The Abdus Salam
International Centre
for Theoretical Physics

2358-16

Joint ICTP-IAEA Workshop on Nuclear Structure Decay Data: Theory and Evaluation

6 - 17 August 2012

Radioactive decay

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Austria

International Atomic Energy Agency

Radioactive decay

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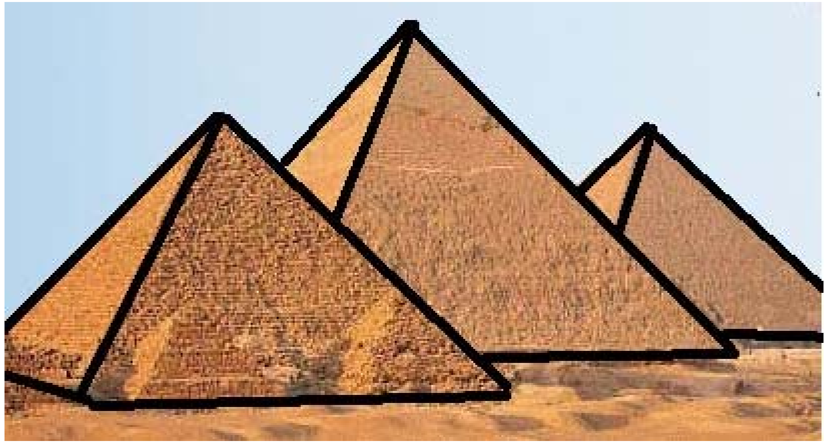
ICTP, ENSDF, August 2012

Outline

- **Stability vs Change**
- **The atom and the atomic nucleus**
  - Mass & Energy
  - When a decay channel is possible?
  - Rutherford & Soddy law of Radioactive decay
- **Radioactive decay**

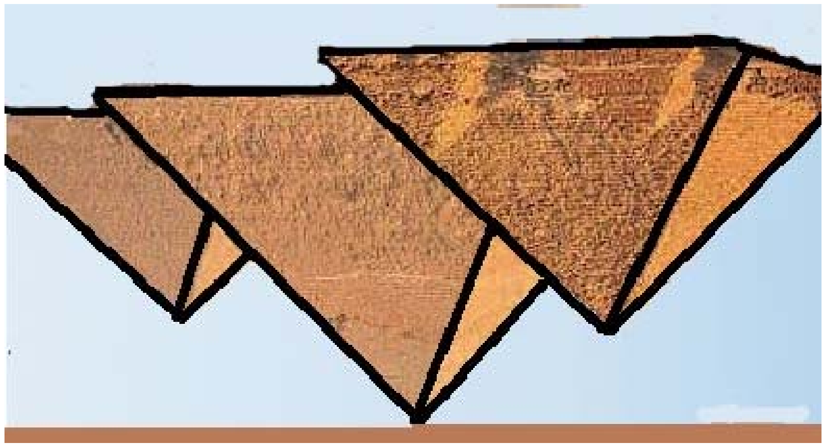
  β, α & γ decay
- **Alpha decay**
- **Conclusions**

What is Stable?

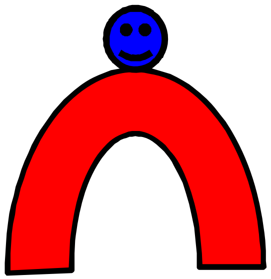


…it depends on

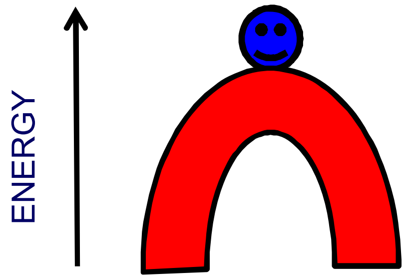
- shape
- position
- environment
- ….

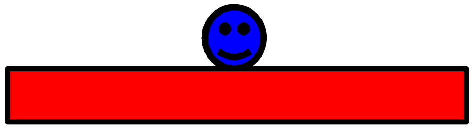


Equilibrium

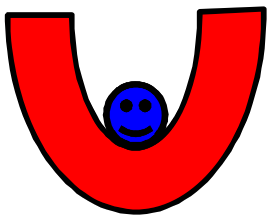


Unstable



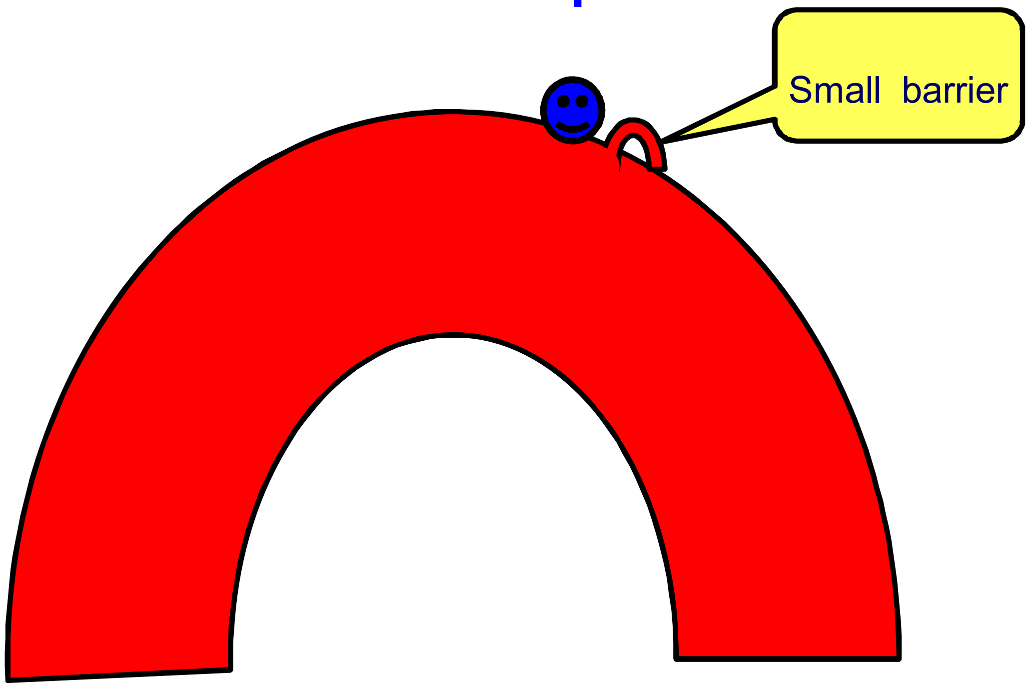


Indiferent



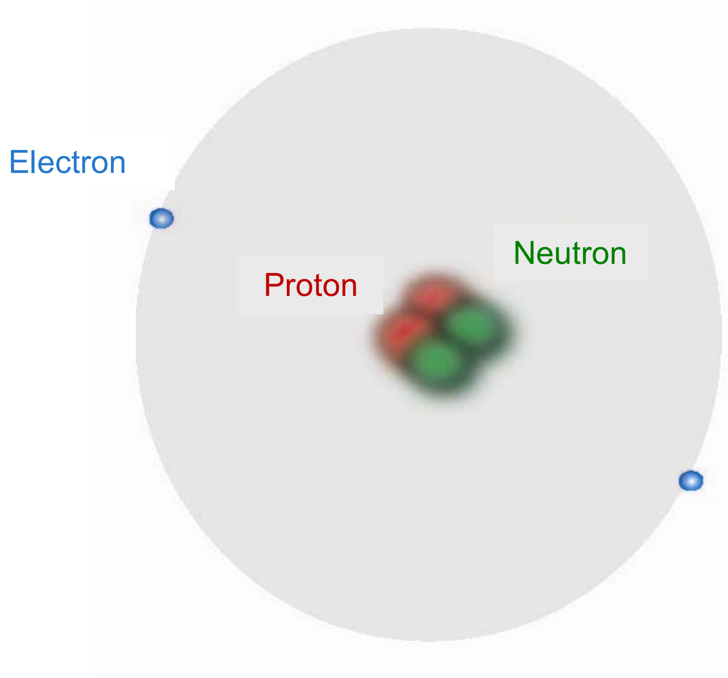
Stable

Equilibrium



Unstable?

The Nucleus



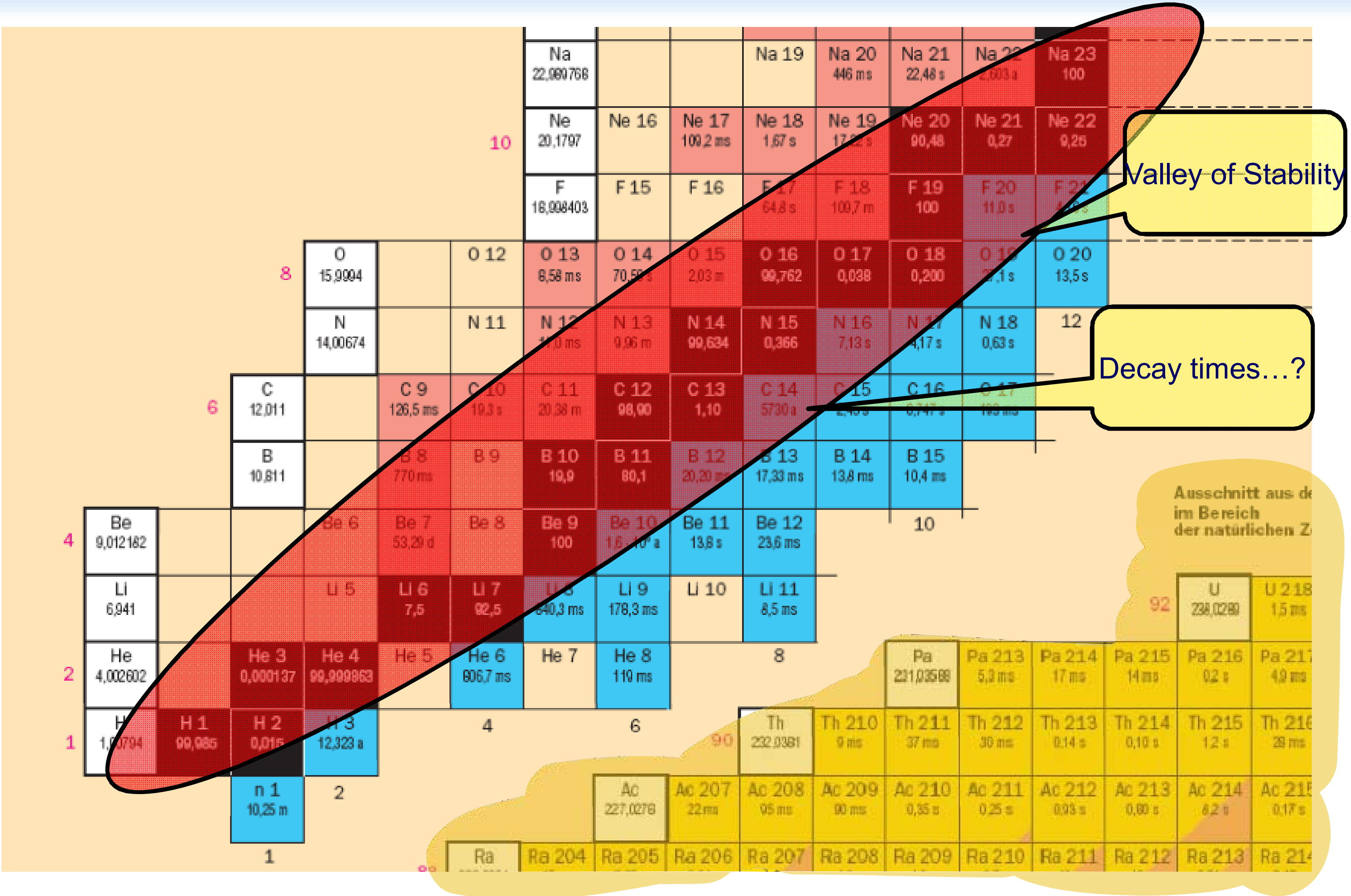
$$^{4}_{2}He_{2}$$

How to specify a nuclide

$$^{A}_{Z}X_{N}$$

- Z: Atomic number (protons)
- N: Neutron number (neutrons)
- A: Mass Number (Z+N)

Chart of nuclides (Table of Isotopes)



What determines the Energy?

Physics' most famous formula:

$$E = m c^2$$

Energies measured in electron-Volts (eV)
MeV=10^6 eV

$$m = E / c^2$$

Masses measured in MeV/c^2

Where to find masses?

NUCLEAR WALLET CARDS

April 2005

Jagdish K. Tuli

National Nuclear Data Center
www.nndc.bnl.gov

Appendix–II Frequently–Used Constants

The frequently used constants are given below in familiar units. Only approximate values are given, see App–III for values to current known precision.

Symbol	Constant	Value
$1/\alpha=\hbar c/e^2$	Fine structure constant	137.0
c	Speed of light in vacuum	2.998×10^{10} cm/s
h	Planck constant	6.626×10^{-27} erg s
$\hbar=h/2\pi$		6.582×10^{-22} MeV s
$\hbar c$		197.3 MeV fm
$k=R/N_A$	Boltzmann constant	8.617×10^{-11} MeV/K
$r_e=e^2/m_ec^2$	Classical e^- radius	2.818 fm
$\lambda_{C,e}=\hbar/m_ec$	Compton wavelength of e^-	386.2 fm
$\lambda_{C,p}=\hbar/m_pc$	Compton wavelength of p	0.210 fm
$\lambda_{C,\pi}=\hbar/m_\pi c$	Compton wavelength of π	1.414 fm
u	Atomic mass unit	931.5 MeV/c^2
m_e	Electron mass	0.511 MeV/c^2
m_n	Neutron mass	939.6 MeV/c^2
m_p	Proton mass	938.3 MeV/c^2

Energy equivalent

$E(m_e) = 0.511$ MeV

…but in order to simplify the writing we simply use:

$m_e = 0.511$ MeV

Isotope mass

One could think that the mass of a nuclide X is calculated adding up the masses of the constituent protons and neutrons:

$$M\left({}^{A}_{Z}X_{N}\right) = Z\,m_{p} + N\,m_{n}$$

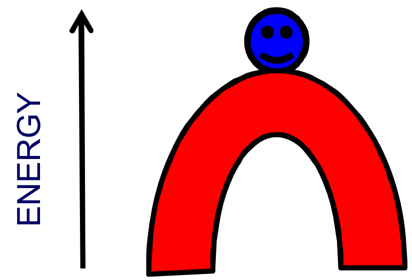
However, when the isotope mass is measured (i.e. with a mass spectrometer) it is seen that ALWAYS:

$$M\left({}^{A}_{Z}X_{N}\right) < Z\,m_{p} + N\,m_{n}$$

$$Z m_p + N m_n$$

Difference of mass (difference of energy)

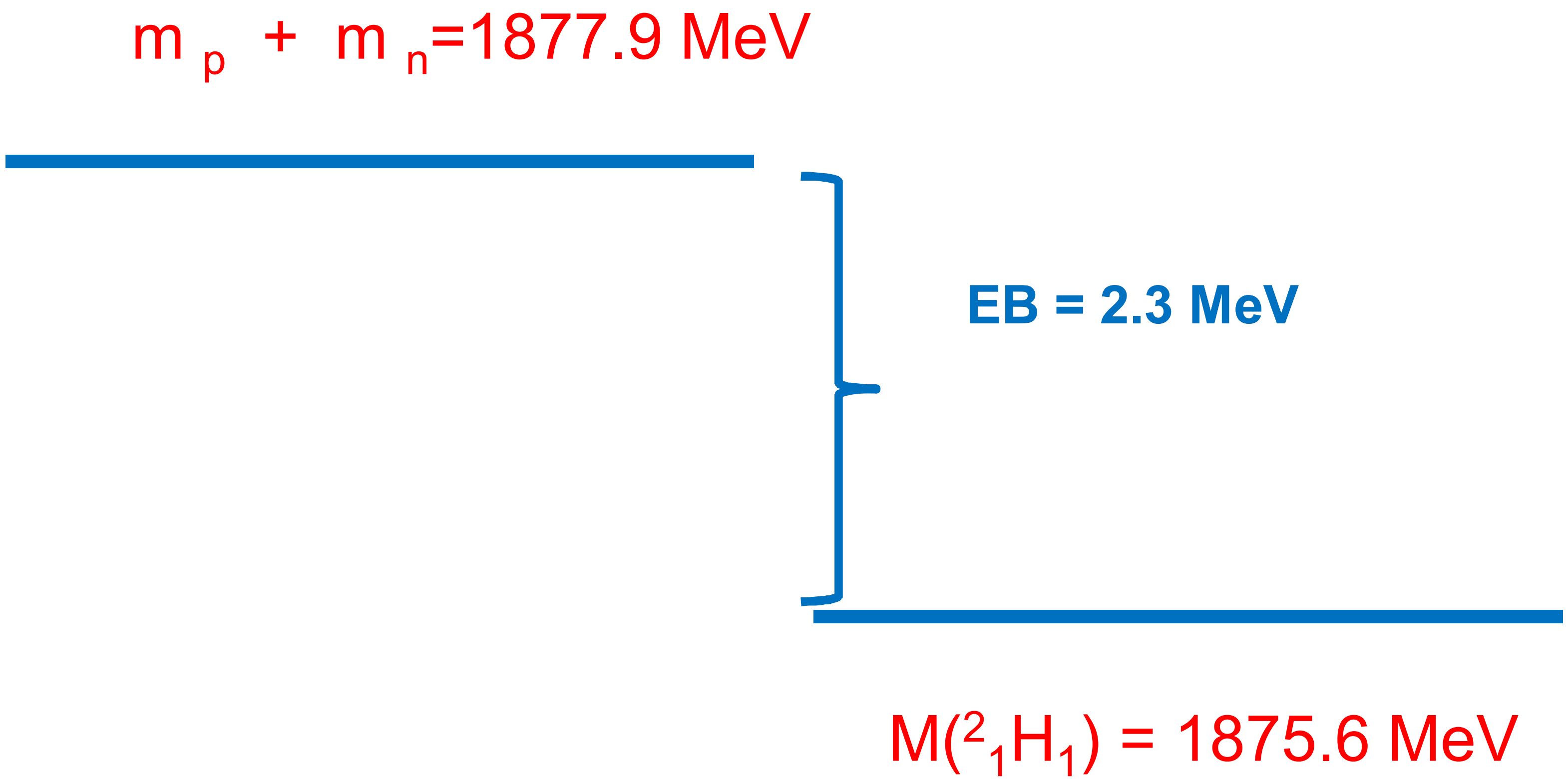
$$M({}^{A}_{Z}X_{N})$$

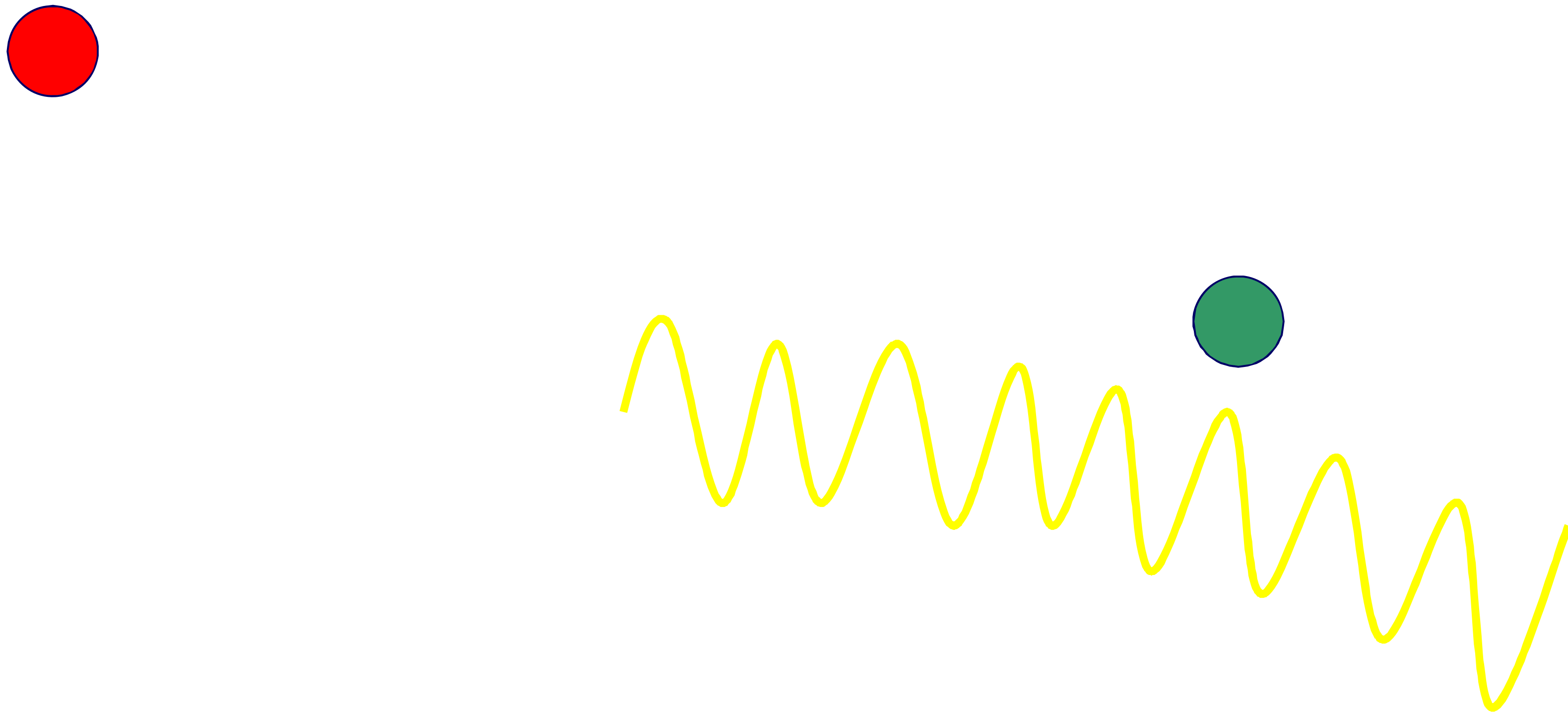


This energy is called BINDING ENERGY (E_B) and is the energy that would be released by the Z protons and N neutrons while joining together to form the isotope ^{A}X

$$E_B = Z\, m_p + N\, m_n - M(\,^{A}_{Z}X_N\,)$$

Example: Mass of Deuteron





Gamma ray of energy
2.3 MeV

Binding energy

This is a nuclear reaction (neutron capture):

n + p => d + γ (2.3 MeV)

