Neutrino Masses in Two Higgs Doublet Models

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Based on: AI, C. Simonetto, JHEP 1111 (2011) 022
D. Hehn, AI, arXiv:1208.3162
J.A. Casas, AI, F. Jimenez-Alburquerque JHEP 0704 (2007) 064

BeNe 2012
19 September 2012

Why neutrino parameters are so different to the quark parameters?

- **Why tiny masses?**
- **Why large mixing angles?**
- **Why mild mass hierarchy?**

Any model of neutrino masses should simultaneously address these three questions, and preferably the model should be testable

A very popular neutrino mass model: the (type I) see-saw model

Add to the Standard Model at least two right-handed neutrinos

$$-\mathcal{L}^{\nu} = (Y_{\nu})_{ij}\bar{l}_{Li}\nu_{Rj}\tilde{\Phi} - \frac{1}{2}M_{\mathrm{M}ij}\bar{\nu}^{C}_{Ri}\nu_{Rj} + \mathrm{h.c.}$$

$$M_{\mathrm{Maj}} \gg M_Z$$

$$-\mathcal{L}^{\nu,\ \mathrm{eff}} = \frac{1}{2}\kappa_{ij}(\bar{l}_{Li}\tilde{\Phi})(\tilde{\Phi}^T l^C_{Lj}) + \mathrm{h.c.}$$

$$\kappa = (Y_{\nu}M_{\mathrm{M}}^{-1}Y_{\nu}{}^{T})$$

$$\mathcal{M}_{\nu} = \frac{v^2}{2}\kappa$$

Very compelling explanation
to the small neutrino masses

Furthermore, this model predicts charged lepton flavour violation:

$\mathrm{BR}(\mu\to e\gamma)\sim 10^{-57}$, in *excellent* agreement with experiments.

Can the see-saw mechanism accommodate a mild neutrino mass hierarchy?

Yes. In fact the see-saw mechanism can accommodate any set of low energy observables ...

$$Y_\nu = \frac{1}{\langle \Phi^0 \rangle} U_{\rm lep}^* \sqrt{D_m} R^T \sqrt{D_M}$$

Casas, AI

… possibly at the price of tunings, unnaturaless, etc.

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A more interesting question is not whether the see-saw can accommodate the data, but whether the see-saw can accommodate the data *with our present (very limited) understanding of the origin of flavour.*

Measured ratios of Yukawa eigenvalues

$m_t/m_c \simeq 140$ $m_b/m_s \simeq 41$ $m_\tau/m_\mu \simeq 17$

$m_c/m_d \simeq 500$ $m_s/m_d \simeq 20$ $m_\mu/m_e \simeq 208$

Hint??

Can the see-saw mechanism accommodate a mild neutrino mass hierarchy *when the neutrino Yukawa couplings are hierarchical*?

Not so easy... The see-saw mechanism tends to produce very large neutrino mass hierarchies Casas, AI, Jimenez-Alburquerque

"Naïve see-saw" (no mixing)

$$m_1 \sim \frac{y_1^2}{M_1}\langle\Phi^0\rangle^2, \qquad m_2 \sim \frac{y_2^2}{M_2}\langle\Phi^0\rangle^2, \qquad m_3 \sim \frac{y_3^2}{M_3}\langle\Phi^0\rangle^2 \qquad\qquad \frac{m_3}{m_2} \sim \frac{y_3^2}{y_2^2}\frac{M_2}{M_3}$$

- Assume hierarchical Yukawa couplings
  $y_1 : y_2 : y_3 \sim 1 : 20 : 20^2$ (down-type quark Yukawas)
  $y_1 : y_2 : y_3 \sim 1 : 300 : 300^2$ (up-type quark Yukawas)

