



2350-1

**Workshop on Quantum Simulations with Ultracold Atoms**

*16 - 20 July 2012*

**An optical-lattice-based quantum simulator for relativistic field theories and topological insulators**

M. Rizzi

*Max Planck Inst. fuer Quantenoptik, Garching, Germany*



Trieste, ICTP, July 17th 2012

# An optical-lattice-based Quantum Simulator for relativistic field theories and topological insulators

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Mazza, MR, Lewenstein, Cirac, PRA **82** 043629 [1007.2344]

Bermudez, Mazza, MR, Goldman, Lewenstein, Martin-Delgado, PRL **105** 190404 [1004.5101]

Mazza, Bermudez, Goldman, MR, Martin-Delgado, Lewenstein, NJP **14** 015007 [1105.0932]



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# Outline

- quantum simulations & cold gases
- topo-insulators & free-Fermi theories
- exp. setup: superlattice + Raman
- proposal at work: towards 3D-TI
- future directions

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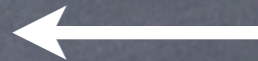
# Quantum simulation paradigm

*iHARD!*

Classical  
computation

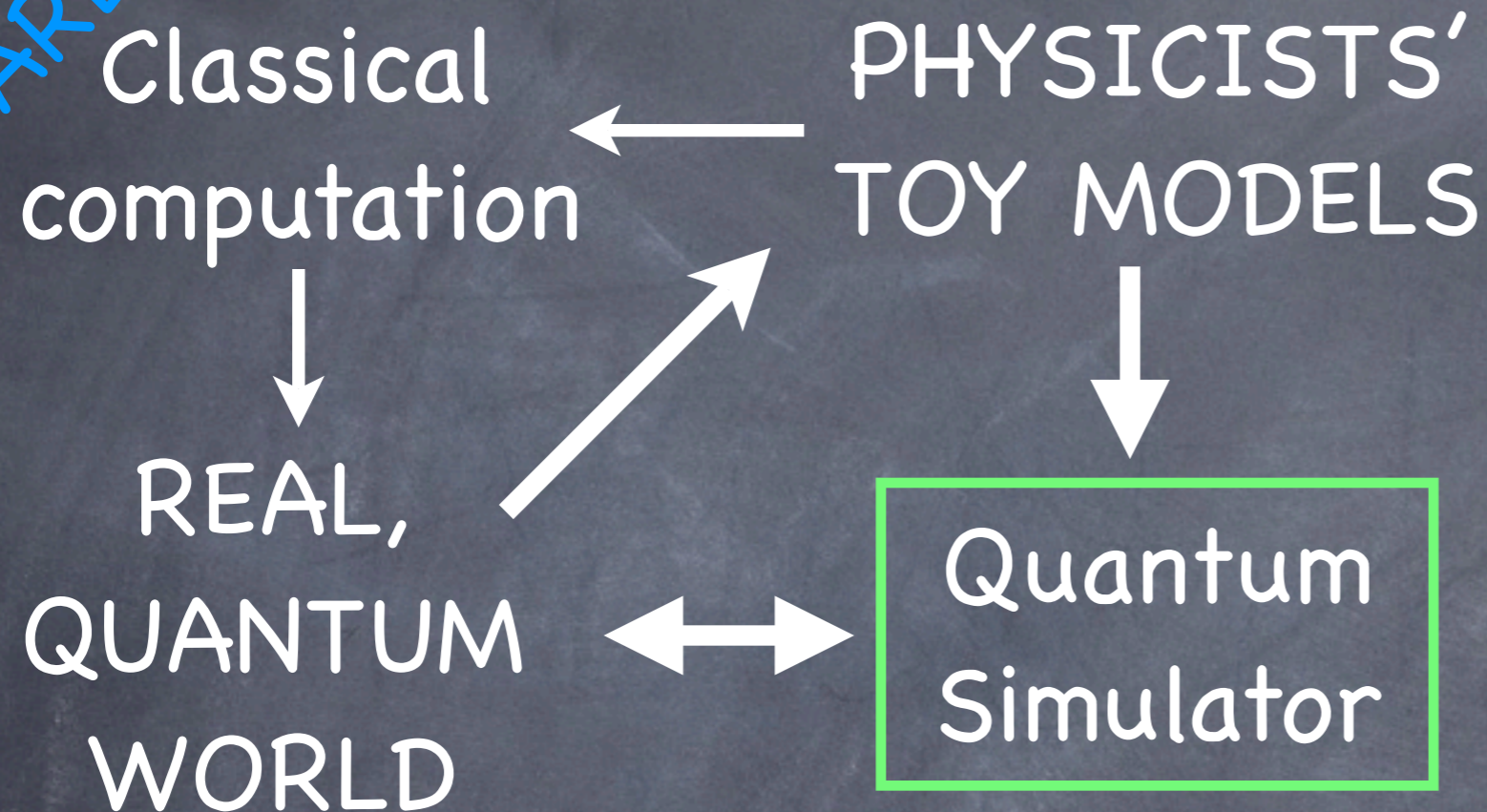
PHYSICISTS'  
TOY MODELS

REAL,  
QUANTUM  
WORLD



# Quantum simulation paradigm

*iHARD!*

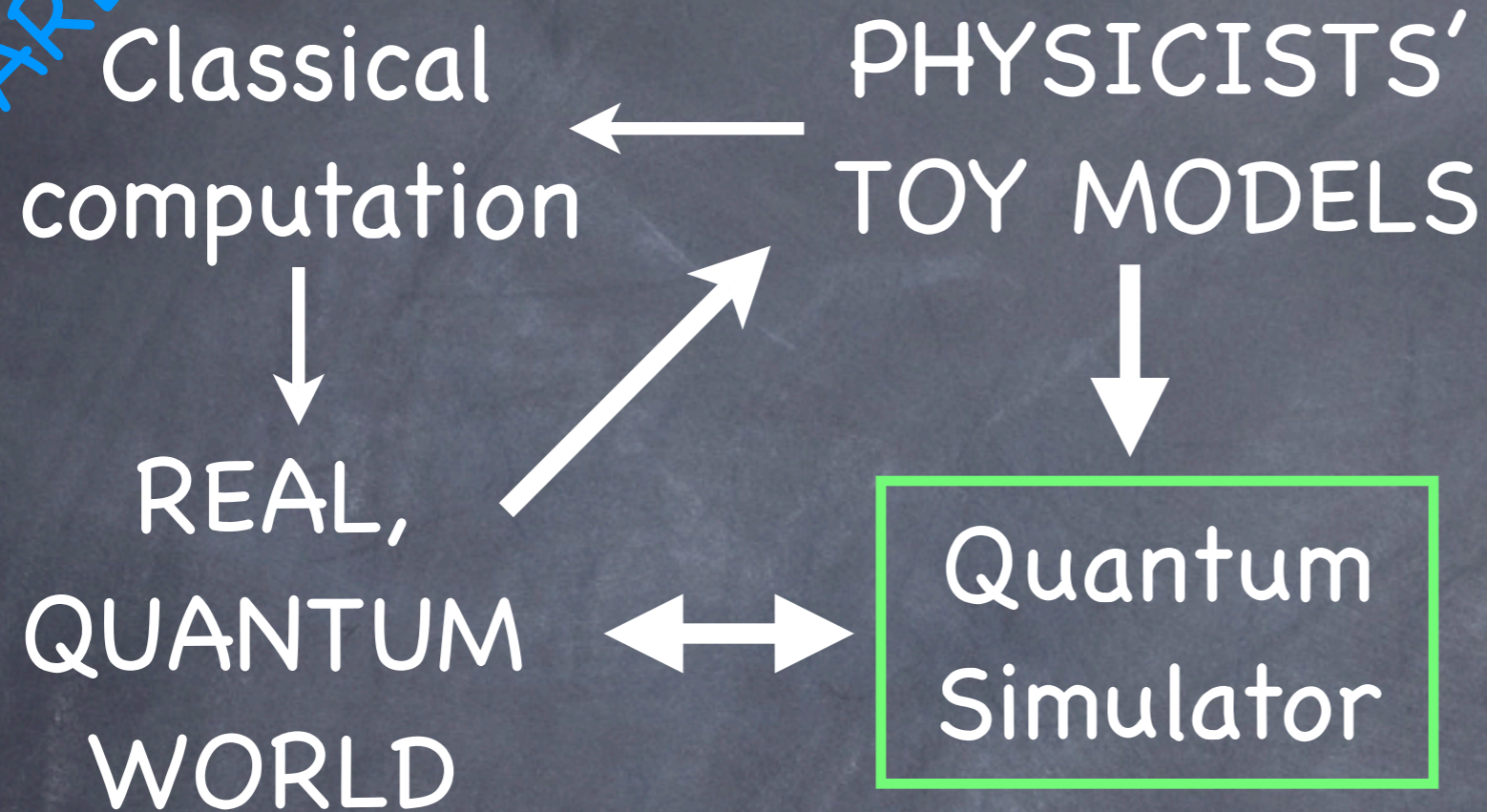


controllable quantum system:

- detailed microscopic knowledge
- high external tunability
- good access for measurements

# Quantum simulation paradigm

*iHARD!*



*GOAL*

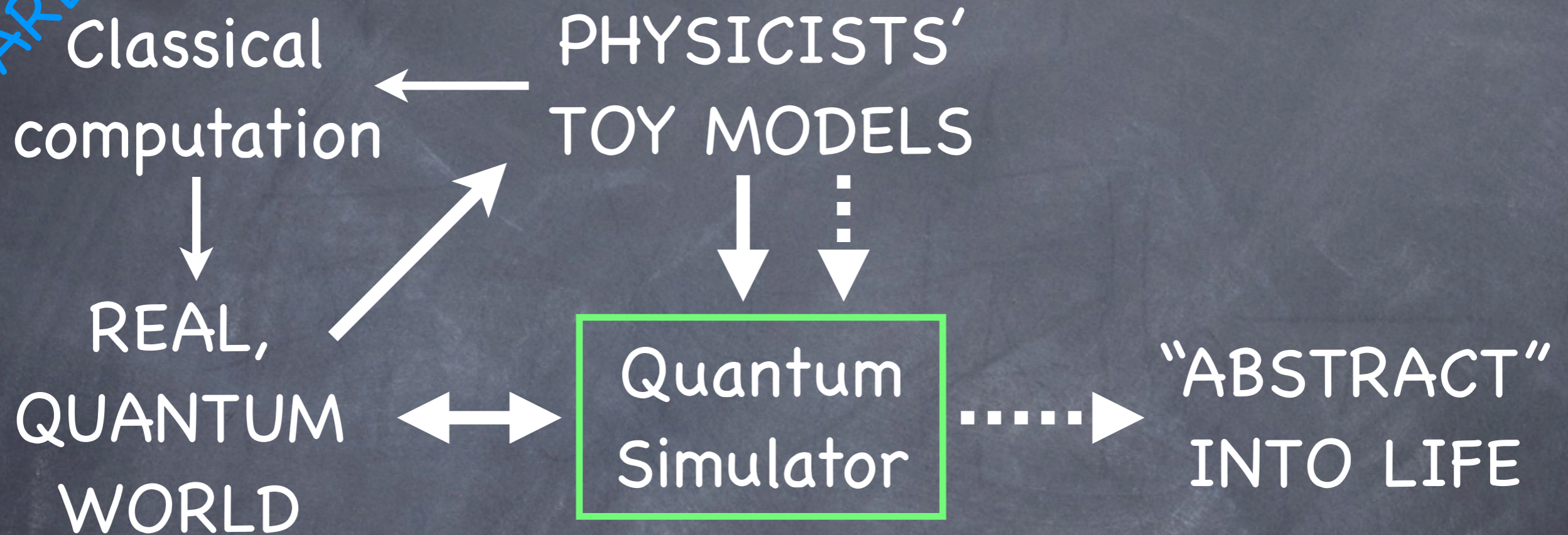
1. map system of interest into the Q-Sim platform (d.o.f., Hamiltonian, observables, ...)
2. let the Q-sim "compute" the quantum solution
3. retrieve the desired informations by measures

*PURPOSE*



# Quantum simulation paradigm

*iHARD!*



*GOAL*

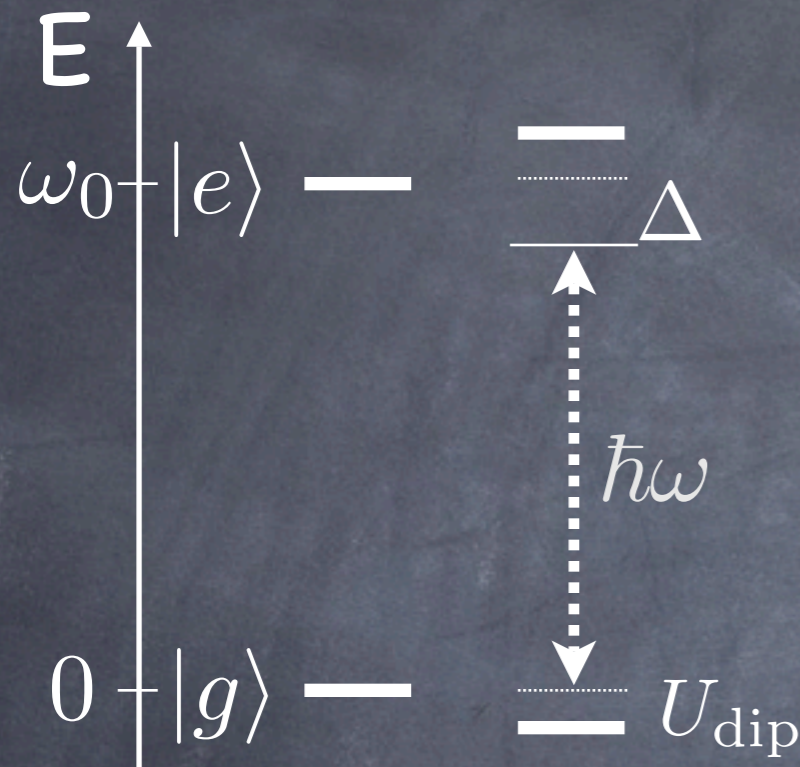
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*PURPOSE  
BUILT*

