B \frac{\partial G_{B,0}^{-1}}{\partial(-1/a)}\right] = \sum_{X,X'} G_B(X,X')\tilde{g}^2 \frac{m}{4\pi\hbar^2}\delta_{X,X'} = \frac{\hbar^2 C}{4\pi m}$$

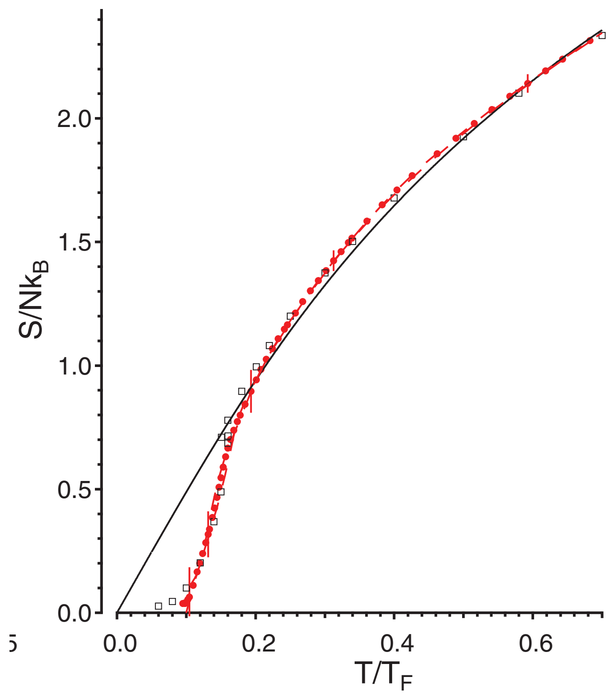
pressure as a function of T/T_F and $1/k_F a$ $(\xi = 0.36)$



comparison with experiments

$$\left(P_0(\mu,T) = \frac{k_B T}{\lambda_T^3} f_{5/2}(z) \right)$$





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Science **335**

p. 563 (2012)

$T_c/T_F \simeq 0.16$ at

$(\mu/k_BT)_c \simeq 2.5$

theory Haussmann/Rantner/Cerrito/Zw. PR **A75** (2007)

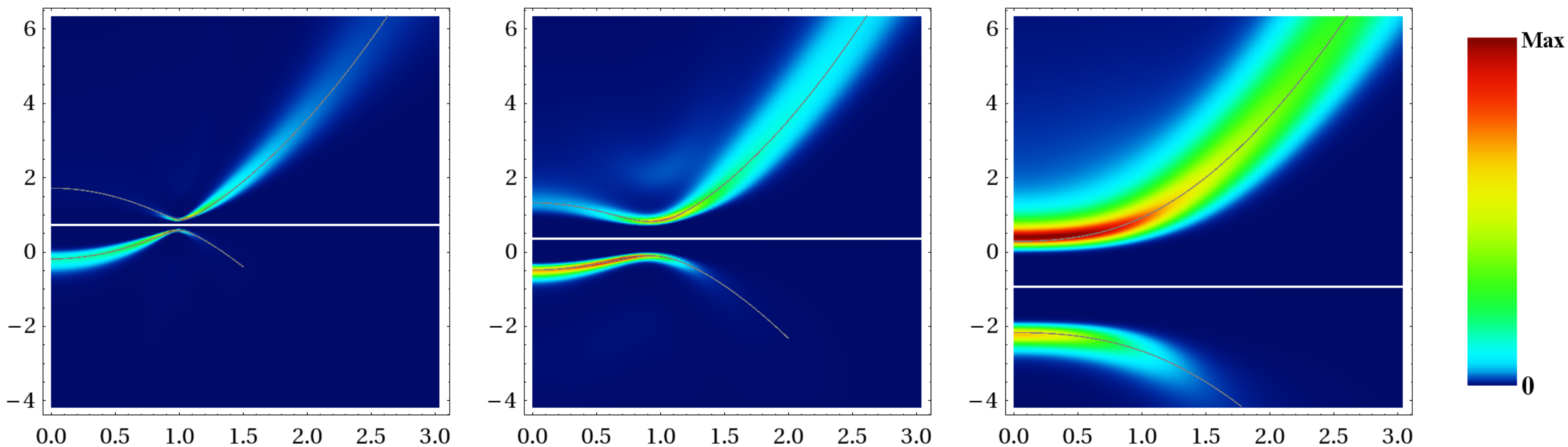
exact LW theory: **bold diagrammatic MC** van Houcke et al 2012

