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2370-16

School and Training Course on Dense Magnetized Plasma as a Source of Ionizing Radiations, their Diagnostics and Applications

8 - 12 October 2012

"Constrained Dynamics" and its manifestationin a low pressure spark experiment

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“Constrained Dynamics” and its manifestation in a low pressure spark experiment

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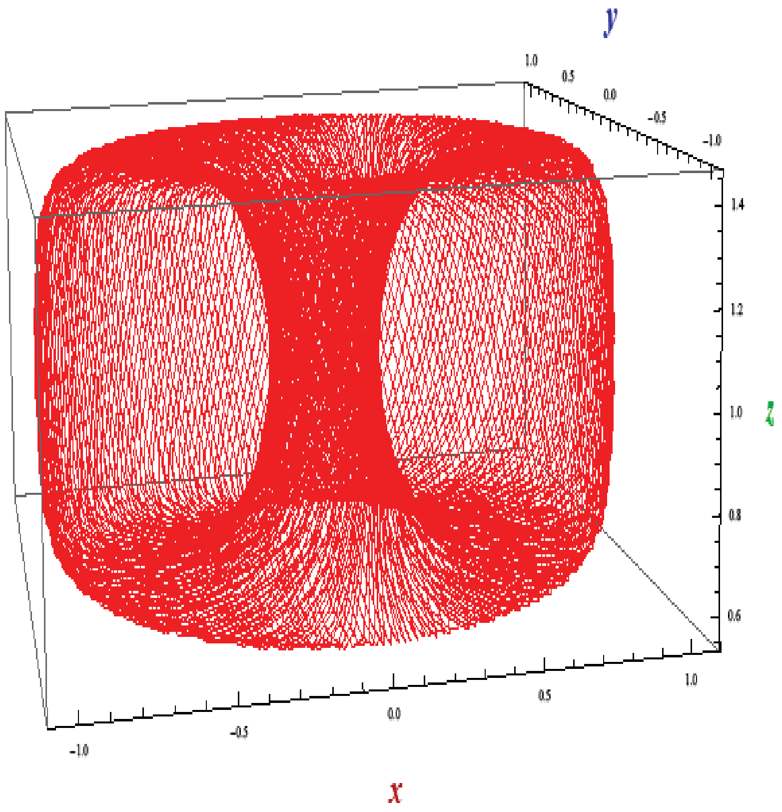
Conclusions from experimental data

- What is the directionality of energetic ions in DPF
  - Energetic ions move along +z direction and also in –z direction
  - They move in loops in r-z plane
  - They have an azimuthal motion
  - A toroidal trajectory agrees with this description



Role of random EM fields in plasma dynamics

- Random EM fields (thermal radiation or turbulence) can drive random currents, which produce random magnetic fields, so that a non-zero average value of $\left\langle \underset{\sim}{\vec{\mathrm{J}}} \times \underset{\sim}{\vec{\mathrm{B}}} \right\rangle$ can arise in the weakly ionized precursor plasma ahead of plasma sheath.
- This should result in small but non-zero value for all 3 components of ion and electron velocities, which can be amplified by compressive radial flow

“Can be”: but will it be?

- The mechanism of random EM fields generating a non-zero seed velocity which can be amplified by compressive flow is ***sufficient*** to generate solenoidal flow (torroidal ion trajectories) in the plasma.
- But is it necessary?
- The plasma may choose not to have any torroidal flow if it had any choice.
- Does it have any choice?

Necessity of solenoidal flow

- Experimental data showing features of solenoidal flow in so many different plasmas suggests that there must be some physical condition which makes it ***necessary*** for the plasma to generate solenoidal flow.
- What is that physical condition?.
- I suggest "constrained dynamics" as a potential candidate for such necessary physical condition

Constrained Dynamics

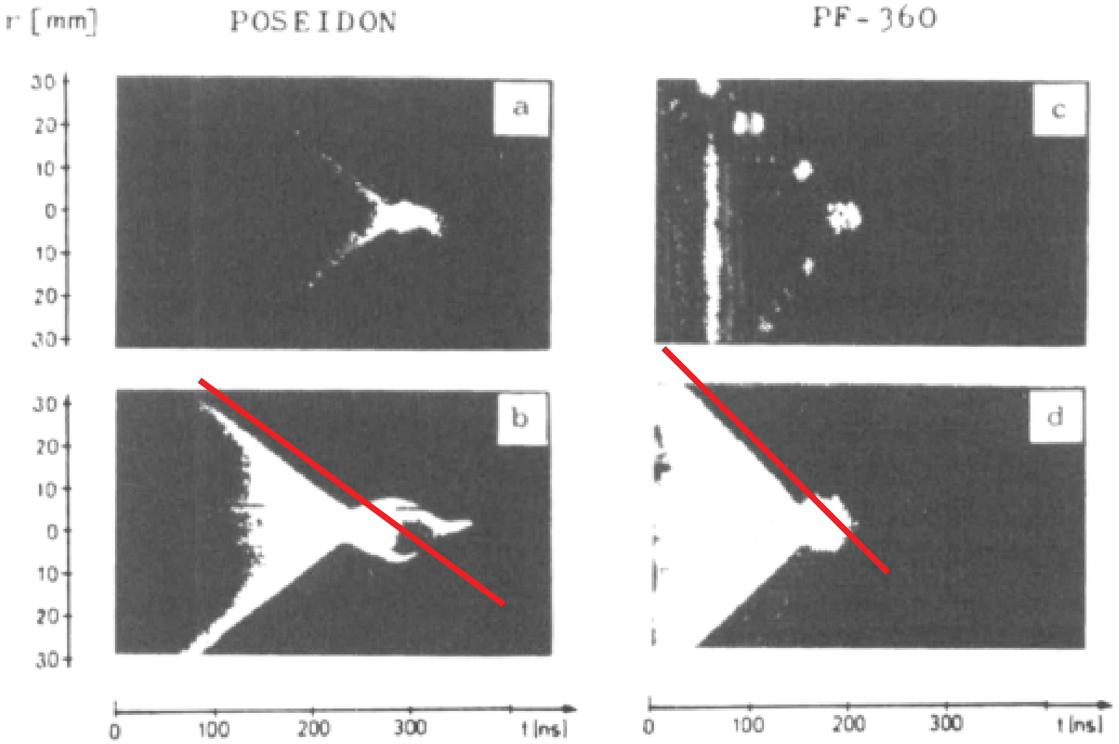
- Terms in the momentum conservation equation are governed by widely dissimilar and independent phenomena
  - Pressure gradient is governed by processes which add or subtract energy (ohmic heating, shock heating, radiative cooling, thermal conduction) or particles (ionization, recombination) and which redistribute particles in space
  - A part of magnetic force is governed by current supplied by external power source and local skin effect.
  - The resultant would ORDINARILY govern plasma acceleration

Constrained Dynamics

- In DPF and many other z-pinches, the current carrying plasma is *constrained* to move through an initially neutral medium
  - The viscous drag offered by the neutral medium to the magnetically accelerated plasma should result in a nearly constant drift velocity and a negligible acceleration.