# Ultracold atoms ~ ideal QS

- true quantum systems  
( $T \approx nK$   $S/N \approx k_B$ )
- highly controllable  
(dimensionality, geometry, interactions, disorder)
- various imaging techniques  
(mom. dist., correlations, densities, single-site)
- diverse statistics  
(fermions, bosons, mixtures)

# Cold gases: optical potentials



$\Gamma$  dipole moment e-g

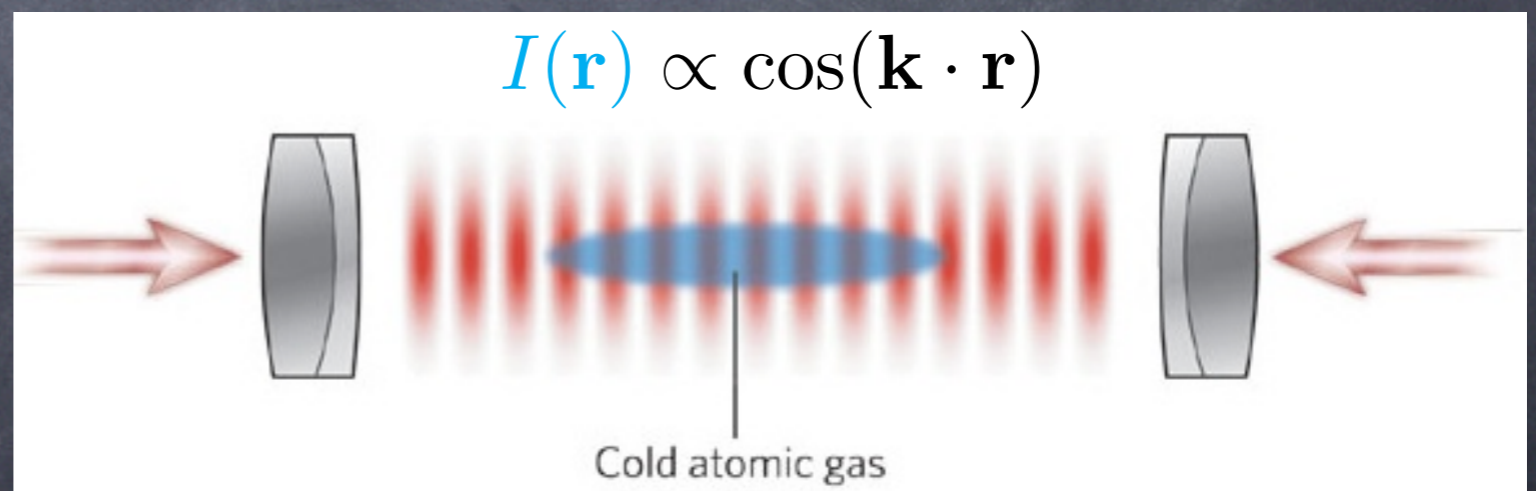
$$U_{\text{dip}}(\mathbf{r}) \propto \frac{\Gamma}{\Delta} I(\mathbf{r}) \quad \text{ac Stark shift}$$

$$\hbar\Gamma_{\text{sc}} \propto \frac{\Gamma^2}{\Delta^2} I(\mathbf{r}) \quad \text{photon scattering}$$

Grimm et al., Adv. At., Mol., Opt. Phys. 42, 95 (2000).

Trapping possibilities:

- spatial dependent  $I(\mathbf{r})$
- spin dependent  $\Delta$
- speckle potentials

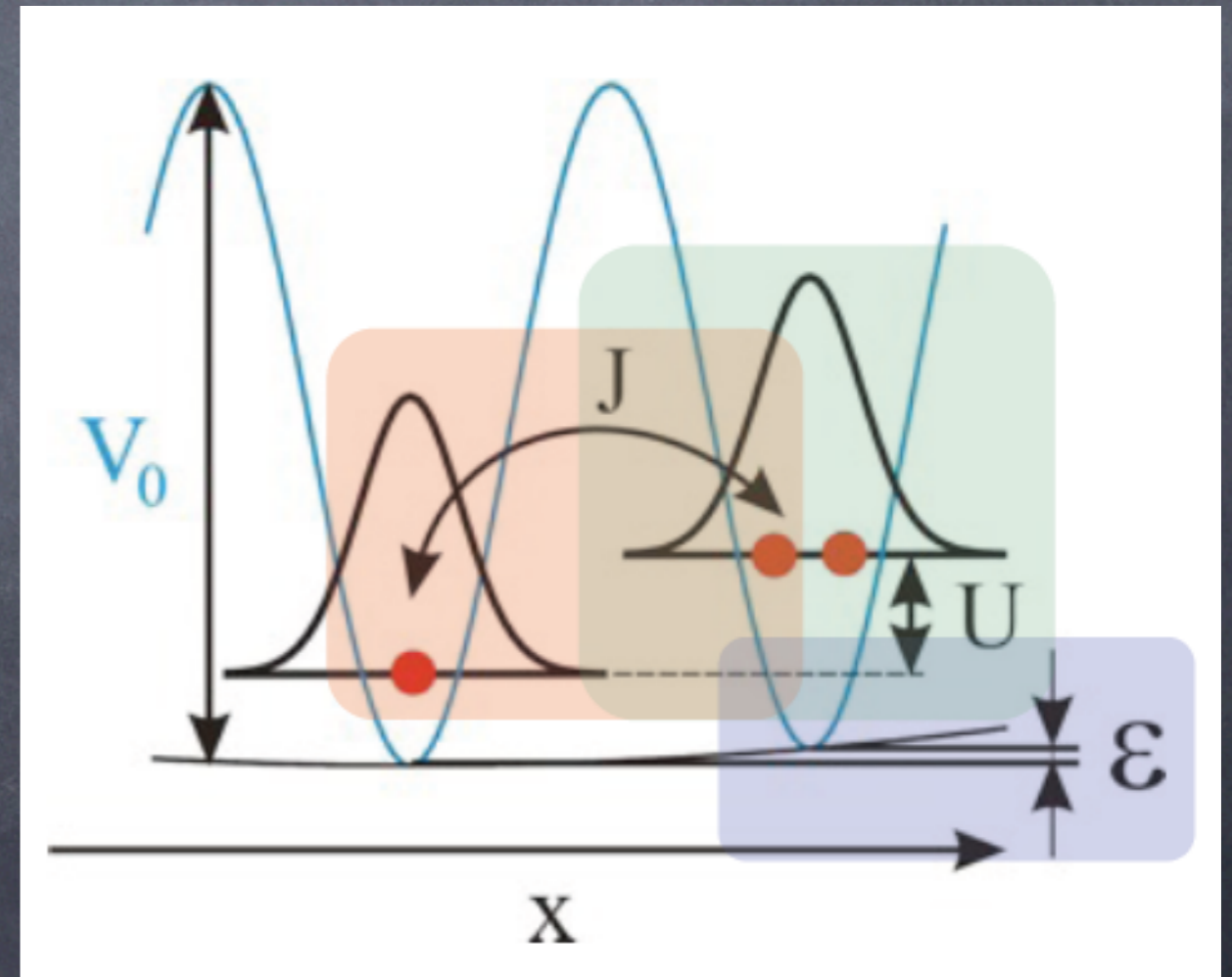
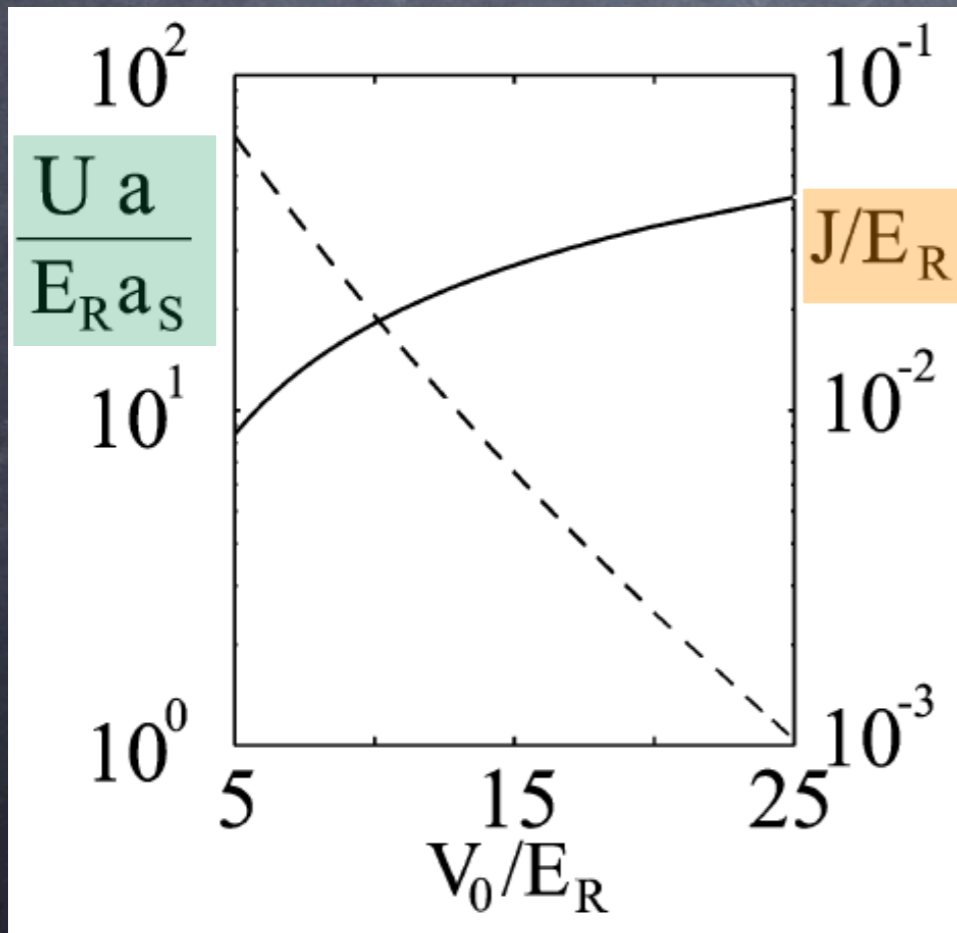


# Cold gases: mapping to Hubbard

Tight-binding model  
out of localized  
(single particle)  
Wannier functions

$$H = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \sum_i \epsilon_i \hat{n}_i + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$$

