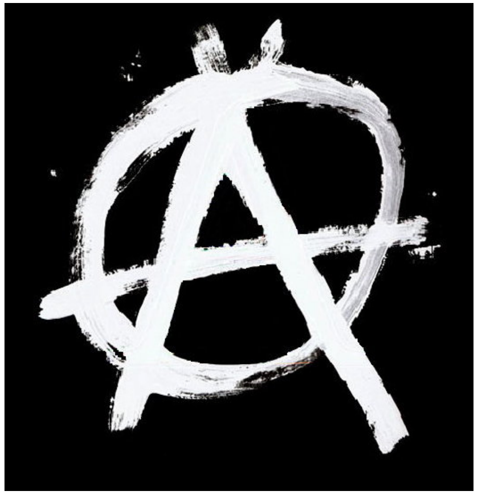
# Leptonic Mixing Anarchy: Alive and Kicking



André de Gouvêa

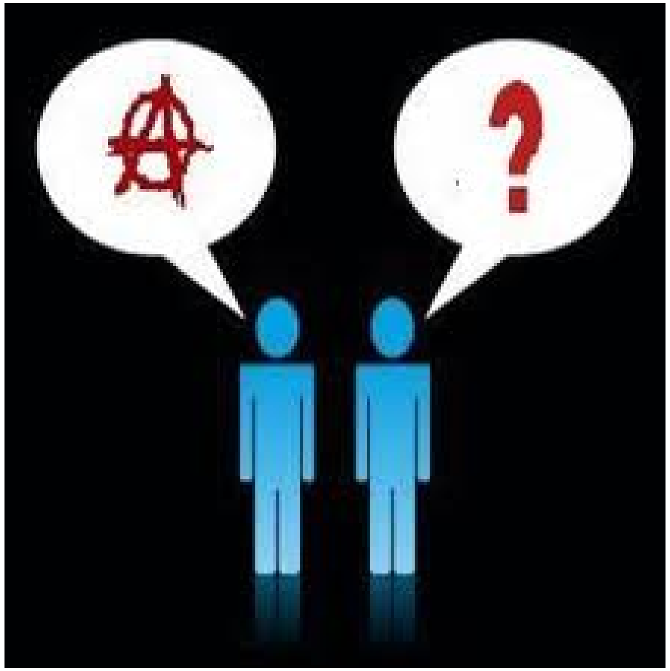
Northwestern University

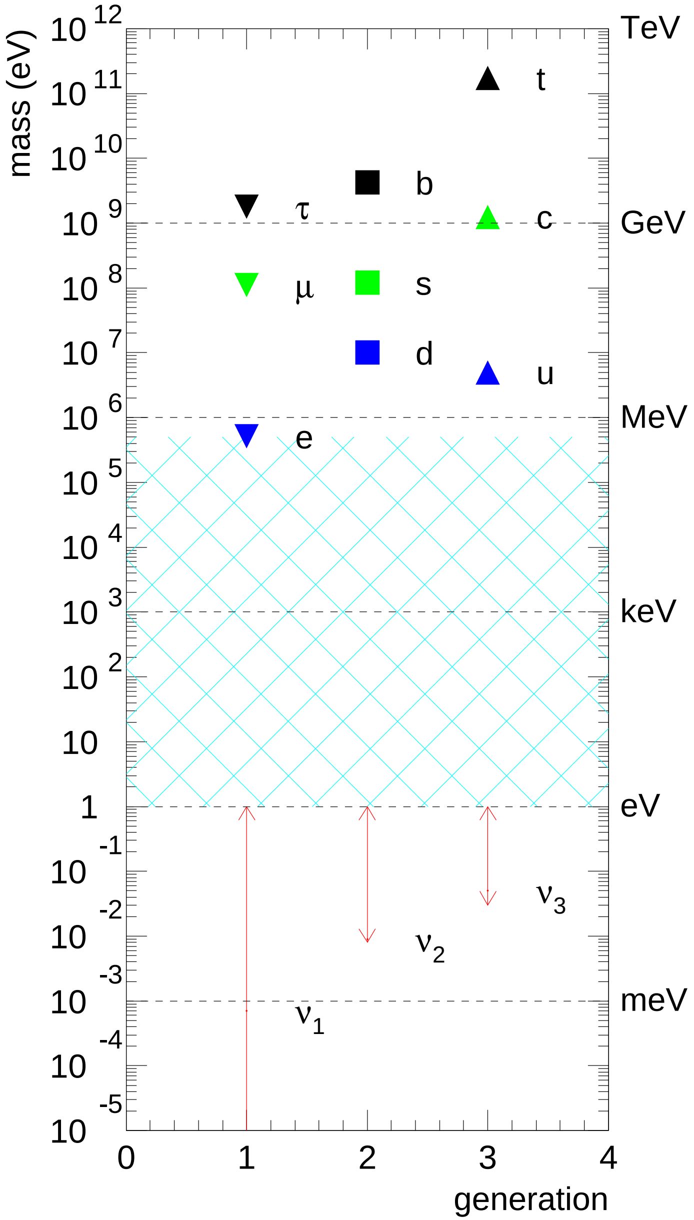
BeNe 2012 – ICTP, Trieste, Italia

September 17–21, 2012

# Outline

1. Introduction and Motivation;
2. The Anarchy Hypothesis: What It Is, and What It Is Not;
3. Hypothesis Test: History, Results, and Predictions;
4. Comments on Model Building: Is Your Model Anarchical?;
5. Final Thoughts.






## What We Are Trying To Understand:

$\Leftarrow$ **NEUTRINOS HAVE TINY MASSES**

$\Downarrow$ **LEPTON MIXING IS "WEIRD"** $\Downarrow$

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

$\boxed{\textbf{What Does It Mean?}}$

# Understanding Fermion Mixing

The other puzzling phenomenon uncovered by the neutrino data is the fact that Neutrino Mixing is Strange. What does this mean?

It means that lepton mixing is very different from quark mixing:

$$V_{MNS} \sim \begin{pmatrix} 0.8 & 0.5 & \mathbf{0.2} \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \qquad V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}$$

$\boxed{\text{WHY?}}$

They certainly look VERY different, but which one would you label as "strange"?

[HINT: it is not $V_{MNS}\ldots$]

**Lepton Mixing Anarchy** is the hypothesis that there is *no symmetry principle* behind the leptonic mixing matrix $U$.

In more concrete terms, it postulates that the observed leptonic mixing matrix can be described as the result of a *random draw* from an unbiased distribution of unitary $3 \times 3$ matrices.

