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Acceleration of Plasma Flows – Theory and Simulation

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Based On:

1. S. Ohsaki, N.L.Shatashvili, Z. Yoshida and S.M. Mahajan. *The Astrophys. J.* **559**, L61 (2001); **570**, 395 (2002)
2. S.M. Mahajan, N.L. Shatashvili, S.V. Mikeladze & K.I. Sigua. *The Astrophys. J.* **634**, 419 (2005)
3. S.M. Mahajan, K.I. Nikol'skaya, N.L. Shatashvili_& Z. Yoshida. *The Astrophys. J.* **576**, L161 (2002)
4. S.M. Mahajan, N.L. Shatashvili, S.V. Mikeladze & K.I. Sigua. *Phys. Plasmas.* **13**, 062902 (2006)
5. Z. Yoshida & N.L. Shatashvili. *AIP Conf. Proc.* **1392**, 73 (2011); *ArXiv:*1105.5281v1 [astro-ph.GA] (2011)

Outline

- **Acceleration of *particles – sources of free energy for acceleration***
- **Acceleration / Generation of *Large Scale Flows* - *sources of energy for acceleration***
- **Magneto-Fluid coupling – model equations – *Quasi-equilibrium approach***
- **Acceleration / Generation of Plasma flows - incompressible plasma case - Catastrophe**
- **Acceleration / Generation of Plasma flows - incompressible plasma case – Reverse Dynamo**
- **Acceleration / Generation of Plasma flows - compressible plasma case**
- **Acceleration / Generation of Plasma flows - compressible rotational plasma case – Beltrami structures in disk-jet system: alignment of flow and generalized vorticity**
- **Summary & Conclusions, *perspectives***

Acceleration of *particles*

- **High-energy particle acceleration is observed in a diverse variety of laboratory devices as well as astrophysical sites ranging from the terrestrial aurorae to the most distant quasars** (*R.D. Blandford. ApJ 1994*).

- **In astrophysical sites particle acceleration is a fairly common channel for the release of large-scale kinetic, rotational and magnetic energy.**

- **Physical mechanisms include electrostatic acceleration, stochastic processes and diffusive shock energization.**

- **The overall acceleration efficiency is controlled by the low energy particle injection.**

Sources of free energy for particle acceleration

- **Bulk kinetic energy of fluid motion in the form of shock front - often consequence of an explosion** (supernova, coronal mass ejection); **relativistic shocks -** may take the form of **a standing shock** as in the termination shock of the Crab pulsar wind, or may travel as **ultra-relativistic jets** of extragalactic radio sources.

- **Rotational energy** - invoked in Jupiter-Jo system; radio pulsars; black holes in both AGN and stellar binary systems.

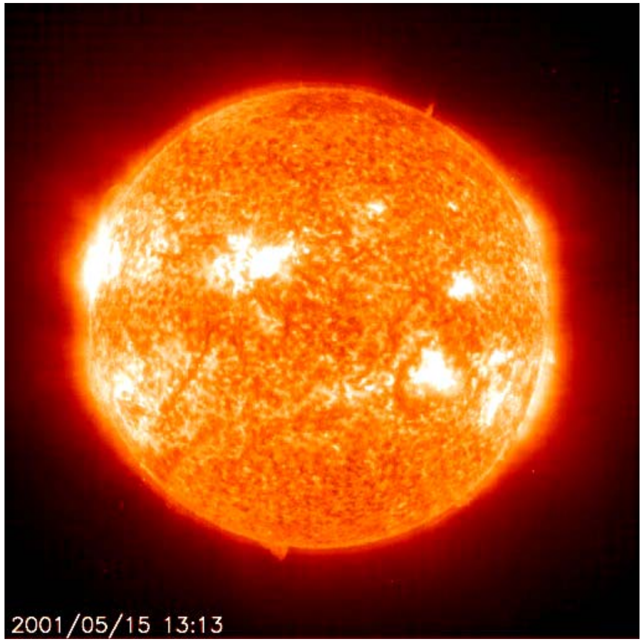
- **Magnetic energy** - Earth's magnetotail; solar flares; is conjectured to occur in coronal regions above accretion disks, and etc.

Acceleration / Generation of *Large Scale Flows*

- **Acceleration of plasma flows** - creation of stellar winds, variety of jets, zonal flows in TOKAMAK-s & etc. – often observed **both in laboratory and astrophysical conditions.**

- **Recent observations - solar corona is a highly dynamic arena replete with multi-species multiple–scale spatiotemporal structures (*e*.g.** *Aschwanden, Poland & Rabin, 2001, ARA&A*).

- **Strong flows are found everywhere in the low atmosphere — in the sub-coronal (*chromosphere*) as well as in coronal regions –** *recent observations from HINODE (De Pontieu et al. 2011,20122).*



Equally important: the plasma flows may complement the abilities of the magnetic field in the creation of the amazing richness observed in the coronal structures.

Sources of energy for plasma flow acceleration

The most obvious process for acceleration (***rotation is ignored***):

the **conversion of**

- **magnetic**
- **and/or the thermal energy**
- **turbulence energy**

===> to plasma kinetic energy

Magnetically driven transient but sudden flow–generation models:

- **Catastrophic models**
- **Magnetic reconnection models**
- **Models based on instabilities**

Quiescent pathway:

- ***Bernoulli mechanism converting thermal energy into kinetic***
- **General magneto-fluid rearrangement of a relatively constant kinetic energy:** going from an initial high density–low velocity to a low density–high velocity state.

Magneto-Fluid coupling – model equations

- **Minimal two-fluid model** – **incompressible, constant density Hall MHD** – ***gravity & rotation are ignored.***

- **Dimensionless system in standard Alfvenic units.**

  Velocities - normalized to the Alfven speed with some appropriate magnetic field.

  Times **-** measured in terms of the (cyclotron time)$^{-1}$, **Lengths** - to collisionless skin depth λ_{i0}.

- **Defining equations are:**

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times \Big[[\boldsymbol{V} - \boldsymbol{\nabla} \times \boldsymbol{B}] \times \boldsymbol{B} \Big], \qquad \boldsymbol{V}_e = \boldsymbol{V} - \boldsymbol{\nabla} \times \boldsymbol{B} \tag{1}$$

$$\frac{\partial \boldsymbol{V}}{\partial t} = \boldsymbol{V} \times (\boldsymbol{\nabla} \times \boldsymbol{V}) + (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} - \boldsymbol{\nabla} \left(P + \frac{V^2}{2} \right) \tag{2}$$

- **The red terms are due to Hall current** and the **blue terms are vorticity forces**.