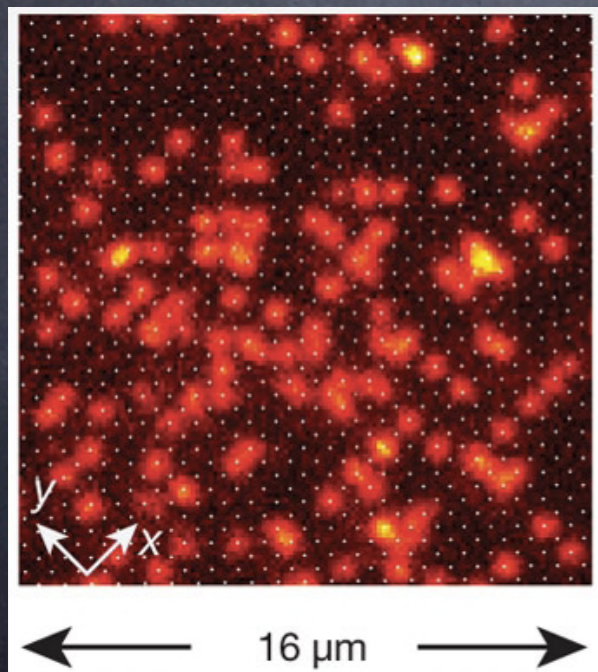
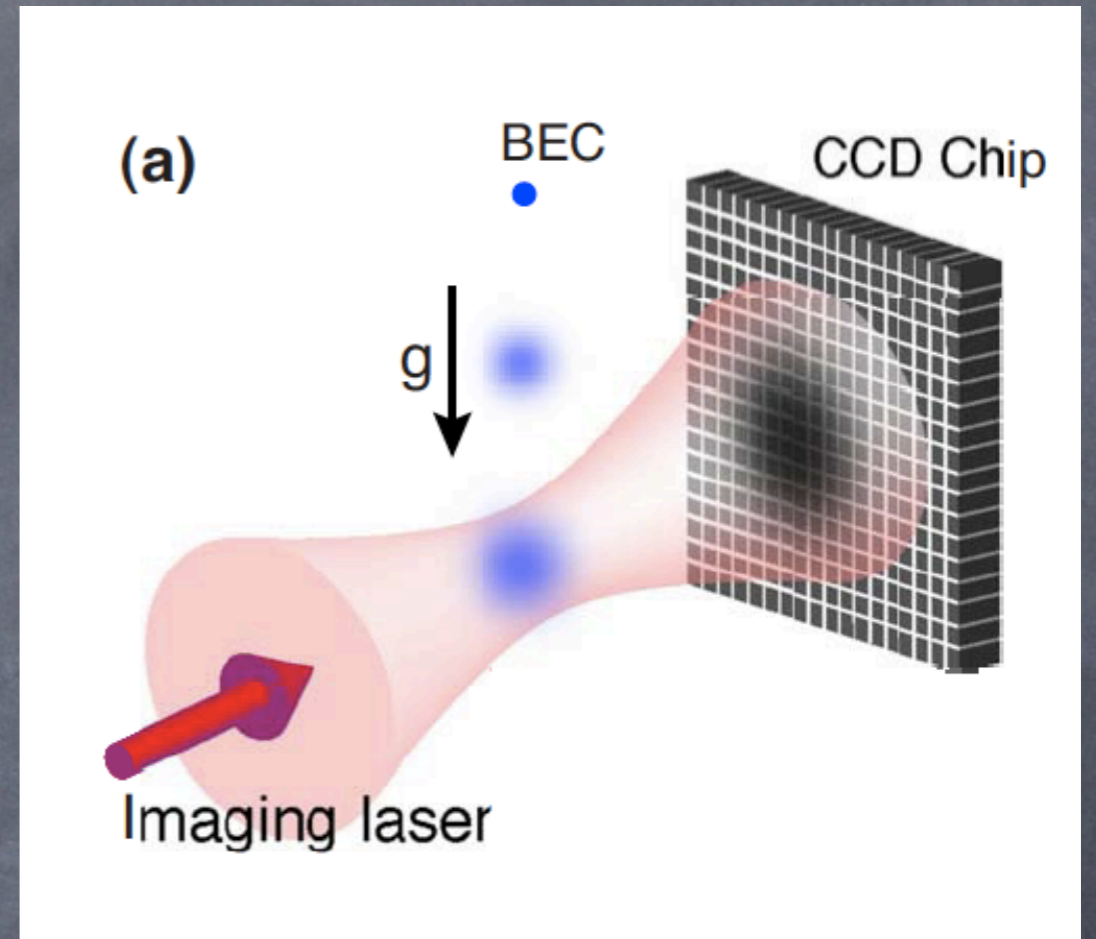
Jaksch et al. PRL **81** 3108 (1998)



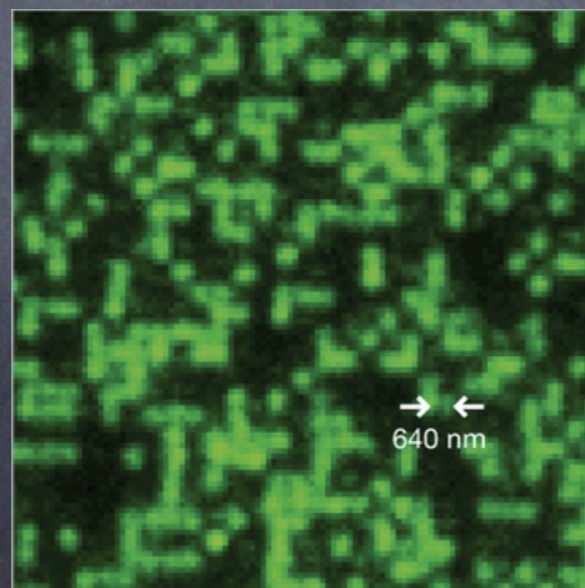
# Cold gases: detection

time-of-flight (TOF) imaging

$$\langle \hat{n}(\mathbf{x}) \rangle_{\text{TOF}} \simeq \langle \hat{n} \left( \mathbf{k} = \frac{m\mathbf{x}}{\hbar t} \right) \rangle_{\text{trap}}$$



Bloch group @ MPQ



Greiner group @ Harvard

Single-site  
imaging/manipulation

# (some) CA-QS in recent years

- Mott-Superfluid transition
- exotic SFs in imbalanced fermions
- antiferromagnetic spin-chains
- itinerant ferromagnetism
- Anderson localization
- equation of state (fermions/bosons)
- ...

# Cold gases: synthetic gauge potentials

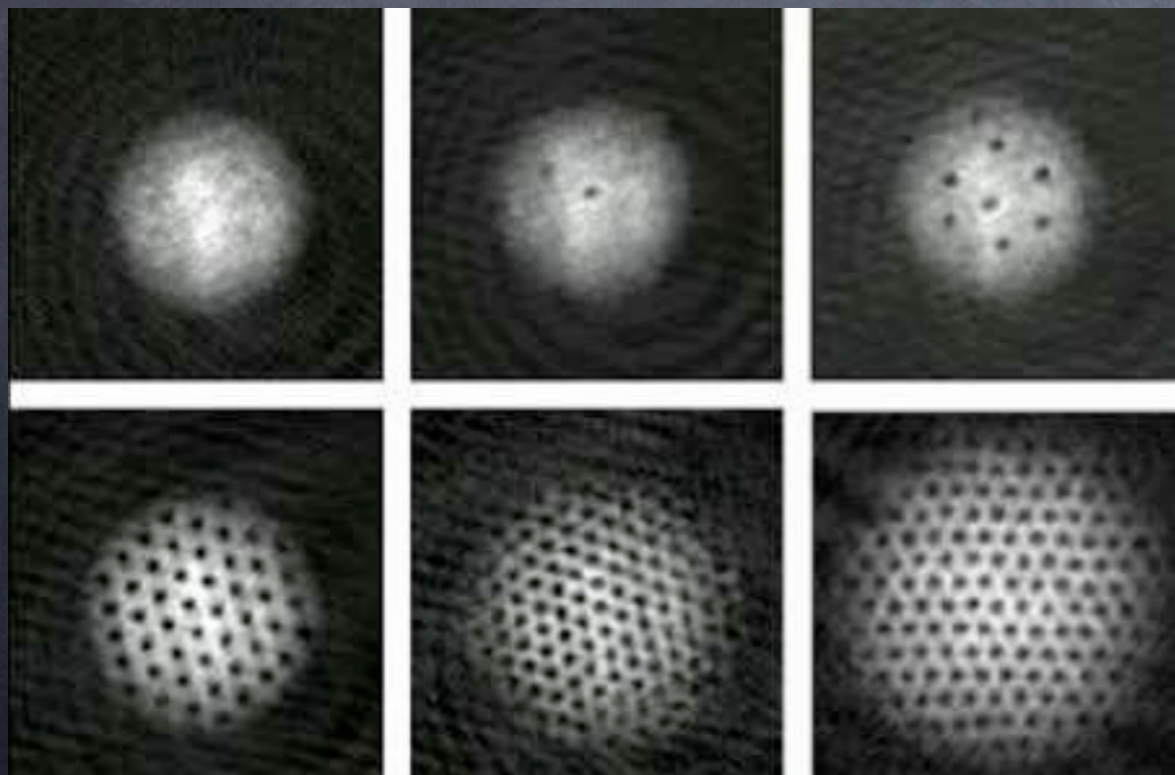
ultracold atoms are neutral !  
real magnetic fields affect only spin d.o.f.



how to access rich "orbital" physics  
of electrons in magn. fields !?  $\mathbf{p} \rightarrow (\mathbf{p} - e\mathbf{A})^2$



rotation of harmonic traps

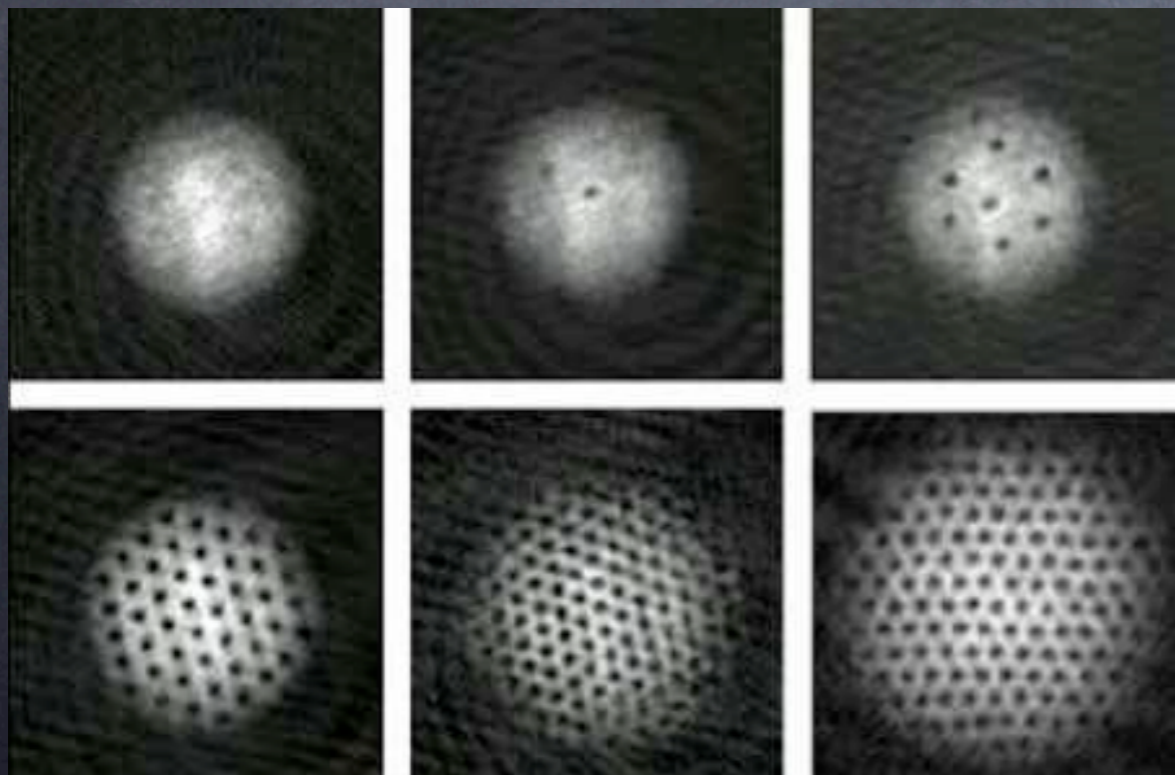


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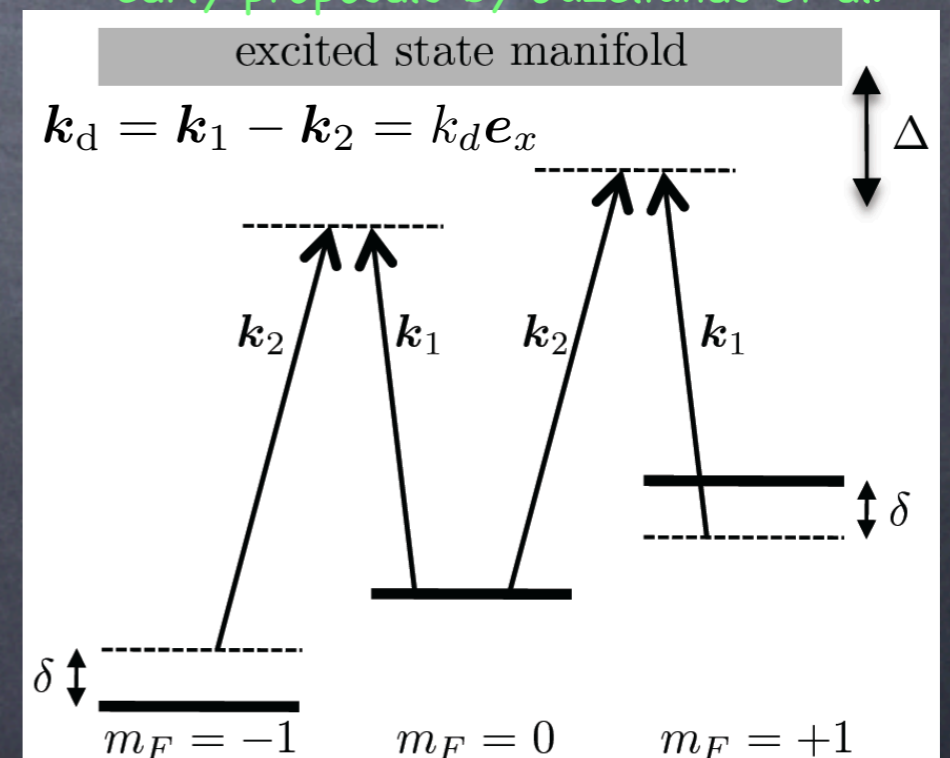
rotation of harmonic traps



A. Fetter, Rev.Mod.Phys. 81 647 ('09)

laser-assisted methods

early proposals by Juzeliunas et al.