This is *not a very ambitious model.* It does not make predictions for the values of any of the mixing parameters, nor does it predict any correlations among the different mixing parameters. It does not, obviously, allow one to reduce the number of mixing parameters compared to those in the lepton mixing sector of the $\nu$SM.

The Anarchy hypothesis, however, *does make some predictions*. It predicts a *probability distribution* for the different parameters that parameterize $U$. The distributions are parameterization dependent, but unique once a parameterization is fixed.

[Murayama et al, hep-ph/9911341, hep-ph/0009174, hep-ph/0301050, 1204.1249]

The probability distributions, first derived by Haba and Murayama, hep-ph/0009174, are easy to obtain. The idea is that they are invariant under a basis redefinition of the neutrino weak eigenstates, i.e, weak-basis independent. They are given by the *invariant Haar measure* of $U(3)$ (assuming that $U$ is a $3\times 3$ unitary matrix).

This is similar to obtaining the probability distribution for picking a point on the surface of a sphere from $\mathrm{d}A = \mathrm{d}\cos\theta \mathrm{d}\phi$. The probability density is flat in $\phi$ and flat in $\cos\theta$.

For unitary $3\times 3$ matrices, using the standard PDG parameterization, one gets that the probability distribution is **flat** in

$$\sin^2\theta_{12} \qquad \sin^2\theta_{23} \qquad \cos^4\theta_{13} \qquad \delta \qquad \phi_{1,2} \text{ (Majorana phases).}$$

These tend to favor large mixing, and large CP-violating effects.

Why? – Because these are most common!

Distribution peaked at $\sin^2 2\theta = 1$

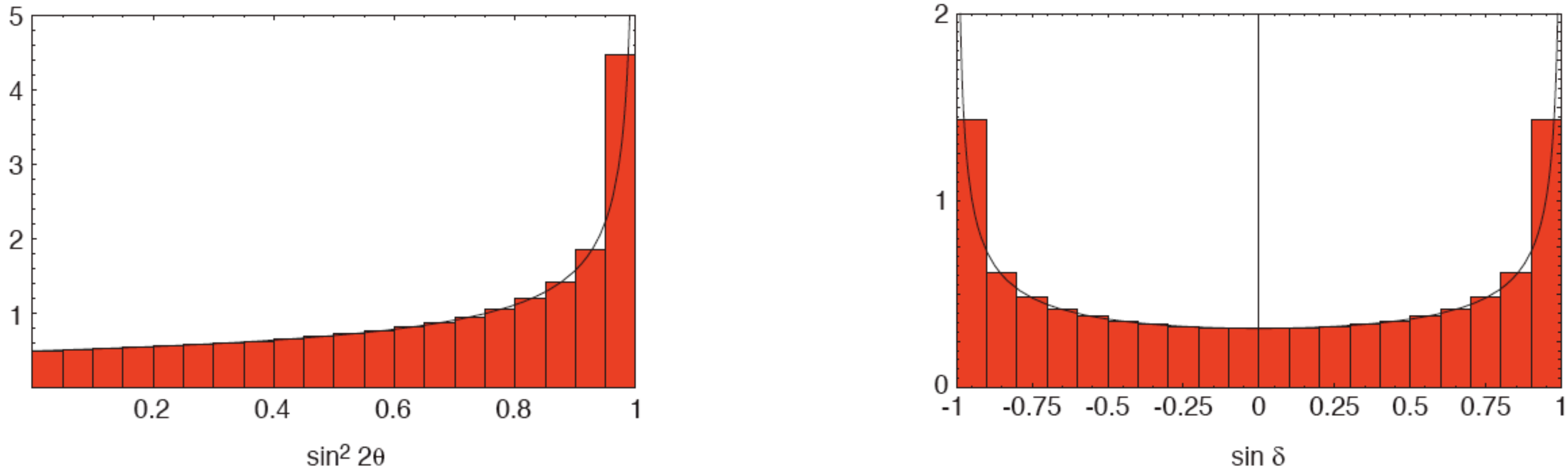



Figure 2: Distributions in (a) $\sin^2 2\theta_{12}$ or $\sin^2 2\theta_{13}$ or $\sin^2 2\theta_{23}$ and (b) $\sin\delta$ for the case of complex mass matrices.

[Haba, Murayama, hep-ph/0009174]

10 anarchical mixing matrices, plus the "real" one

$$\left(\begin{array}{ccc} |U_{e1}|^2 & |U_{e2}|^2 & |U_{e3}|^2 \\ \cdots & \cdots & |U_{\mu 3}|^2 \\ \cdots & \cdots & |U_{\tau 3}|^2 \end{array}\right) = \left(\begin{array}{ccc} 0.69 & 0.29 & 0.02 \\ \cdots & \cdots & 0.40 \\ \cdots & \cdots & 0.58 \end{array}\right), \left(\begin{array}{ccc} 0.36 & 0.35 & 0.29 \\ \cdots & \cdots & 0.68 \\ \cdots & \cdots & 0.03 \end{array}\right),$$

$$\left(\begin{array}{ccc} 0.83 & 0.11 & 0.06 \\ \cdots & \cdots & 0.87 \\ \cdots & \cdots & 0.07 \end{array}\right), \left(\begin{array}{ccc} 0.71 & 0.13 & 0.16 \\ \cdots & \cdots & 0.20 \\ \cdots & \cdots & 0.64 \end{array}\right), \left(\begin{array}{ccc} 0.24 & 0.47 & 0.29 \\ \cdots & \cdots & 0.58 \\ \cdots & \cdots & 0.13 \end{array}\right),$$

$$\left(\begin{array}{ccc} 0.16 & 0.35 & 0.49 \\ \cdots & \cdots & 0.13 \\ \cdots & \cdots & 0.38 \end{array}\right), \left(\begin{array}{ccc} 0.63 & 0.24 & 0.13 \\ \cdots & \cdots & 0.73 \\ \cdots & \cdots & 0.14 \end{array}\right), \left(\begin{array}{ccc} 0.12 & 0.35 & 0.53 \\ \cdots & \cdots & 0.12 \\ \cdots & \cdots & 0.35 \end{array}\right),$$

$$\left(\begin{array}{ccc} 0.22 & 0.55 & 0.23 \\ \cdots & \cdots & 0.12 \\ \cdots & \cdots & 0.65 \end{array}\right), \left(\begin{array}{ccc} 0.21 & 0.37 & 0.42 \\ \cdots & \cdots & 0.08 \\ \cdots & \cdots & 0.50 \end{array}\right), \left(\begin{array}{ccc} 0.54 & 0.44 & 0.02 \\ \cdots & \cdots & 0.54 \\ \cdots & \cdots & 0.44 \end{array}\right).$$

# What Anarchy is NOT ('secondo me')

Nowhere did I mention neutrino or charged-lepton **masses**.