Quasi-equilibrium Approach

Model: recently developed magneto-fluid theory (*Mahajan & Yoshida, PRL 1998*)

Assumption (gravity included): there exists fully ionized & magnetized plasma structures ==> the quasi–equilibrium two–fluid model will capture the essential physics of flow acceleration

Simplest two–fluid equilibria: $T = \mathrm{const} \longrightarrow \mathrm{n}^{-1}\nabla \mathrm{p} \rightarrow \mathrm{T}\nabla \ln \mathrm{n}.$

Generalization to homentropic fluid: $p = \mathrm{const}\cdot \mathrm{n}^{\gamma}$ **is straightforward.**

The dimensionless equations for compressible case: *Mahajan et al. 2001, PoP*

$$\frac{1}{n}\nabla\times\mathbf{b}\times\mathbf{b}+\nabla\left(\frac{r_{A0}}{r}-\beta_0\ln\ n-\frac{V^2}{2}\right)+\mathbf{V}\times(\nabla\times\mathbf{V})=0, \tag{3}$$

$$\nabla\times\left[\left(\mathbf{V}-\frac{\alpha_0}{n}\nabla\times\mathbf{b}\right)\times\mathbf{b}\right]=0 \tag{4}$$

$$\nabla\cdot(n\mathbf{V})=0 \tag{5}$$

$$\nabla\cdot\mathbf{b}=0 \tag{6}$$

Model details

Parameters:

$$r_{A0} = GM/V_{A0}^2 R_0 = 2\beta_0\, r_{c0}, \quad \alpha_0 = \lambda_{i0}/R_0, \quad \beta_0 = c_{s0}^2/V_{A0}^2,$$

c_{s0} **- sound speed,** R_0 **- the characteristic scale length**

$\lambda_{i0} = c/\omega_{i0}$ **- the collisionless ion-skin depth** **are defined with** $n_0,\ T_0,\ B_0$

Hall current contributions are significant when $\alpha_0 > \eta,$ ($\boldsymbol{\eta}$ - inverse Lundquist number)

Important in: interstellar medium, turbulence in the early universe, white dwarfs, neutron stars, stellar atmosphere.

Typical solar plasma: condition is easily satisfied.

Hall currents modifying the dynamics of the microscopic flows/fields - have a profound impact on the generation of macroscopic magnetic fields *(Mininni et al 2001,2003)* **& macroscopic flows** *(Mahajan et al 2002,2005).*

The double Beltrami solutions are

$$\mathbf{b} + \alpha_0 \nabla \times \mathbf{V} = d\ n\ \mathbf{V}, \qquad \mathbf{b} = a\ n\ \left[\mathbf{V} - \frac{\alpha_0}{n} \nabla \times \mathbf{b}\right], \tag{7}$$

a and ***d*** — **dimensionless constants related to ideal invariants:**
the Magnetic & the Generalized helicities (*Mahajan & Yoshida 1998; Mahajan et al. 2001*)

$$h_1 = \int (\mathbf{A} \cdot \mathbf{b})\ d^3x, \tag{8}$$

$$h_2 = \int (\mathbf{A} + \mathbf{V}) \cdot (\mathbf{b} + \nabla \times \mathbf{V}) d^3x. \tag{9}$$

obeying the **Bernoulli Condition**

$$\nabla \left(\frac{2\beta_0 r_{c0}}{r} - \beta_0 \ln n - \frac{V^2}{2} \right) = 0, \tag{10}$$

relating the density with the flow kinetic energy & gravity.

Quasi-equilibrium → Eruptive and Explosive events, Flaring

The parameters of the DB field change – *assumption*

- **the parameter change is sufficiently slow / adiabatic.**
- **at each stage, the system can find its local DB equilibrium.**
- **in slow evolution the dynamical invariants: h_1, h_2, & the total (magnetic + fluid) energy E are conserved.**

The General equilibrium solution for incompressible case is shown to be

(G_λ - *solution of Beltrami equation*)

$$\boldsymbol{b} = C_\mu \boldsymbol{G}_\mu(\mu) + C_\lambda \boldsymbol{G}_\lambda(\lambda), \tag{11}$$

$$\boldsymbol{V} = \left(\frac{1}{a} + \mu\right) C_\mu \boldsymbol{G}_\mu(\mu) + C_\lambda \left(\frac{1}{a} + \lambda\right) \boldsymbol{G}_\lambda(\lambda). \tag{12}$$

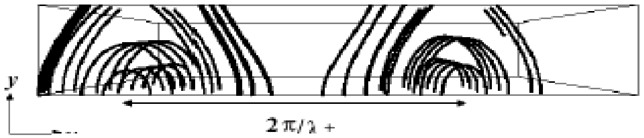
The catastrophic loss of equilibrium may occur in one of the following two ways:

1. **the roots** (λ - large-scale, μ - short-scale) **of the quadratic equation**, ***determining the length scales for the field variation***, **go from being real to complex**.

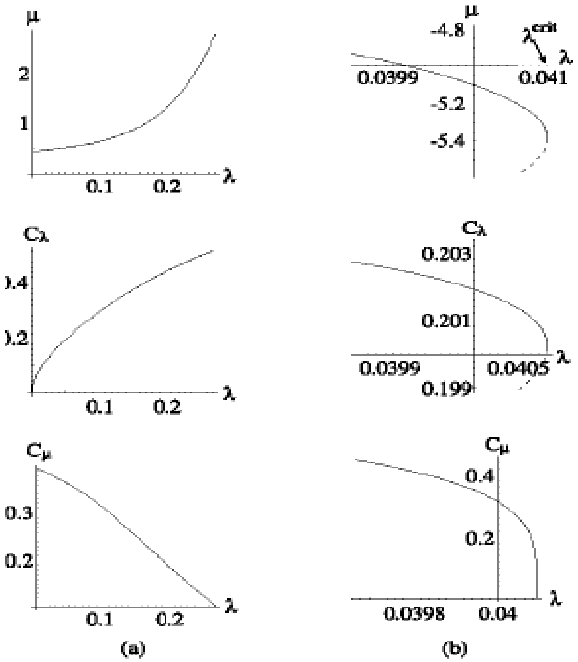
2. **amplitude of either of the 2 states** $(C_{\mu/\nu})$ **ceases to be real**.

- **Large scale λ – control parameter — observationally motivated choice.**
- ***Example*: Study of the structure–structure interactions working with simple 2D Beltrami ABC field with periodic boundary conditions. Choosing real λ, μ for quasi–equilibrium structures.**
- **There are two scenarios of losing equilibrium:**
  (1) Either of $(C_{\mu/\nu})^2$ **$\rightarrow 0$** (starting from positive values) **for real** $\lambda_{\mu/\nu}$
  (2) The roots $\lambda_{\mu/\nu}$ **coalesce** $(\lambda_\mu \leftrightarrow \lambda_\nu)$ **for real** $\lambda_{\mu/\nu}$ **and** $C_{\mu/\nu}$.
- **Conditions for catastrophic changes in Slowly Evolving Solar Structures (sequence of DB magneto-fiuid states) leading to a fundamental transformation of the initial state are derived as:**
- **For** $E > E_c = 2\left(h_1 \pm \sqrt{h_1 h_2}\right)$ **, the DB equilibrium suddenly relaxes to a SB state corresponding to the large macroscopic size.**
- **All of the short–scale magnetic energy is catastrophically transformed to the flow kinetic energy. Seeds of destruction lie in the conditions of birth.**
- **The proposed mechanism for the energy transformation work in all regions of Solar atmosphere with different dynamical evolution depending on the *I&B C-s* for a given region.**

