2357-21

Innovations in Strongly Correlated Electronic Systems: School and Workshop

6 - 17 August 2012

Quench Dynamics in One-Dimension: A Renormalization Group Approach

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Quench dynamics in one-dimension: A renormalization group approach

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Supported by NSF-DMR

A quantum quench

Start initially in a state $\left|\Phi_{H_i}\right\rangle$ which is the ground state of some Hamiltonian Hi

Drive the system out of equilibrium by a sudden change in parameters of the Hamiltonian   Hi → Hf

Explore the time-evolution and the long-time behavior.

a). Is the system thermal at long times?
b). What does it mean to be thermal for an isolated quantum system in a pure state?
c). "Glassy behavior" with intermediate long-lived metastable states?
d). New kinds of nonequilibrium phase transitions?

Some experimental motivation first:

Cold atomic gases

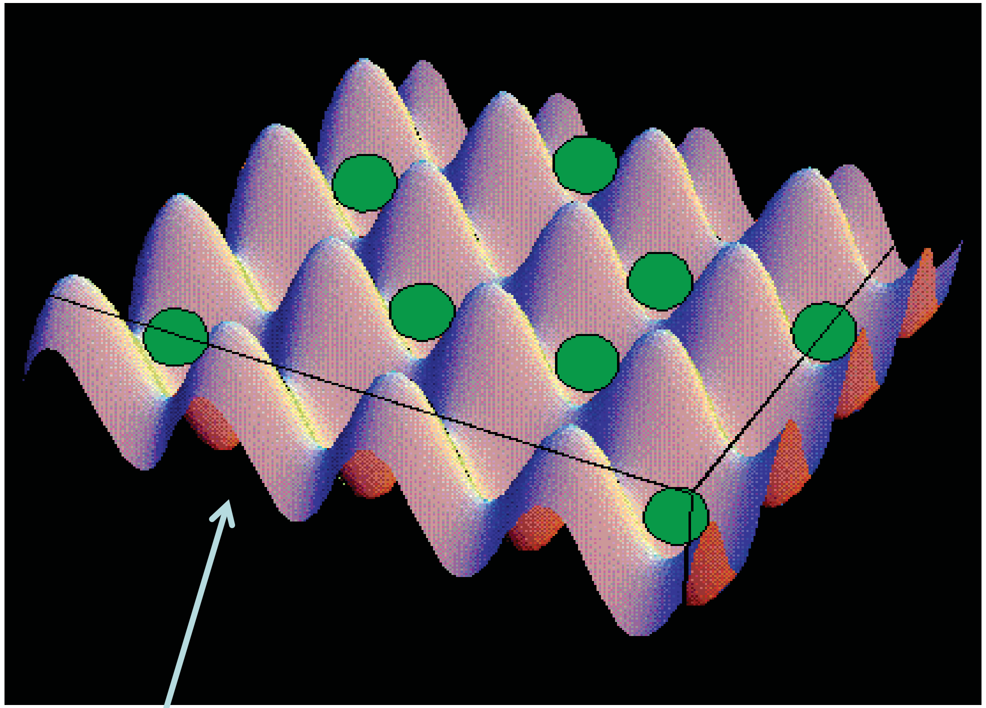
$W_{cold-atoms} \approx 1\mu K$

$W_{solids} \approx 10^4 K$

Alkali atoms:

Bosons: $^{87}Rb, ^{23}Na, ^{7}Li$

Fermions: $^{40}K, ^{6}Li$



Electric fields in a laser induce a dipole moment which interacts with the field: schematic of a potential felt by the atoms

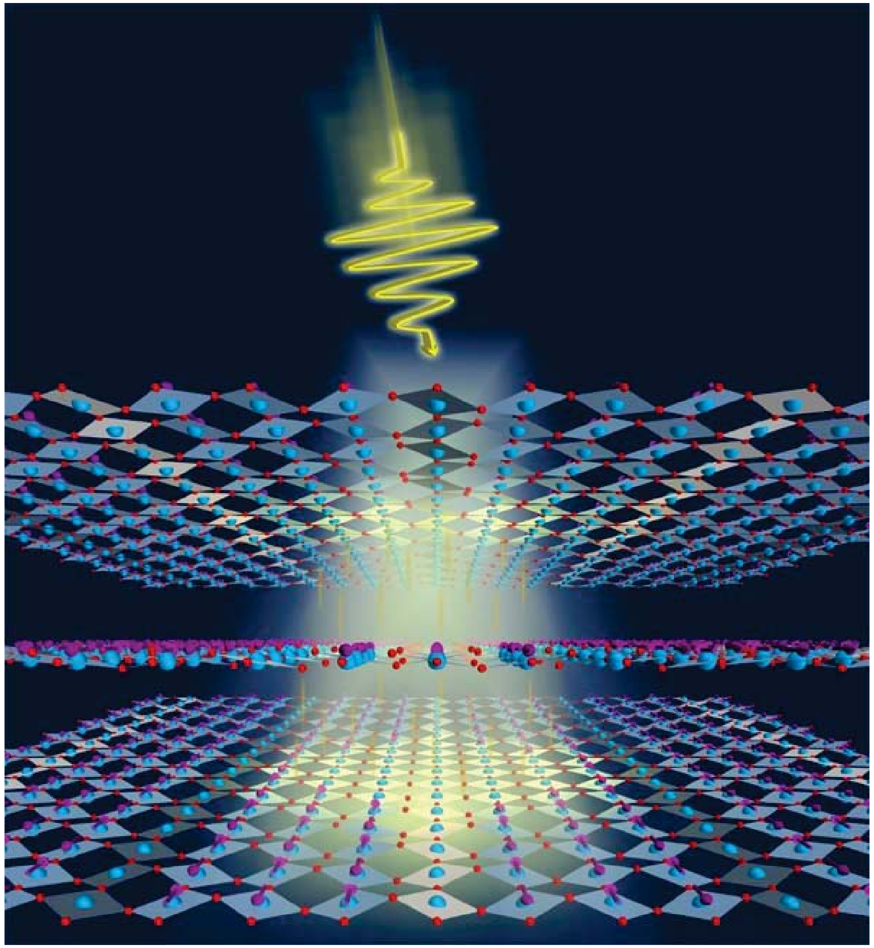
Unique features:

1. **Possible to realize almost ideal (isolated from the surroundings) condensed matter systems. More often than not the systems are out of equilibrium. Easier to study dynamics as they occur on much lower energy-scales.**

2. **Highly tunable systems where the interaction between particles and the external potentials acting on them can be tuned easily and rapidly, the former by using Feshbach resonances.**

Quench= Unitary time-evolution from a nonequilibrium initial state

Ultra-fast Optical Pump Probe methods:



Fausti et al, Science 2011 (Hamburg)

Ultra-fast lasers can probe dynamics
on femto-second time scales,
much faster than times needed to
thermalize via coupling to a reservoir
such as lattice vibrations (pico seconds).

Optically induced phase-transitions

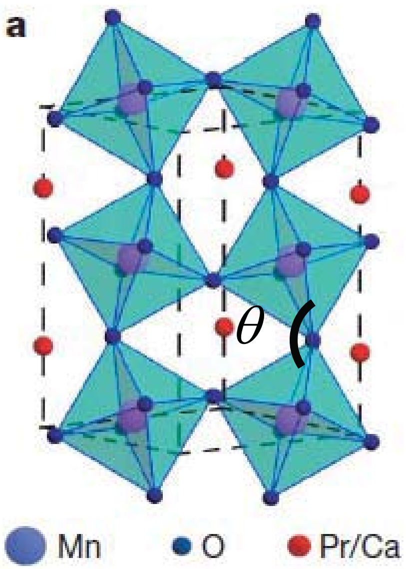
Control of the electronic phase of a manganite by mode-selective vibrational excitation

Matteo Rini[1], Ra'anan Tobey[2], Nicky Dean[2], Jiro Itatani[1,3], Yasuhide Tomioka[4], Yoshinori Tokura[4,5], Robert W. Schoenlein[1] & Andrea Cavalleri[2,6] **Nature, 2007**

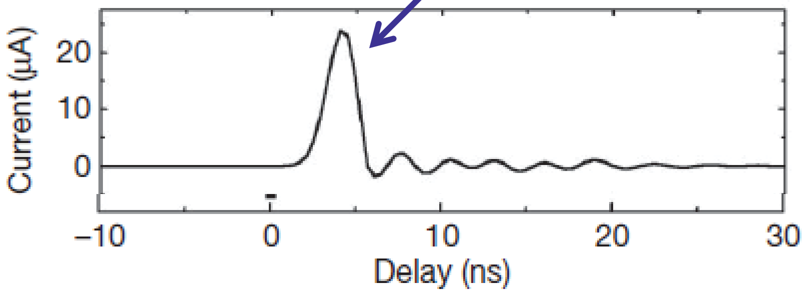
Electron kinetic energy: $W = f(\theta)$

Optical control over: $\dfrac{U}{W}$

Buckling angle : θ



Metastable metallic state



By optically exciting the Mn-O stretching mode, the band-width W is modified via the buckling angle, and a transition to a metallic phase is observed.
The phase persists for ~100 ps

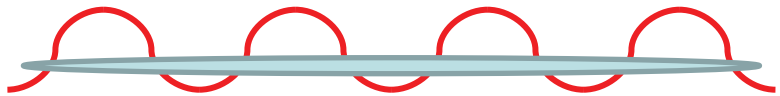
Other examples:
Optically induced magnetic-paramagnetic phase transitions, Rasing et al, PRL 2009

Road-map for the talk

1. Quenches involving free theories:
Interaction quench in a 1D Bose gas (Luttinger liquid) .

Result: Nonequilibrium steady state.

Lancaster and Mitra, PRE 2010
Mitra and Giamarchi, PRL 2011



2. How does the nonequilibrium state of the Luttinger liquid respond to a periodic potential? Results in the superfluid phase: role of irrelevant operators.

Mitra and Giamarchi, PRL 2011, PRB 2012
Tavora and Mitra (in preparation).

3. Quenches from the superfluid to the Mott-insulator phase:
A new kind of dynamical phase transition, one that occurs as a function of time.