There is also the inverse reaction (deuteron photo-disintegration):

d + γ (2.3 MeV) => n + p

2.3 MeV is a relatively "small" energy, it is said that the deuteron is weakly bounded.

Its Binding energy per nucleon (E_B / A) = 2.3 MeV/2 = 1.15 MeV/A

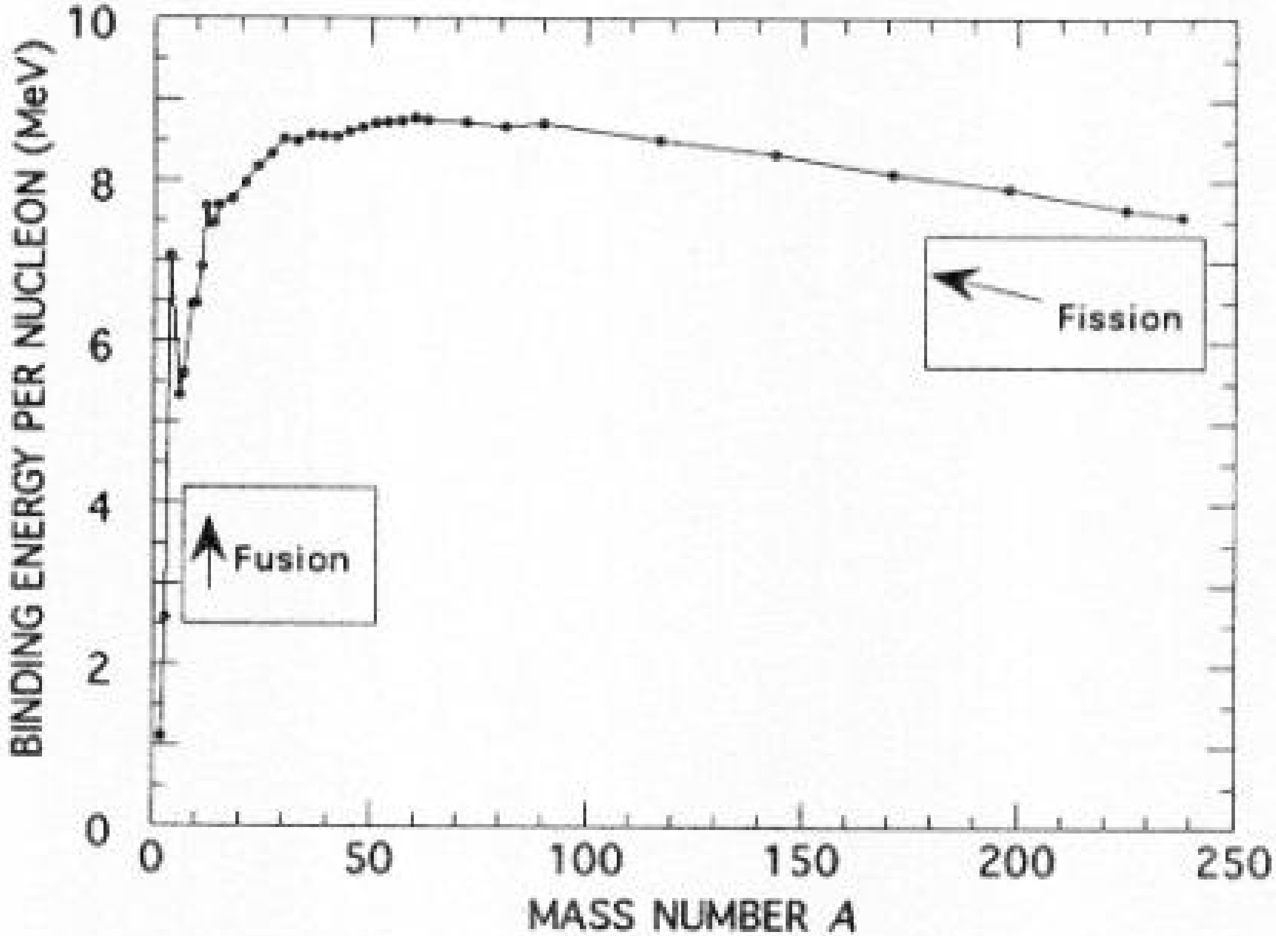
Binding energy

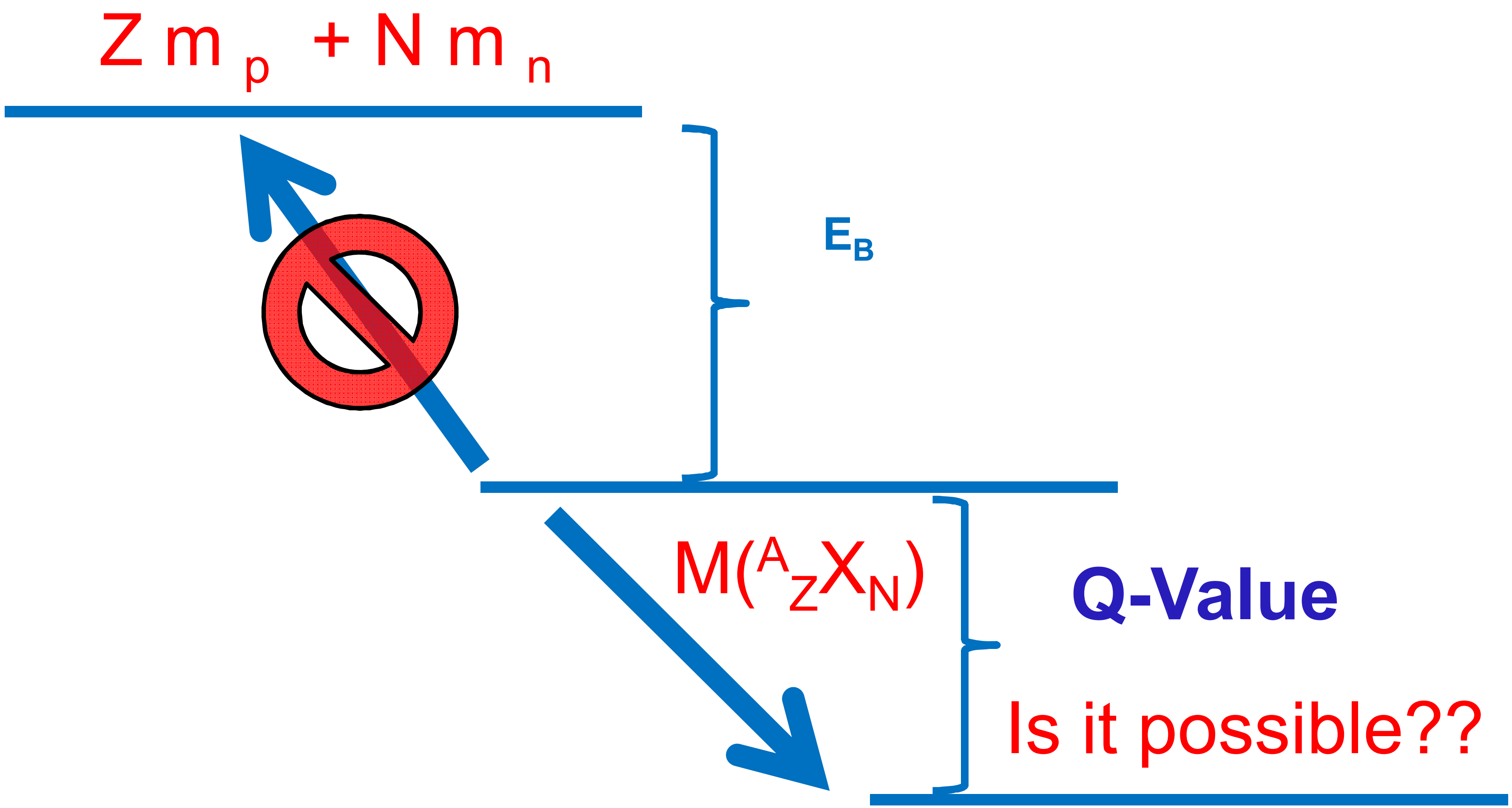
The Binding Energy for ^{12}C is:

$$E_B = 6\ m_p + 6\ m_n - M(^{12}C) = 92.16\ \text{MeV}$$

$$E_B / A = 92.16 / 12\ \ \text{MeV/A}\ \ = 7.68\ \text{MeV / A}$$

Binding energy





$Z m_p + N m_n$
E_B
$M(^A_Z X_N)$
Q-Value
Is it possible??

Rutherford & Soddy 1902

Radioactive Decay law

$$N=N_0\, e^{-\lambda t}$$

N_0 : initial # of atoms

λ : decay constant [1\s]

N_0 Initial number of radioactive nuclei

λ: probability of disintegration per atom per unit of time t

If ΔN= atoms disintegrate in Δt

$\Delta N = -\lambda N \Delta t \longrightarrow \Delta N/N = -\lambda \Delta t$ If $\Delta t \longrightarrow 0$

$dN/N = -\lambda dt \longrightarrow \int dN/N = -\lambda dt$

$$\ln N = -\lambda t \longrightarrow N = N_0 e^{-\lambda t}$$

A similar formula is obtained for the number of disintegrations per second: **Activity**

$$N = N_0 e^{-\lambda t} \longrightarrow A = A_0 e^{-\lambda t}$$

A_0 is the Inicial Activity:

$$A_0 = \lambda \; N_0$$

The unit of Activity (1 disintegracion per second) is called **Becquerel (Bq)**

Curie

1Ci=Activity of 1 gram of the radium isotope ^{226}Ra

1Ci=3.7 10^{10} Bq

Half-life

$T_{1/2}$: the time required for half the atoms in a radioactive substance to disintegrate

Mean life (Average lifetime)

$$\tau = \frac{T_{1/2}}{\ln 2} = \frac{1}{\lambda}$$

$$\Gamma = \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-16}\,[\mathrm{eV}]\,[s]}{\tau\,[s]}$$

Γ : Total width

$$T_{i \to f} = \frac{2\pi}{\hbar} \left| \langle f | H' | i \rangle \right|^2 \rho,$$

Matrix element

Phase space factor

Q = available kinetic energy

large Q → large phase space → higher rate

Beta decay of the free neutron

Conservation laws OK?

n

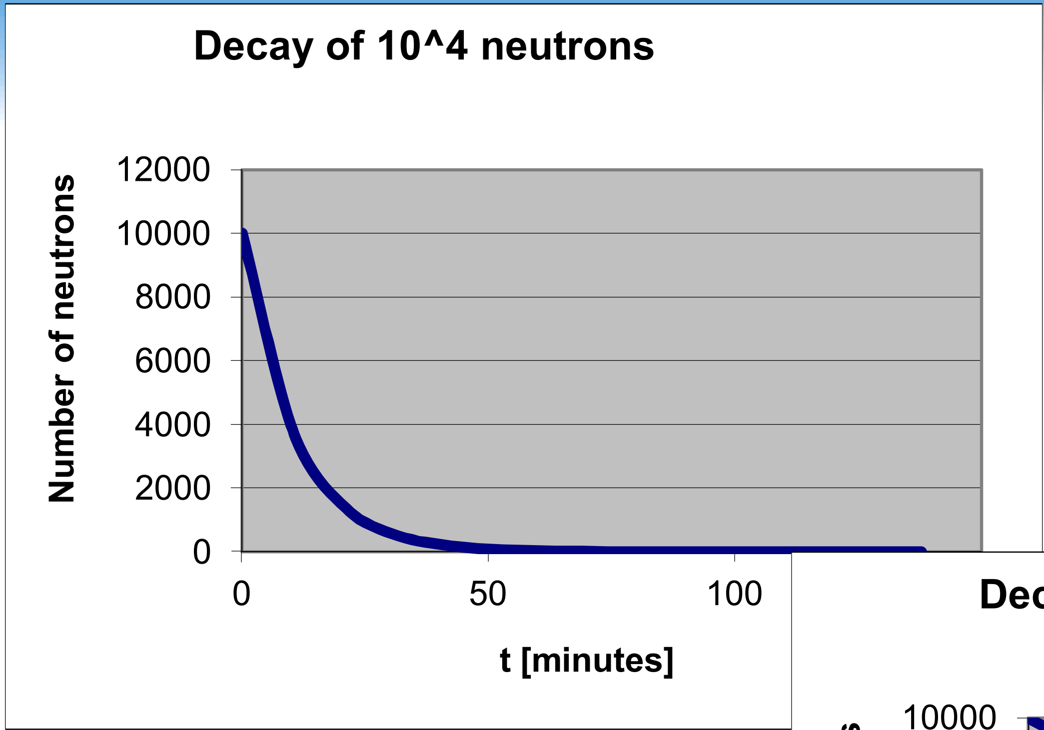
$m_n = 939.6$ MeV

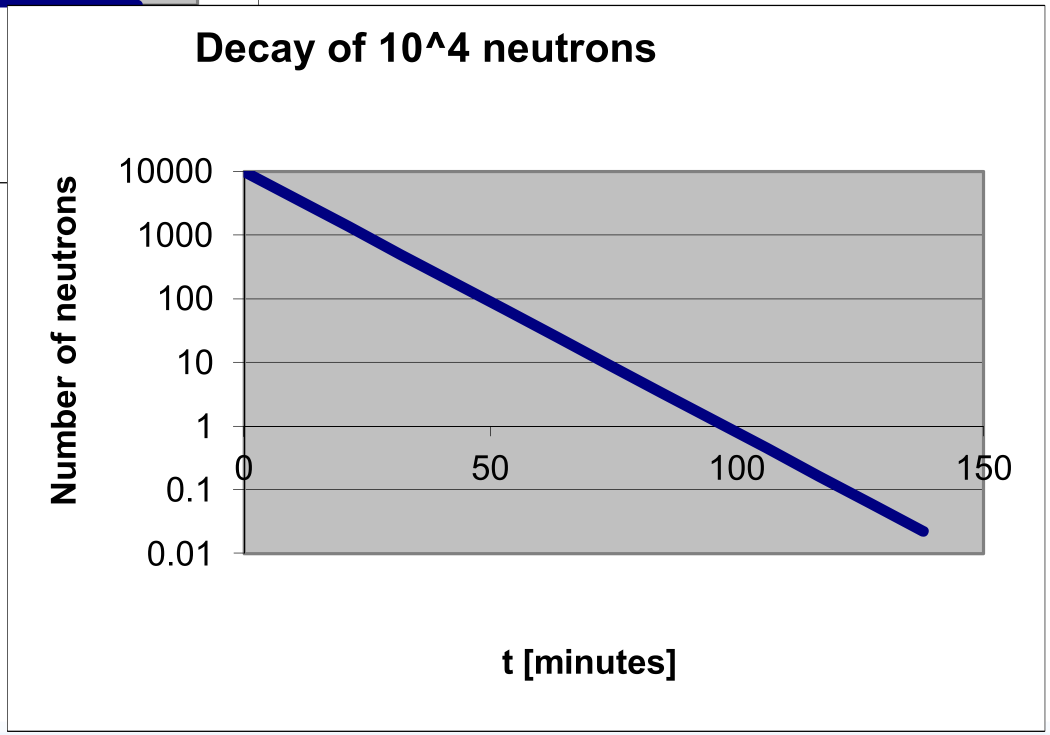
$p + e + \bar{\nu}$

0.79 MeV

$m_p + m_e = 938.3 + 0.511$ MeV

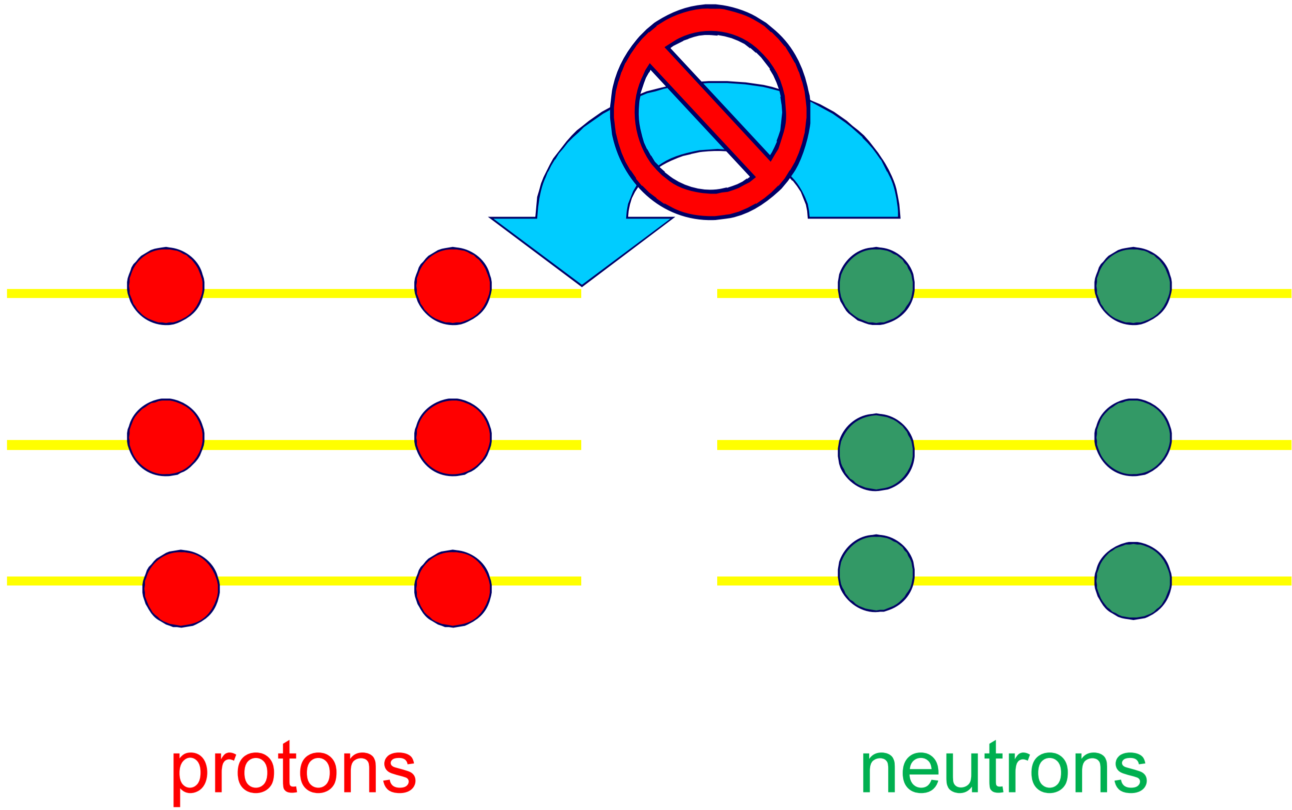
$T_{1/2} = 10.6$ minutes


Decay of 10^4 neutrons
Number of neutrons
12000
10000
8000
6000
4000
2000
0
0
50
100
t [minutes]


Decay of 10^4 neutrons
Number of neutrons
10000
1000
100
10
1
0.1
0.01
0
50
100
150
t [minutes]

Why don't all the neutrons inside a nucleus decay?

