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**2369-3**

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# Plasma confinement and turbulent transport in tokamak

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Joint-ICTP-IAEA College on Plasma Physics
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# Plasma Confinement and Turbulent transport in Tokamak

M. Kikuchi, JAEA

# 3-1. Dynamical system

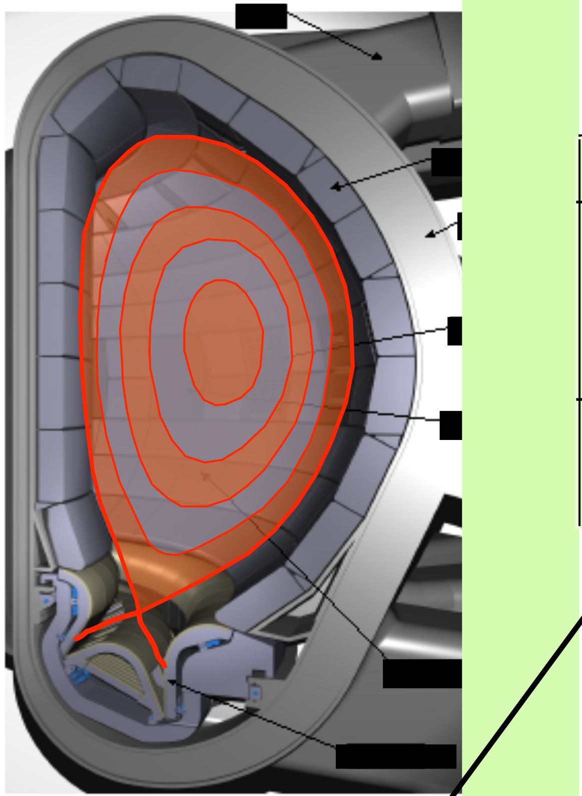
Concept of "attractor" in dynamical system is interesting to understand turbulence in high temperature plasma.

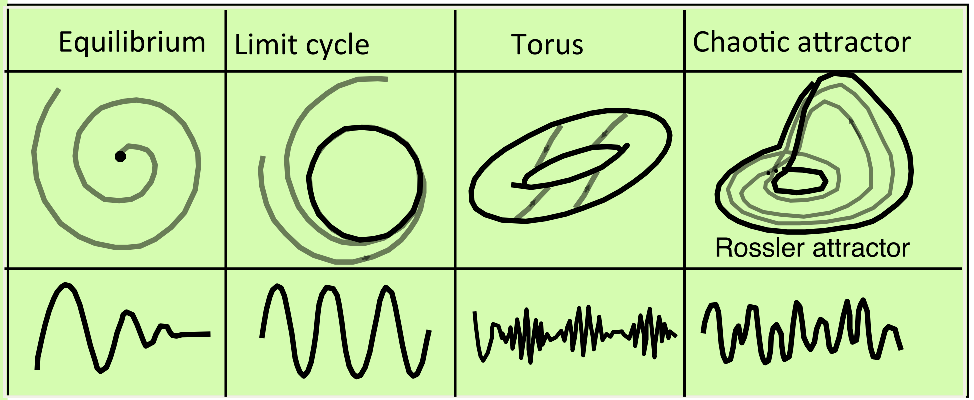
$$\dot{\mathbf{x}} = F(\mathbf{x}, \mu)$$

dynamical systems with some conserved quantity is called "**conservative system**" while the system with some quantity is being dissipated is called "**dissipative system**". Motion of the dissipative system is settled down to a specific trajectory or point after enough time. Stable state after the transient is called attractor. There are

[1] Equilibrium point: the motion converges to a point in phase space.
[2] Limit cycle: repeat the periodic motion in phase space.
[3] Torus: Motion in phase space wound the torus.
[4] Chaotic attractor: phase space orbit does not close forever.

# 3-2. To achieve good Plasma Confinement, suppression of turbulence is necessary

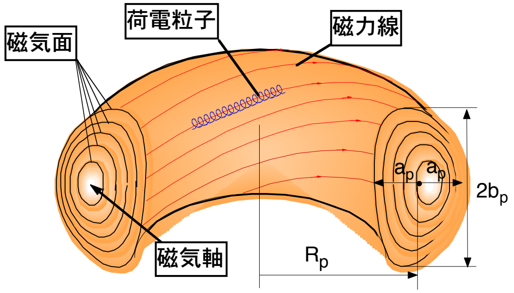





Phase space structure

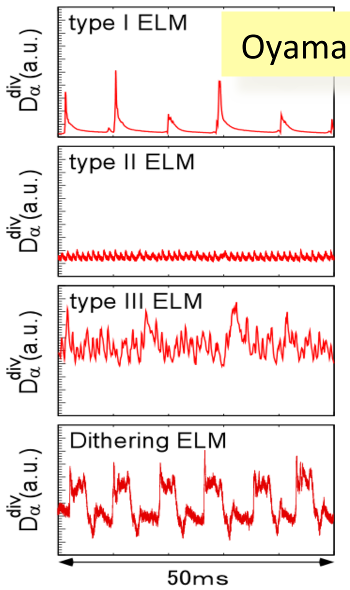
Wave form

Equilibrium, transition



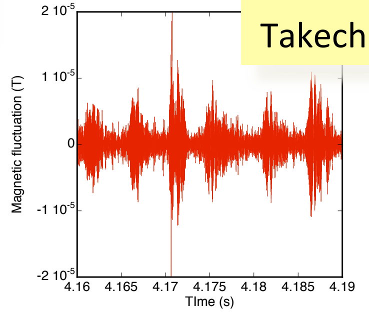

ELM?

Oyama



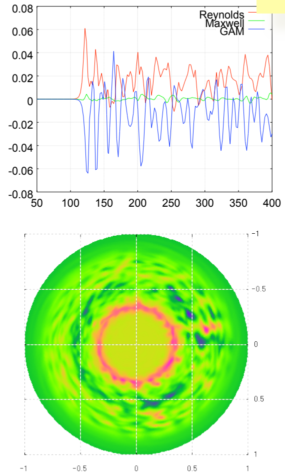

TAE？

Takechi




Turbulent transport？

Miyato



# 3-2 Self-organized critical transport (SOC)

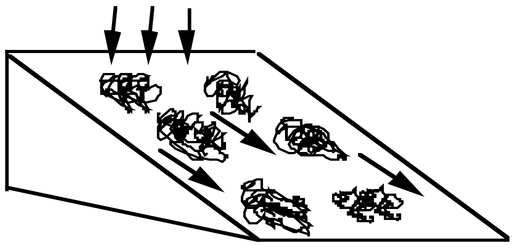
Equilibrium statistical physics⇔ Complexity Science

Gauss distribution ⇔ Power law

We frequently see power law in Nature.

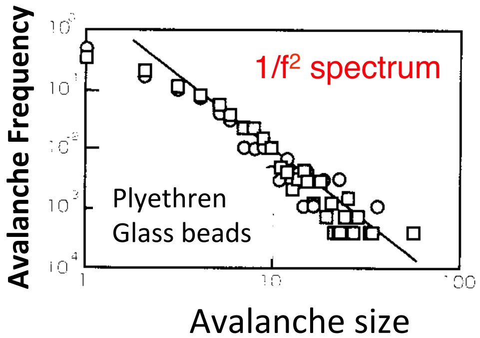

**P. Bak, Phys. Rev. A38(1988)364**

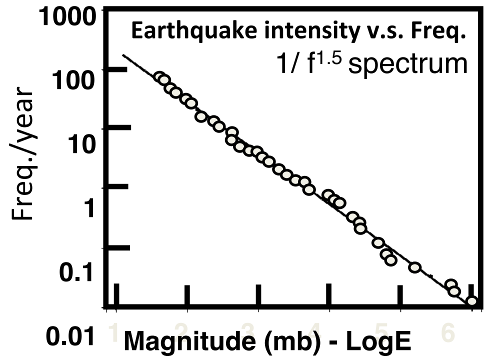
**Physics of Sand pile: critical gradient**



SOC : **Self-organized criticality**

Bak et.al. Phys. Rev. A 38, 364 1988



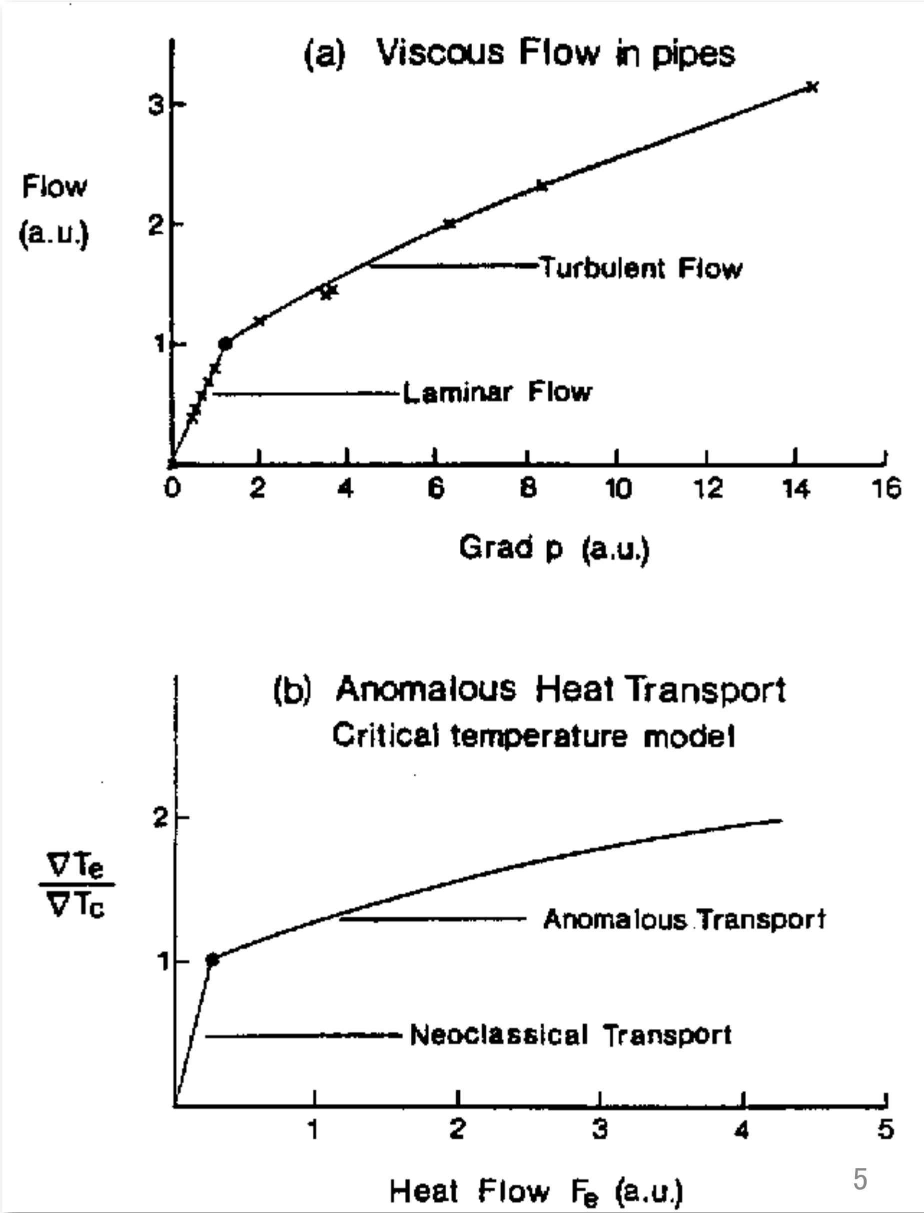

Gutenberg-Lichter Law

1/f noize

Self-organized criticality : system will become critical by itself( Per Bak)