A. Mitra, arXiv: 1207.3777

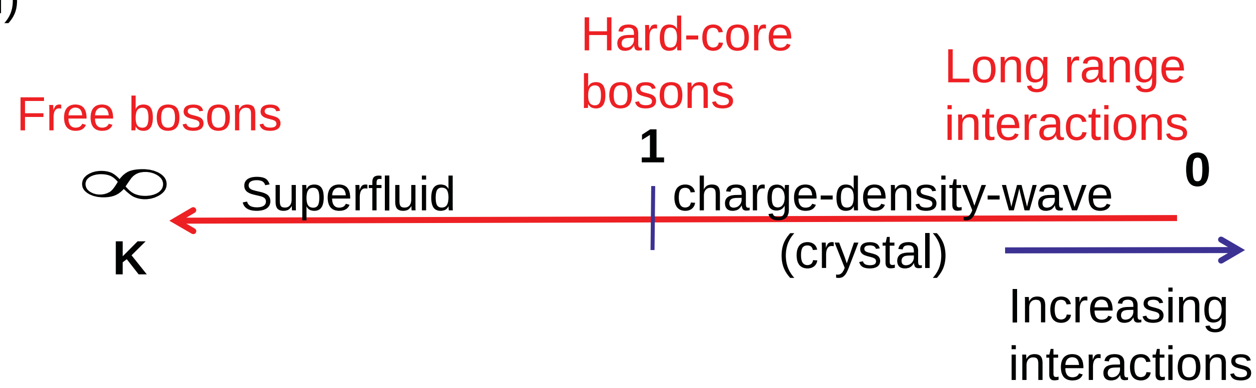
4. The situation with fermions: Magnetic field quench in an XX spin-chain (free fermions on a lattice). Modified GGE. Effect of weak interactions.

Lancaster and Mitra, PRE 2010
Lancaster, Giamarchi and Mitra, PRB 2011

Equilibrium and low-energy properties of 1D Interacting Bose gas

$$H_f = \frac{u}{2\pi}\int dx\left[K\left(\pi\Pi(x)\right)^2 + \frac{1}{K}\left(\partial_x\phi(x)\right)^2\right]$$
$$= \sum_{p\neq 0} u|p|\gamma_p^\dagger\gamma_p$$

1D interacting Bose gas
characterized by an interaction parameter K:
K = 1/(Interaction Strength)



ψ Boson creation operator

$\rho(r)$ Boson density at r=(x,t)

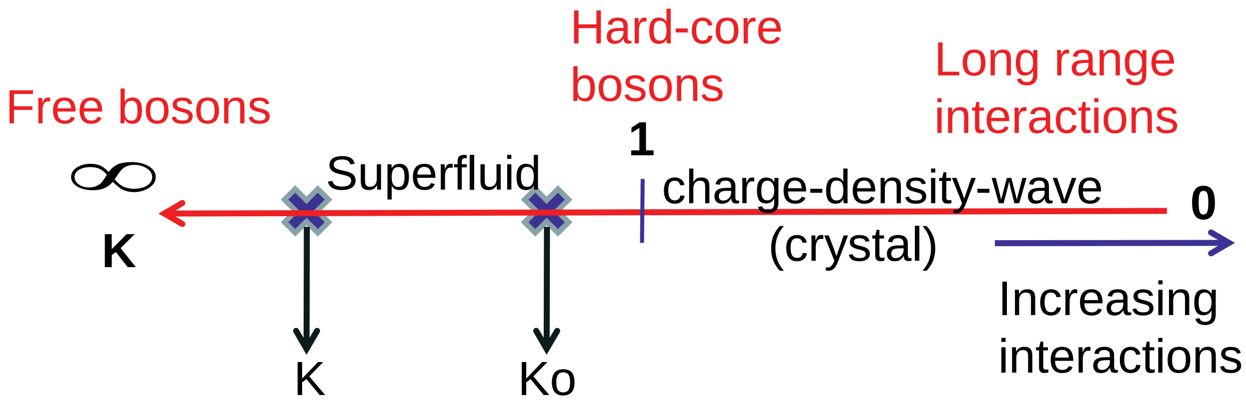
Due to quantum-fluctuations, only quasi-long range order

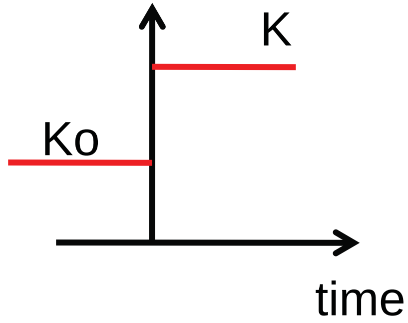
Density-density correlator: $\langle\rho(x)\rho(0)\rangle \approx \frac{1}{x^{2K}}\cos(2\pi\rho_0 x)$

Dual fields

Boson propagator (superfluid order parameter): $\langle\psi(r)\psi^{+}(0)\rangle \approx \frac{1}{r^{1/2K}}$

Generating an out of equilibrium state via an interaction quench





Hi : Bosons with interaction Ko

Hf : Bosons with interaction K

$\left|\Phi_{H_i,H_f}\right\rangle$ **Ground-state of Hi, Hf**

$$\left\langle \Phi_{H_i} \middle| e^{iH_f t} A e^{-iH_f t} \middle| \Phi_{H_i} \right\rangle \xrightarrow{t \to \infty}$$

What connection does this have with

$$\left\langle \Phi_{H_f} \middle| A \middle| \Phi_{H_f} \right\rangle$$

Equal time correlations at long times after the quench from Ko→K

Ko=1, Iucci and Cazalilla, 2009

Density-density correlator:

$$C_{\varphi\varphi} = \left\langle e^{i\gamma\varphi(r,t)} e^{-i\gamma\varphi(0,t)} \right\rangle \xrightarrow{t\to\infty} \frac{1}{r^{2K_{neq}}}$$

Dual fields

Boson propagator:

$$C_{\theta\theta} = \left\langle e^{i\gamma\theta(r,t)} e^{-i\gamma\theta(0,t)} \right\rangle \xrightarrow{t\to\infty} \frac{1}{r^{2K^{\theta}_{neq}}}$$

$$K_{neq} = \frac{\gamma^2 K_0}{8}\left(1+\frac{K^2}{K_0^2}\right) \longrightarrow K_{eq} = \frac{\gamma^2 K}{4}$$

Compare with equilibrium (K=Ko)

$$K_{neq} > K_{eq}$$

$$K^{\theta}_{neq} > K^{\theta}_{eq}$$

$$K^{\theta}_{neq} = \frac{\gamma^2}{8K_0}\left(1+\frac{K_0^2}{K^2}\right) \longrightarrow K^{\theta}_{eq} = \frac{\gamma^2}{4K}$$

All correlations always decay faster after the quench as compared to the decay in the ground state of Hf.
In some sense like an effective-temperature,
yet decay is still a power-law

REASON BEHIND NEW EXPONENTS: Infinite number of conserved quantities

Initial state is ground state of $\longrightarrow H_i = \sum_p \epsilon_p^a a_p^\dagger a_p \longrightarrow$ Density modes of the Bose gas with interaction Ko

Time-evolution is due to $\longrightarrow H_f = \sum_p \epsilon_p^b b_p^\dagger b_p \longrightarrow$ Density modes of the Bose gas with interaction K

$$b_p = \cosh \Theta_p a_p + \sinh \Theta_p a_{-p}^\dagger$$

Initial state a ground state of Hi $\langle a_p^+ a_p \rangle = 0$

Hence the initial distribution function which is also conserved during the dynamic is $\langle b_p^\dagger b_p \rangle = \sinh^2 \Theta_p$

Generalized Gibbs Ensemble can recover new exponents

M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, *Phys. Rev. Lett.* 98, 050405 (2007).

$$\rho_{GGE} = \frac{1}{Z_{GGE}} e^{-\sum_p \beta_p \varepsilon_p b_p^+ b_p}$$

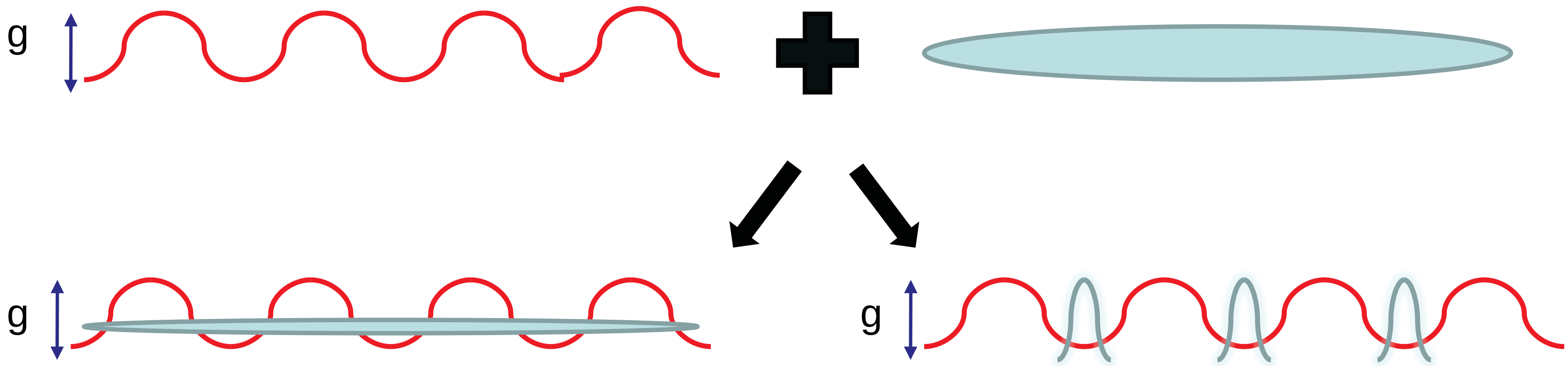
where

$$\langle \Phi_i | b_p^+ b_p | \Phi_i \rangle = Tr\left[\rho_{GGE} b_p^+ b_p\right]$$

NEXT: What happens in the presence of non-linearities that take the system away from exact solvability?

I will consider a non-linearity in the form of a commensurate periodic potential.