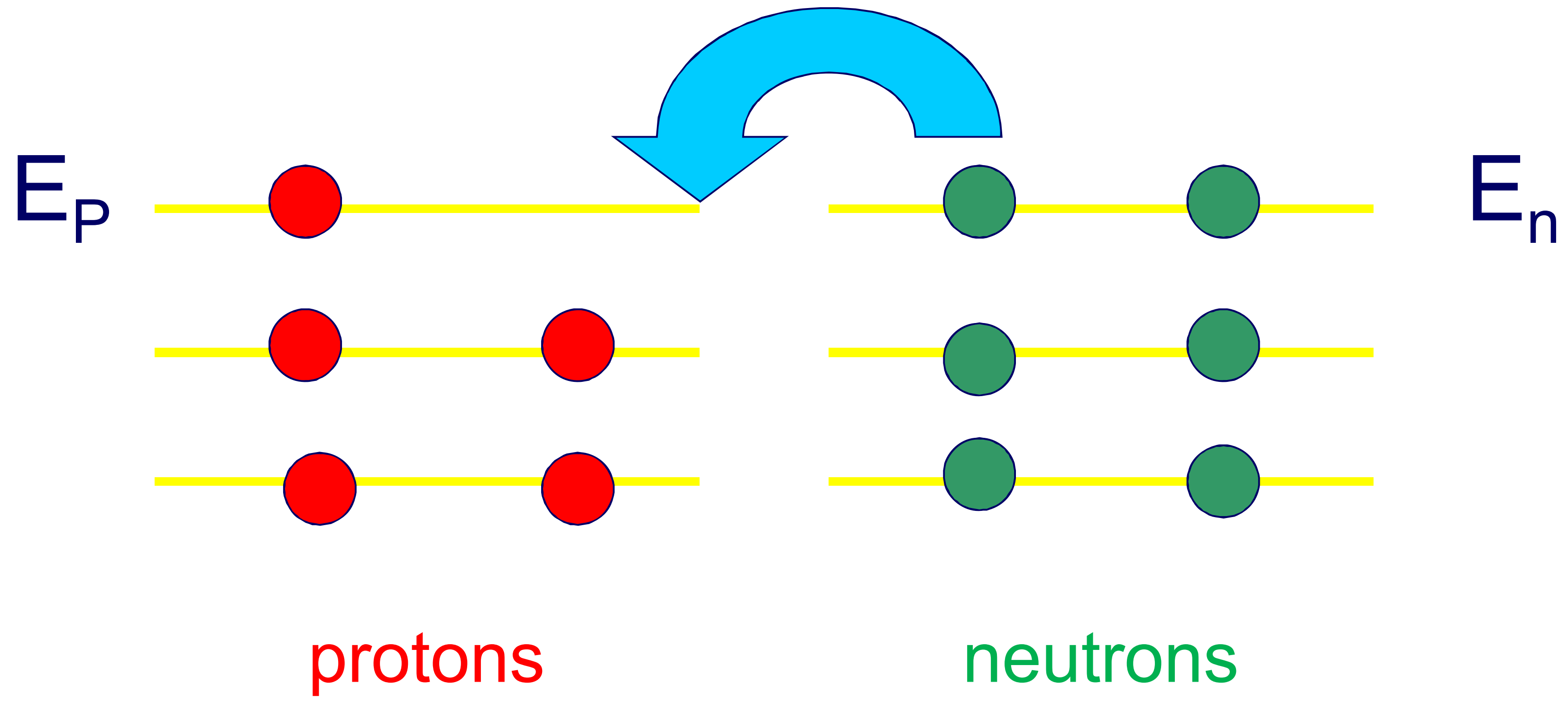
neutrons and protons are in specific quantized levels->



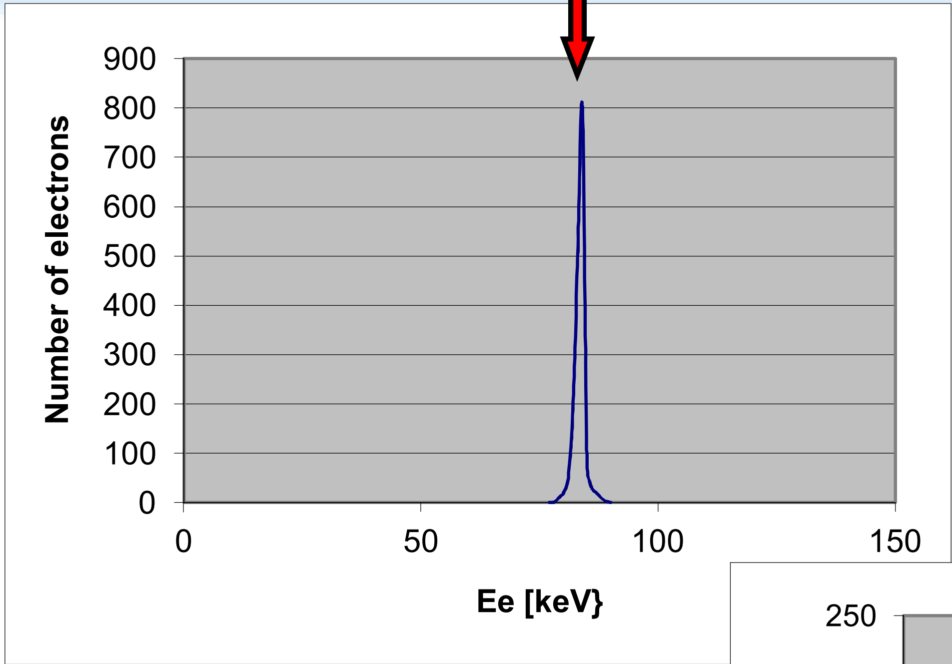
Pauli's exclusion principle

In some nuclei there is an empty proton orbit->

The energy or electron "should be": $E_e = E_n - E_p$



$E_e = E_n - E_p$

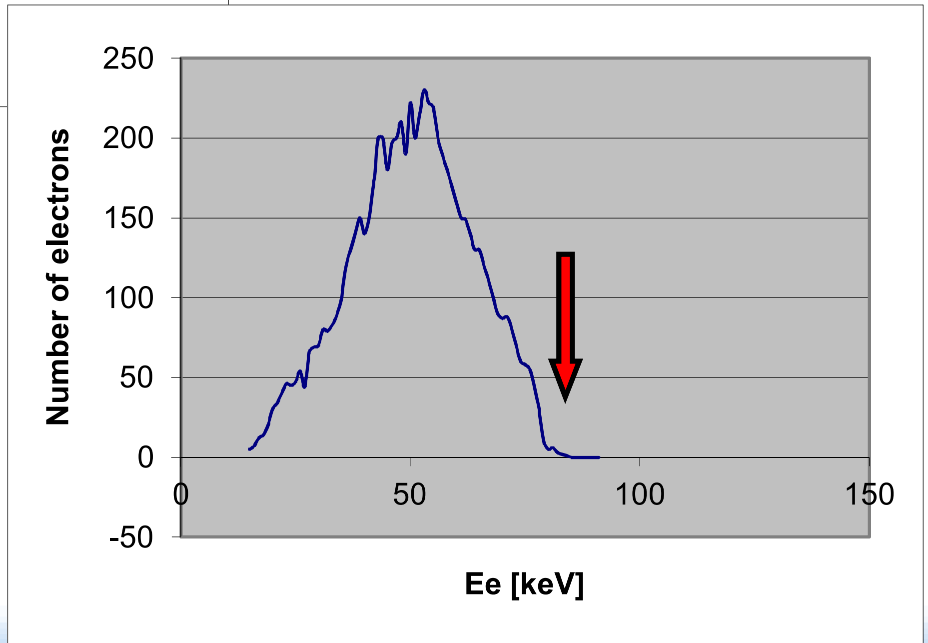


Expected

Shape predicted by Fermi's GR

$p^2 (Ee - E)^2 dp$

Observed:



E. Fermi (1934) there is a third particle (neutral) that goes undetected and carries part of the energy: neutrino ν (actually later was called antineutrino $\bar{\nu}$)

$$n \xrightarrow{\beta^-} P + e + \bar{\nu}$$

Also possible p decay, but only inside a nucleus

$$P \xrightarrow{\beta^+} n + e^+ + \nu$$

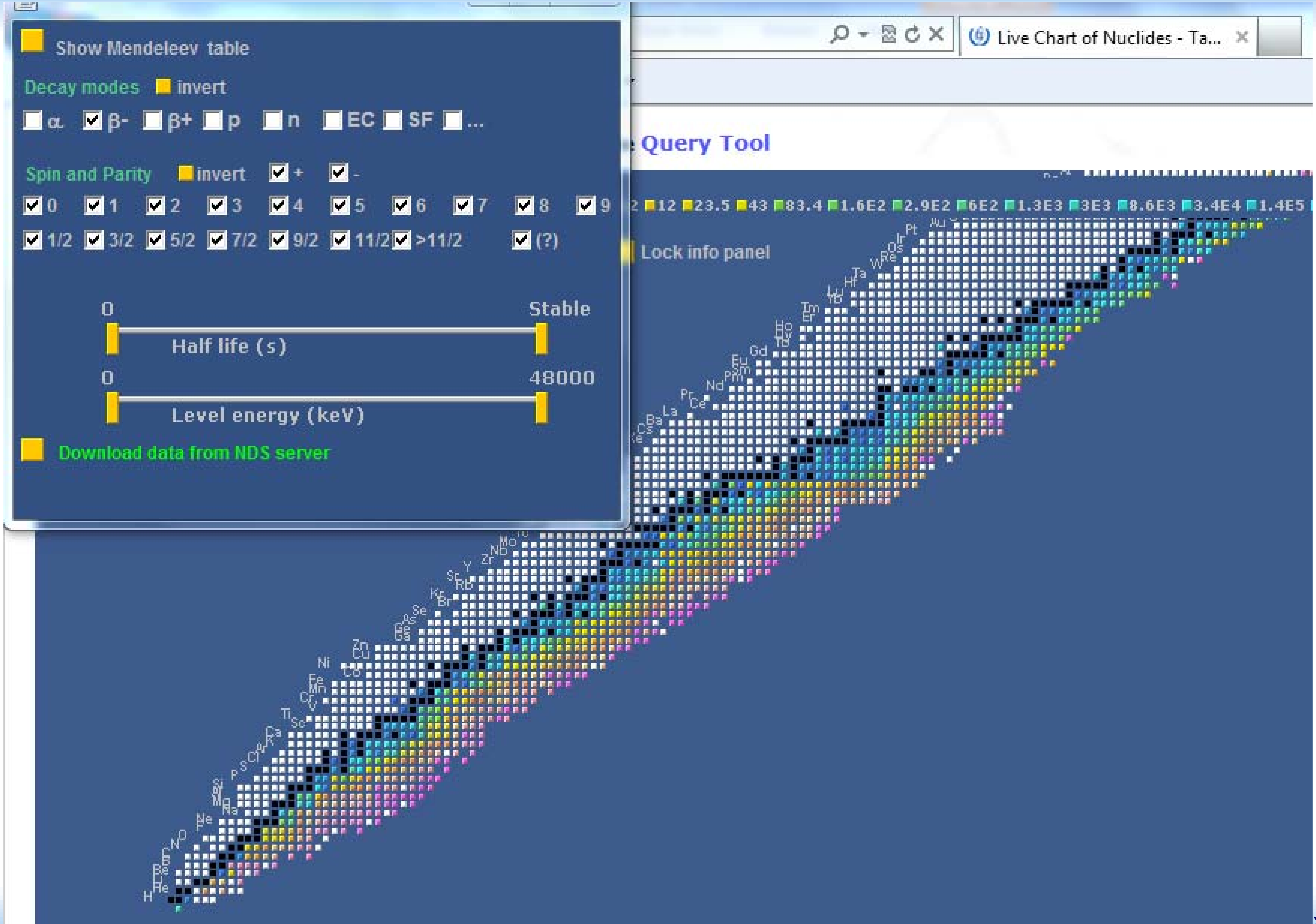
$${}^{A}_{Z}X_{N} \xrightarrow{\beta^-} {}^{A}_{Z+1}Y_{N-1} + e + \bar{\nu}$$

$${}^{14}_{6}C_{8} \xrightarrow{\beta^-} {}^{14}_{7}N_{7} + e + \bar{\nu}$$

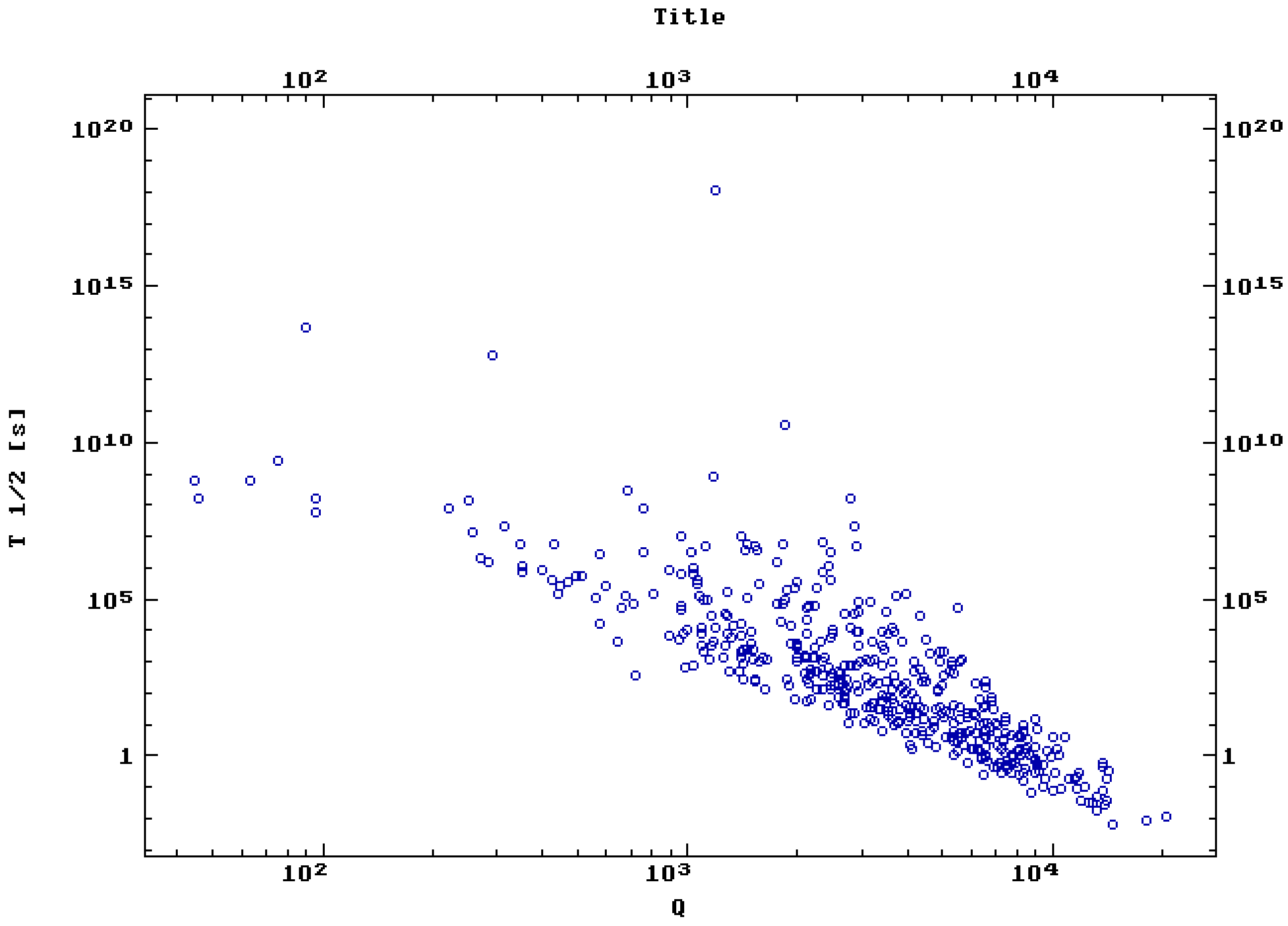
$$M({}^{14}_{6}C_{8})$$

0.157 MeV

$$M({}^{14}_{7}N_{7} + e + \bar{\nu})$$


Live Chart of Nuclides - Ta...
Query Tool
Show Mendeleev table
Decay modes
invert
α β- β+ p n EC SF ...
Spin and Parity
invert
+ -
0 1 2 3 4 5 6 7 8 9
1/2 3/2 5/2 7/2 9/2 11/2 >11/2 (?)
Lock info panel
0
Stable
Half life (s)
0
48000
Level energy (keV)
Download data from NDS server
12 23.5 43 83.4 1.6E2 2.9E2 6E2 1.3E3 3E3 8.6E3 3.4E4 1.4E5

All Beta decays I>80%



Also there is BETA + decay:

$$ {}^{A}_{Z}X_{N} \xrightarrow{\beta^{+}} {}^{A}_{Z-1}Y_{N+1} + e^{+} + \nu $$

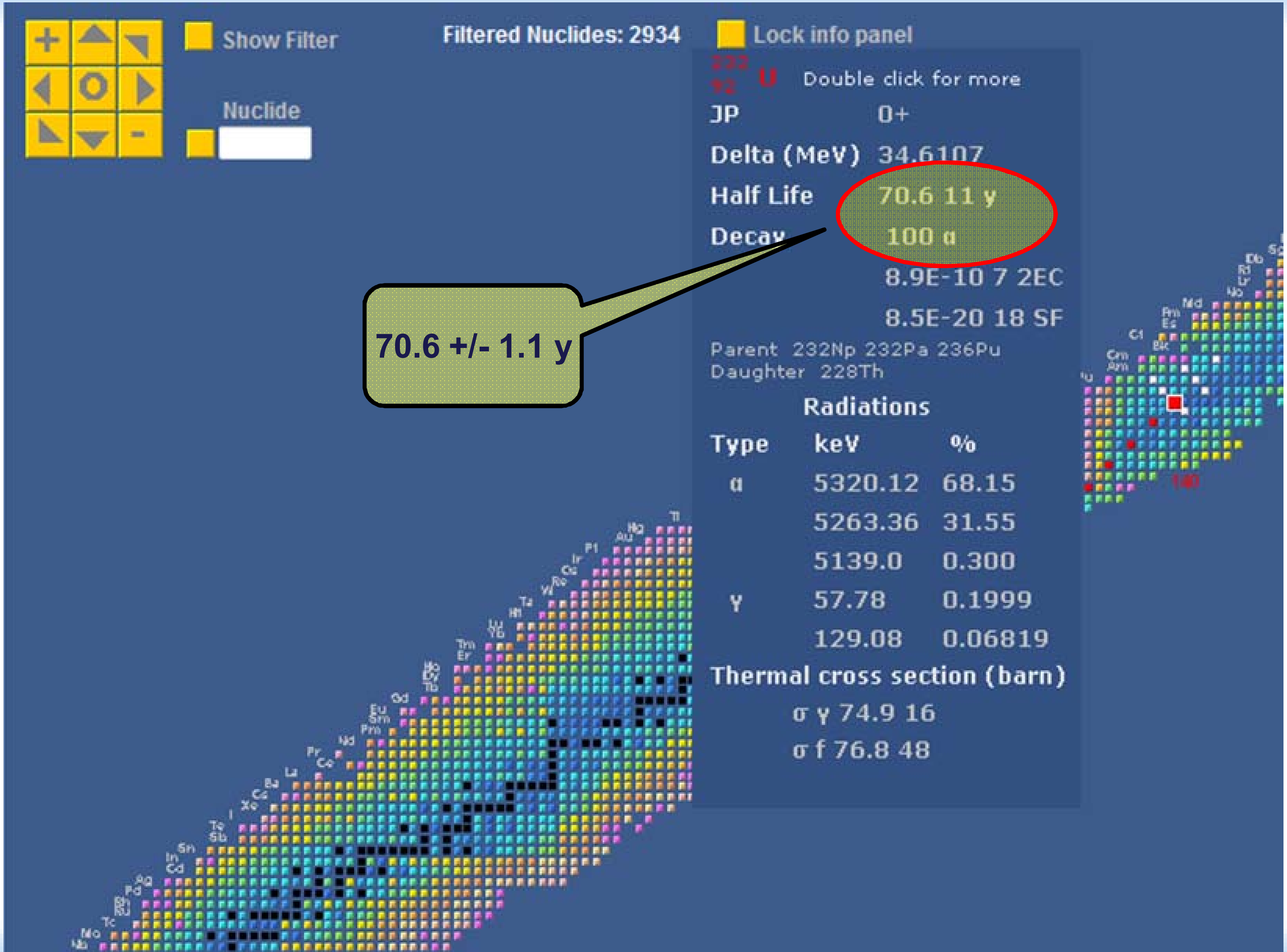
And alpha decay:

$$ {}_{Z}X_{N} \xrightarrow{\alpha} {}^{A-4}_{Z-2}Y_{N+2} + \alpha $$

M(?)