# 3-3. Self-organized critical transport (Rebut)

**Laminar flow**
**and turbulent flow**




**P.H. Rebut(former JET director) is first to explain turbulent plasma transport by critical temperature gradient transport (1988).**

# 3.4 Drift wave : universal wave in confined plasma.

Sound wave ($\omega^2 = k_{//}^2 C_s^2$) coupled to drift motion ($\omega$*) is called "Drift Wave".

$$\omega(\omega - \omega_*) = k_\parallel^2 C_s^2$$

$$\omega_* = -(k_\perp T/eB)(\mathrm{d}\ln n_e/\mathrm{d}r)$$

$$C_s = ((Z_i T_e + \gamma_i T_i)/m_i)^{1/2}$$

Including polarization drift $\boldsymbol{v}_{pa} = \dfrac{m_a}{e_a B^2}\dfrac{\mathrm{d}\boldsymbol{E}}{\mathrm{d}t}$ , dispersion relation becomes

$$\omega^2(1 + \tau k_\perp^2 \rho_i^2/2) - \omega\omega_* = k_\parallel^2 C_s^2$$

If we include ion/e temperature gradient, we obtain following dispersion relation.

$$\omega(\omega - \omega_*) = k_\parallel^2 C_s^2 \left[1 + \frac{\omega_*}{\omega}\frac{Z_i}{\gamma_i \tau + Z_i}\left(\eta_i - \frac{\gamma_i - Z_i}{Z_i}\right)\right]$$

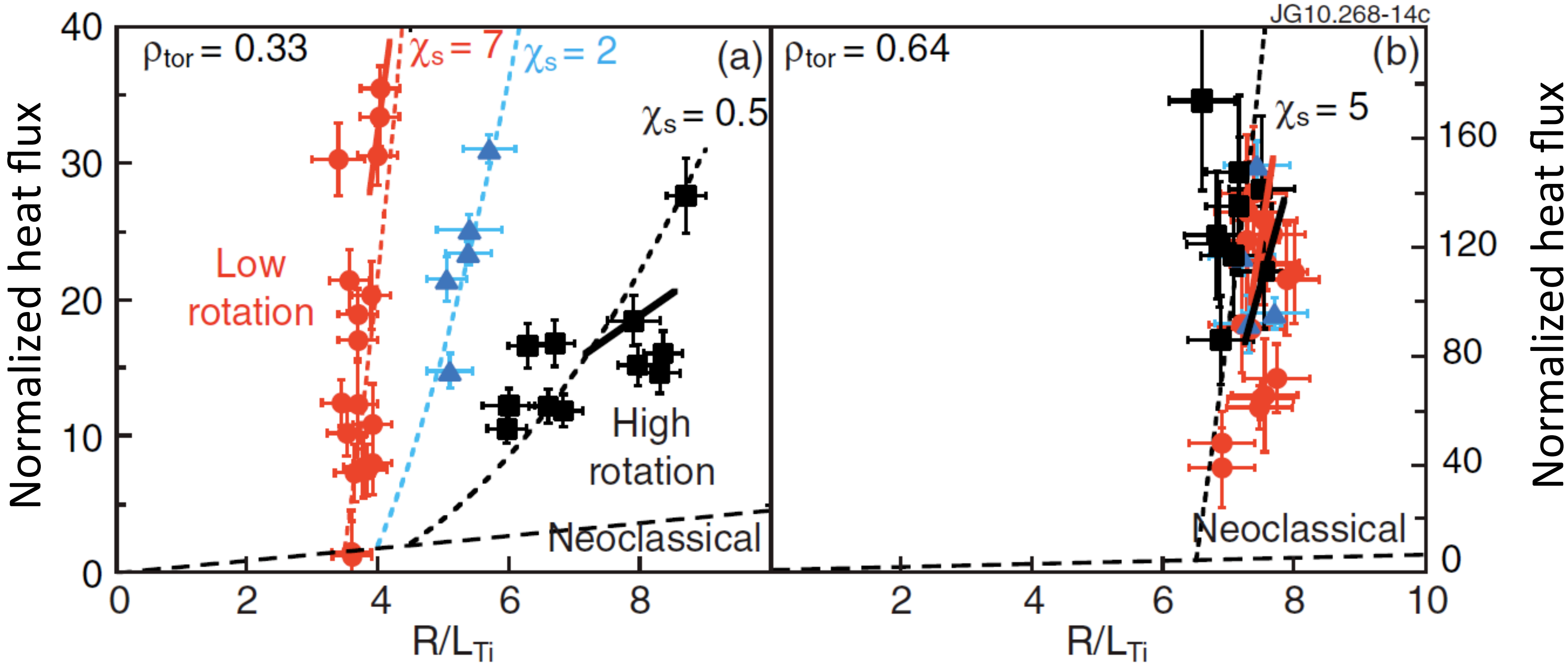
For the case of $Z_i = 1, \gamma_i = 5/2, \omega \sim k_\parallel C_s \ll \omega_*$,

$$\omega^2 \sim -\frac{k_\parallel^2 C_s^2}{2.5\tau + 1}\left(\eta_i - \frac{2}{3}\right)$$

**Critical temperature gradient**

# 3.5 Critical temperature gradient transport (ion)

Ion heat transport is governed by critical temperature gradient transport.
Note : high toroidal rotation (shear) reduce ion thermal diffusivity.



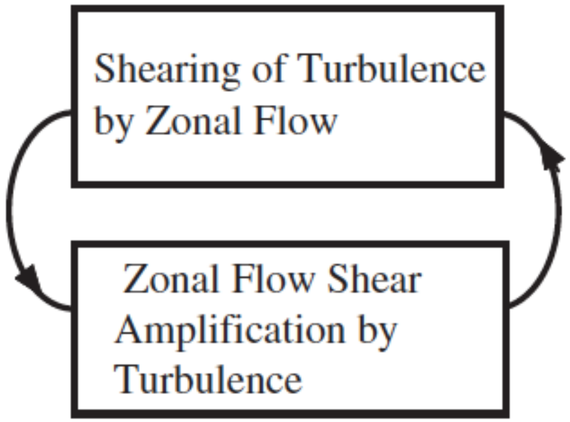

$L_{Ti}=T_i/dT_i/dr$

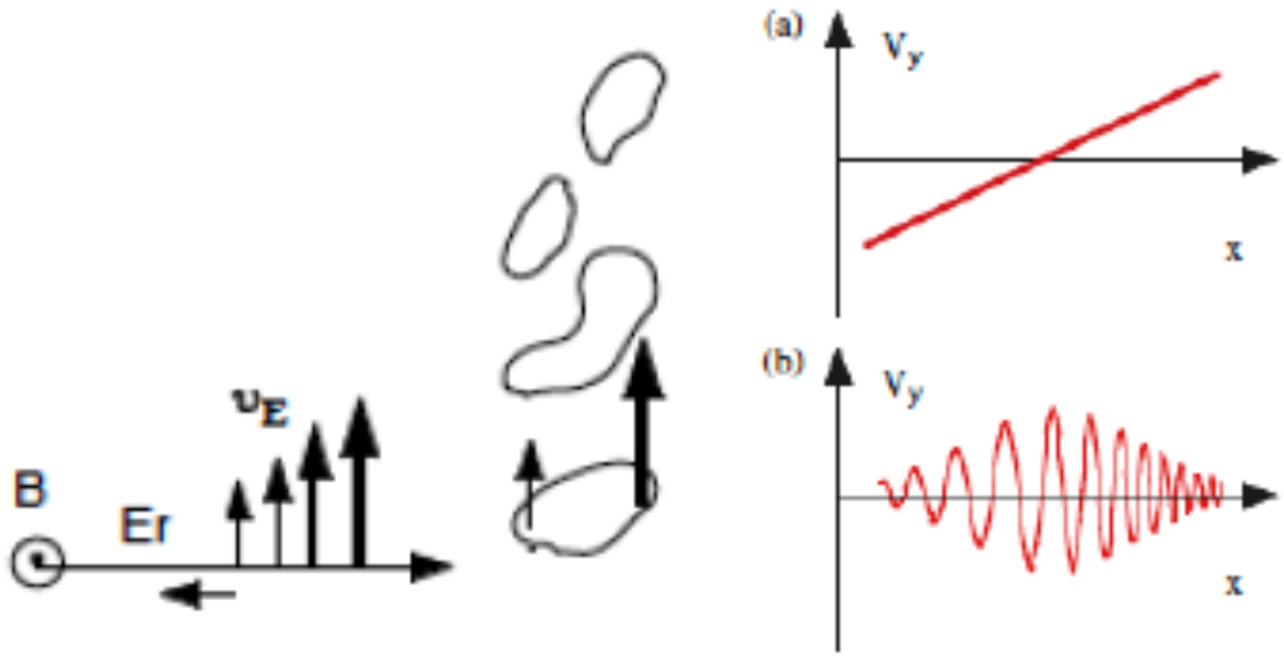
P. Mantica, PRL2009, PRL2011

# 3.6 Turbulence suppression by E x B drift and Zonal flow

Shear in Er x B drift can stabilize turbulence.
Basic picture is shown below.

