BeNe, Trieste, 18 September '12

Hierarchy vs Anarchy in Neutrino Masses and Mixing

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in
Neutrino Masses and Mixing

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Based on: GA, Feruglio, Merlo, Stamou, ArXiv:1205.4670
GA, Feruglio, Merlo, ArXiv:1205.5133
GA, Feruglio, Masina, Merlo, ArXiv: 1207.0587

Recent developments in neutrino mixing

- Are sterile neutrinos coming back?

  Not pursued here: we assume 3 ν's in the following

  Schwetz' talk

- θ_{13} measured (~ 10σ from zero, near the previous bound)

  T2K, MINOS, DoubleCHOOZ, Daya Bay, RENO

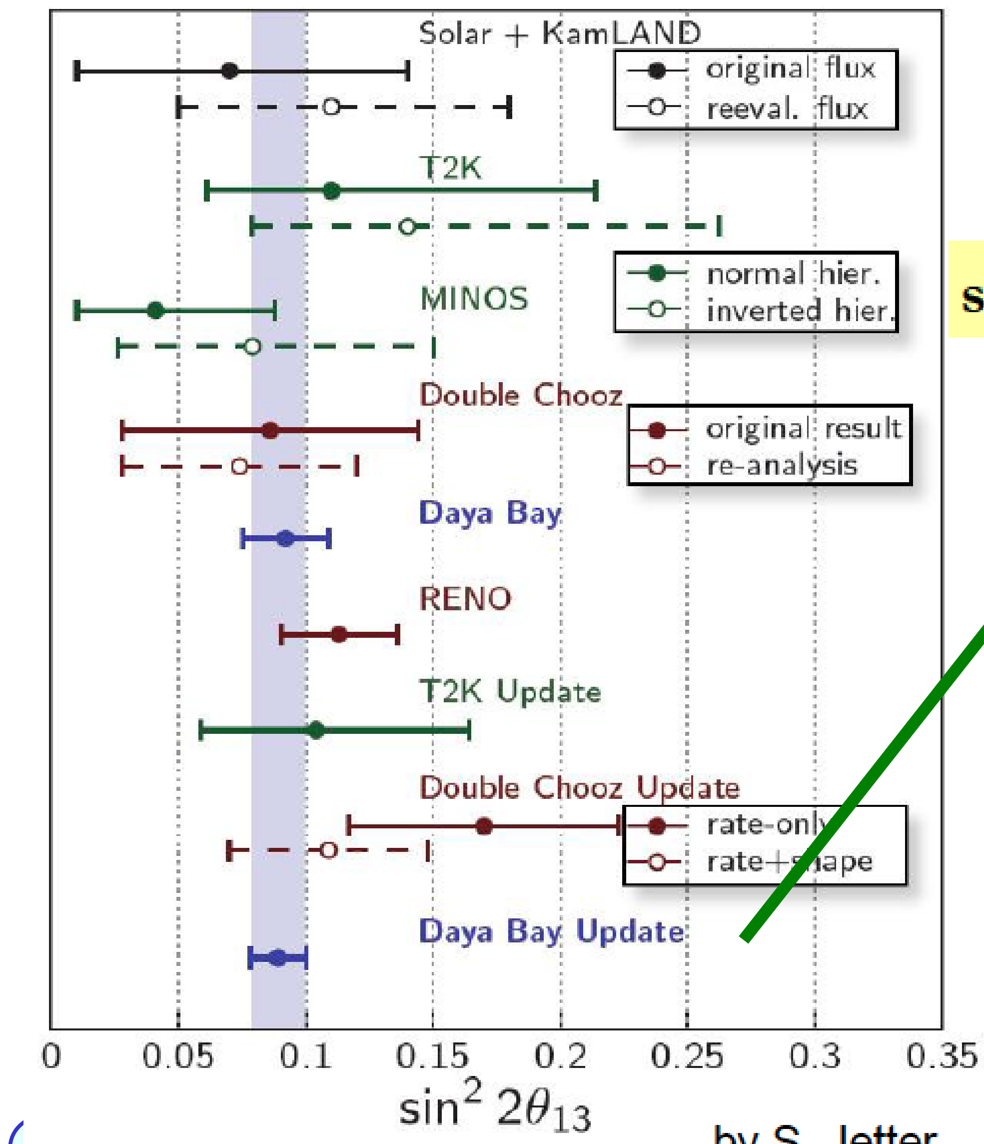
- Indication of θ_{23} non maximal, in 1st octant

  Indication of $\cos\delta_{CP} < 0$

  Related to θ_{13} large, from MINOS and T2K

  Fogli et al '12, Gonzalez-Garcia et al '12

Now we have a good measurement of θ_{13}!!



by S. Jetter

~8σ from zero

Daya Bay

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010(\text{stat}) \pm 0.005(\text{syst})$$

$\sin\theta_{13} = 0.15 \pm 0.01$

$\sin^2\theta_{13} = 0.023 \pm 0.003$

$\theta_{13} = 8.7^{\circ} \pm 0.6^{\circ}$

A large impact on model building!

Parameter	Best fit	1σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.33 – 2.49
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.31 – 2.49
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31
δ/π (NH)	1.08	0.77 – 1.36
δ/π (IH)	1.09	0.83 – 1.47

Fogli et al '12

θ_{23} non maximal

$\cos\delta < 0$?

$\sin^2\theta_{12}$	0.30 ± 0.013
$\theta_{12}/^\circ$	33.3 ± 0.8
$\sin^2\theta_{23}$	$0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$
$\theta_{23}/^\circ$	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.2}_{-1.3}$
$\sin^2\theta_{13}$	0.023 ± 0.0023
$\theta_{13}/^\circ$	$8.6^{+0.44}_{-0.46}$
$\delta_{CP}/^\circ$	240^{+102}_{-74}
$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	7.50 ± 0.185
$\dfrac{\Delta m^2_{31}}{10^{-3}\ \mathrm{eV}^2}$ (N)	$2.47^{+0.069}_{-0.067}$
$\dfrac{\Delta m^2_{32}}{10^{-3}\ \mathrm{eV}^2}$ (I)	$-2.43^{+0.042}_{-0.065}$

Gonzalez-Garcia et al '12

In spite of this progress viable models still span a wide range that goes from very little structure to a lot of symmetry

At one extreme are models dominated by chance
Some examples:

Anarchy
$\mu\tau$-anarchy
$U(1)_{\text{Froggatt-Nielsen}}$ charges
••••••

On the other hand the range for each mixing angle has narrowed and precise special patterns can be tentatively identified as starting approximations that, if significant, lead to specified discrete symmetries:

TriBimaximal (TB), BiMaximal (BM),.......
Discrete non abelian flavour groups A4, S4,.....

θ_{13} near the previous bound and θ_{23} non maximal both go in the direction of Anarchy (a great success for Anarchy!)

Anarchy: no order for leptons

In the lepton sector no symmetry, no dynamics is needed; only chance

Hall, Murayama, Weiner '00

.....

de Gouvea, Murayama '12

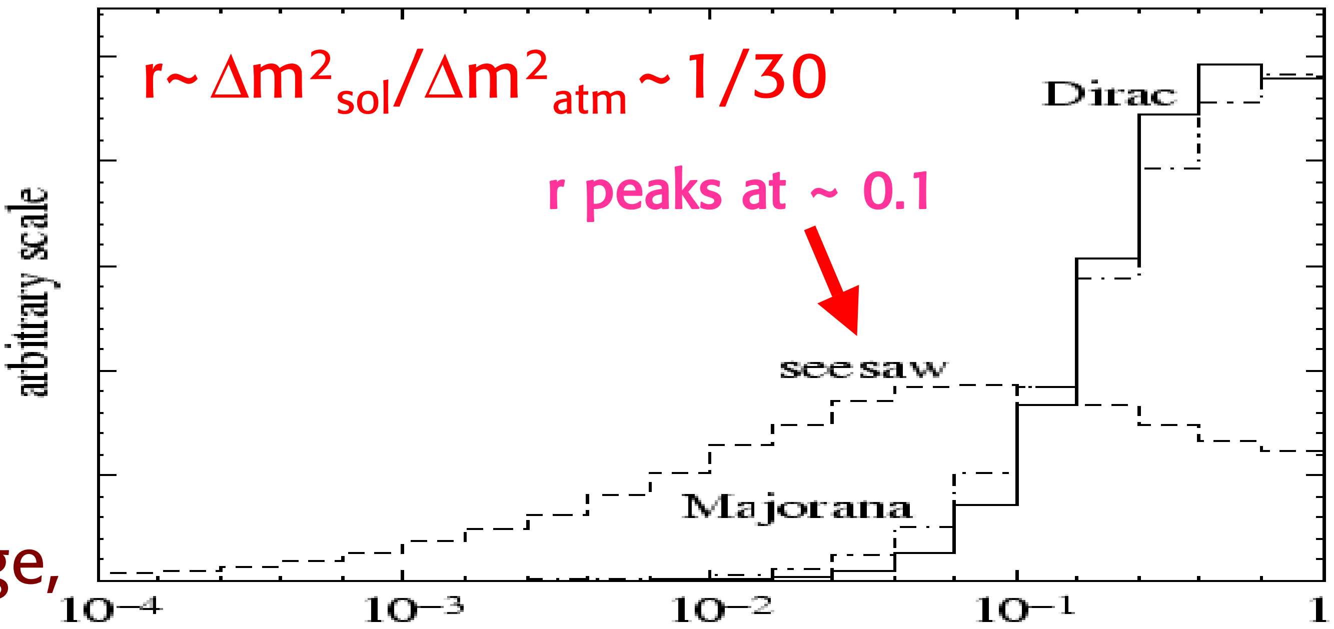
$\theta_{12}, \theta_{13}, \theta_{23}$ are just 3 random angles, the value of $r = \Delta m^2_{sun} / \Delta m^2_{atm}$ is also determined by chance