M(?) + m(α)

Example: 232U



$$^{232}_{92}U_{140} \xrightarrow{\alpha} {}^{228}_{90}Th_{138} + \alpha$$

Q-Value

5.41 MeV

T=70.6 y → $\lambda = 0.693/T$

$$A_0(232g) = \lambda N_0 = 3.14\ 10^{-10}\ 1/s\ \ 6.02\ 10^{23}$$

$$A_0 = 1.9\ 10^{14}\ \ Bq$$

$A_0 = 1.9\ 10^{14}$ Bq

232 g of 232U emit $1.9\ 10^{14}$ alpha particles 5.3 MeV each per second

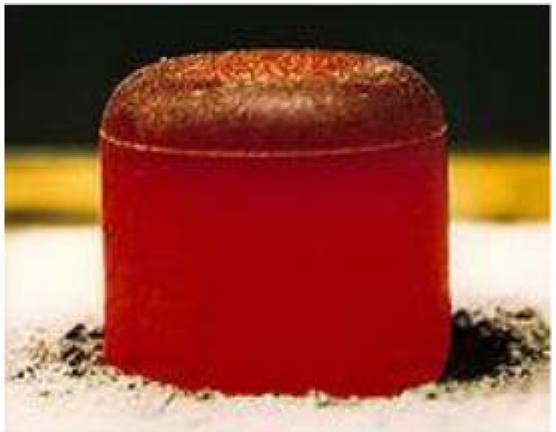
1 MeV = $1.6\ 10^{-13}$ Joules

Then the energy liberated is

= 161.1 Joules

Since a Joule per second is a Watt the total power released is 161.1 Watts

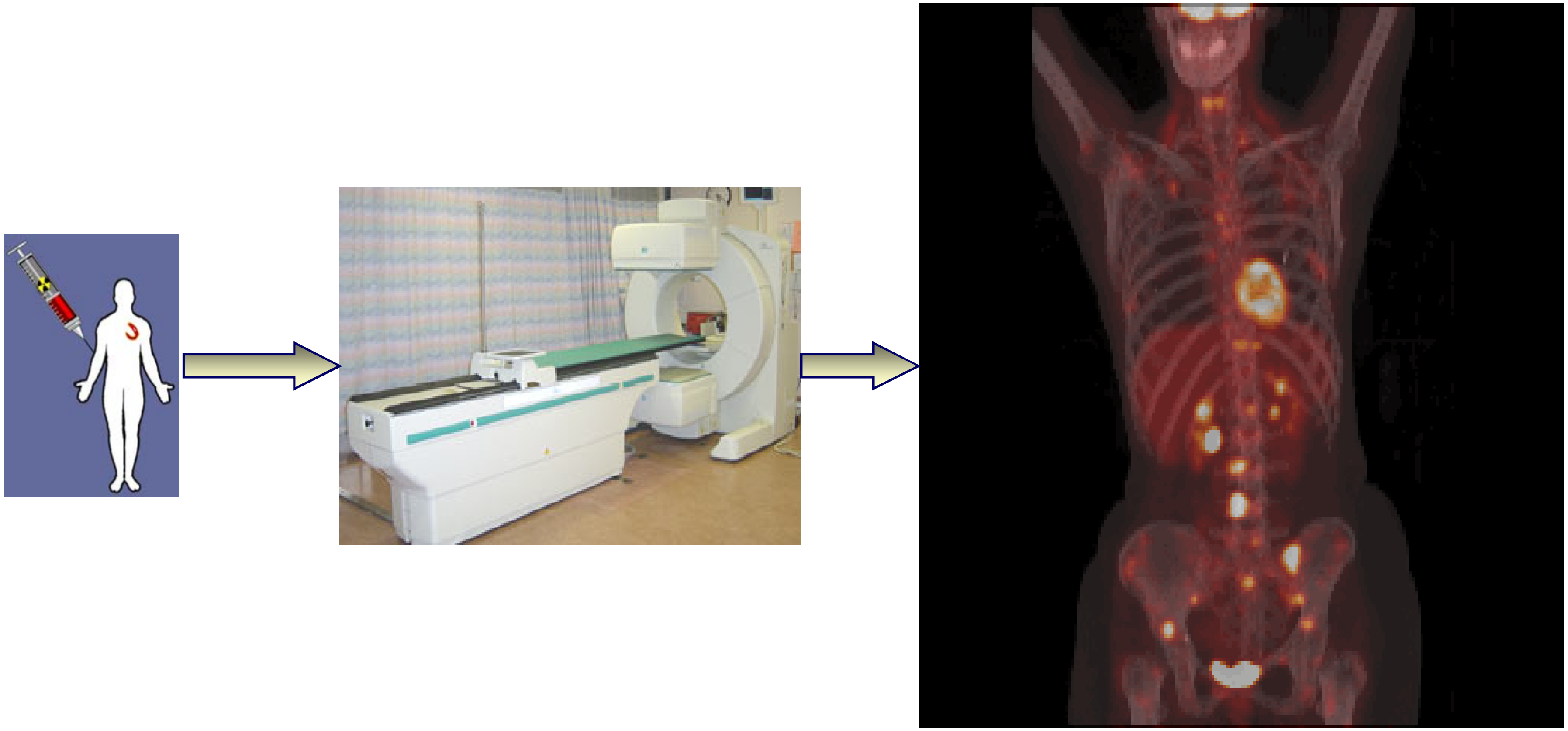
238Pu glowing red hot



Used in Radioisotope thermoelectric generators (RTG)

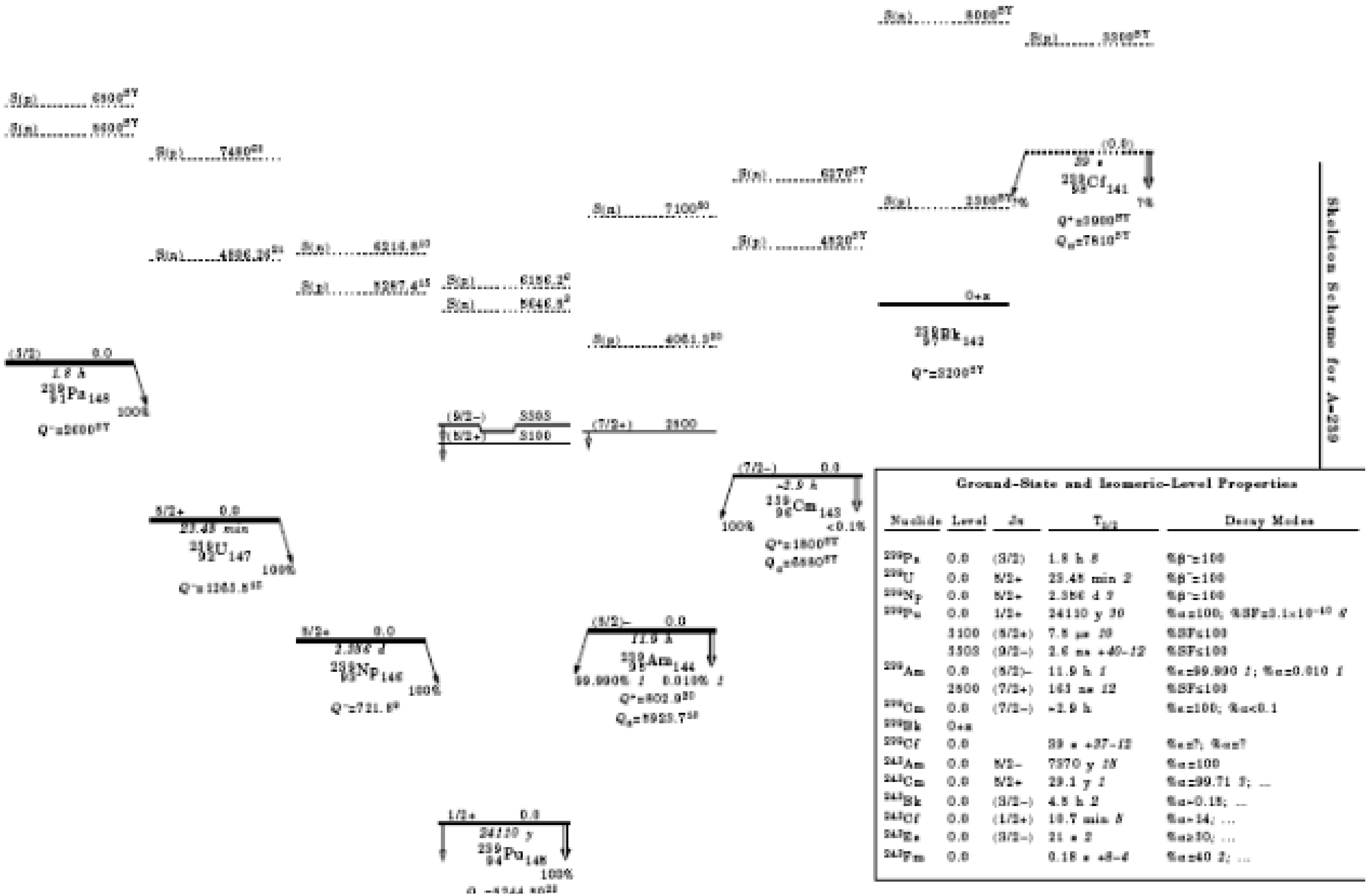
Used as in the MARS explorer Curiosity (August 2012)

Medical - diagnosis



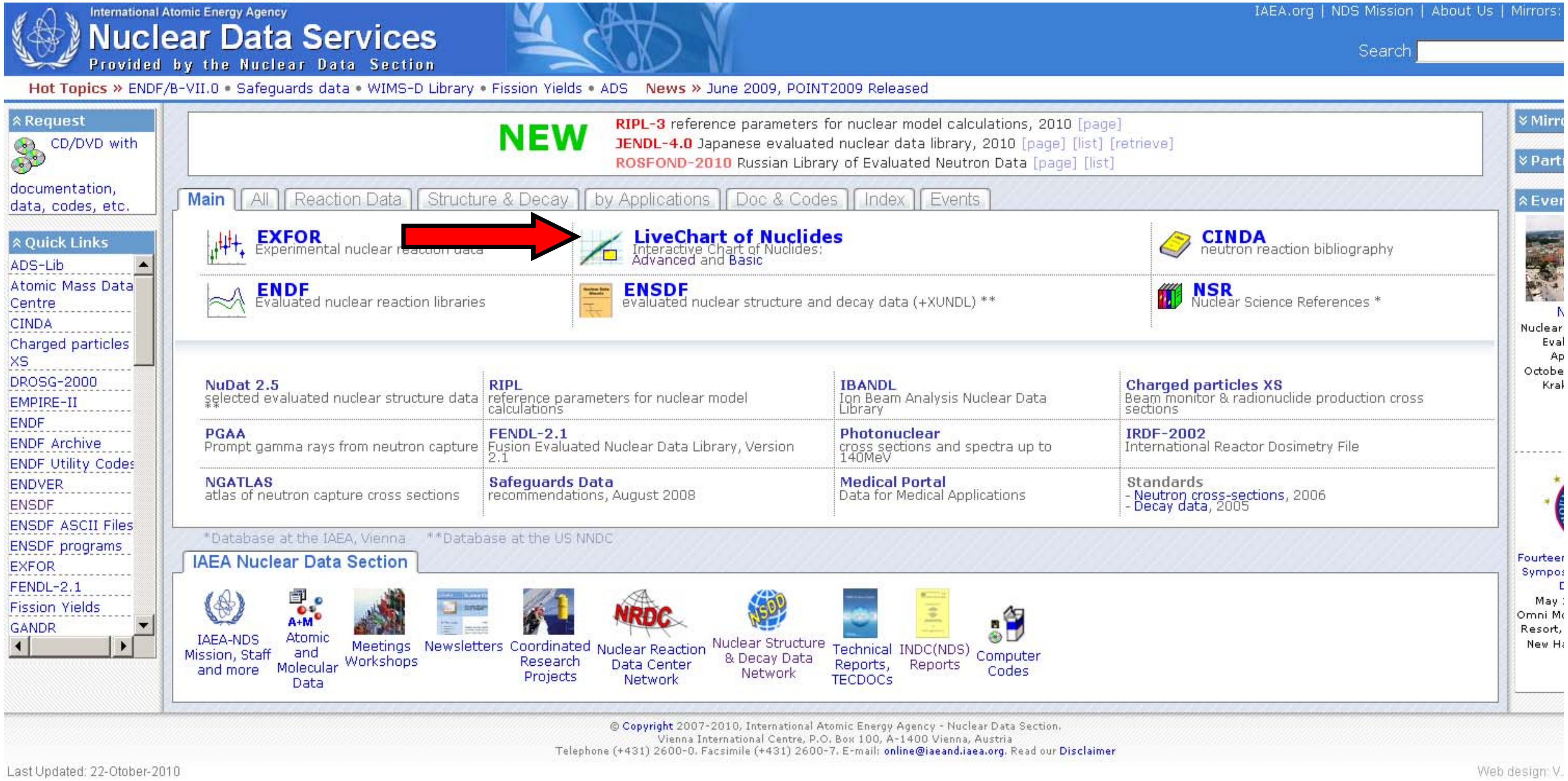
Medical - therapy

ENSDF

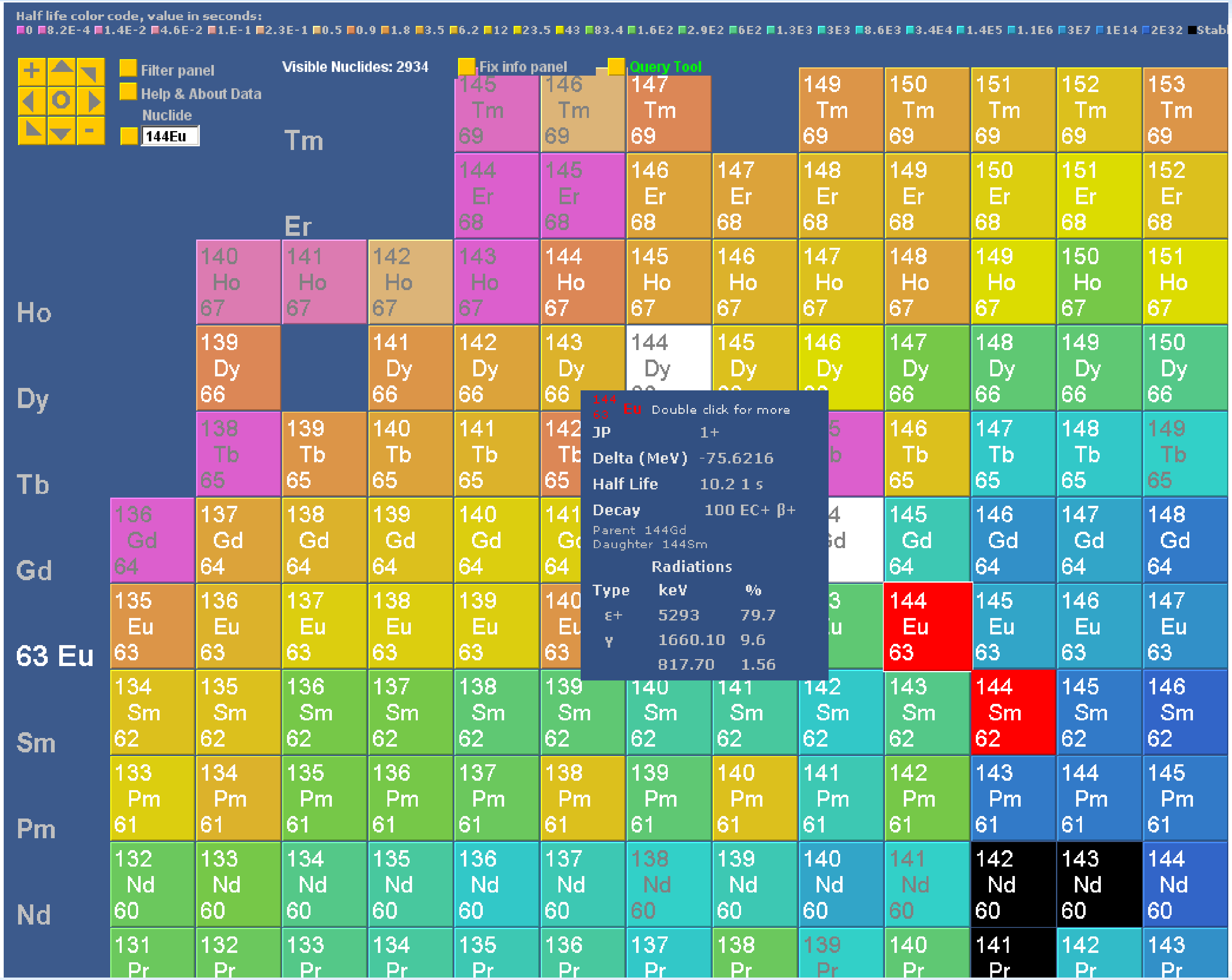


Nuclide	Level	Jπ	$T_{1/2}$	Decay Modes
^{239}Pa	0.0	(3/2)	1.8 h 5	%β^-=100
^{239}U	0.0	5/2+	23.45 min 2	%β^-=100
^{239}Np	0.0	5/2+	2.356 d 3	%β^-=100
^{239}Pu	0.0	1/2+	24110 y 30	%α=100; %SF=3.1×10^{-10} 6
	3100	(5/2+)	7.5 μs 10	%SF≤100
	3500	(9/2−)	2.6 ns +40−12	%SF≤100
^{239}Am	0.0	(5/2)−	11.9 h 1	%ε=99.990 1; %α=0.010 1
	2500	(7/2+)	163 ns 12	%SF≤100
^{239}Cm	0.0	(7/2−)	≈2.9 h	%ε=100; %α<0.1
^{239}Bk	0+x			
^{239}Cf	0.0		39 s +37−12	%α?; %ε=?
^{243}Am	0.0	5/2−	7370 y 15	%α=100
^{243}Cm	0.0	5/2+	29.1 y 1	%α=99.71 3; ...
^{243}Bk	0.0	(3/2−)	4.5 h 2	%α≈0.15; ...
^{243}Cf	0.0	(1/2+)	10.7 min 5	%α≈14; ...
^{243}Es	0.0	(3/2−)	21 s 2	%α≥30; ...
^{243}Fm	0.0		0.18 s +8−4	%α≈40 2; ...