There are two mechanisms to create Er.
- One is equilibrium radial electric field Er
- Other is Zonal flow created by turbulence.

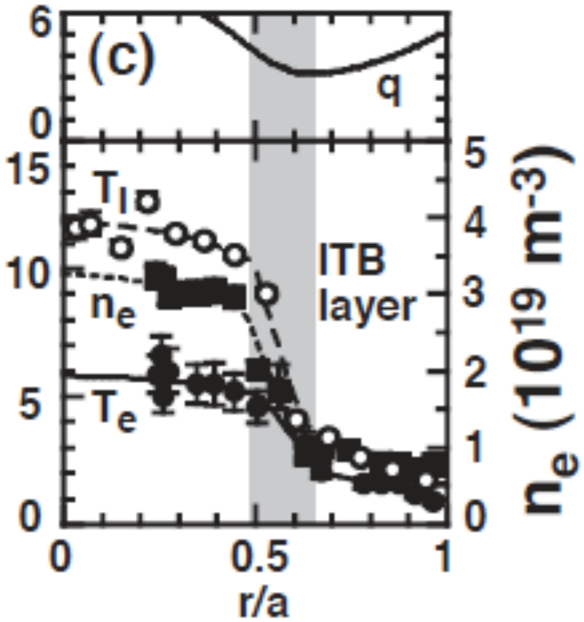


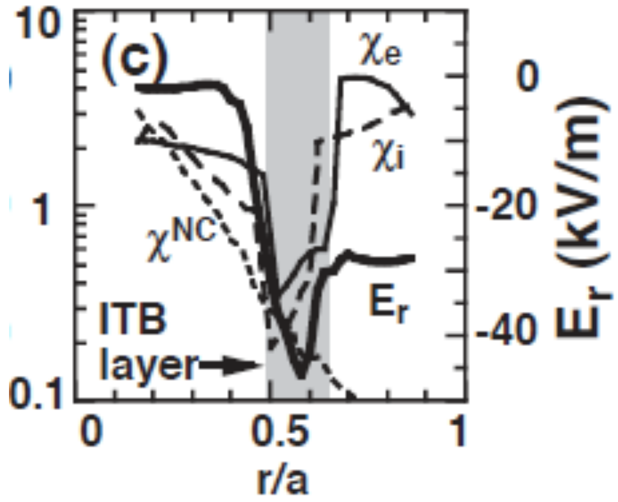




1) dEr/dr by equilibrium flow shear

2) dEr/dr by Zonal flow shear

# 3.6 Turbulence reduction by equilibrium flow shear




Shirai, Kikuchi, NF 1999

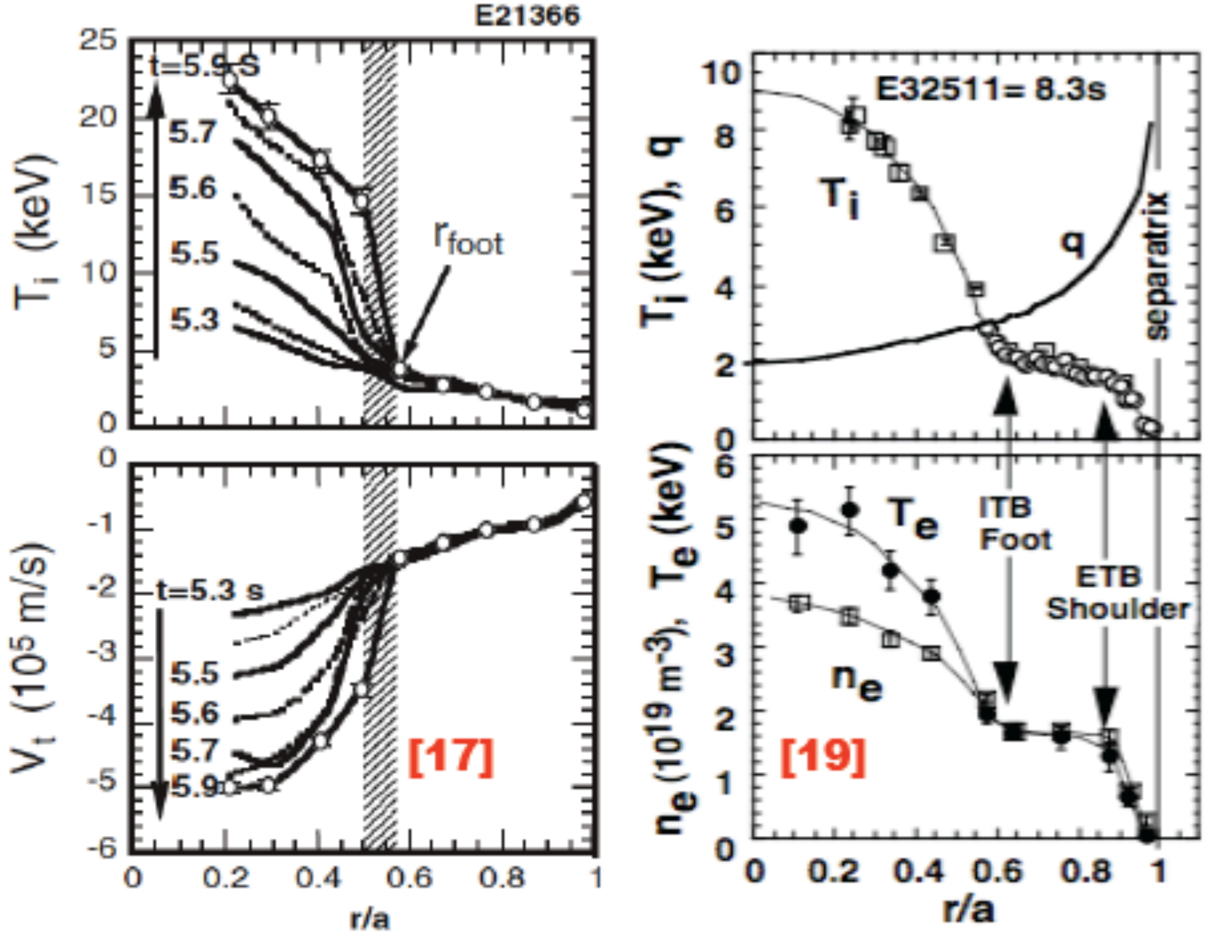
Er can be estimated from measurements.
Existence of Er shear at ITB(Internal transport barrier)
ITB: thin thermal insulation layer inside the plasma.

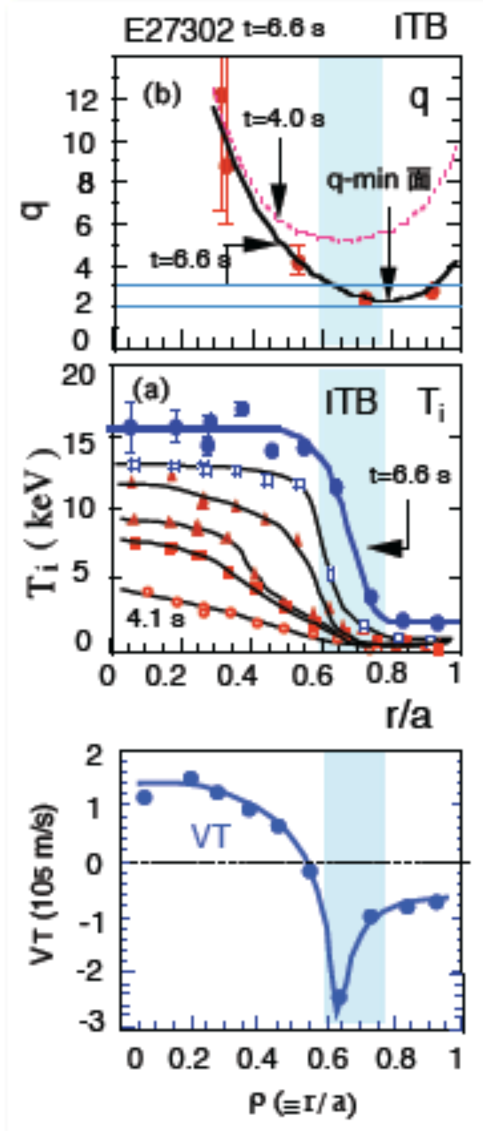
$$E_r = -\frac{\mathrm{d}\Phi}{\mathrm{d}\rho} = \frac{1}{\sum_{j=1}^{4} \alpha^*_{I,j}} \left( - \frac{\langle B^2 \rangle}{(RB_\phi)^2} \frac{\mathrm{d}\psi}{\mathrm{d}\rho} R u_{I\phi} - \sum_{j=1}^{4} \alpha^*_{I,j} \frac{1}{e_j n_j} \frac{\mathrm{d}P_j}{\mathrm{d}\rho} + \sum_{j=1}^{3} \alpha^*_{I,j+4} \frac{1}{e_j} \frac{\mathrm{d}T_j}{\mathrm{d}\rho} \right)$$