Anarchy (or accidental hierarchy): No structure in the neutrino sector

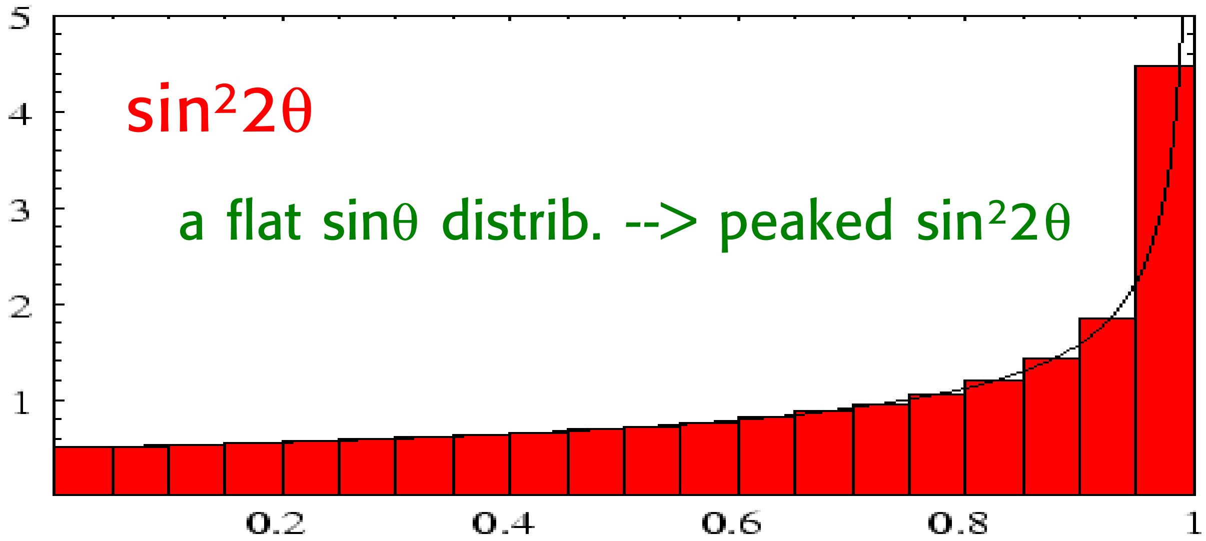
Hall, Murayama, Weiner '00

See-Saw:
$m_\nu \sim m^T M^{-1} m$
produces hierarchy from random m, M

could fit the data on r



All mixing angles should be not too large, not too small



Predicts θ_{13} near old bound and
θ_{23} sizably non maximal

successful!

Anarchy and its variants can be embedded in a simple GUT context based on

$SU(5)xU(1)_{flavour}$

Froggatt Nielsen '79

Offers a simple description of hierarchies for quarks and leptons, but only orders of magnitude are predicted (large number of undetermined o(1) parameters c_{ab})

The typical order parameter is $o(\lambda_C)$ and the entries of mass matrices are suppressed by $m_{ab} \sim c_{ab} \, (\lambda_C)^{nab}$

The exponenents n_{ab} are fixed by the charge imbalance

Anarchy can be realised in SU(5) by putting all the flavour structure in $T \sim 10$ and not in $F^{bar} \sim 5^{bar}$

$m_u \sim 10 . 10$ — strong hierarchy $m_u : m_c : m_t$

$m_d \sim 5^{bar} . 10 \sim m_e^T$ — milder hierarchy $m_d : m_s : m_b$ or $m_e : m_\mu : m_\tau$

Experiment supports that down quark & charged lepton hierarchy is roughly the square root of up quark hierarchy

$m_\nu \sim \nu_L^T m_\nu \nu_L \sim 5^{barT} . 5^{bar}$ or for see saw $(5^{bar}.1)^T\ (1.1)\ (1.5^{bar})$

For example, for the simplest flavour group, $U(1)_F$

1st fam. 2nd 3rd

Anarchy
$$\begin{cases} T : & (3, 2, 0) \\ F^{bar} : & (0, 0, 0) \\ 1 : & (0, 0, 0) \end{cases}$$

A milder ansatz - μ–τ anarchy: no structure only in 23

Consider a matrix like $m_\nu \sim L^T L \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & 1 & 1 \\ \varepsilon^2 & 1 & 1 \end{pmatrix}$

$q(\bar{5}) \sim (2, 0, 0)$

with coeff.s of o(1) and det23~o(1)

Note: $\theta_{13} \sim \varepsilon^2$

$\theta_{23} \sim 1$

["semianarchy", while $\varepsilon \sim 1$ corresponds to anarchy]

After 23 and 13 rotations $m_\nu \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^2 & 0 \\ \varepsilon^2 & \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$r = \Delta m^2_{sun} / \Delta m^2_{atm}$

Normally two masses are of o(1) or r ~1 and $\theta_{12} \sim \varepsilon^2$

But if, accidentally, $\eta \sim \varepsilon^2$, then r is small and θ_{12} is large.

The advantage over anarchy is that θ_{13} is naturally small and a single accident is needed to get both θ_{12} large and r small

Ramond et al, ...

recently reanalysed by Buchmuller et al, '11

With see-saw one can do better

G.A., Feruglio, Masina'02

1st fam. 2nd 3rd

$q(10)$: $(5, 3, 0)$
$q(\bar{5})$: $(2, 0, 0)$
$q(1)$: $(1,-1, 0)$

Needed: not all charges positive

$q(H) = 0$, $q(\bar{H}) = 0$
$q(\theta) = -1$, $q(\theta') = +1$

In first approx., with $<\theta>/M \sim \lambda \sim \lambda' \sim 0.35 \sim o(\lambda_C)$

$10_i 10_j$

$$m_u \sim v_u \begin{bmatrix} \lambda^{10} & \lambda^8 & \lambda^5 \\ \lambda^8 & \lambda^6 & \lambda^3 \\ \lambda^5 & \lambda^3 & 1 \end{bmatrix},$$

$10_i \bar{5}_j$

$$m_d = m_e^T \sim v_d \begin{bmatrix} \lambda^7 & \lambda^5 & \lambda^5 \\ \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^2 & 1 & 1 \end{bmatrix}$$

"lopsided"

$\bar{5}_i 1_j$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda' & 1 \\ \lambda & \lambda' & 1 \end{bmatrix},$$

$1_i 1_j$

$$M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix}$$

Note: coeffs. 0(1) omitted, only orders of magnitude predicted with $\lambda \sim \lambda'$

$$\bar{5}_i 1_j \qquad\qquad 1_i 1_j$$

$$m_{\nu D} \sim v_u \begin{bmatrix} \lambda^3 & \lambda & \lambda^2 \\ \lambda & \lambda & 1 \\ \lambda & \lambda & 1 \end{bmatrix}, \qquad M_{RR} \sim M \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda^2 & \lambda \\ \lambda & \lambda & 1 \end{bmatrix}$$

see-saw $\quad m_\nu \sim m_{\nu D}{}^T M_{RR}{}^{-1} m_{\nu D}$

$$m_\nu \sim v_u{}^2/M \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix},$$

$\det_{23} \sim \lambda^2$

lopsided m_D
and M_{33} non zero
guarantees
det 23 suppressed

$r \sim \lambda^4$, $\theta_{13} \sim \lambda^2$, θ_{12}, $\theta_{23} \sim 1$

In this model all small parameters are naturally explained in terms of suitable suppression factors fixed by the charges

Called $PA_{\mu\tau}$ in the following

SU(5)xU(1)

1st fam. 2nd 3rd

Ψ_{10}: (5, 3, 0)
Ψ_5: (2, 0, 0)
Ψ_1: (1,-1, 0)

Equal 2,3 ch. for lopsided

Recall: $m_u \sim 10\ 10$
$m_d = m_e^T \sim \bar{5}\ 10$
$m_{\nu D} \sim \bar{5}\ 1$; $M_{RR} \sim 1\ 1$

Model	Ψ_{10}	Ψ_5	Ψ_1
Anarchy (A)	(3,2,0)	(0,0,0)	(0,0,0)
Semianarchy $\mu\tau$-Anarchy ($A_{\mu\tau}$)	(3,2,0)	(1,0,0)	(2,1,0)
Pseudo $\mu\tau$-Anarchy ($PA_{\mu\tau}$)	(5,3,0)	(2,0,0)	(1,-1,0)
Hierarchy (H) new	(5,3,0)	(2,1,0)	(2,1,0)

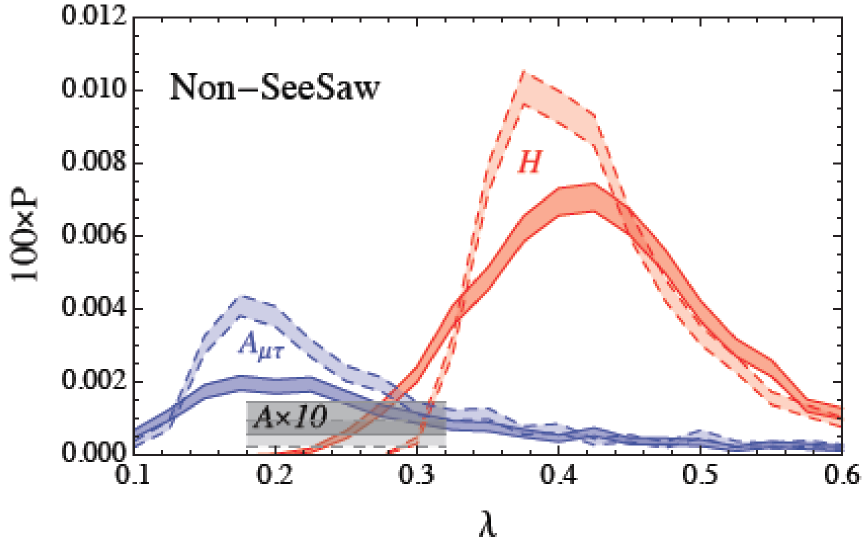
No structure for leptons (Anarchy)

No automatic det23 = 0 (Semianarchy): all charges non negative

Automatic det23 = 0 (Pseudo $\mu\tau$-Anarchy): charges of both signs

With suitable charge assignments many relevant patterns can be obtained

here r, θ_{23} are suppressed (Hierarchy)