NDS (IAEA) Web page



LiveChart


Half life color code, value in seconds:
0 8.2E-4 1.4E-2 4.6E-2 1.E-1 2.3E-1 0.5 0.9 1.8 3.5 6.2 12 23.5 43 83.4 1.6E2 2.9E2 6E2 1.3E3 3E3 8.6E3 3.4E4 1.4E5 1.1E6 3E7 1E14 2E32 Stabl
Filter panel
Help & About Data
Nuclide
144Eu
Visible Nuclides: 2934
Fix info panel
Query Tool
Tm
Er
Ho
Dy
Tb
Gd
63 Eu
Sm
Pm
Nd
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148 Gd 64
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144 Nd 60
131 Pr
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134 Pr
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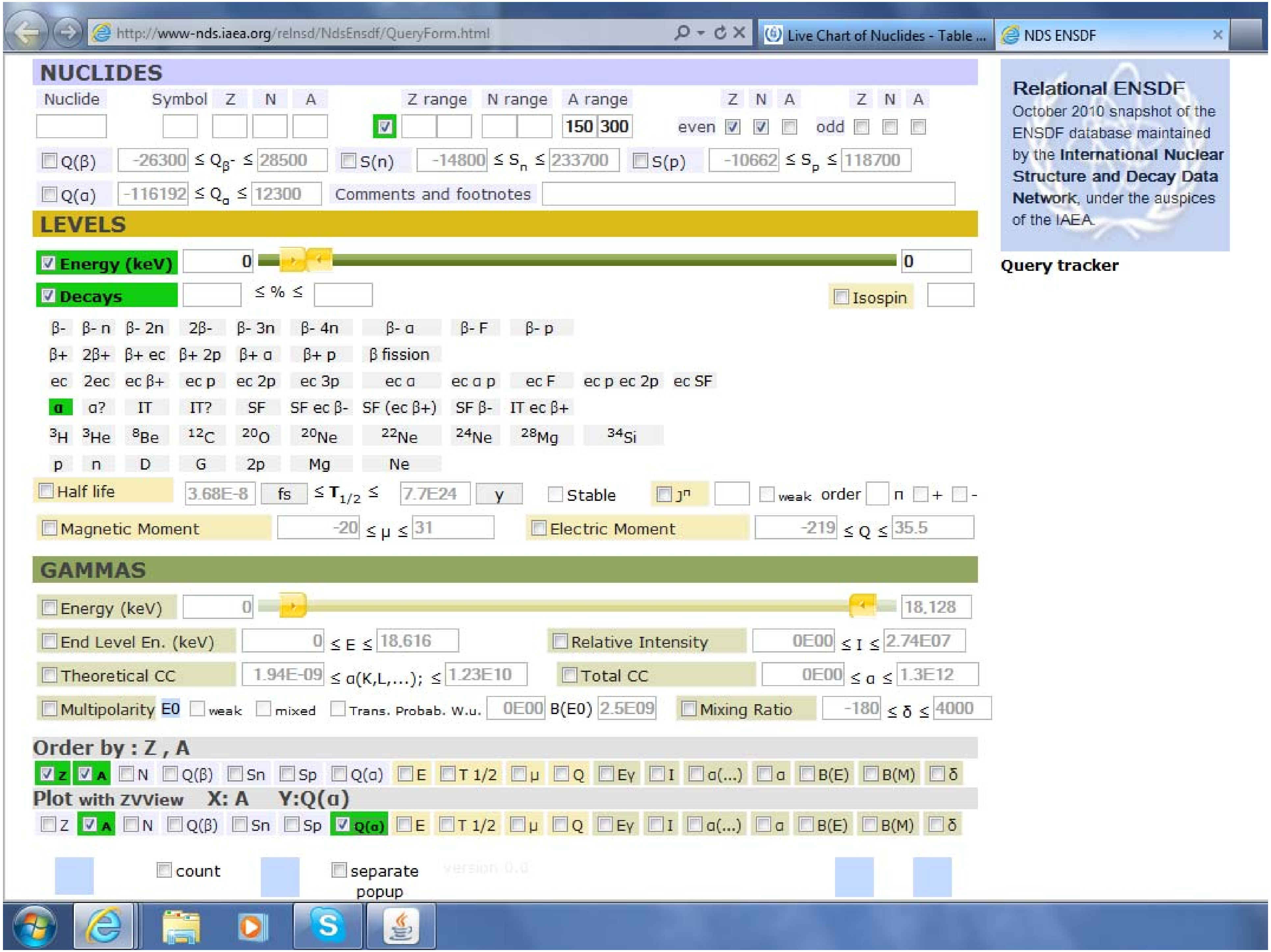
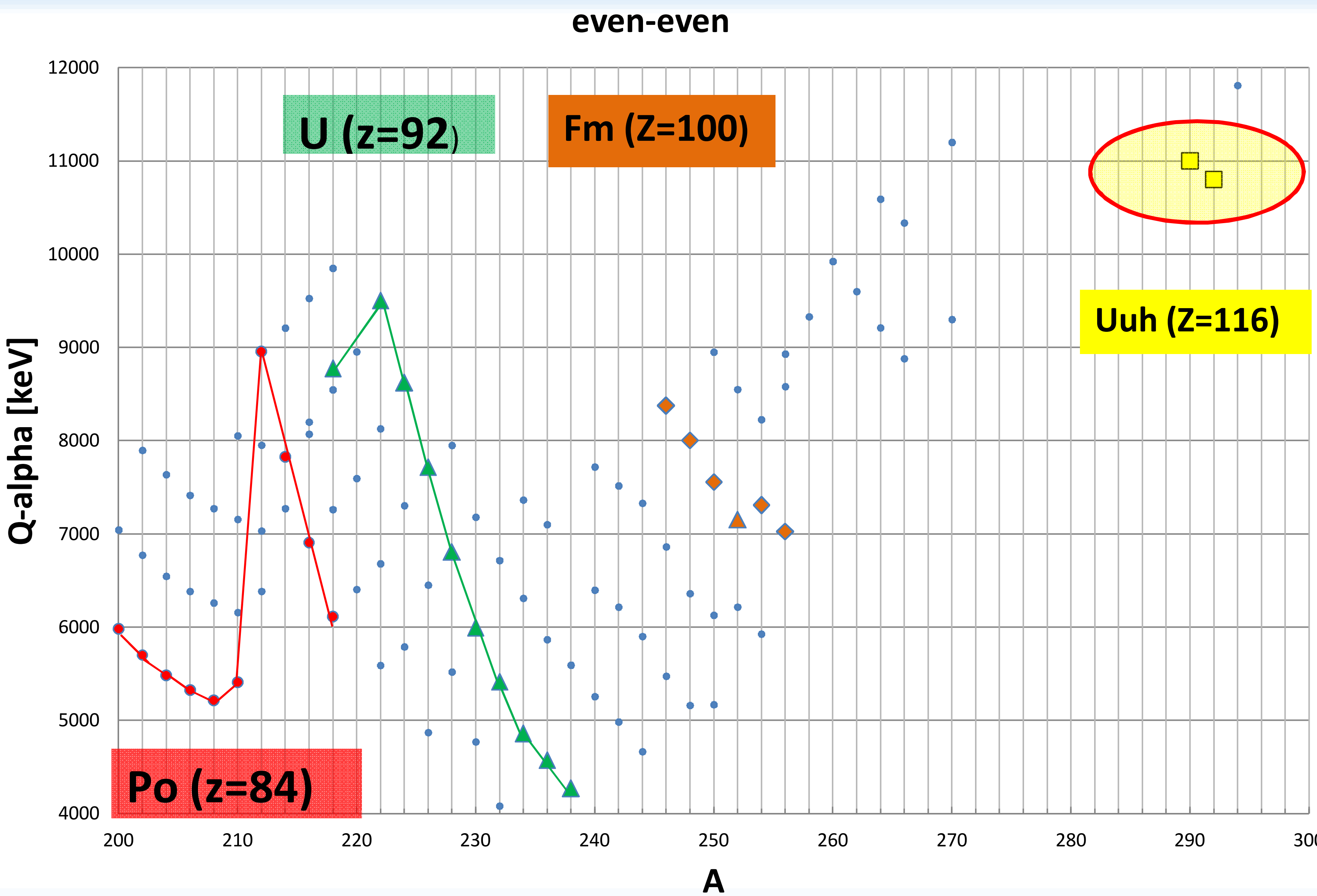
136 Pr
137 Pr
138 Pr
139 Pr
140 Pr
141 Pr
142 Pr
143 Pr
144 63 Eu Double click for more
JP 1+
Delta (MeV) -75.6216
Half Life 10.2 1 s
Decay 100 EC+ β+
Parent 144Gd
Daughter 144Sm
Radiations
Type keV %
ε+ 5293 79.7
γ 1660.10 9.6
817.70 1.56

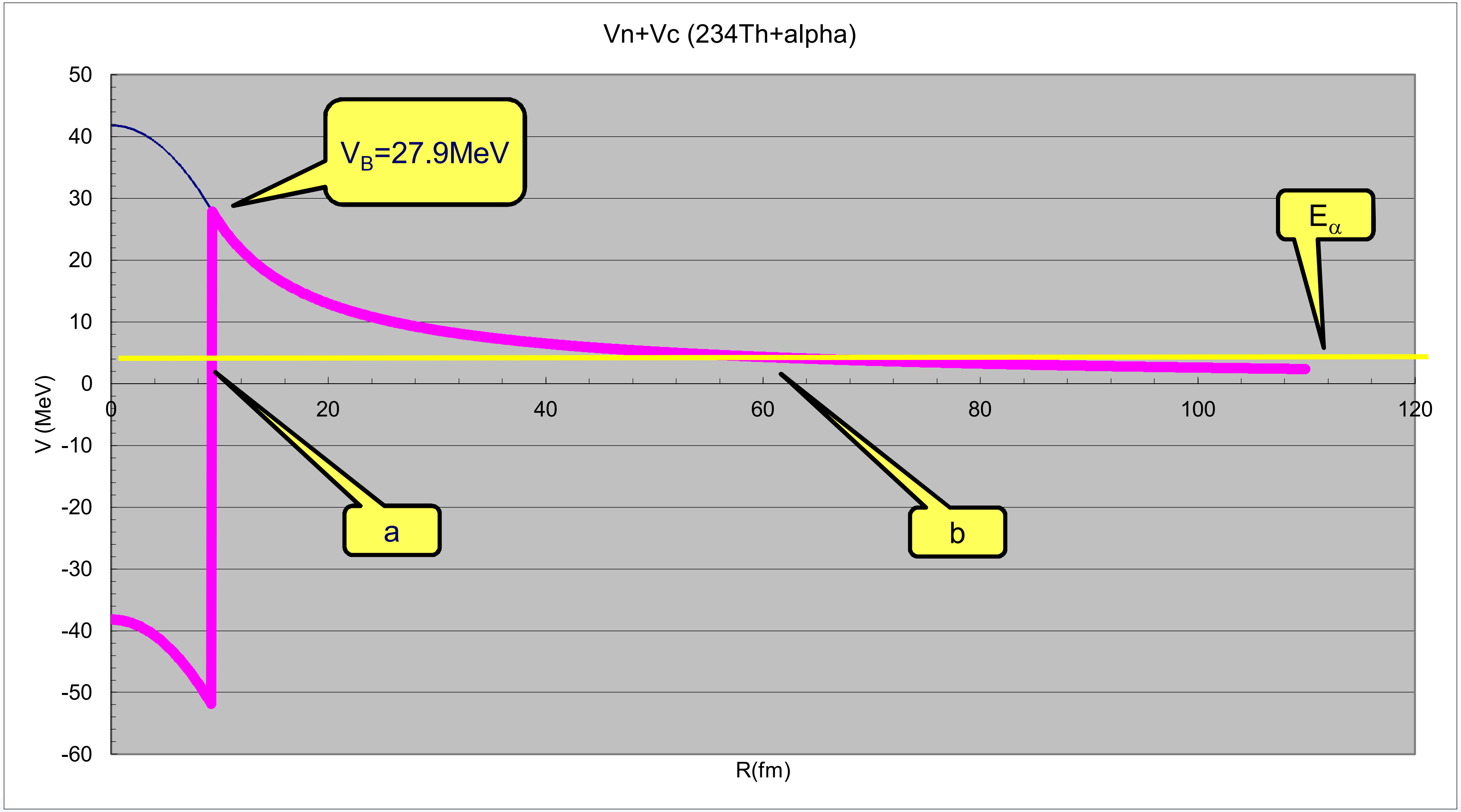

http://www-nds.iaea.org/relnsd/NdsEnsdf/QueryForm.html
Live Chart of Nuclides - Table ...
NDS ENSDF
NUCLIDES
Nuclide Symbol Z N A
Z range N range A range
150 300
Z N A even
Z N A odd
Q(β) -26300 ≤ Qβ- ≤ 28500
S(n) -14800 ≤ Sn ≤ 233700
S(p) -10662 ≤ Sp ≤ 118700
Q(α) -116192 ≤ Qα ≤ 12300
Comments and footnotes
LEVELS
Energy (keV) 0 0
Decays ≤ % ≤
Isospin
β- β- n β- 2n 2β- β- 3n β- 4n β- α β- F β- p
β+ 2β+ β+ ec β+ 2p β+ α β+ p β fission
ec 2ec ec β+ ec p ec 2p ec 3p ec α ec α p ec F ec p ec 2p ec SF
α α? IT IT? SF SF ec β- SF (ec β+) SF β- IT ec β+
3H 3He 8Be 12C 20O 20Ne 22Ne 24Ne 28Mg 34Si
p n D G 2p Mg Ne
Half life 3.68E-8 fs ≤ T1/2 ≤ 7.7E24 y
Stable
Jπ weak order π + -
Magnetic Moment -20 ≤ μ ≤ 31
Electric Moment -219 ≤ Q ≤ 35.5
GAMMAS
Energy (keV) 0 18,128
End Level En. (keV) 0 ≤ E ≤ 18,616
Relative Intensity 0E00 ≤ I ≤ 2.74E07
Theoretical CC 1.94E-09 ≤ α(K,L,...); ≤ 1.23E10
Total CC 0E00 ≤ α ≤ 1.3E12
Multipolarity E0 weak mixed Trans. Probab. W.u. 0E00 B(E0) 2.5E09
Mixing Ratio -180 ≤ δ ≤ 4000
Order by : Z , A
Z A N Q(β) Sn Sp Q(α) E T 1/2 μ Q Eγ I α(...) α B(E) B(M) δ
Plot with ZVView X: A Y:Q(α)
Z A N Q(β) Sn Sp Q(α) E T 1/2 μ Q Eγ I α(...) α B(E) B(M) δ
count
separate popup
Relational ENSDF
October 2010 snapshot of the ENSDF database maintained by the International Nuclear Structure and Decay Data Network, under the auspices of the IAEA.
Query tracker


even-even
Q-alpha [keV]
A
U (z=92)
Fm (Z=100)
Uuh (Z=116)
Po (z=84)
12000
11000
10000
9000
8000
7000
6000
5000
4000
200
210
220
230
240
250
260
270
280
290
300

Uuh Livermorium (Z=116)

```
29016    ADOPTED LEVELS: TENTATIVE                              05NDS    200512
29016  H TYP=FUL$AUT= M. GUPTA, THOMAS W. BURROWS$CIT=NDS 106, 251 (2005)$
290162 H CUT=12-Aug-2005$
29016  Q             8330   SY 1820   SY 11000     80 2003AU03,2004OG12
29016 cQ SN$estimated uncertainty=1380 keV
29016 cQ SP$estimated uncertainty=1330 keV
29016 cQ QA$from E|a=10.85 MeV {I8} (2004Og12,2004OgZZ). Other: 11.30 MeV {I35}
290162cQ (2003Au03. Syst.)
29016 c  2002OgZX, 2003OgZZ: by the complete fusion reaction
290162c  {+249}Cf({+48}Ca,3n) at an energy of 265 MeV. {+290}116 is the expected
290163c  daughter produced by the |a-decay of Z=118 in this reaction. See
290164c  the {+294}118 Adopted Levels for details of experimental apparatus.
290165c  The optimal cross-section and the highest yield of evr's is expected
290164c  for the above channel by theory (2002Za19,2002Za16,2002Za01) and
290165c  systematic extrapolations from the radioactive properties of
290166c  neighboring even-even nuclei such as {+292}116, {+288}114, {+284}112
290167c  and {+280}Ds created with {+244}Pu and {+248}Cm targets. The beam of
290168c  {+48}Ca ions was provided by the JINR U400 cyclotron and EVR's were
290169c  separated by DGFRS. A fission fragment calibration was performed using
29016Ac  SF fragments from {+252}No with a known average energy release of about
29016Bc  176 MeV.
```

Barrier Penetration
(l=0)



$$Q_\alpha = (M_{parent} - M_{daughter} - M_{alpha})c^2$$

$$T_\alpha = E_\alpha = Q_\alpha \frac{M_{daughter}}{M_{parent}}$$

$$T_{recoil} = Q_\alpha \frac{M_{alpha}}{M_{parent}}$$

$$\lambda = fT$$

Transmission factor

$$f = \frac{v}{2R}$$

Collisions per second

$$E_\alpha = 1/2 M_\alpha v^2 \approx \frac{931.5 MeV}{2c^2} v^2$$

$$T \approx e^{-2G}$$

Gamow factor

$$G = \frac{zZe^2k}{\hbar v} f(x)$$

$$e^2k = 1.44 MeVfm$$

$$G = \frac{zZe^2k}{\hbar c \frac{v}{c}} f(x)$$

$X=E_\alpha/V_B$

For Parent 238U Daughter 234Th

$R=1.2(A_\alpha^{1/3}+A_d^{1/3})=9.3$fm

b=60.7fm

$$\frac{v}{c} = \mathrm{sqrt}(2E_\alpha/M_\alpha)=0.048$$

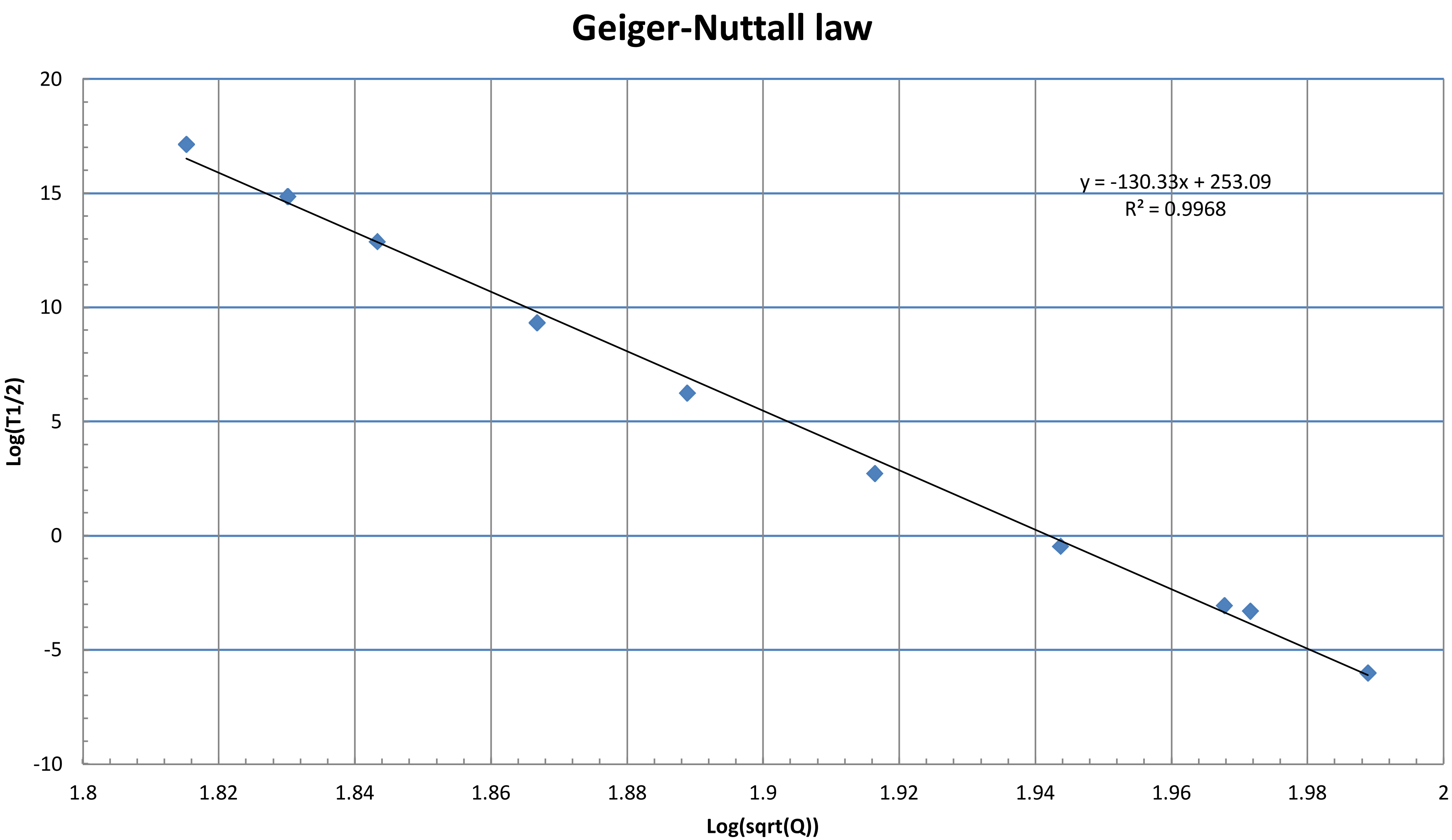
$$f = \frac{v}{2R} = 0.77\ 10^{21}\ \ 1/s$$

$$G = \frac{zZe^2k}{\hbar c \frac{v}{c}} f(x) \qquad f(x) = \cos^{-1}\sqrt{x} - \sqrt{x(1-x)}.......(x = E_\alpha / V_B)$$

$$G = \frac{2*90}{197*0.048} f(x) \approx 29.9 \qquad \lambda = f\ e^{-2G} = 2.9\ 10^{-39} s^{-1}$$

$$T1/2=0.693/\lambda = 3.1\ 10^{17} s$$

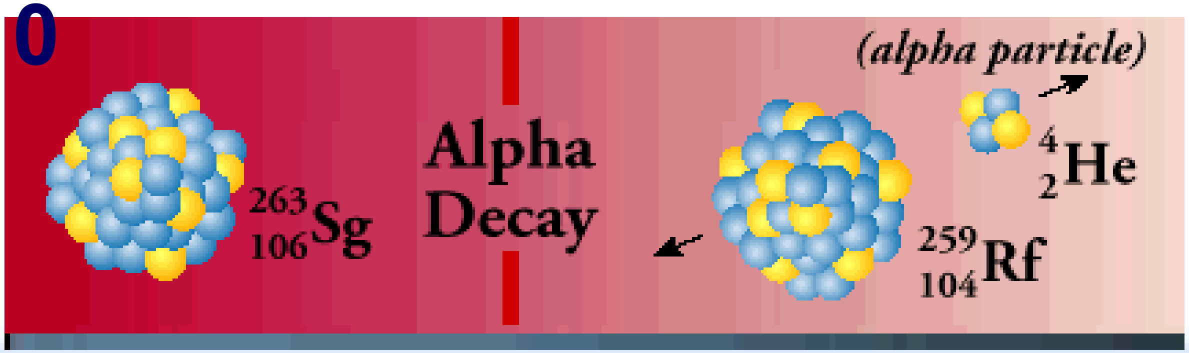
Geiger-Nuttall law



http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/alpdec.html

Alpha Decay

- All nuclei with mass numbers greater than A of 150 are thermodynamically unstable against alpha emission (Q_α is positive)
- Even so alpha emission is dominant decay process only for heaviest nuclei A≥210



