Models of Flavor Symmetries and the Stability of their Predictions

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Based on work done in collaboration

with Maximilian Fallbacher, Michael Ratz, Christian Staudt, arXiv:1208.2947 [hep-ph] with K.T. Mahanthappa, Phys. Lett. B681 (2009) 444; Phys. Lett. B652 (2007) 34

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Where Do We Stand?

- Exciting Time in v Physics: recent hints of large θ_{13} from T2K, MINOS, Double Chooz, and Daya Bay, RENO
- Latest 3 neutrino global analysis (including recent results from reactor experiments):

$$P(\nu_a \to \nu_b) = \left| \left\langle \nu_b | \nu, t \right\rangle \right|^2 \simeq \sin^2 2\theta \, \sin^2 \left(\frac{\Delta m^2}{4E} L \right)$$

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, 2012

Parameter	Best fit	1σ range	2σ range	3σ range
$\delta m^2/10^{-5} \text{ eV}^2 \text{ (NH or IH)}$	7.54	7.32-7.80	7.15-8.00	6.99 - 8.18
$\sin^2 \theta_{12} / 10^{-1}$ (NH or IH)	3.07	2.91-3.25	2.75-3.42	2.59-3.59
$\Delta m^2/10^{-3} \text{ eV}^2 \text{ (NH)}$	2.43	2.33-2.49	2.27-2.55	2.19 - 2.62
$\Delta m^2 / 10^{-3} \text{ eV}^2 \text{ (IH)}$	2.42	2.31-2.49	2.26-2.53	2.17 - 2.61
$\sin^2 \theta_{13} / 10^{-2}$ (NH)	2.41	2.16-2.66	1.93-2.90	1.69 - 3.13
$\sin^2 \theta_{13} / 10^{-2}$ (IH)	2.44	2.19-2.67	1.94-2.91	1.71 - 3.15
$\sin^2 \theta_{23} / 10^{-1}$ (NH)	3.86	3.65 - 4.10	3.48-4.48	3.31 - 6.37
$\sin^2 \theta_{23} / 10^{-1}$ (IH)	3.92	3.70-4.31	$3.53 - 4.84 \oplus 5.43 - 6.41$	3.35 - 6.63
δ/π (NH)	1.08	0.77-1.36		
δ/π (IH)	1.09	0.83-1.47	—	_

Origin of Mass Hierarchy and Mixing

- In the SM: 22 physical quantities which seem unrelated
- Question arises whether these quantities can be related
- No fundamental reason can be found in the framework of SM
- less ambitious aim \Rightarrow reduce the # of parameters by imposing symmetries
 - SUSY Grand Unified Gauge Symmetry
 - GUT relates quarks and leptons: quarks & leptons in same GUT multiplets
 - one set of Yukawa coupling for a given GUT multiplet ⇒ intra-family relations
 - seesaw mechanism naturally implemented
 - Family Symmetry
 - relate Yukawa couplings of different families
 - inter-family relations \Rightarrow further reduce the number of parameters

⇒ Experimentally testable correlations among physical observables



Origin of Flavor Mixing and Mass Hierarchy

- Several models have been constructed based on
 - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F
- Family Symmetries G_F based on continuous groups:
 - U(1)
 - SU(2)
 - SU(3)



GUT Symmetry SU(5), SO(10), ...

- Recently, models based on discrete family symmetry groups have been constructed
 - A₄ (tetrahedron)
 - T´ (double tetrahedron)
 - S₃ (equilateral triangle)
 - S₄ (octahedron, cube)
 - A₅ (icosahedron, dodecahedron)
 - <u>∆</u>27
 - Q4

Motivation: Tri-bimaximal (TBM) neutrino mixing

For anomaly constraints: see talk by Michael Ratz this afternoon

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Tri-bimaximal Neutrino Mixing

• Neutrino Oscillation Parameters
$$P(\nu_a \to \nu_b) = |\langle \nu_b | \nu, t \rangle|^2 \simeq \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{4E}L\right)$$

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

• Latest Global Fit (3σ)

