

Lorentz-symmetry violation and neutrino physics



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BeNe 2012, 17 September 2012, ICTP, Trieste

Outline:

A. Motivations

→ Possible origins of LV

B. Low- E effects; EFT

→ SME test framework for ν 's

C. Phenomenology

→ qualitatively new effects for ν 's

A. Why test Spacetime Symmetries?

Motivation (i): philosophical necessity

physics is an experimental science

→ solid **experimental confirmation** of foundations of physics is **crucial**

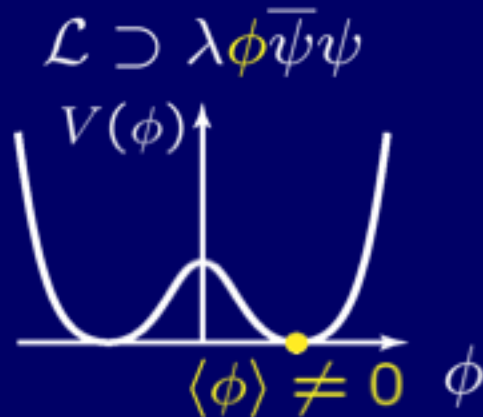
Motivation (ii): discovery potential

various approaches to physics beyond the Standard Model (e.g., "quantum gravity") can accommodate tiny **violations of Relativity**

Possible origins for Lorentz/CPT violation

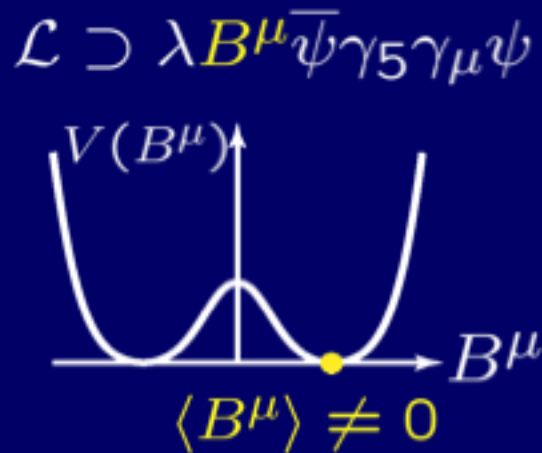
(1) Spontaneous Lorentz breaking in string theory

conventional
case:
gauge symmet.



$$\mathcal{L} \supset \underbrace{\lambda \langle \phi \rangle}_{m = \text{const.}} \bar{\psi} \psi$$

string theory:
Lorentz
symmetry

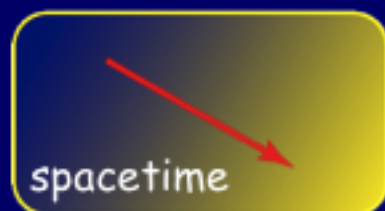


$$\mathcal{L} \supset \underbrace{\lambda \langle B^\mu \rangle}_{b^\mu = \text{const.}} \bar{\psi} \gamma_5 \gamma_\mu \psi$$

Kostelecký, Perry, Potting, Samuel '89; '90; '91; '95; '00

(2) Cosmol. varying scalars (e.g., fine-structure parameter)

intuitive
argument:



small scalar



large scalar

gradient of the
scalar selects
pref. direction

mathematical argument:

$\xi = \xi(x)$... varying coupling
 ϕ, Φ ... dynamical fields

$$\mathcal{L} \supset \xi \partial^\mu \phi \partial_\mu \Phi$$

Integration by parts:

$$\mathcal{L}' \supset -(\partial^\mu \xi) \phi \partial_\mu \Phi$$

slow variation of ξ :
 $K^\mu \equiv (\partial^\mu \xi) \simeq \text{const.}$

$$\mathcal{L}' \supset -K^\mu \phi \partial_\mu \Phi$$

Kostelecký, R.L., Perry '03; Arkani-Hamed et al. '03

Other mechanisms for Lorentz violation

Noncommutative field theory

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$$

Seiberg-Witten: $\hat{x}^\mu \rightarrow$ usual Minkowski coordinates x^μ

\rightarrow SME terms emerge: $\mathcal{L}_{\text{photon}} \supset \frac{1}{8} q \theta^{\alpha\beta} F_{\alpha\beta} F^{\mu\nu} F_{\mu\nu}$

e.g. Mocioiu *et al.* '00; Carroll *et al.* '01

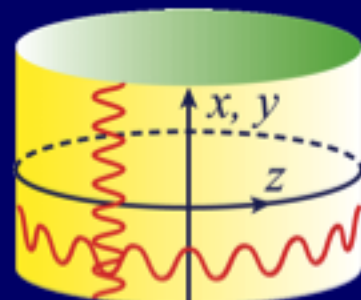
Topology (in usual $4d$ spacetime: 1 large compact dim.)