Lin, Spielman, et al. Nature 462, 628 ('09)



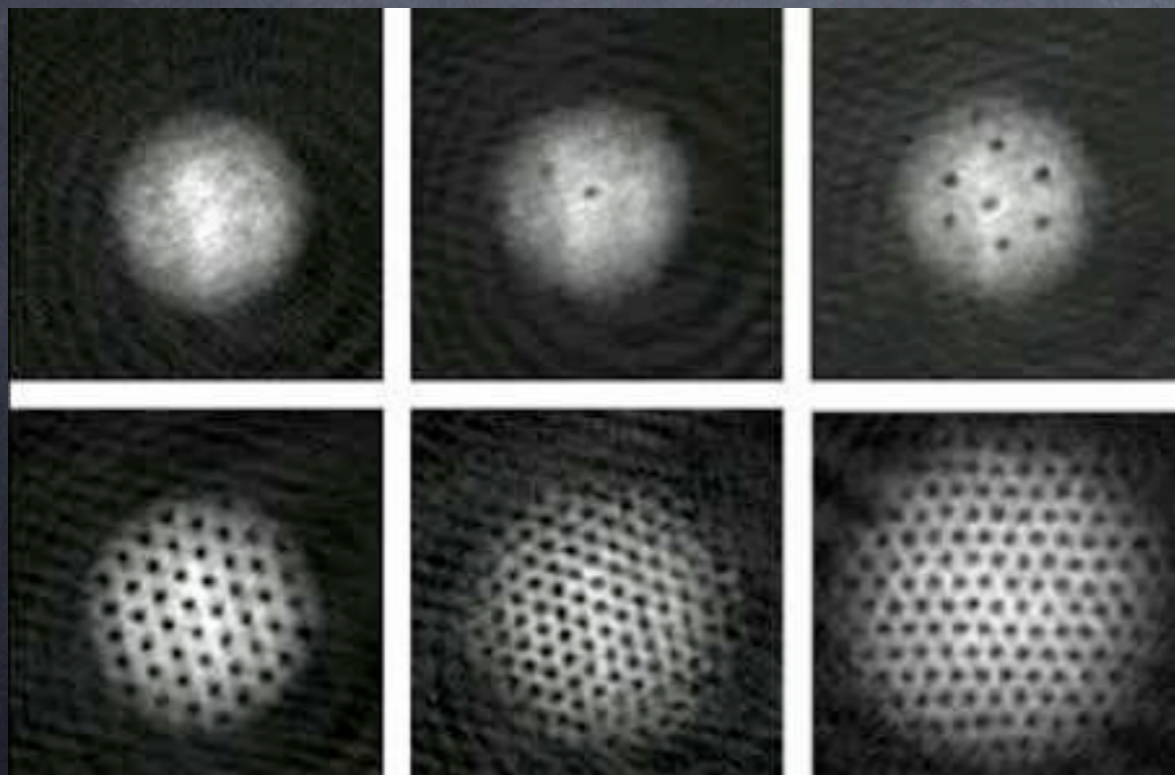
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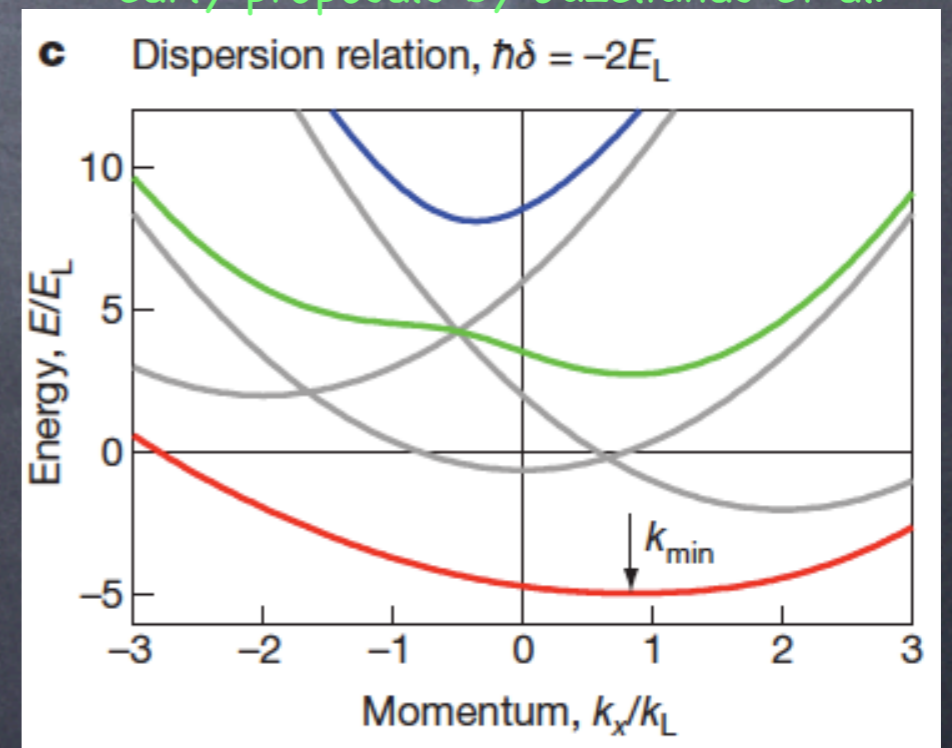
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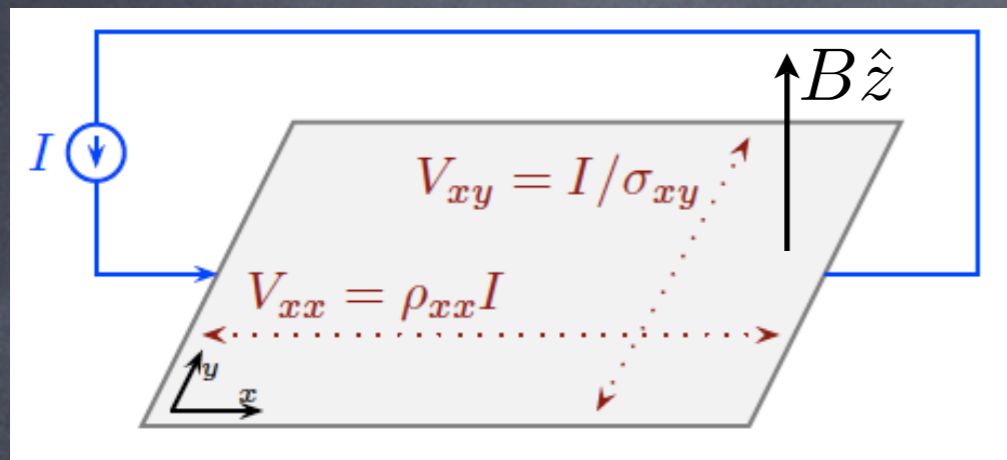
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# Quantum Hall physics

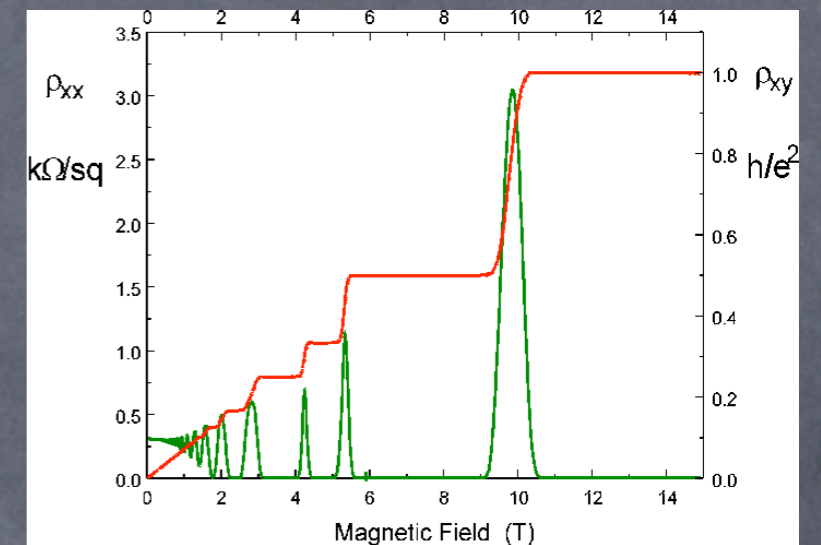
- 2D electron gas @ low T & intense B



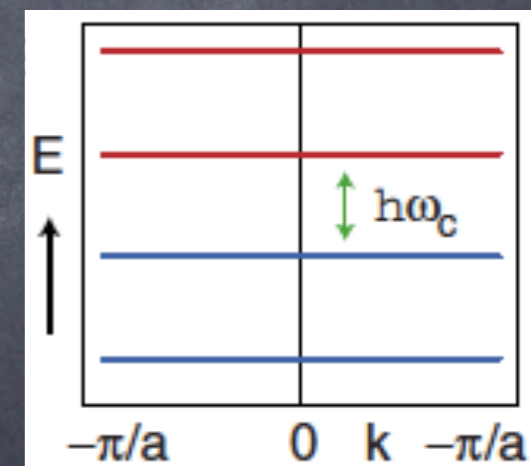
$$\sigma_{xy} = \nu \frac{e^2}{h}$$

$$\nu = \frac{n_e \phi_0}{B}$$

$$\nu = \frac{p}{q}$$

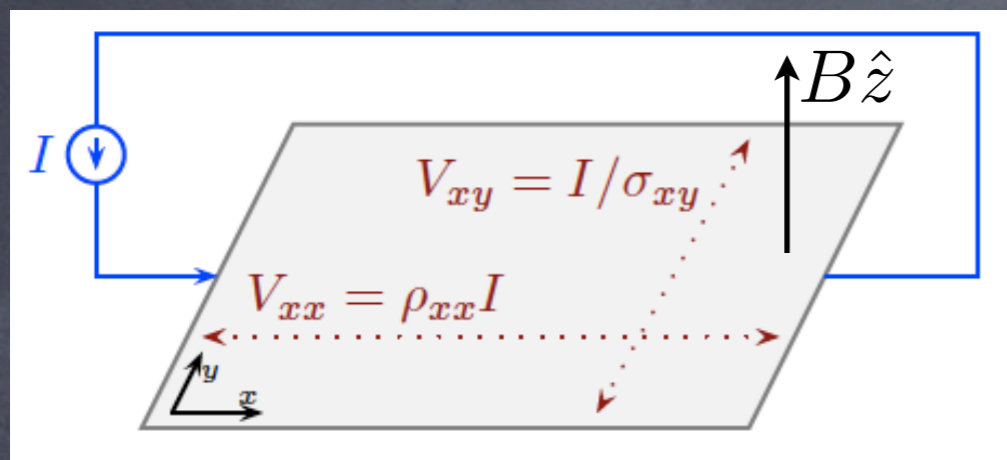


- precise & robust quantization of transport properties
- spectrum: (flat) Landau levels with gaps



# Quantum Hall physics

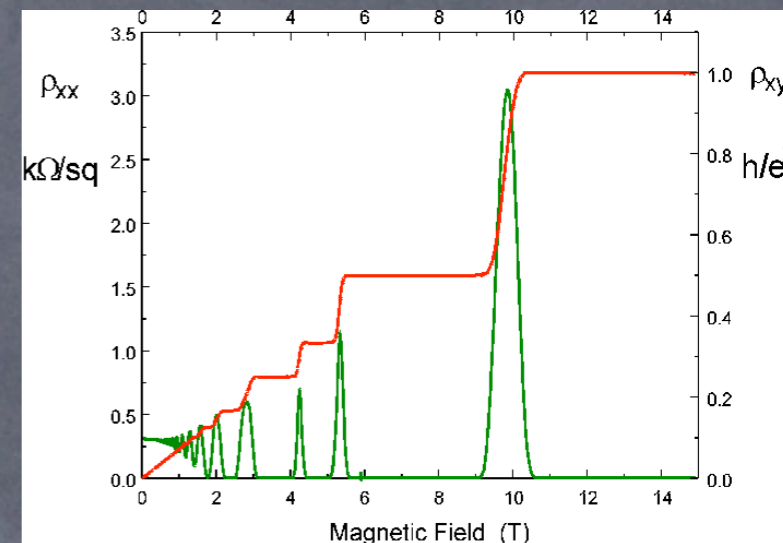
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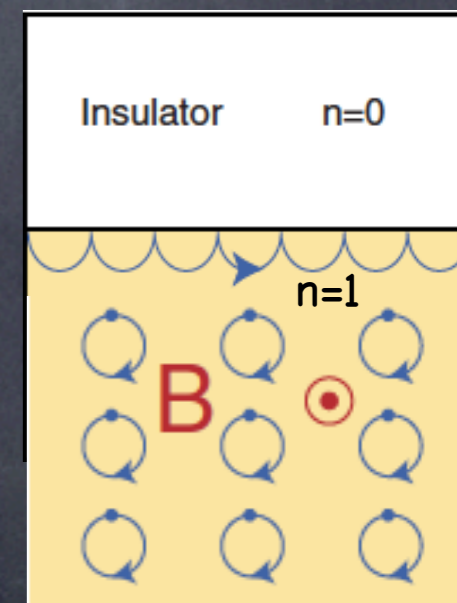
- spectrum: (flat) Landau levels with gaps