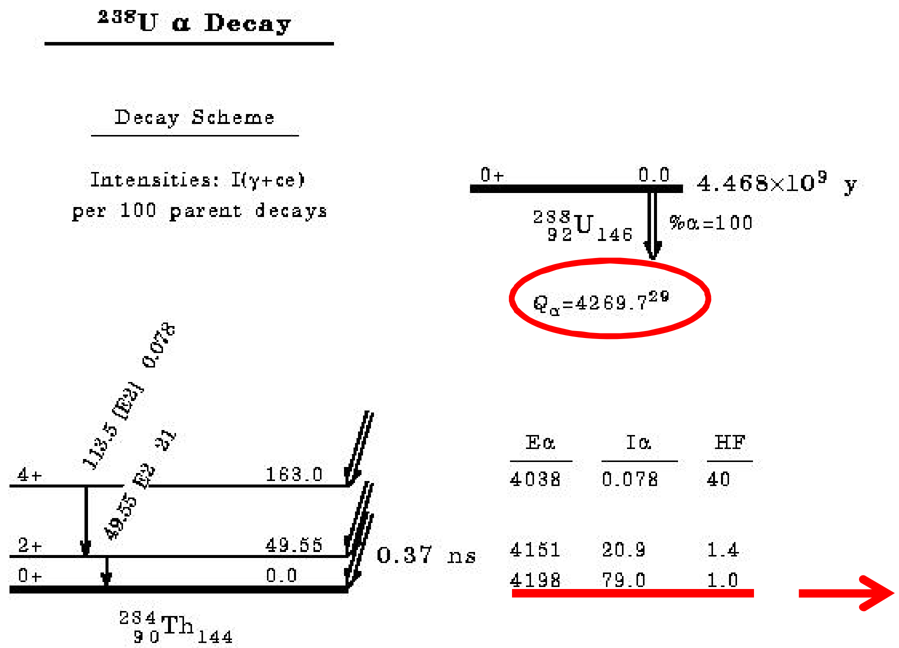
Energetics of alpha decay

- ^{238}U (mass excess Δ=+47.3070)→ ^{234}Th (mass excess Δ=+40.612 MeV) + α (Δ=+2.4249 MeV)

$$Q_a = 47.3070 - (40.612 + 2.4249) = 4.270 \text{ MeV}$$

Q_a is shared among α-particle and recoil daughter nucleus:

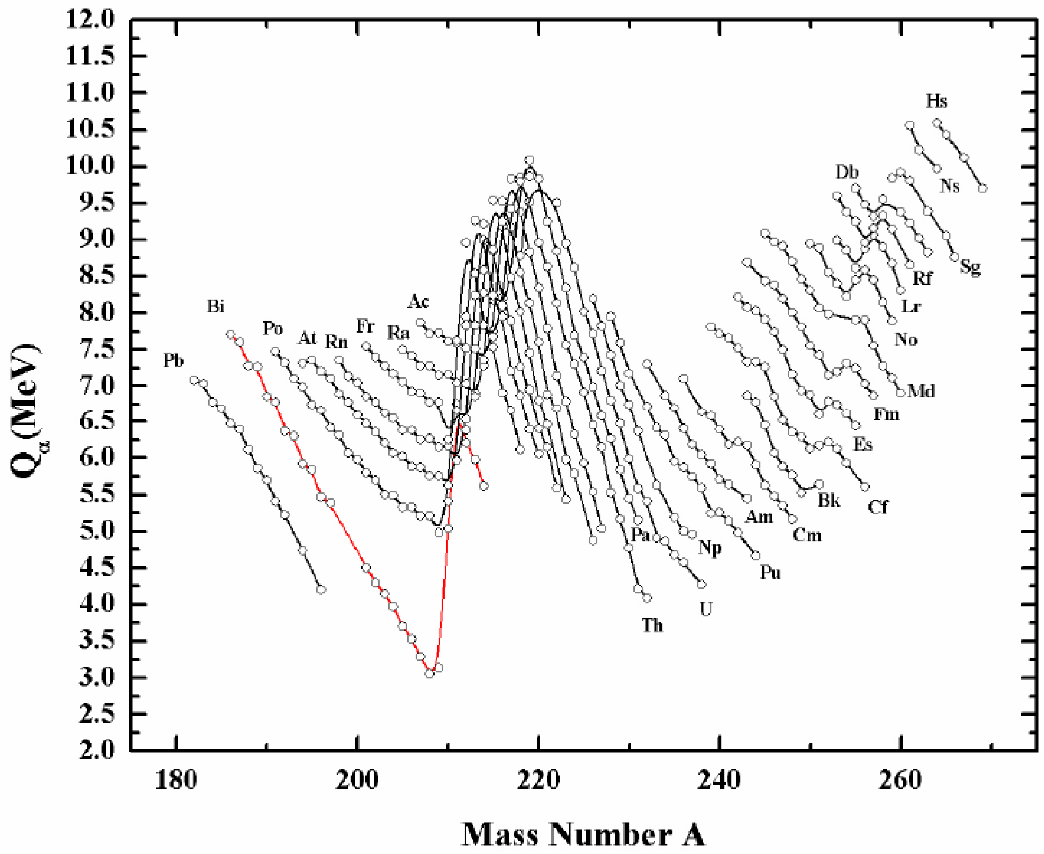
$$\alpha\text{-particle kinetic energy } T_a = \frac{234}{238} Q_a = 4.198 \text{ MeV}$$



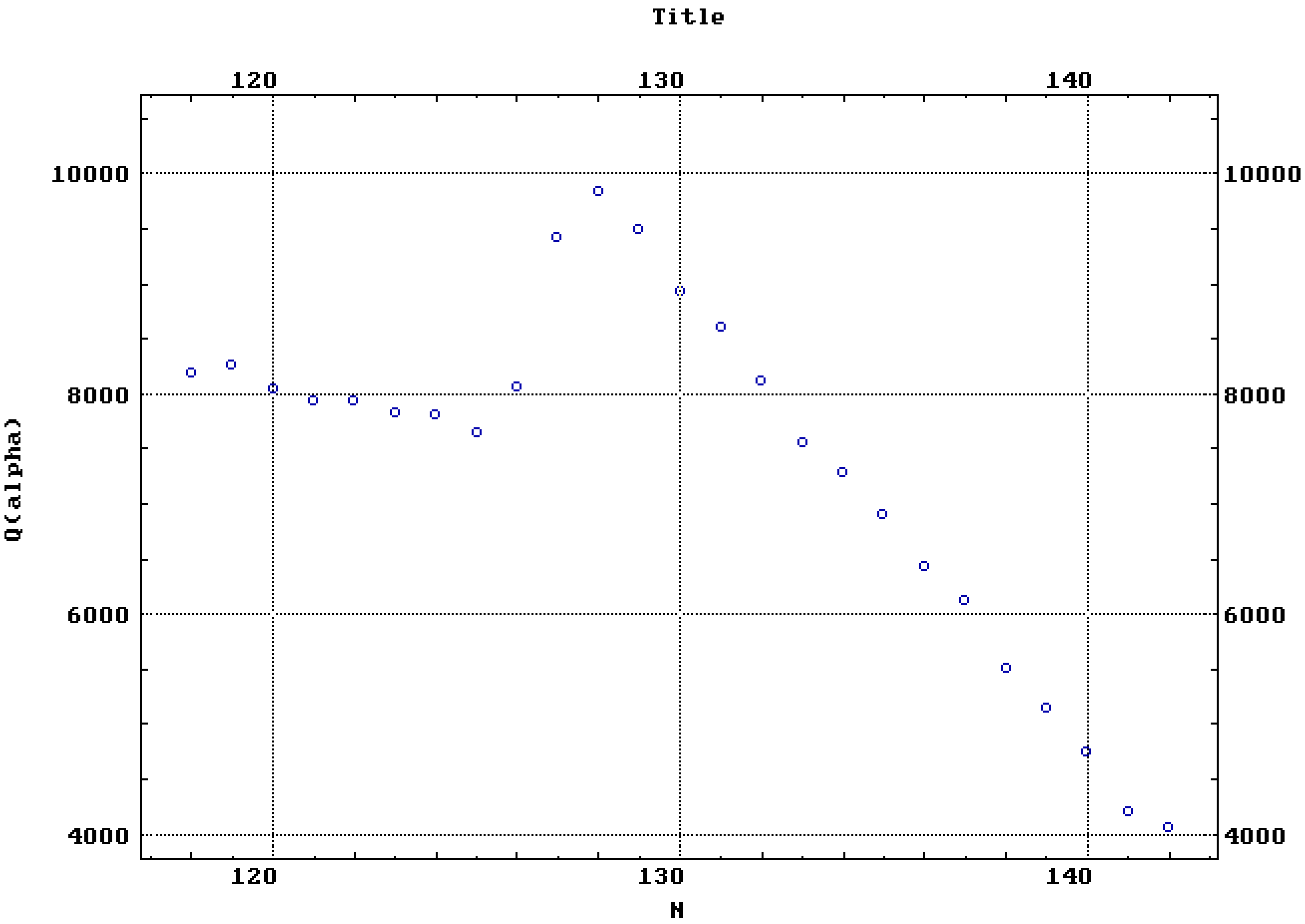
$0^+ \rightarrow 0^+$

$$ {}^{A}Z \rightarrow {}^{(A-4)}(Z-2) + {}^{4}He + Q_a $$

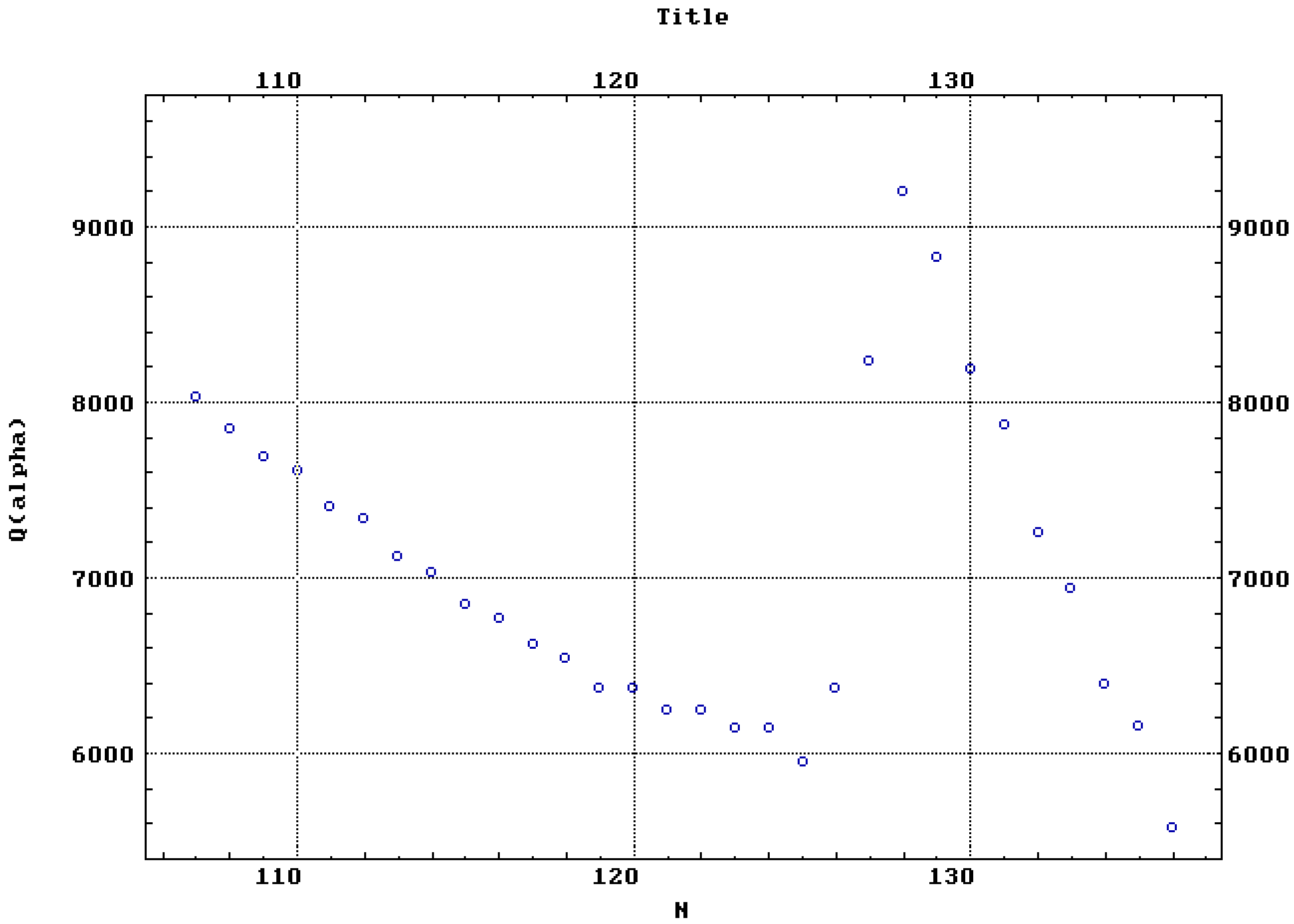
- **Q values generally increase wit A**
- **Variation due to shell effects**
- **Peaks near N=126 shell**
- **short-lived α-emitters near doubly magic ^{100}Sn**
  - **^{107}Te, ^{108}Te, ^{111}Xe**
- **alpha emitters have been identified along the proton drip line above A=100**



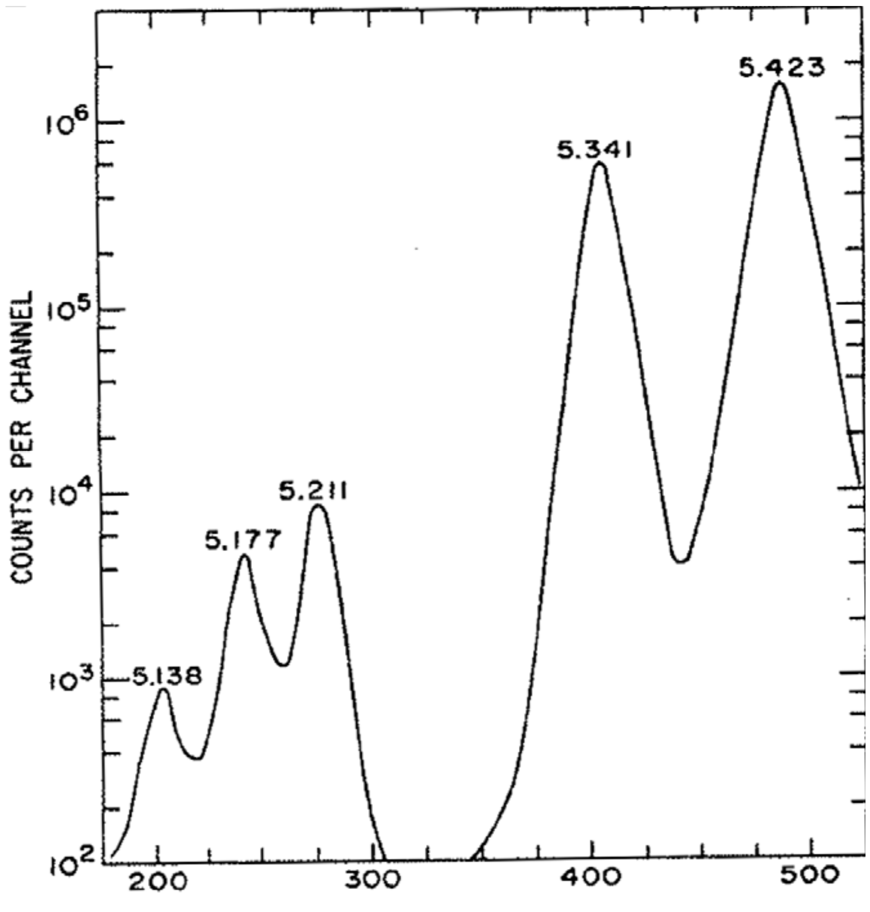
Qa near N=126 (Z=90)

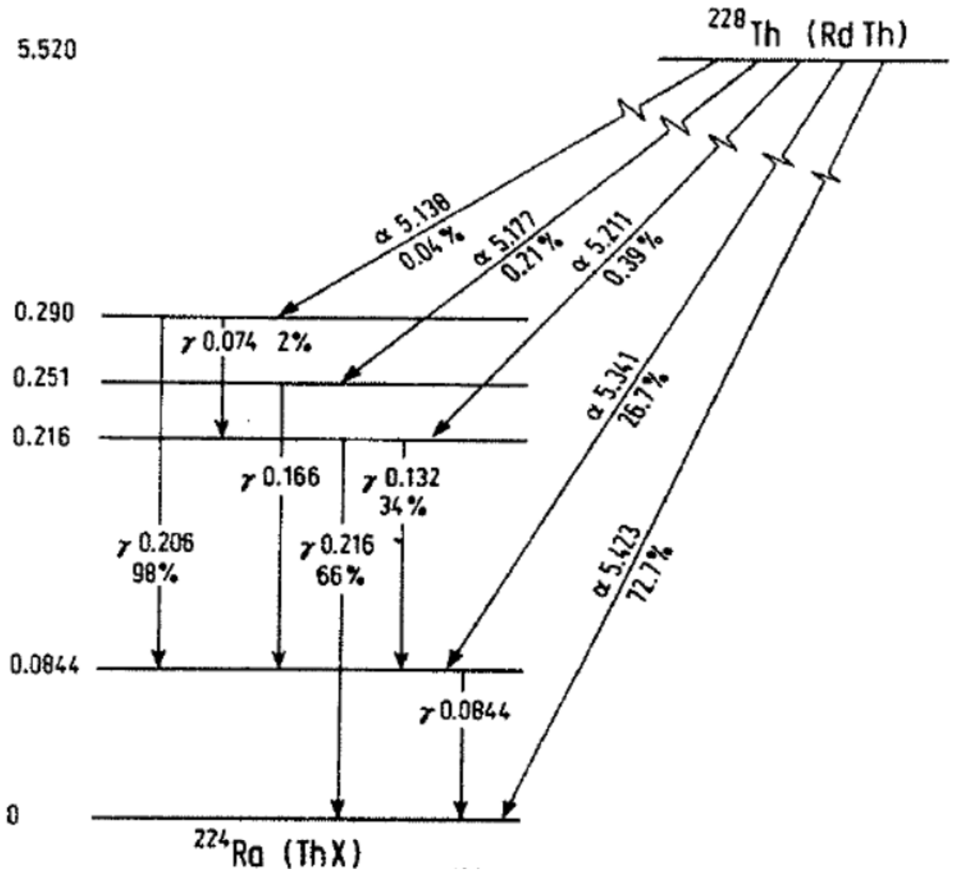


Qa near N=126 (Z=86)