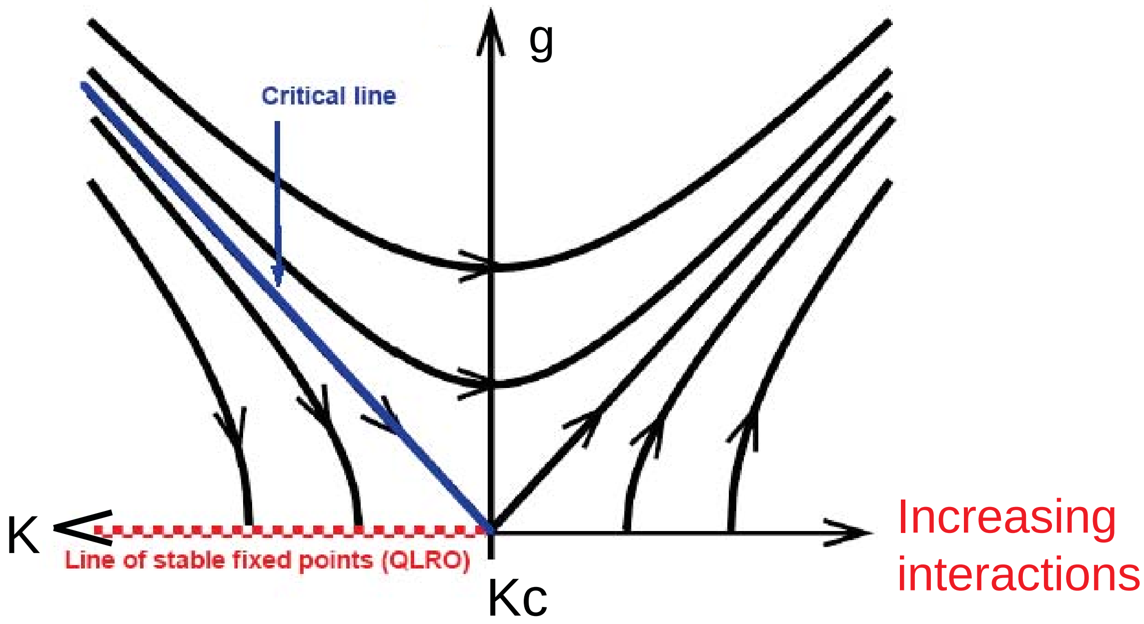
Ground state properties: Interacting bosons in a periodic potential



Superfluid

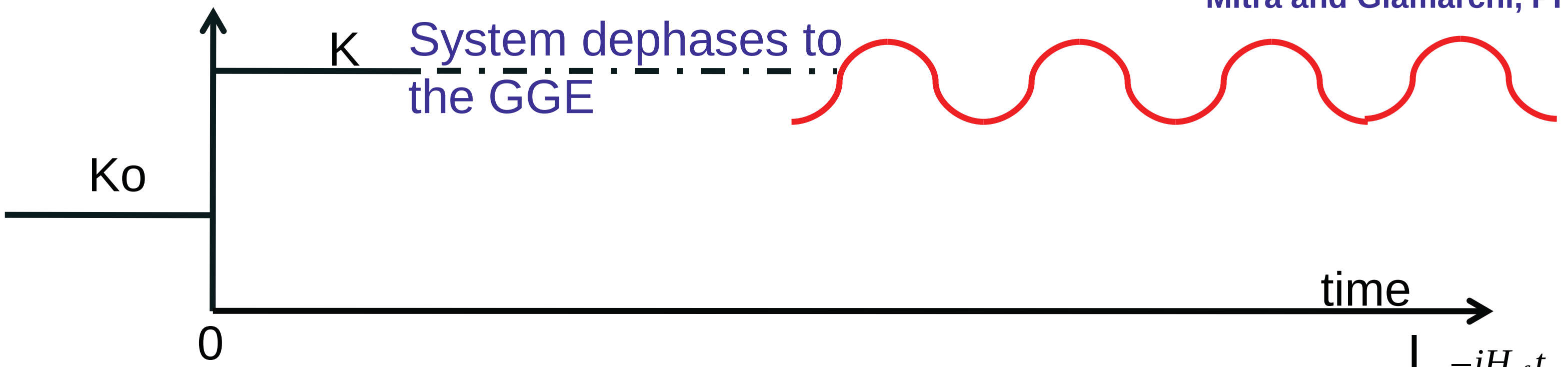
Mott-Insulator

Berezinskii-Kosterlitz-Thouless Flows



Approach 1: After an initial quench from Ko ---> K, assume bosons have reached a nonequilibrium steady-state characterized by a GGE. Perturbatively study the effect of the periodic potential on the GGE. Technically simpler as the system is time-translationally invariant.

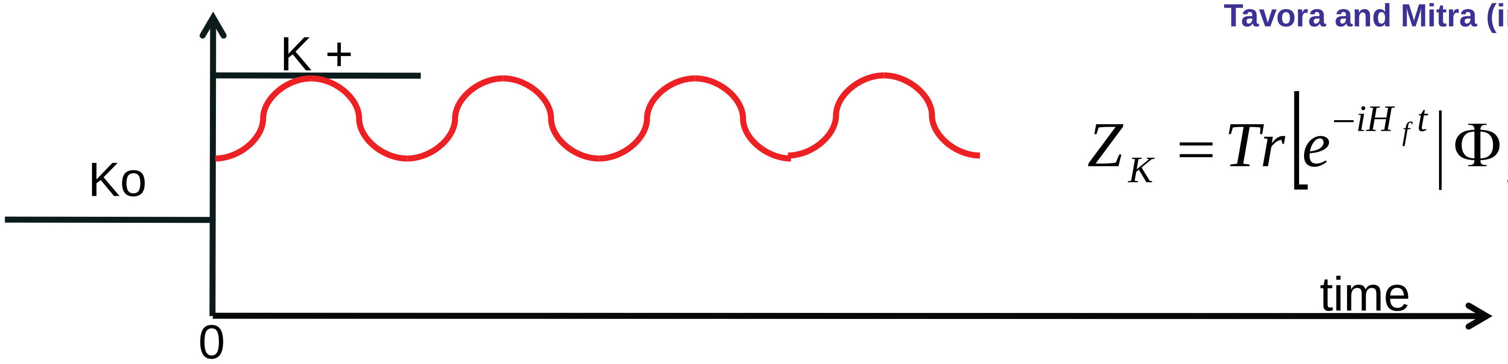
Mitra and Giamarchi, PRL 2011, PRB 2012



$$Z_K = \underset{t\to\infty}{Lt}\, Tr\left[e^{-iH_f t}\rho_{GGE}e^{iH_f t}\right]$$

Approach 2: At an initial time, not only the interaction is being quenched from Ko → K, but also the lattice potential is being switched on suddenly. Study time evolution from the initial pure state. Problem no longer time-translationally invariant.

A. Mitra, arXiv: 1207.3777
Tavora and Mitra (in preparation)



$$Z_K = Tr\left[e^{-iH_f t}\left|\Phi_i\right\rangle\left\langle\Phi_i\right|e^{iH_f t}\right]$$

RG procedure to study the effect of a periodic potential: Approach 1, initial state is GGE.

$$Z_K = Tr\left[e^{-iH_f t}\rho_{GGE}e^{iH_f t}\right]$$

Keldysh Action

$$Z_K = \int \mathcal{D}\left[\phi_{cl}, \phi_q\right] e^{i(S_0+S_{sg})}$$

GGE implies oscillations due to $e^{-iu|q|(t+t')}$ have been averaged out

Quadratic Part

$$S_0 = \sum_{q,\omega} \begin{pmatrix}\phi_{cl}^*(q,\omega) & \phi_q^*(q,\omega)\end{pmatrix} \frac{1}{\pi K u} \begin{pmatrix} 0 & (\omega - i\delta)^2 - u^2q^2 \\ (\omega + i\delta)^2 - u^2q^2 & 4i|\omega|\delta\frac{K_0}{2K}\left(1+\frac{K^2}{K_0^2}\right)\end{pmatrix} \begin{pmatrix}\phi_{cl}(q,\omega) \\ \phi_q(q,\omega)\end{pmatrix}$$

Cosine Potential

$$S_{sg} = g\int dx \int dt \left[\cos\gamma\phi_- - \cos\gamma\phi_+\right]$$

Split fields into slow and fast modes in momentum space

$$\phi_\pm = \phi_\pm^< + \phi_\pm^>$$

$$G_{0,\Lambda} = G^<_{0,\Lambda-d\Lambda} + G^>_{\Lambda-d\Lambda,\Lambda}$$

Go=Correlators for the slow and fast fields

$$G^>_{\Lambda-d\Lambda,\Lambda} = d\Lambda\frac{dG_{0,\Lambda}}{d\Lambda}$$

Expand Zk to quadratic order in g and integrate fast modes.

Generation of dissipation and noise

$$S_0 = \sum_{q,\omega} \left(\phi^*_{cl}(q,\omega) \;\; \phi^*_q(q,\omega)\right) \;\; \frac{1}{\pi K u} \begin{pmatrix} 0 & \omega^2 - i\eta\omega - u^2q^2 \\ \omega^2 + i\eta\omega - u^2q^2 & 2i\eta|\omega|\frac{K_0}{2K}\left(1+\frac{K^2}{K_0^2}\right) \end{pmatrix} \begin{pmatrix} \phi_{cl}(q,\omega) \\ \phi_q(q,\omega) \end{pmatrix}$$

$$2i\eta\omega\coth\left(\frac{\omega}{2T}\right) \xrightarrow{\omega\to 0} 4i\eta T$$

$$S_0 \quad = \quad \int dR \int d(uT) \frac{1}{\pi K} \left[\phi_q\left(\partial_R^2 - \partial_{uT+}^2\right)\phi_{cl} + \phi_{cl}\left(\partial_R^2 - \partial_{uT-}^2\right)\phi_q\right]$$

Under RG usual corrections to K and g. In addition the following terms generated:

$$\delta S = \int dR \int d(uT) \frac{1}{\pi K}\left[-2\frac{\eta}{u}\phi_q\partial_{uT}\phi_{cl} \;\; + i\frac{4T_{eff}\eta}{u^2}\frac{K_0}{2K}\left(1+\frac{K^2}{K_0^2}\right)\phi_q^2\right]$$