The anarchy hypothesis is silent regarding the values of these parameters, or even their probability distributions.

There is no model-independent way of defining a probability distribution for an extensive parameter. This is easy to understand. Say one would like to postulate that some Yukawa coupling $y$ is distributed according to a flat distribution:

$$f(y) = 1/(\max - \min) \quad y \in [\min,\ \max].$$

max and min not well defined! Should one pick min$= 0$, max$= 1$? Perhaps max$= 4\pi\ldots$

Even the *measure* is not well-defined. Should the distribution for $y$ be flat, or that for $\log y$? How about $y^2$, or $1/y$ ...?

# According to Thomas (1209.3023), this is the result of Nature's draw:

| | Free Fluxes + RSBL | | Huber Fluxes, no RSBL | |
|---|---|---|---|---|
| | bfp $\pm 1\sigma$ | $3\sigma$ range | bfp $\pm 1\sigma$ | $3\sigma$ range |
| $\sin^2\theta_{12}$ | $0.30 \pm 0.013$ | $0.27 \to 0.34$ | $0.31 \pm 0.013$ | $0.27 \to 0.35$ |
| $\theta_{12}/^\circ$ | $33.3 \pm 0.8$ | $31 \to 36$ | $33.9 \pm 0.8$ | $31 \to 36$ |
| $\sin^2\theta_{23}$ | $0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$ | $0.34 \to 0.67$ | $0.41^{+0.030}_{-0.029} \oplus 0.60^{+0.020}_{-0.026}$ | $0.34 \to 0.67$ |
| $\theta_{23}/^\circ$ | $40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.2}_{-1.3}$ | $36 \to 55$ | $40.1^{+2.1}_{-1.7} \oplus 50.7^{+1.1}_{-1.5}$ | $36 \to 55$ |
| $\sin^2\theta_{13}$ | $0.023 \pm 0.0023$ | $0.016 \to 0.030$ | $0.025 \pm 0.0023$ | $0.018 \to 0.033$ |
| $\theta_{13}/^\circ$ | $8.6^{+0.44}_{-0.46}$ | $7.2 \to 9.5$ | $9.2^{+0.42}_{-0.45}$ | $7.7 \to 10.$ |
| $\delta_{\rm CP}/^\circ$ | $240^{+102}_{-74}$ | $0 \to 360$ | $238^{+95}_{-51}$ | $0 \to 360$ |
| $\frac{\Delta m^2_{21}}{10^{-5}\ \rm eV^2}$ | $7.50 \pm 0.185$ | $7.00 \to 8.09$ | $7.50^{+0.205}_{-0.160}$ | $7.04 \to 8.12$ |
| $\frac{\Delta m^2_{31}}{10^{-3}\ \rm eV^2}$ (N) | $2.47^{+0.069}_{-0.067}$ | $2.27 \to 2.69$ | $2.49^{+0.055}_{-0.051}$ | $2.29 \to 2.71$ |
| $\frac{\Delta m^2_{32}}{10^{-3}\ \rm eV^2}$ (I) | $-2.43^{+0.042}_{-0.065}$ | $-2.65 \to -2.24$ | $-2.47^{+0.073}_{-0.064}$ | $-2.68 \to -2.25$ |

**Table 1:** Three-flavour oscillation parameters from our fit to global data after the Neutrino 2012 conference. For "Free Fluxes + RSBL" reactor fluxes have been left free in the fit and short baseline reactor data (RSBL) with $L \lesssim 100$ m are included; for "Huber Fluxes, no RSBL" the flux prediction from [42] are adopted and RSBL data are not used in the fit.

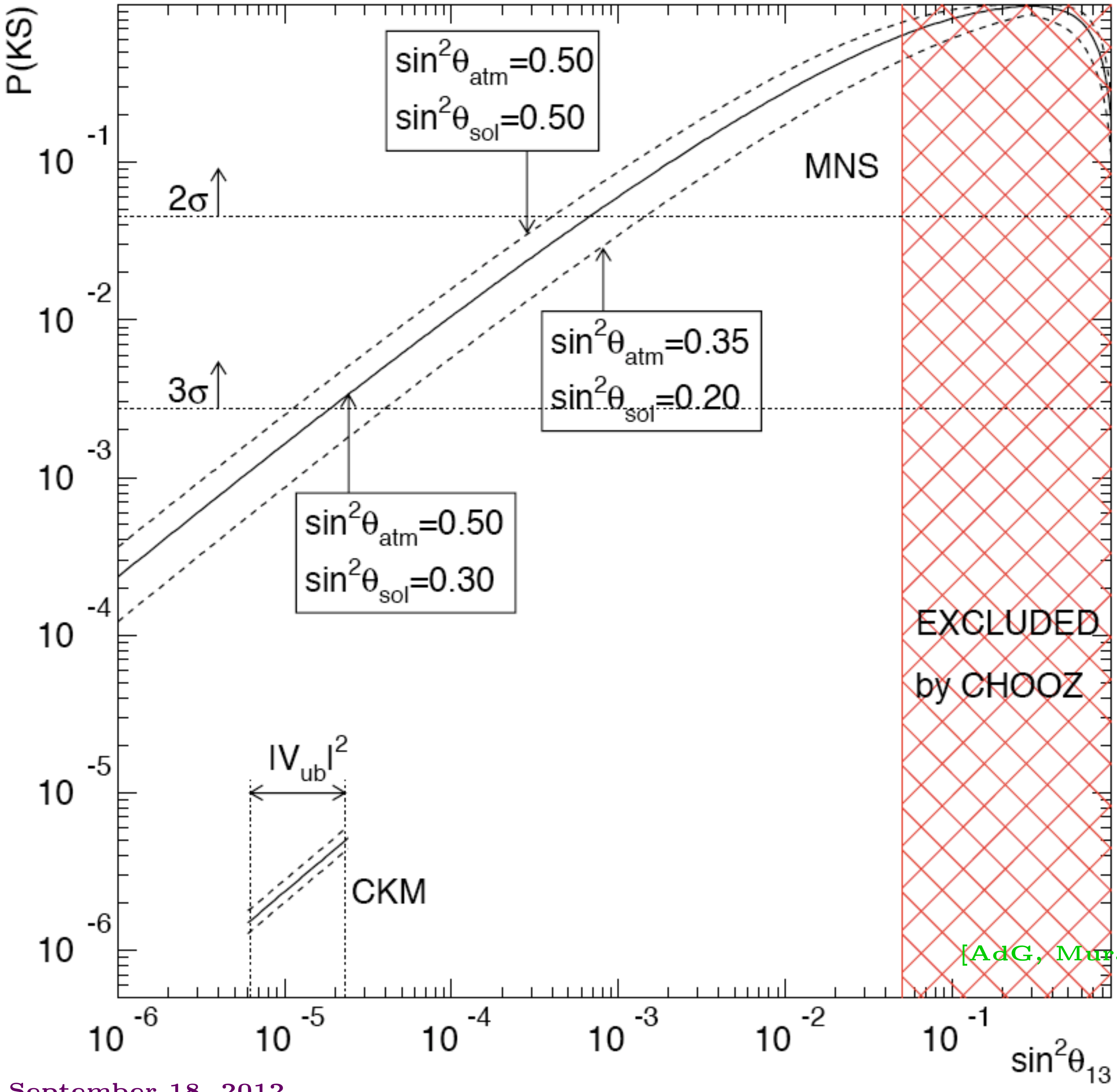
**Is it consistent with the anarchy hypothesis?**