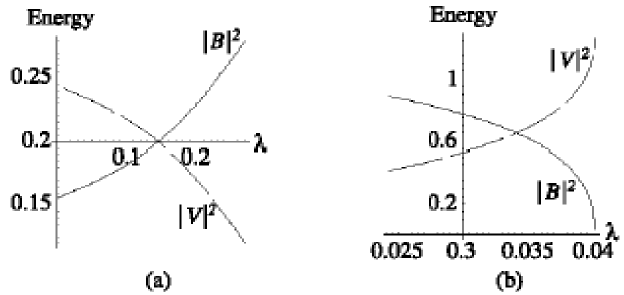
Plasma flow acceleration – catastrophe ($n = const$)



2D Beltrami ABS Field with periodic boundary conditions



a) No catastrophe initial conditions
b) Catastrophe initial conditions



a) No catastrophe initial conditions

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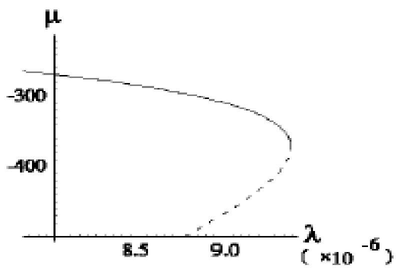
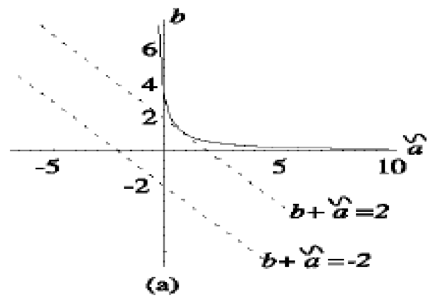
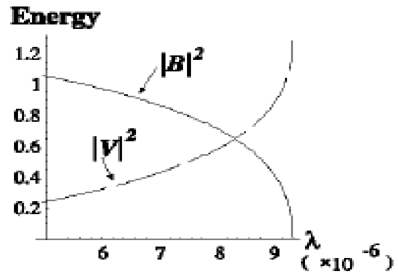
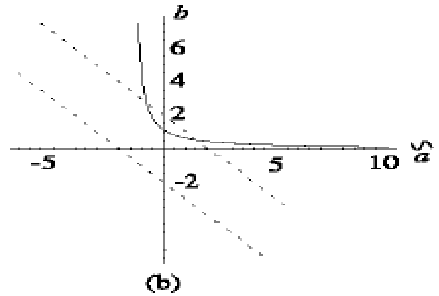








Solar Atmosphere:
Almost all initial magnetic energy (short scale) is transferred to flow

Root coalescence:
No separation between roots at the transition!

Quasi-equilibrium → Eruptive and Explosive events, Flaring
Compressible case

Closed HMHD system of equilibrium equations (*g(r)* = r_{c0} / *r*) ===>

$$\frac{\alpha_0^2}{n}\nabla\times\nabla\times\mathbf{V}+\alpha_0\,\nabla\times\left[\left(\frac{1}{a\,n}-d\right)n\,\mathbf{V}\right]+\left(1-\frac{d}{a}\right)\mathbf{V}=0, \tag{13}$$

$$n=\exp\left(-\left[2g_0-\frac{V_0^2}{2\beta_0}-2g+\frac{V^2}{2\beta_0}\right]\right) \tag{14}$$

1D simulation - a variety of boundary conditions: *Mahajan et al. ApJL 2002*

Flow with 3.3 km/s ends up with ~ 100 km/s at (Z - Z_0) ~ 0.09 R_0.

For small α_0 there exists some height where density drops sharply with a corresponding sharp rise in the flow speed --->

- **There is a catastrophe in the system.**
- **The distance over which it appears is determined by the strength of gravity *g(z)* .**
- **Amplification of flow is determined by local β_0.**

If density fall is at a much slower rate than the slow scale and $n \gg (a\,d)^{-1}$

1D problem solution gives (*a* ~ *d* = 100; *(a –d)/ad* ~10^{-6}) :

$$|V_{max}| = \frac{1}{d\,n_{min}} \tag{15}$$

Principal results (*Mahajan et al 2006*):

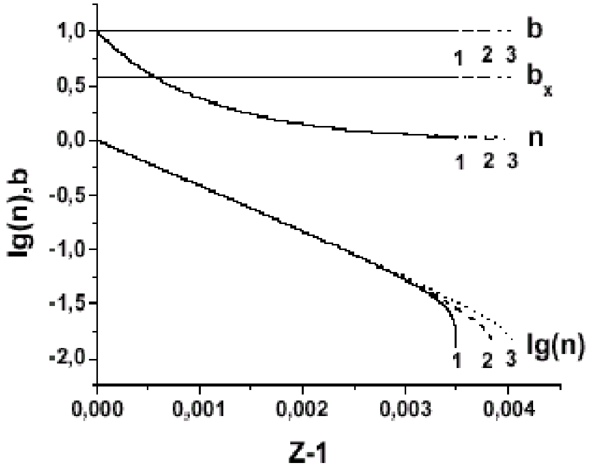
- **the transverse components of the magnetic field vary keeping** $b_x^2 + b_y^2 = b_{0\perp}^2 = const$
- **The density and the velocity fields are related approximately by** $|V|^2 = 1/d^2 n^2$

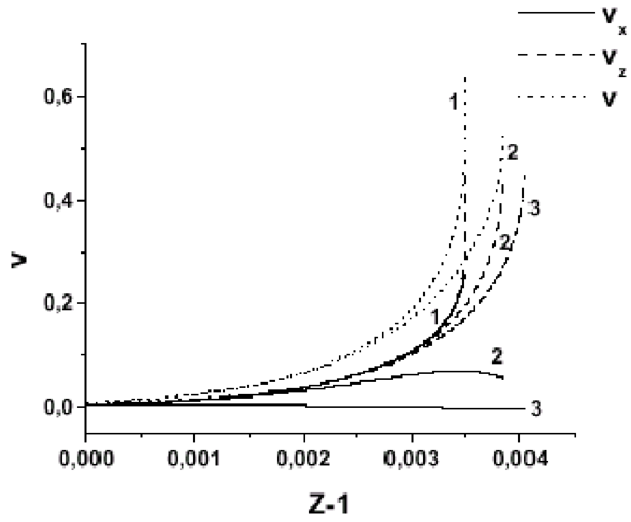
  magnetic energy does not change $|\mathbf{b}|^2 = const$ **to leading order**.
- **The Bernoulli condition transforms to the defining differential equation for density ==>**

$$n_{min} = (2\beta_0)^{-1/2} d^{-1}$$

- ***Note:*** **similar results found for** ***a* ~ *d*** **<< 1** *when the inverse micro scale* **~ *(a - a*$^{-1}$*)* >> 1** *with* ***(dn – a)*** **<< 1** ; **also when we assume an equation of state & temperature is allowed to vary.**

Plasma flow acceleration – catastrophe ($n \neq const$)





Sub-Alfvénic flows: Boundary conditions at:

$Z_0 > (1+2.8\bullet10^{-3})R_s$ – the influence of ionization can be neglected

$|b_0| = 1$, $V_0 = 0.01V_{A0}$ (with $V_{x0} = V_{y0} = V_{z0}$)

DB parameters:

$d \sim a \sim 100$, $(a-d)/a^2 \sim 10^{-6}$

3 sets of curves labeled by α_0 for parameters versus height (Z- 1).