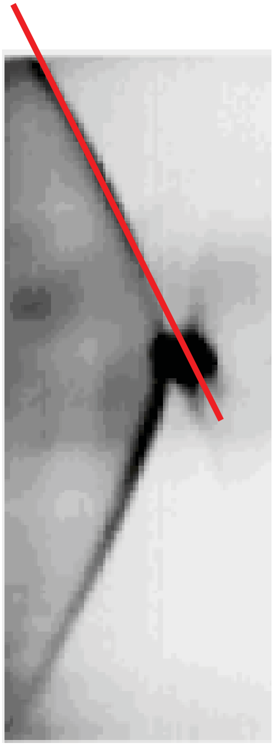
  - This can be seen in streak camera pictures in many DPF installations

Streak camera pictures of DPF



Herold et al Nuclear fusion, 29 (1989) p1255

POSEIDON, PF-360 and PF-1000



PF-1000

Scholz et al NUCLEONIKA, 51, 2006, p79-84

How can momentum be conserved?

- Since externally applied magnetic force, pressure and acceleration are governed by widely dissimilar and independent phenomena, momentum conservation is possible only if there exists a *free parameter in the momentum conservation equation*, which is not controlled by any external phenomenon.

Two-fluid model

$$\frac{\partial \vec{v}_e}{\partial t} + \left(\vec{v}_e \cdot \vec{\nabla}\right)\vec{v}_e = -\frac{e}{m_e}\left(\vec{E} + \vec{v}_e \times \vec{B}\right) - \frac{\vec{\nabla} p_e}{m_e n_e}$$

$$\frac{\partial \vec{v}_i}{\partial t} + \left(\vec{v}_i \cdot \vec{\nabla}\right)\vec{v}_i = \frac{e}{m_i}\left(\vec{E} + \vec{v}_i \times \vec{B}\right) - \frac{\vec{\nabla} p_i}{m_i n_i}$$

- Combine ion and electron velocities

Center of mass velocity

$$\vec{v} = \left(n_i m_i \vec{v}_i + n_e m_e \vec{v}_e\right) / \left(n_i m_i + n_e m_e\right) \approx \left(m_i \vec{v}_i + m_e \vec{v}_e\right) / m_i$$

Current density

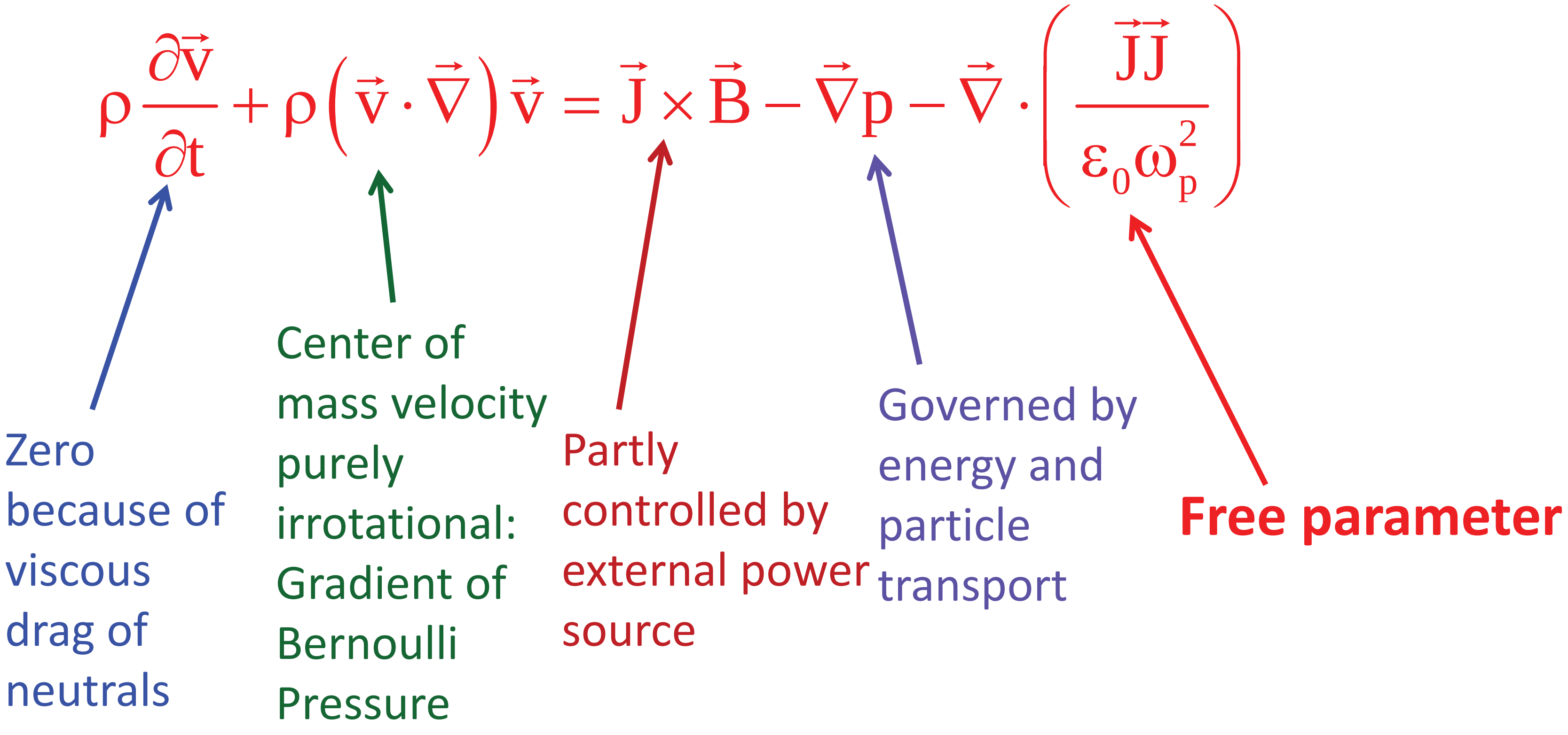
$$J = e\left(n_i \vec{v}_i - n_e \vec{v}_e\right) \approx e n_e \left(\vec{v}_i - \vec{v}_e\right)$$

- Invert and substitute for ion and electron velocities

S. K. H. AULUCK , *J. Plasma Phys*. Vol 36 p. 211-234, 1986.

Two-fluid model

- This leads to a generalized Ohm’s Law (not considered here) and equation for conservation of momentum for center of mass



$$\rho\frac{\partial\vec{v}}{\partial t}+\rho\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}=\vec{J}\times\vec{B}-\vec{\nabla}p-\vec{\nabla}\cdot\left(\frac{\vec{J}\vec{J}}{\varepsilon_0\omega_p^2}\right)$$

Zero because of viscous drag of neutrals

Center of mass velocity purely irrotational: Gradient of Bernoulli Pressure

Partly controlled by external power source

Governed by energy and particle transport

Free parameter

Free currents

- ***The momentum balance equation must be obeyed at all times and at all points***
- Current density is defined at every point
- But only that portion of current density, whose flux at the electrode surface equals the external current, is governed by the external power source.
- The remaining current, whose flux at the electrode surface is zero, is **free current**, flowing in closed loops.

