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Interplay between superconductivity, magnetism and nematic order in the iron pnictides - Part I

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Interplay between superconductivity, magnetism, and nematic order in the iron pnictides

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Columbia University &

Los Alamos National Laboratory

ICTP Workshop

Aug 10th, 2012



postdoc positions available!!!

Rafael M. Fernandes

University of Minnesota

ICTP Workshop

Aug 10th, 2012

Outline

1. The superconducting state (Lecture I)

- unconventional superconductivity
- 2. The magnetic state (Lectures I & II)
 - itinerant or localized?

3. The nematic state (Lecture II)

- a new electronic phase that emerges from magnetism and triggers structural and orbital order
- 4. Competing orders (Lecture II)
 - competition between SC, magnetism, and nematics

Superconductivity reaches the iron age!



first high-temperature superconductors since the cuprates!



stone age



copper age



Iron pnictides: typical phase diagram

magnetic, structural, and superconducting order



- How these different ordered states interact with each other?
- Is there a primary degree of freedom?



Iron pnictides: multi-band systems

- Parent compounds are metals
- Before adding any interactions, we need to understand the non-interacting properties of the tetragonal phase.





Iron pnictides: band structure

DFT calculations: Fermi surface has multiple sheets
 > 3d⁶ configuration: several orbitals cross the Fermi level



Paglione and Greene, Nature Phys (2010)

Zhang and Singh, PRB (2009)

Iron pnictides: band structure

• A simplified (yet useful) theorist-view of the Fermi surface



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• Bardeen, Cooper, and Schrieffer (1957): attractive electron-phonon interaction induces superconductivity







Nobel Priz

John Bardeen

Leon Cooper Robert Schrieffer

• Bardeen, Cooper, and Schrieffer (1957): attractive electron-phonon interaction induces superconductivity

 \succ interaction is repulsive at **r**=0, t=0 (Coulomb repulsion)





figures from Monthoux, Pines, and Lonzarich, Nature (2010)

• Bardeen, Cooper, and Schrieffer (1957): attractive electron-phonon interaction induces superconductivity

> as the particle moves, it polarizes the lattice, leaving an attractive interaction at \mathbf{r} =0, t>0

 $V < 0 \Rightarrow$ attraction



figures from Monthoux, Pines, and Lonzarich, Nature (2010)

• Bardeen, Cooper, and Schrieffer (1957): attractive electron-phonon interaction induces superconductivity

> as the particle moves, it polarizes the lattice, leaving an attractive interaction at \mathbf{r} =0, t>0



• Bardeen, Cooper, and Schrieffer (1957): attractive electron-phonon interaction induces superconductivity

bound state: Cooper pairs



New quantum ground state characterized by coherent Cooper pairs

$$T_c = 1.13\,\omega_D \exp\left(-\frac{1}{V\rho_F}\right)$$

Coupling constant can be calculated from first-principles, but NOT T_c ...

• Bardeen, Cooper, and Schrieffer (1957): attractive electron-phonon interaction induces superconductivity

> Let's look a bit more at technical points...

• Bardeen, Cooper, and Schrieffer (1957): attractive electron-phonon interaction induces superconductivity

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k'}} \frac{V_{\mathbf{k}-\mathbf{k'}} \Delta_{\mathbf{k'}}}{2E_{\mathbf{k'}}} \tanh\left(\frac{E_{\mathbf{k'}}}{2T}\right)$$

$$1 = \lambda \int_{0}^{\omega_{D}} \frac{d\varepsilon}{\sqrt{\varepsilon^{2} + \Delta^{2}}} \tanh\left(\frac{\sqrt{\varepsilon^{2} + \Delta^{2}}}{2T}\right)$$

Bardeen, Cooper, and Schrieffer (1957): attractive electron-phonon interaction induces superconductivity
 > linearizing the equation close to T_c

$$1 = \lambda \int_{0}^{\omega_{D}} \frac{d\varepsilon}{\varepsilon} \tanh\left(\frac{\varepsilon}{2T_{c}}\right)$$

$$T_c = \left(\frac{2\mathrm{e}^{\gamma}}{\pi}\right) \omega_D \,\mathrm{e}^{-1/\lambda}$$

Are the iron pnictides electronphonon superconductors?

• First-principle calculations of the electron-phonon interaction predict a maximum T_c of 0.8K

Before studying other sources of pairing, let's review the possible symmetries of the SC state in a tetragonal system



Boeri et al, PRL (2008)

• In general, the symmetry of the superconducting state is a subgroup of the symmetry of the normal state



U(1) gauge symmetry is always broken

in conventional (electron-phonon) superconductors, usually no other symmetry is broken

in unconventional superconductors, besides U(1), other symmetries can be broken as well

• In general, the symmetry of the superconducting state is a subgroup of the symmetry of the normal state



in crystals with inversion symmetry, two possible spin states:



singlet (anti-symmetric)



triplet (symmetric)

• The total wave-function must be anti-symmetric with respect to the exchange of two electrons





singlet (anti-symmetric)

Cooper pair wave-function

$$\Psi(-\mathbf{k}) = \Psi(\mathbf{k})$$

even parity (s-wave, d-wave...)



triplet (symmetric)

$$\Psi(-\mathbf{k}) = -\Psi(\mathbf{k})$$

odd parity (p-wave, f-wave...)