Dissipation

Dissipation*Temperature

RG equations

$$\frac{dg}{d\ln l} = \left[2 - \frac{\gamma^2}{8}K_0(1 + K^2/K_0^2)\right] g$$

$$\frac{dK^{-1}}{d\ln l} = \frac{\pi g^2}{4\alpha^4}\left(\frac{\gamma^2}{2}\right)^2 \frac{K_0}{2}\left(1 + \frac{K^2}{K_0^2}\right) I_K$$

$$\frac{1}{Ku}\frac{du}{d\ln l} = \frac{\pi g^2}{4\alpha^4}\left(\frac{\gamma^2}{2}\right)^2 \frac{K_0}{2}\left(1 + \frac{K^2}{K_0^2}\right) I_u$$

$$\frac{d\eta}{d\ln l} = \eta + \frac{\pi g^2 Ku}{2\alpha^4}\left(\frac{\gamma^2}{2}\right)^2 \frac{K_0}{2}\left(1 + \frac{K^2}{K_0^2}\right) I_\eta$$

$$\frac{d(T_{eff}\eta)}{d\ln l} = 2T_{eff}\eta + \frac{\pi g^2 u^2 K^2}{4\alpha^4}\left(\frac{\gamma^2}{2}\right)^2 I_T$$

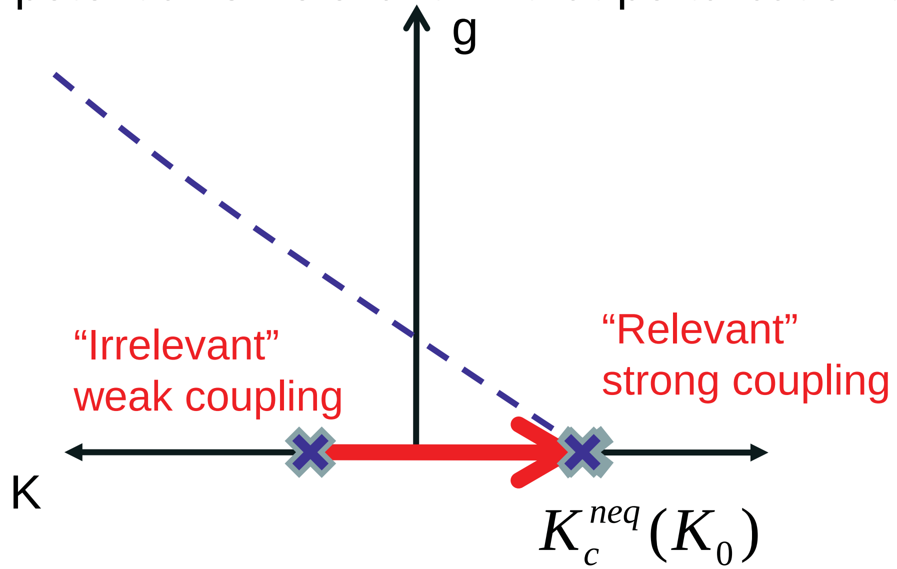
New location
of the critical point

When K=Ko, usual BKT
flow equations

$$I_{T,\eta} = 0$$

EFFECT 1: CHANGE IN THE LOCATION OF THE CRITICAL POINT.

Phase diagram is still separable into two regimes, one where the periodic potential is “irrelevant” in the sense that perturbation theory is valid. And another phase where the periodic potential is “relevant” in that perturbation theory breaks down.



$$K_{neq} > K_{eq}$$

$$K_{neq} = \frac{K_0}{2}\left(1+\frac{K^2}{K_0^2}\right)$$

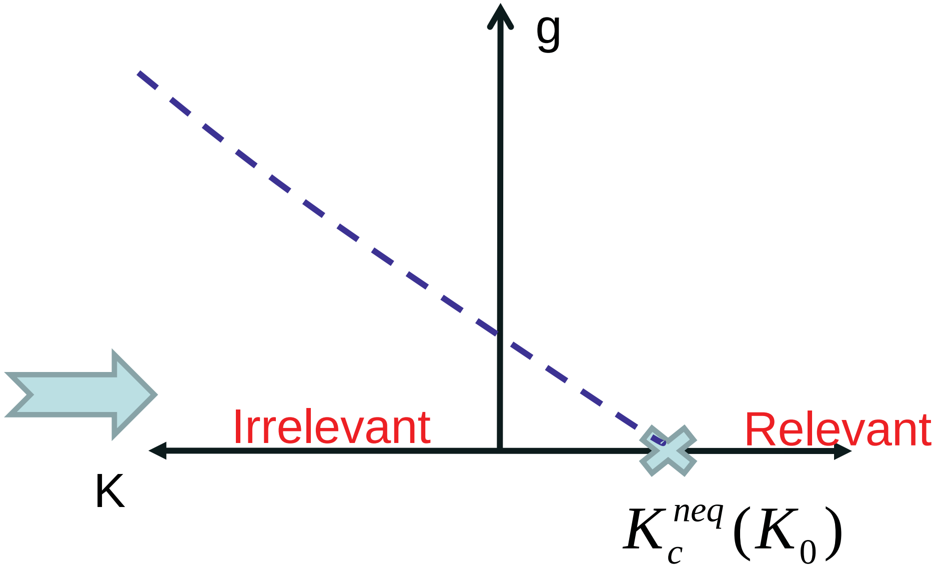
$$\langle \rho(r)\rho(0)\rangle \;\; t\to\infty \;\; \frac{1}{r^{2K_{neq}}}$$

However, since the nonequilibrium system is more disordered (faster decay of correlations) ,the periodic potential is less effective in localizing the system.

Thus critical point for the Mott transition in the nonequilibrium system is shifted to larger values of interactions.

Naïve expectation when the periodic potential is irrelevant: The same quadratic theory but with slightly renormalized parameters:

$$S = \frac{1}{2\pi K^*}\int dx\left[\frac{1}{u}\left(\partial_t\varphi\right)^2 - u\left(\partial_x\varphi\right)^2\right]$$



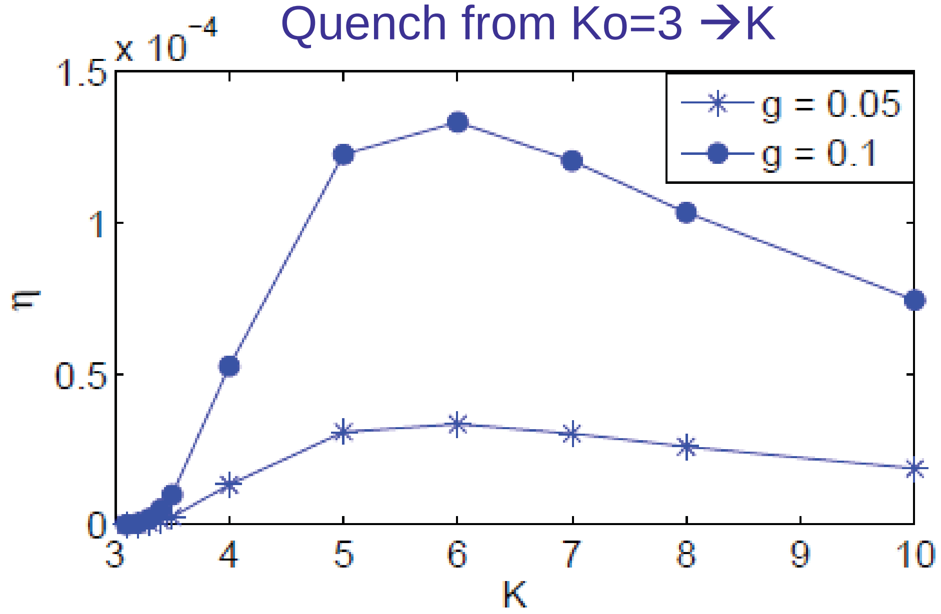
Instead a quadratic theory with qualitatively different features: Generation of dissipation (over-damped boson density modes):

$$\frac{1}{K^*}\left[\frac{1}{u}\partial_t^2\varphi - u\partial_x^2\varphi - \eta^*\partial_t\varphi\right] = 0$$

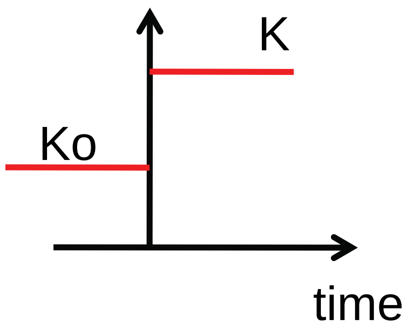
and also a temperature, which is strictly speaking defined in the classical limit of mode frequency << temperature

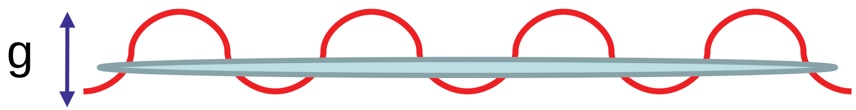
Low frequency, long-wavelength properties near the nonequilibrium fixed-point:

$$S^* = \sum_{q,\omega} \left(\phi^*_{cl} \quad \phi^*_q\right) \frac{1}{\pi K^* u} \begin{pmatrix} 0 & -i\eta^*\omega - u^2q^2 \\ i\eta^*\omega - u^2q^2 & 4iT^*_{eff}\eta^*\frac{K_0}{2K^*}\left(1+\frac{K^{*2}}{K_0^2}\right) \end{pmatrix} \begin{pmatrix} \phi_{cl} \\ \phi_q \end{pmatrix}$$



Non-monotonic dependence of
the dissipation on quench amplitude

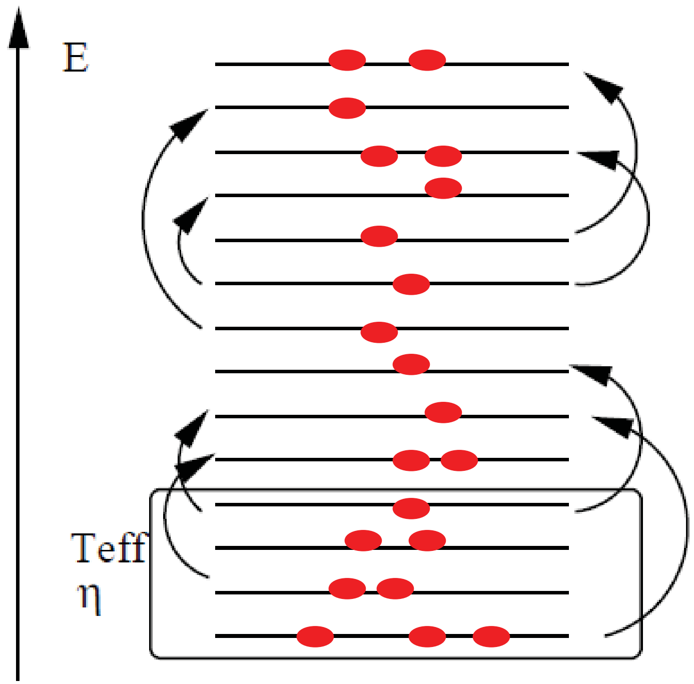




Dissipation => Inelastic scattering results in energy exchange between low frequency modes and high frequency modes.