Necessity of solenoidal ion flow

- The free current is solenoidal (because of conservation of charge) and the center of mass velocity has no solenoidal component (no external solenoidal force) $m_i \vec{v}_i^{SOL} + m_e \vec{v}_e^{SOL} = 0$
- Therefore there must *necessarily* exist solenoidal ion flow

$$\vec{v}_i^{sol} = -\frac{m_e}{m_i} \vec{v}_e^{sol} \approx \frac{m_e}{m_i} \frac{\vec{J}_{free}}{ne}$$

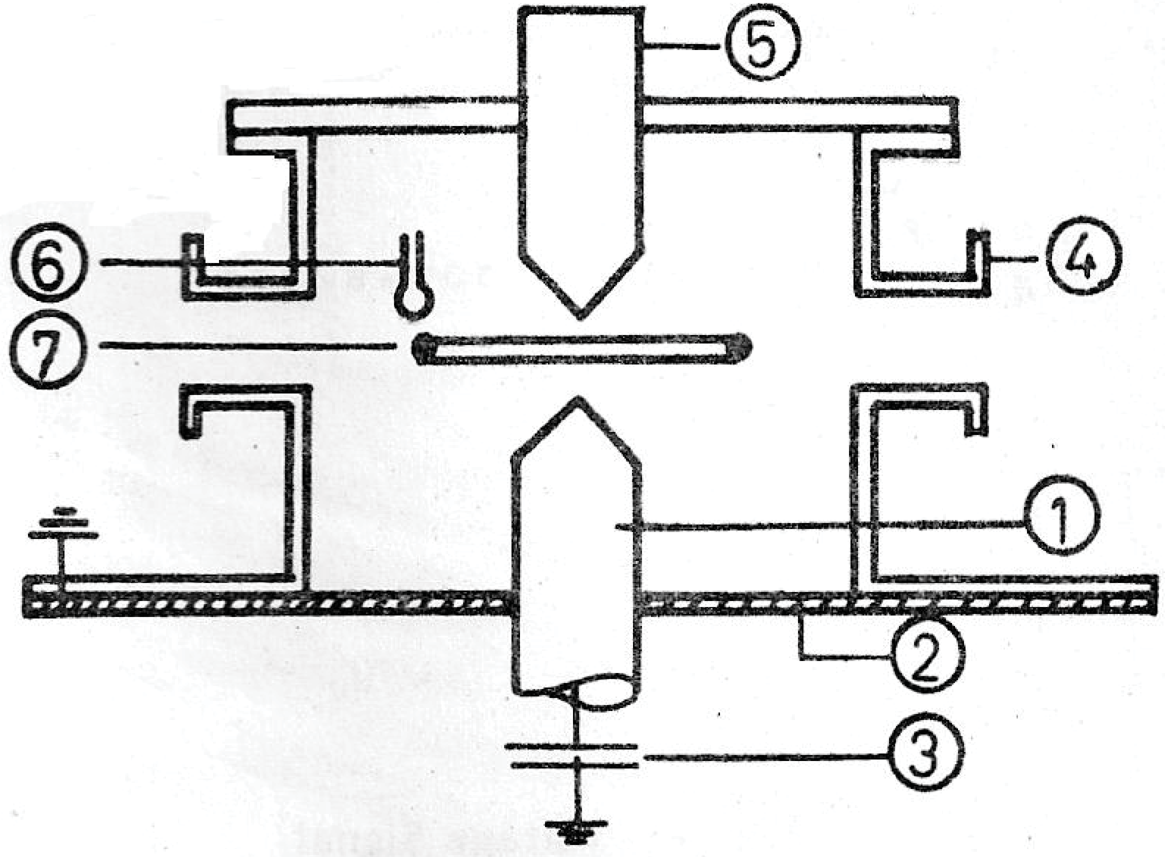
- The principle of constrained dynamics shows that solenoidal ion flow must *necessarily* be present. But it is *not sufficient* to calculate the details of the flow

Experimental demonstration

- create as large a mismatch between the externally controlled magnetic pressure and the plasma thermal and Bernoulli pressures
- force the current to undergo oscillations and the pressure to be limited to very small values

Low pressure spark experiment

Spark discharge in ~100 hPa He with a low circuit inductance.
Voltage 2-5 kV,
Length 1-5 cm



Schematic of the experiment: (1) High Voltage Electrode (2) Insulator (3)Capacitor (4) Vacuum Chamber (5) Ground electrode (6) dI/dt probe (7) Diamagnetic loop for axial flux.