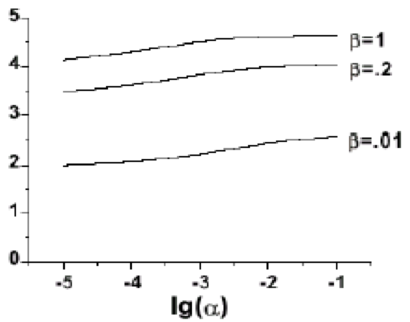
1- 2- 3 correspond to: $\alpha_0 = 0.000013; 0.005; 0.1$

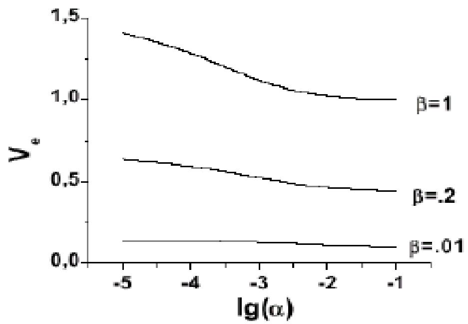
(a) (b)

(a) Blowup distance

(b) velocity

vs.. α_0





Following are the (n_0 ; B_0 ; T_0 ; V_{A0}):

$10^{11}cm^{-3}$; 100G; 5eV; 600km/s; $\beta_0 \sim 0.007 << 1$

$|b|^2 \sim const$; Density fall → Velocity increase

Catastrophe!

Acceleration is determined by local β_0

Steady flow generation /acceleration – *Reverse Dynamo*

- **The Dynamo mechanism - generic process of generating macroscopic magnetic fields from an initially turbulent system -** *biggest industry in plasma astrophysics + fusion.*

- **Standard Dynamo – generation of macro-fields from (primarily microscopic) velocity (*Flow Dominated Dynamo - FDD*) & magnetic (*Magnetically Dominated Dynamo - MDD*) fields.**

- **Latest understanding - coupling of FDD & MDD at different heights (going from lower scale structures to larger scale structures).**

- The **relaxations** observed in the **reverse field pinches** is an illustration of **MDD in action.**

- **Kinematic dynamo – the velocity field is externally specified & is not a dynamical variable!**

- **"Higher" theories – MHD, Hall MHD, two fluid etc - the velocity field evolves just as the magnetic field does – the fields are in mutual interaction.**

A question – A possible inference:

If short–scale turbulence can generate long–scale magnetic fields,
then short-scale turbulence should also be able to generate macroscopic velocity fields.

Process of conversion of short–scale kinetic energy to large–scale magnetic → "Dynamo" (D)

The mirror image process - conversion of short–scale magnetic energy to large–scale kinetic energy → "Reverse Dynamo" (RD)

Extending the definitions:

- **Dynamo (D) process -** Generation of macroscopic magnetic field from **any mix** of short–scale energy (magnetic & kinetic).
- **Reverse Dynamo (RD) process -** Generation of macroscopic flow from **any mix** of short–scale energy (magnetic & kinetic).

Theory and simulation show

(1) D & RD processes operate simultaneously — whenever a large scale magnetic field is generated there is a concomitant generation of a long scale plasma flow

(2) **The composition of the turbulent energy determines the ratio of the macroscopic flow / macroscopic magnetic field**

Micro and Macro Fields

The total fields in Eq.-s (1), (2) are broken into ambient & generated fields (Mahajan *et al* 2005).

The **generated fields -** further split into **macro** & **micro** fields**:**

$$\boldsymbol{B} = \boldsymbol{b}_0 + \boldsymbol{H} + \boldsymbol{b}$$

$$\boldsymbol{V} = \boldsymbol{v}_0 + \boldsymbol{U} + \boldsymbol{v}$$

$\boldsymbol{b}_0$, $\boldsymbol{v}_0$ **- equilibrium,** $\boldsymbol{H}$, $\boldsymbol{U}$ **- macroscopic,** $\boldsymbol{b}$, $\boldsymbol{v}$ **- microscopic fields.**

Traditional dynamo theories - the short scale velocity field $\boldsymbol{v}_0$ **is dominant.**

We shall not introduce any initial hierarchy between $\boldsymbol{v}_0$ **&** $\boldsymbol{b}_0$ **.**

We shall develop the natural unified Flow–Field theory.

Equilibrium – Initial State

Departure from the standard dynamo approach - our choice of the initial plasma state.

Equilibrium fields - the DB pair obeying Bernoulli condition $\nabla(p_0 + \boldsymbol{v}_0{}^2/2) = const$

which may be solved in terms of the Single Beltrami (SB) states $(\nabla \times \boldsymbol{G}(\mu) = \mu \boldsymbol{G}(\mu))$ **given by (11) & (12) for equilibrium fields.**

Below: ***λ*** **- micro-scale,** ***μ*** **- macro-scale;**

$|b| \ll b_0,$ **at the same scale** $|v| < v_0$ **and** $\boldsymbol{v}_{e0} \equiv \boldsymbol{v}_0 - \nabla \times \boldsymbol{b}_0$

Primary interest – to create macro fields from the ambient microfields.

Constructing the closure model of the Hall MHD eq-s & assuming that the original equilibrium is predominantly short-scale (from the DB fields we keep only the λ - part)

$$\boldsymbol{v}_0 = \boldsymbol{b}_0 \left(\lambda + a^{-1}\right) \qquad \boldsymbol{v}_{e0} = \boldsymbol{v}_0 - \nabla \times \boldsymbol{b}_0 = \boldsymbol{b}_0 \; a^{-1} \qquad (16)$$

Straightforward algebra for isotropic ABC initial flow ===>

$$\ddot{U} = bN_1 + \frac{\lambda}{2}\frac{b_0^2}{3}\boldsymbol{\nabla}\times\left[\left(\left(\lambda+\frac{1}{a}\right)^2\right)\boldsymbol{U}-\lambda\boldsymbol{H}\right] \quad (17)$$

$$\ddot{H} = bN_2 - \lambda\frac{b_0^2}{3}\left(1-\frac{\lambda}{a}-\frac{1}{a^2}\right)\boldsymbol{\nabla}\times\boldsymbol{H} \quad (18)$$

N_1 & N_2 **- time derivatives of the nonlinear terms – do not change on short-scale averaging.**

b_0^2 **– the ambient micro scale energy.**

H **evolves independently of** ***U*** **but evolution of** ***U*** **does require knowledge of** ***H*** **.**

Working out the nonlinear solution in linear clothing (neglecting NLN terms) we find:

$$\boldsymbol{U} = \frac{q}{(s+r)}\boldsymbol{H} \quad (19)$$

q, s, r **– fully defined by DB parameters & initial turbulent energy.**

A few remarkable features of linear solution:

• A choice of ***a; d*** (& hence of ***λ***) **fixes relative amounts of microscopic energy in ambient fields**
==> also **fixes the relative amount of energy in the generated macroscopic fields *U* & *H*.**

• **The linear solution makes NLN terms strictly zero – it is an exact (a special class) solution of the NLN system ==> remains valid even as *U* & *H* grow to larger amplitudes**
(*appears in Alfvénic systems: MHD - nonlinear Alfvén wave:* Walen 1944,1945; in HMHD - Mahajan & Krishan, 2005)

Analytical Results — An Almost Straight Dynamo

$a \sim d \gg 1$, **inverse micro scale micro-scales fields are** $\lambda \sim a \gg 1$ ===>

$\boldsymbol{v}_0 \sim a\,\boldsymbol{b}_0 \gg \boldsymbol{b}_0$ **the ambient micro-scales fields are primarily kinetic.**

Generated macro-fields have opposite ordering - $\boldsymbol{U} \sim a^{-1}\boldsymbol{H} \ll \boldsymbol{H}$

super-Alfvénic "turbulent flows" lead to steady flows that are equally sub–Alfvénic

Analytical Results — An Almost straight Reverse Dynamo

$a \sim d \ll 1$ $\lambda \sim a - a^{-1} \gg 1$ $\boldsymbol{v}_0 \sim a\,\boldsymbol{b}_0 \ll \boldsymbol{b}_0$

The ambient energy is mostly magnetic; from a strongly sub-Alfvénic turbulent flow

the system generates a strongly super-Alfvénic macro–scale flow $\boldsymbol{U} \sim a^{-1}\boldsymbol{H} \gg \boldsymbol{H}$

D, RD Summary:

- **Dynamo and "Reverse Dynamo" mechanisms have the same origin – are manifestation of the magneto-fluid coupling**

- ***U*** and ***H*** are generated simultaneously and proportionately. **Greater the macro-scale magnetic field (generated locally), greater the macro-scale velocity field (generated locally)**

- **Growth rate of macro-fields is defined by DB parameters (by the ambient magnetic and generalized helicities) and scales directly with ambient turbulent energy**

$$\sim b_0^2 \ (v_0^2)$$

- ***Larger the initial turbulent magnetic energy, the stronger the acceleration of the flow.***

D, RD Summary:

Impacts: on the evolution of large-scale magnetic fields and their opening up with respect to fast particle escape from stellar coronae; on the dynamical and continuous kinetic energy supply of plasma flows observed in astrophysical systems, TOKAMAK core region.

Caution: Initial and final states have finite helicities (magnetic and kinetic).

The helicity densities are dynamical parameters that evolve self-consistently during the flow acceleration.

Rotation, dissipation & heat flux as well as compressibility effects were neglected!

A simulation Example for Dynamical Acceleration

2.5D numerical simulation of the general two-fluid equations in Cartesian Geometry.
Code: Mahajan et al. PoP 2001, Mahajan et al, 2005

Simulation system contains:

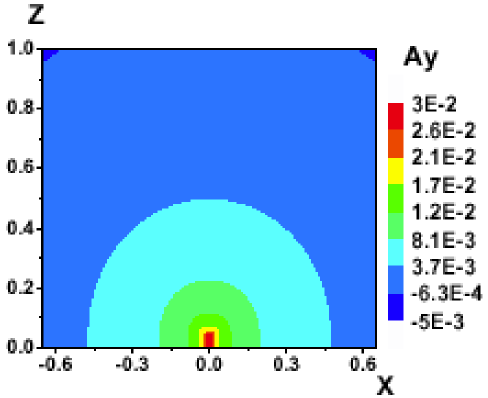
- **an ambient macroscopic field**
- **effects not included in the analysis**:
  1. dissipation and heat flux
  2. plasma is compressible embedded in a gravitational field →
  extra possibility for micro-scale structure creation.

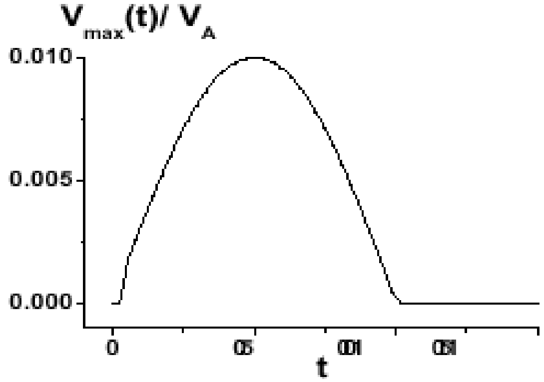
Transport coefficients are taken from Braginskii and are local.

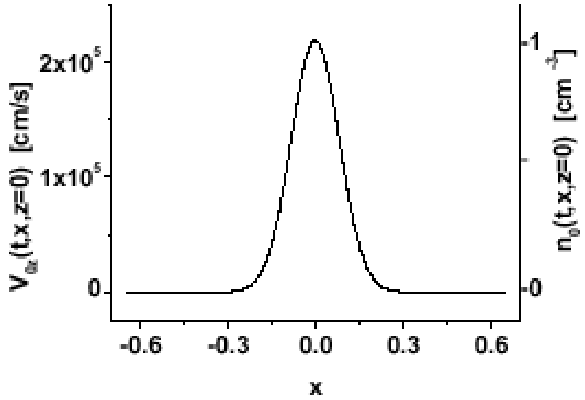
Diffusion time of magnetic field > duration of interaction process (would require T ≤· a few eV -s).

Study of trapping and amplification of a weak flow impinging on a single closed-line magnetic structure ===>