In **[AdG, Murayama, hep-ph/0301050]**, a Kolmogorov-Smirnov (KS) test was proposed as a test statistic for the anarchy hypothesis.

The KS test allows one to address the following question: if I make $N$ draws from the same probability distribution, how likely is it that I would get a less typical result upon repeating this procedure?

If the probability is largish, then I tend to agree that the probability distribution in question provides a good description to my $N$ results. If it is smallish, not so much.

For the anarchy hypothesis, we have $N = 1$ (alas, only one observable leptonic mixing matrix!). We further use the KS probability to define goodness-of-fit, and to make predictions.




In 2003:

$P(KS) = 64\%$

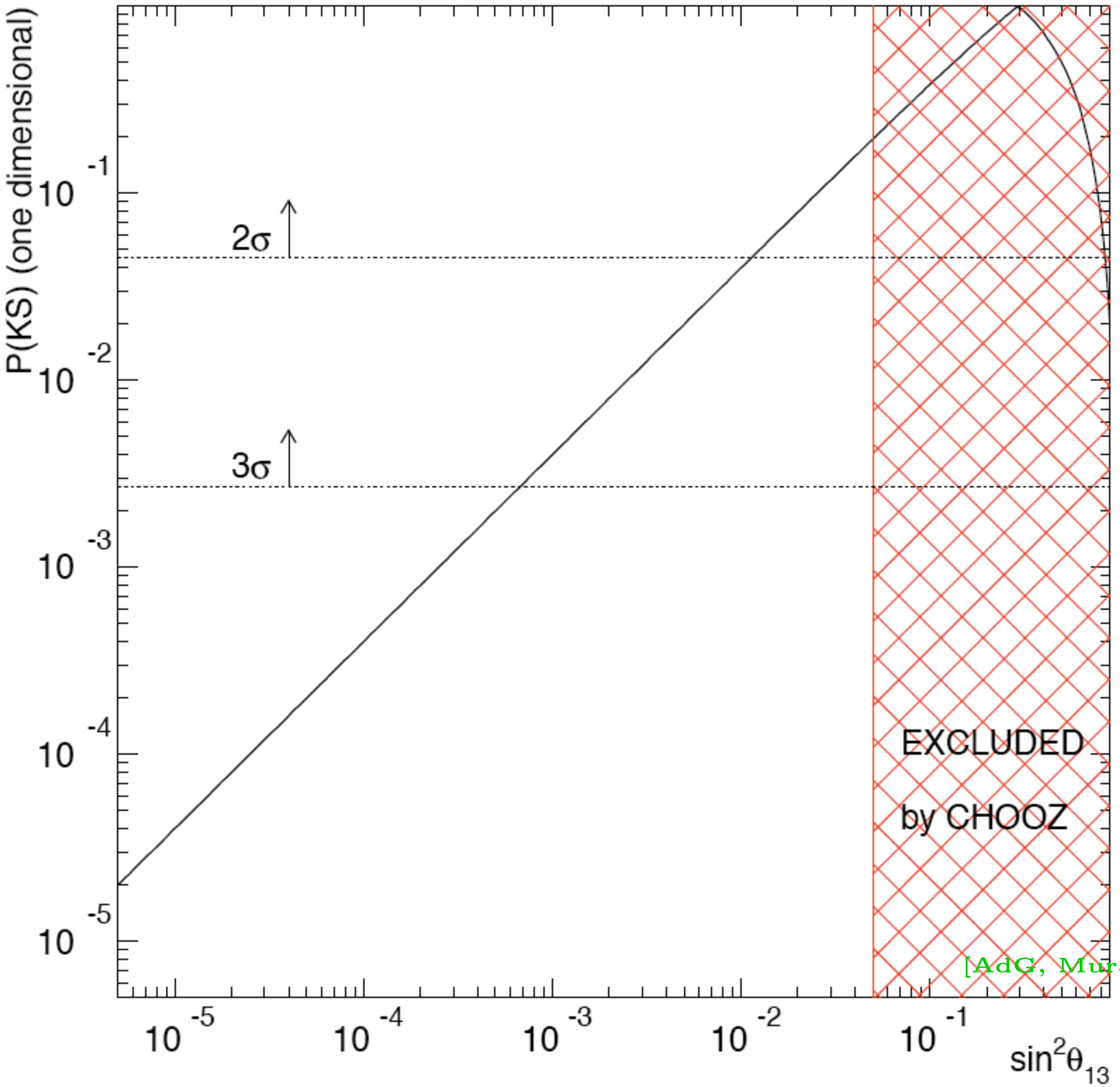
Anarchy in great shape,

unless $\theta_{13}$ small.

[Anarchy fails miserably

in the quark sector.]