• The possible shapes of the wave-function depend on the irreducible representations of the crystal point group

consider the simple case of a tetragonal lattice



we will restrict our analysis to a **singlet state** with **no time-reversal symmetry breaking** and **2 dimensions**

Tetragonal



• Possible symmetries of the Cooper-pair wave-function in the tetragonal lattice

Wave- function name	Group- theoretic notation, T_j	Residual symmetry	Basis function	Nodes
s wave	A_{1g}	$D_{4h} \times T$	$1,(x^2+y^2),z^2$	none
g	A_{2g}	$D_4[C_4] \times C_i \times T$	$xy(x^2-y^2)$	line
$d_{x^{2}-y^{2}}$	B_{1g}	$D_4[D_2] \times C_i \times T$	$x^2 - y^2$	line
d_{xy}	B_{2g}	$D_4[D'_2] \times C_i \times T$	xy	line

Kirtley and Tsuei, RMP (2000)











	Wave- function name	Group- theoretic notation, T_j	Resid symn	dual netry	Basis function	Nodes
	s wave g $d_{x^2-y^2}$ d_{xy}	A_{1g} A_{2g} B_{1g} B_{2g}	D_{4h} $D_4[C_4]$ $D_4[D_2]$ $D_4[D_2]$	$\times T$ $\times C_i \times T$ $\times C_i \times T$ $\times C_i \times T$	$ \begin{array}{c} 1,(x^2+y^2),z^2\\xy(x^2-y^2)\\x^2-y^2\\xy\end{array} $	none line line line
	S C d _{xy}		d _x 2-y2	wh vai Bri Re	ny would the go nish in some re illouin zone? epulsive intera	ap function egions of th actions!
R Klemm		g _{xy(x²-y²)}	2)			

Superconductivity due to repulsive interactions

- Several approaches: (non-extensive list)
 - RVB theory: slave-boson, Gutzwiller projection, variational Monte Carlo
 Anderson, Science (1987)

Recent numerical solutions of the Hubbard model (DMFT) cf. Tremblay's talk

Pairing mediated by spin fluctuations

Berk and Schrieffer, PRL (1966)

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> Pairing mediated by spin fluctuations

Berk and Schrieffer, PRL (1966)



motivation: proximity to a magnetic instability

back to the blackboard...

Superconductivity mediated by spin fluctuations

• Effective interaction in the singlet channel:

$$V_{\rm pair} \propto \chi_s({f r})$$



spin fluctuations are the glue

purely electronic mechanism

Superconductivity mediated by spin fluctuations

• Effective interaction in the singlet channel:

$$V_{\rm pair} \propto \chi_s({f r})$$

> for an antiferromagnetic instability, $\chi_s(\mathbf{r})$ is maximum and positive at the origin, but oscillates



regions of attractive and repulsive interaction

Cooper-pair wave-function changes in space to match the attractive regions

Border of antiferromagnetism

figure from Monthoux, Pines, and Lonzarich, Nature (2010)

 Antiferromagnetic susceptibility in single-band tetragonal system: real space

$$V_{\rm pair} \propto \chi_s({f r})$$



• Problem simplifies if we look in **momentum-space**

$$V_{\mathbf{k}-\mathbf{k}'} \approx \chi_s(\mathbf{Q}) \delta_{\mathbf{k}-\mathbf{k}',\mathbf{Q}}$$
 in
magnetic ordering vector

a well-defined repulsive interaction in momentum-space

• Problem simplifies if we look in **momentum-space**

$$V_{\mathbf{k}-\mathbf{k}'} \approx \chi_s(\mathbf{Q}) \delta_{\mathbf{k}-\mathbf{k}',\mathbf{Q}}$$

typical Fermi surface for the cuprates



Chubukov, Pines, Schmalian (2002)

a repulsive interaction strongly peaked in momentum-space

BCS-like gap equation:

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k'}} \frac{V_{\mathbf{k}-\mathbf{k'}}\Delta_{\mathbf{k'}}}{2E_{\mathbf{k'}}} \tanh\left(\frac{E_{\mathbf{k'}}}{2T}\right)$$

with
$$E_{\mathbf{k}} = \sqrt{\varepsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2} > 0$$

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the main contribution to the momentum sum comes from: $k'\!\approx k+Q$

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BCS-like gap equation:

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since $\chi_s(\mathbf{Q}) > 0$, it must follow:

$$\operatorname{sign} \Delta_{\mathbf{k}} = -\operatorname{sign} \Delta_{\mathbf{k}+\mathbf{Q}}$$
$$\implies d_{x^2-y^2} - \operatorname{wave}$$

• Problem simplifies if we look in **momentum-space**

$$V_{\mathbf{k}-\mathbf{k}'} \approx \chi_s(\mathbf{Q}) \delta_{\mathbf{k}-\mathbf{k}',\mathbf{Q}}$$

typical Fermi surface for the pnictides



magnetic susceptibility



a repulsive interaction strongly

peaked in momentum-space

Mazin et al, PRL (2008)