Thus as the high frequency modes are gradually integrated out via RG, they act as a reservoir for the low-frequency modes giving it a damping.

System effectively acts as its own reservoir.



Classical analog of the Fluctuation-Dissipation-Theorem is obeyed. Low-frequency part is subjected to a “noise” due to the integrated out high-frequency modes:

$$2\eta\omega\coth\left(\frac{\omega}{2T}\right) \xrightarrow{\omega\to 0} 4\eta T_{eff}$$

Consequences

Density-density correlators now decay exponentially fast (as compared to a power-law):

Unequal positions:

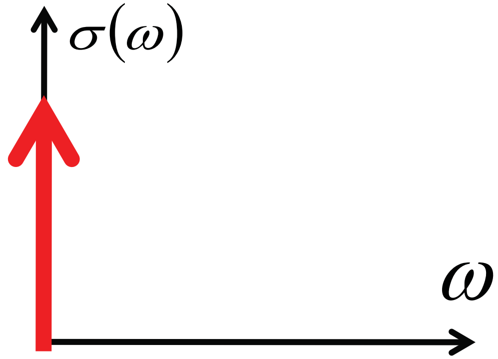
$$\langle \rho(x)\rho(y)\rangle \approx e^{-T_{eff}\frac{K^*}{u}|x-y|}$$

Unequal times:

$$\langle \rho(t)\rho(0)\rangle \approx e^{-T_{eff}\frac{K^*}{\sqrt{\eta}}\sqrt{|t|}}$$

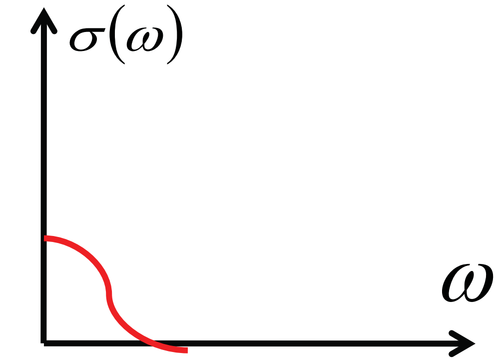
Dissipation=0, but finite temperature would have implied an infinitely long-lived current carrying state, and hence an infinite dc conductivity.

$$\sigma(\omega) = D\delta(\omega) + \sigma_{reg}$$



Dissipation implies finite dc conductivity

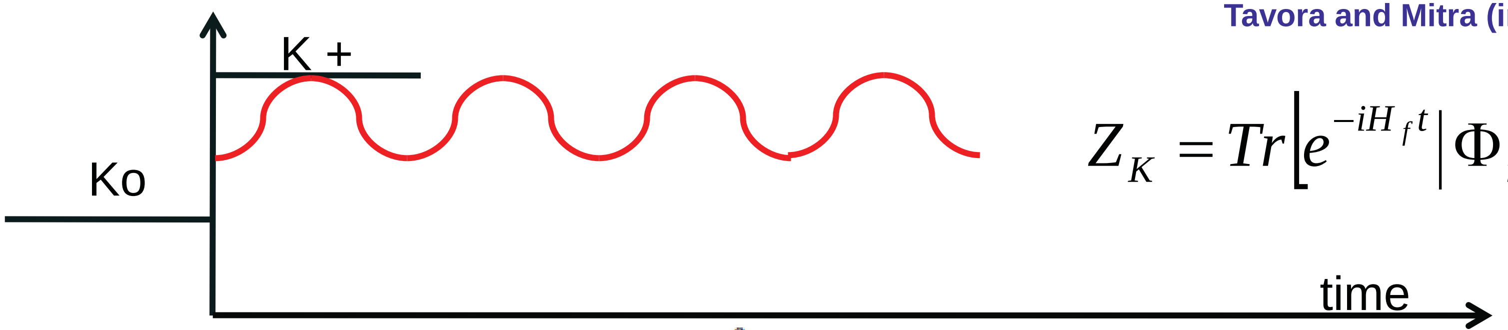
$$\sigma(\omega) = \frac{D}{\pi}\frac{\eta}{\eta^2+\omega^2} + \sigma_{reg}$$



Dissipation is also generated in equilibrium and finite temperature, see example Sirker et al, PRL 2009

Approach 2: At an initial time, not only the interaction is being quenched from Ko → K, but also the lattice potential is being switched on suddenly. Problem no longer time-translationally invariant. Study the time-evolution perturbatively in the periodic potential.

A. Mitra, arXiv: 1207.3777
Tavora and Mitra (in preparation)



$$Z_K = Tr\left[e^{-iH_f t} \left| \Phi_i \right\rangle \left\langle \Phi_i \right| e^{iH_f t} \right]$$

$$Z_K = \int \mathcal{D}\left[\phi_{cl}, \phi_q\right] e^{i(S_0 + S_{sg})}$$

$$S_0 = \int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \int_0^t dt_1 \int_0^t dt_2 \left(\phi_{cl}^*(1) \quad \phi_q^*(2)\right) \begin{pmatrix} 0 & G_A^{-1}(1,2) \\ G_R^{-1}(1,2) & -\left[G_R^{-1} G_K G_A^{-1}\right](1,2) \end{pmatrix} \begin{pmatrix} \phi_{cl}(2) \\ \phi_q(2) \end{pmatrix}$$

$$S_{sg} = \frac{gu}{\alpha^2} \int_{-\infty}^{\infty} dx_1 \int_0^t dt_1 \left[\cos\{\gamma\phi_-(1)\} - \cos\{\gamma\phi_+(1)\}\right]$$

Split fields into slow and fast modes in momentum space

$$\phi_\pm = \phi_\pm^< + \phi_\pm^>$$

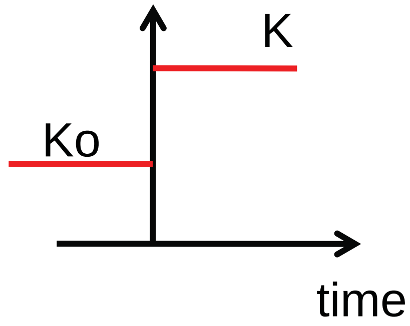
$$G_{0,\Lambda} = G_{0,\Lambda-d\Lambda}^< + G_{\Lambda-d\Lambda,\Lambda}^>$$

$$G_{\Lambda-d\Lambda,\Lambda}^> = d\Lambda \frac{dG_{0,\Lambda}}{d\Lambda}$$

Go=Correlators for the slow and fast fields

Expand Zk to quadratic order in g and integrate fast modes. Corrections to the action that now depend on time after the quench.

$$C_{ab}(1,2) = \langle e^{i\gamma\phi_a(1)} e^{-i\gamma\phi_b(2)} \rangle \qquad 1(2) = R + (-)\frac{r}{2}, T_m + (-)\frac{\tau}{2}$$



The correlator depends both on the time-difference τ as well as the mean time T_m after the quench, and as expected is always translationally invariant in space.

$$\boxed{\begin{aligned} C_{+-}(r, T_m, \tau) = & \left[\frac{\alpha}{\sqrt{\alpha^2 + (u\tau + r)^2}} \frac{\alpha}{\sqrt{\alpha^2 + (u\tau - r)^2}} \right]^{K_{neq}} \\ & \times \left[\frac{\sqrt{\alpha^2 + \{2u(T_m + \tau/2)\}^2}}{\sqrt{\alpha^2 + (2uT_m + r)^2}} \frac{\sqrt{\alpha^2 + \{2u(T_m - \tau/2)\}^2}}{\sqrt{\alpha^2 + (2uT_m - r)^2}} \right]^{K_{tr}} \\ & \times e^{-iK_{eq}\left[\tan^{-1}\left(\frac{u\tau + r}{\alpha}\right) + \tan^{-1}\left(\frac{u\tau - r}{\alpha}\right)\right]} \qquad (3) \end{aligned}}$$

$$\boxed{K_{neq} = \frac{\gamma^2}{8} K_0 \left(1 + \frac{K^2}{K_0^2}\right)}$$

$$\boxed{K_{tr} = \frac{\gamma^2}{8} K_0 \left(1 - \frac{K^2}{K_0^2}\right)}$$

$$\boxed{K_{eq} = \frac{\gamma^2 K}{4}}$$

At short times Tm<<1, power-law in space with exponent Ko
At long times, Tm>>1, power-law in space and time, with exponent Kneq
The crossover between these two limits determined by Ktr
SCALING DIMENSION OF THE LATTICE IS TIME-DEPENDENT

RG equations that depend explicitly on the time Tm after the quench

$$\frac{dg}{d\ln l} = g\left[2 - \left(K_{neq} + \frac{K_{tr}}{1+4T_m^2}\right)\right]$$

$$\frac{dK^{-1}}{d\ln l} = \frac{\pi g^2\gamma^2}{8} I_K(T_m)$$

$$\frac{dT_m}{d\ln l} = -T_m$$

$$\frac{1}{Ku}\frac{du}{d\ln l} = \frac{\pi g^2\gamma^2}{8} I_u(T_m)$$

$$\frac{d\eta}{d\ln l} = \eta + \frac{\pi g^2\gamma^2 K}{4} I_\eta(T_m)$$

$$\frac{d(\eta T_{eff})}{d\ln l} = 2\eta T_{eff} + \frac{\pi g^2\gamma^2 K}{8} I_{T_{eff}}(T_m)$$

Scaling dimension of the lattice is time-dependent. At short times it is Ko, at long times it is Kneq

Time after the quench acts as an additional inverse energy scale in the problem

Dissipation and noise whose strengths are now time-dependent.

$I_{K,u,\eta,T_{eff}}$ reach steady state values at $T_m \gg 1$, whereas for short times, they vanish as $T_m \to 0$ as expected since the effect of the lattice potential vanishes at T_m=0.