[AdG, Murayama, hep-ph/0301050]




In 2003: Prediction

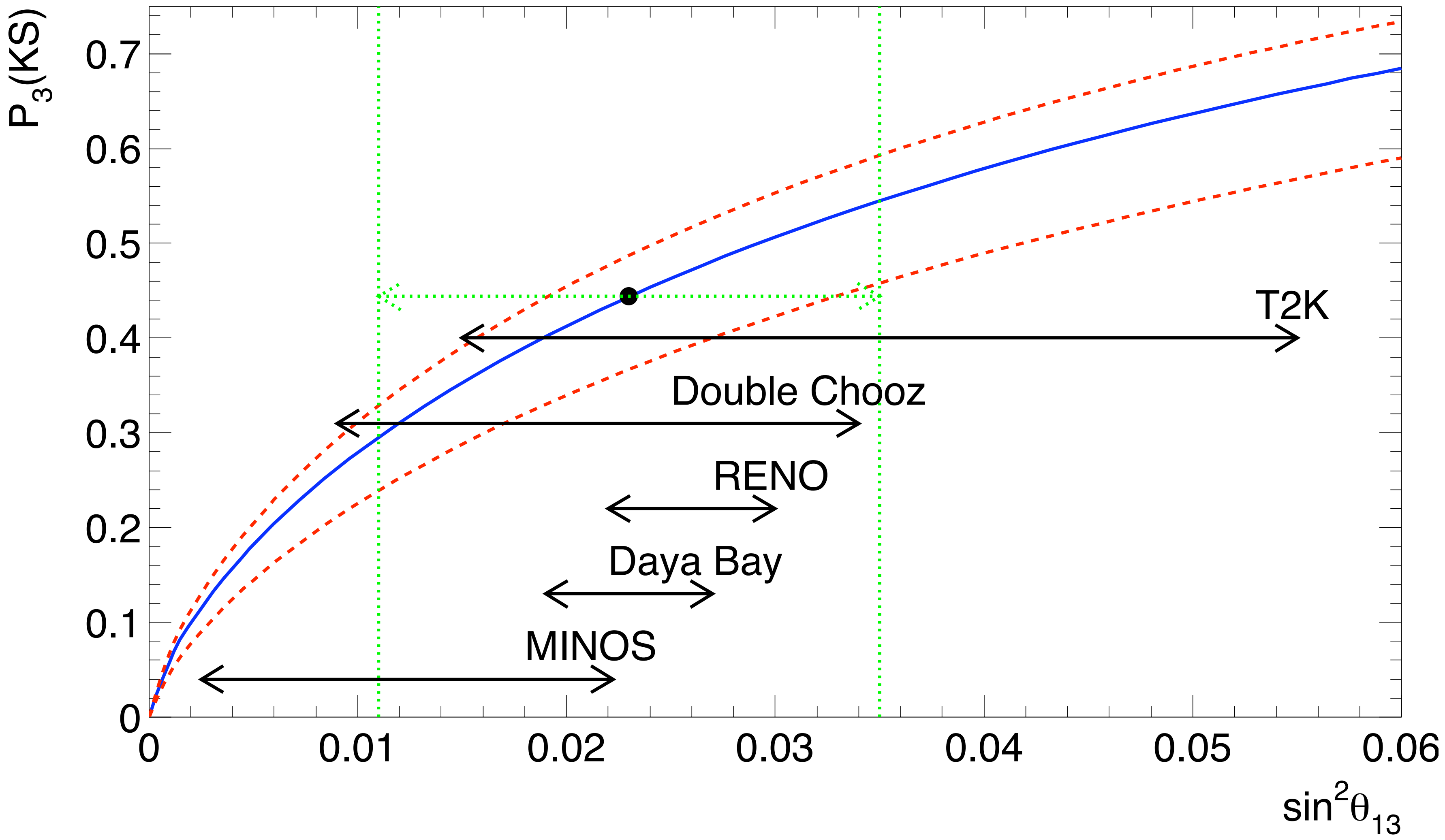
$\sin^2\theta_{13} \geq 0.011$ at 95.5% C.L. "(2$\sigma$)"

(CONFIRMED!)

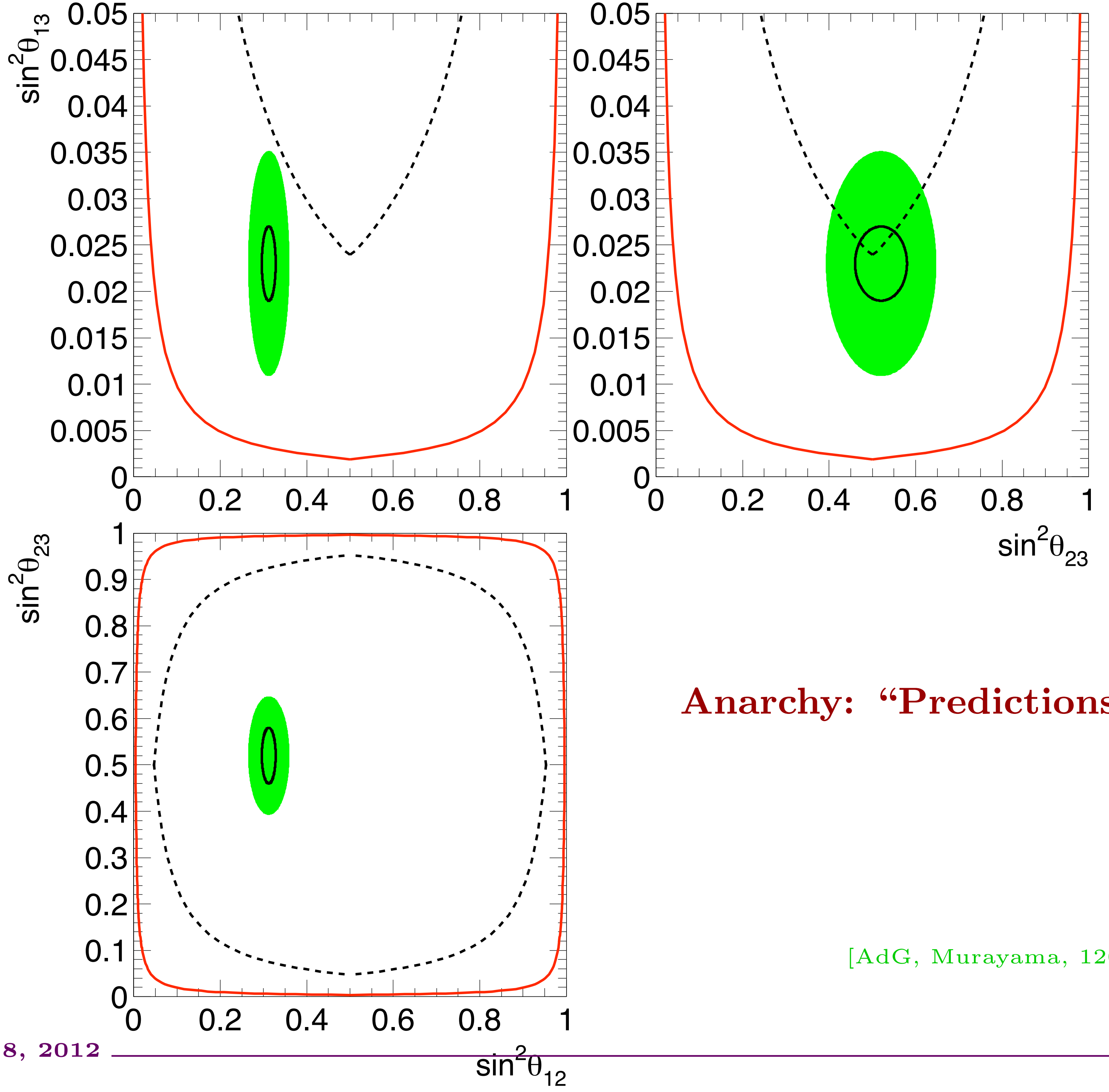
[1d $P(KS)$:marginalize over all other parameters]