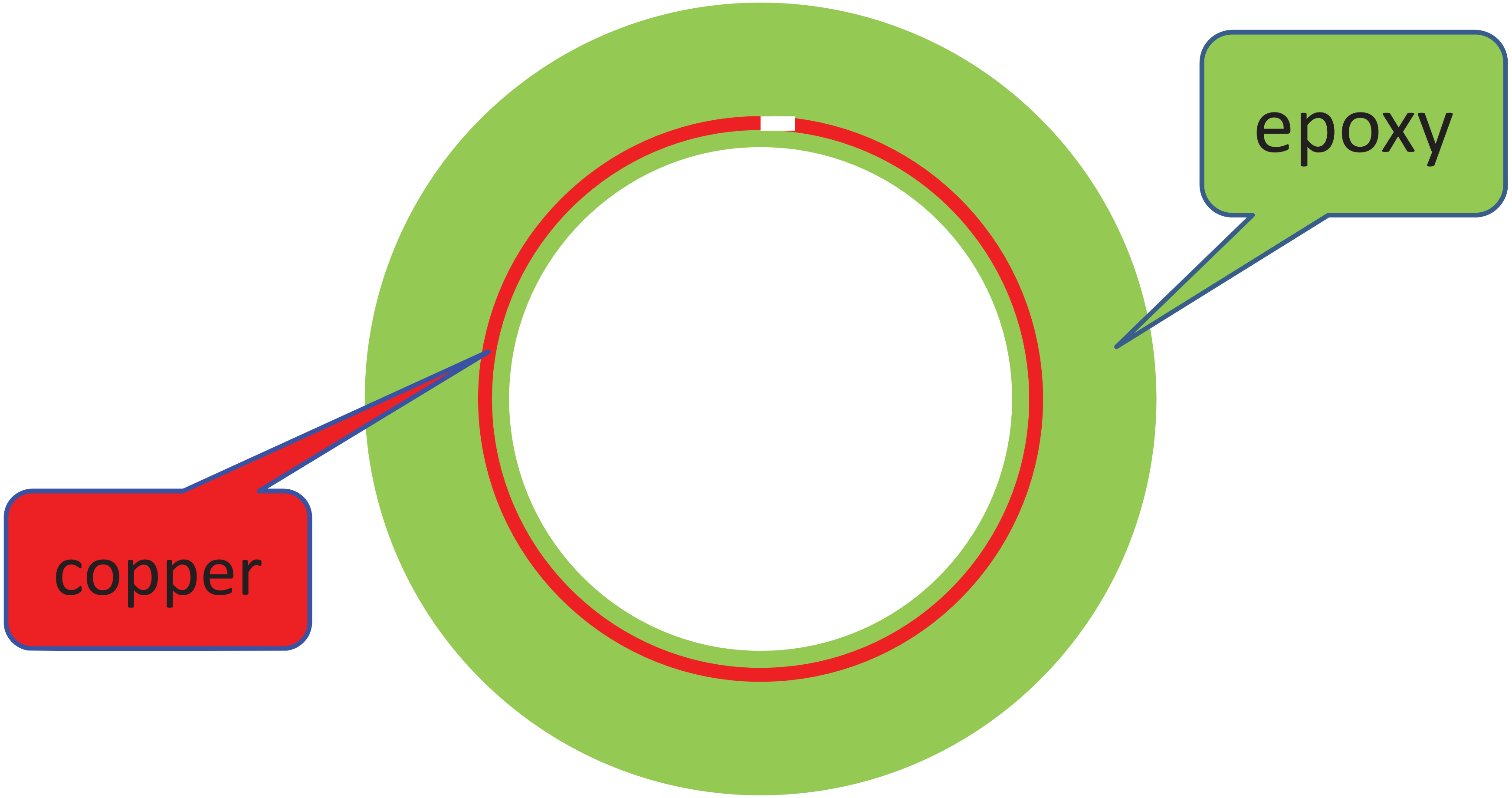
Diagnostics

- Framing camera:
  - Straight cylindrical shape
  - Initial radius ~4 mm, height ~3-5 cm
  - expanding @ 3 mm/μs
- Visible spectroscopy:
  - Temperature ~0.5 eV for 100 hPa pressure
  - Saha equation gives degree of ionization ~6×10^{-7}
- Magnetic Probe for dI/dt
- Diamagnetic loop

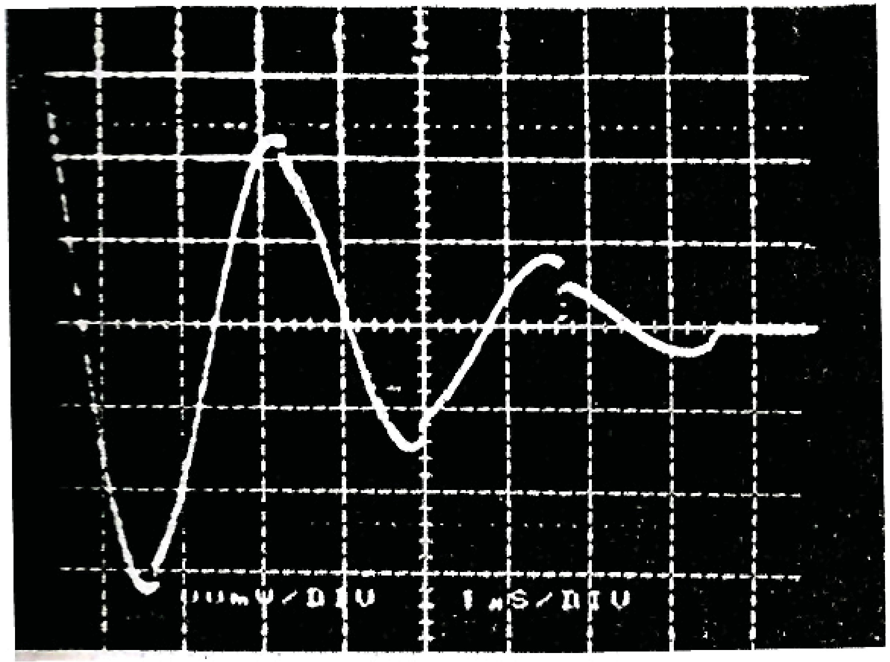
Diagnostics

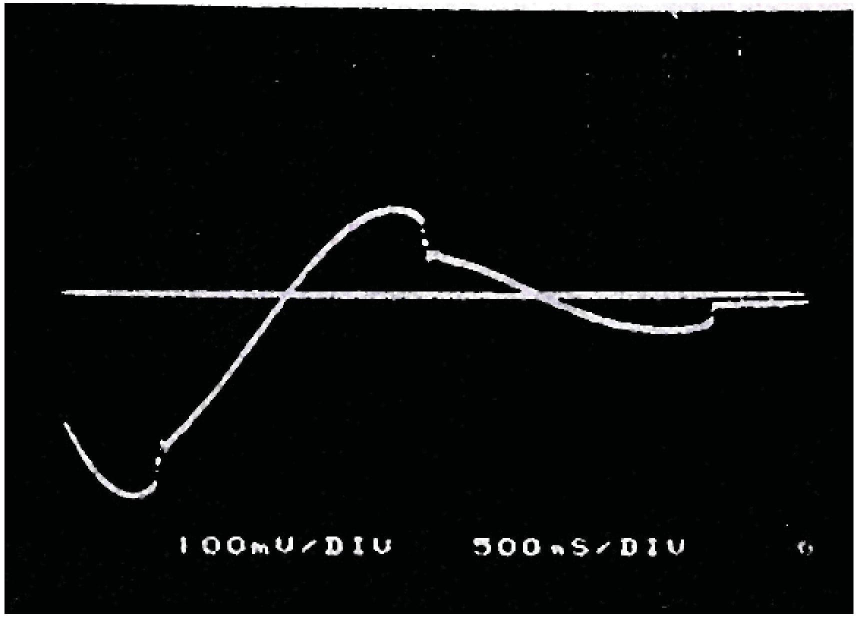
- Magnetic probe was a 3 mm diameter single turn of 0.5 mm dia copper wire, insulated with epoxy, directly connected to RG174 cable, kept at 70 mm from axis.
- Diamagnetic loop was machined out of copper-laminated epoxy sheet. Its OD had a snug fit to inner wall of chamber, ID =76 mm. A 2 mm width, 80 mm ID, 15 μm thick copper ring was machined out of copper lamination. A 0.5 mm gap was created in the circumference, across which a cable was soldered with minimum lead length. It was coated with epoxy.