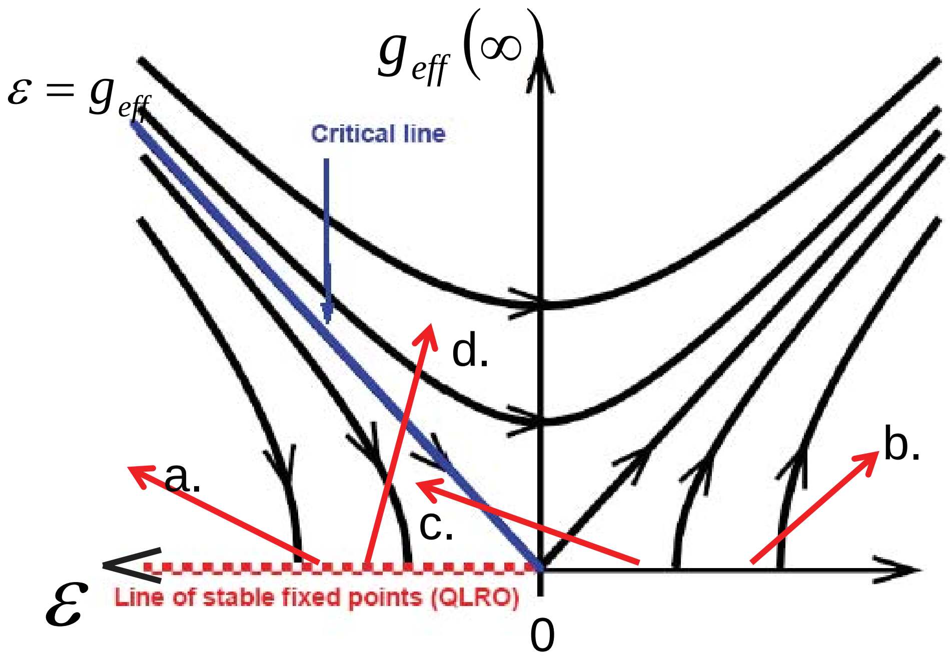
$$\boxed{K_{neq} = \frac{\gamma^2}{8} K_0\left(1 + \frac{K^2}{K_0^2}\right)}$$

$$\boxed{K_{tr} = \frac{\gamma^2}{8} K_0\left(1 - \frac{K^2}{K_0^2}\right)}$$

For times $T_m < \frac{1}{\eta}$ we may neglect dissipation and noise.

$$\frac{dg}{d\ln l} = g\left[2 - \left(K_{neq} + \frac{K_{tr}}{1+4T_m^2}\right)\right]$$
$$\frac{dK^{-1}}{d\ln l} = \frac{\pi g^2 \gamma^2}{8} I_K(T_m)$$
$$\frac{dT_m}{d\ln l} = -T_m$$

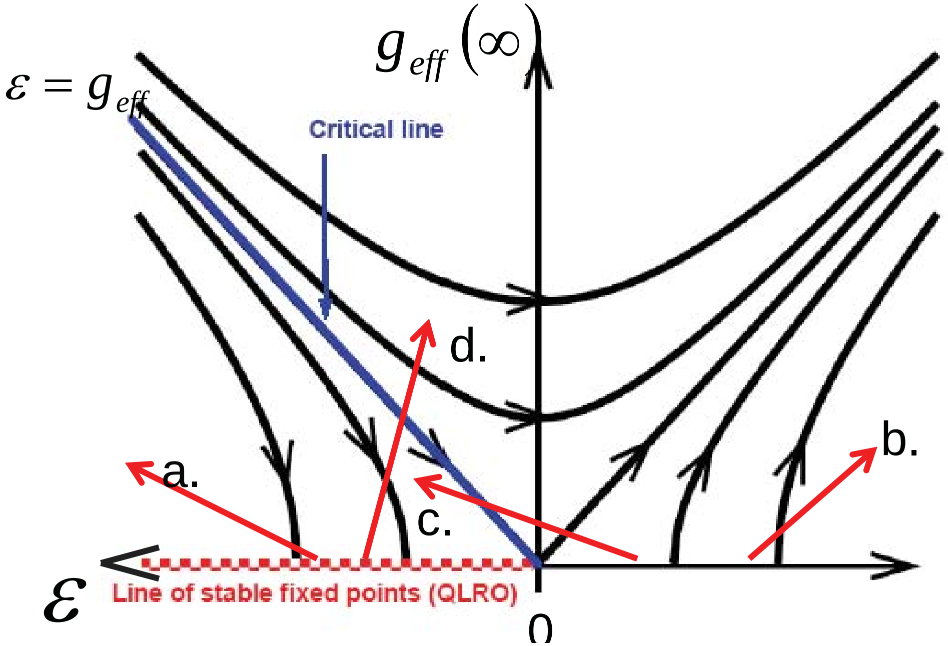
Convenient to define an effective coupling $g_{eff} \approx g\sqrt{I_K(T_m)}$ that vanishes at Tm=0, and reaches a steady state value at Tm >> 1



Arrows connect Hamiltonians before and after the quench

$$\varepsilon = K_{neq} + \frac{K_{tr}}{1+4T_m^2} - 2$$

a. Periodic potential irrelevant at all times.
b. Periodic potential relevant at all times
c. Periodic potential relevant at short times, irrelevant at long times.
d. Periodic potential irrelevant at short times, relevant at long times. This case shows a dynamical phase transition



At strong coupling

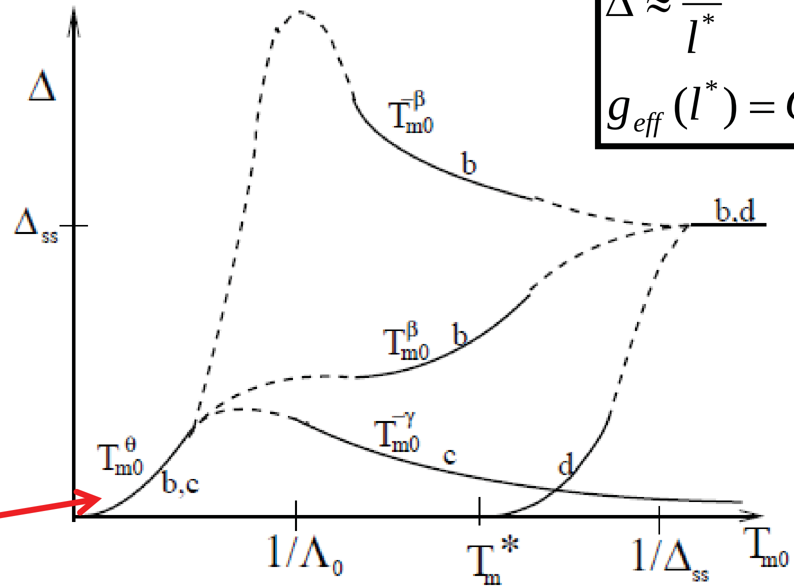
$$g\cos(\varphi) \approx 1 - \frac{g}{2}\varphi^2$$

$$\boxed{\begin{array}{l} \Delta \approx \dfrac{1}{l^*} \\ g_{eff}(l^*) = O(1) \end{array}}$$

$$\beta = \theta\left(\left|\frac{\gamma^2 K_0}{4} - K_{neq}\right|\right),\ \gamma = 1 + A\theta$$

$$A = \sqrt{\epsilon_0^2 - g_{eff,0}^2}$$

$$\theta = \frac{1}{2 - \frac{\gamma^2 K_0}{4}}$$



Results agree with a lattice quench
at the exactly solvable Luther-Emery point (Iucci and Cazalilla)

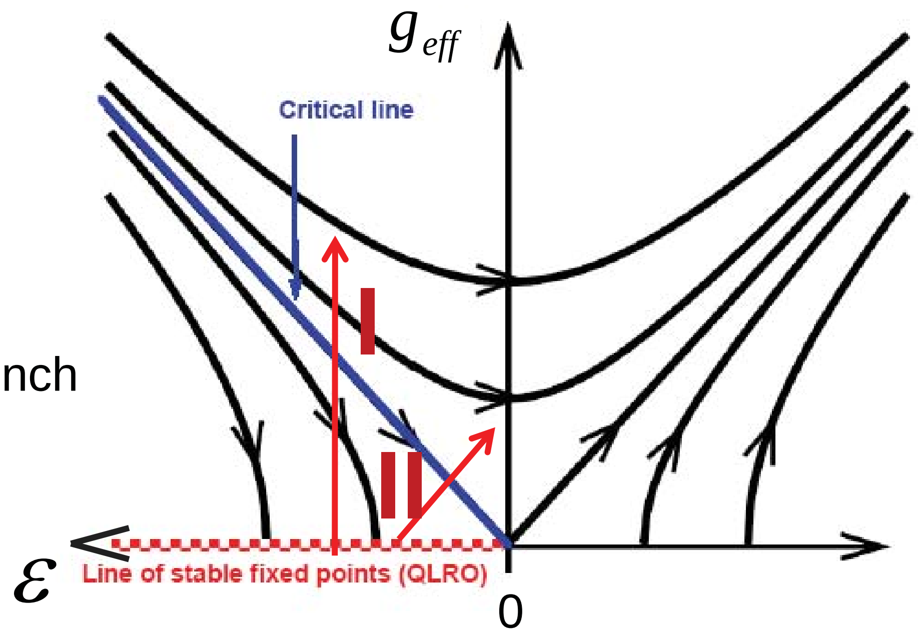
Dynamical Phase Transition

At a time Tm* such that $\epsilon(T_m^*) = g_{eff}(T_m^*)$ a non-analytic behavior in the solution of the RG equations.

$$\Delta \simeq \frac{1}{l^*} = \theta(T_{m0} - T_m^*) e^{-\frac{f}{\sqrt{T_{m0} - T_m^*}}}$$

I: Pure lattice quench

II: Lattice and interaction quench



Non-analytic behavior at a critical time in the Loschmidt echo in the transverse-field Ising model after a quench: Heyl, Polkovnikov, Kehrein, arXiv:1206.2505

For times $T_m > \frac{1}{\eta}$ when the lattice potential is irrelevant: A quantum kinetic equation approach.