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, 2012

• Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix} \qquad \sin^2 \theta_{\rm atm, TBM} = 1/2 \qquad \sin^2 \theta_{\odot, TBM} = 1/3 \\ \sin \theta_{13, TBM} = 0.$$

• Generally: TBM (from symmetry) + holomorphic Corrections/contributions

Tri-bimaximal Neutrino Mixing

	$ heta_{12}$	$ heta_{13}$	$ heta_{23}$
TBM prediction:	$\arctan\left(\sqrt{0.5}\right) \approx 35.3^{\circ}$	0	45°
Best fit values $(\pm 1\sigma)$:	$(33.6^{+1.1}_{-1.0})^{\circ}$	$\left(8.93^{+0.46}_{-0.48}\right)^{\circ}$	$(38.4^{+1.4}_{-1.2})^{\circ}$

Fogli, Lisi, Marrone, Montanino, Palazzo, Rotunno, 2012

An Example: Non-Abelian Finite Family Symmetry A₄

- TBM mixing matrix: can be realized with finite group family symmetry based on A₄ Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)
- A₄: even permutations of 4 objects
 S: (1234) → (4321)
 - T: (1234) → (2314)
- a group of order 12



Invariant group of tetrahedron

[Animation Credit: Michael Ratz]

$$\mathcal{L}_{\rm FF} = \frac{1}{M_x \Lambda} \left[\begin{array}{c} \mathcal{L}_{\rm FF} = \frac{1}{M_z \Lambda} \left[\lambda_1 H_5 H_5 \overline{F} \, \overline{F} \, \overline{F} \\ \lambda_1 H_5 H_5 \overline{F} \, \overline{F} \, \eta \right], \\ \lambda_2 H_5 H_5 \overline{F} \, \overline{F} \, \eta \\ \lambda_2 H_5 H_5 \overline{F} \, \overline{F} \, \eta \\ \end{array} \right], \tag{6}$$

where M_x is the cutoff scale at which the lepton number violation operator $HH\overline{F}\overline{F}$ is generated, where M_x is the cutoff scale at which the lepton number violation operator $HH\overline{F}\overline{F}$ is generated, $(z_1, z_2 \sqrt{h}) \delta^2 4\Lambda$ is the cutoff scale; above which the $({}^{(d)}T$ symmetry is exact. The parameters y's and while Λ is the cutoff scale, above which $z_2 Tz_3 y_{1} Ty_{1} = y_{1} (1 + 2) Tz_{2} Tz_{3} y_{2} Tz_{3} y_{3} = y_{1} + 2) \delta^2 y_{3} = 0$ are the coupling constants. The vacuum expectation values (VEV's) of various SU(5) singlet scalar are the coupling constants. The vacuum expectation values (VEV's) of various SU(5) singlet scalar $\underbrace{\textbf{Eutrino}}_{z=x_5+3}\underbrace{\textbf{W}\otimes 3=3\oplus 3\oplus 1\oplus 1'\oplus 1'}_{\oplus}$ $\frac{z = x_{5} + 3 \otimes 3}{z = x_{5} + 3 \otimes 3} = 3 \oplus 3 \oplus 1^{\prime} \oplus 1^{\prime} \oplus 1^{\prime} \oplus 1^{\prime}$ • fermion charge assignments $(\xi) = \chi_{0} = \chi_{1} = \chi_{1} = \chi_{1} = \chi_{1} = \chi_{2} = \chi_{1} = \chi_{1} = \chi_{2} = \chi_{2} = \chi_{2} = \chi_{1} = \chi_{1} = \chi_{1} = \chi_{2} =$ (7)(8)(9)(10)(11) where Groups is given by [9] 1'''_{H} (3 + 3) 3_{R}(1') sentation is given by [9] 1''_{H} (3 + 3) 3_{R}(1') $TST^{2} = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & TST^{2} = \frac{1}{3} \\ 2 & 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 2 & 2 \\ 2 & TST^{2} = \frac{1}{3} \\ 2 & 2 & -1 \end{pmatrix} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} ,$ • product rules. devlotes the student per the state of th (12)(12)

while G_T and G_S denote subgroup generated by the elements T and S respectively. (Our notation while G_T and G_S denote subgroup generated by the elements T and S, respectively. (Our notation is the same as in Ref [10]). The details concerning vacuum alignment of these VEV's will be