Initial Characteristics of Magnetic Field and flow







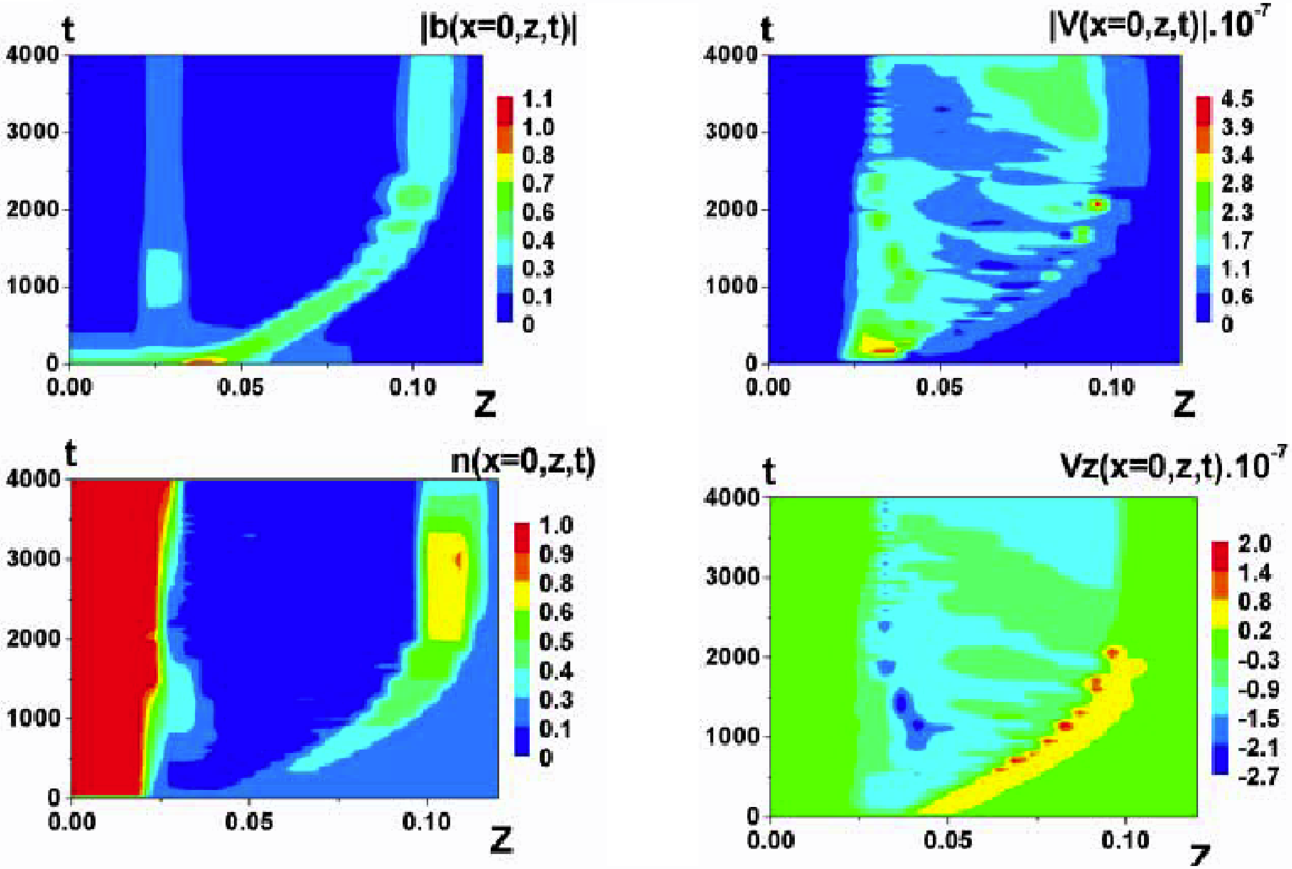
$\mathbf{B} = \nabla \times \mathbf{A} + B_z \hat{\mathbf{z}}$ $\mathbf{A}(0; A_y; 0)$; $\mathbf{b}=\mathbf{B}/B_{0z}$; $b_x(t, x, z \neq 0) \neq 0$ $B_{0z}=100G$ - uniform it time.

Weak symmetric up-flow (pulse-like): $|V|_{0max} \ll C_{s0}$ **C_{s0} - initial sound speed; time duration - t_0=100s**

Initially Gaussian; peak is located in the central region of a single closed magnetic field structure.

Initial flow parameters: $V_{0max}(x\_o, z=0)=V_{0z}=2.18 \cdot 10^5$ cm/s; $n_{0max}=10^{12}$ cm^{-3}; $T(x,z=0)=const=T_0$ =10eV

Dynamo and Reverce Dynamo Phenomena
In the center of the original closed magnetic field structure



Dynamical Emergence of the new magnetic field in region different from original; flux moves to the upper heights with time!

Accelerated flow follows the maximum field localization area – RD! D & RD phenomena have oscillating/pulsating character.

Generated field maximum ~$0.5b_0$; accelerated flow max. radial speed ~200km/s ; at ~2000sec time flow converts to down-flow!

Simulation Summary:

- **Dissipation present: Hall term *(through the mediation of micro-scale physics)* plays a crucial role in acceleration/heating processes**

- **Initial fast acceleration in the region of maximum original magnetic field + the creation of new areas of macro-scale magnetic field localization with simultaneous transfer of the micro-scale magnetic energy to flow kinetic energy = manifestations of the combined effects of the D and RD phenomena**

- **Continuous energy supply from fluctuations (dissipative, Hall, vorticity) ===> maintenance of quasi-steady flows for significant period**

- **Simulation: actual h_1, h_2 are dynamical.**
  Even if they are not in the required range initially, their evolution could bring them in the range where they could satisfy conditions needed to efficiently generate flows ===> several phases of acceleration

Beltrami structures in disk-jet system: alignment of flow & generalized vorticity

- Compressible case with dissipation and rotation included

An accretion disk (AD) often combines with spindle-like jet of ejecting gas, & constitutes a typical structure that accompanies a massive object of various scales, ranging from young stars to AGN. ***The mechanism that rules each part of different systems - not universal.***

Since early 70s (the discovery of radio galaxies & quasars) the main evidence from detecting **jets in different classes of astrophysical systems** observed to produce collimated jets near the massive central object - **the direct association with an AD** ***(reflecting different accretion regimes)*** -

the opposite is not true in some objects for which AD-s do not require collimated jets ***(viscous transport/disk winds play the similar role in the energy balance)*****.**

The macroscopic disk-jet geometry - a marked similarity despite the huge variety of the scaling parameters (*Lorentz factor, Reynolds number, Lundquist number, ionization fractions, etc.)*

The basic properties of disk-jet systems

In the disk region:

- **transport processes of mass, momentum & energy depend strongly on the scaling parameters.**
- the **classical *(collisional)* processes are evidently insufficient to account for the accretion rate**, thus **turbulent transports *(involving magnetic perturbations)* must be invoked**.
- **winds may also remove the angular momentum from the disk.**

The connection of the disk & the jet is more complicated:

- **mass & energy of jet are fed by the accreting flow**, the mechanism & process of transfer are still not clear.
- **the major constituent of jets is the material of an AD surrounding the central object**.

For the **fastest outflows** the contributions to the total mass flux may come **from outer regions as well**. In AGN one may think of taking some energy from the central black hole.

The basic properties of disk-jet systems - 2

Livio (1997):

(i) **powerful jets are produced by systems in which on top of an AD** threaded by a vertical field, **there exists an additional source of energy/wind,** possibly **associated with the central object;**

(ii) **launching of an outflow from an AD requires a hot corona** ***or some additional source of energy;***

(iii) **extensive hot atmosphere around the compact object can provide additional acceleration.**

Role of Magnetic Field

Magnetic fields are considered to play an important role in defining the local accretion.

- **When magnetic field is advected inwards by accreting material or/and generated locally** by some mechanism, the centrifugal force due to rotation may boost **jet along the magnetic field lines** up to a super-Alfvénic speed.