Vacuum fluctuations along this dim.

have periodic boundary conditions

\rightarrow preferred direction in vacuum

\rightarrow calculation: $k^\mu A^\nu \tilde{F}_{\mu\nu} \subset \mathcal{L}_{\text{SME}}$



Klinkhamer '00

...

B. Effective field theory test framework

Why test framework?

- prediction of experimental effects
- analysis and comparison of tests
- theoretical insight

How to obtain test framework?

Purpose: broad experimental searches independent of details of underlying physics

→ construct gen. model compatible with key phys. principles

Ingredients for test framework?

- established physics → all feasible tests can be described
- effective field theory (well established & versatile tool)
- Lorentz/CPT violation (preferred directions; prev. Sec)

Actual construction of the effective field theory (SME)

$$\mathcal{L}_{\text{SME}} = \underbrace{\mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{EH}}}_{\text{present physics}} + e k_{\mu} A_{\nu} \tilde{F}^{\mu\nu} + \frac{e}{2\kappa} s^{\mu\nu} R_{\mu\nu} + \dots$$

- $k^{\mu}, s^{\mu\nu}, \dots$ coefficients for Lorentz violation
- minimal SME \rightarrow fermion 44, photon 23, ...
- generated by underlying physics (Sec A)
- amenable to ultrahigh-precision tests (Sec C)



Remarks:

- $k^{\mu}, c^{\mu\nu}, \dots$ coefficients usually taken as spacetime constant
- situations involving gravity require more care
- many theor. SME studies \rightarrow thus far no inconsistencies

Neutrinos with Lorentz/CPT violation

consider 6-dim. Hilbert space $(|e\rangle, |\mu\rangle, |\tau\rangle, |e\rangle^C, |\mu\rangle^C, |\tau\rangle^C)$:
 q.m. ν propagation (incl. oscillations) governed by 6x6 Hamiltonian

$$H = \underbrace{\begin{pmatrix} |\vec{p}| + \frac{m_l m_l^\dagger}{2|\vec{p}|} & 0 \\ 0 & |\vec{p}| + \frac{m_l^\dagger m_l}{2|\vec{p}|} \end{pmatrix}}_{\text{usual LI piece}} + \frac{1}{|\vec{p}|} \underbrace{\begin{pmatrix} \hat{a} - \hat{c} & \hat{H} - \hat{g} \\ \hat{H}^\dagger - \hat{g}^\dagger & -\hat{a}^T - \hat{c}^T \end{pmatrix}}_{\text{LV \& CPTV correction}}$$

Lorentz-/CPT-violating a , c , g , and H contain:

- (1) **preferred directions** arising from LV/CPTV background in $\delta\mathcal{L}$
- (2) powers $(p^\mu)^{(d-3)}$ arising from operators of **mass-dimension** d
- (3) polarization vectors ε^μ arising from the underlying **spin d.o.f.**

sample structure: $\hat{H} \supset \sim \epsilon_\mu \epsilon_\nu^* H^{(d)\mu\nu}_{\alpha_1 \dots \alpha_{d-3}} p^{\alpha_1} \dots p^{\alpha_{d-3}}$

Kostelecký and Mewes, [arXiv:1112.6395](#)