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Tri-bimaximal Neutrino Mixing from A₄

• Neutrino Masses: triplet flavon contribution

$$3_{S} = \frac{1}{3} \begin{pmatrix} 2\alpha_{1}\beta_{1} - \alpha_{2}\beta_{3} - \alpha_{3}\beta_{2} \\ 2\alpha_{3}\beta_{3} - \alpha_{1}\beta_{2} - \alpha_{2}\beta_{1} \\ 2\alpha_{2}\beta_{2} - \alpha_{1}\beta_{3} - \alpha_{3}\beta_{1} \end{pmatrix} \qquad 1 = \alpha_{1}\beta_{1} + \alpha_{2}\beta_{3} + \alpha_{3}\beta_{2}$$

• Neutrino Masses: singlet flavon contribution

 $1 = \alpha_1 \beta_1 + \alpha_2 \beta_3 + \alpha_3 \beta_2$

• resulting mass matrix:

$$M_{\nu} = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix} \qquad U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$
$$V_{\nu}^{\text{T}} M_{\nu} V_{\nu} = \text{diag}(u + 3\xi_0, u, -u + 3\xi_0) \frac{v_u^2}{M_x}$$

Form diagonalizable:

- -- no adjustable parameters
- -- neutrino mixing from CG coefficients!

Altarelli, Feruglio (2005)

$$\frac{HHLL}{M}\left(\frac{\langle \xi \rangle}{A} + \frac{\langle \eta \rangle}{A}\right)$$

$$\frac{HLL}{M}\left(\frac{\langle \xi \rangle}{A} + \frac{\langle \eta \rangle}{A}\right)$$

$$\frac{HHLL}{M}\left(\frac{\langle \xi \rangle}{A} + \frac{\langle$$

$$(v_S + \delta v_{S1}, v_S + \delta v_{S2}, v_S + \delta v_{S3}), \quad \langle \varphi_T \rangle = (v_T + \delta v_{T1}, \delta v_{T2}, \delta v_{T3}),$$

An Example: Non-Abelian Finite Family Symmetry A₄

- TBM mixing matrix: can be realized with finite group family symmetry based on A₄
- Deficiencies:
 - does NOT give quark mixing
 - does NOT explain mass hierarchy
 - all CG coefficients real
- SU(5) GUT compatible \Rightarrow T' Symmetry (double covering of A₄)

see talk by K.T. Mahanthappa on Thursday

- large θ_{13} possible
- complex CG coefficients of $T' \Rightarrow$ novel origin of CP violation

M.-C.C, Mahanthappa, Phys. Lett. B681, 444 (2009)

Flavor Model Structure: A₄ Example



- interplay between the symmetry breaking patterns in two sectors lead to lepton mixing (BM, TBM, ...)
- symmetry breaking achieved through flavon VEVs
- each sector preserves different residual symmetry
- full Lagrangian does not have these residual symmetries
- general approach: include high order terms in holomorphic superpotential
- possible to construct models where higher order holomorphic superpotential terms vanish to ALL orders
- quantum correction?

⇒ uncertainty due to Kähler corrections

Leurer, Nir, Seiberg (1993); Dudas, Pokorski, Savoy (1995); Dreiner, Thormeier (2003); King, Peddie (2003); King, Peddie, Ross, Velaso-Sevilla, Vives (2004)

Kähler Corrections

M.-C.C., Fallbacher, Ratz, Staudt (2012)