Anarchy (A): both r and θ_{13} small by accident

$\mu\tau$-anarchy ($A_{\mu\tau}$): only r small by accident

H, $PA_{\mu\tau}$: no accidents

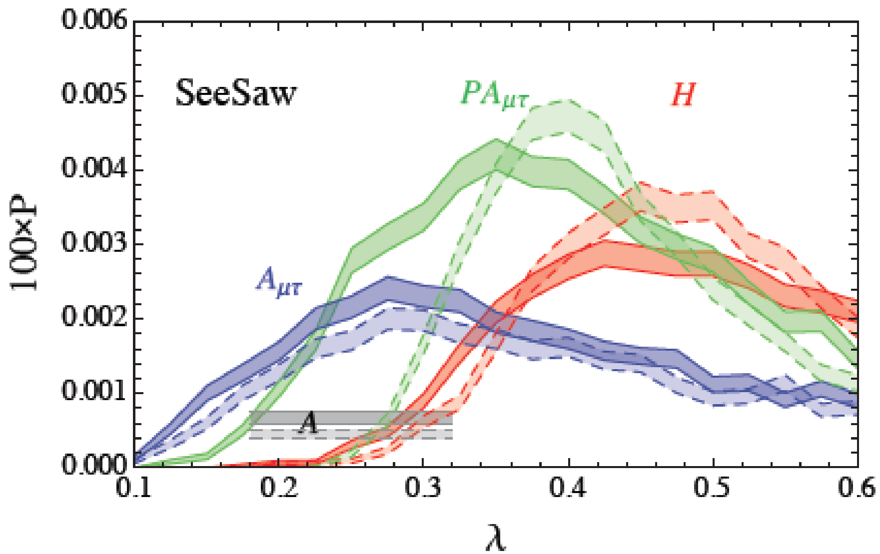
GA, Feruglio, Masina '02,'06
GA, Feruglio, Masina, Merlo '12

Optimal values of $\lambda \sim o(\lambda_C)$

$A_{\mu\tau}$: λ ~ 0.2 (non SS), 0.3 (SS)

$PA_{\mu\tau}$: λ ~ 0.35-0.4

H: λ ~ 0.4 (non SS), 0.45 (SS)



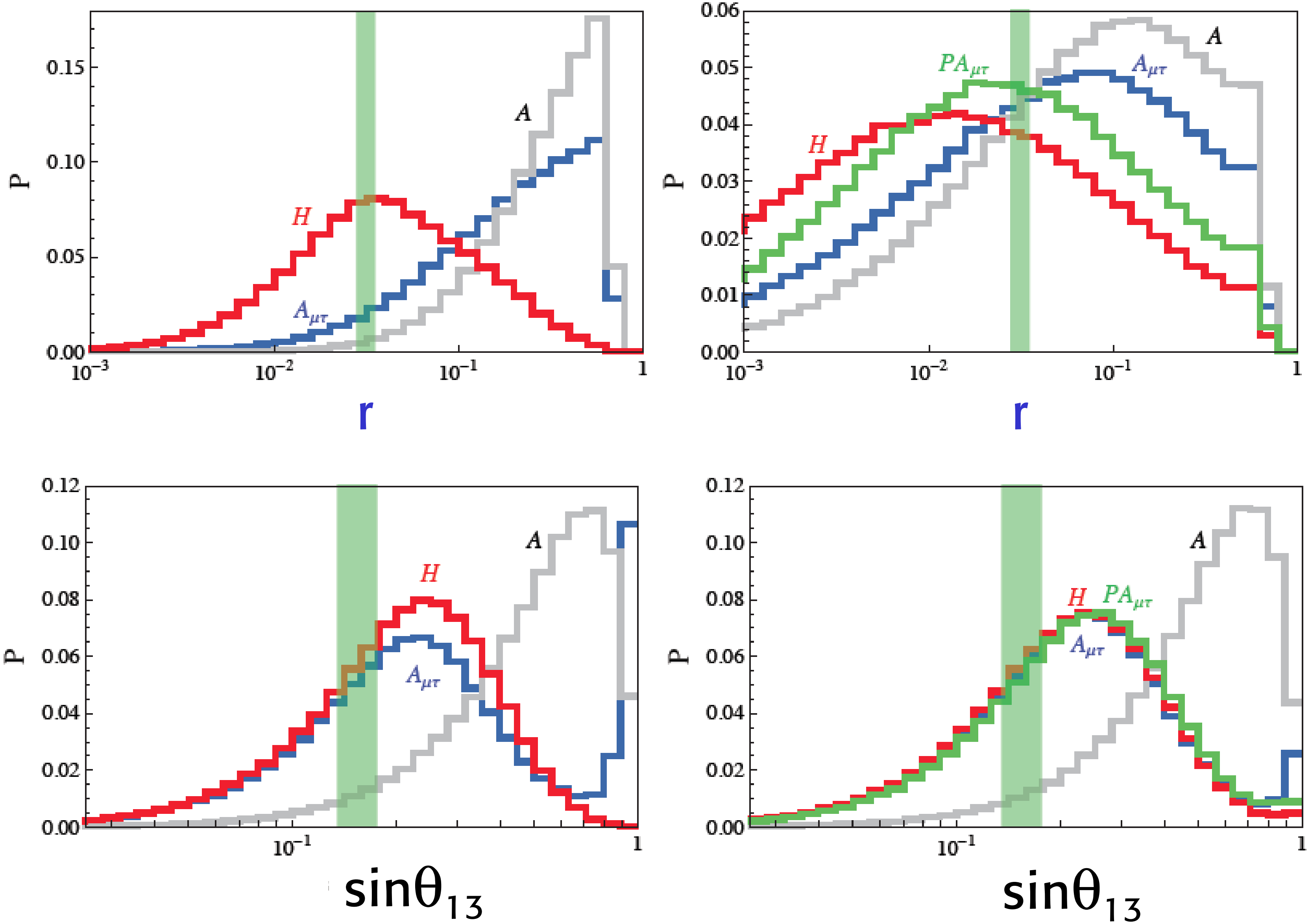
extraction range:
solid [0.5-2.0] dashed [0.8-1.2]

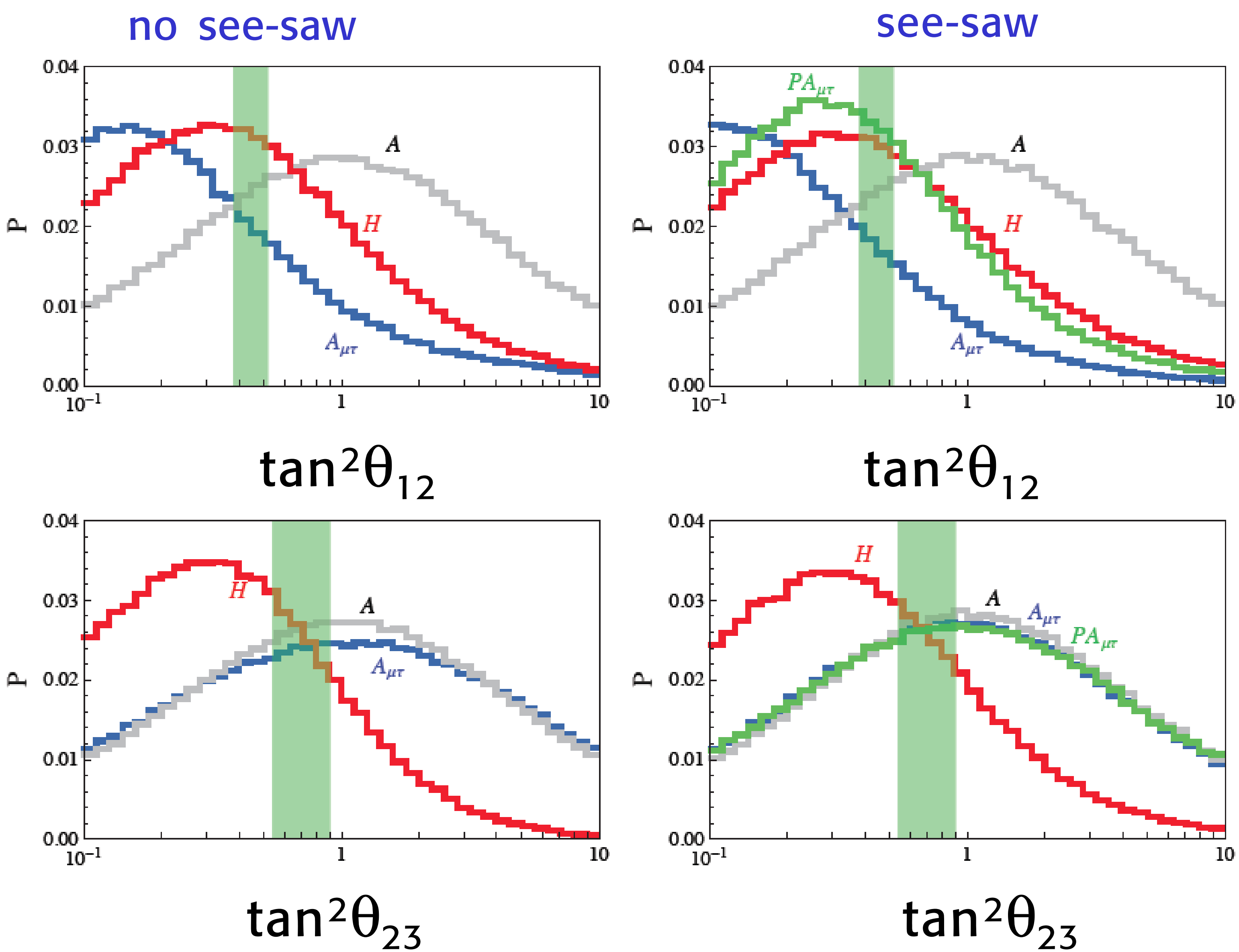
no see-saw

$$O_5 = \ell^T \frac{\lambda^2}{M} \ell HH \rightarrow \nu_L{}^T m_\nu \nu_L$$

when all charges are positive see-saw only affects r

see-saw





Thus: If we embed anarchy in GUT's and explain quark hierarchies in terms of FN charges, then more effective variants of anarchy can be built, where chance is somewhat mitigated

At the other extreme from Anarchy

models with a maximum of order:
based on non abelian discrete flavour groups

(reviews: G.A., Feruglio, Rev.Mod.Phys. 82 (2010) 2701
G.A., Feruglio, Merlo, ‘12)

A number of “coincidences” could be hints pointing to the underlying dynamics

TB Mixing

TB mixing is close to the data:
θ_{12}, θ_{23} agree within ~ 2σ
θ_{13} is the smallest angle

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

At 1σ: Fogli et al '12

$\sin^2\theta_{12} = 1/3$: 0.291- 0.325
$\sin^2\theta_{23} = 1/2$: 0.36 - 0.41
$\sin\theta_{13} = 0$: 0.14 - 0.16

A coincidence or a hint?

Called:
Tri-Bimaximal mixing

Harrison, Perkins, Scott '02

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

θ_{13} largish and θ_{23} non maximal tend to move away from TB

LQC: Lepton Quark Complementarity

$$\theta_{12} + \theta_C = (46.4 \pm 0.8)^\circ \sim \pi/4$$

Suggests Bimaximal mixing corrected by diagonalisation of charged leptons

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

A coincidence or a hint?