- q=1 -> Integer QHE: single particle physics



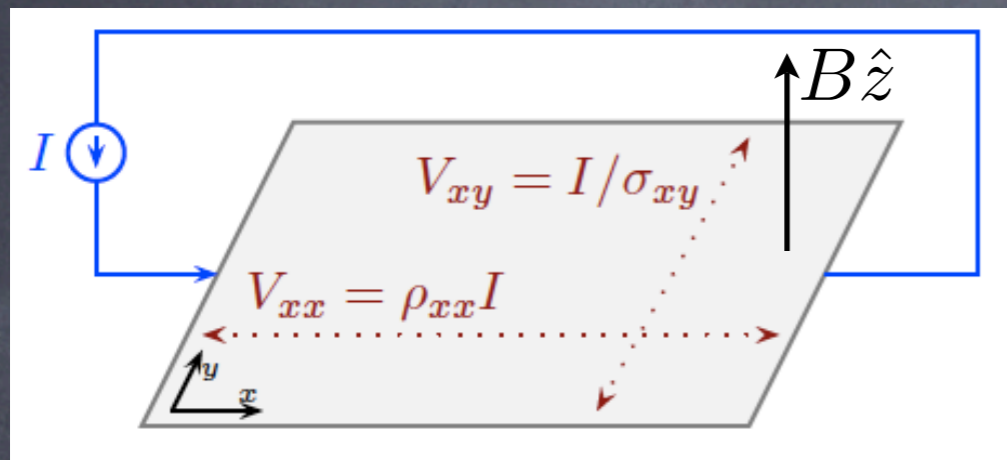
- related to a topological invariant (TKNN)

- presence of zero-energy edge modes



# Quantum Hall physics

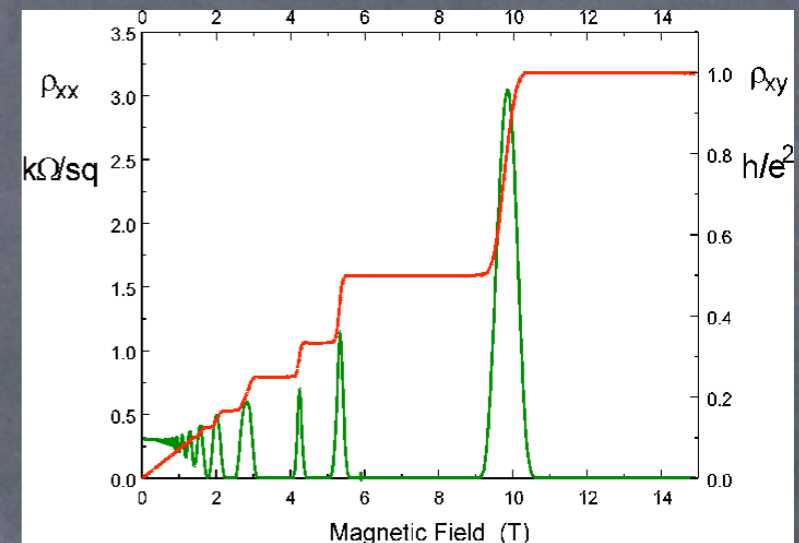
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$$\nu = \frac{n_e \phi_0}{B}$$

$$\nu = \frac{p}{q}$$



- precise & robust quantization of transport properties

- spectrum: (flat) Landau levels with gaps

- q ≠ 1 → Fractional QHE: many-body physics



- interactions generate strong correlations

- non-perturbative, e.g. Laughlin  $\nu = 1/3$  :  $\Psi = \prod_{j < l} (z_j - z_l)^3$

# Topological insulators

**IQHE** is a paradigmatic example of:

- **gapped bulk** systems showing “exotic” properties  
--> transport (or response to EM probes)
- bulk characterized by a **topological invariant**  
--> integral on the toroidal Brillouin zone (TKNN)
- **gapless edge states** carry informations on the bulk  
--> holographic principle

# Topological insulators

Full characterization  
via free-Fermi Hamiltonians  
with symmetries:

- $T$  = time-reversal
- $C$  = particle-hole
- $S = CT$  = chiral

Class	Name	$T$	$C$	$S$	$d = 1$	$d = 2$	$d = 3$
A	Unitary	0	0	0		$\mathbb{Z}$ [IQHE]	
AIII	Chiral unitary	0	0	1	$\mathbb{Z}$		$\mathbb{Z}$
AI	Orthogonal	+1	0	0			
BDI	Chiral Orthogonal	+1	+1	1	$\mathbb{Z}$ [1D polyac.]		
AII	Symplectic	-1	0	0		$\mathbb{Z}_2$ [QSHE]	$\mathbb{Z}_2$ [3DTI]
CII	Chiral Symplectic	-1	-1	1	$2\mathbb{Z}$		$\mathbb{Z}_2$

A. Altland, and M. R. Zirnbauer, Phys. Rev. B **55**, 1142 ('97)

M. Hasan, and C.L. Kane, Rev. Mod. Phys. **82** 3045 ('10)

# Target Hamiltonian

Quantum simulator for non-interacting Fermions:

$$H_{\text{sys}} = \sum_{\mathbf{r}} \sum_{\tau\tau'} \left[ \sum_{\nu} t_{\nu} c_{\mathbf{r}+\nu\tau'}^{\dagger} [U_{\nu}]_{\tau'\tau} c_{\mathbf{r}\tau} + \Omega c_{\mathbf{r}\tau'}^{\dagger} [\Lambda]_{\tau'\tau} c_{\mathbf{r}\tau} + \text{H.c.} \right]$$

with control over all parameters  $D, \Omega, \Lambda, t, U$



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with control over all parameters  $D, \Omega, \Lambda, t, U$

IDEA! cold gases in an optical lattice setup that is:

- bichromatic
- spin-independent
- Raman assisted

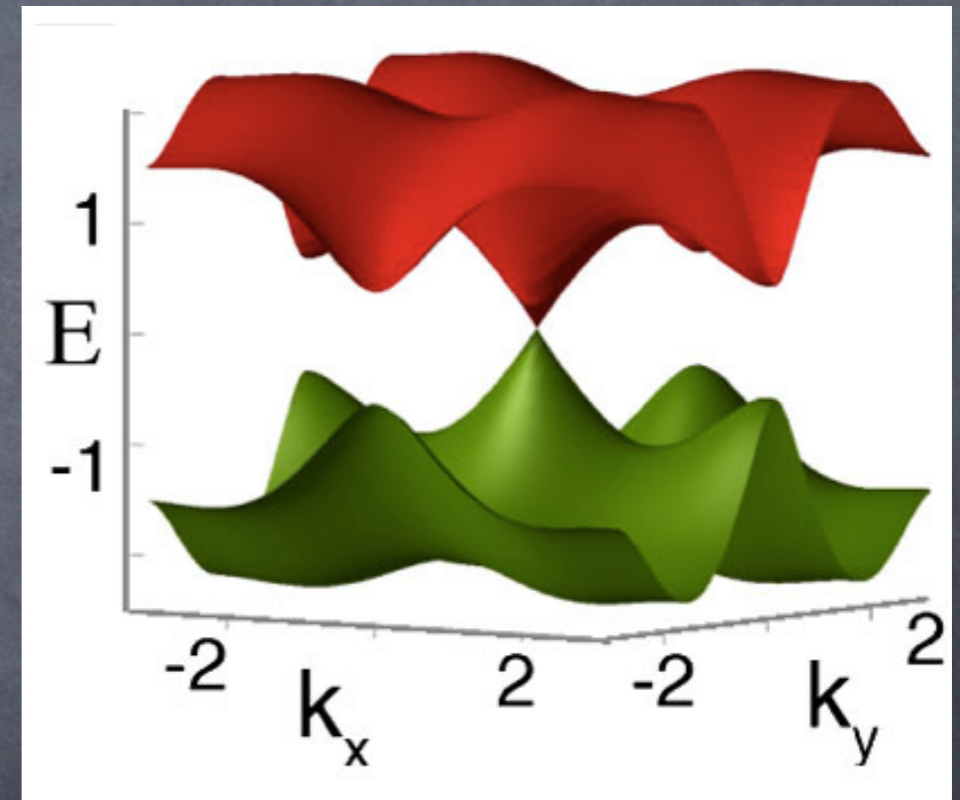
# Not many-body but rich

$$H_{\text{sys}}(\{t_\nu, U_{\mathbf{r}\nu}, \Lambda, \Omega\}) \rightarrow H_{\text{eff}} \approx H_{\text{rel}}, H_{\text{top}}$$

$H_{\text{rel}}$  emerging relativistic fermions  
with/-out mass in  $D=1,2,3$   
→ as graphene BUT richer ←

↓  
tailorable to get isolated  
boundary massless fermions

↓  
Topological Insulators !  $H_{\text{top}}$



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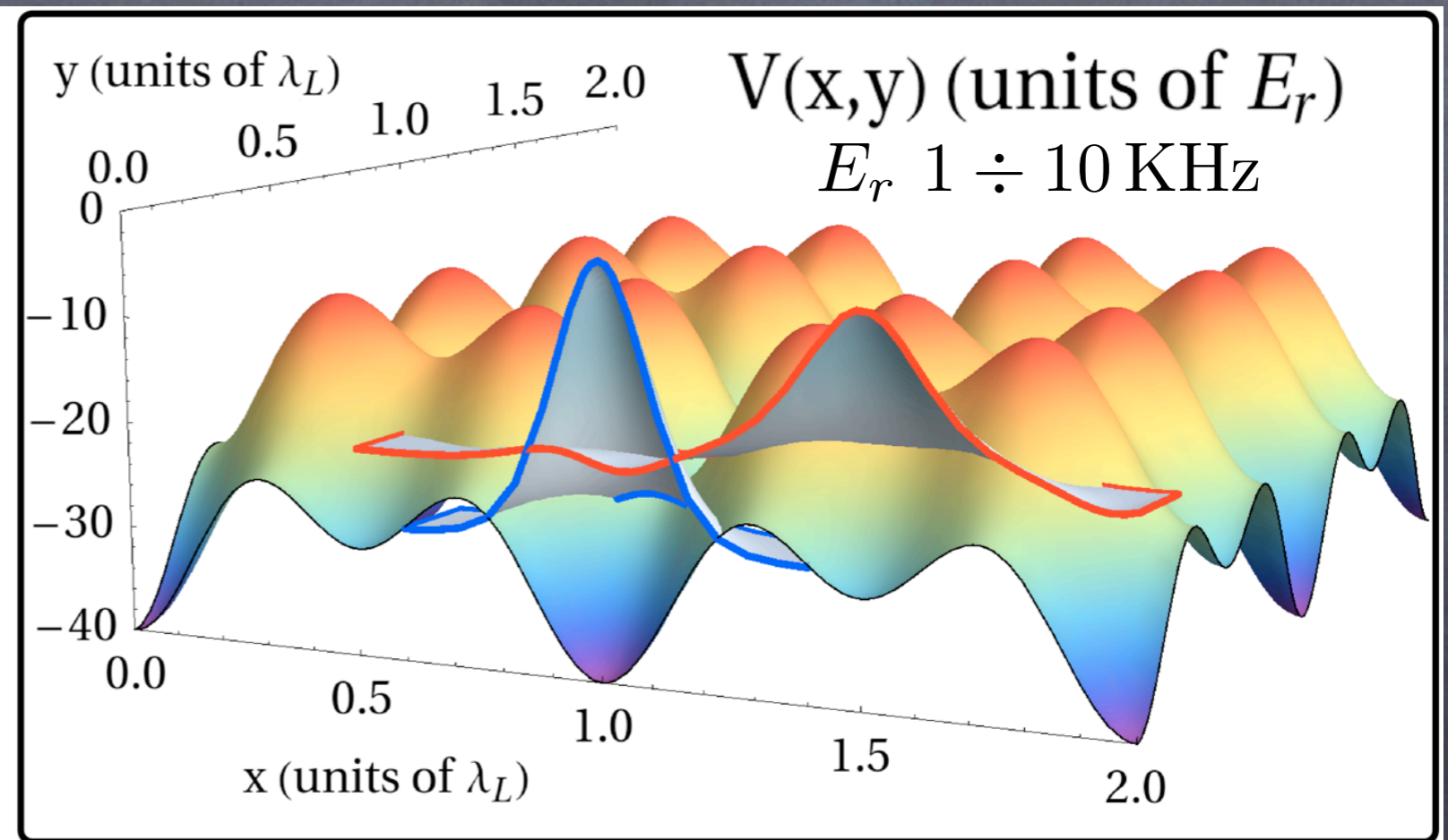
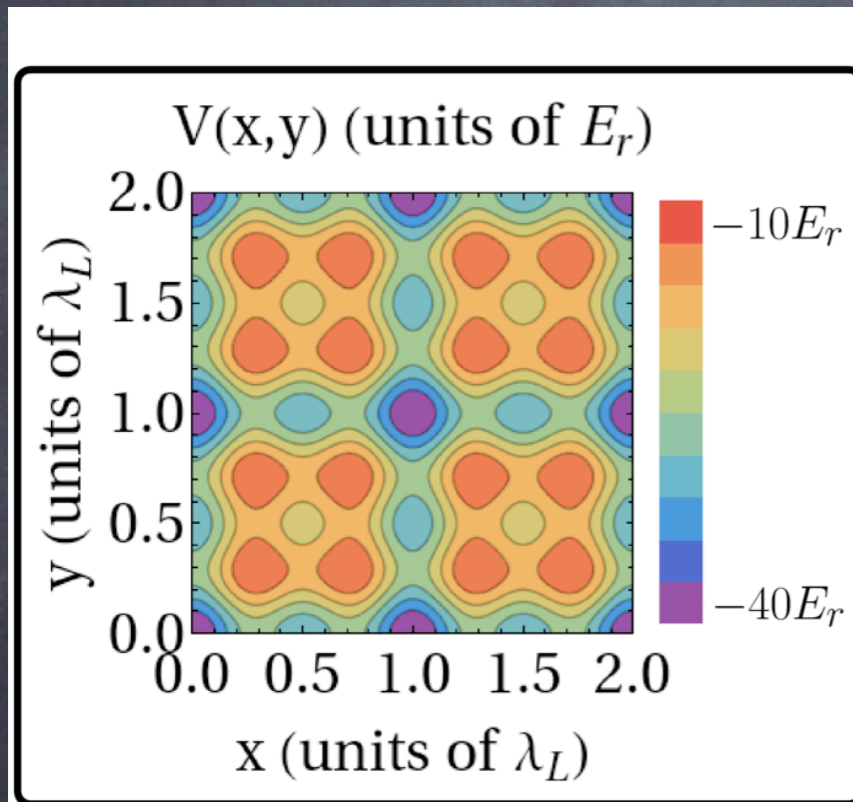
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# Sketch of the proposal

$$V(\mathbf{x}) = -V_0 \sum_{j \in \{1,2,3\}} [\cos^2(qx_j) + \xi \cos^2(2qx_j)]$$