Schematic of diamagnetic probe



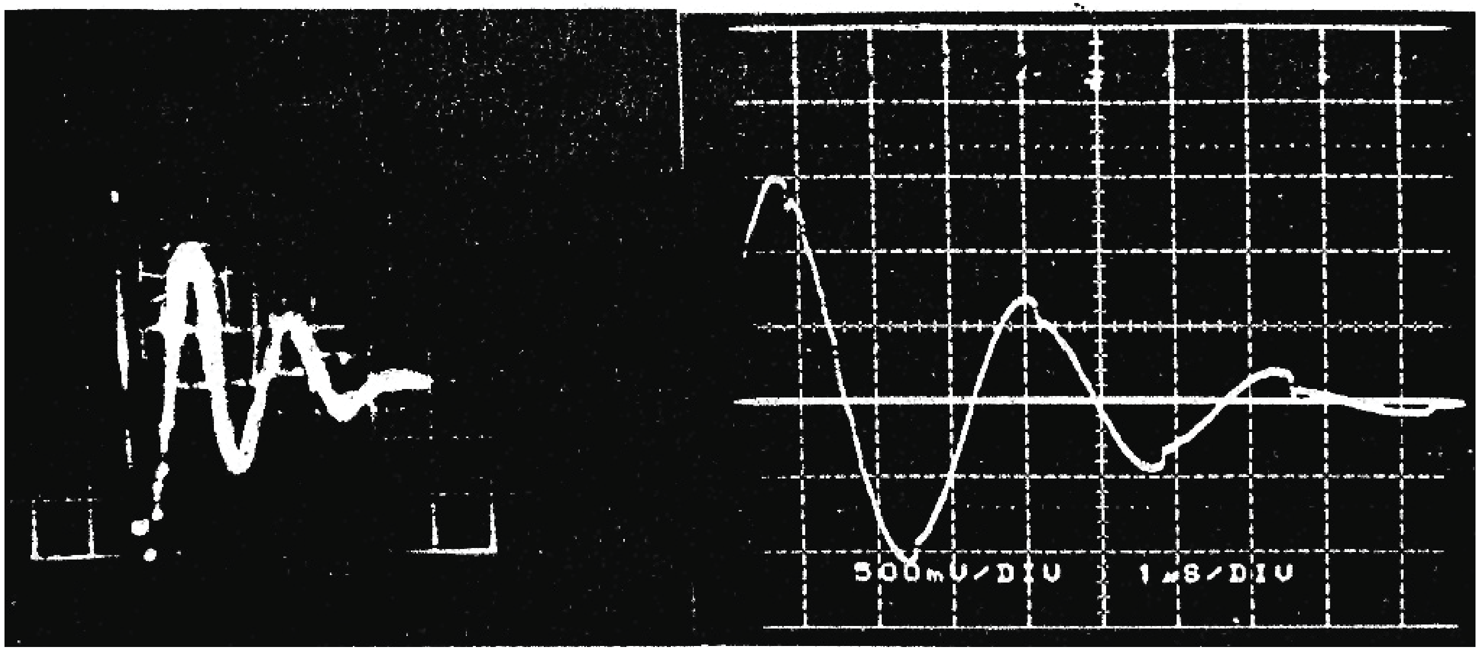
dI/dt signals





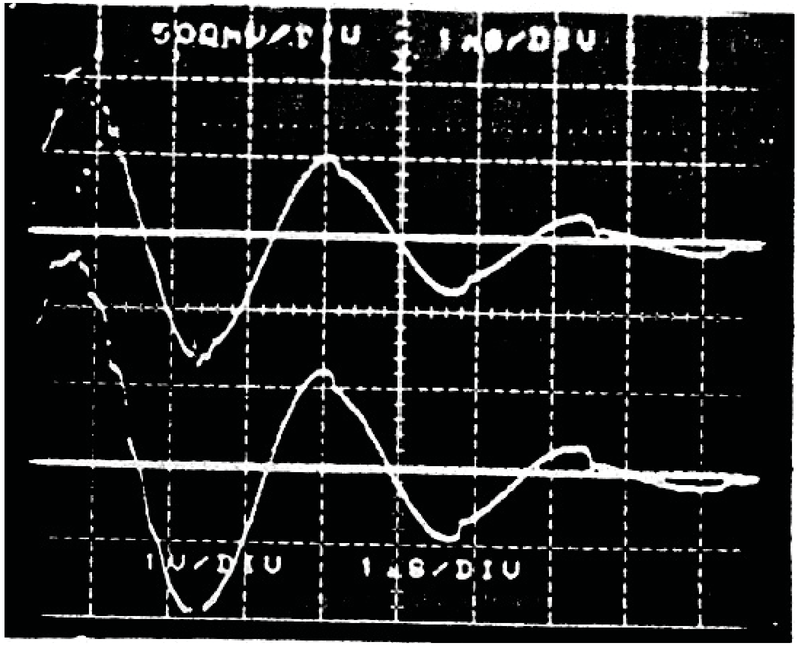
- Signals reproducible under identical discharge conditions

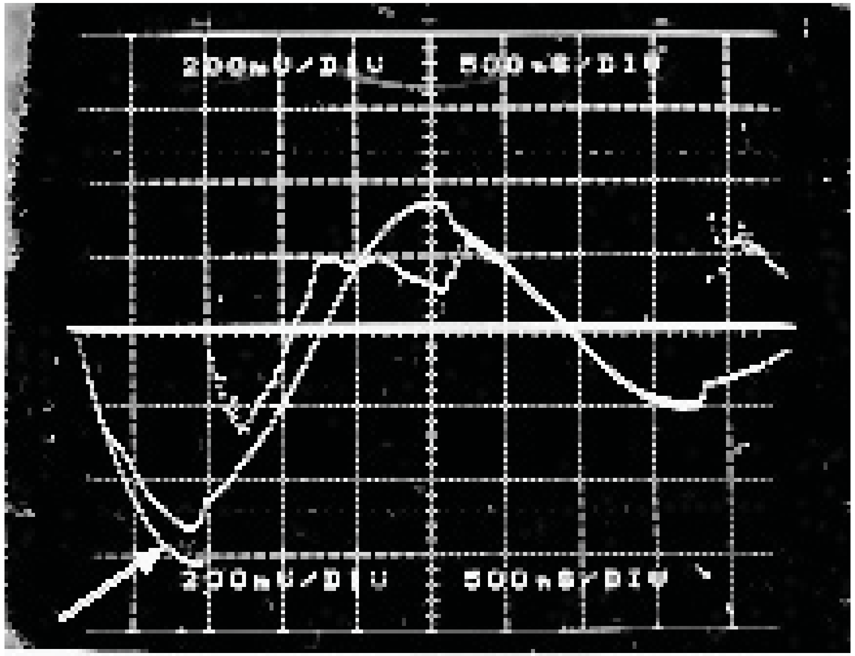
Voltage and dI/dt from same shot



- Voltage signal has no dips, while dI/dt has. This shows that inductance must be a function of I.