Impurity Charge-Exchange RS

Kikuchi, Frontier in Fusion Research, Springer,2011

## 3.6 Typical examples of ITB





[17] Y. Koide,S. Ishida et al., IAEA Seville vol1(1994)199

[18] Y. Koide, T. Takizuka et al., PPCF38(1996)1011: Citation=48

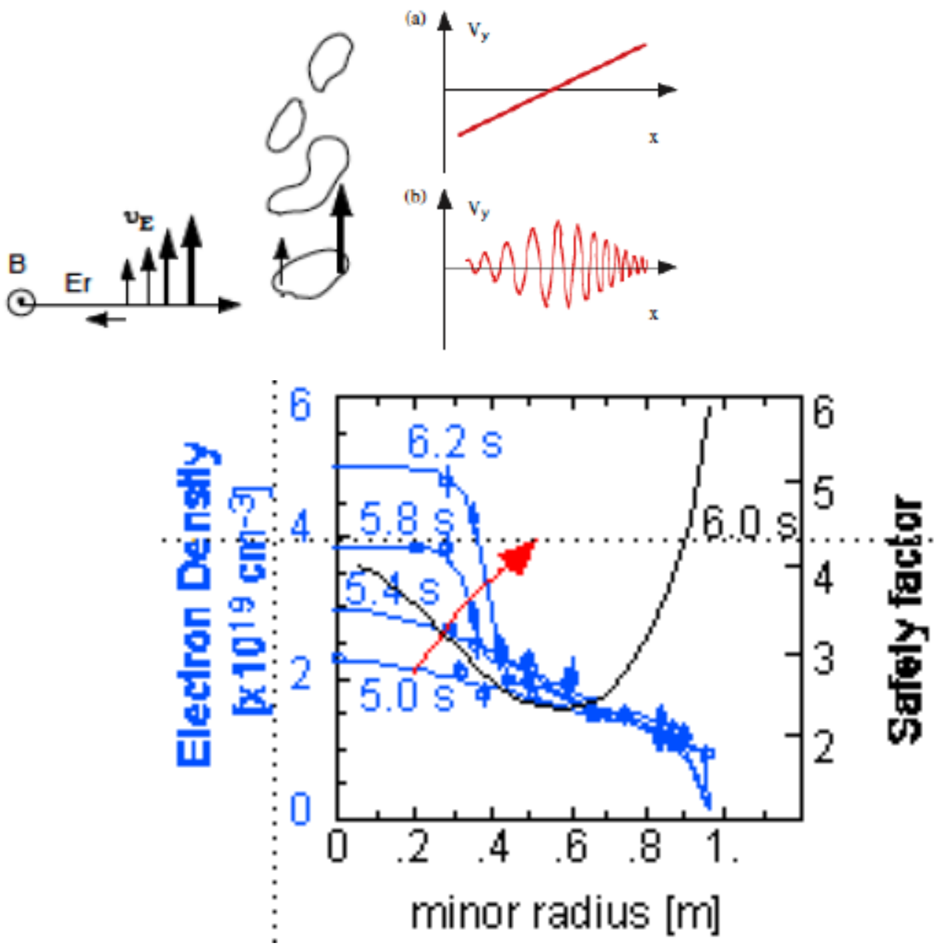
[19] Y. Kamada, PPCF42(2000)A65 : Citation=31

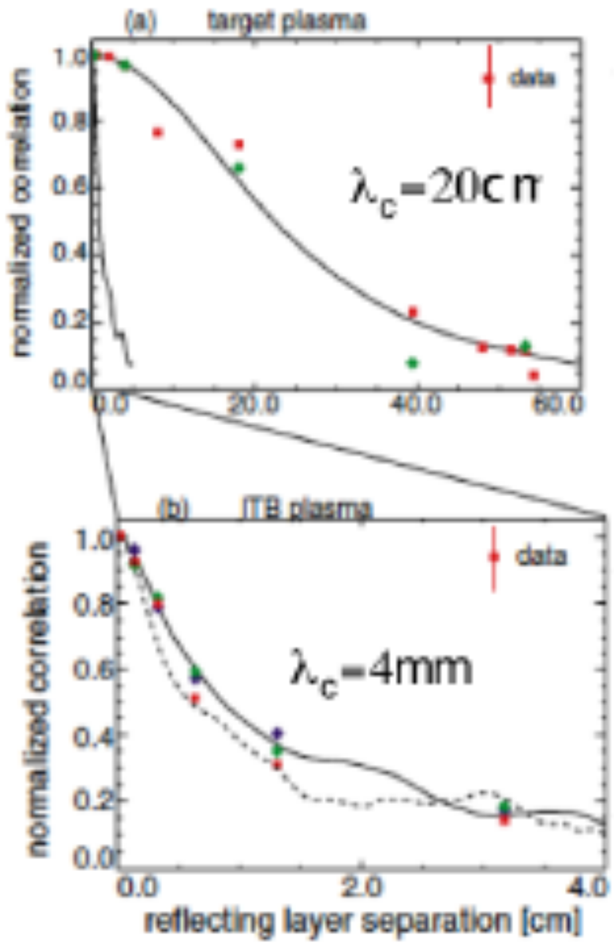
[20] Y. Koide, M. Mori et al., PPCF40(1998)641 : Citation=23


Fujita NF38(1998)207

# 3.6 Reduction of turbulent radial correlation

**as measured by correlation reflectometer**



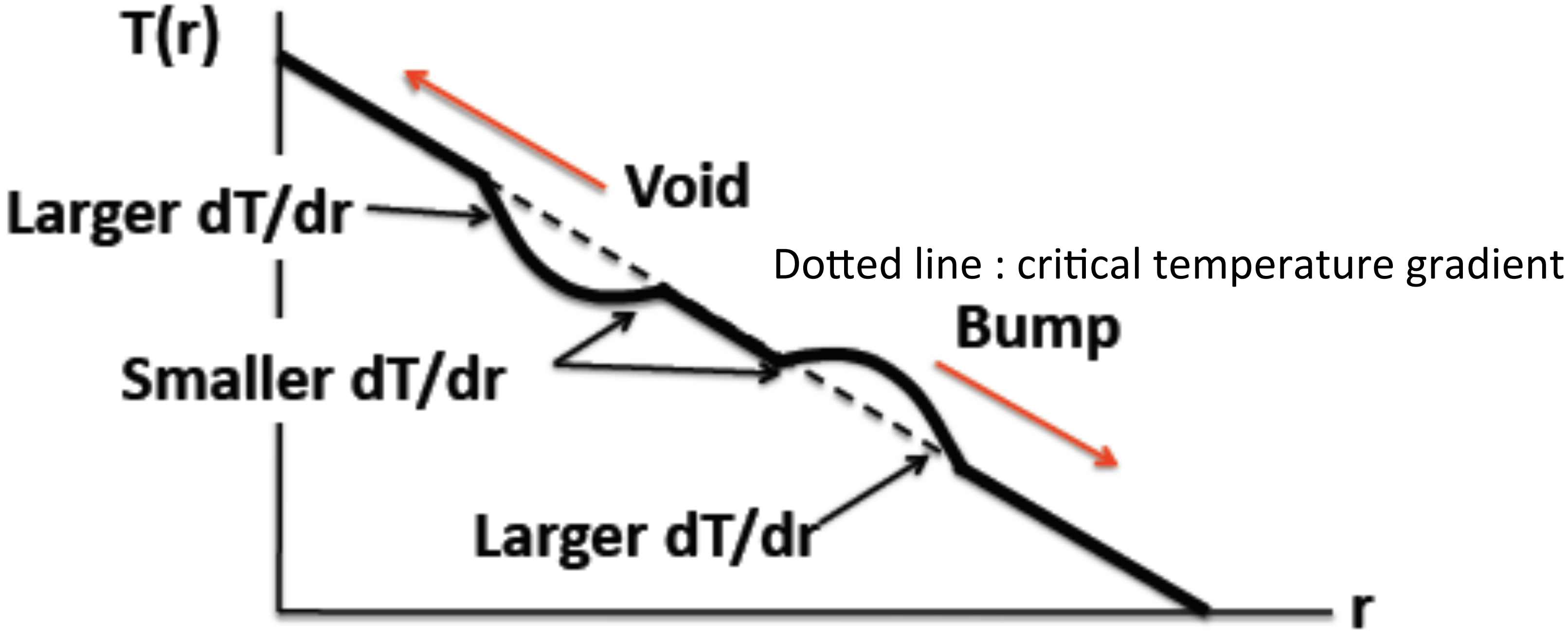




**R. Nazikian, K. Shinohara et al., PRL94(2005)135002, "Turbulence de-correlation during ITB formation"**

# 3.7 Avalanche dynamics of $(dT/dr)_c$ transport

Hwa’s Joint Reflection Symmetry:

Flux $\Gamma(\delta P)$ is invariant for x -> -x, $\delta$P -> -$\delta$P transformation.




Profile resilience: Temperature gradient stays near critical value

P.H. Diamond, T.S. Hahm, On the dynamics of turbulent transport near marginal stability, PoP 2(1995)3640

# 3.7 Avalanche dynamics with Er shear

0th order force balance equation $u_{\zeta i} = \frac{1}{B_\theta}\left[E_r - \frac{1}{eZ_i n_i}\frac{dP_i}{dr} - \frac{K_1}{eZ_i}\frac{dT_i}{dr}\right]$

When we have perturbation in T, Er will change.

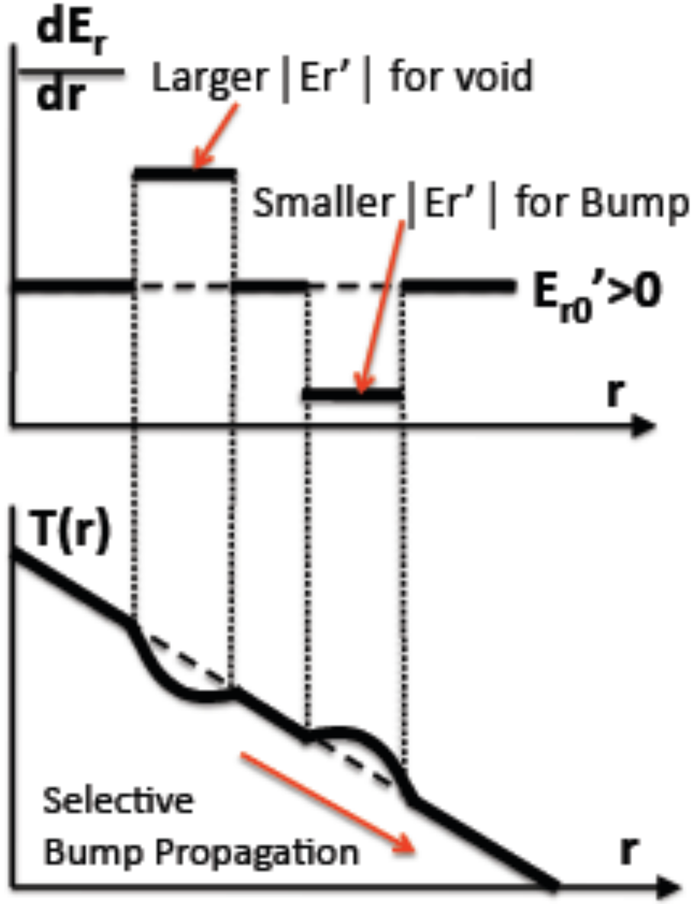
$$dE_r/dr \sim c d^2 T_i/dr^2$$

Void : $d^2T_i/dr^2 > 0$

Larger Er shear => turbulence is stabilized.

Bump : $d^2T_i/dr^2 < 0$

Lower Er shear => turbulence is destabilized.