S. Folling, et al.,  
Nature 448, 1029  
(2007).



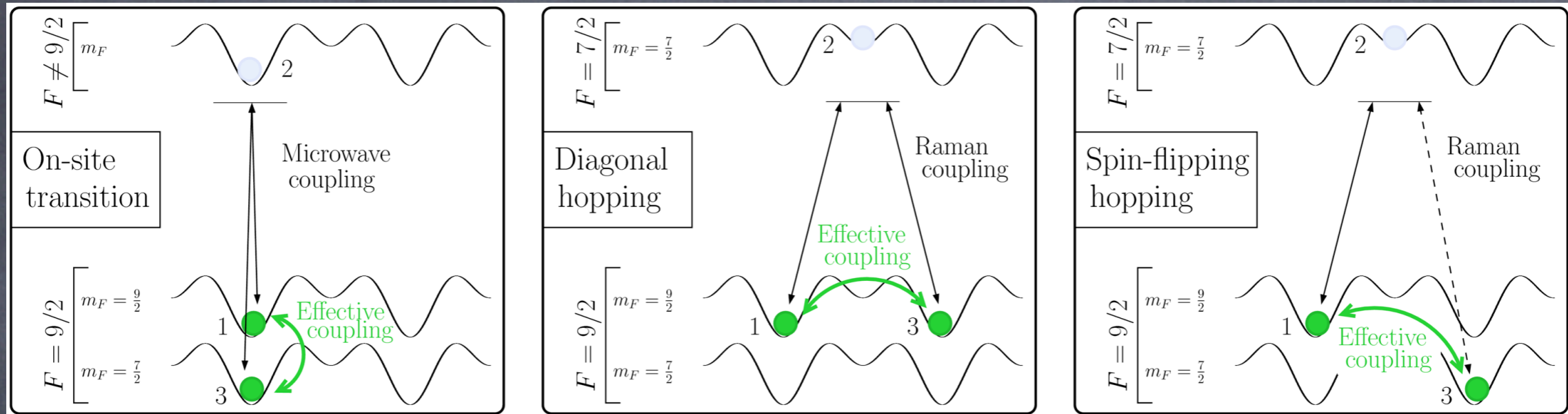
lowest band <--> simulation sites

higher band(s) <--> auxiliary buses

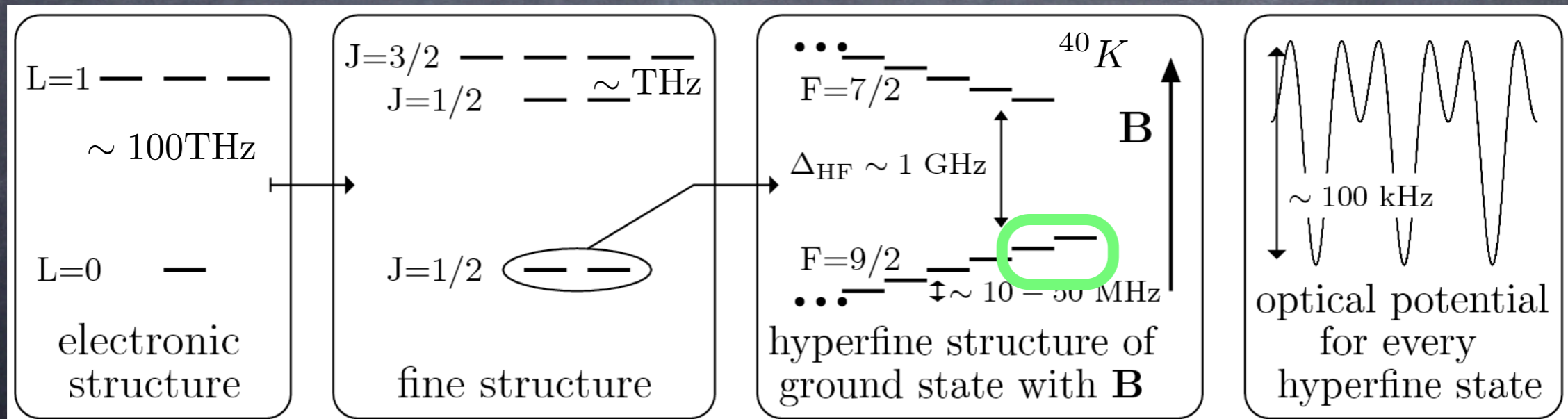
hyp.fine state <--> spin d.o.f.

# Sketch of the proposal

adiabatic elimination of different Raman couplings

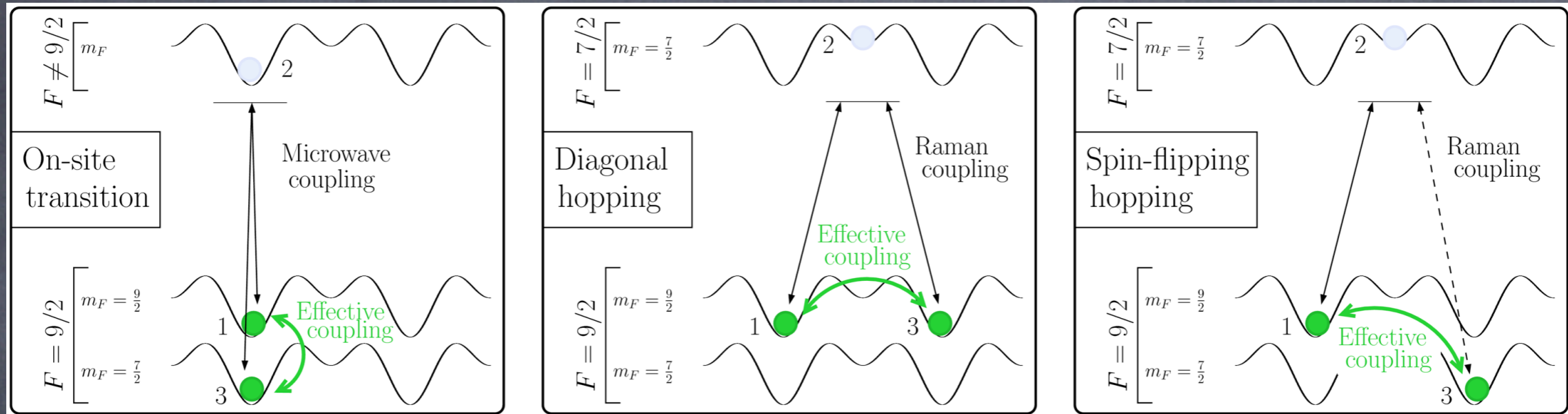


hierarchy of energy scales to address transitions

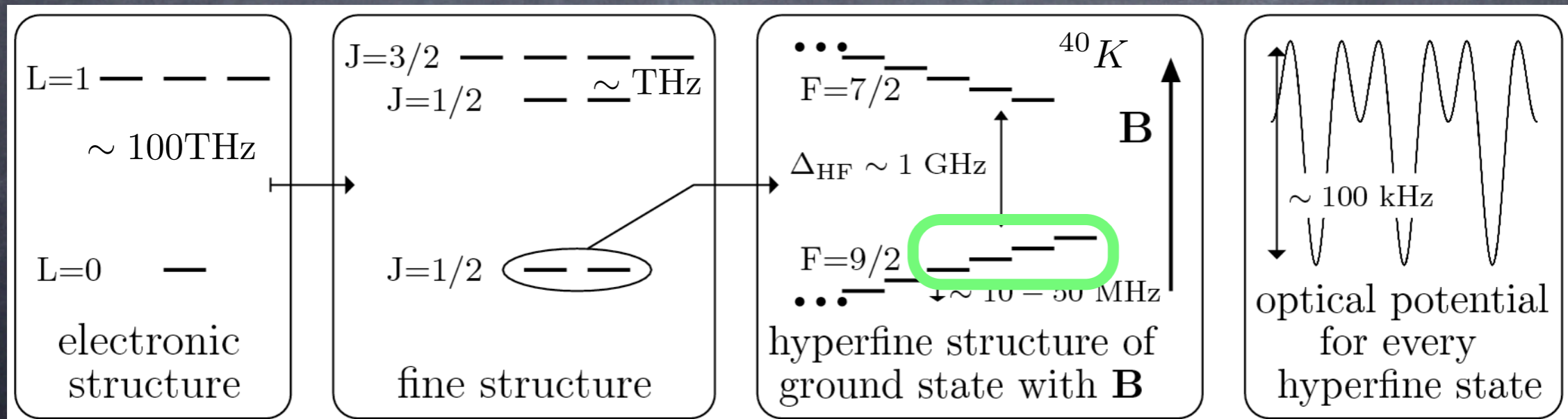


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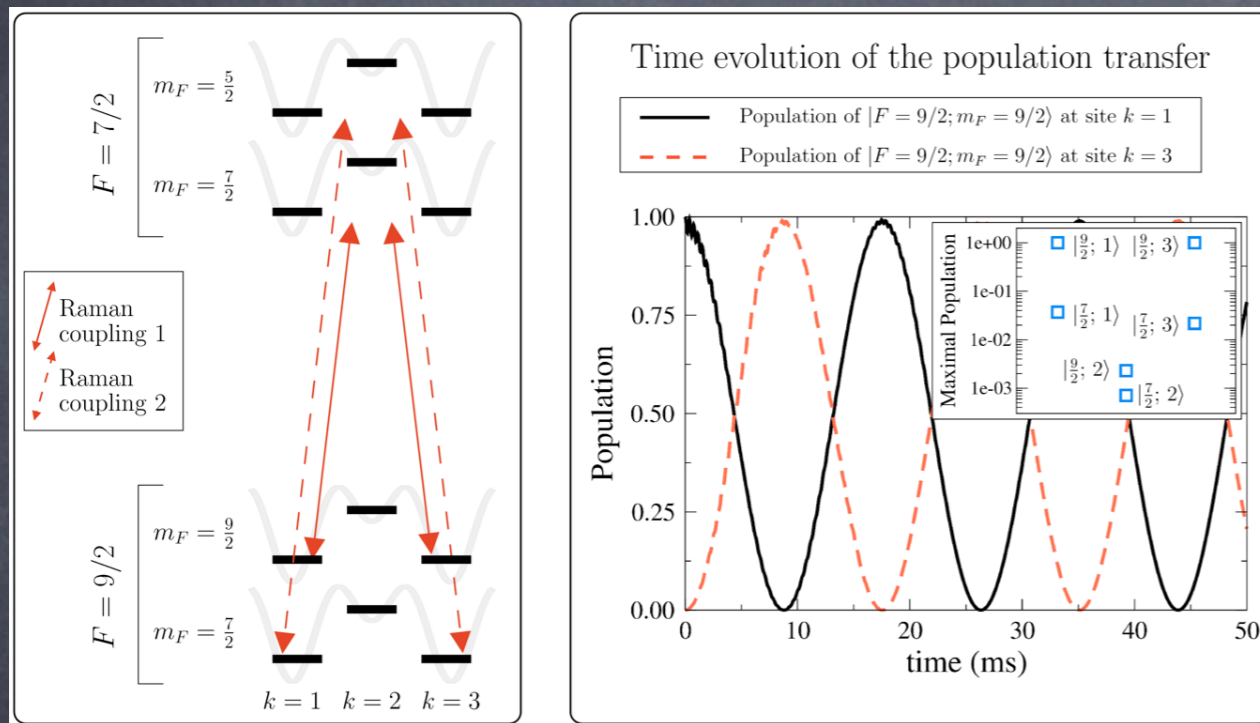