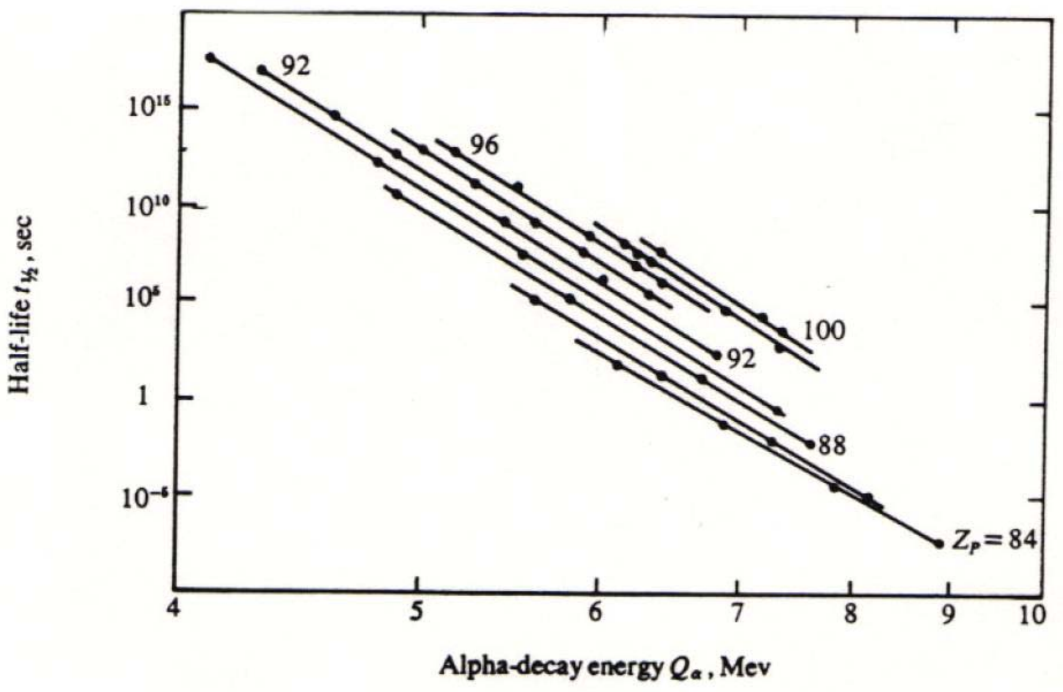
Fine structure for alpha decay





Theory of alpha decay I

- Geiger Nuttall law of alpha decay
  - $\text{Log } t_{1/2}=A+B/(Q_a)^{0.5}$
  - constants A and B have a Z dependence
- simple relationship describes the data on α-decay
- over 20 orders of magnitude in decay constant or half-life



Theory of alpha decay II

- The alpha emission rate is expressed in terms of two factors

  - rate *f* at which an alpha particle appears at the inside wall of the nucleus

  - probability that the alpha particle tunnels through the barrier

  $$\lambda_a = f \cdot T$$

  where *f* is frequency factor, T is transmission coefficient

- Pre-formation factor: an additional factor that describes probability of preformation of alpha particle inside the parent nucleus

  - no clear way to calculate such a factor

  - empirical estimates have been made

  - theoretical estimates of the emission rates are higher than observed rates

  - preformation factor can be estimated for each measured case

  - uncertainties in the theoretical estimates contribute to the differences

Alpha decay calculations

- **Frequency for an alpha particle to reach edge of a nucleus estimated as velocity divided by the distance across the nucleus**
- **On the order of 10^{21} s^{-1}**

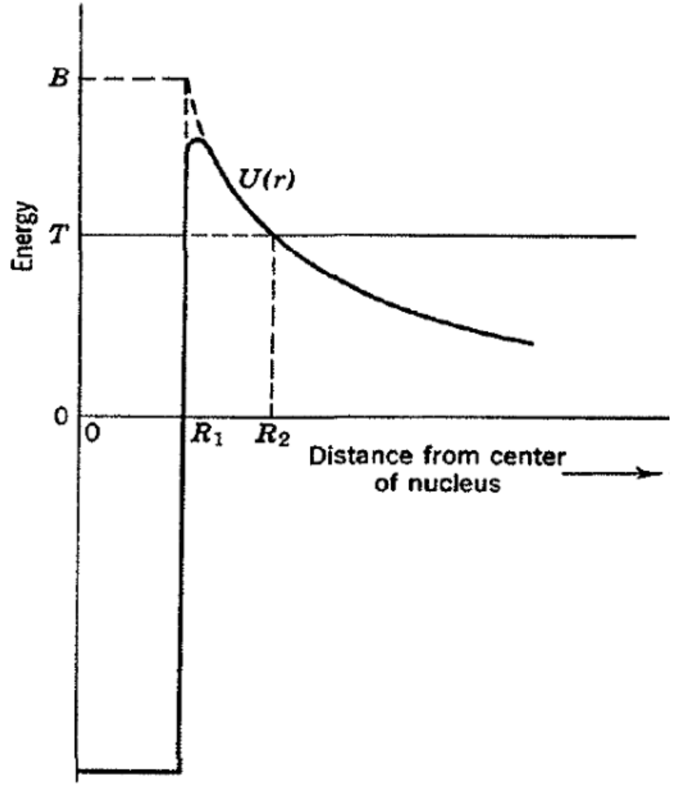
$$f = \frac{\nu}{2R} \approx \frac{\sqrt{2(V_o + Q)/\mu}}{2R}$$

- Alpha particle barrier penetration from Gamow

$$T = e^{-2G}$$

where Gamow factor

$$2G = \frac{4\pi}{h} \int_{R_1}^{R_2} \left[2\mu \left(\frac{Z_a Z_D e^2}{r} - Q_a \right) \right]^{1/2}$$



Gamow calculations

- From Gamow

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{f\mathrm{T}} = \frac{\ln 2}{(2(V_0+Q_a)/\mu)^{1/2}} e^{-2\mathrm{G}}$$

- Calculated emission rate typically one order of magnitude larger than observed rate
- observed half-lives are longer than predicted
- observation suggest probability to find a *preformed* alpha particle on order of 10^{-1}
- **Even-even nuclei undergoing ℓ =0 decay**
  - -average preformation factor is ~ 10^{-2}
  - -neglects effects of angular momentum
  - -Assumes α-particle carries off no orbital angular momentum (ℓ = 0)
- **If α decay takes place to or from excited state some angular momentum may be carried off by the α-particle**

Hindrance Factors

- Previous derivation only holds for even-even nuclei
- odd-odd, even-odd, and odd-even nuclei have longer half-lives than predicted due to hindrance factors
- **Hindrance** obtained from difference between calculated and measured half-life

  -Hindrance factors (HF) between 1 and 3E4

  -Determined by ratio of measured alpha-decay half life over calculated alpha-decay half life

$$HF = T_{1/2}(\text{exp})/T_{1/2}(\text{calc})$$

- **Favoured transitions: HF < 4**

  -Takes place between levels with the same spin and parity. The assumption is that 0^+ to 0^+ αlpha transitions from even-even nuclei are the fastest (HF=1)

**Current theories of alpha radioactivity include: microscopic models, cluster models, combined models

Presentation of E. Browne (IAEA-ICTP Workshop on ENSDF 2008):

- **Energy (keV)**
  Most measurements are relative to a line from a standard radionuclide. Include this information in a comment.
  Use Ritz's (At. Data and Nucl. Data Tables 47, 205 (1991)) evaluated E_α and I_α when no new values are available.
- **Intensity**
  Give intensities preferably "per 100 α decays" (NB=1), and a branching factor BR to convert them to "per 100 decays of the parent nucleus.
- **Hindrance factor**
  HF= experimental $T_{1/2}(\alpha)$/theoretical $T_{1/2}(\alpha)$. The theoretical value is from 1947Pr17 (M.A. Preston). The assumption is that 0^+ to 0^+ α transitions from even-even nuclei are the fastest (HF=1). These transitions are used to determine the radius parameter r_0
(See1998Ak04,
Y.A. Akovali). Use program ALPHAD.

Presentation of E. Browne cont'd: The Radius Parameter r0 (Y. Akovali, Oak Ridge)

- **Odd-N nucleus (Z, A)**

  r0(Z, N) = [r0(Z, N-1) + r0(Z, N+1)]/2

- **Odd-Z nucleus (Z, A)**

  r0(Z, N) = [r0(Z-1, N) + r0(Z+1, N)]/2

- **Odd-Odd nucleus (Z, A)**

  r0(Z, N) = [r0(Z, N-1) + r0(Z, N+1)]/2 =

  [r0(Z-1, N+1)+r0(Z-1, N-1)+r0(Z+1, N+1) +r0(Z+1, N-1)]/4

Presentation of E. Browne 2008 cont'd: Example

- **219Rn ⇒215Po (Odd-N)**

  r0 (Z=84, N=131) = [r0(84, 130) + r0(84, 132)] /2

From 1998Ak04:

r0(84,214) = 1.559 *8*

r0(84,216) = 1.5555 *2,* therefore

r0 (Z=84, N=131) = 1.557

Use Table 1 – "Calculated r0 for even-even nuclei" (1998Ak04). Insert R0= … in *comment* record:

CA HF R0=…

Run program ALPHAD to calculate hindrance factors.

Beta decay and EC

$$\beta^- : n \rightarrow p + e^- + \bar{\nu}$$

$$\beta^+ : p \rightarrow n + e^+ + \nu$$

$$EC : p + e^- \rightarrow n + \nu$$

Beta decay

$J_i\ \pi_i$

${}_{Z}X_{N}$

$J_f\ \pi_f$

$E\ \beta-$

${}_{Z+1}X_{N-1}$

$L_\beta = n = 0,1,2$

Allowed ($n=0$ and $\pi_i=\pi_f$),

1^{st} forbidden, $n=1$

2^{nd} forbidden…

$J_i = J_f + L_\beta + S_\beta$

$S_\beta = s_e + s_\nu$

$0\quad (\uparrow\downarrow)\quad$ F — **Fermi**

$1\quad (\uparrow\uparrow, \downarrow\downarrow)\quad$ GT — **Gamow-Teller**

$\pi_i\ \pi_f = (-1)^{L\beta}$

Beta decay

Half-life

$$\log ft = \log f + \log t$$

partial half-life of a given β⁻ (β⁺,EC) decay branch:

$$t \equiv T_{1/2}^{\beta_i} = \frac{T_{1/2}}{BR_i}$$

statistical rate function (phase-space factor): the energy & nuclear structure dependences of the decay transition:

$$f \equiv f_{\beta} \equiv f_n, n = 0,1,2...$$

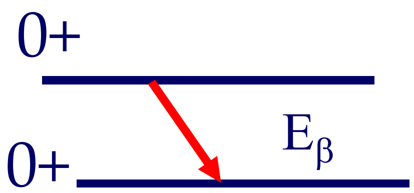
Decay Mode	Type	log *f*
β–	allowed	$\log f_0^-$
β–	1st-forb	$\log f_0^- + \log(f_1^- / f_0^-)$
EC+β+	allowed	$\log(f_0^{EC} + f_0^+)$

Beta decay

Classification of allowed decay

$$(\pi_i \pi_f = +1)$$

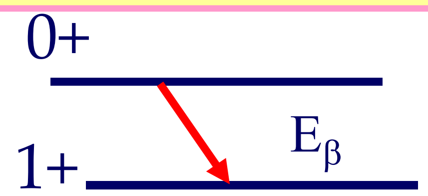
Fermi



$$\Delta J = |J_i - J_f| \equiv 0$$

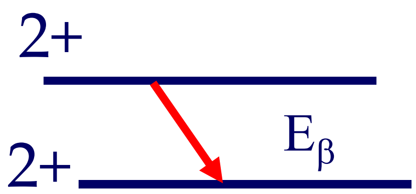
$$L_\beta = 0 \quad S_\beta = 0 \downarrow\uparrow$$

Gamow-Teller



$$\Delta J = |J_i - J_f| \equiv 1$$

$$L_\beta = 0 \quad S_\beta = 1 \uparrow\uparrow or \downarrow\downarrow$$



mixed Fermi & Gamow-Teller

$$\Delta J = |J_i - J_f| \equiv 0 \quad J_i \neq 0$$

Logft

(logft values are only indicative)

	Type	ΔJ	$\Delta \pi$	logft
Super allowed	F	0	N	$\leq$ 3.6
allowed	F/GT	0,1	N	3.6-5.9
1FNU	F/GT	0,1	Y	5.9-8.5
1FU	GT	2	Y	≥ 8.5
2FNU	F/GT	2	N	≥ 11
2FU	GT	3	N	

Logft

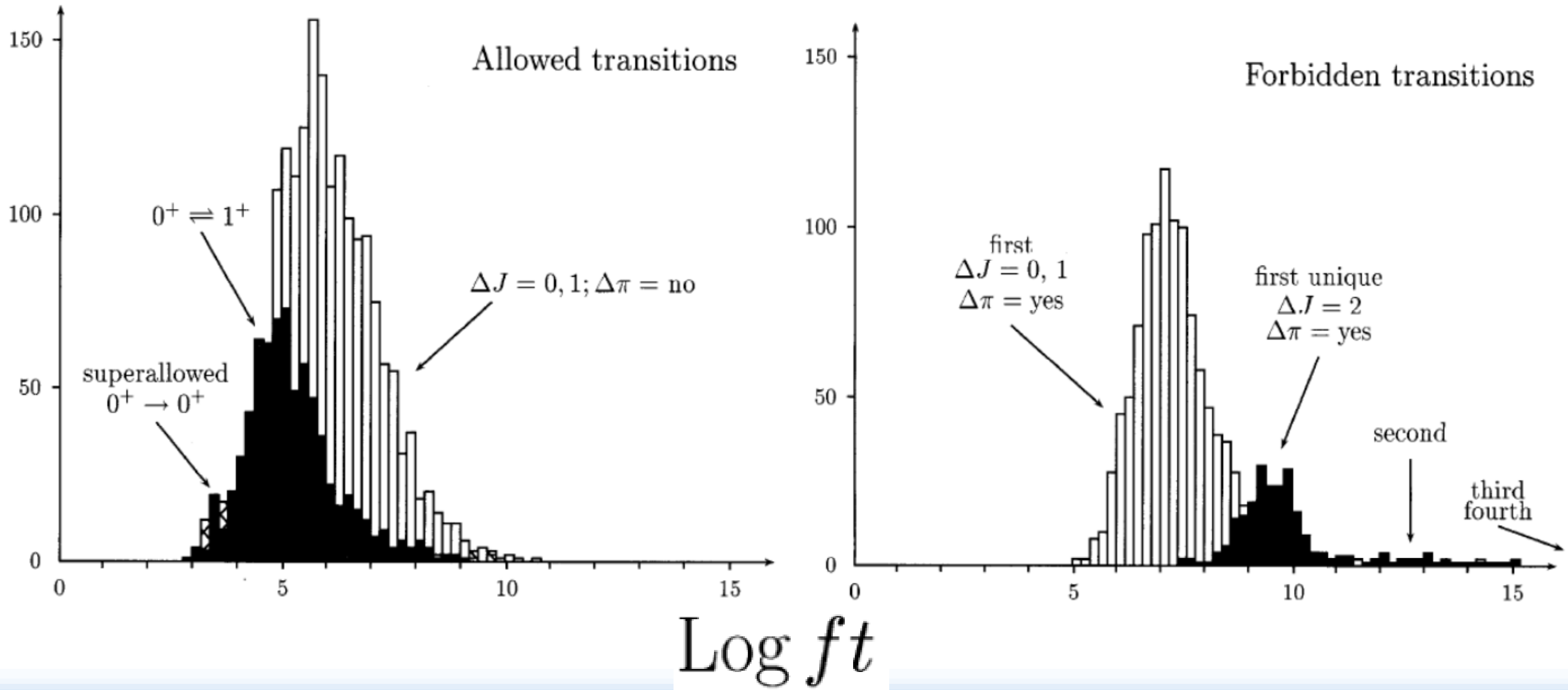
Log *ft* values –

Nuclear Data Sheets 84, 487 (1998)
Article No. DS980015

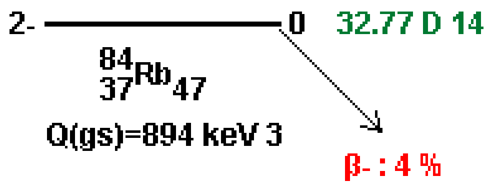
~3900 cases -> gives centroids and widths