- For the right-handed neutrino masses, we don't know

Hierarchy in ν_R as in Y_ν — $\frac{m_3}{m_2} \sim 20 - 300$

Degenerate ν_R — $\frac{m_3}{m_2} \sim 400 - 90000$

far from $\frac{m_3}{m_2} \lesssim 6$

A more rigorous analysis shows that generically

$$\frac{m_3}{m_2} \gtrsim \frac{y_3^2}{y_2^2}\frac{M_3}{M_2}$$

Hierarchical ν_R: $\frac{m_3}{m_2} \gtrsim \mathcal{O}(10^{3-7})$

Degenerate ν_R: $\frac{m_3}{m_2} \gtrsim \mathcal{O}(10^{2-5})$

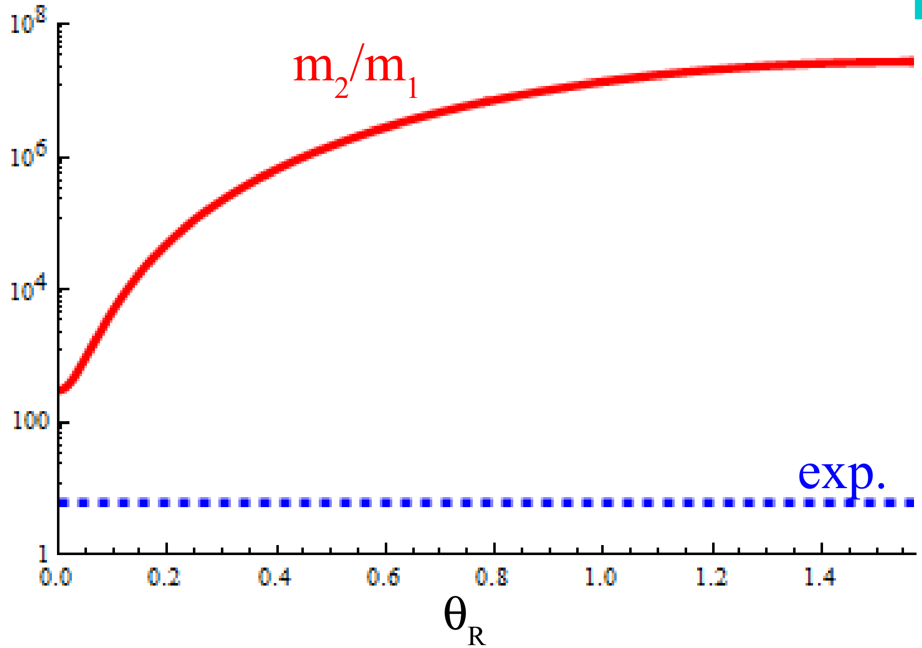
far from $\frac{m_3}{m_2} \lesssim 6$

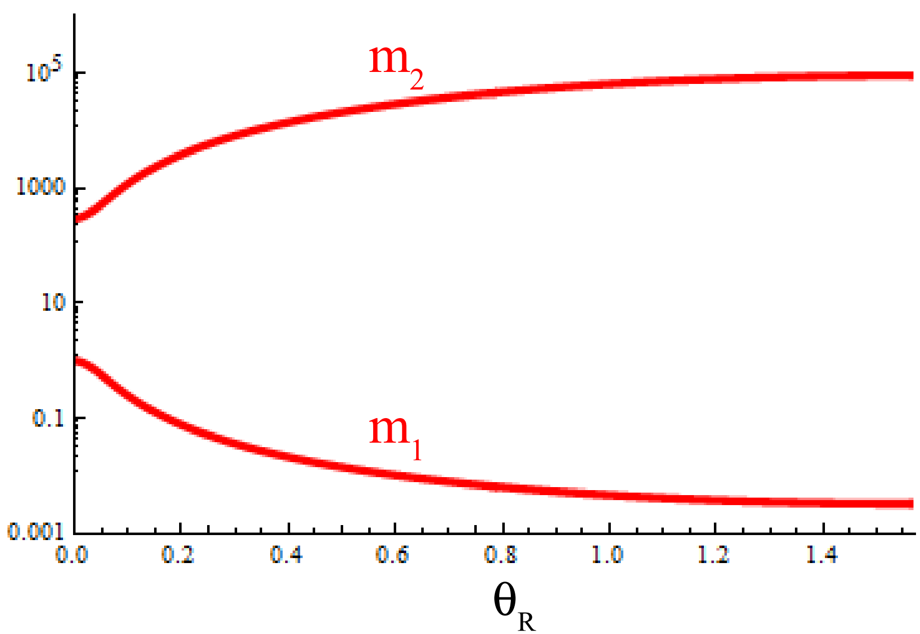
Consider a model with just two right-handed neutrinos

Assume
$$\begin{aligned} y_1 : y_2 &= 1 : 300 \\ M_1 : M_2 &= 1 : 300 \end{aligned}$$
(inspired by the hierarchy in the up-quark sector)

The neutrino mass hierarchy depends just on the mixing angle in the right-handed sector, θ_R, and on the Majorana phase α.