Momentum resolved rf-spectroscopy measures

hole spectral function $A_-(\boldsymbol{k}, \varepsilon_{\boldsymbol{k}} - \hbar\omega)$ Stewart, Gaebler, Jin '08

$A(\boldsymbol{k}, \varepsilon)$ from $\mathcal{G}(\boldsymbol{k}, \tau)$ via $\mathcal{G}(\boldsymbol{k}, \omega_n) = \int d\varepsilon \, \frac{A(\boldsymbol{k}, \varepsilon)}{-i\hbar\omega_n + \varepsilon - \mu}$ (**Maxent**)

numerical spectral functions $A(\boldsymbol{k}, \varepsilon)$ at $T = 0$ (PR **A80** '09)



$(k_F a)^{-1} = -1$ **unitarity** $(k_F a)^{-1} = +1$

II) The unitary gas as a 'perfect fluid' (Kovtun Son Star. '05)

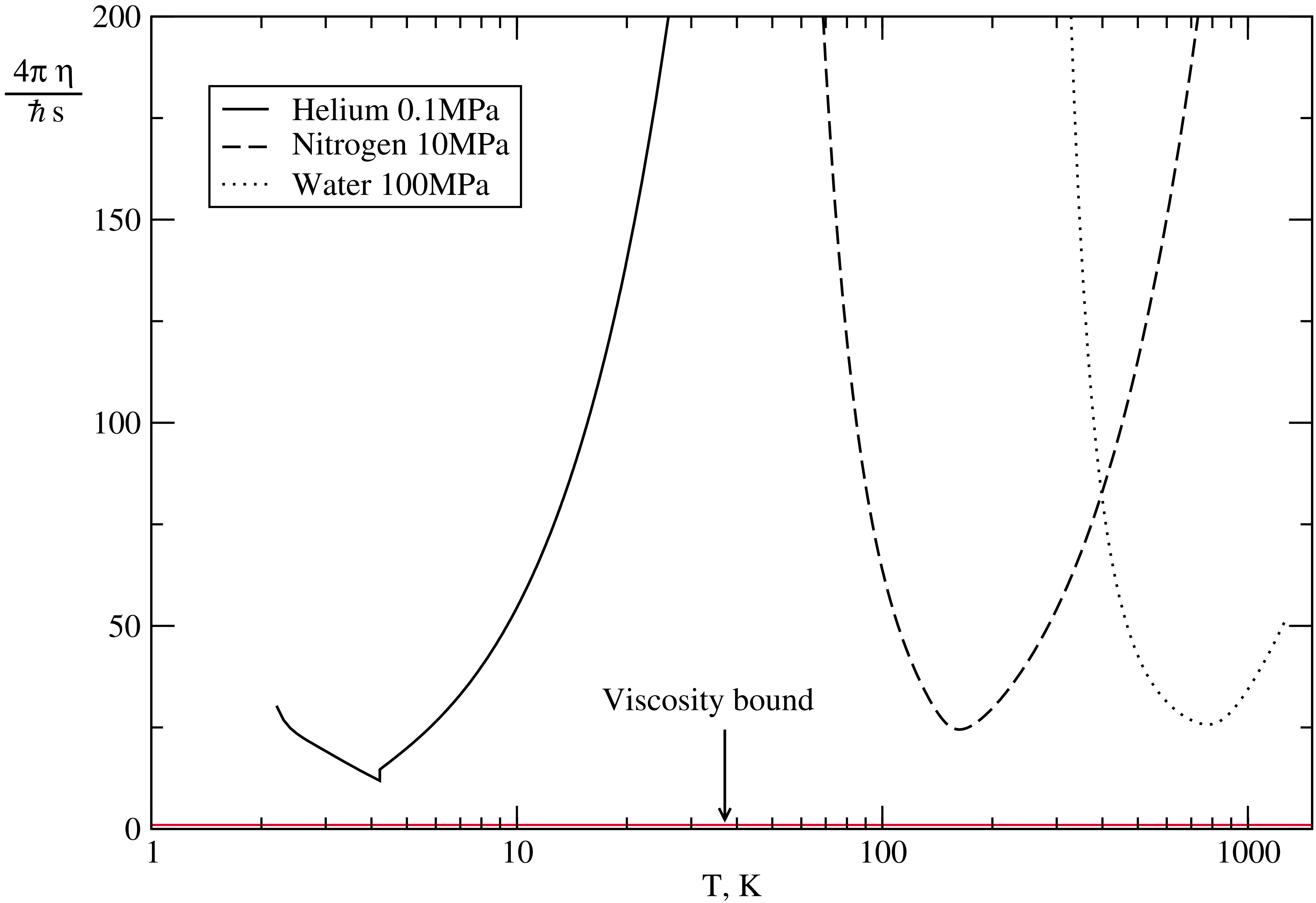
AdS/CFT $\mathcal{N} = 4$ SSYM-Theory in the **t'Hooft limit**