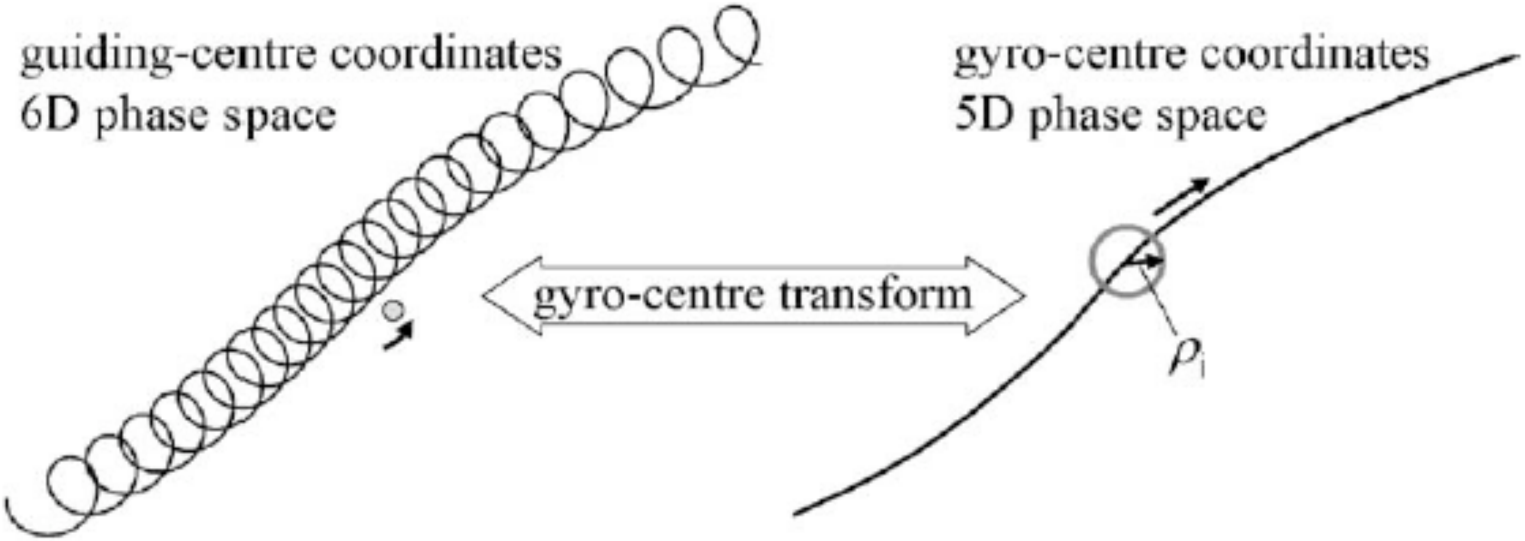


# 3.8 Gyrokinetic simulations

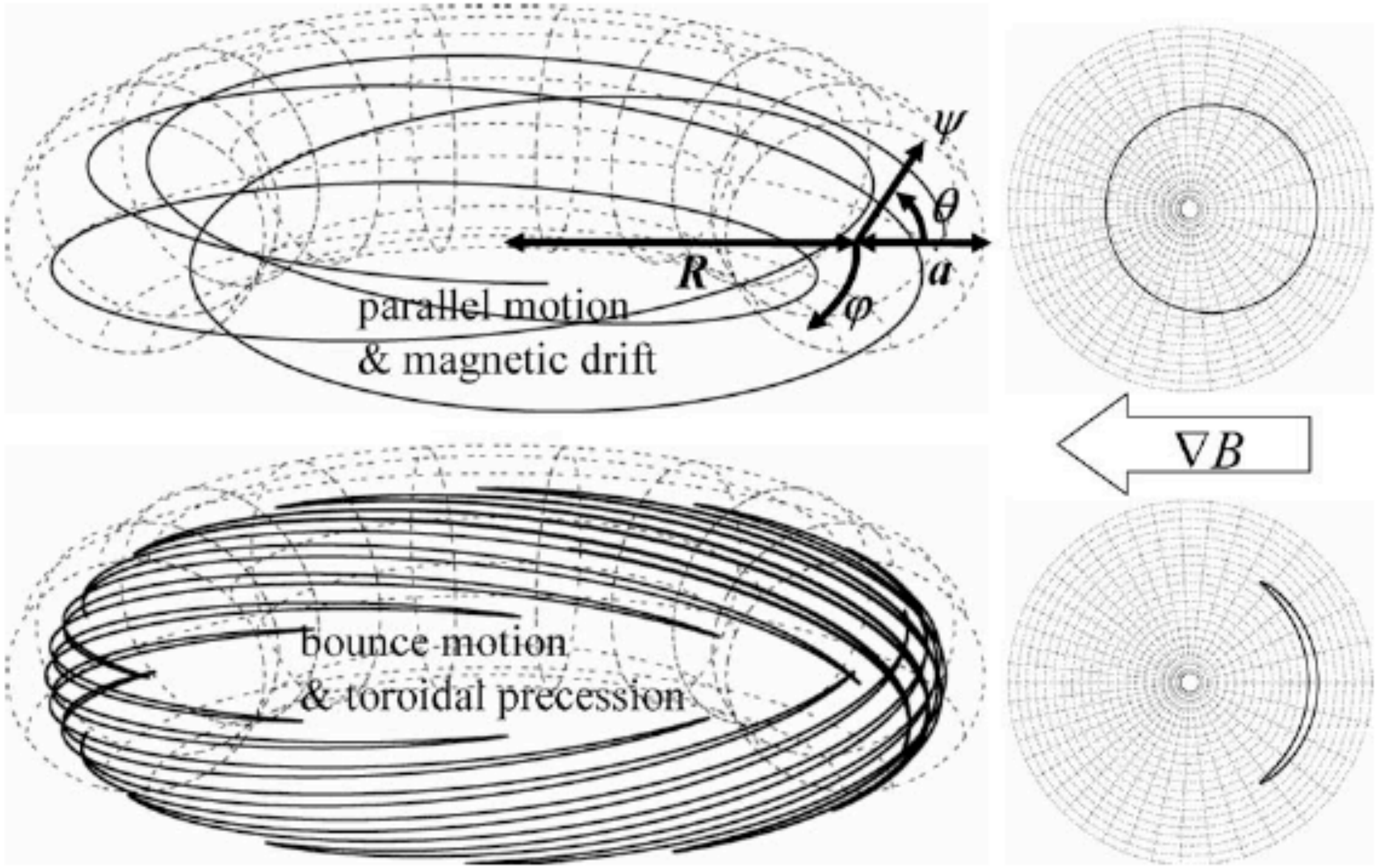
**Gyro kinetic/fluid simulation clarified self-organized criticality in ITG**

Lagrange-Hamilton mechanics of charged particle motion : see my Springer book

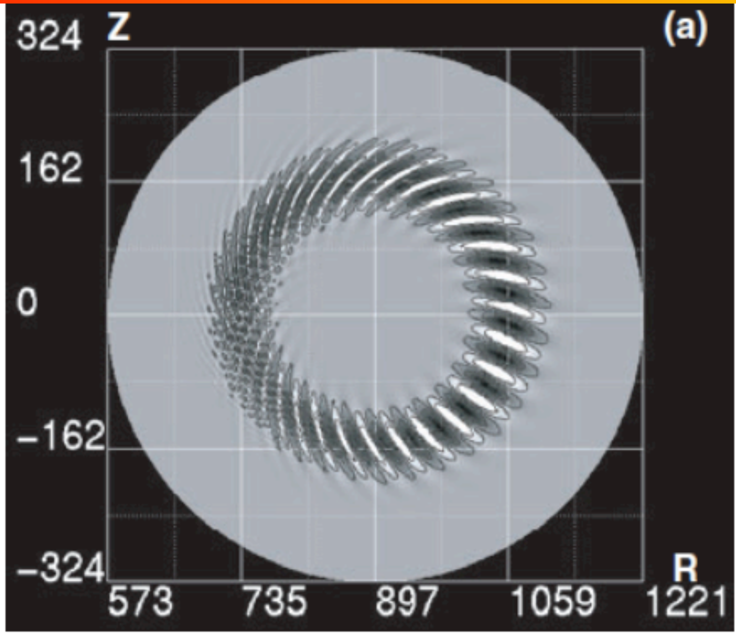
$$\frac{D\bar{f}_s}{Dt} \equiv \frac{\partial \bar{f}_s}{\partial t} + \{\bar{f}_s, \bar{H}_s\} = \frac{\partial \bar{f}_s}{\partial t} + \{\bar{\mathbf{R}}, \bar{H}\} \cdot \frac{\partial \bar{f}_s}{\partial \bar{\mathbf{R}}} + \{\bar{u}, \bar{H}\} \frac{\partial \bar{f}_s}{\partial \bar{u}} = 0$$



# 3.8 Gyrokinetic simulation



# 3.9 Ion Temperature Gradient (ITG) Mode

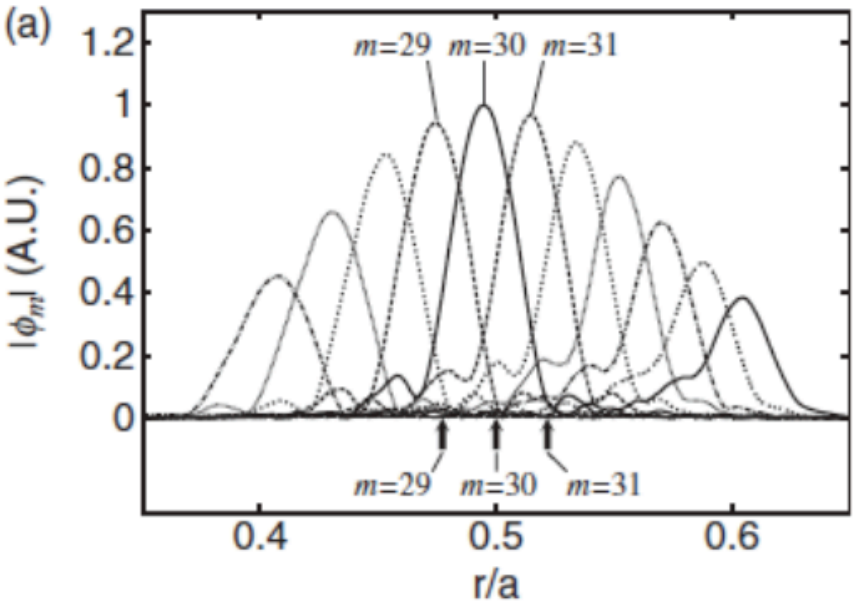



Equi-contour of electro static potential

Linea Eigenmode structure of toroidal ITG mode

Mode is radially elongated and tilted in poloidal direction.