• Superpotential: holomorphic

$$\mathscr{W}_{\text{leading}} = \frac{1}{\Lambda} (\Phi_e)_{gf} L^g R^f H_d + \frac{1}{\Lambda \Lambda_\nu} (\Phi_\nu)_{gf} L^g H_u L^f H_u$$

$$\longrightarrow \mathscr{W}_{\text{eff}} = (Y_e)_{gf} L^g R^f H_d + \frac{1}{4} \kappa_{gf} L^g H_u L^f H_u$$

order parameter <flavon vev> / $\Lambda \sim \theta c$

• Kähler potential: non-holomorphic

$$K = K_{\text{canonical}} + \Delta K$$

- Canonical Kähler potential $K_{\text{canonical}} \supset (L^f)^{\dagger} \delta_{fg} L^g + (R^f)^{\dagger} \delta_{fg} R^g$
- Correction

$$\Delta K = \left(L^f\right)^{\dagger} (\Delta K_L)_{fg} L^g + \left(R^f\right)^{\dagger} (\Delta K_R)_{fg} R^g$$

- can be induced by flavon VEVs
- important for order parameter $\sim \theta c$
- can lead to non-trivial mixing

Kähler Corrections

M.-C.C., Fallbacher, Ratz, Staudt (2012)

• Consider infinitesimal change, x :

$$K = K_{\text{canonical}} + \Delta K = L^{\dagger} (1 - 2x P) L$$

• rotate to canonically normalized L':

$$L \rightarrow L' = (1 - x P) L$$

 \Rightarrow corrections to neutrino mass matrix

$$\mathcal{W}_{\nu} = \frac{1}{2} (L \cdot H_{u})^{T} \kappa_{\nu} (L \cdot H_{u})$$

$$\simeq \frac{1}{2} [(\mathbb{1} + xP)L' \cdot H_{u}]^{T} \kappa_{\nu} [(\mathbb{1} + xP)L' \cdot H_{u}]$$

$$\simeq \frac{1}{2} (L' \cdot H_{u})^{T} \kappa_{\nu} L' \cdot H_{u} + x (L' \cdot H_{u})^{T} (P^{T} \kappa_{\nu} + \kappa_{\nu} P)L' \cdot H_{u}]$$

with
$$\kappa \cdot v_u^2 = 2m_
u$$

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Kähler Corrections

M.-C.C., Fallbacher, Ratz, Staudt (2012)

• Consider infinitesimal change, x :

$$K = K_{\text{canonical}} + \Delta K = L^{\dagger} (1 - 2x P) L$$

rotate to canonically normalized L':

$$L \rightarrow L' = (1 - x P) L$$

 \Rightarrow corrections to neutrino mass matrix

$$m_{\nu}(x) \simeq m_{\nu} + x P^T m_{\nu} + x m_{\nu} P$$

 \Rightarrow differential equation

$$\frac{\mathrm{d}m_{\nu}}{\mathrm{d}x} = P^T m_{\nu} + m_{\nu} P$$

- same structure as the RG evolutions for neutrino mass operator
- analytic understanding of evolution of mixing parameters

S. Antusch, J. Kersten, M. Lindner, M. Ratz (2003)

• size of Kähler corrections can be substantially larger (no loop suppression)

M.-C.C., Fallbacher, Ratz, Staudt (2012)

• Kähler corrections due to flavon field:

Inear in flavon:

$$\Delta K_{\text{linear}} = \sum_{i \in \{a,s\}} \left(\frac{\kappa_{\Phi_{\nu}}^{(i)}}{\Lambda} \Delta K_{L^{\dagger}(L \otimes \Phi_{\nu})\mathbf{s}_{i}}^{(i)} + \frac{\kappa_{\Phi_{e}}^{(i)}}{\Lambda} \Delta K_{L^{\dagger}(L \otimes \Phi_{e})\mathbf{s}_{i}}^{(i)} \right) + \frac{\kappa_{\xi}}{\Lambda} \Delta K_{\xi L^{\dagger}L} + \text{h.c.}$$

possible to forbid these terms with additional symmetries

M.-C.C., Fallbacher, Ratz, Staudt (2012)

- Kähler corrections due to flavon field:
 - quadratic in flavon



- such terms cannot be forbidden by any (conventional) symmetry
- Kähler corrections once flavon fields attain VEVs
- additional parameters κ_{d0}^{X} diminish predictivity of the scheme
- possible to forbid all contributions from RH sector as well as $(L\Phi_{\nu})^{\dagger}(L\Phi_{e})$ with additional symmetries in the particular A₄ model