Golden Ratio

$$\sin^2\theta_{12} = \frac{1}{\sqrt{5}\phi} = \frac{2}{5+\sqrt{5}} \approx 0.276$$

$$U_{GR} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta_{12}}{\sqrt{2}} & -\frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

A coincidence or a hint?

Cannot all be true hints, perhaps none

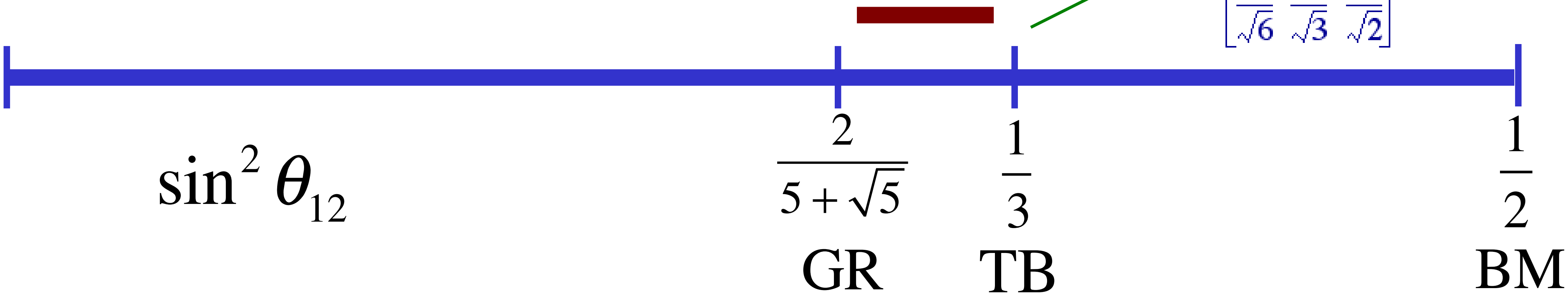
Neutrino mixing

$\sin^2\theta_{23} \sim 1/2$

$\sin^2\theta_{13} \sim 0$

Exp

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$



TB: Group A4, S4..... A vast literature (Ma, Rajasekaran '01.....)

GR: Golden Ratio - Group A5 Feruglio, Paris '11

BM: Group S4 GA, Feruglio, Merlo '09

A recent review of discrete flavour groups:
GA, F. Feruglio, ArXiv:1002.0211 (Review of Modern Physics)

TB Mixing naturally leads to discrete flavour groups (similarly for GR, BM....)

TB Mixing: $$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

This is a particular rotation matrix with specified fixed angles

Why and how discrete groups, in particular A4, work?

TB mixing corresponds to m in the basis where charged leptons are diagonal

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

Crucial point 1:
m is the most general matrix invariant under
SmS = m and $A_{23} m A_{23} = m$ with:

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad A_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

2-3 symmetry

$S^2 = A_{23}^2 = 1$

Crucial point 2:

Charged lepton masses:
a generic diagonal matrix
is defined by invariance under T
(or ηT with η a phase):

$$m_l = v_T \frac{v_d}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$m_l^+ m_l = T^+ m_l^+ m_l T$$

a possible T is

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

An essential observation is that

$\omega^3=1$ --> $T^3=1$

S, T and A_{23} are all contained in S4

$S^4=T^3=(ST^2)^2=1$ define S4

Thus S4 is the reference group for TB mixing

Lam

A4: a vast literature (Ma, Rajasekaran '01.....)

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has $4!/2 = 12$ elements.

A4 is a subgroup of S4

$S^2=T^3=(ST)^3=1$ define A4

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

$3, 1, 1', 1''$ (promising for 3 generations!)

Ch. leptons $l \sim 3$ $e^c, \mu^c, \tau^c \sim 1, 1'', 1'$

Invariance under S and T is automatic in A4 while
A_{23} is not contained in A4 (2<->3 exchange is an odd perm.)
But 2-3 symmetry happens in A4 if $1'$ and $1''$ symm. breaking
flavons are absent or have equal VEV's [2 of S4 = $1' + 1''$ of A4].

Crucial point 3: A4 must be broken: the alignment

Before SSB the model is invariant under the flavour group A4

There are flavons ϕ_T, ϕ_S, ξ... with VEV's that break A4:

ϕ_T breaks A4 down to G_T, the subgroup generated by
1, T, T^2, in the charged lepton sector

ϕ_S, ξ break A4 down to G_S, the subgroup generated by
1, S, in the neutrino sector

$$\boxed{\begin{aligned} \langle\varphi_T\rangle &= (v_T, 0, 0) \\ \langle\varphi_S\rangle &= (v_S, v_S, v_S) \\ \langle\xi\rangle = u \quad &, \quad \langle\tilde{\xi}\rangle = 0 \end{aligned}}$$

ϕ_T, ϕ_S ~ 3
ξ ~ 1

The 2-3 symmetry occurs in A4 if 1′ and 1″ flavons are absent

This aligment along subgroups of A4 must naturally occur in a good model

At LO TB mixing is exact

$r \sim \Delta m^2_{sol}/\Delta m^2_{atm}$

The only fine-tuning needed is to account for $r^{1/2} \sim 0.2$

[In most A4 models $r^{1/2} \sim 1$ would be expected as l, $\nu^c \sim 3$]

When NLO corrections are included from operators of higher dimension in the superpotential each mixing angle receives generically corrections of the same order $\delta\theta_{ij} \sim o(VEV/\Lambda)$

As the maximum allowed corrections to θ_{12} (and also to θ_{23}) are numerically $o(\lambda_C^2)$, we need $VEV/\Lambda \sim o(\lambda_C^2)$ and we typically expect:

$$\theta_{13} \sim o(\lambda_C^2)$$

Exp: $\sin\theta_{13} \sim 0.151 \sim 3 \sin\theta_C^2$ or $0.7 \sin\theta_C$

Of course the generic prediction can be altered in special versions e.g. Lin '09 discussed a A4 model where $\theta_{13} \sim o(\lambda_C)$

We now compare

"Typical" A4 models

"Special" A4 models $\longrightarrow$ with extra symmetry to separate θ_{13} from θ_{12}

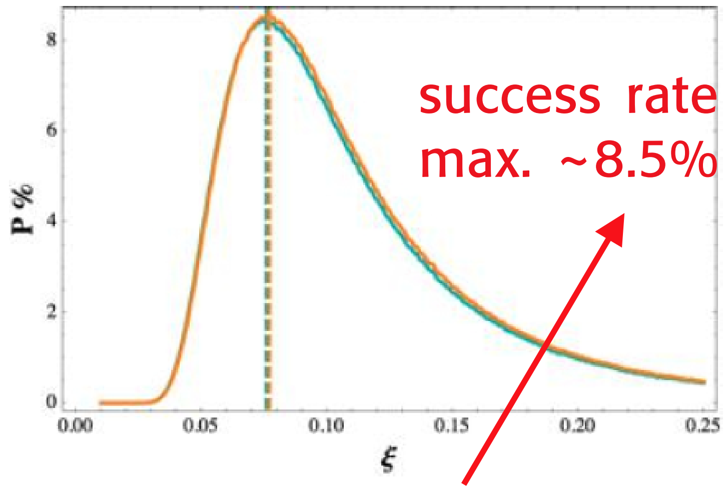
Bimaximal models

At LO the mixing angles are fixed at either TB or BM

Higher order operators lead to departures of o(VEV/Λ).
But the coeffs of these operators are not fixed.

