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NETWORK RECONSTRUCTION WITH HIDDEN NODES

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Abstract

Developing methods for reconstructing the couplings between elements in a network and applying them to real data have attracted a lot of attention in recent years. This work, however, does not take into account the fact that in real life applications we only have access to observations from a small fraction of the elements in a network. In this talk, we study how the effect of the hidden nodes can be taken into account in network reconstruction.

Let us denote the joint distribution of the observed and hidden nodes given the network connectivity by $p(\{s_{observed}\},\{s_{hidden}\}|\{J_{observed}\},\{J_{hidden}\})$. Here $\{s_{observed}\}$ represents the state of the observed nodes, $\{s_{hidden}\}$, the state of the hidden ones, $\{J_{observed}\}$, the set of couplings between observed nodes and, $\{J_{hidden}\}$, the set of all coupling that involve a hidden node. To take into account the effect of hidden nodes in inferring the connections between the observed ones, ideally one should integrate out the configuration of the hidden nodes and also integrate in a prior over the distribution of the couplings that involve these hidden nodes. This means calculating the likelihood of the observed data as

$$L=\int d\{J_{hidden}\}\, p(\{J_{hidden}\})\ Tr_{\{s_{hidden}\}}\, p(\{s_{observed}\},\{s_{hidden}\}|\{J_{observed}\},\{J_{hidden}\})$$

A major problem here is that the trace is exponentially hard in the size of the hidden part of the network. Considering the case of the kinetic Ising model, we describe mean-field methods for integrating out the configuration of the hidden variables from the likelihood and for including prior distribution over the connections involving hidden part of the network. We show that one can identify two ways of deriving these mean-field equations. In the first one, starting from the likelihood of the joint distribution of hidden and observed nodes, we integrate out the hidden nodes using dynamical mean-field theory. In the second one, one derives learning rules for the whole network, assuming momentarily that the state of the hidden nodes is known. One then approximates these learning rules using mean-field dynamics for the hidden nodes. We show that in mean-field architectures the effect of hidden nodes to the first order can be considered as a multiplicative factor that can be easily corrected for. Applying these mean-field approaches to various architectures we find good performance for reconstructing the connectivity.