Tavora and Mitra (in preparation)

$$F^\dagger g^{-1} - g^{-1} F = \Sigma^K - \Sigma^R \circ F + F^\dagger \circ \Sigma^A$$

F: Boson distribution function

$$\Sigma \propto O\left(g^2\right) \approx e^{-\gamma^2 \langle \phi\phi \rangle}$$

Even to leading order in the potential, multi-particle scattering processes are involved.

May also be generalized to the "relevant" regime where $\cos(\varphi) \approx 1 - a\varphi^2 + b\varphi^4$

QUENCH IN THE XX CHAIN (FREE FERMIONS):
Playing with the initial condition can generate interesting non-equilibrium states such as those that carry current:

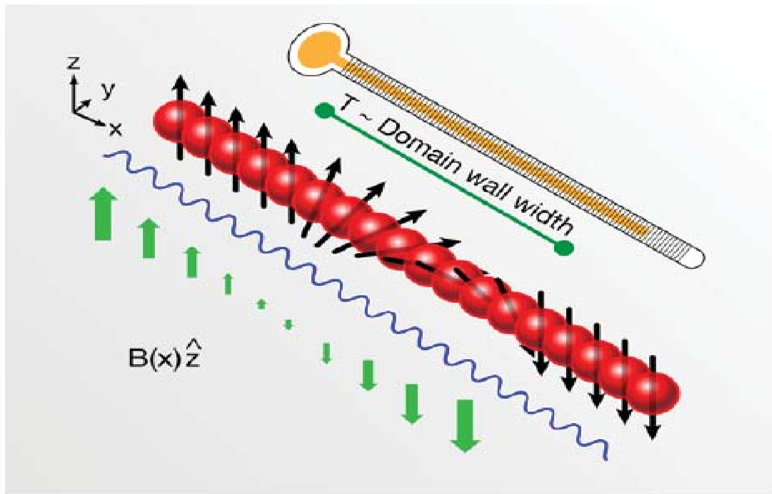
Lancaster and Mitra, PRE 2010

$$H_{xx} = -J\sum_j \left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y - \frac{h_j}{J} S_j^z\right]$$

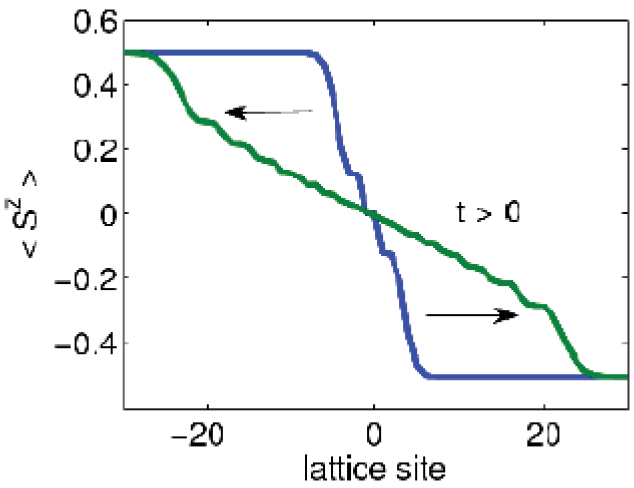
$$h_j = \mathcal{F} j a$$

Jordan-Wigner transformation

$$H_{xx} = -\frac{J}{2}\sum_j \left[c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j\right] + \sum_j j\mathcal{F} a c_j^\dagger c_j$$



A. M. Rey *Physics* **2**, 103 (2009)

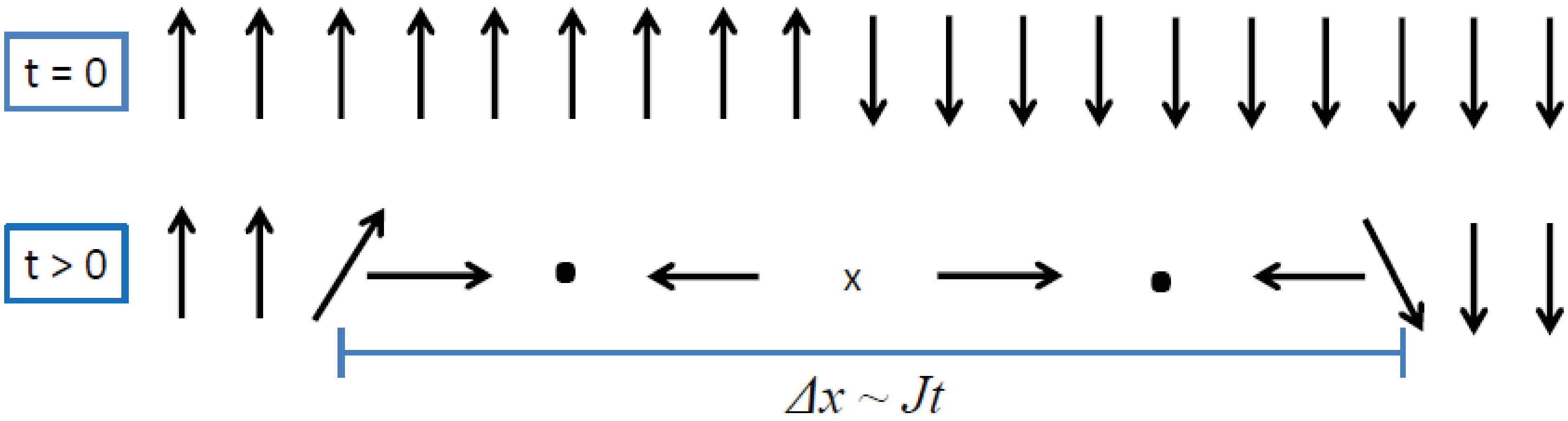


Ballistic expansion of domain wall

$$\left\langle \hat{S}^z_j \hat{S}^z_{j+n} \right\rangle \xrightarrow{t\to\infty} C^{zz}_{\text{eq.}}(n) = -\frac{\sin^2\left[\frac{\pi n}{2}\right]}{\pi^2 n^2}$$

$$\boxed{\left\langle \hat{S}^x_j \hat{S}^x_{j+n} \right\rangle \xrightarrow{t\to\infty} \cos\left(\frac{\pi n}{2}\right) C^{xx}_{\text{eq.}}(n) = \frac{\kappa^2 \cos\left(\frac{\pi n}{2}\right)}{\sqrt{8n}}}$$

JLL and A. Mitra, Phys. Rev. E **81**, 061134 (2010).



$$\partial_t \hat{S}^z_n = -(j_{n+1} - j_n)$$

$$\langle j_n \rangle = J\,\mathrm{Im}\left\langle \hat{S}^+_n \hat{S}^-_{n+1} \right\rangle$$

Within central subsystem

$$|\Psi(t)\rangle \to |\Psi_{\text{NESS}}\rangle$$

(Locally) equivalent to ground state of

$$\hat{H}^{\text{eff}}_{\text{sub}} = \hat{H}_{xx} - \lambda \hat{J}$$

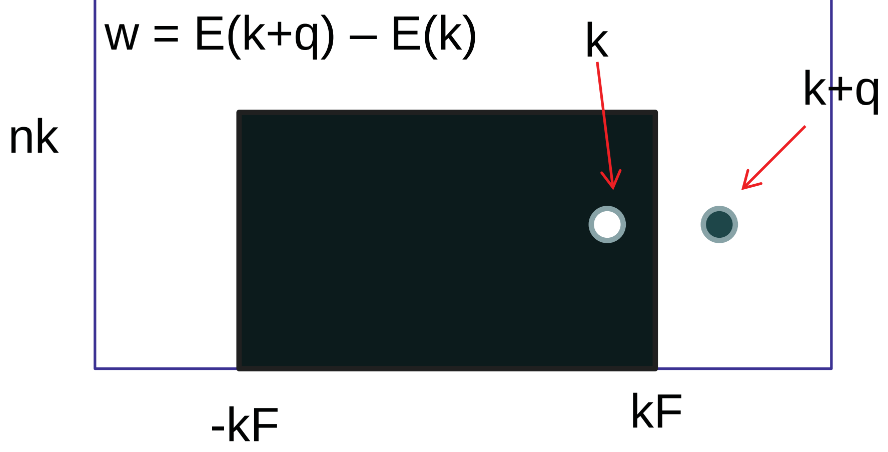
Quench from a spatially inhomogeneous state can lead to current carrying steady states. These states for a free theory can be described by a GGE with a suitable Lagrange multiplier that imposes current flow via the boundary conditions.

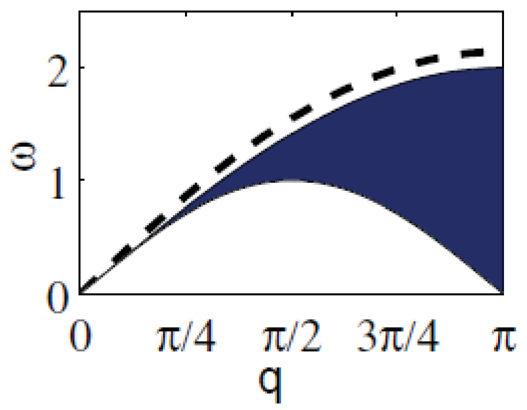
CAN "FRICTION" APPEAR FOR INTERACTING FERMIONS THAT ARE OUT OF EQUILIBRIUM?

In equilibrium: Sound modes of an interacting Fermi gas = weakly interacting bosons

In general one expects that a sound mode (or boson) at (w,q) can decay via the creation of electronic (particle-hole) excitations.