In a typical A4 model

GA, Feruglio, Merlo, Stamou '12



Optimal value $\xi = 0.076$

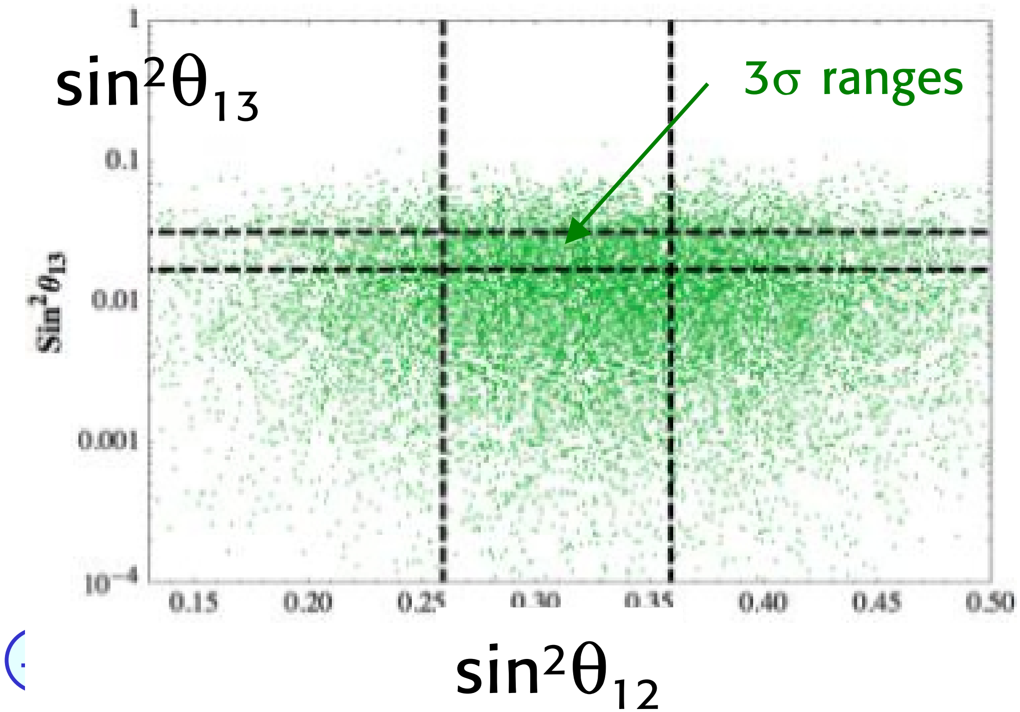
c^e: ch. lept. c^ν: neutrinos

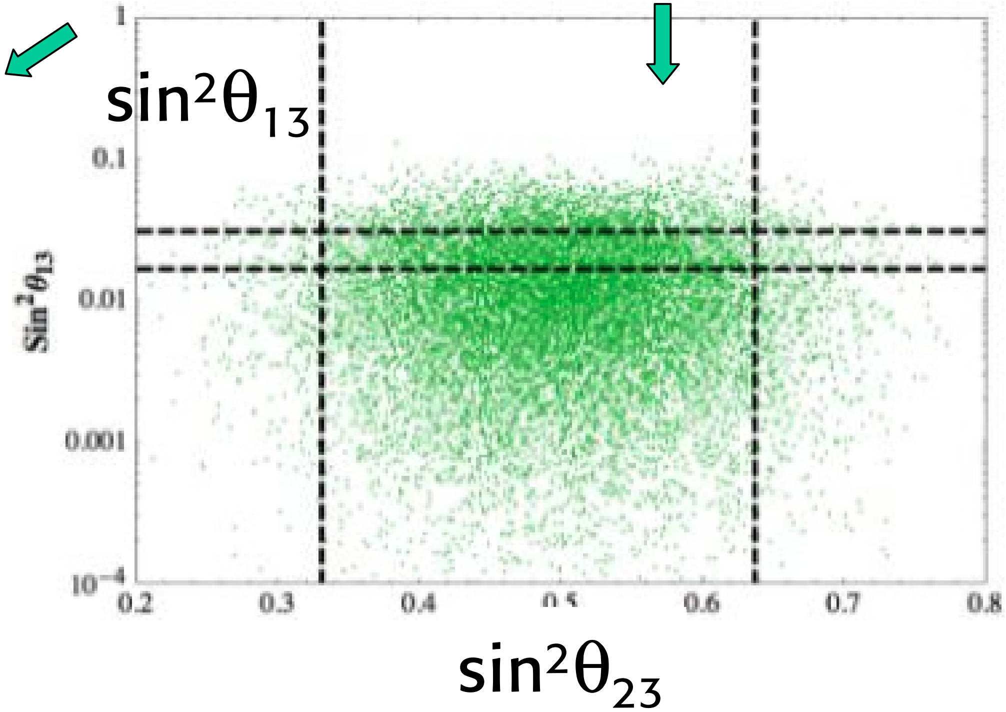
$$\sin^2\theta_{23} = \frac{1}{2} + \mathcal{R}e(c^e_{23})\,\xi + \frac{1}{\sqrt{3}}\left(\mathcal{R}e(c^\nu_{13}) - \sqrt{2}\,\mathcal{R}e(c^\nu_{23})\right)\xi$$

$$\sin^2\theta_{12} = \frac{1}{3} - \frac{2}{3}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi + \frac{2\sqrt{2}}{3}\mathcal{R}e(c^\nu_{12})\,\xi$$

$$\sin\theta_{13} = \frac{1}{6}\left|3\sqrt{2}\,(c^e_{12} - c^e_{13}) + 2\sqrt{3}\left(\sqrt{2}\,c^\nu_{13} + c^\nu_{23}\right)\right|\xi\,.$$

c^a_{ij}: random complex with abs. value gaussian around 1 with variance 0.5





Improving A4 to make θ_{13} naturally large

Lin '09 (before T2K...)
GA, Feruglio, Merlo, Stamou '12

Group theory can help: A_{23} is not in A4, it is an accidental symmetry

If A_{23} is broken by a larger term (ξ') than the rest of A4 (ξ):

SmS=m $\longrightarrow$ $$m_\nu = \begin{pmatrix} x & y-w & y+w \\ y-w & x+z+w & y-z \\ y+w & y-z & x+z-w \end{pmatrix}$$ w breaks A_{23}

Modified mixing for $\xi = 0$

$$U_{TB}\,V = \begin{pmatrix} \sqrt{2/3}\,\alpha & 1/\sqrt{3} & \sqrt{2/3}\,\xi' \\ -\alpha/\sqrt{6}+\xi'^*/\sqrt{2} & 1/\sqrt{3} & -\alpha^*/\sqrt{2}-\xi'/\sqrt{6} \\ -\alpha/\sqrt{6}-\xi'^*/\sqrt{2} & 1/\sqrt{3} & +\alpha^*/\sqrt{2}-\xi'/\sqrt{6} \end{pmatrix}$$

$\xi' \sim o(w)$
$\alpha \sim 1$

In the Lin version of A4 ch. leptons (ξ) and ν's (ξ') are kept separate also at NLO.

Thus a separate minimisation allows for different scales

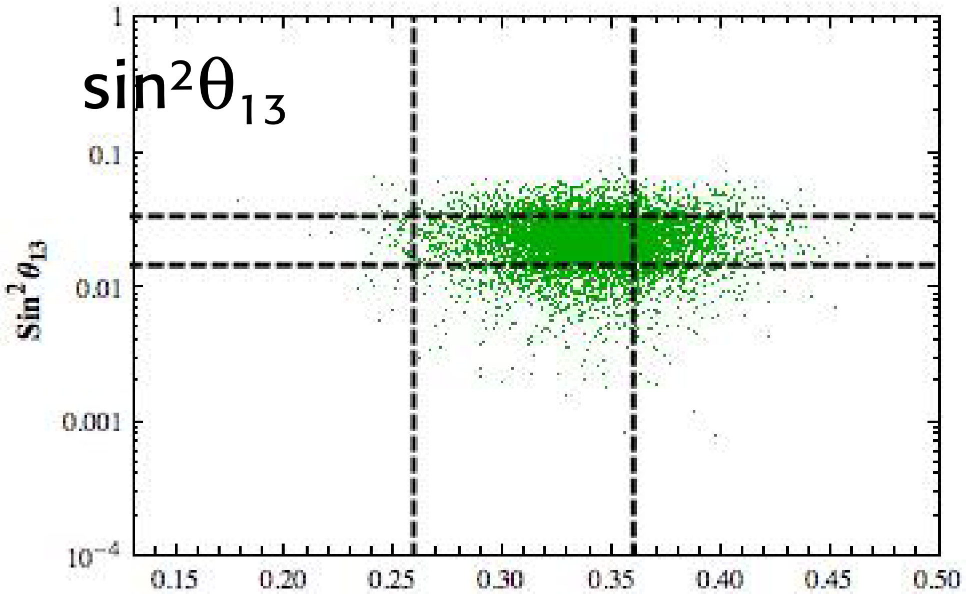
$|\xi'|\sim 0.184$ and $\xi\sim 0.005\text{-}0.06$

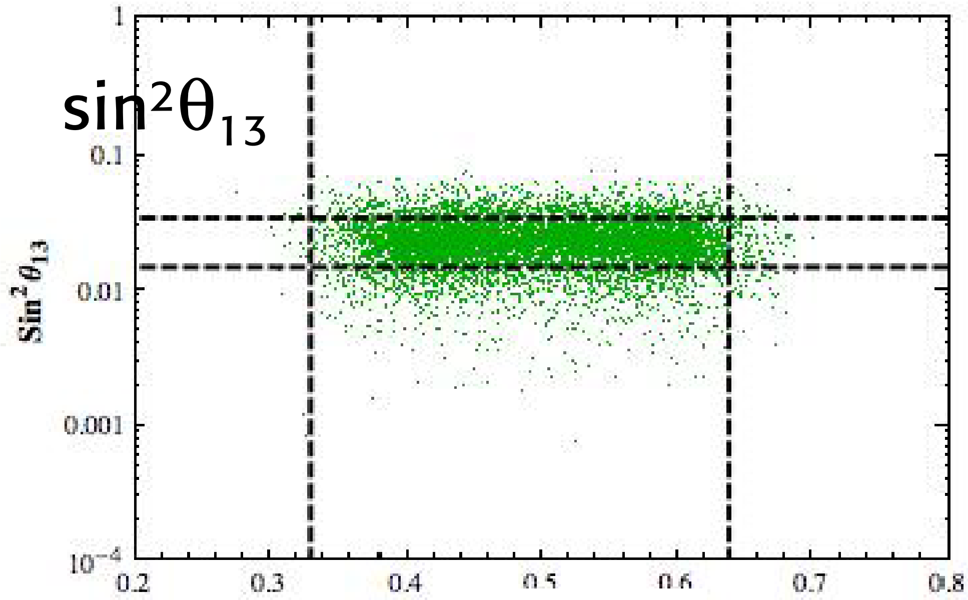
$$\sin\theta_{13} = \left|\sqrt{\frac{2}{3}}\xi' + \frac{c^e_{12} - c^e_{13}}{\sqrt{2}}\xi\right|$$

$$\sin^2\theta_{12} = \frac{1}{3-2|\xi'|^2} - \frac{2}{3}\mathcal{R}e(c^e_{12}+c^e_{13})\,\xi$$

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{3}}|\xi'|\,\cos\delta + \mathcal{R}e(c^e_{23})\,\xi$$