[AdG, Murayama, hep-ph/0301050]

# Neutrino Mixing Anarchy: Alive and Kicking!




[AdG, Murayama, 1204.1249]




## Anarchy: "Predictions"

[AdG, Murayama, 1204.1249]

These tend to favor large mixing, and large CP-violating effects.

Why? – Because these are most common!

Prediction: $\delta$ CP-violation "large"

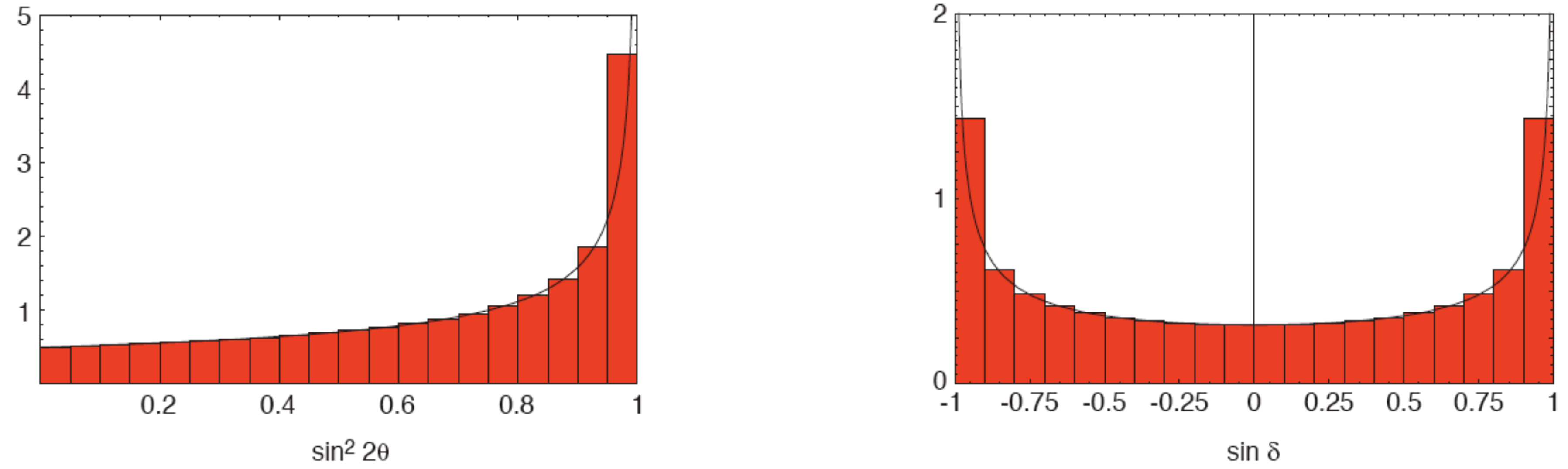
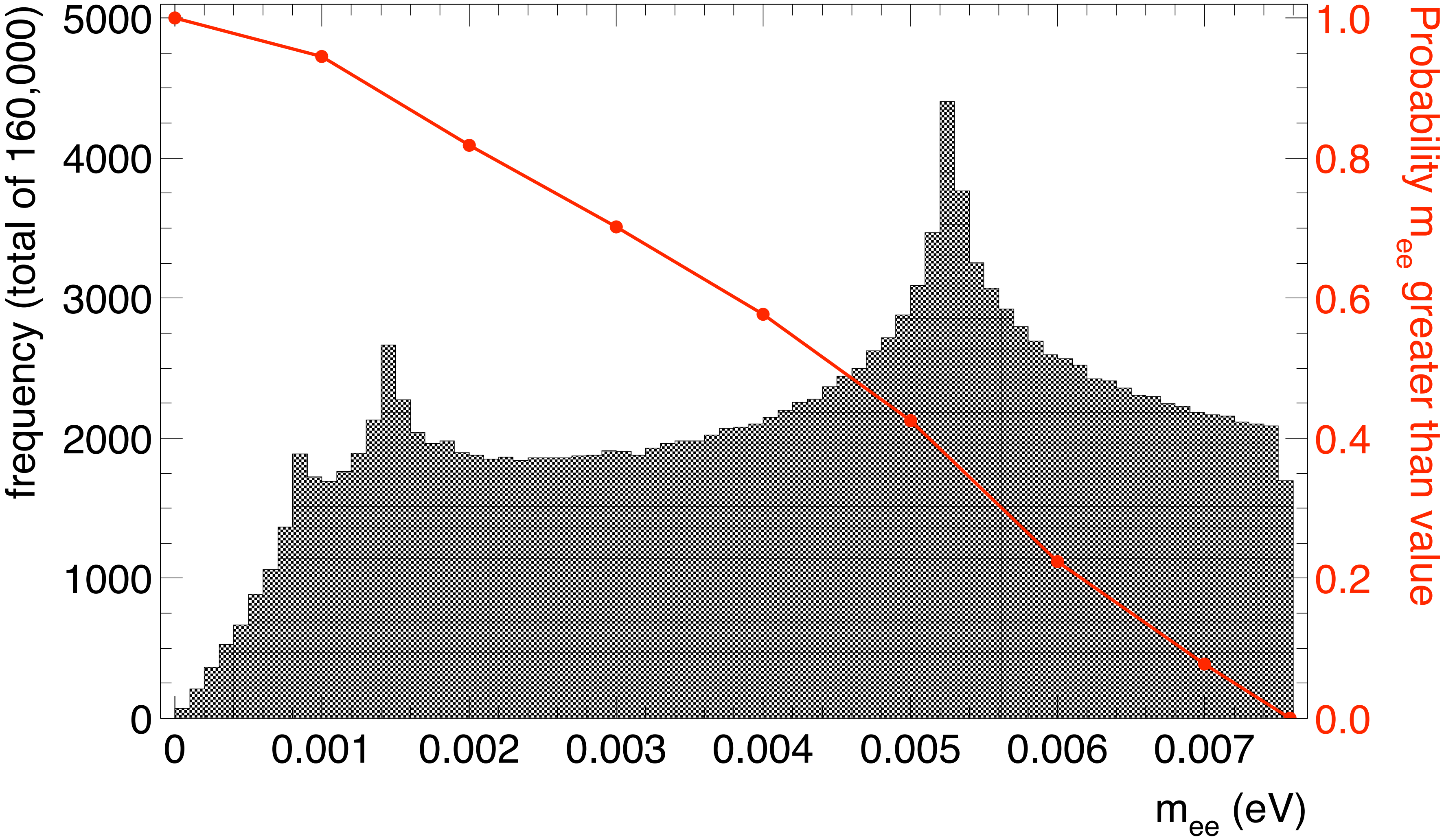