$\lambda = g^2 N \to \infty$ is equivalent to a **classical** theory of gravity

AdS-metric $$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + d\boldsymbol{x}^2 + dz^2\right)$$ $$\boxed{\frac{L}{\ell_P} = \lambda^{1/4} \to \infty}$$

'radial' coord. z is effectively an RG-scale (McGreevy '09)

Conjecture: **All** (relativistic, scale invariant) fluids have $$\boxed{\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}}$$



4π η / ħ s
Helium 0.1MPa
Nitrogen 10MPa
Water 100MPa
Viscosity bound
200
150
100
50
0
1
10
100
1000
T, K

Why does string theory apply to water ?

assume a Lennard-Jones fluid $V(r) = 4\varepsilon\left[(\sigma/r)^{12} - (\sigma/r)^{6}\right]$

reduced density $n^\star = n\sigma^3$ and temp. $T^\star = k_B T/\varepsilon$

critical point at $n_c^\star = 0.36$ and $T_c^\star = 1.36$

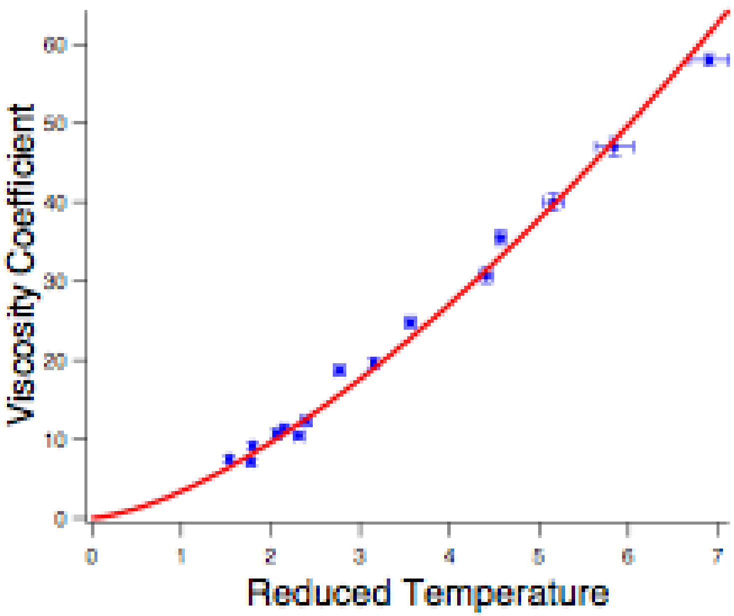
time scale for classical dynamics $\tau = \sqrt{m\sigma^2/\varepsilon} \rightarrow$