Less fine tuning
Larger success rate ~55%



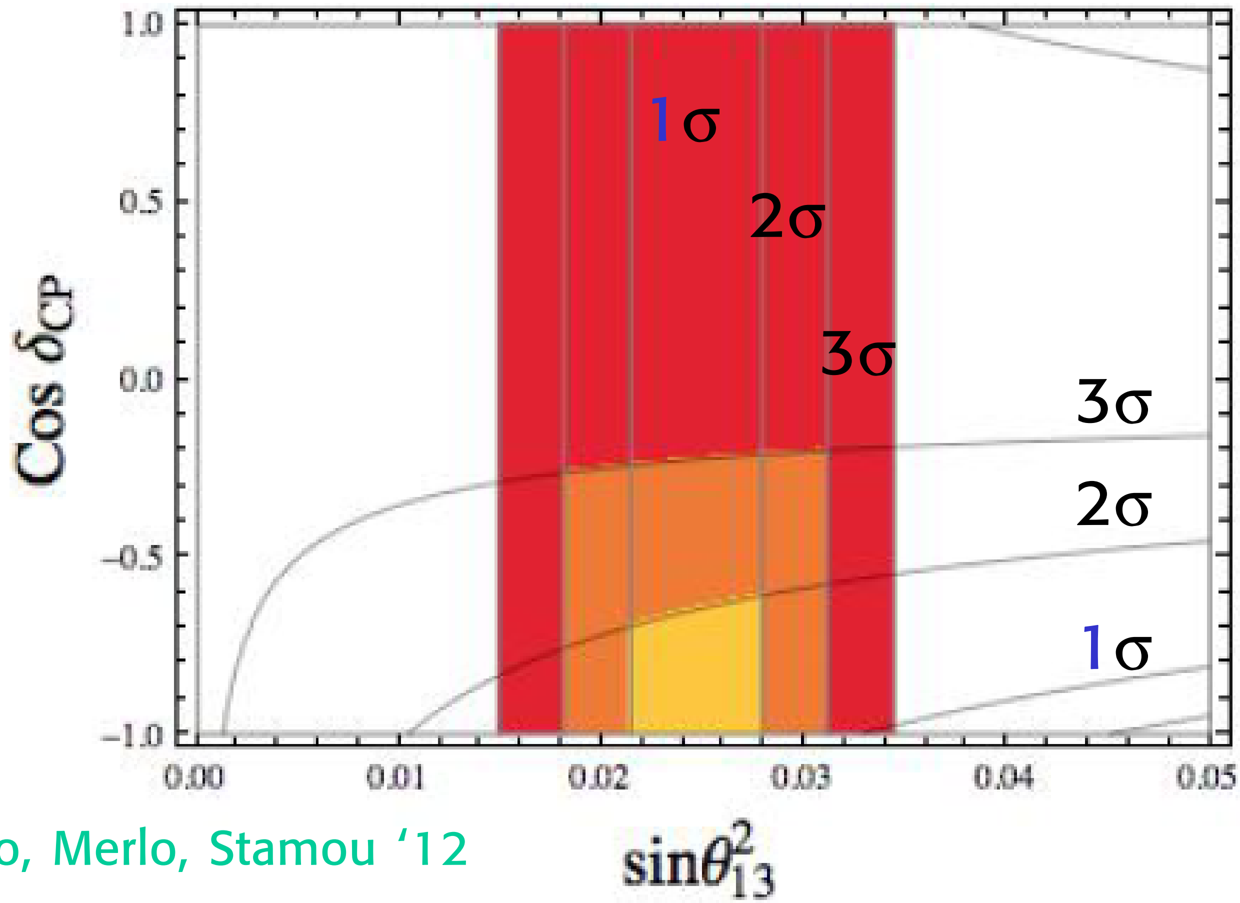


GA, Feruglio, Merlo, Stamou '12

In Lin model by neglecting the small corrections proportional to ξ a sum rule is obtained:

$$\sin^2\theta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}} \sin\theta_{13} \cos\delta_{CP}$$

which is in agreement with exp. indication of $\cos\delta_{CP} < 0$



GA, Feruglio, Merlo, Stamou '12

Bimaximal Mixing

Taking the "complementarity" relation seriously:

$\theta_{12} + \theta_C = (47.0 \pm 1.7)^\circ \sim \pi/4$ Raidal'04

leads to consider models that give $\theta_{12} = \pi/4$ but for corrections from the diag'tion of charged leptons

$$U_{PMNS} = U_\ell^\dagger U_\nu$$

Recall:

$$\lambda_C \approx 0.22 \text{ or } \sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$$

Normally one obtains $\theta_{12} + o(\theta_C) \sim \pi/4$ "weak compl." rather than $\theta_{12} + \theta_C \sim \pi/4$

Bimaximal mixing

$$U_{BM} = \begin{pmatrix} \sqrt{\frac{1}{2}} & -\sqrt{\frac{1}{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\sqrt{\frac{1}{2}} \\ \frac{1}{2} & \frac{1}{2} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

Most general mass matrix

$$m_\nu^{(0)} = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

invariant under

$$S^T m_\nu^{(0)} S = m_\nu^{(0)}$$

$$A_{23}^T m_\nu^{(0)} A_{23} = m_\nu^{(0)}$$

$$S = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

adding

$$T = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix}$$

for invariance of diagonal charged leptons

One is led to S4: $S^2 = (ST)^3 = T^4 = 1$

The large deviations from BM mixing could arise from charged lepton diagonalisation

In this case both θ_{12} and θ_{13} need shifts of $o(\lambda_C)$

$$U_e^\dagger U_{BM} \quad \left\{ \begin{aligned} \bar{U}_{12} &= -\frac{e^{-i(\alpha_1+\alpha_2)}}{\sqrt{2}} + \frac{s^e_{12} e^{-i\alpha_2} + s^e_{13} e^{i\delta_e}}{2} \\ \bar{U}_{13} &= \frac{s^e_{12} e^{-i\alpha_2} - s^e_{13} e^{i\delta_e}}{\sqrt{2}} \\ \bar{U}_{23} &= -e^{-i\alpha_2} \frac{1 + s^e_{23} e^{i\alpha_2}}{\sqrt{2}} \end{aligned} \right.$$

GA, Feruglio, Masina
Frampton et al
King
Antusch et al........

Corr.'s from s^e_{12}, s^e_{13} to U_{12} and U_{13} are of first order (2nd order to U_{23})

We have built a S4 model where NLO corrections are non generic and s^e_{23} is negligible

GA, Feruglio, Merlo '09
D. Meloni '11

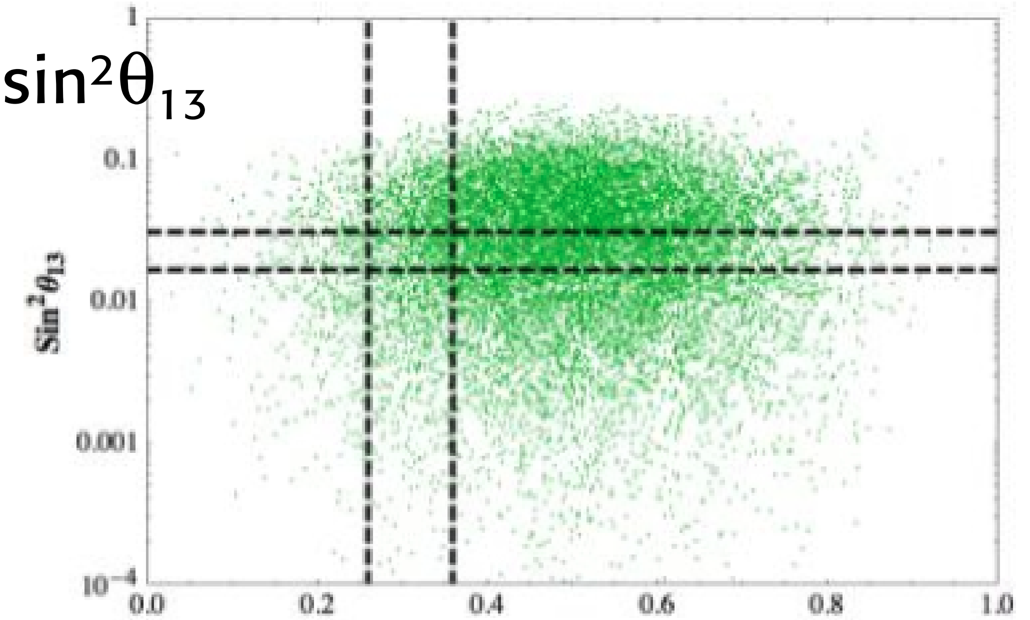
s^e_{23} is negligible

$$U_e = \begin{pmatrix} 1 & c^e_{12}\,\xi & c^e_{13}\,\xi \\ -c^{e*}_{12}\,\xi & 1 & 0 \\ -c^{e*}_{13}\,\xi & 0 & 1 \end{pmatrix}$$

$\xi \sim$ 0.172

$$\delta_{CP} = \pi + \arg\left(c^e_{12} - c^e_{13}\right)$$
$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\left|c^e_{12} - c^e_{13}\right|\xi$$
$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi$$
$$\sin^2\theta_{23} = \frac{1}{2}\,.$$

$\sin^2\theta_{13}$



$\sin^2\theta_{12}$

In a random generation of coefficients the success rate is small (2.6%).
The main problem here is to get $\sin^2\theta_{12}$ right by chance

GA, Feruglio, Merlo, Stamou '12

$$\delta_{CP} = \pi + \arg\left(c^e_{12} - c^e_{13}\right)$$

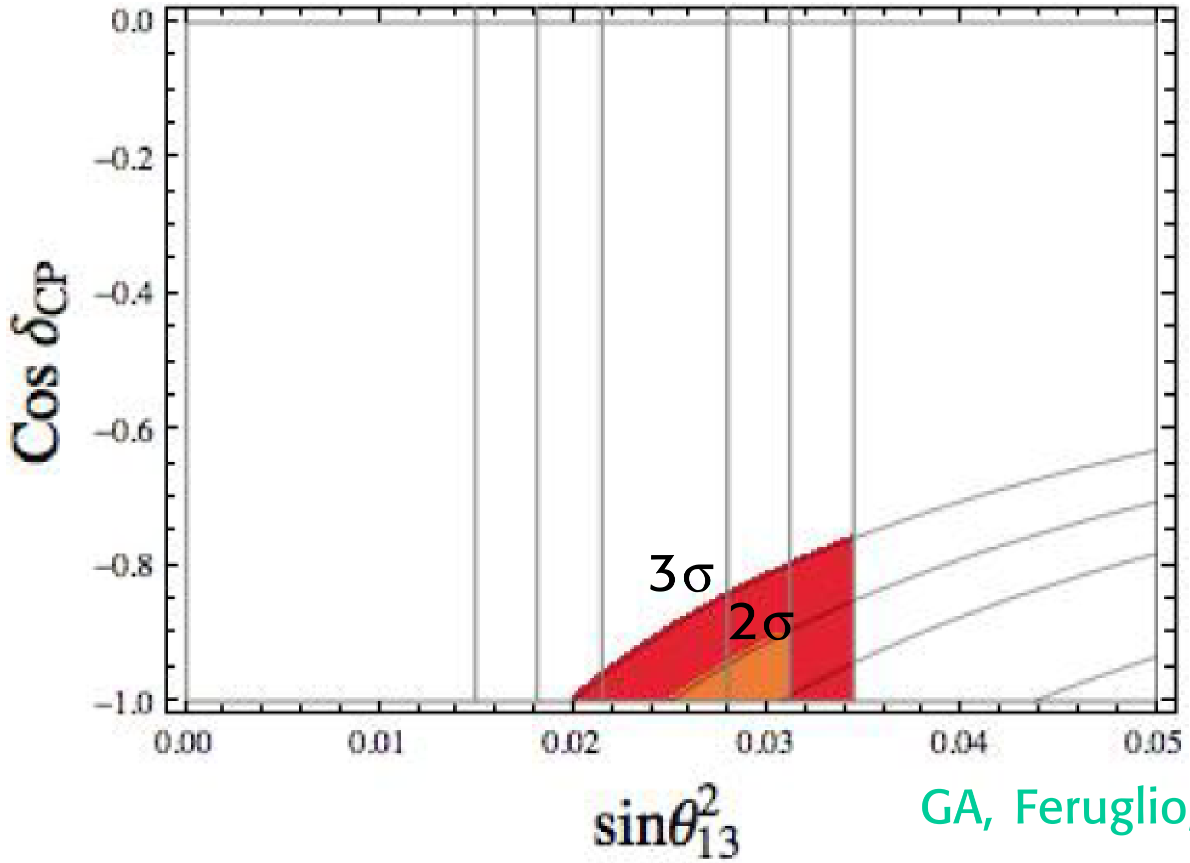
$$\sin\theta_{13} = \frac{1}{\sqrt{2}}\left|c^e_{12} - c^e_{13}\right|\xi$$

$$\sin^2\theta_{12} = \frac{1}{2} - \frac{1}{\sqrt{2}}\mathcal{R}e(c^e_{12} + c^e_{13})\,\xi$$

$$\sin^2\theta_{23} = \frac{1}{2}\,.$$

For dominance of a single c^e, e.g. $c^e{}_{13}$=0 we have a sum rule

$$\sin^2\theta_{12} = \frac{1}{2} + \sin\theta_{13}\,\cos\delta_{CP}$$



Then
$\cos\delta_{CP} \sim -1$
is predicted

GA, Feruglio, Merlo, Stamou '12

Constraints from lepton flavour violation (LFV)