$\alpha=0$





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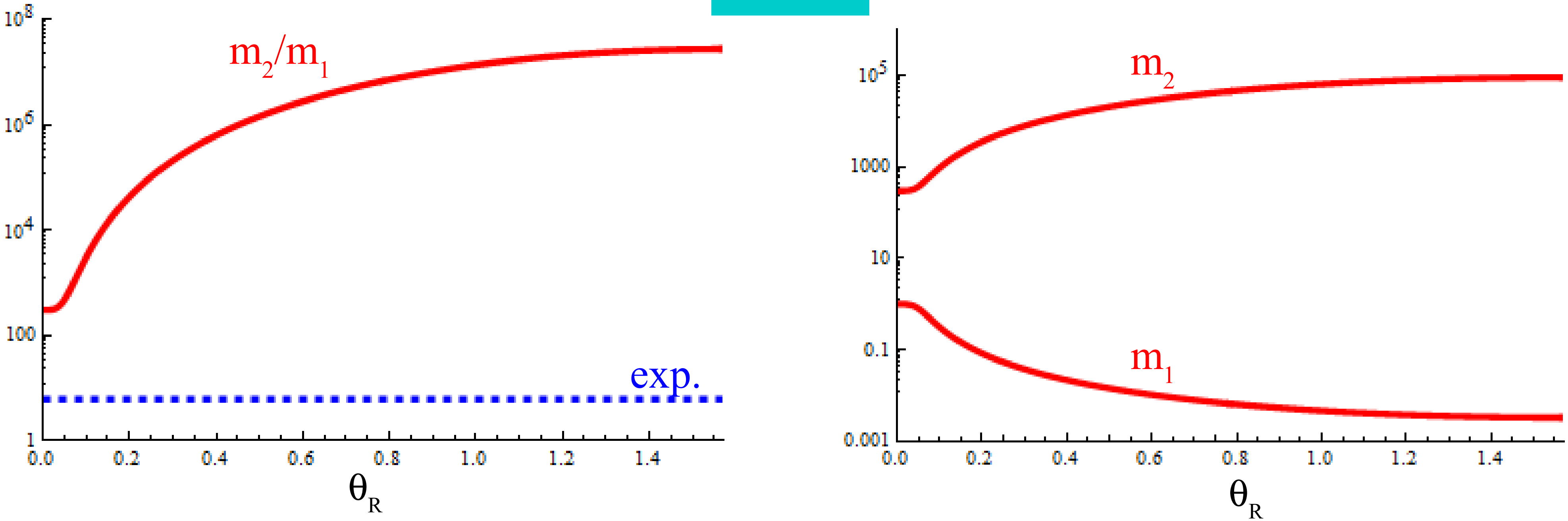

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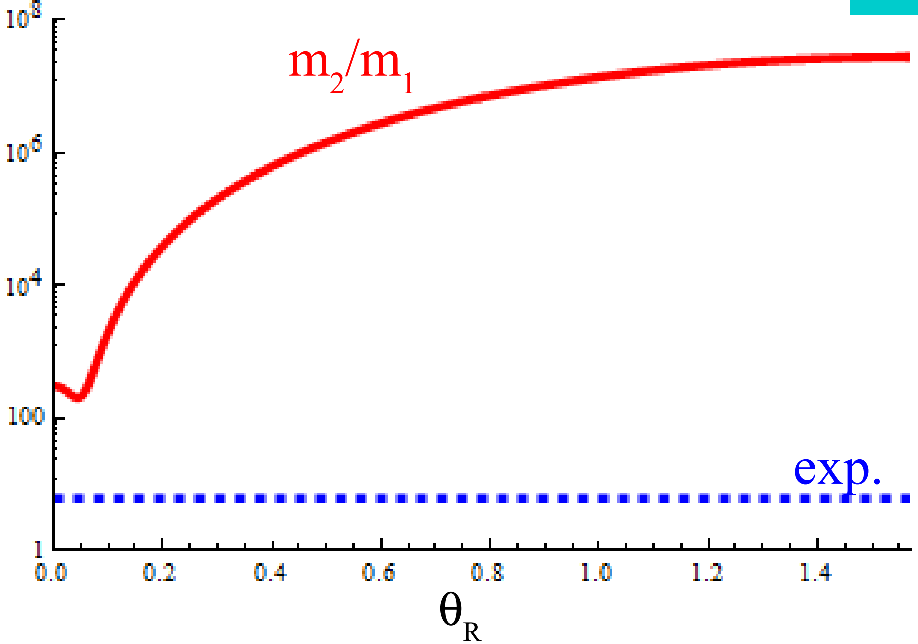