- **AGN - there is an alternative idea** suggested by *Blandford & Znajek (1977)* **based on electro-dynamical processes extracting energy from a rotating black hole.**

- **Extra-galactic radio jets might be accelerated by highly disorganized magnetic fields that are strong enough to dominate the dynamics until the terminal Lorentz factor is reached.**

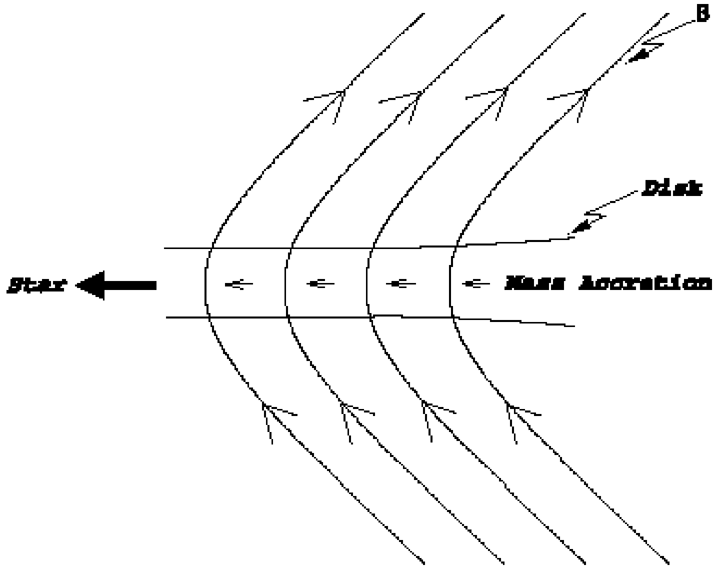


FIG. 1.—Schematic diagram of accretion flow in a disk threaded by magnetic flux accumulated by the process of star formation.

In addition to the energetics to account for the acceleration of ejecting flow, we have to explain how the streamlines/magnetic-field lines change the topology through the disk-jet connection.

Momentum Equation – Simplest MHD Model

Despite the diversity & complexity of holistic processes, there must yet be a simple and universal principle that determines the geometric similarity of disk-jet compositions!

The momentum balance equations for the ion & electron fluids are ($P = \rho V$ momentum density)

$$\partial_t \boldsymbol{P} + \nabla \cdot (\rho \boldsymbol{V} \otimes \boldsymbol{V}) = \epsilon^{-1} \rho (\boldsymbol{V} \times \boldsymbol{B} - \partial_t \boldsymbol{A}) - \rho \nabla (\phi + \varphi) - \nabla p_i - \nu \boldsymbol{P} \tag{20}$$

$$0 = \epsilon^{-1} \rho (\boldsymbol{V}_e \times \boldsymbol{B} - \partial_t \boldsymbol{A}) - \rho \nabla \varphi + \nabla p_e \tag{21}$$

ν is the effective friction coefficient, Φ - the gravity potential.

More generally the last term on r.h.s. should be written as the viscosity term: $-\nabla \cdot \boldsymbol{\Pi}$

Since the flow V is primarily in azimuthal (θ) direction, in a Keplerian thin disk $V \approx V_0\, r^{-1/2}\, \boldsymbol{e}_\theta$ the viscosity force can be approximated as

$$\nabla \cdot \boldsymbol{\Pi} = (\nabla \times (\rho\, \eta \nabla \times V)) \sim \rho\, \eta\, V_0\, r^{-5/2}\, \boldsymbol{e}_\theta \longrightarrow \quad -\nabla \cdot \boldsymbol{\Pi} = -\nu(r)\, \boldsymbol{P}$$

Momentum Equation – Simplest MHD Model

Energy densities are normalized by unit kinetic energy $\mathcal{E}_0 = \frac{\rho_0 V_0^2}{2}.$

Scale parameter is $\epsilon = \frac{\delta_i}{L_0}$ **,** ***L_0*** **- system size,** $\delta_i = mc/\sqrt{4\pi e^2 \rho_0}$

V_0 **and** ρ_0 **are the representative flow velocity and mass density in the disk.**

Stationary Solutions

$\partial_t = 0.$ **macroscopic structure of an astronomical system - we may assume** $\epsilon \ll 1.$

Expanding $\boldsymbol{B} = \boldsymbol{B}^{(0)} + \epsilon \boldsymbol{B}^{(1)} + \cdots,$

Stationary Solutions - 2

(21) ==> $$0 = \epsilon^{-1}\rho \boldsymbol{V} \times \boldsymbol{B}^{(0)} + \rho\left[\boldsymbol{V} \times \boldsymbol{B}^{(1)} - \rho^{-1}(\nabla \times \boldsymbol{B}^{(0)}) \times \boldsymbol{B}^{(0)}\right] - \rho\nabla\varphi + \nabla p_e + O(\epsilon).$$

We get: $\boldsymbol{B}^{(0)} = \mu \boldsymbol{P},$ ---> $\nabla \cdot (\mu \boldsymbol{P}) = \boldsymbol{P} \cdot \nabla\mu = 0.$

***(20)* ==>** $$\nabla \cdot (\rho \boldsymbol{V} \otimes \boldsymbol{V}) = [\nabla \times (\mu \boldsymbol{P})] \times (\mu \boldsymbol{P}) - \rho\nabla\phi - \nabla p - \nu \boldsymbol{P}, \qquad p = p_i + p_e.$$

To derive a term that balances with the viscosity (friction) term, we decompose the "inertia term"

$\rho = \rho_1 \rho_2$ ===> $\boldsymbol{P}_1 = \rho_1 \boldsymbol{V},$ $\boldsymbol{P}_2 = \rho_2 \boldsymbol{V}.$

Final set is given as $(\nabla h = \rho^{-1}\nabla p)$ **:**

$$\boldsymbol{P}_2 \times \boldsymbol{\Omega} = \frac{1}{2}\nabla P_2^2 + \rho_2^2\nabla(\phi + h) + \rho_2\nu \boldsymbol{P}_2 + \frac{\rho_2}{\rho_1}(\nabla \cdot \boldsymbol{P}_1)\boldsymbol{P}_2, \qquad (22)$$

$$\boldsymbol{\Omega} = \nabla \times \boldsymbol{P}_2 - \mu\rho_2\nabla \times (\mu \boldsymbol{P}) = \nabla \times \boldsymbol{P}_2 - \mu\rho_2\nabla \times \boldsymbol{B}^{(0)} \qquad (23)$$

Disk-Jet Structure

We assume toroidal symmetry ($\partial_\theta = 0$ in the r-θ-z coordinates).

A massive central object (a singularity at origin) neglecting mass in a disk ==> $\phi = -MG/r$

Then $\boldsymbol{V} \approx V_\theta \boldsymbol{e}_\theta$ **with Keplerian velocity** $V_\theta \propto r^{-1/2}$ **in disk region**

$\nabla \times \boldsymbol{V} = \Omega_z \boldsymbol{e}_z$ **with** $\Omega_z \propto r^{-3/2}$

The momentum is strongly localized in the thin disk & the vorticity diverges near the axis.

This singular configuration allows only a special geometric structure to emerge:

(i) In disk radial flow (<< V_θ) is caused by viscosity (friction).

The mass conservation ($\nabla \cdot \boldsymbol{P}$ =0) and balance of the friction and the partial inertia term demand

$$\boldsymbol{V} \cdot \nabla \log \rho_2 = \nu, \quad \text{determines parameter } \rho_2 \qquad (24)$$

$$\boldsymbol{V} \cdot \nabla \log \rho_1 = -\nabla \cdot \boldsymbol{V} - \nu. \qquad (25)$$

(ii) **the remaining terms in (22) must not have an azimuthal (toroidal) component.**
R.h.s. have only poloidal (*r - z* plane) components ==> l.h.s. have a toroidal component.