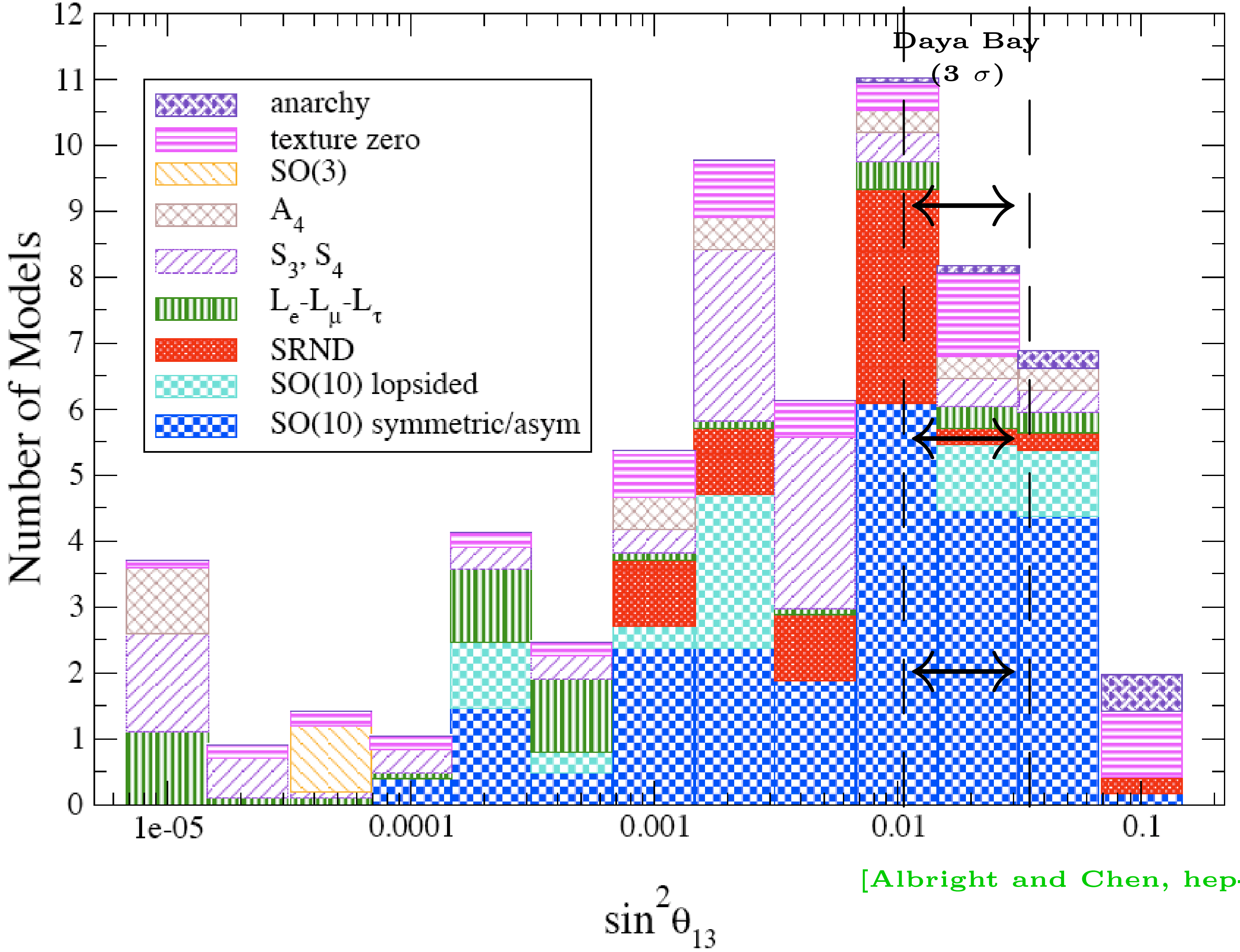

Figure 2: Distributions in (a) $\sin^2 2\theta_{12}$ or $\sin^2 2\theta_{13}$ or $\sin^2 2\theta_{23}$ and (b) $\sin\delta$ for the case of complex mass matrices.