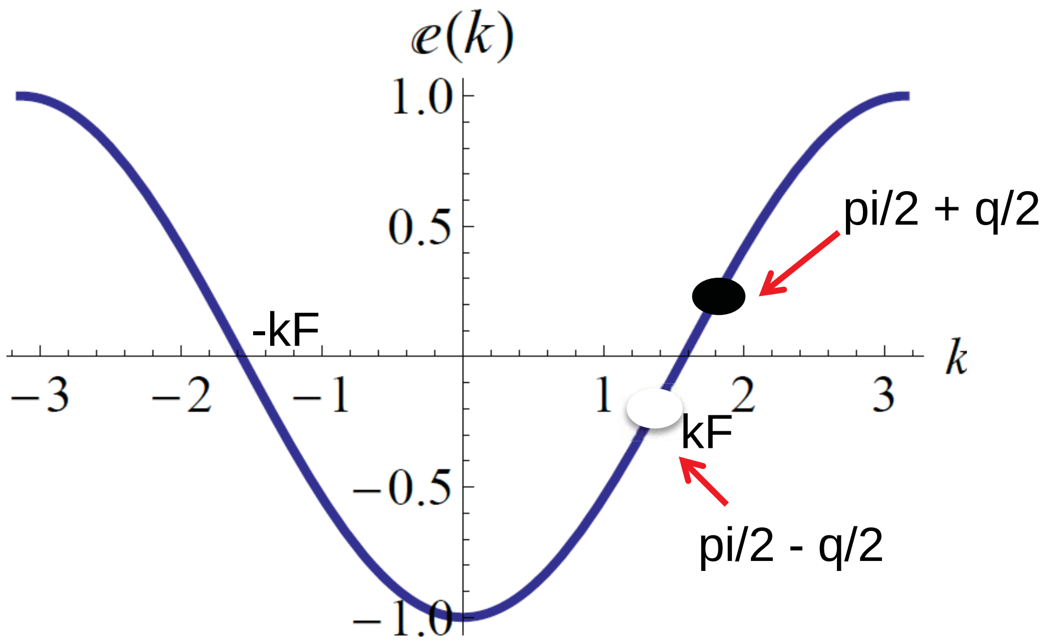
Fermion distribution function at T=0

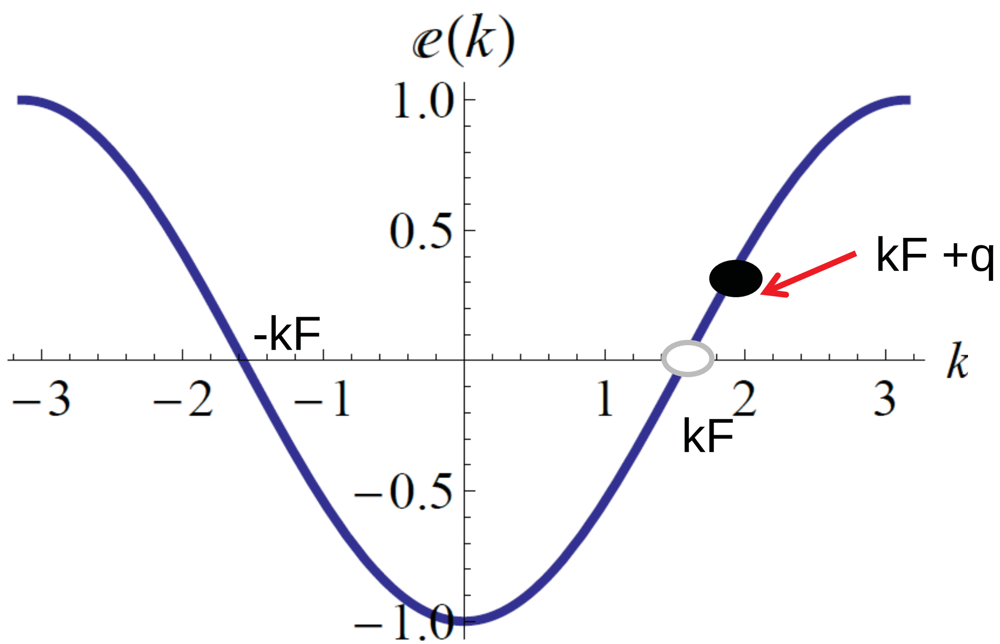




In 1D and in equilibrium, these processes do not occur due to phase space restrictions: sound modes are long lived.

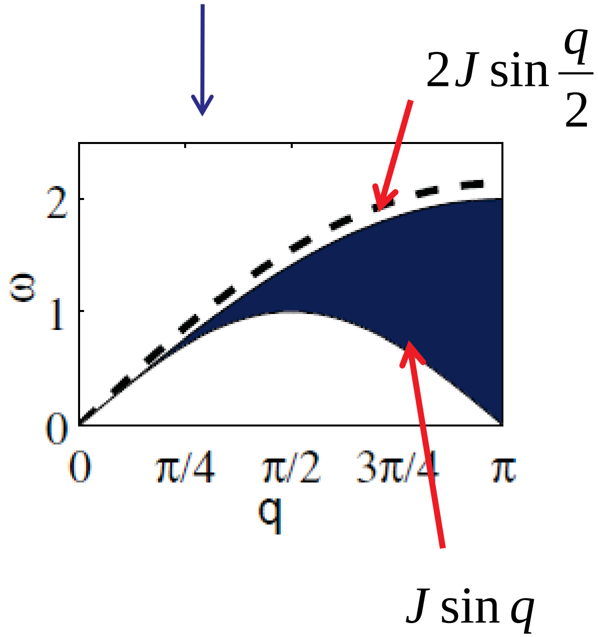
By quenching a parameter of the Hamiltonian, we generate non-interacting but out of equilibrium fermions. We study how they respond to weak interactions





Recall: Particle-hole continuum for spinless lattice fermions

Particle-hole continuum and collective mode for the XXZ chain from RPA



QUENCH FROM ISING CHAIN TO XX CHAIN

$$H_i' = -J\sum_j \left[2S_j^x S_{j+1}^x - S_j^z\right] \longrightarrow \begin{aligned} H_i' &= \sum_k \epsilon_k^i \eta_k^\dagger \eta_k \\ \epsilon_k^i &= -2J sgn(\cos k - 1)|\sin \tfrac{k}{2}| \end{aligned}$$

JORDAN-WIGNER TRANSFORMATION

$$H_f' = -J\sum_j \left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right] \longrightarrow \begin{aligned} H_f' &= \sum_k \epsilon_k c_k^\dagger c_k \\ \epsilon_k &= -J\cos k \end{aligned}$$

QUENCH FROM ISING CHAIN WITH DZYALOSHINSKII-MORIYA INTERACTIONS TO XX CHAIN

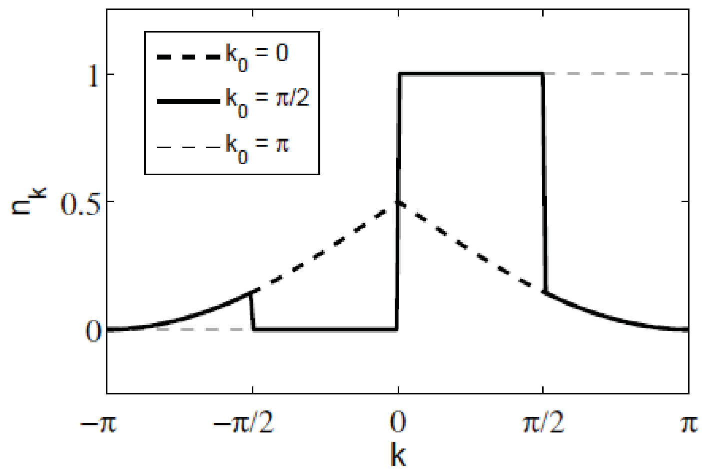
$$H_i' = -J\sum_j \left[2S_j^x S_{j+1}^x - S_j^z\right] \underbrace{-\lambda \sum_j [\hat{S}_j^y \hat{S}_{j+1}^x - \hat{S}_j^x \hat{S}_{j+1}^y]}_{\text{Produces current carrying states after the quench}} \longrightarrow \begin{aligned} \hat{H}_i(\lambda) &= \sum_k \epsilon_k' \eta_k^\dagger \eta_k, \\ \epsilon_k' &= \epsilon_k^i - \lambda \sin k \end{aligned}$$

$$H_f' = -J\sum_j \left[S_j^x S_{j+1}^x + S_j^y S_{j+1}^y\right]$$

$$\begin{aligned} H_f' &= \sum_k \epsilon_k c_k^\dagger c_k \\ \epsilon_k &= -J\cos k \end{aligned}$$

Distribution function of fermions are far out of equilibrium :

k0 is related to current strength



Effect of SzSz
(or nearest neighbour) interactions?

Random Phase Approximation to find the collective (sound) modes.

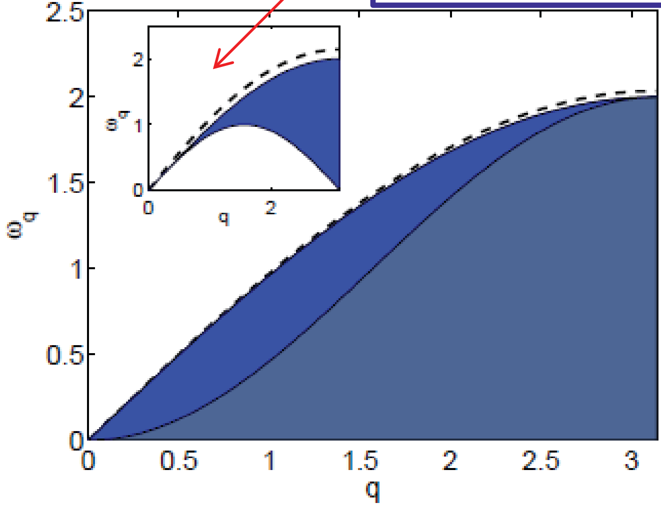
NON-EQUILIBRIUM FERMIONS: OVERDAMPED SOUND MODES

Highly broadened fermion distribution function. Phase space constraints are lifted, and collective modes can decay into particle-hole excitations.

In equilibrium

$$\omega(q) \simeq J|q|\sqrt{1+\frac{V_q}{\pi J}}.$$

Thus for nonequilibrium fermions, we find over-damped modes for attractive interactions.



Collective mode for repulsive interactions are still undamped on the lattice.

$$\omega_q \simeq J|q|\sqrt{1+\frac{V_0^2}{32J^2}}.$$

Conclusions

- Quantum quenches in free theories can lead to interesting nonequilibrium states that may or may not be described by a generalized Gibbs ensemble (GGE).
- In the presence of non-linearities, an analytic approach to study dynamics is presented that is valid in the thermodynamic and long-time limit where numerical studies are still hard to do.
- Even when the periodic potential is “irrelevant”, its effect is non-trivial as it generates a dissipation and a noise.
- When the periodic potential is relevant, a new kind of non-equilibrium phase transition is identified which corresponds to non-analytic behavior during the time-evolution. In particular an order-parameter is found to be zero at all times $t<t^*$, and non-zero after this time.
- The RG makes predictions for how an order-parameter evolves in time. The results are in agreement with a lattice quench at the exactly solvable Luther-Emery point, and generalizes the results to the case where the model is not exactly solvable.