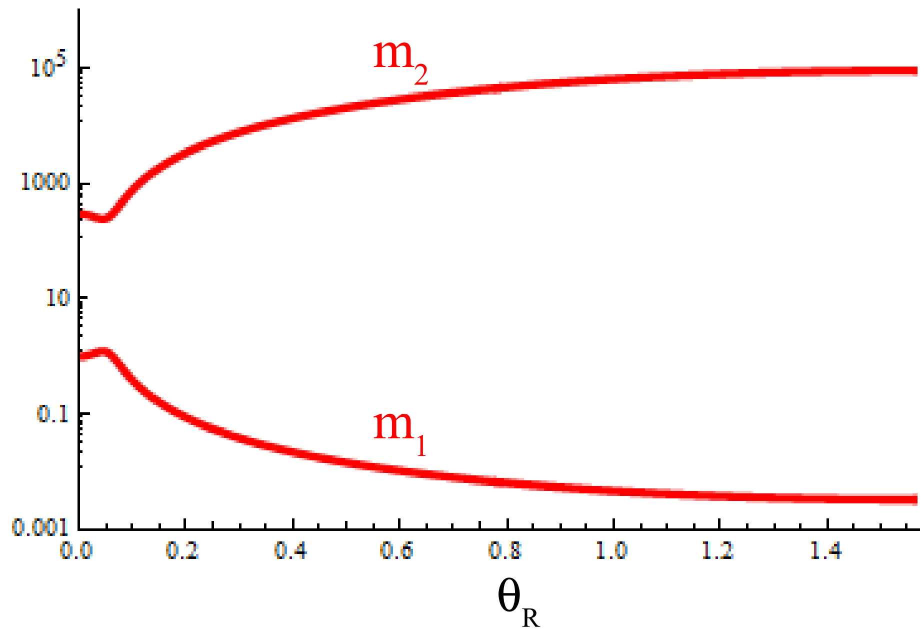

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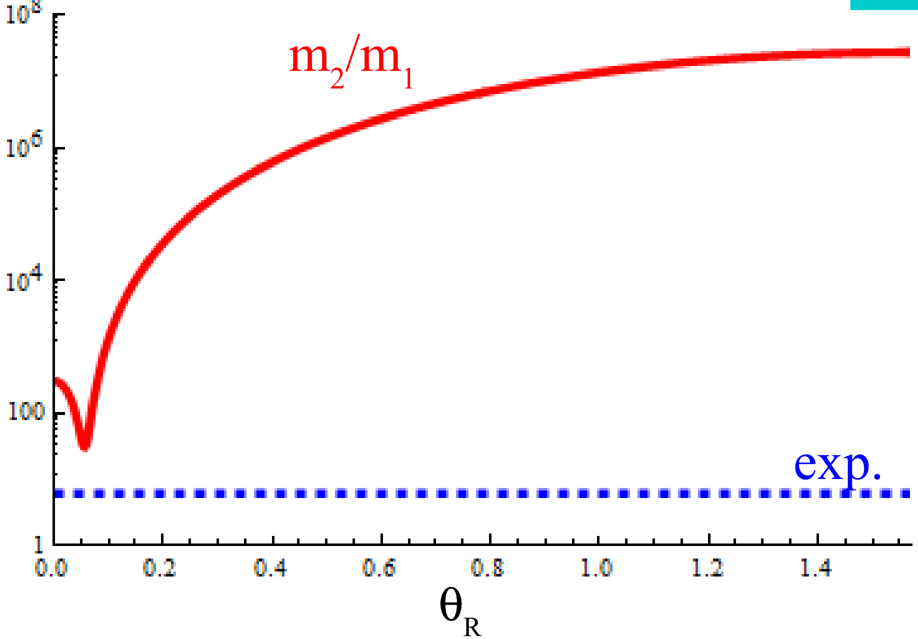