The mode can be decomposed as overlapping of many poloidal harmonics.



# 3.10 Critical temperature gradient transport (electron)

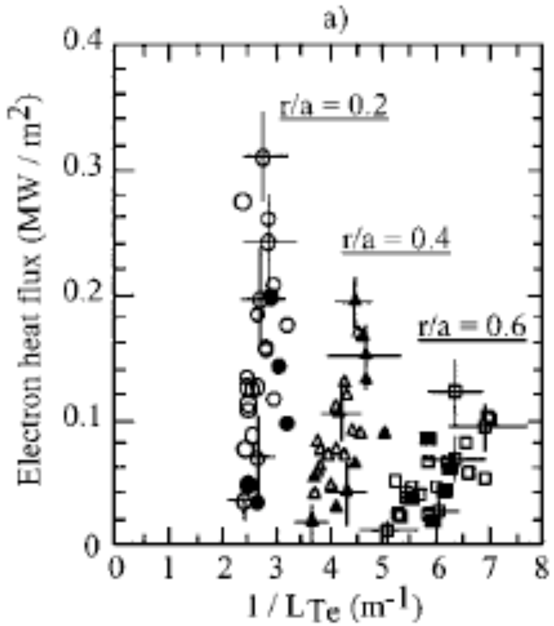
Existence of $dT_{ec}/dr$

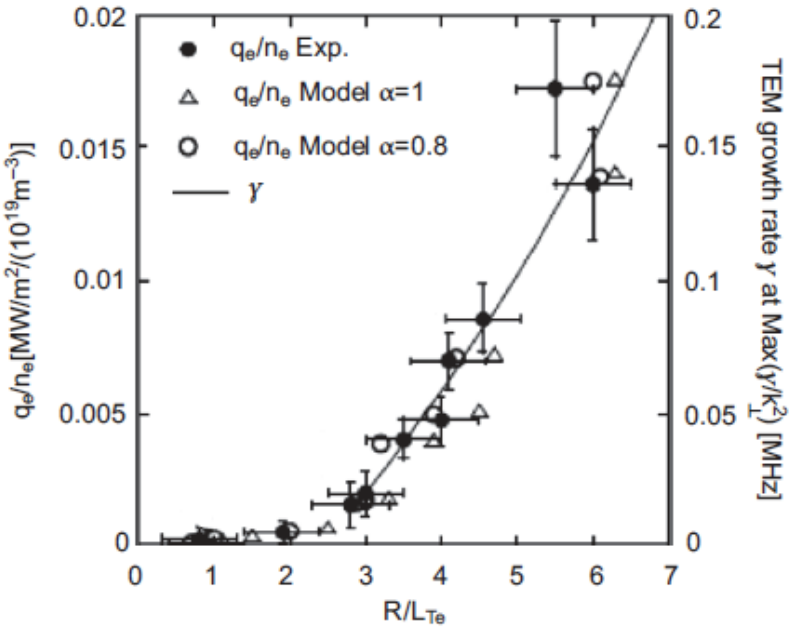
$$\frac{R}{L_c} = 5(\pm 1) + 10(\pm 2)\frac{|s|}{q}$$

ETG: Jenko formula on shear dependence

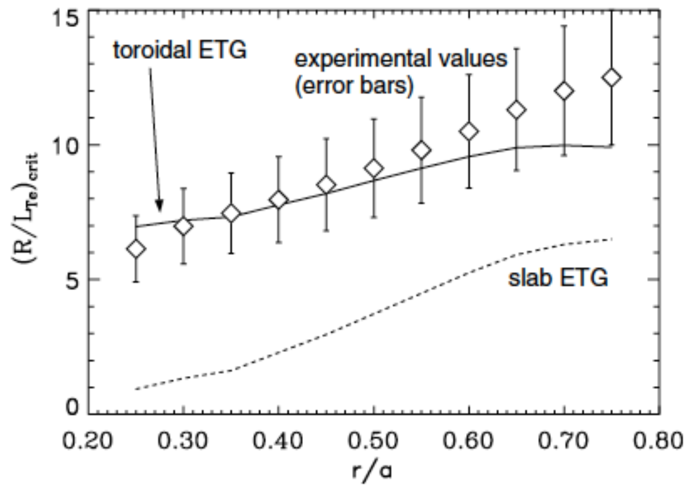
$$(R/L_{Te})_{crit} = (1+\tau_e)(1.33 + 1.91 s/q)$$

$$\tau_e = Z_{eff} T_e / T_i$$



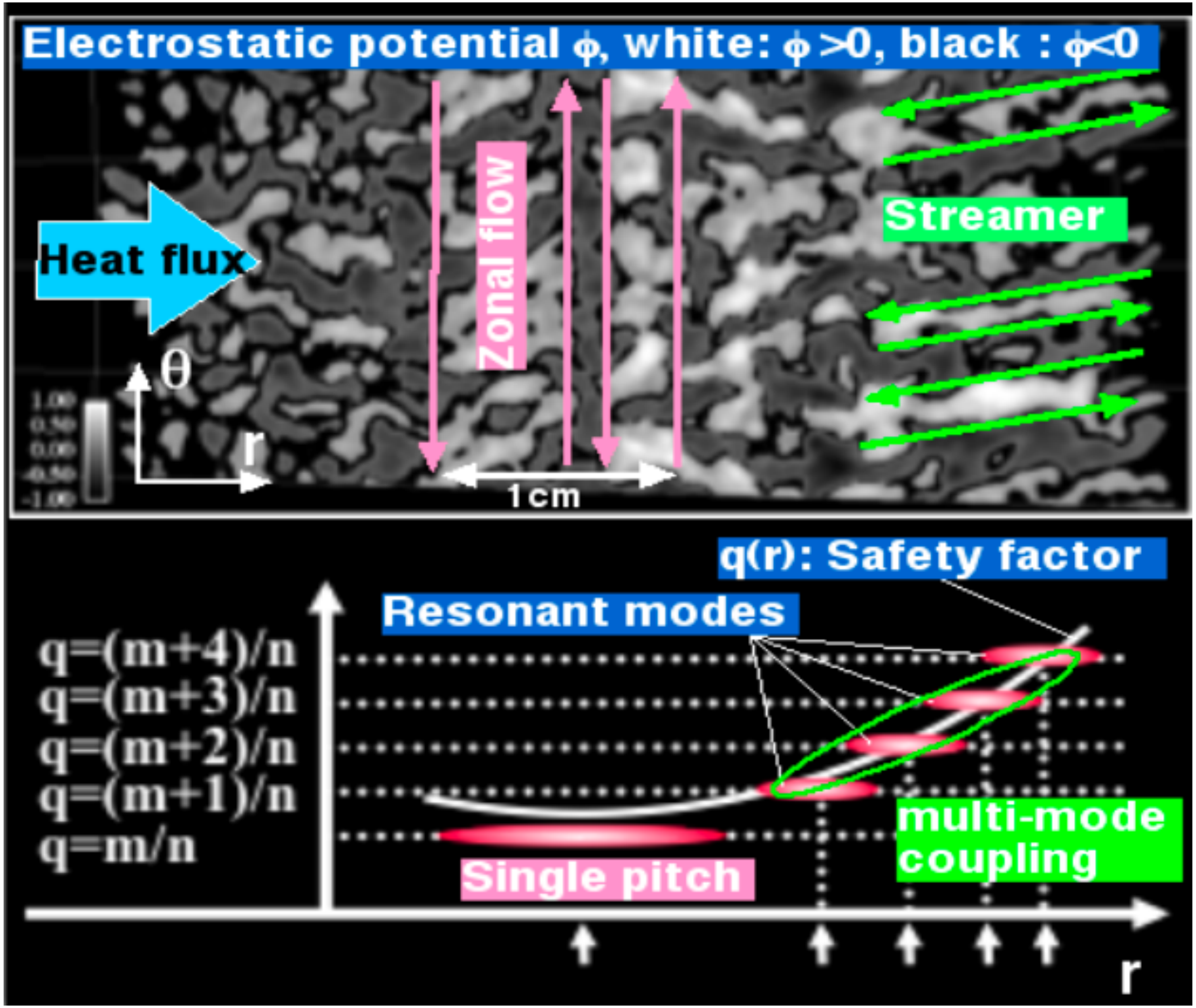




Ryter et al. , PRL95(2005)085001 (TEM)