M.-C.C., Fallbacher, Ratz, Staudt (2012)

• Kähler corrections due to flavon field χ : Δ

$$\Delta K \supset \sum_{i=1}^{6} \kappa^{(i)} \Delta K^{(i)}_{(L\chi)_{\boldsymbol{X}}^{\dagger}(L\chi)_{\boldsymbol{X}}} + \text{h.c.}$$

six possible contractions:

$$\begin{split} \Delta K_{(L\chi)_{1}^{\dagger}(L\chi)_{1}}^{(1)} &= (L_{1}^{\dagger}\chi_{1}^{\dagger} + L_{2}^{\dagger}\chi_{3}^{\dagger} + L_{3}^{\dagger}\chi_{2}^{\dagger})(L_{1}\chi_{1} + L_{2}\chi_{3} + L_{3}\chi_{2}), \\ \Delta K_{(L\chi)_{1'}(L\chi)_{1'}}^{(2)} &= (L_{3}^{\dagger}\chi_{3}^{\dagger} + L_{1}^{\dagger}\chi_{2}^{\dagger} + L_{2}^{\dagger}\chi_{1}^{\dagger})(L_{3}\chi_{3} + L_{1}\chi_{2} + L_{2}\chi_{1}), \\ \Delta K_{(L\chi)_{1''}(L\chi)_{1''}}^{(3)} &= (L_{2}^{\dagger}\chi_{2}^{\dagger} + L_{1}^{\dagger}\chi_{3}^{\dagger} + L_{3}^{\dagger}\chi_{1}^{\dagger})(L_{2}\chi_{2} + L_{1}\chi_{3} + L_{3}\chi_{1}), \\ \Delta K_{(L\chi)_{3_{1}}(L\chi)_{3_{1}}}^{(4)} &= (L_{1}^{\dagger}\chi_{1}^{\dagger} + \omega^{2}L_{2}^{\dagger}\chi_{3}^{\dagger} + \omega L_{3}^{\dagger}\chi_{2}^{\dagger})(L_{1}\chi_{1} + \omega L_{2}\chi_{3} + \omega^{2}L_{3}\chi_{2}) \\ &+ (L_{3}^{\dagger}\chi_{3}^{\dagger} + \omega^{2}L_{1}^{\dagger}\chi_{2}^{\dagger} + \omega L_{2}^{\dagger}\chi_{1}^{\dagger})(L_{3}\chi_{3} + \omega L_{1}\chi_{2} + \omega^{2}L_{2}\chi_{1}) \\ \Delta K_{(L\chi)_{3_{2}}(L\chi)_{3_{2}}}^{(5)} &= (L_{1}^{\dagger}\chi_{1}^{\dagger} + \omega L_{2}^{\dagger}\chi_{3}^{\dagger} + \omega^{2}L_{3}^{\dagger}\chi_{2}^{\dagger})(L_{1}\chi_{1} + \omega^{2}L_{2}\chi_{3} + \omega L_{3}\chi_{2}) \\ &+ (L_{3}^{\dagger}\chi_{3}^{\dagger} + \omega L_{1}^{\dagger}\chi_{2}^{\dagger} + \omega^{2}L_{3}^{\dagger}\chi_{2}^{\dagger})(L_{1}\chi_{1} + \omega^{2}L_{2}\chi_{3} + \omega L_{3}\chi_{2}) \\ \Delta K_{(L\chi)_{3_{1}}(L\chi)_{3_{2}}}^{(6)} &= (L_{1}^{\dagger}\chi_{1}^{\dagger} + \omega^{2}L_{2}^{\dagger}\chi_{3}^{\dagger} + \omega L_{3}^{\dagger}\chi_{2}^{\dagger})(L_{1}\chi_{1} + \omega^{2}L_{2}\chi_{3} + \omega L_{3}\chi_{2}) \\ &+ (L_{3}^{\dagger}\chi_{3}^{\dagger} + \omega L_{1}^{\dagger}\chi_{2}^{\dagger} + \omega L_{3}^{\dagger}\chi_{1}^{\dagger})(L_{2}\chi_{2} + \omega^{2}L_{1}\chi_{3} + \omega L_{3}\chi_{2}) \\ &+ (L_{3}^{\dagger}\chi_{3}^{\dagger} + \omega^{2}L_{1}^{\dagger}\chi_{3}^{\dagger} + \omega L_{3}^{\dagger}\chi_{1}^{\dagger})(L_{2}\chi_{2} + \omega^{2}L_{1}\chi_{3} + \omega L_{3}\chi_{2}) \\ &+ (L_{3}^{\dagger}\chi_{3}^{\dagger} + \omega^{2}L_{1}^{\dagger}\chi_{3}^{\dagger} + \omega L_{3}^{\dagger}\chi_{1}^{\dagger})(L_{2}\chi_{2} + \omega^{2}L_{1}\chi_{2} + \omega L_{3}\chi_{2}) \\ &+ (L_{3}^{\dagger}\chi_{3}^{\dagger} + \omega^{2}L_{1}^{\dagger}\chi_{3}^{\dagger} + \omega L_{3}^{\dagger}\chi_{1}^{\dagger})(L_{2}\chi_{2} + \omega^{2}L_{1}\chi_{2} + \omega L_{2}\chi_{1}) \\ &+ (L_{3}^{\dagger}\chi_{3}^{\dagger} + \omega^{2}L_{1}^{\dagger}\chi_{3}^{\dagger} + \omega L_{3}^{\dagger}\chi_{1}^{\dagger})(L_{2}\chi_{2} + \omega^{2}L_{1}\chi_{2} + \omega L_{2}\chi_{1}) \\ &+ (L_{2}^{\dagger}\chi_{2}^{\dagger} + \omega^{2}L_{1}^{\dagger}\chi_{3}^{\dagger} + \omega L_{3}^{\dagger}\chi_{1}^{\dagger})(L_{2}\chi_{2} + \omega^{2}L_{1}\chi_{3} + \omega L_{3}\chi_{1}) \end{split}$$