Review Of Log*ft* Values In β Decay*

B. Singh, J.L. Rodriguez, S.S.M. Wong & J.K. Tuli



Logft



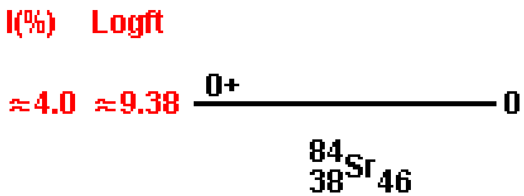
Total half-life of level

$T_{1/2}$: 2.831 10^6 s

Partial half-life of β branch

$T^{\beta-}{}_{1/2}$: 2.831 10^6 s / 0.04

= 7.08 10^7 s

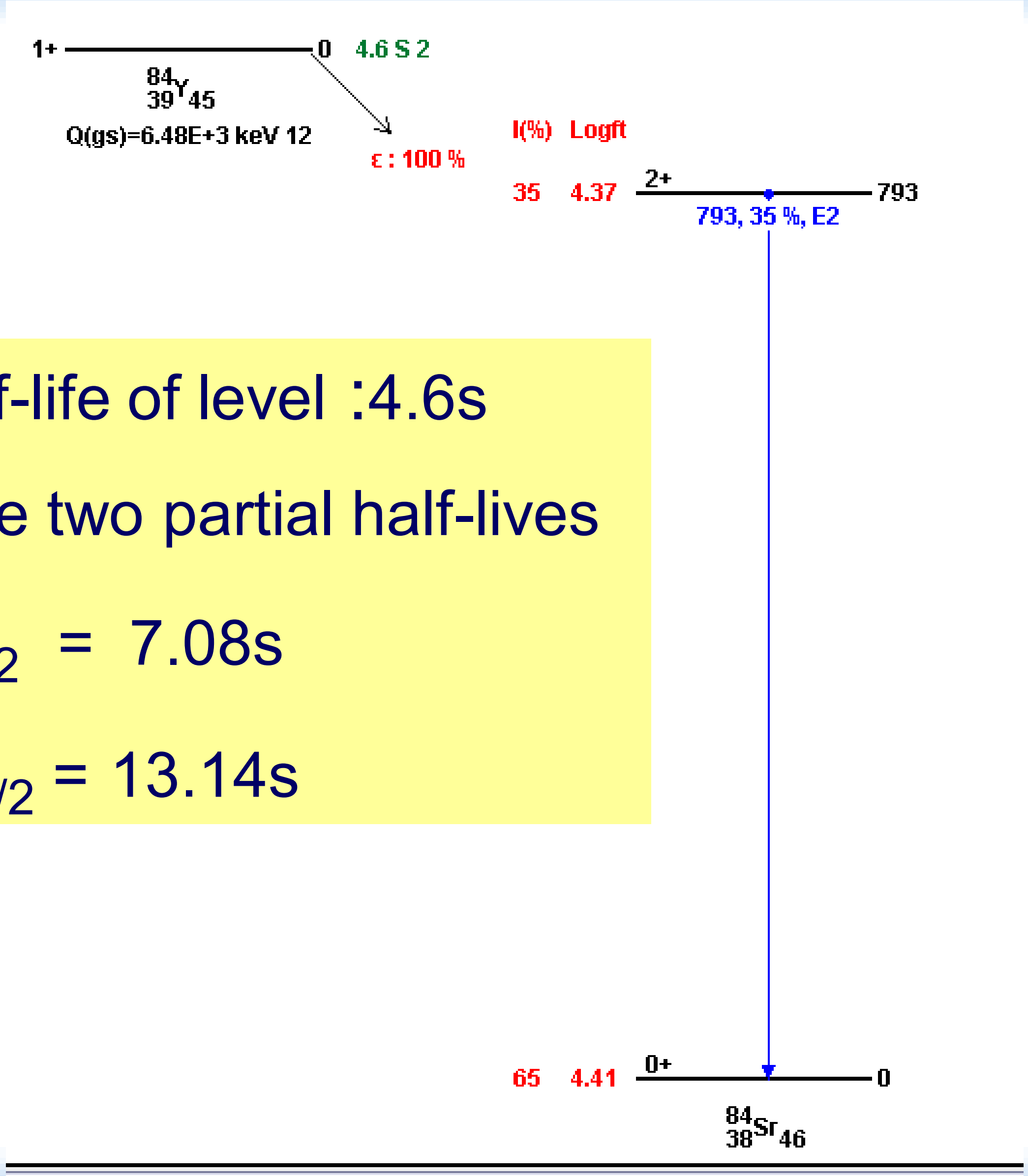


Logft

```
84SR    84RB B- DECAY              1958BE81               97NDS    199706
 84RB  P 0         2-             32.77 D   14           894       3
 84SR  N                  0.04      25
 84SR  L 0         0+
0
0       TRANSITION(KEV)=     894  3, T1/2(SEC)=     2.831E6 12, BRANCHING(%)=     4.0 AP, PARTIAL
T1/2(SEC)=      7.08E7  3
                LOG PARTIAL T1/2 =  7.8499     19
                                    FIRST-FORBIDDEN-UNIQUE
                 LOG(F1/F0) =  0.480 FOR BETAS, + OR -
                E=  894.00     LOG F1= 1.534+- 0.008
 LOG F1T =  9.384+- 0.008     F1T= 0.24228E+10
+                              AVERAGE BETA(+-) ENERGY=  332.85+-  1.214    EBAR/E =  0.3723

0 84SR  B          4       AP           9.38AP                    1U     OLD CARD
 84SR  B          4       AP           9.4  AP                  1U         NEW CARD
 84SRS B EAV=332.8 13                                                      NEW CARD
 84SRS B EAV=   332.8 13$
```

Logft



Total half-life of level :4.6s

There are two partial half-lives

$T^{\beta+(0+)}_{1/2} = 7.08s$

$T^{\beta+(2+)}_{1/2} = 13.14s$

Logft

```
 84SR    84Y EC DECAY (4.6 S)         1976IA01               97NDS    199706
 84Y   P 0         1+              4.6 S     2            6.48E+3   12
 84SR  N 0.35      10          1.0       1.0
 84SR  L        0    0+

                          NO POSITRON INTENSITY GIVEN
0       TRANSITION(KEV)=  6.48E3 12, T1/2(SEC)=       4.60 20, BRANCHING(%)=      65 10, PARTIAL T1/2(SEC)=        7.1 12
               LOG PARTIAL T1/2 =     0.85       7
               CAPTURE TO POSITRON RATIO = 1.111E-02+- 7.63E-04         LOG(E/B+)= -1.954         K/B+= 9.729E-03
               POSITRON INTENSITY = 6.43E+01+-  9.9E+00 ,          ELECTRON CAPTURE INTENSITY = 7.14E-01+-  1.2E-01 ,
               K/(EC+B+)=9.6218E-03+- 6.46E-04     L/(EC+B+)=1.1219E-03+- 7.54E-05     MNO/(EC+B+)=2.4444E-04+- 1.64E-05
               E= 6480.00      LOG F0= 3.559+- 0.044
 LOG F0T =  4.409+- 0.082      F0T= 0.25659E+05
+                                 AVERAGE BETA(+-) ENERGY= 2535.76+- 58.336    EBAR/E =  0.4646

0 84SR  E              6410    0.7112    4.41 9                 65  10        OLD CARD
 84SR  E              64   10  0.71  12  4.41   9                65  10          NEW CARD
 84SRS E EAV=2536 59$CK=0.0096 7$CL=0.00112 8$CM+=0.000244 17                           NEW CARD
 84SRS E EAV=  254E+1  6$CK=  0.0096  7$CL= 0.00112  8$CM+=0.000244 17$
+                                                                          CHECK OLD SECOND CARD
0
 84SR  L    793.0   32+

0       TRANSITION(KEV)=  5.69E3 12, T1/2(SEC)=       4.60 20, BRANCHING(%)=      35 10, PARTIAL T1/2(SEC)=         13  4
               LOG PARTIAL T1/2 =     1.12       13
               CAPTURE TO POSITRON RATIO = 1.768E-02+- 1.43E-03         LOG(E/B+)= -1.753         K/B+= 1.548E-02
               POSITRON INTENSITY = 3.44E+01+-  9.8E+00 ,          ELECTRON CAPTURE INTENSITY = 6.08E-01+-  1.8E-01 ,
               K/(EC+B+)=1.5211E-02+- 1.20E-03     L/(EC+B+)=1.7747E-03+- 1.40E-04     MNO/(EC+B+)=3.8668E-04+- 3.05E-05
               E= 5687.00      LOG F0= 3.247+- 0.052
 LOG F0T =  4.366+- 0.136      F0T= 0.23201E+05
+                                 AVERAGE BETA(+-) ENERGY= 2151.59+- 57.970    EBAR/E =  0.4612

0 84SR  E              3410    0.6118    4.3714                 35  10        OLD CARD
 84SR  E              34   10  0.61  18  4.37  14                35  10          NEW CARD
 84SRS E EAV=2152 58$CK=0.0152 12$CL=0.00177 14$CM+=0.00039 3                           NEW CARD
 84SRS E EAV=  215E+1  6$CK=  0.0152 12$CL= 0.00177 14$CM+= 0.00039  3$
```

Electromagnetic transitions

Electric (quadrupole) properties

- **Partial γ-ray half-life:**

$$T^{\gamma}_{1/2}(\mathrm{E}\lambda) = \ln 2 \left\{ \frac{8\pi}{\hbar} \frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} B(\mathrm{E}\lambda) \right\}^{-1}$$

- **Electric quadrupole transitions:**

$$B(\mathrm{E}2; I_\mathrm{i} \to I_\mathrm{f}) = \frac{1}{2I_\mathrm{i}+1} \sum_{M_\mathrm{i}} \sum_{M_\mathrm{f}\mu} \left| \left\langle I_\mathrm{f} M_\mathrm{f} \right| \sum_{k=1}^{A} e_k r_k^2 Y_{2\mu}(\theta_k, \varphi_k) \left| I_\mathrm{i} M_\mathrm{i} \right\rangle \right|^2$$

- **Electric quadrupole moments:**

$$eQ(I) = \left\langle IM = I \right| \sqrt{\frac{16\pi}{5}} \sum_{k=1}^{A} e_k r_k^2 Y_{20}(\theta_k, \varphi_k) \left| IM = I \right\rangle$$

Electromagnetic transitions

Magnetic (dipole) properties

- **Partial γ-ray half-life:**

$$T^{\gamma}_{1/2}(\mathrm{M}\lambda) = \ln 2 \left\{ \frac{8\pi}{\hbar} \frac{\lambda+1}{\lambda[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} B(\mathrm{M}\lambda) \right\}^{-1}$$

- **Magnetic dipole transitions:**

$$B(\mathrm{M1}; I_\mathrm{i} \to I_\mathrm{f}) = \frac{1}{2I_\mathrm{i}+1} \sum_{M_\mathrm{i}} \sum_{M_\mathrm{f}\mu} \left| \left\langle I_\mathrm{f} M_\mathrm{f} \left| \sum_{k=1}^{A} \left(g_k^l l_{k,\mu} + g_k^s s_{k,\mu}\right) \right| I_\mathrm{i} M_\mathrm{i} \right\rangle \right|^2$$

- **Magnetic dipole moments:**

$$\mu(I) = \left\langle IM = I \left| \sum_{k=1}^{A} \left(g_k^l l_{k,z} + g_k^s s_{k,z}\right) \right| IM = I \right\rangle$$

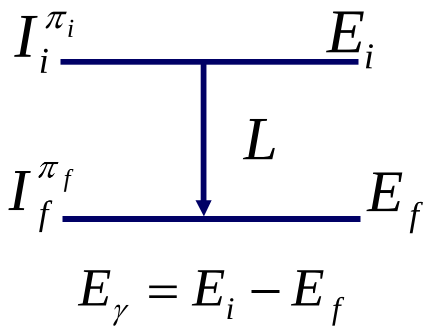
Electromagnetic transitions

γ-ray decay

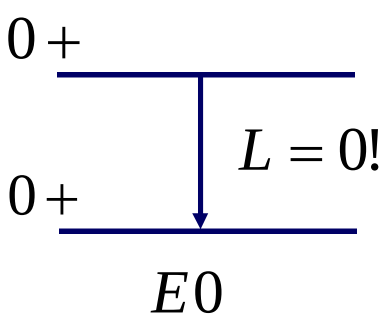
$$|I_i - I_f| \le L \le |I_i + I_f|$$

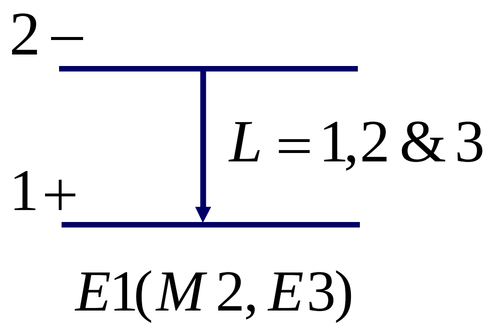
$$\Delta\pi(EL) = (-1)^L \qquad \Delta\pi(ML) = (-1)^{L+1}$$

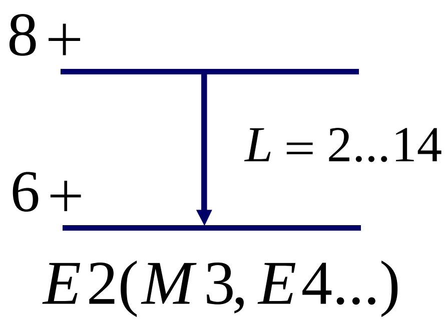
electric multipole — magnetic multipole

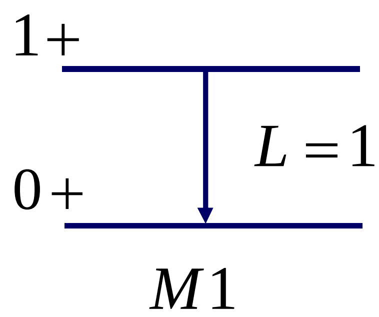


dipole	quadrupole	octupole	hexadecapole	
E1:L=1,yes	E2:L=2,no	E3:L=3,yes	E4:L=4,no	E5:L=5,yes
M1:L=1,no	M2:L=2,yes	M3:L=3,no	M4:L=4,yes	M5:L=5,no









Electromagnetic transitions

Hindrance Factor in γ-ray decay

$$F_{W(N)} = \frac{B(XL)_{Theory}}{B(XL)_{Exp}} = \frac{T^{\gamma}_{1/2}(XL)_{Exp}}{T^{\gamma}_{1/2}(XL)_{Theory}}$$

... usually an upper limit, but ...

Hindrance Factor: Weisskopf (W): based on spherical shell model potential

Nilsson (N): based on deformed Nilsson model potential

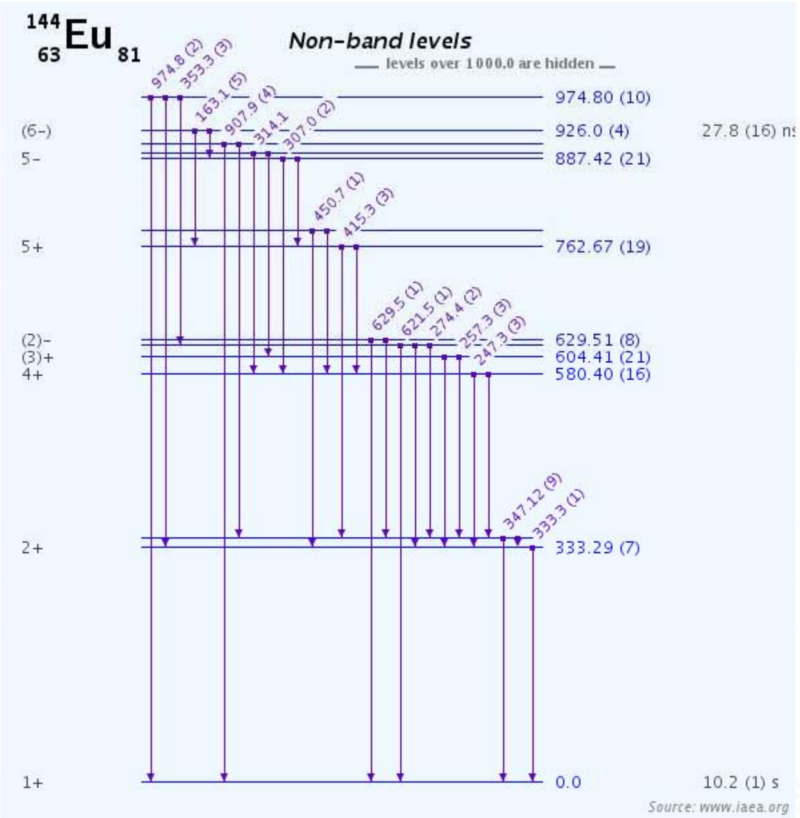
EL	$B(EL)_W, e^2\, fm^{2L}$	$T^{\gamma}_{1/2}(EL)_W$, sec	ML	$B(ML)_W, \mu_N^2\, fm^{2L-2}$	$T^{\gamma}_{1/2}(ML)_W$, sec
E1	$0.06446A^{2/3}$	$6.762A^{-2/3}E_\gamma^{-3}\times 10^{-15}$	M1	1.7905	$2.202E_\gamma^{-3}\times 10^{-14}$
E2	$0.0594A^{4/3}$	$9.523A^{-4/3}E_\gamma^{-5}\times 10^{-9}$	M2	$1.6501A^{2/3}$	$3.100A^{-2/3}E_\gamma^{-5}\times 10^{-8}$
E3	$0.0594A^{2}$	$2.044A^{-2}E_\gamma^{-7}\times 10^{-2}$	M3	$1.6501A^{4/3}$	$6.655A^{-4/3}E_\gamma^{-7}\times 10^{-2}$
E4	$0.06285A^{8/3}$	$6.499A^{-8/3}E_\gamma^{-9}\times 10^{4}$	M4	$1.7458A^{2}$	$2.116A^{-2}E_\gamma^{-9}\times 10^{5}$
E5	$0.06929A^{10/3}$	$2.893A^{-10/3}E_\gamma^{-11}\times 10^{11}$	M5	$1.9247A^{8/3}$	$9.419A^{-8/3}E_\gamma^{-11}\times 10^{11}$

Nuclear levels (from LiveChart)

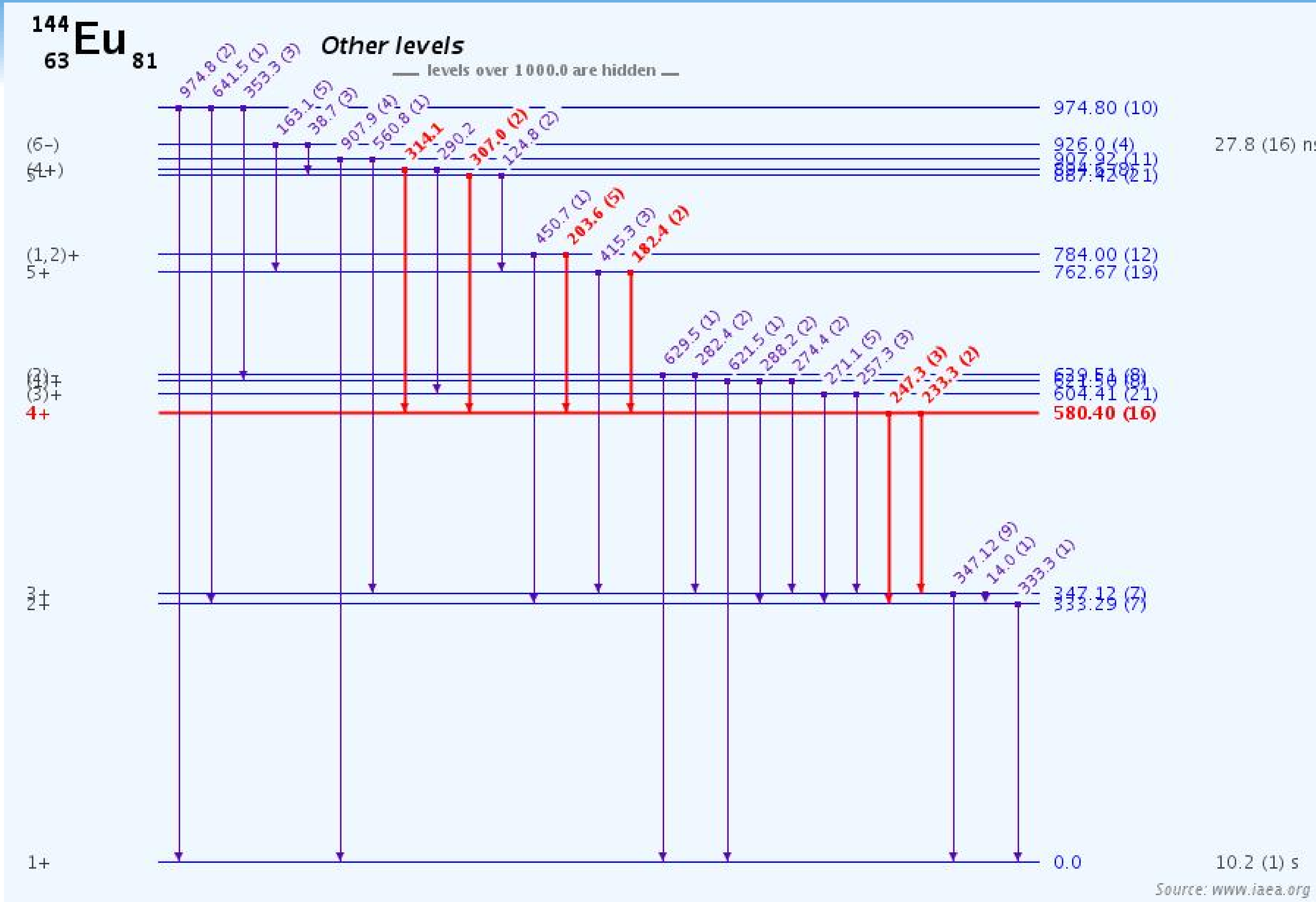
0.0 < Energy < 1000.0 | Image height: 600 | Level width: 300 | Band spacing: 20 | Emphasise level: 0.0 and/or gamma 0.0

☑ Show level energy ☑ Show spin-parity ☑ Show half life ☑ Show gamma ☐ Show all (possible overlapping) ☐ Show legend

☑ Non-band ☐ Band 1 ☐ Band 2 ☐ Band 3 ☐ Band 4



See extract from ENSDF Datasets of ^{144}Eu (snapshot October 2009).



144 63 Eu 81
Other levels
levels over 1000.0 are hidden
974.80 (10)
(6–)
926.0 (4)
27.8 (16) ns
(1,2)+
784.00 (12)
5+
762.67 (19)
4+
580.40 (16)
604.41 (21)
3+
347.12 (7)
2+
333.29 (7)
1+
0.0
10.2 (1) s
Source: www.iaea.org

Thank you