[Haba, Murayama, hep-ph/0009174]




[AdG, Murayama, 1204.1249]

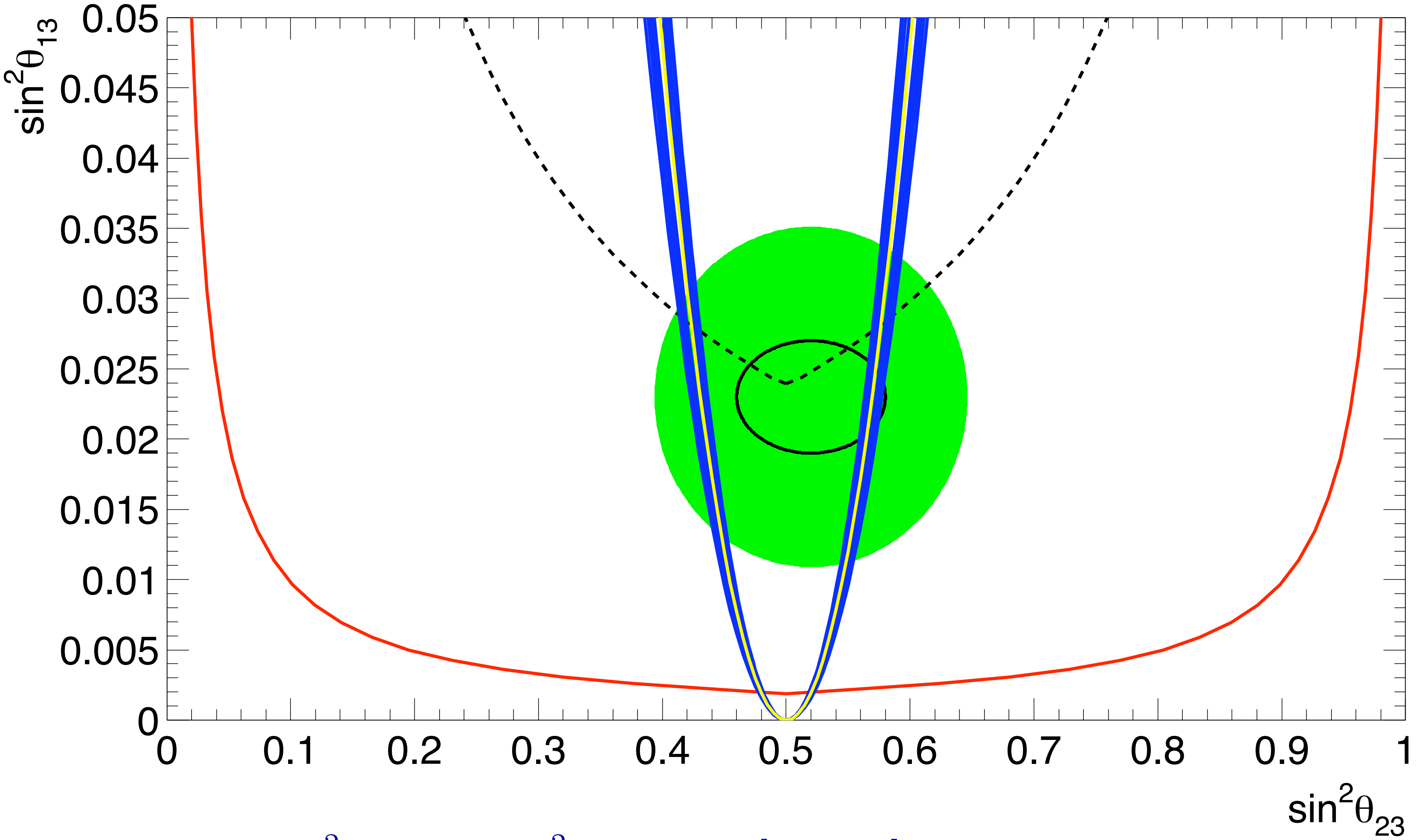
Prediction for $m_{ee}$: How likely is it that it vanishes? $m_1 = 0.005$ eV fixed at "worst case" scenario, assuming a normal mass hierarchy.




[Albright and Chen, hep-ph/0608137]

"Left-Over" Predictions: $\delta$, mass-hierarchy, $\cos 2\theta_{23}$. More important: CORRELATIONS!

## Anarchy vs. Order — more precision required!




Order: $\sin^2\theta_{13} = C\cos^2 2\theta_{23}$, $C \in [0.8, 1.2]$

[AdG, Murayama, 1204.1249]

# Some Models that Satisfy the Anarchy Hypothesis

- Assume all entries of $m_\nu$ are $\mathcal{O}(1)$ in some units [Unit doesn't matter]. What matters is that the different elements are uncorrelated and drawn from the same random distribution.

  By the way, most models for the underlying physics have random, $\mathcal{O}(1)$ parameters that need to be adjusted in order to fit the data. They are anarchical in spirit (even if their predictions are often far from it).

- It is possible that there is an underlying symmetry at some high energy scale and the anarchy hypothesis still holds. This would happen if, for example, the low-energy remnants of the underlying symmetry are "erased" (too many fields integrated out, renormalization group effects, etc).

- It is possible that there is an underlying symmetry, but a subset of the lepton fields is blind to it.

  E.g. imagine a Froggatt-Nielsen model with Dirac neutrinos. Assign flavor charges such that all $L$s (lepton-doublets) have the same charge, but the $E$s (charged-lepton singlets) and $N$s (right-handed neutrinos) don't.

In this case, one generates Dirac mass matrices (for both the charged and neutral leptons) proportional to

$$\left(\begin{array}{ccc} \mathrm{L} & \mathrm{L} & \mathrm{L} \\ M & M & M \\ \mathrm{S} & \mathrm{S} & \mathrm{S} \end{array}\right).$$

In this case, the mass eigenvalues are $L, M, S$, and the "right-handed" mixing matrices are hierarchical. The "left-handed" mixing matrices are anarchical and so is $U$!

Incidently, this type of scenario is compatible with $SU(5)$ GUTs, since it postulates large mixing among left-handed leptons and right-handed down-type quarks. The latter is not observable (unless there is more physics beyond the SM). [Masiero, Murayama, hep-ph/0205111]

# Final Thoughts

1. The Anarchy Hypothesis fits the lepton mixing data very well.

2. This does NOT mean that the Anarchy Hypothesis is true. It just means that the data are unable to falsify it. This statement is most likely to remain true for the foreseeable future.

3. The Anarchy Hypothesis may or may not fit the data "better" than different Order Hypotheses. I certainly hope that any worthwhile Order Hypothesis fits the data better than the Anarchy Hypothesis — after all, this is what Order Hypotheses are built to do! Comparisons, however, are very tricky! [Altarelli, next talk]

4. The "flavor problem" has been around for 40 years or so. It has frustrated generations of particle physicists. I am not implying that we don't need to worry about flavor in the lepton mixing sector. Symmetry models, however, have the "burden of proof." They need to do better than the Anarchy Hypothesis. Qualitatively better!

# In Conclusion . . .

# In Conclusion . . .


?