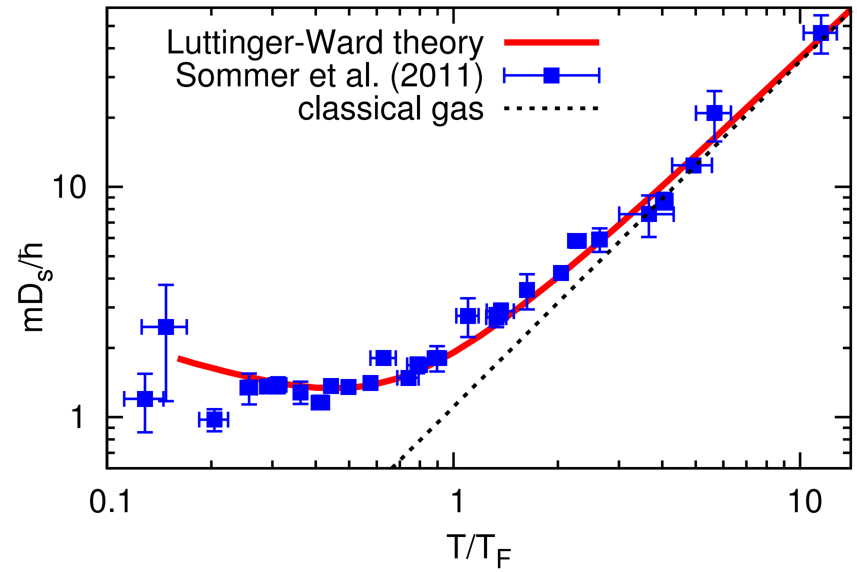
dim. analysis $\eta_{\mathrm{LJ}} = \frac{\varepsilon\tau}{\sigma^3}\eta^\star(n^\star, T^\star) \rightarrow \eta_{\mathrm{LJ}}^{\mathrm{min}} = \mathrm{const}\,\frac{\sqrt{m\varepsilon}}{\sigma^2}$

quantum viscosity $\eta^{\mathrm{min}} = \alpha_\eta\, \hbar n$ with $\alpha_\eta = \mathrm{const}/\Lambda_{\mathrm{DB}} \gtrsim \mathcal{O}(1)$

because the de Boer par. $\Lambda_{\mathrm{DB}} = \hbar/\sigma\sqrt{m\varepsilon}$ cannot be $\gg 1$!

measurements of **viscosity** and **spin diffusion** of the unitary gas





Cao ... Science **331** (2011) and Sommer ... Nature **472** (2011)

shear viscosity of the unitary gas

Boltzmann-limit $\eta(T \gg T_F) = 2.8\,\hbar n (T/T_F)^{3/2} = 4.2\,\frac{\hbar}{\lambda_T^3}$

(density drops out!), well defined quasipart. $\hbar/\tau_\eta \ll k_B T$

superfluid below $T_c \simeq 0.16\,T_F$ has **finite** viscosity due to

a) phonon interactions: $\eta(T) \sim T^{-5}$ as $T \ll T_c$ Rupak/Schäfer '07

b) fermionic qp's: $\eta(T) \to$ const as $T \to 0$ Pethick/Smith '75

$T \ll T_c$ inaccessible since mean free path $\simeq$ trap size

transport coefficients of the unitary gas from Luttinger-Ward

Kubo formula $\quad Re\,\eta(\omega) = \dfrac{Im\,\chi_{xy}^{ret}(\omega)}{\omega}$

perturbation $\quad \hat{H}' = h_\ell(t) \cdot \hat{\Pi}_\ell \quad (\ell = 0, 2 \rightarrow$ bulk, shear$)$

euclidean time $\tau \;\rightarrow\; \chi_\ell(\tau) = \int d^3x \left\langle \tilde{T}\, \hat{\Pi}_\ell(\boldsymbol{x}, \tau) \hat{\Pi}_\ell(\boldsymbol{0}, 0) \right\rangle$

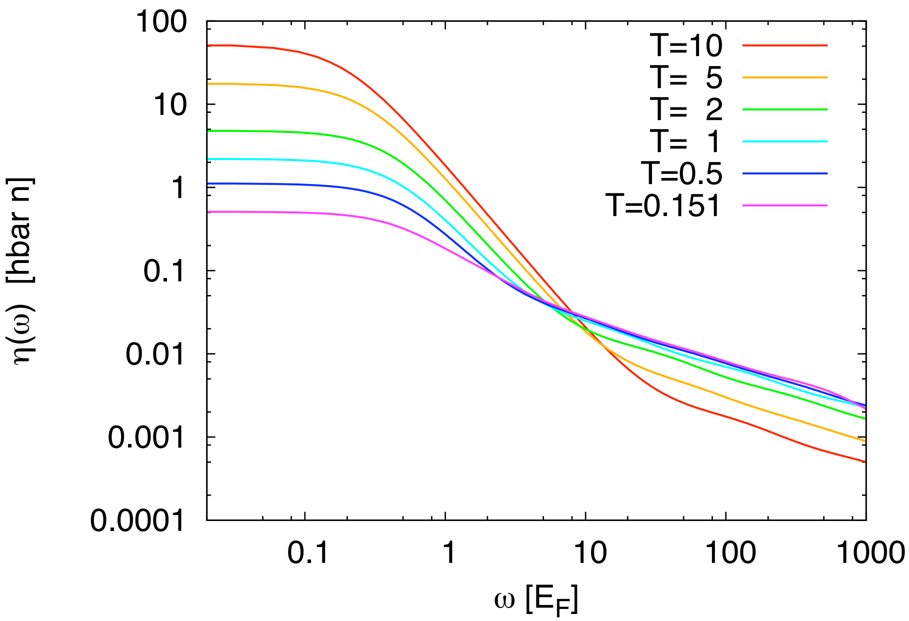
from $\quad \chi_\ell(\tau) = -\dfrac{\delta^2 \Omega}{\delta h_\ell(\tau) \delta h_\ell(0)}\Big|_{h=0} \;\rightarrow\; \chi_{xy}(i\omega_m)$