hierarchy of energy scales to address transitions



# Numerical Validation

## Runge-Kutta evolution of the full link model

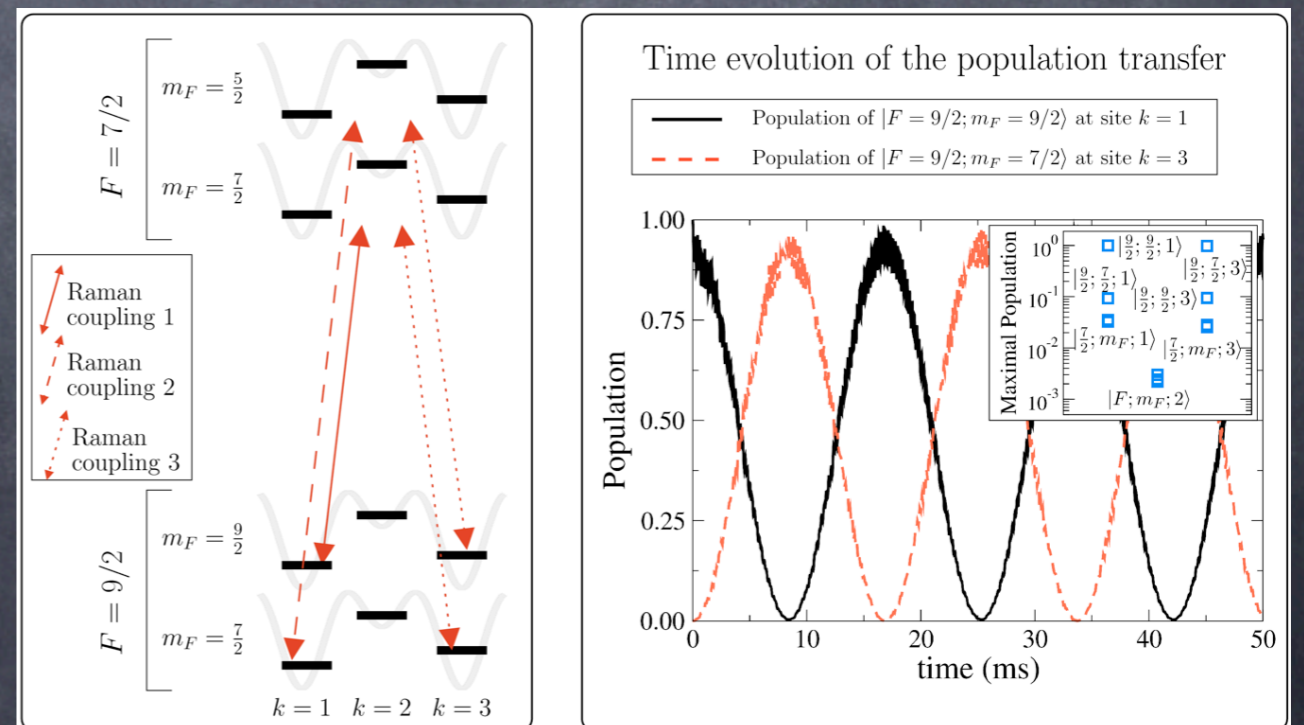


### DIAGONAL

- eff. hopping  $\sim 100\text{Hz}$
- "fidelity"  $> 97\%$
- Zeeman helps tuning different elements

### OFF-DIAGONAL

- "fidelity"  $> 88\%$
- additional staggering to avoid on-site flips

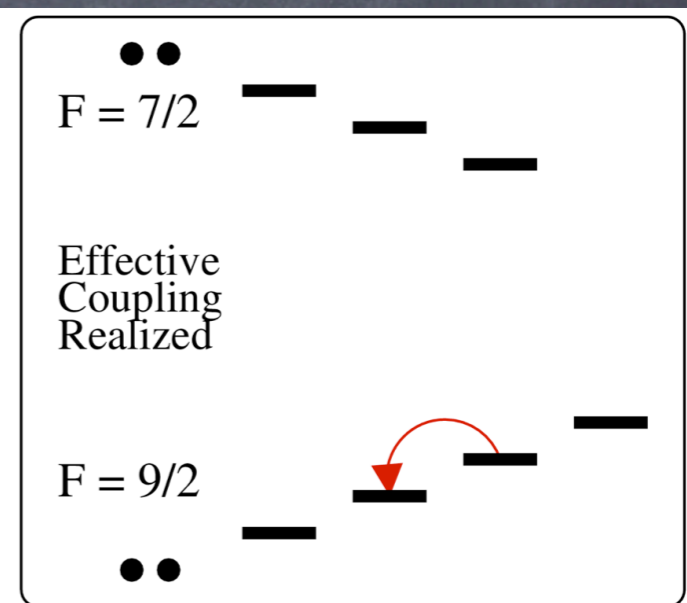
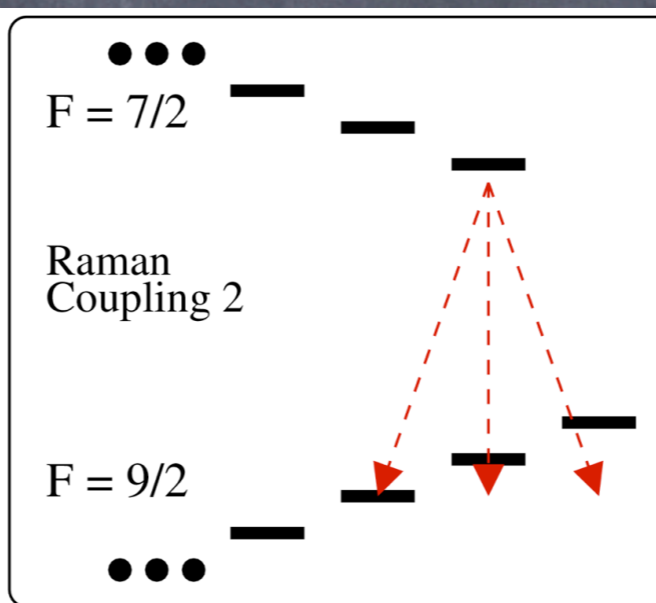
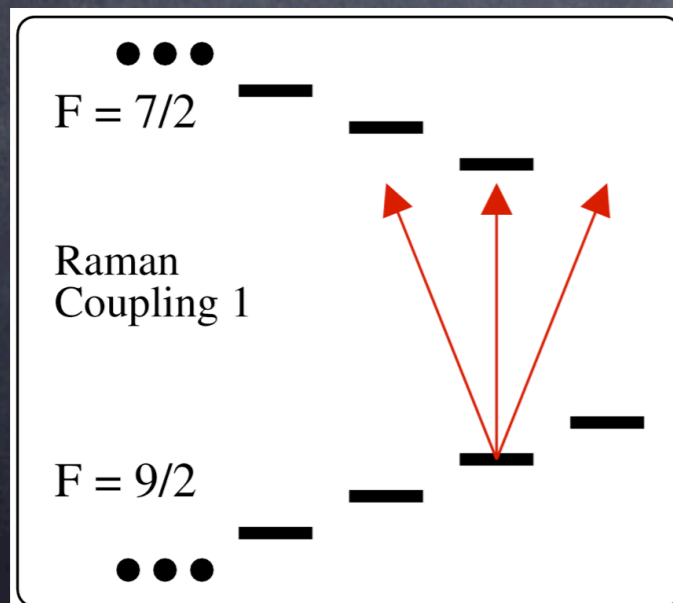


# A proposal for all dimensions

- need of momentum transfer  
--> directionality selection rule !  
(laser aligned to x/y/z rules over x/y/z only)

$$|S_{1,2}| \neq 0 \quad \text{iff} \quad |(\mathbf{p}_1 - \mathbf{p}_2) \cdot \boldsymbol{\nu}| \neq 0$$

- polarization selection rule is reliable in 1D only,  
but hierarchy of energies helps in 2- & 3-D





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# Relativistic field theory

our superlattice+Raman setup gives control on

$$H_{\text{sys}} = \sum_{\mathbf{r}} \sum_{\tau\tau'} \left[ \sum_{\nu} t_{\nu} c_{\mathbf{r}+\nu\tau'}^{\dagger} [U_{\nu}]_{\tau'\tau} c_{\mathbf{r}\tau} + \Omega c_{\mathbf{r}\tau'}^{\dagger} [\Lambda]_{\tau'\tau} c_{\mathbf{r}\tau} + \text{H.c.} \right]$$

to simulate Dirac equation

$$H_{\text{DI}} = c\boldsymbol{\alpha} \cdot \mathbf{p} + mc^2\beta \quad \begin{cases} \{\alpha_{\nu}, \alpha_{\mu}\} = 2\delta^{\nu\mu} \\ \{\alpha_{\nu}, \beta\} = 0 \end{cases}$$

via low-energy effective description

$$H_{\text{sys}}(\{t_{\nu}, U_{\mathbf{r}\nu}, \Lambda, \Omega\}) \rightarrow H_{\text{eff}} \approx H_{\text{rel}} = \int d\mathbf{r} \Psi(\mathbf{r})^{\dagger} H_{\text{DI}} \Psi(\mathbf{r})$$

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$D$	$\alpha_x$	$\alpha_y$	$\alpha_z$	$\beta$	$\Gamma$
1	$\sigma_x$			$\sigma_z$	$\mathbf{i}\sigma_x$
2	$\sigma_x$	$\sigma_y$		$\sigma_z$	$\mathbf{i}\sigma_y$
3	$\sigma_z \otimes \sigma_x$	$\sigma_z \otimes \sigma_y$	$\sigma_z \otimes \sigma_z$	$\sigma_x \otimes \mathbb{I}_2$	$-\mathbf{i}\alpha_x\alpha_y\alpha_z$

number of hyperfine states  
needed for the simulation

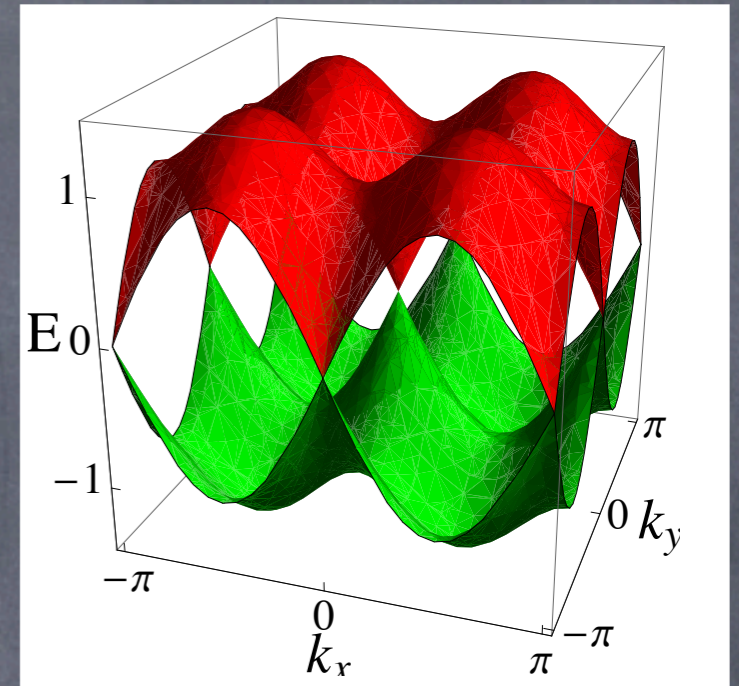
$$\dim\Psi(\mathbf{r}) = N_D = \begin{cases} 2 & D = 1, 2 \\ 4 & D = 3 \end{cases}$$

# Shaping dispersion relations

- step 1: naive massless Dirac fermions

$$\begin{array}{l} U_\nu = i\alpha_\nu \\ \Lambda = 0 \end{array} \longrightarrow H = \sum_{\mathbf{k} \in \text{BZ}} \Psi_{\mathbf{k}}^\dagger \left( \sum_\nu 2t_\nu \sin k_\nu \alpha_\nu \right) \Psi_{\mathbf{k}} \longrightarrow$$

$$\begin{array}{l} c = 2t \\ \mathbf{p}_d = \mathbf{k} - \mathbf{K}_d \end{array} \quad H_{\text{eff}} = \sum_d \sum_{\mathbf{p}_d} \Psi^\dagger(\mathbf{p}_d) [c\boldsymbol{\alpha}^d \cdot \mathbf{p}_d] \Psi(\mathbf{p}_d)$$

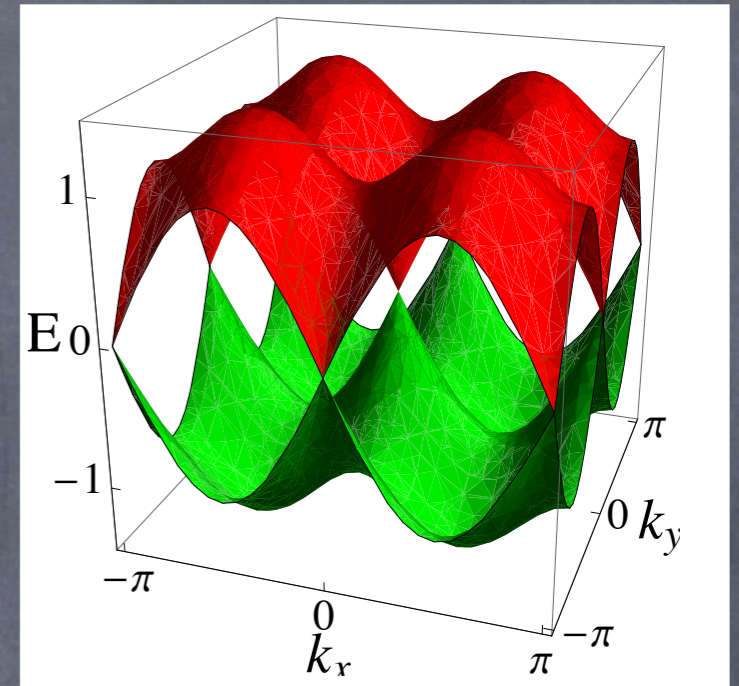


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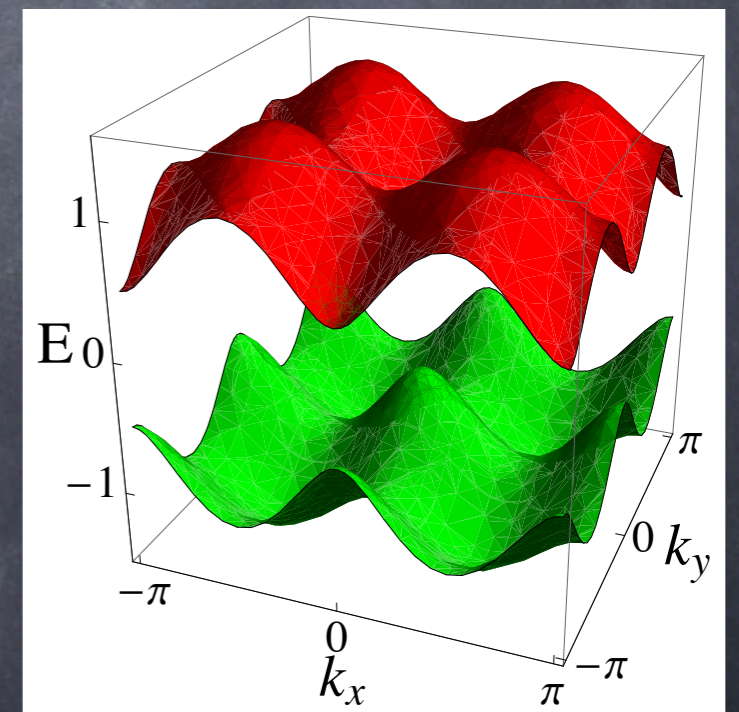
$$\begin{aligned}
 & \boxed{c = 2t} \\
 & \mathbf{p}_d = \mathbf{k} - \mathbf{K}_d
 \end{aligned}
 \quad
 H_{\text{eff}} = \sum_{\mathbf{d}} \sum_{\mathbf{p}_d} \Psi^\dagger(\mathbf{p}_d) [c\alpha^{\mathbf{d}} \cdot \mathbf{p}_d] \Psi(\mathbf{p}_d)$$