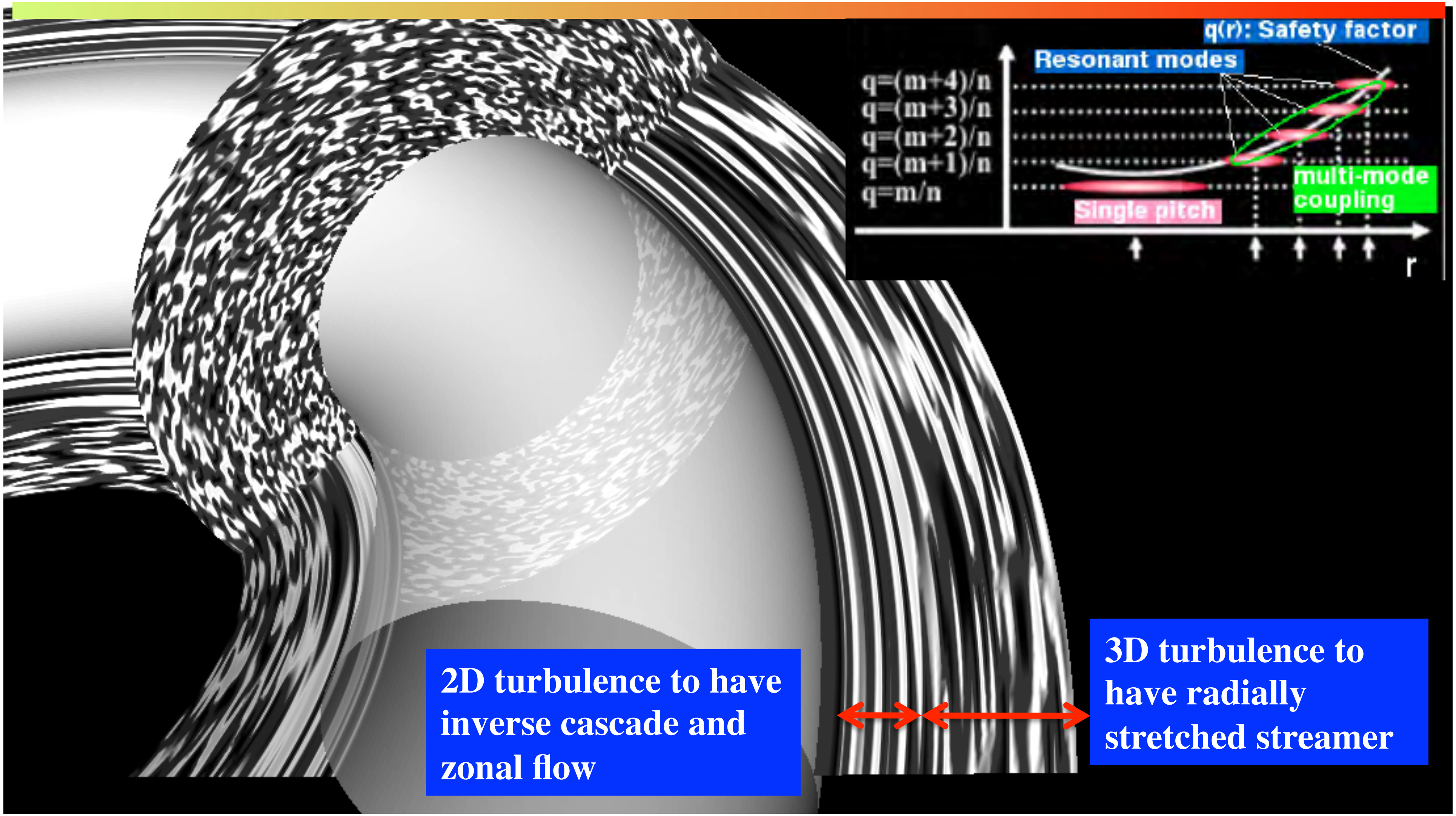


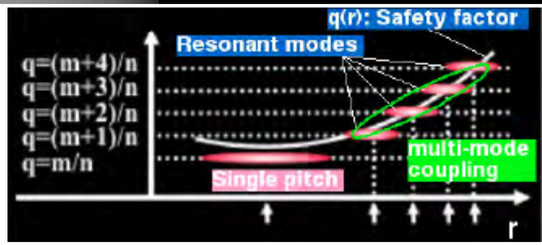

Hoang et al., PRL87(2001)125001(ETG)
Jenko, PRL 89(2002)

# 3.10 Zonal flow and streamer in ETG turbulence



# 3.10 2D and 3D turbulences in tokamak w and w/o mag. shear



## 3.11 Difference between 2D and 3D turbulences

3D turbulence : $\mathbf{u}\cdot\nabla\mathbf{u}$ produces wave with wave number 2 times larger.

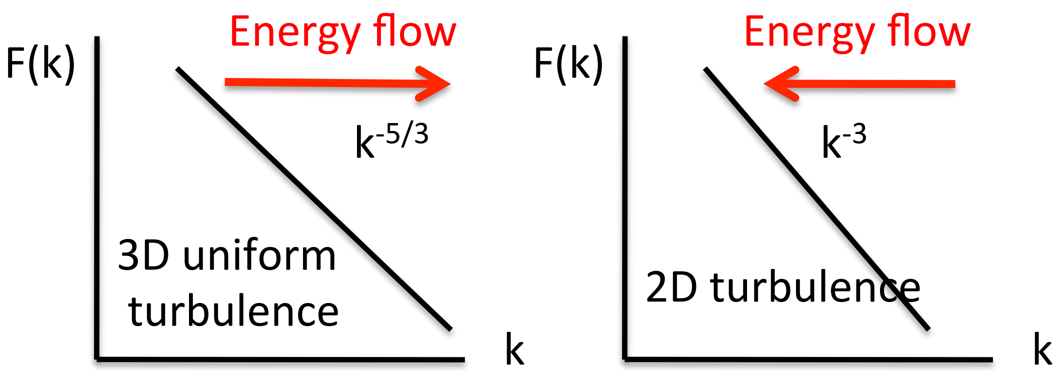
So, energy flow is in the direction of high k.

And energy dissipates at high k regime.

For isotropic 3D turbulence, Kolmogolov spectrum $F(k) \sim k^{-5/3}$.

2D turbulence : Energy flow is from high k to low k, opposite to 3D turbulence.

Spectrum in isotropic 2D turbulence : spectrum $F(k) \sim k^{-3}$.




See : P. Diamond, SI Itoh, K. Itoh
Modern plasma physics : volume 1,
Physical kinetics of turbulent plasmas

For your reference, M. Kikuchi "Frontier in Fusion Research", Springer, London, 2011

Mitsuru Kikuchi

**Frontiers in Fusion Research**

Physics and Fusion

*Frontiers in Fusion Research* provides a systematic overview of the latest physical principles of fusion and plasma confinement. It is primarily devoted to the principle of magnetic plasma confinement, that has been systematized through 50 years of fusion research.

*Frontiers in Fusion Research* begins with an introduction to the study of plasma, discussing the astronomical birth of hydrogen energy and the beginnings of human attempts to harness the Sun's energy for use on Earth. It moves on to chapters that cover a variety of topics such as:

- charged particle motion,
- plasma kinetic theory,
- wave dynamics,
- force equilibrium, and
- plasma turbulence.

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Engineering

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Kikuchi

Frontiers in Fusion Research

Mitsuru Kikuchi

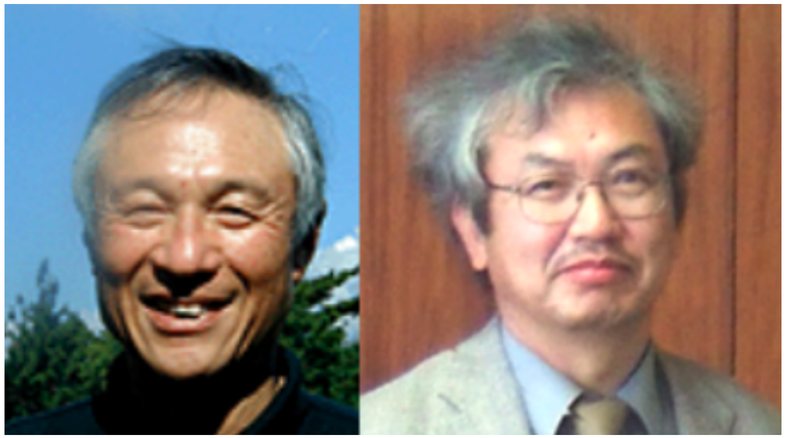
# Frontiers in Fusion Research

## Physics and Fusion

Springer

# End of 3rd lectures
# Below : appendix

Hasegawa-Mima equation (2D turbulence)



A Hasegawa and K. Mima

# 2D turbulence: Hasegawa-Mima equation

Turbulence is assumed electrostatic $\boldsymbol{E} = -\nabla_\perp \phi$

$\boldsymbol{B}$ and $T_e$ are uniform $\boldsymbol{B} = B\mathbf{z}$

Density gradient to drive drift wave $n = \tilde{n}(x, y, t) + n_0(x)$

Boltzmann relation for electron $\dfrac{\tilde{n}}{n_0} = \dfrac{e\phi}{T_e}$

Continuity equation $\dfrac{\partial n}{\partial t} + \nabla \cdot (n\boldsymbol{V}_\perp) = 0$

$$\frac{\partial}{\partial t}\frac{\tilde{n}}{n_0} + \boldsymbol{V}_\perp \cdot \frac{\nabla n_0}{n_0} + \nabla \cdot \boldsymbol{V}_\perp + \frac{1}{n_0}\nabla \cdot \left(\tilde{n}\boldsymbol{V}_\perp\right) = 0$$

$=O(\varepsilon^3)$

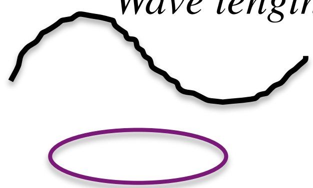
P. Diamond,
A Hasegawa,
K. Mima
Alfven Prize
winner 2011.

Ion flow is $\boldsymbol{E} \times \boldsymbol{B}$ and polarization drifts $\boldsymbol{V}_\perp = \boldsymbol{V}_E + \boldsymbol{V}_{pi}$

$\boldsymbol{E} \times \boldsymbol{B}$ drift $\boldsymbol{V}_E = -\dfrac{\nabla_\perp \phi \times \boldsymbol{B}}{B^2}$

Polarization drift $\boldsymbol{V}_p = \dfrac{m_a}{e_a B^2}\dfrac{d\boldsymbol{E}}{dt}$ *(important for ion)*

Fluctuation is small and low frequency $\dfrac{e\phi}{T_e} = O(\varepsilon), \dfrac{1}{\Omega_i}\dfrac{\partial}{\partial t} = O(\varepsilon)$




$$\frac{d}{dt} = \frac{\partial}{\partial t} + \boldsymbol{V}_E \cdot \nabla_\perp \longrightarrow \boldsymbol{V}_{pi} = -\frac{m_i}{eB^2}\left[\frac{\partial}{\partial t}\nabla_\perp \phi + (\boldsymbol{V}_E \cdot \nabla_\perp)\nabla_\perp \phi\right]$$

*Doppler shift*

# 2D turbulence: Hasegawa-Mima equation

$$\frac{\partial}{\partial t}\frac{\tilde{n}}{n_0} + \boldsymbol{V}_\perp \cdot \frac{\nabla n_0}{n_0} + \nabla \cdot \boldsymbol{V}_\perp + \frac{1}{n_0}\nabla \cdot \left(\tilde{n}\boldsymbol{V}_\perp\right) = 0$$

$= O(\varepsilon^3)$

***E x B*** nonlinearity is zero. $\nabla_\perp \cdot (\tilde{n}\boldsymbol{V}_E) = 0$ Since $\nabla_\perp \cdot (\phi \nabla_\perp \phi \times \mathbf{z}) = 0$

2nd order polarization term is small $\nabla_\perp \cdot (\tilde{n}\boldsymbol{V}_{pi}) = O(\varepsilon^3)$

$$\frac{\partial}{\partial t}\frac{e\phi}{T_e} - \frac{\nabla_\perp \phi \times \mathbf{z}}{B} \cdot \frac{\nabla n_0}{n_0} + \nabla_\perp \cdot \left[ -\frac{\nabla_\perp \phi \times \mathbf{z}}{B} - \frac{m_i}{eB^2}\left( \frac{\partial}{\partial t}\nabla_\perp \phi + \left(\frac{\nabla_\perp \phi \times \mathbf{z}}{B} \cdot \nabla_\perp\right)\nabla_\perp \phi \right) \right] = 0$$