Two examples of diamagnetic loop signals





- The difference in shapes of probe and diamagnetic loop signals shows negligible cross-sensitivity

Are these real plasma phenomena?

- Dielectric breakdown in probes?
  - The voltage across the probe is directly measured to be ~1 Volt. The probe is covered with epoxy.
- semiconducting oxide layers at joints in circuit acting as nonlinear circuit elements?
  - At pressures ~0.6 to 1 Pa, the usual damped sinusoidal waveform is seen without distortion
- Sheath phenomena at electrodes?
  - Sheaths cannot produce axial magnetic field!!

Is the axial flux signal an artifact?

- The diamagnetic loop was extremely planar, centered with and perpendicular to the axis.
- The signal that was measured at the cable was the integration of the electric field along the copper track, whose two ends were *at the same axial location*. Only azimuthal component of electric field could therefore contribute to the signal.
- In at least some shots, probe and loop signals had distinctly different shape
  - Rules out cross-sensitivity between θ and z components

Unstable helical modes can create axial magnetic field!!

- Instability is non-periodic by definition.
  - The observation of a repeated train of sharp transients does not fit the picture of an instability.
- The transient phenomenon occurs repeatedly near the zeros of current and vanishes at higher currents

Interpretation in terms of Constrained Dynamics

- The plasma is known to be very weakly ionized. Therefore, the motion of the ion component is expected to be mainly governed by collisions with the neutrals
  - Acceleration not correlated with magnetic force.
- From the estimated electron density, the collision-less electron skin depth $c/\omega_{pe} \approx 4\ \mathrm{mm}$ of the same order as the initial radius of the luminous column.

Interpretation in terms of Constrained Dynamics

- Radial component of Momentum balance equation for plasma

$$\underbrace{\frac{J_\theta^2}{r\varepsilon_0\omega_{pe}^2}+J_\theta B_z}_{\text{Free parameter}}-\left(\underset{\substack{\uparrow\\ \text{Driven by LCR circuit}}}{J_z B_\theta}+\underset{\substack{\uparrow\\ \text{Weakly ionized plasma has negligible pressure}}}{\frac{\partial p}{\partial r}}\right)=\underbrace{\rho\left(\frac{\partial\vec{v}}{\partial t}+\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}\right)_r}_{\text{Dominated by slow hydrodynamics of neutrals}}$$

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}\right)_r\approx-\frac{\partial p_{\text{neutral}}}{\partial r}$$

Interpretation in terms of Constrained Dynamics

$$\frac{J_\theta^2}{r\varepsilon_0\omega_{pe}^2} + J_\theta B_z - \left(J_z B_\theta + \frac{\partial}{\partial r}\left(p + p_{\text{neutral}}\right) \right) = 0$$

- A simple model:
  - Bennett profile for density
  - Temperatures and drift velocity uniform

$$p + p_{\text{neutral}} = \frac{p_0}{\left(1 + b^2 r^2\right)^2}; J_z = \frac{Ib^2}{\pi}\frac{1}{\left(1 + b^2 r^2\right)^2}$$

$$J_z B_\theta + \frac{\partial}{\partial r}\left(p + p_{\text{neutral}}\right) = \frac{\mu_0 b^4 r}{2\pi^2\left(1 + b^2 r^2\right)^3}\left\{ I^2 - \frac{8\pi^2 p_0}{\mu_0 b^2} \right\}$$

Interpretation in terms of Constrained Dynamics

$$-\frac{\partial B_z}{\partial r} = \mu_0 J_\theta = \frac{\left(r\omega_{pe}^2\right)}{2c^2}\left\{\sqrt{{B_z}^2 + \frac{B_0^2}{\left(1+b^2r^2\right)^3}} - B_z\right\}$$

$$B_0^2 \equiv \frac{2{\mu_0}^2c^2b^4}{\pi^2\omega_{pe}^2}\left(I^2 - I_B^2\right); I_B^2 \equiv \frac{8\pi^2 p_0}{\mu_0 b^2}$$

- Boundary condition: Flux of B_z within conducting boundary must be conserved ≅0
- $\therefore B_z$ ***must*** reverse its sign within the chamber