M.-C.C., Fallbacher, Ratz, Staudt (2012)

- Contributions from Flavon VEVs (1,0,0) and (1,1,1)
 - five independent "basis" matrices

$$P_{\rm I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\rm II} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\rm III} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_{\rm IV} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \qquad P_{\rm V} = \begin{pmatrix} 0 & i & -i \\ -i & 0 & i \\ i & -i & 0 \end{pmatrix}$$

- RG correction: essentially along $P_{III} = diag(0,0,1)$ direction due to y_{τ} dominance
- Kähler corrections can be along different directions than RG

Enhanced θ_{13}

M.-C.C., Fallbacher, Ratz, Staudt (2012)

- consider change due to correction along Pv direction
- Kähler metric:

$$\mathcal{K}_L = 1 - 2 x P$$
 with $P_V = egin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \ -\mathrm{i} & 0 & \mathrm{i} \ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}$

- Contributions of flavon VEV: $\langle \Phi \rangle = (1, 1, 1) \upsilon$
- Corrections to the leading order TBM prediction ($m_e \ll m_\mu \ll m_ au$)

$$\Delta \theta_{13} \simeq \kappa_{\rm V} \cdot \frac{v^2}{\Lambda^2} \cdot 3\sqrt{6} \frac{m_1}{m_1 + m_3}$$

- Complex matrix $P \Rightarrow CP$ violation induced
- for the example considered: $\delta pprox \pi/2$







Conclusion

- Kähler corrections induced (and determined) by flavon VEVs (with order parameter $\sim \theta_c)$
 - while similar in structure to RG corrections, can be along different directions than RG
 - size of Kähler corrections generically dominate RG corrections (no loop suppression, contributions from copies heavy states)
 - non-zero CP phases can be induced
 - additional parameters (Kähler coefficients) introduced
- robustness of model predictions diminished given the presence of these potentially sizable corrections and new parameters
- theoretical understanding of Kähler corrections crucial for achieving precision compatible with experimental accuracy