Reasoning leading to this result:

define $\Psi_A = (\psi_e, \psi_\mu, \psi_\tau, \psi_e^C, \psi_\mu^C, \psi_\tau^C)^T$, $A = 1, \dots, 6$

most general free, local, unitary, transl.-inv. EFT Lagrangian

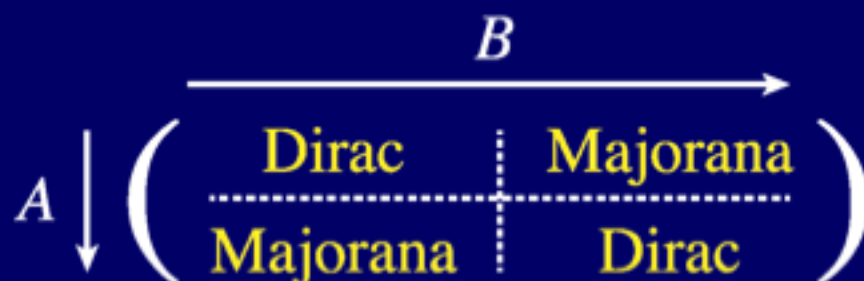
$$\delta\mathcal{L}_{\text{EFT}} = \bar{\Psi}_A (\hat{S}_{AB} + \hat{P}_{AB}\gamma_5 + \hat{V}_{AB}^\mu\gamma_\mu + \hat{A}_{AB}^\mu\gamma_5\gamma_\mu + \hat{T}_{AB}^{\mu\nu}\sigma_{\mu\nu}) \Psi_B$$

S, P, V, A, T consist of LV/CPTV backgrounds & arbitrary # of ∂ 's

Example:
$$\hat{S}_{AB} \equiv \sum_{d=3}^{\infty} S_{AB}^{(d)} \alpha_1 \alpha_2 \dots \alpha_{d-3} \partial_{\alpha_1} \partial_{\alpha_2} \dots \partial_{\alpha_{d-3}}$$

Remarks:

- still some redundancies and LI pieces (removable)
- need to add h.c. for hermitian Lagrangian



- assumptions:
- LI masses are **type-I seesaw** compatible
 - treat LV/CPTV as **perturbation**

LI unitary transformation #1:

- separate spinors into **light** Ψ_L and **heavy** Ψ_R fields
- **effect**: linear mixture of LV/CPTV terms

heavy Ψ_R don't propagate at low E can be **projected out**

- may now use 2-component Weyl spinors ξ for remaining Ψ_L
- **effect**: entries of flavor-space H reduce from **4x4 $\gamma \rightarrow 2x2 \sigma$**

LI unitary transformation (ultrarel. limit) #2:

- block-diagonalize to disentangle LI Dirac & Majorana m terms
- **effects**: linear mixture of LV/CPTV terms & **p^{-1} dependence**

remove C -conjugation redundancy by **projecting onto positive E** :

- **effect 1**: $\xi^\dagger \sigma^\mu \xi$ introduces **polarizations ε^μ**
- **effect 2**: entries of flavor-space H reduce from **2x2 $\sigma \rightarrow \mathbb{C}$**

→ prev. displayed **most general** q.m. ν -propagation Hamiltonian in $(|e\rangle, |\mu\rangle, |\tau\rangle, |e\rangle^C, |\mu\rangle^C, |\tau\rangle^C)$ space **compatible** with EFT:

$$\delta H_{\text{SME}}^{\text{propagation}} = \frac{1}{|\vec{p}|} \begin{pmatrix} \hat{a} - \hat{c} & \hat{H} - \hat{g} \\ \hat{H}^\dagger - \hat{g}^\dagger & -\hat{a}^T - \hat{c}^T \end{pmatrix}$$

where a, c, g, H are lin. combinations of the **LV/CPTV** $S, P, V^\mu, A^\mu, T^{\mu\nu}$ quantities in the EFT Lagrangian

note: ν and **anti- ν** coefficients **cannot** be chosen **independently**

Ex. 1: **no room** for **mass difference** between ν and **anti- ν** in SME

Ex. 2: special choice: diagonalizable, isotropic, $c = 0$

→ $g = 0, H = 0$, and a takes special form involving $\overset{\circ}{a}_e^{(d)}$

neutrino disp. rel. $\overset{\circ}{E}_{\nu_e} = |\vec{p}| + \frac{m_e}{2|\vec{p}|} + \sum_d |\vec{p}|^{d-3} \overset{\circ}{a}_e^{(d)}$

antineutrino disp. rel. $\overset{\circ}{E}_{\bar{\nu}_e} = |\vec{p}| + \frac{m_e}{2|\vec{p}|} - \sum_d |\vec{p}|^{d-3} \overset{\circ}{a}_e^{(d)}$