$C_s = (T_e/m_i)^{1/2}$, $\rho_s = C_s/\Omega_i$, $V_{de} = -T_e(\partial n_0/\partial x)/eBn_0$

***E x B*** convection of Polarization drift is key to nonlinear drift wave coupling (See DII Appendix A)

$$\hat{\phi} = \frac{e\phi}{T_e} \qquad \left[1 - \rho_s^2 \nabla_\perp^2\right]\frac{\partial \hat{\phi}}{\partial t} + V_{de}\frac{\partial \hat{\phi}}{\partial y} + \rho_s^3 c_s \nabla_\perp \cdot \left[ (\nabla_\perp \hat{\phi} \times \mathbf{z} \cdot \nabla_\perp)\nabla_\perp \hat{\phi} \right] = 0$$

Since $\nabla_\perp \cdot \left[ (\nabla_\perp \hat{\phi} \times \mathbf{z} \cdot \nabla_\perp)\nabla_\perp \hat{\phi} \right] = \nabla_\perp \hat{\phi} \times \mathbf{z} \cdot \nabla_\perp \nabla_\perp^2 \hat{\phi}$ in 2D, we obtain following **H-M equation**.

$$\left[1 - \rho_s^2 \nabla_\perp^2\right]\frac{\partial \hat{\phi}}{\partial t} + V_{de}\frac{\partial \hat{\phi}}{\partial y} + \rho_s^3 c_s \nabla_\perp \hat{\phi} \times \mathbf{z} \cdot \nabla_\perp \nabla_\perp^2 \hat{\phi} = 0$$

# MHD Vorticity Equation

MHD fluid eq. $m_i\left[\frac{\partial \boldsymbol{V}}{\partial t}+\boldsymbol{V}\cdot\nabla\boldsymbol{V}\right]=eZ_i(\boldsymbol{E}+\boldsymbol{V}\times\boldsymbol{B})-\frac{1}{n}\nabla P$

We define MHD vorticity as $\boldsymbol{\omega}_{\mathrm{m}}=\boldsymbol{\omega}+\frac{eZ_i}{m}\boldsymbol{B}=\nabla\times(\boldsymbol{V}+\frac{eZ_i}{m}\boldsymbol{A})$

MHD fluid eq. $\frac{\partial \boldsymbol{V}}{\partial t}=\frac{eZ_i}{m_i}\boldsymbol{E}+\boldsymbol{V}\times\boldsymbol{\omega}_{\mathrm{m}}-\frac{1}{2}\nabla V^2-\frac{1}{m_i n}\nabla P$ Since $\boldsymbol{V}\cdot\nabla\boldsymbol{V}=\frac{1}{2}\nabla V^2-\boldsymbol{V}\times(\nabla\times\boldsymbol{V})$

Take its rotation $\nabla\times$

$$\frac{\partial \boldsymbol{\omega}}{\partial t}=-\frac{eZ_i}{m_i}\frac{\partial \boldsymbol{B}}{\partial t}+\nabla\times(\boldsymbol{V}\times\boldsymbol{\omega}_{\mathrm{m}})-0+\frac{1}{m_i n^2}\nabla n\times\nabla P$$

Since $\nabla\times(\boldsymbol{V}\times\boldsymbol{\omega}_{\mathrm{m}})=\boldsymbol{V}\nabla\cdot\boldsymbol{\omega}_{\mathrm{m}}-\boldsymbol{\omega}_{\mathrm{m}}\nabla\cdot\boldsymbol{V}+(\boldsymbol{\omega}_{\mathrm{m}}\cdot\nabla)\boldsymbol{V}-(\boldsymbol{V}\cdot\nabla)\boldsymbol{\omega}_{\mathrm{m}}$

$$\frac{d\boldsymbol{\omega}_m}{dt}+\boldsymbol{\omega}_{\mathrm{m}}\nabla\cdot\boldsymbol{V}=(\boldsymbol{\omega}_{\mathrm{m}}\cdot\nabla)\boldsymbol{V}+\frac{1}{m_i n^2}\nabla n\times\nabla P$$

or

$$n\frac{d}{dt}\left(\frac{\boldsymbol{\omega}_m}{n}\right)=(\boldsymbol{\omega}_{\mathrm{m}}\cdot\nabla)\boldsymbol{V}+\frac{1}{m_i n^2}\nabla n\times\nabla P$$

Since $\nabla\cdot\boldsymbol{V}=-\frac{dn}{dt}/n$

# 2D turbulence: Vorticity Equation

$$\frac{d\boldsymbol{\omega}_m}{dt} + \boldsymbol{\omega}_{\rm m}\nabla\cdot\boldsymbol{V} = (\boldsymbol{\omega}_{\rm m}\cdot\nabla)\boldsymbol{V} + \frac{1}{m_i n^2}\nabla n\times\nabla P \qquad \nabla n \sim \nabla P$$

2D turbulence

Only z component is important for 2D turbulence: $\boldsymbol{\omega}_m=\omega_m\mathbf{z}=(\omega+\Omega_i)\mathbf{z}$ , $\Omega_i=eB/m_i$

Vorticity $\omega = \boldsymbol{\omega}\cdot\mathbf{z} = \nabla\times\boldsymbol{V}\cdot\mathbf{z} \sim \nabla_\perp\times\boldsymbol{V}_E\cdot\mathbf{z} = -\left(\nabla_\perp\times\frac{\nabla_\perp\phi\times\mathbf{z}}{B}\right)\cdot\mathbf{z} = \frac{1}{B}\nabla_\perp^2\phi$

$$\hat{\phi} = \frac{e\phi}{T_e}$$

$$\frac{d\omega}{dt} = \frac{\partial\omega}{\partial t} + \frac{1}{B}\nabla_\perp\phi\times\mathbf{z}\cdot\nabla_\perp\omega = \rho_s^2\Omega_i\left[\underbrace{\frac{\partial}{\partial t}\nabla_\perp^2\hat{\phi}}_{1} + \underbrace{\rho_s C_s\left(\nabla_\perp\hat{\phi}\times\mathbf{z}\cdot\nabla_\perp\right)\nabla_\perp^2\hat{\phi}}_{2}\right]$$

$$\boldsymbol{\omega}_{\rm m}\nabla\cdot\boldsymbol{V} = -(\omega+\Omega_i)\frac{dn/dt}{n} \sim -\Omega_i\frac{dn/dt}{n} \sim -\Omega_i\left(\frac{dn_0/dt}{n_0} + \frac{d}{dt}\frac{e\phi}{T_e}\right)$$

$$\boldsymbol{\omega}_{\rm m}\nabla\cdot\boldsymbol{V} \sim -\Omega_i\left(\frac{1}{n_0 B}\nabla_\perp\phi\times\mathbf{z}\cdot\nabla_\perp n_0 + \frac{\partial}{\partial t}\hat{\phi}\right) = -\Omega_i\left[\frac{\partial}{\partial t}\hat{\phi} + \frac{T_e(\partial n_0/\partial x)}{e n_0 B}\frac{\partial\hat{\phi}}{\partial y}\right]$$

**HM eq.** $$\underbrace{\left[1-\rho_s^2\nabla_\perp^2\right]}_{1}\frac{\partial\hat{\phi}}{\partial t} + V_{de}\frac{\partial\hat{\phi}}{\partial y} + \underbrace{\rho_s^3 c_s\nabla_\perp\hat{\phi}\times\mathbf{z}\cdot\nabla_\perp\nabla_\perp^2\hat{\phi}}_{2} = 0$$

# 2D turbulence: Vorticity Equation

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2D turbulence

Only z component is important for 2D turbulence: $\boldsymbol{\omega}_m = \omega_m \mathbf{z} = (\omega + \Omega_i)\mathbf{z}$ , $\Omega_i = eB/m_i$

Vorticity $\quad \omega = \boldsymbol{\omega}\cdot\mathbf{z} = \nabla\times\boldsymbol{V}\cdot\mathbf{z} \sim \nabla_\perp\times\boldsymbol{V}_E\cdot\mathbf{z} = -\left(\nabla_\perp\times\frac{\nabla_\perp\phi\times\mathbf{z}}{B}\right)\cdot\mathbf{z} = \frac{1}{B}\nabla_\perp^2\phi$

$$\boxed{\hat{\phi} = \frac{e\phi}{T_e}}$$

$$\frac{d\omega}{dt} = \frac{\partial\omega}{\partial t} + \frac{1}{B}\nabla_\perp\phi\times\mathbf{z}\cdot\nabla_\perp\omega = \rho_s^2\Omega_i\left[\underbrace{\frac{\partial}{\partial t}\nabla_\perp^2\hat{\phi}}_{1} + \underbrace{\rho_s C_s\left(\nabla_\perp\hat{\phi}\times\mathbf{z}\cdot\nabla_\perp\right)\nabla_\perp^2\hat{\phi}}_{2}\right]$$

$$\boldsymbol{\omega}_{\rm m}\nabla\cdot\boldsymbol{V} = -(\omega + \Omega_i)\frac{dn/dt}{n} \sim -\Omega_i\frac{dn/dt}{n} \sim -\Omega_i\left(\frac{dn_0/dt}{n_0} + \frac{d}{dt}\frac{e\phi}{T_e}\right)$$

$$\boldsymbol{\omega}_{\rm m}\nabla\cdot\boldsymbol{V} \sim -\Omega_i\left(\frac{1}{n_0 B}\nabla_\perp\phi\times\mathbf{z}\cdot\nabla_\perp n_0 + \frac{\partial}{\partial t}\hat{\phi}\right) = -\Omega_i\left[\frac{\partial}{\partial t}\hat{\phi} + \frac{T_e(\partial n_0/\partial x)}{e n_0 B}\frac{\partial\hat{\phi}}{\partial y}\right]$$

**HM eq.** $\quad \underbrace{\left[1 - \rho_s^2\nabla_\perp^2\right]}_{1}\frac{\partial\hat{\phi}}{\partial t} + V_{de}\frac{\partial\hat{\phi}}{\partial y} + \underbrace{\rho_s^3 c_s\nabla_\perp\hat{\phi}\times\mathbf{z}\cdot\nabla_\perp\nabla_\perp^2\hat{\phi}}_{2} = 0$