- step 2: naive massive Dirac fermions

$$\begin{aligned}
 & \boxed{\Lambda = \beta}
 \end{aligned}
 \longrightarrow
 \text{extra } \sum_{\mathbf{k} \in \text{BZ}} \Psi_{\mathbf{k}}^\dagger \Omega \beta \Psi_{\mathbf{k}} \longrightarrow$$

$$\begin{aligned}
 & \boxed{mc^2 = \Omega}
 \end{aligned}
 \quad
 H_{\text{eff}} = \dots [c\alpha^{\mathbf{d}} \cdot \mathbf{p}_d + mc^2 \beta] \dots$$



# Shaping dispersion relations

- naive Dirac fermions suffer “doubling” due to chiral (D=1,3)/time reversal (D=2) symmetry  $\Gamma$  (Nielsen–Ninomiya theorem)



- step 3: Wilson fermions avoid “doubling”

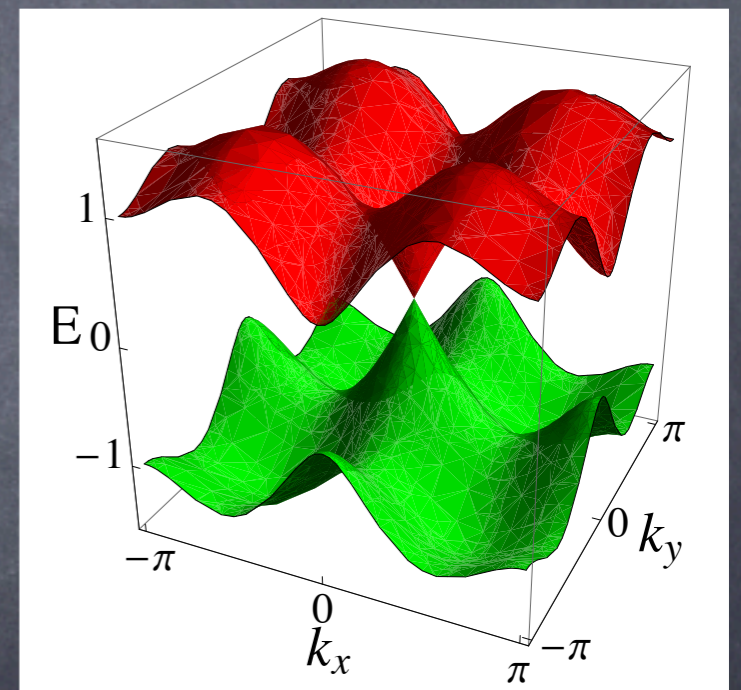
$$\tilde{U}_\nu = -\beta$$

$$\tilde{H} = \sum_{\mathbf{k} \in \text{BZ}} \Psi_{\mathbf{k}}^\dagger \left( \sum_\nu -2\tilde{t}_\nu \cos k_\nu \beta \right) \Psi_{\mathbf{k}}$$



$$m_\nu c^2 = 2\tilde{t}_\nu$$

$$m_{\mathbf{K}_d} = m - \sum_\nu (-1)^{d_\nu} m_\nu$$



- step 3b: Kaplan fermions  $m_{\mathbf{K}_0} < 0$  & boundary theories

# A fractional capacitor for 3D TI

• 4 components gas

• chirality  $\gamma_5 = \sigma_z \otimes \mathbb{I}$

• on-site  $\Lambda = \beta = \sigma_x \otimes \mathbb{I}$

• hopping  $U_\nu = \alpha_\nu = \sigma_z \otimes \sigma_\nu$   
 $\tilde{U}_\nu = -\beta$

# A fractional capacitor for 3D TI

• 4 components gas

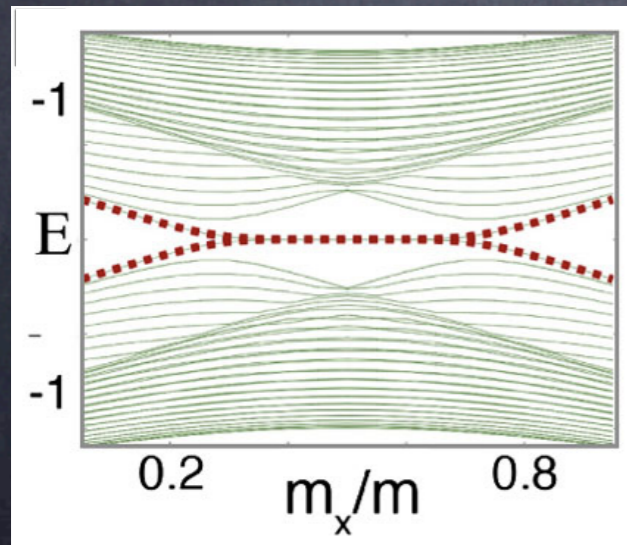
• on-site  $\Lambda = \beta = \sigma_x \otimes \mathbb{I}$

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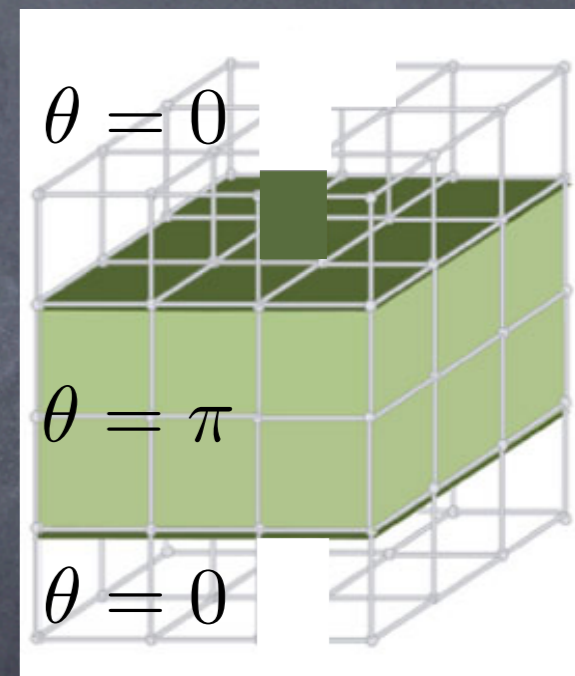
• hopping  $U_\nu = \alpha_\nu = \sigma_z \otimes \sigma_\nu$   
 $\tilde{U}_\nu = -\beta$

• create finite regions with negative mass by focusing on-site corrections  $\Lambda$

• complex mass gives axions...



• zero-energy edge states for a wide parameter window

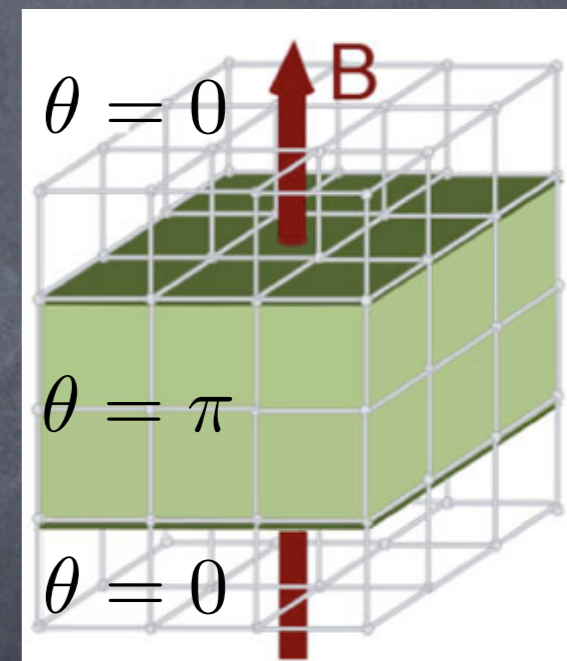




# A fractional capacitor for 3D TI

- peculiar response to artificial magnetic field  
(engineerable in cold gases)

Magneto-Electric effect  $\nabla \cdot \mathbf{E} = 4\pi\rho - (e^2/\pi c)\nabla\theta \cdot \mathbf{B}$



# A fractional capacitor for 3D TI

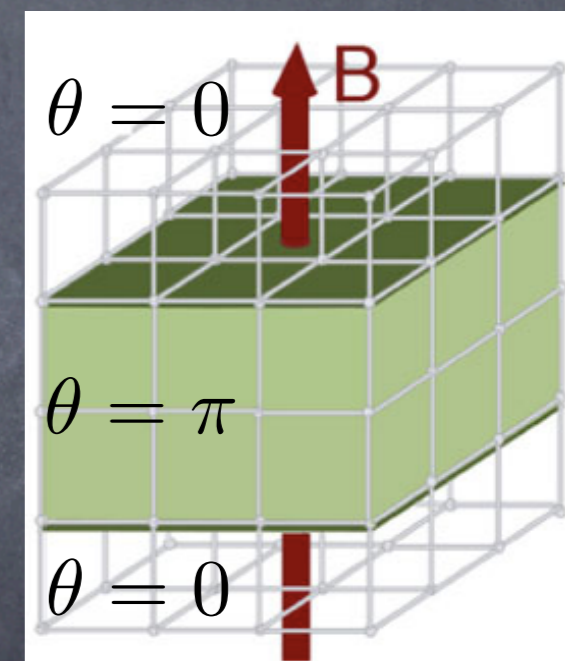
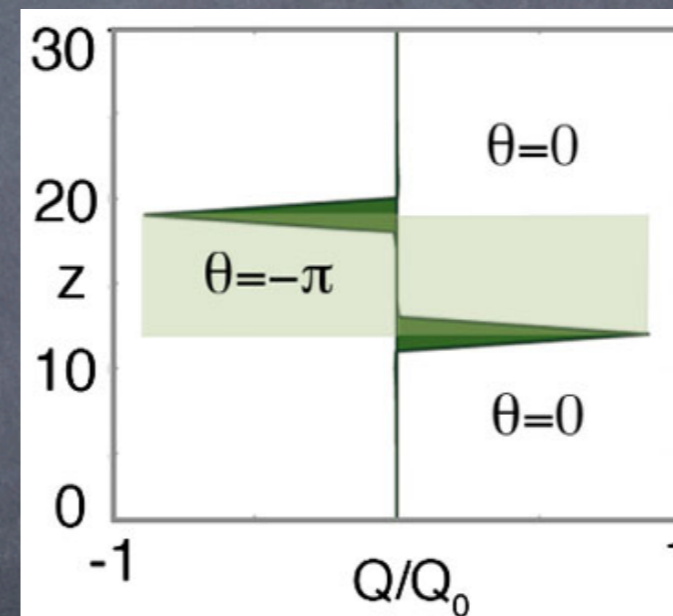
- peculiar response to artificial magnetic field  
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Magneto-Electric effect  $\nabla \cdot \mathbf{E} = 4\pi\rho - (e^2/\pi c)\nabla\theta \cdot \mathbf{B}$

- Fractional charge:

$$Q = e/2 [\delta(z - z_1) - \delta(z - z_2)]$$

detectable by in-situ  
imaging of cold gases !



# Outline

- quantum simulations & cold gases
- topo-insulators & free-Fermi theories
- exp. setup: superlattice + Raman
- proposal at work: towards 3D-TI
- future directions

# Future directions

- beyond single-particle physics for TI's:
  - is the Q-simulator robust?
  - how does the periodic table change?
  - are there other invariants?
- topologically non-trivial flat-bands
  - geometry of the lattice to shape flat-bands
  - interactions/dissipations lead to FTI's as FQHE !?
  - dipolar terms (long-range) add something?
- topological superconductors & anyons
  - alkali-earth paired with condensates

# Team



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Thanks for your  
attention !

# Outline

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