• Problem simplifies if we look in **momentum-space**

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typical Fermi surface for the pnictides

a repulsive interaction strongly peaked in momentum-space

BCS-like gap equation

$$\begin{cases} \Delta_1 = -V\Delta_2 \sum_{\mathbf{k}} \frac{1}{2E_{2,\mathbf{k}}} \tanh\left(\frac{E_{2,\mathbf{k}}}{2T}\right) \\ \Delta_2 = -V\Delta_1 \sum_{\mathbf{k}} \frac{1}{2E_{1,\mathbf{k}}} \tanh\left(\frac{E_{1,\mathbf{k}}}{2T}\right) \end{cases}$$

with
$$E_{i,\mathbf{k}} = \sqrt{\varepsilon_{i,\mathbf{k}}^2 + \Delta_i^2} > 0$$



• Linearized gap equations (close to T_c)

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \begin{pmatrix} 0 & -V \\ -V & 0 \end{pmatrix} = \frac{1}{\rho_F \ln(W/T_c)} \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix}$$

two solutions:

$$T_c = W \mathrm{e}^{-\lambda \rho}$$

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \propto \begin{pmatrix} +1 \\ +1 \end{pmatrix} \qquad \lambda = -V$$

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \propto \begin{pmatrix} +1 \\ -1 \end{pmatrix} \qquad \lambda = V$$

• Pairing due to conventional attractive electron-phonon interaction (enhanced by charge/orbital fluctuations)

V < 0

Kontani & Onari, PRL (2010)



• Linearized gap equations (close to T_c)

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \begin{pmatrix} 0 & -V \\ -V & 0 \end{pmatrix} = \frac{1}{\rho_F \ln(W/T_c)} \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix}$$

1

two solutions:

$$T_c = W \mathrm{e}^{-\lambda \rho}$$

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \propto \begin{pmatrix} +1 \\ +1 \end{pmatrix} \qquad \lambda = -V$$

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix} \propto \begin{pmatrix} +1 \\ -1 \end{pmatrix} \qquad \lambda = V$$

 Pairing due to purely repulsive electronic interaction (enhanced by spin fluctuations)

V > 0

Mazin et al, PRL (2008) Kuroki et al, PRL (2008) Chubukov et al, PRB (2008) Cvetkovic et al, EPL (2009) Wang et al, PRL (2009) Sknepnek et al, PRB (2009) Graser et al, NJP (2009)



• Problem simplifies if we look in **momentum-space**

$$V_{\mathbf{k}-\mathbf{k}'} \approx \chi_s(\mathbf{Q}) \delta_{\mathbf{k}-\mathbf{k}',\mathbf{Q}}$$

typical Fermi surface for the pnictides

a repulsive interaction strongly peaked in momentum-space

BCS-like gap equation

$$\begin{cases} \Delta_1 = -V\Delta_2 \sum_{\mathbf{k}} \frac{1}{2E_{2,\mathbf{k}}} \tanh\left(\frac{E_{2,\mathbf{k}}}{2T}\right) \\ \Delta_2 = -V\Delta_1 \sum_{\mathbf{k}} \frac{1}{2E_{1,\mathbf{k}}} \tanh\left(\frac{E_{1,\mathbf{k}}}{2T}\right) \end{cases}$$

with
$$E_{i,\mathbf{k}} = \sqrt{\varepsilon_{i,\mathbf{k}}^2 + \Delta_i^2} > 0$$



- The presence of intra-band repulsion can lead to accidental nodes (i.e. not enforced by symmetry)
- RPA, RG, FLEX, and fRG calculations give an s⁺⁻ state, but also show that a d-wave state may happen for certain dopings



Hirschfeld et al, RPP (2011)

How to probe spin fluctuations experimentally?

 Nuclear magnetic resonance (NMR): measures the momentum-integrated dynamic magnetic susceptibility
 spin-lattice relaxation rate

$$\frac{1}{T_1} \propto T \lim_{\omega \to 0} \sum_{\mathbf{q}} \frac{\mathrm{Im} \, \chi(\mathbf{q}, \omega)}{\omega}$$



$$\frac{1}{T_1 T} \propto \frac{1}{T + \theta}$$



 $\theta \approx -T_{\rm AFM}$

Ning et al, PRL (2008)





Ning et al, PRL (2008)

How to probe spin fluctuations experimentally?

• Inelastic neutron scattering (INS): directly measurement of the spin-spin correlation function

 \succ cross section for the scattered neutrons:

 $I(\mathbf{q},\omega) \propto \frac{\mathrm{Im}\,\chi(\mathbf{q},\omega)}{1}$

How to probe spin fluctuations experimentally?

- Inelastic neutron scattering (INS): directly measurement of the spin-spin correlation function
 - near a metallic AFM transition:









 Resonance mode is an indirect consequence that the SC gap changes sign across the Fermi surface



+ Q_{AFM} +

resonance mode

no resonance

figure from Schmalian, Physik Journal (2011)