(iii) **In the vicinity of the axis threading the central object the flow** $\boldsymbol{V} = \boldsymbol{P}/\rho$
must align to the generalized vorticity $\boldsymbol{\Omega}$ (dominated by $\nabla \times \boldsymbol{V} \propto r^{-3/2}\boldsymbol{e}_z$)
producing the collimated jet structure, to minimize the Coriolis force $\boldsymbol{V} \times \boldsymbol{\Omega}$

The alignment condition - Beltrami condition (λ - scalar function) — $\boldsymbol{\Omega} = \lambda \boldsymbol{P}$ (26)

(iv) remaining potential forces must balance ===> ***the Bernoulli condition***

$$\frac{1}{2\rho_2^2}\nabla P_2^2 + \nabla(\phi + h) = \nabla\left(\frac{1}{2}V^2 + \phi + h\right) + V^2\nabla \log \rho_2 \quad = 0. \qquad (27)$$

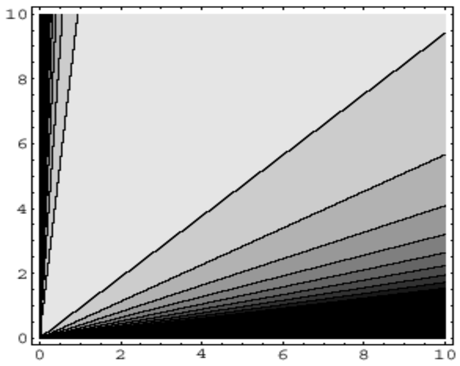
The system of determining equations:

Eq. (24) determines "artificial ingredient" $\boldsymbol{\rho_2}$ **for given** $\boldsymbol{v}$ **.**

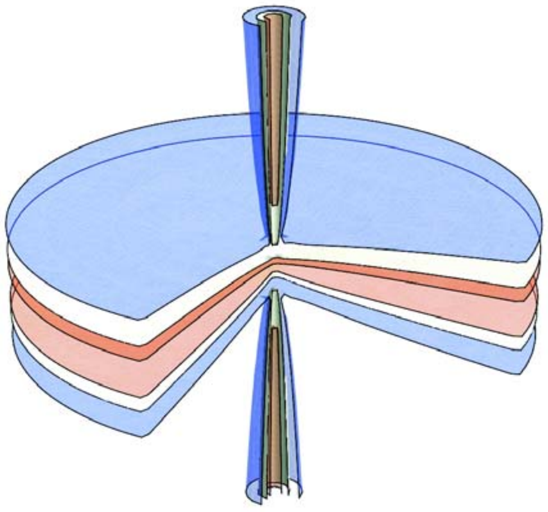
$\boldsymbol{V}(= \boldsymbol{P}_2/\rho_2)$ **is governed by (26)**

After determining V **and** $\boldsymbol{\rho_2}$ **we solve (27) to determine enthalpy** $\boldsymbol{h}$ **.**

A similarity solution modeling fundamental disk-jet structure (*μ ≈ 0* case)



The momentum field (streamlines of poloidal component of *P*) of the similarity solution

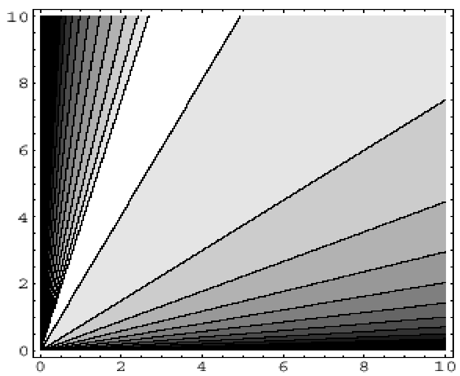


The density ρ in the similarity solution (*log* scale)

We assume $\rho_{||}(\sigma) \approx z^{-2/3}$; **and** $\rho_2 \approx r^{-1/2}$.

A levelset surface of ρ is shown in the domain

$r < 5$ $|z| < 5.$



The distribution of $\rho_\perp$ of the similarity solution

Summary for Disk-Jet Problem

Invoking the simplest (minimum) model of MHD we have shown that:

- **the combination of a thin disk and narrowly-collimated jet is the unique structure that is amenable to the singularity of the Keplerian vorticity - the Beltrami condition - the alignment of flow and generalized vorticity - characterizes such a geometry.**

The conventional vorticity is generalized to combine with magnetic field *(or, electromagnetic vorticity)* as well as to subtract the viscosity (friction) force causing the accretion and the centrifugal force of the Keplerian velocity.

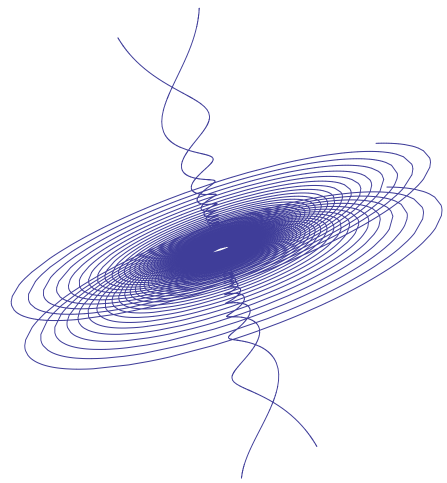
- **we have found an analytic solution in which the generalized vorticity is purely kinematic ($\boldsymbol{\Omega} = \nabla \times \boldsymbol{P}_2$ with the momentum $\boldsymbol{P}_2$ modified by the viscosity effect).**

- **The principal force that ejects the jet is the hydrodynamic pressure dominated by $V_\theta^2/2$**

- **Additional magnetic force may contribute to jet acceleration if the self-consistently generated large-scale magnetic field is sufficiently large – such structures can be described by the generalized model.**

The Generalized Beltrami Vortex

- **Identifying the disk-jet structure as a generalized Beltrami vortex, we will be able to understand the self-organization process in terms of the "generalized helicity" ==>**



- ***Important lesson*** **- the helicity of the generalized vorticity is the key parameter that characterizes the self-organizing of a disk-jet system.**

The streamlines of "generalized Beltrami Flow" – in a disk-jet system the accreting flow and jet align parallel to *a generalized vorticity*

Disk-Jet problem Simulation results

- **The Keplerian velocity / similarity solution has a singularity at the origin which disconnects the disk part & the jet part.**
- **To "connect" both subsystems we need a singular perturbation that dominates the small-scale hierarchy on which the disk and jet regions are connected smoothly.**
- **The connection point must switch the topology of the flow - the topological difference of 2 sub-systems' vector fields demands "decoupling" of them.**
- **Instead of disconnecting them by a singularity, we will have to consider a small-scale structure in which the topological switch can occur.**
- **Hall effect (scaled by ε) or viscosity/resistivity (scaled by reciprocal Reynolds number) yields a singular perturbation ===> the generalized magneto-Bernoulli mechanism may effectively accelerate the jet-flow.**
- **The mechanism of singular perturbation & the local structure of the disk-jet connection point may differ depending on the plasma condition near the central object.**

THANK YOU !