C. Phenomenology and Tests

Novel qualitative features in **neutrino oscillations**

eff. Hamiltonian for 3 left-handed ν 's in flavor basis ($a, b \dots e, \mu, \tau$):

$$(H + \delta H)_{ab} = \frac{1}{2|\vec{p}|} (m_l m_l^\dagger)_{ab} + \overbrace{(a^\mu)_{ab} \hat{p}_\mu}^{p_\mu \dots \nu \text{ propagation direction} \rightarrow \text{directional dependence}} - \overbrace{(c^{\mu\nu})_{ab} \hat{p}_\mu \hat{p}_\nu E}^{\text{unconventional } E \text{ dependence} \rightarrow U = U(E)} + \dots$$

$(a^\mu)_{ab} \xrightarrow{\bar{\nu}} -(a^\mu)_{ab}^* \rightarrow \text{CPT violation}$

→ great flexibility for matching observations

models based on this general effective Hamiltonian:

- "bicycle model" (Kostelecký *et al.*, 2004)
- "tandem model" (Katori *et al.*, 2006)
- "BMW model" (Barger *et al.*, 2007)
- "puma model" (Díaz *et al.*, 2010)
- "isotropic bicycle model" (Barger *et al.*, 2011) ...

phenomenology: example "puma model"

	3 ν SM	"puma model"
established results (with $\theta_{13} = 0$)	needs 5 parameters	needs only 2 parameters
LSND anomaly MiniBooNE low- E excess	its 6 parameters are insufficient	needs 6 parameters

- most recent exp. results: [arXiv:0801.0287v5](#), January 2012 edition
- 12 pages of tables for $d = 2, 3, 4, 5, 6$, and $d > 6$
 - analyses by MINOS, IceCube, MiniBooNE, LSND, ...
 - sample section of table:

Table XIII. Neutrino sector, $d = 3$

Combination	Result	System	Ref.
$ \tilde{a}^{(3)} $	$< 1.9 \times 10^{-7}$ GeV	IceCube meson threshold [13]*	
$ \text{Re}(a_L)_{\mu\tau}^X $	$< 5.9 \times 10^{-23}$ GeV	MINOS	[103]
$ \text{Im}(a_L)_{\mu\tau}^X $	$< 2.2 \times 10^{-20}$ GeV	"	[103]
$ \text{Re}(a_L)_{\mu\tau}^Y $	$< 6.1 \times 10^{-23}$ GeV	"	[103]
$ \text{Im}(a_L)_{\mu\tau}^Y $	$< 2.2 \times 10^{-20}$ GeV	"	[103]
a_L^X, a_L^Y	$< 1.8 \times 10^{-23}$ GeV	IceCube	[104]
$ (a_L)_{\mu\tau}^X $	$< 5.9 \times 10^{-23}$ GeV	MINOS FD	[105]
$ (a_L)_{\mu\tau}^Y $	$< 6.1 \times 10^{-23}$ GeV	"	[105]
$ a_L^X , a_L^Y $	$< 3.0 \times 10^{-20}$ GeV	MINOS ND	[106]
$ \text{Re}(a_{\text{eff}}^{(3)})_{00}^{e\mu} $	$< 1.3 \times 10^{-18}$ GeV	LSND	[13]*
"	$< 1.5 \times 10^{-19}$ GeV	MiniBooNE	[13]*
"	$< 9.2 \times 10^{-20}$ GeV	MiniBooNE $\bar{\nu}$	[13]*
$ \text{Re}(a_{\text{eff}}^{(3)})_{00}^{e\mu} $	$< 1.3 \times 10^{-18}$ GeV	LSND	[13]*

Summary

presently **no** compelling exp. evidence for Relativity violations, but:

(1) various theoretical approaches to **quantum gravity** can cause such violations



(2) at **low E** , such violations are described by **SME** test framework (eff. field theory + background fields)



(3) for free ν 's, **all** Lorentz-/CPT-violating operators compatible with local, unitary EFT have been **classified**

(4) Lorentz/CPT violation predicts a wealth of **qualitatively novel features**; provides ample ground for phenomenological **model building**