These SUSY GUT models with A4 or S4 flavour symmetry imply LFV, thru non diagonal lepton mass terms

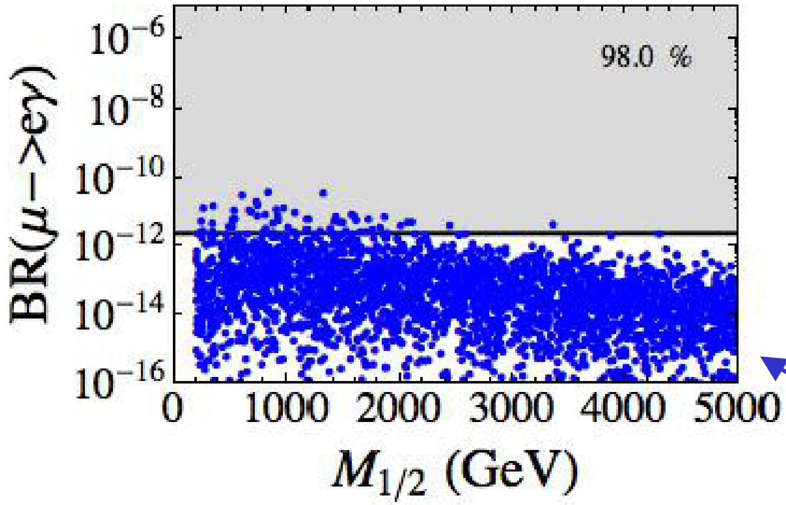
Existing bounds on LFV, e.g. from μ -> e γ, τ -> μ γ, lead to constraints that are particularly strong for the S4 model of Bi-mixing with (large) corrections from charged leptons

The MEG recent bound on Br(μ -> e γ) $< 2.4\ 10^{-12}$

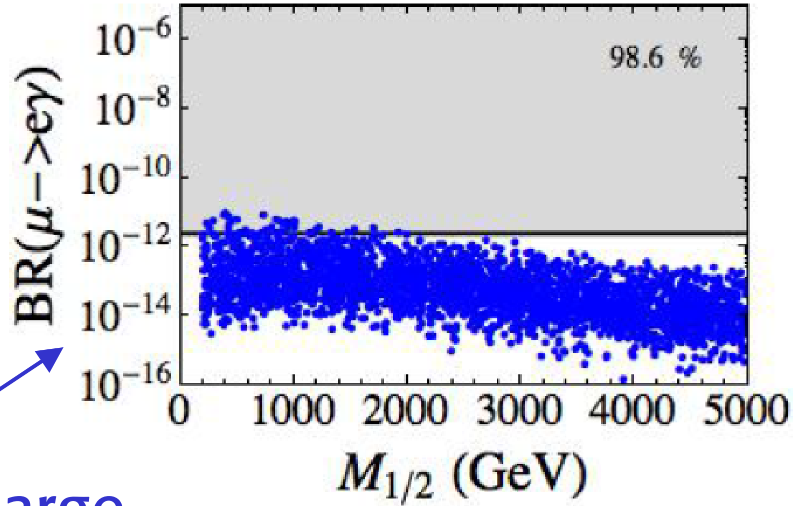
poses a serious constraint on SUSY models with non diagonal mass matrices at the GUT scale

Br(μ -> e γ) < 2.4 10^{-12}: a serious constraint

CMSSM



Typical A4, ξ = 0.076

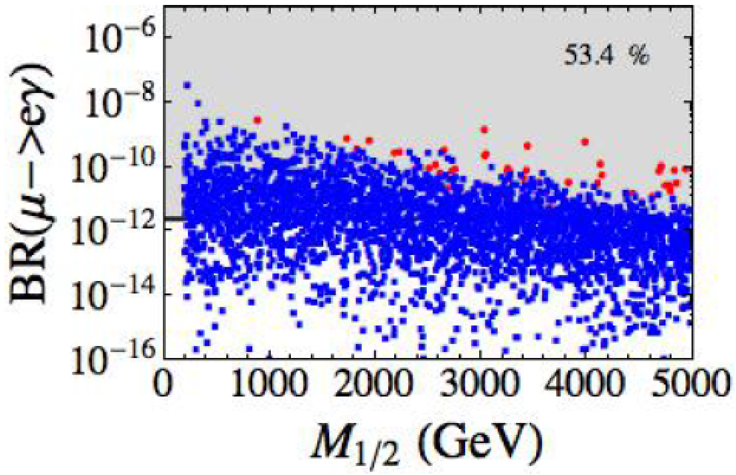


Lin-type A4, ξ' = 0.184

[main effect o(ξ'^2)]

m_0 ~ 5 TeV large

tanβ ~ 2



S4, ξ = 0.172

S4 is disadvantaged as large off diagonal ch. lepton mass terms are needed (of o(λ_C))

Needs either m_0 or $M_{1/2}$ heavy

GA, Feruglio, Merlo, Stamou '12

Conclusion

Data on mixing angles are much better now but models of neutrino mixing still span a wide range from anarchy to discrete flavour groups (in the near future it will not be easy todecide from the data which ideas are right)

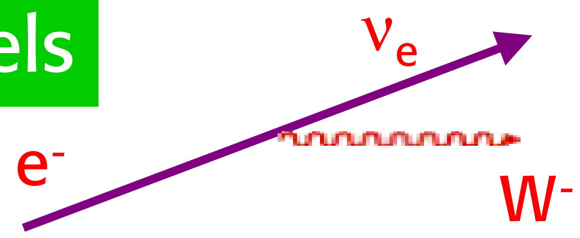
Anarchy has passed the θ_{13} test but in the SU(5)xU(1) context unequal FN charges for lepton families can still do better

Among discrete symmetry models typical A4 and S4 models need some fine tuning, while A4 models of the Lin type can naturally reproduce all the data. LFV pose strong constraints on models with o(λ_C) corrections to ν mixing from charged leptons

So far no real illumination came from leptons to be combined with the quark sector for a more complete theory of mass and mixing

EXTRA

Still the main framework: 3-ν Models



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^+ \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

flavour mass

$U = U_{PMNS}$

Pontecorvo
Maki, Nakagawa, Sakata

In basis where e^-, μ^-, τ^- are diagonal:

δ: CP violation

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim$$

s = solar: large

CHOOZ: $|s_{13}|$ small

$$\sim \begin{bmatrix} c_{13}\,c_{12} & c_{13}\,s_{12} & s_{13}e^{-i\delta} \\ \dots & \dots & c_{13}\,s_{23} \\ \dots & \dots & c_{13}\,c_{23} \end{bmatrix}$$

atm.: ~ max

(some signs are conventional)

In general: $U = U^+{}_e U_\nu$

Are sterile ν's coming back? A number of "hints"

(they do not make an evidence but
pose an experimental problem that needs clarification)

- LSND and MiniBoone
- Reactor flux & anomaly
- Gallium ν_e disappearance vs ν_e^{bar} reactor limits

If all true (unlikely) then need at least 2 sterile ν's

Important information also from

- Neutrino counting from cosmology

Feruglio

from LBL experiments searching for ν_μ -> ν_e conversion

T2K: muon neutrino beam produced at JPARC [Tokai] E=0.6 GeV and sent to SK 295 Km apart [1106.2822]

MINOS: muon neutrino beam produced at Fermilab [E=3 GeV] sent to Soudan Lab 735 Km apart [1108.0015]

$$P(\nu_\mu \to \nu_e) = \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \sin^2\frac{\Delta m^2_{32} L}{4E} + \dots$$

both experiments favor $\sin^2\vartheta_{13}$ ~ few %

from SBL reactor experiments searching for anti-ν_e disappearance

Double Chooz (far detector): $\sin^2\vartheta_{13} = 0.022 \pm 0.013$

Daya Bay (near + far detectors): $\sin^2\vartheta_{13} = 0.024 \pm 0.004$

RENO (near + far detectors): $\sin^2\vartheta_{13} = 0.029 \pm 0.006$

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\vartheta_{13} \sin^2\frac{\Delta m^2_{32} L}{4E} + \dots$$

SBL reactors are sensitive to ϑ_{13} only
LBL experiments anti-correlate $\sin^2 2\vartheta_{13}$ and $\sin^2\vartheta_{23}$
also breaking the octant degeneracy ϑ_{23} <->($\pi-\vartheta_{23}$)

Hierarchy for masses and mixings via horizontal $U(1)_{FN}$ charges.

Froggatt, Nielsen '79

The simplest flavour symmetry

Principle:

A generic mass term

$\bar{R}_1 m_{12} L_2 H$

is forbidden by U(1)

if $q_1+q_2+q_H$ not 0

q_1, q_2, q_H:
U(1) charges of
$\bar{R}_1, L_2, H$

U(1) broken by vev of "flavon" field θ with U(1) charge $q_\theta = -1$.