Interpretation in terms of Constrained Dynamics

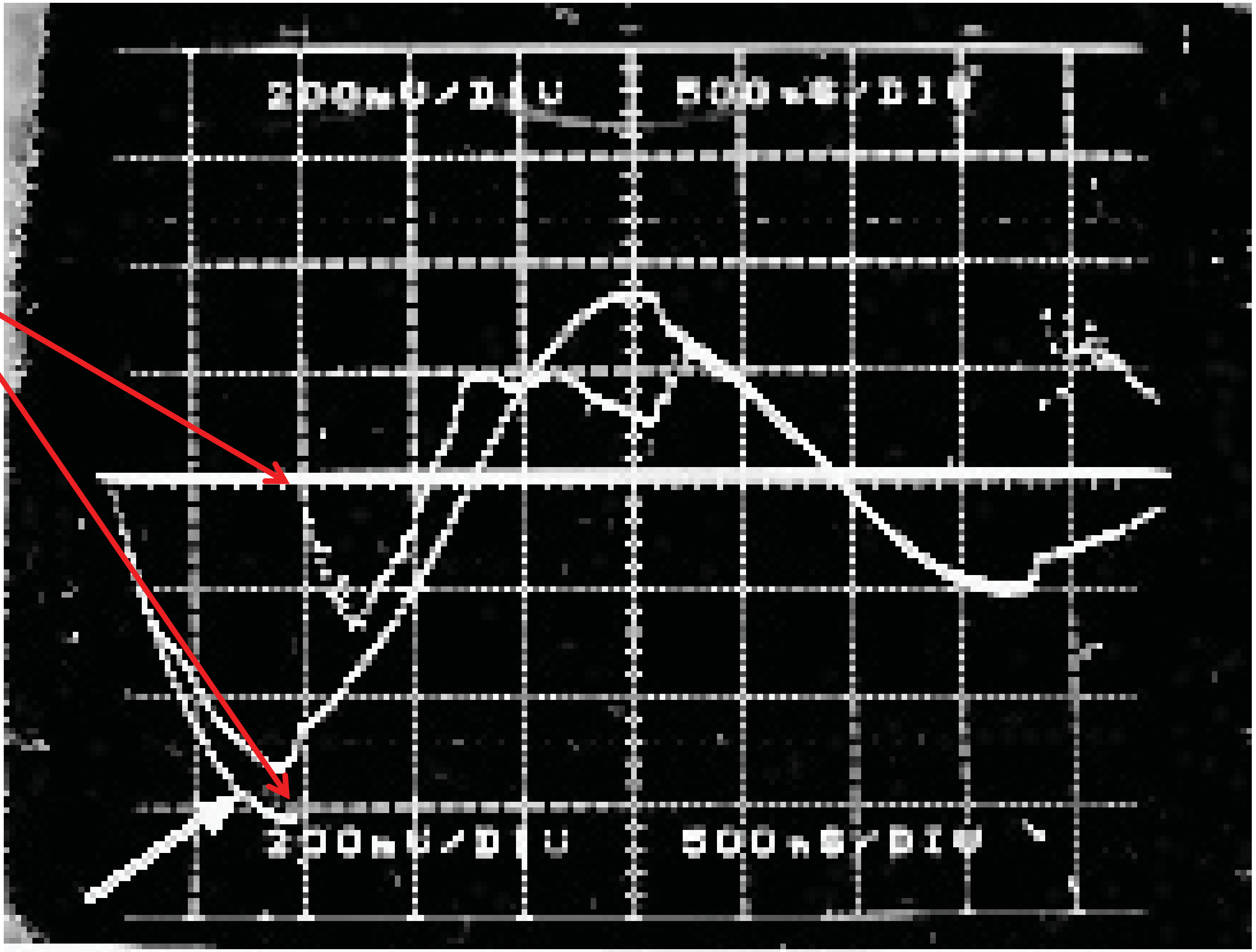
- B_z cannot pass through zero unless $I^2 - I_B^2 \geq 0$

$$-\frac{\partial B_z}{\partial r} = \mu_0 J_\theta = \frac{\left(r\omega_{pe}^2\right)}{2c^2}\left\{\sqrt{B_z{}^2 + \frac{B_0^2}{\left(1+b^2r^2\right)^3}} - B_z\right\}$$

$$B_0^2 \equiv \frac{2\mu_0{}^2c^2b^4}{\pi^2\omega_{pe}^2}\left(I^2 - I_B^2\right); I_B^2 \equiv \frac{8\pi^2 p_0}{\mu_0 b^2}$$

- Therefore as current oscillates and $I^2 - I_B^2 < 0$, B_z must become zero so that flux remains zero

Look at the signal



$\dot{\Phi}_z$

makes a sudden transition to zero and again recovers

Circuit model

- Inductance of the circuit can be defined in terms of total magnetic energy.
  - B_z must contribute to inductance

Flux-linkage $\Phi = L_0 I + L_1 I\sqrt{1 - I_B^2 / I^2}$

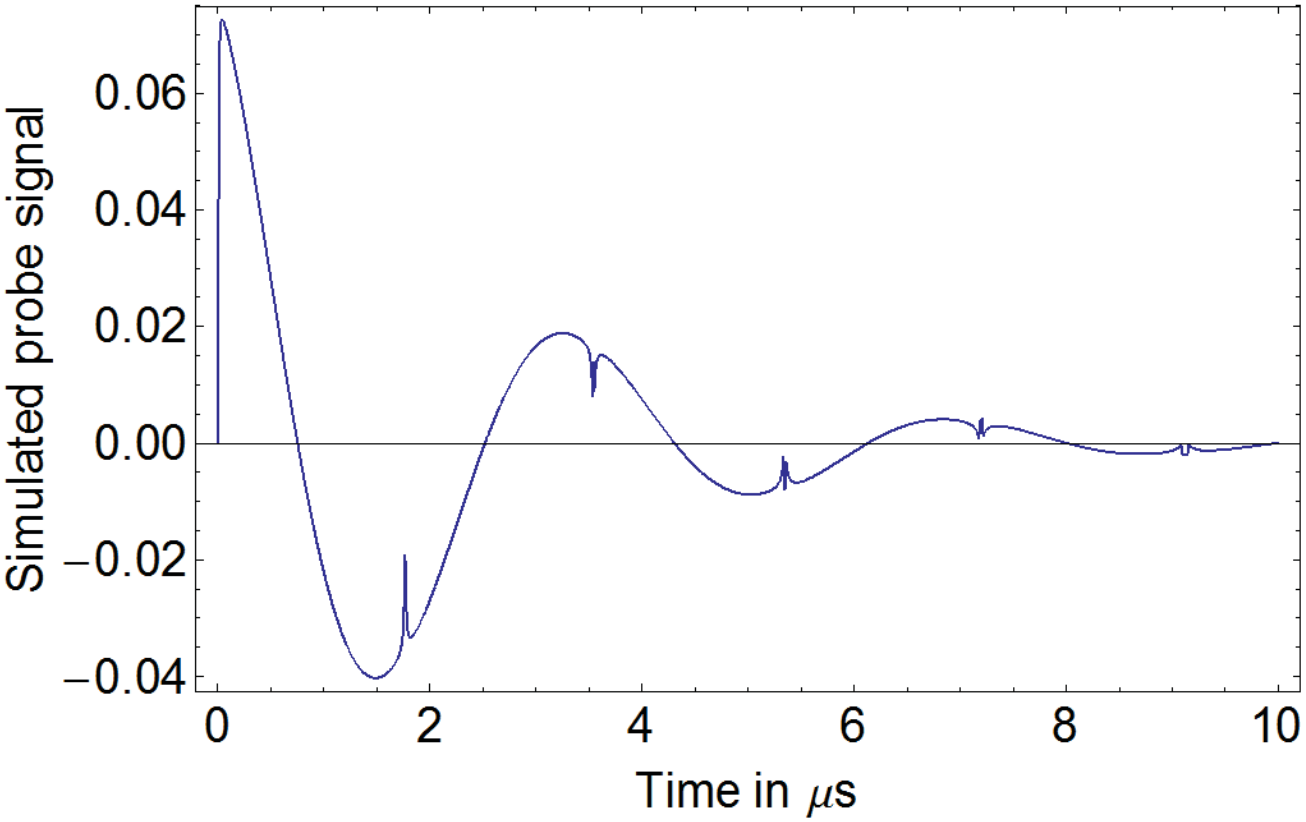
$$L_{eff} \frac{dI}{dt} = V_0 - C^{-1}\int_0^t I dt - R_{eff} I$$

$$L_{eff} \equiv \left(L_0 + L_1\sqrt{1 - I_B^2 / I^2} + \frac{L_1 I_B^2}{\sqrt{I^2 - I_B^2}} \right) \text{ for } I^2 - I_B^2 > 0,\ L_0 \text{ otherwise}$$

$$R_{eff} = R_0 + R_1\sqrt{1 - I_B^2 / I^2} \text{ for } I^2 - I_B^2 > 0,\ R_0 \text{ otherwise}$$