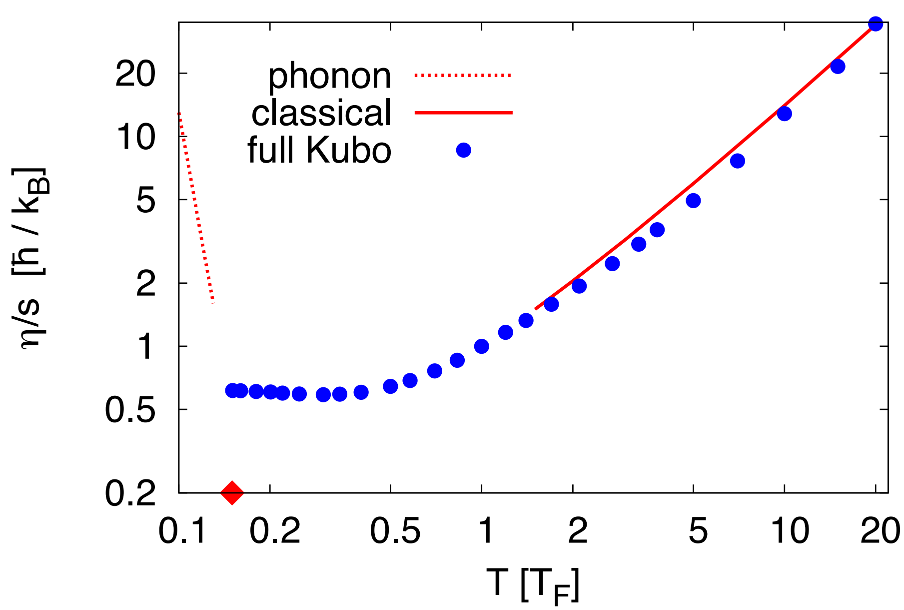
requires contin. to real frequencies ω (Pade, Ansatz)

spin diffusion $(\ell = 1)$ minimum value $D_s \simeq 1.3\,\hbar/m$ near $T = 0.5\,T_F$

Ward-identities due to scale and translation inv.

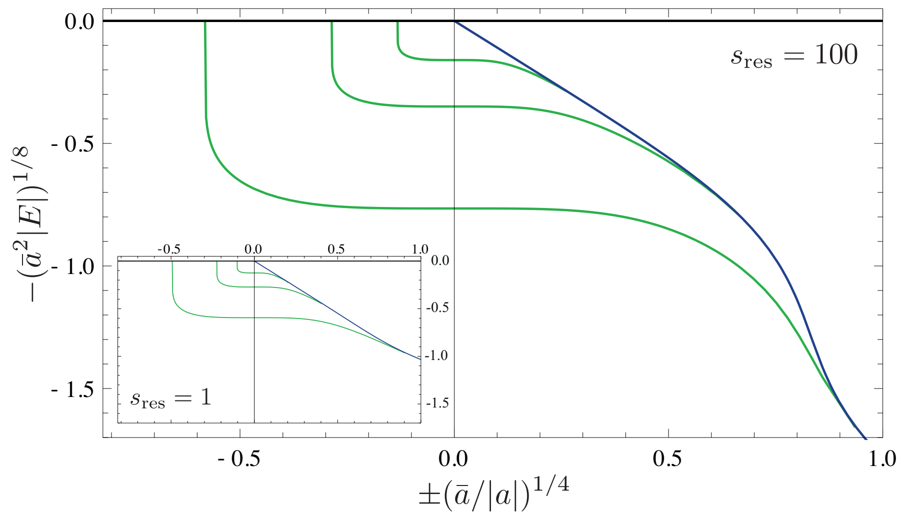
- guarantee that $\zeta(\omega) \equiv 0$
- sum rule $$\frac{2}{\pi}\int_0^\infty d\omega \left[Re\,\eta(\omega) - \frac{\hbar^{3/2}C}{15\pi\sqrt{m\omega}} \right] \equiv p$$
- Boltzmann-limit $\eta \to 4.2\,\frac{\hbar}{\lambda_T^3} \sim T^{3/2}$; $D_s \to 1.1\,\hbar/m\,(T/T_F)^{3/2}$