If vev $\theta = w$, and $w/M=\lambda$ we get for a generic interaction:

Δ_{charge}

$\bar{R}_1 m_{12} L_2 H \, (\theta/M)^{q1+q2+qH}$ $\qquad$ $m_{12} \rightarrow m_{12}\, \varepsilon^{q1+q2+qH}$

Hierarchy: More Δ_{charge} -> more suppression ($\varepsilon = \theta/M$ small)

One can have more flavons ($\varepsilon, \varepsilon'$, ...)
with different charges (>0 or <0) etc -> many versions

TB mixing

Harrison, Perkins, Scott

A simple mixing matrix compatible with all present data

$$m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U\mathrm{diag}(m_1, m_2, m_3)U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \rightarrow \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \rightarrow \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \rightarrow \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

Compare with quark mixings $\lambda_C \sim (m_d/m_s)^{1/2}$

Bimaximal Mixing

Taking the "complementarity" relation seriously:

$\theta_{12} + \theta_C = (46.4 \pm 0.8)^o \sim \pi/4$

leads to consider models that give $\theta_{12} = \pi/4$ but for corrections from the diag'tion of charged leptons

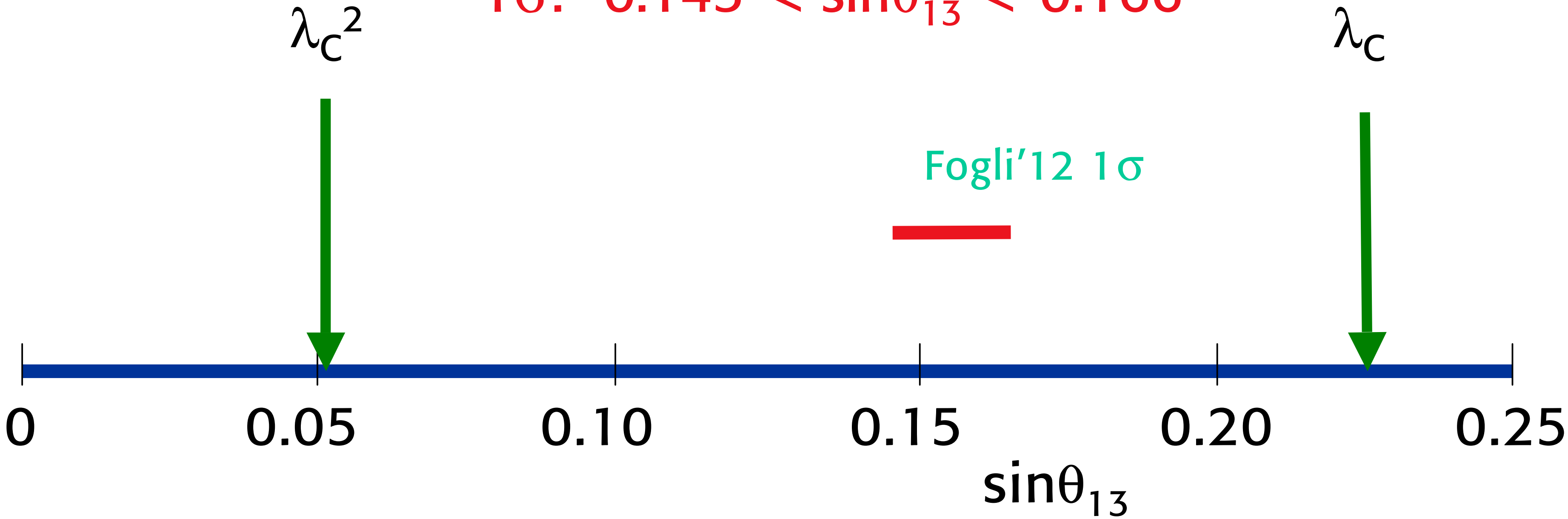
$$U_{PMNS} = U_\ell^\dagger U_\nu$$

Recall:

$\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

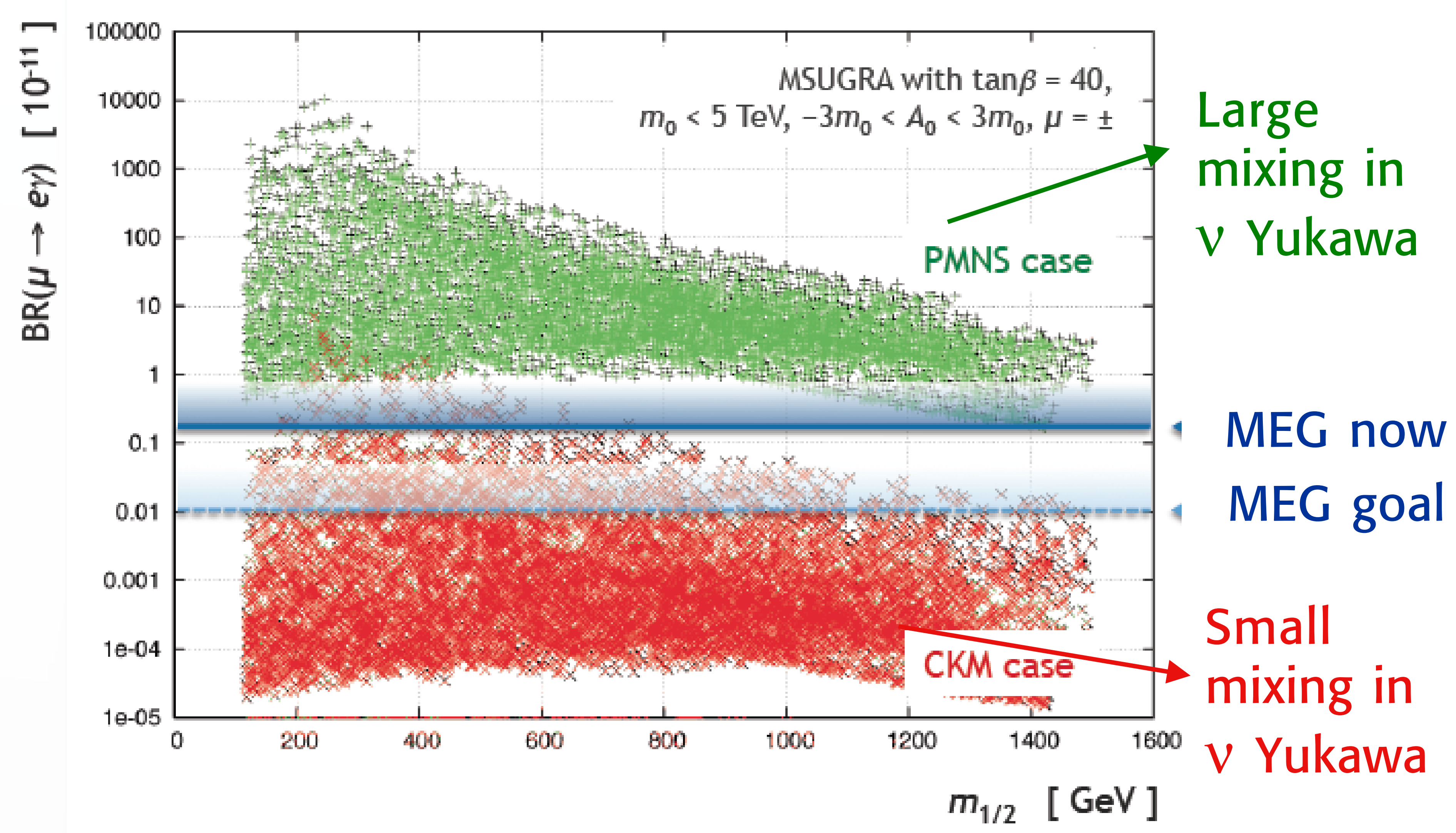
Normally one obtains $\theta_{12} + o(\theta_C) \sim \pi/4$ "weak compl." rather than $\theta_{12} + \theta_C \sim \pi/4$

1σ: $0.143 < \sin\theta_{13} < 0.166$

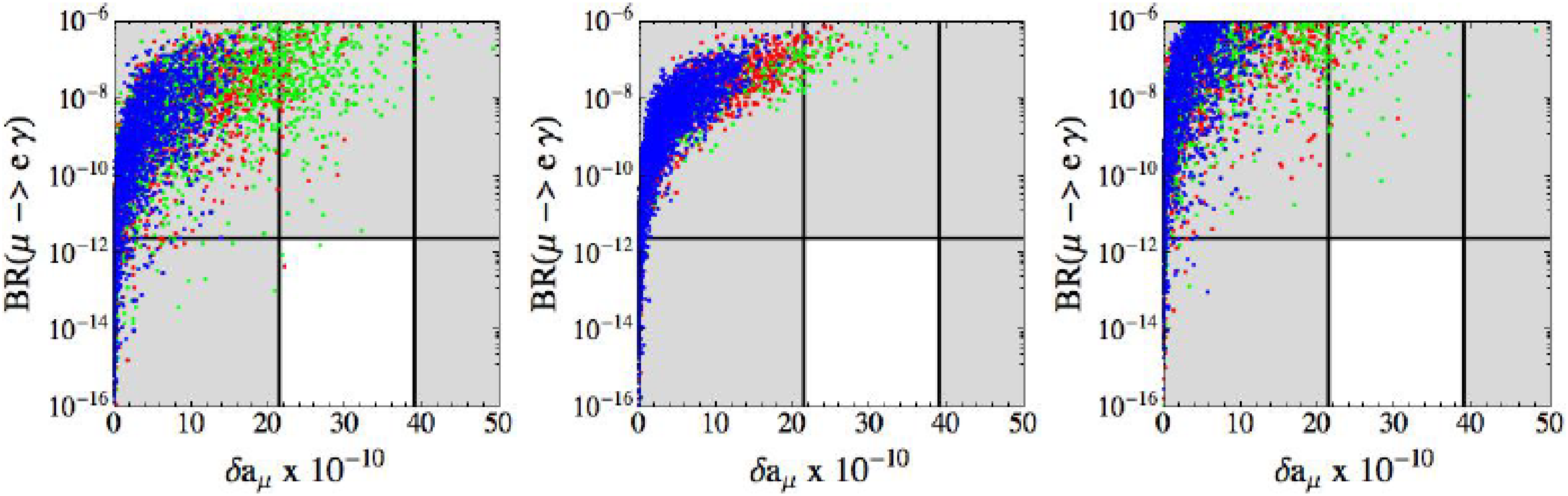


MEG new limit on Br($\mu \to e\,\gamma$) < 2.4 10^{-12}

a serious constraint on SUSY models with non diagonal mass matrices at the GUT scale



But all points that satisfy the $\mu \to e\,\gamma$ bound cannot accommodate $(g\text{-}2)_\mu$



(a) Typical A_4. $\xi = 0.075$. (b) Special A_4. $|\xi'| = 0.183$. (c) S_4. $\xi = 0.174$.

or the tension between LHC SUSY limits, m_H=125 GeV and the CMSSM is also manifest in these models