Simulated circuit model

- Bandwidth limitation can be modelled by a probe with some integration time constant

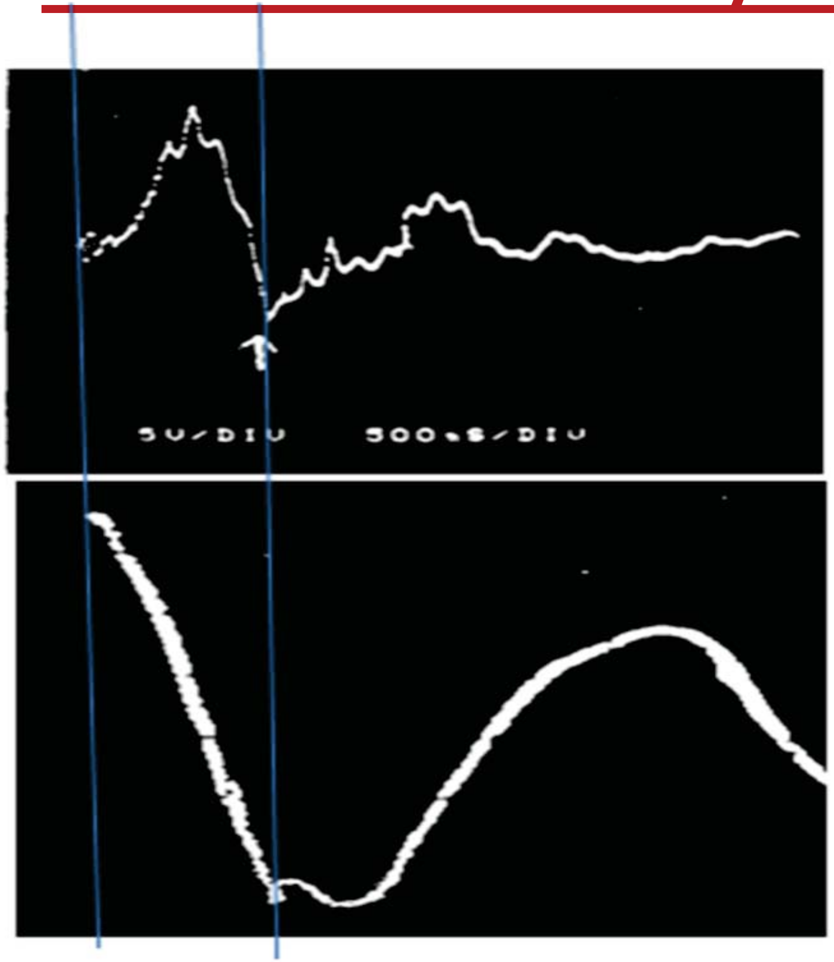


parameters :L_0=100 nH, L_1=4 nH, I_B=30 A, V_0=4 kV, C_0=2.8 μF, R_0=50 mΩ, R_1=40 mΩ.

Discussion

- Some strange structure is observed in experimental signals, which does not appear to be an artifact.
  - Needs more experimentation!!!
- “Constrained Dynamics” provides ONE way of understanding why such structure should be expected from fundamental plasma physics.
- If investigated further and confirmed, it provides an insight into why DPF produces solenoidal ions motion and so high fusion reactivity

Constrained Dynamics in DPF???



- Diamagnetic loop OUTSIDE THE CATHODE in DPF consistently shows a signal which begins at the start of current and continues for a long time

What is the suspense all about?

- The idea of constrained dynamics seems to suggest that the neutral gas ahead of the plasma sheath is a silent but active player in the plasma dynamics.
- This is very unexpected and intriguing. If it is true, it has enormous implications for the future of DPF research.
- BUT IS IT TRUE? ONLY EXPERIMENTS CAN TELL!

What this implies for DPF research

- Do free currents exist in the plasma sheath?
  - Place a diamagnetic loop OUTSIDE THE CATHODE.
  - Take all possible care to rule out artifacts.
  - If you consistently see a signal, find its characteristics
    - Structure, amplitude, reproducibility, integration, correlation with other properties, long-time behavior
  - See how it varies at different r & z.
  - Look for correlation with current factor in Lee model
  - Does the integrated signal last much beyond the current signal? This may indicate self-sustained structures in the post pinch plasma.

What this implies for DPF research

- Role of the neutral gas just ahead of sheath
  - Put an optical collimator through the anode, take out the light through a fiber and put it to a spectrograph.
  - Before the dense plasma comes into the field of view, record the H_α /H_β or He_α /He_β lines from the partially ionized precursor plasma. Interpret line shapes and positions
  - See how the line shapes and positions change as continuum radiation increases. Do you see Doppler shift beyond what can be explained by sheath motion?

What this implies for DPF research

- Look at the process of ingestion of neutral gas
  - How different ionized lines (say of Neon) appear as the sheath passes over the collimator in the anode.
- Look for rotational features in optical spectra
  - Place optical collimators in a plane perpendicular to the axis looking to left and right of the axis from opposite sides. Put these four optical channels to the slit of the spectrograph at the same time. Do the time-gated spectra look different? If so, that indicates existence of solenoidal flow.

Other Dense Magnetized Plasmas

- A gas embedded z-pinch with a hollow initial current profile should also experience constrained dynamics.
  - Put a diamagnetic loop outside the current return path.
  - Neutron TOF spectrum at 0° and 90° may have very different width
- GIT-12 experiments should also put a diamagnetic loop outside the cathode and look for azimuthal electric field.

Could this be a fusion breakthrough?

- A metallic tube filled with deuterium (or DT) is designed so that it explodes near the peak of current on a multi-mega-ampere fast capacitor bank.
- The current will be transferred to a shock-heated plasma layer, which is driven through neutral gas ahead of it. If constrained dynamics operates, this may produce a very large neutron yield.

Thank you

for your kind attention