III) Efimov physics beyond universality   Schmidt, Rath, Zw. '12

Bosons form trimers at $a_-^{(n)} < 0$ **universality** $a_-^{(n+1)}/a_-^{(n)} \to 22.69...$



scale in $(a, E)-$ plane set by

three-body parameter

exp. observation $a_- \approx -9.45\, l_{\rm vdW}$

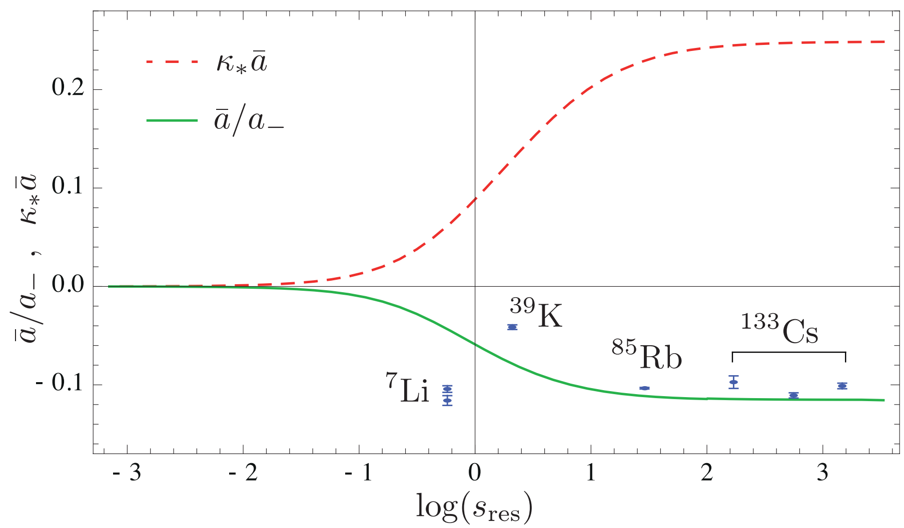
for the **first** Efimov trimer

Feshbach coupling $\hat{H}' = \frac{g}{2}\int \chi(\boldsymbol{r}_2 - \boldsymbol{r}_1)\phi(\frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2})\psi^*(\boldsymbol{r}_1)\psi^*(\boldsymbol{r}_2)$

with finite range $\chi(r) \sim \exp -r/\bar{a}$ $\bar{a} =$ **mean scatt. length** exact solution of RG-flow for atom-dimer vertex $\lambda_3^{(k)}(q_1, q_2; E)$

poles of $\lambda_3^{(k=0)}$ give **Efimov spectrum** which is fixed by $\bar{a}$

and the dimensionless resonance strength $s_{\rm res} = 0.956\, l_{\rm vdw}/r^\star$



crossover from open-channel dominated limit $s_{\rm res} \gg 1$ to $s_{\rm res} \ll 1$ where $a_- = -10.3\, r^\star$

(Petrov '04, Gogolin .... '08)

non-universal ratios $a_-^{(1)}/a_- = 17.1$ exp. 19.7 O'Hara Jochim '09

The unitary gas is a benchmark for many-body physics. It

- realizes a high-temperature fermionic superfluid

  $T_c/T_F \simeq 0.16$ and a scale-invariant many-body problem

  with universal ratios $p/p_F = \xi \simeq 0.37$ and $S/Nk_B|_c \simeq 0.7$

- is the most perfect non-relativistic fluid with η/s close to

  the KSS bound and quantum-limited spin-diffusion $D_s \simeq 1.3\,\hbar/m$

The Efimov spectrum for cold atoms is fixed by l_{vdw} and $r^\star$

in the absence of 3-body forces