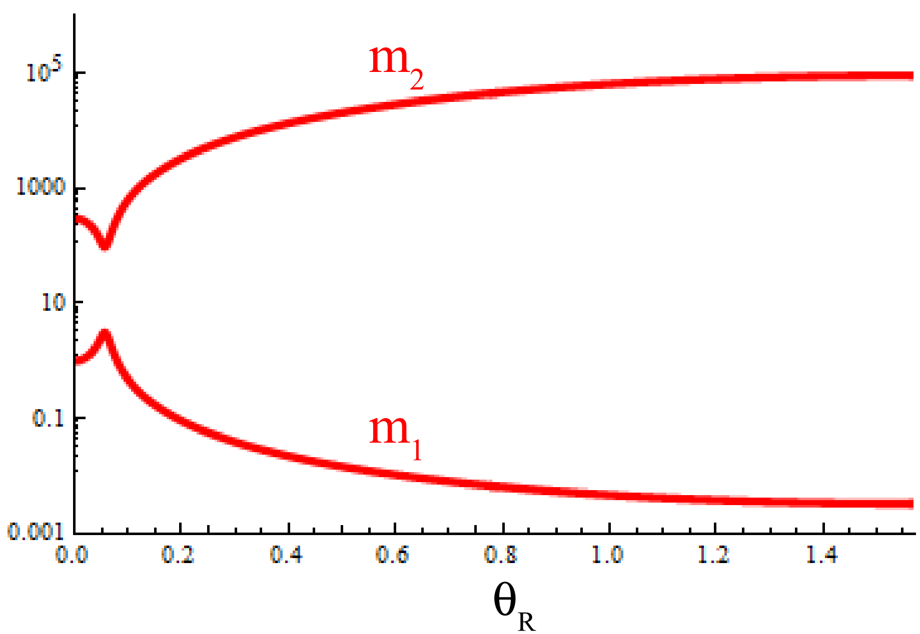

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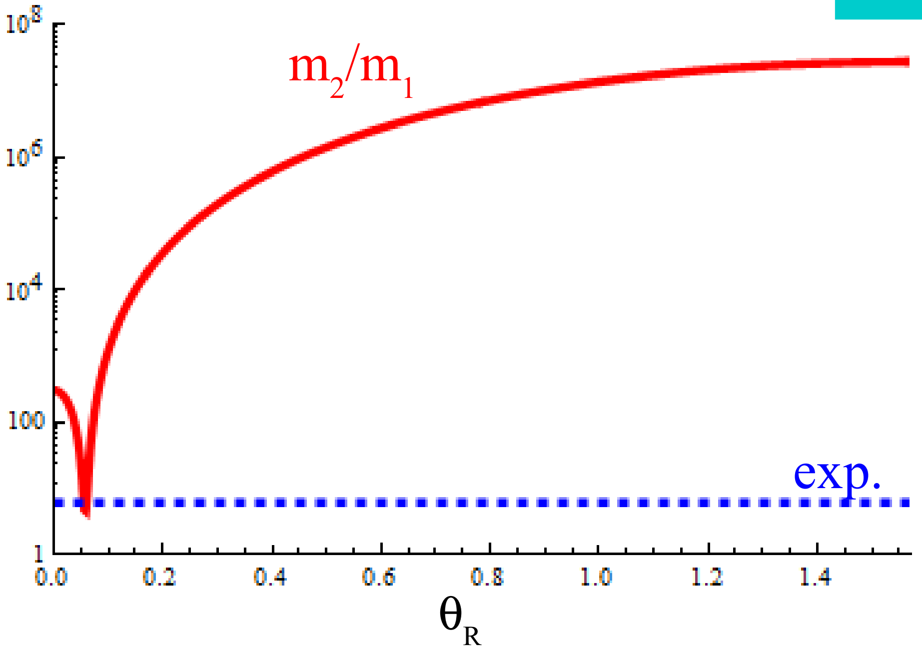

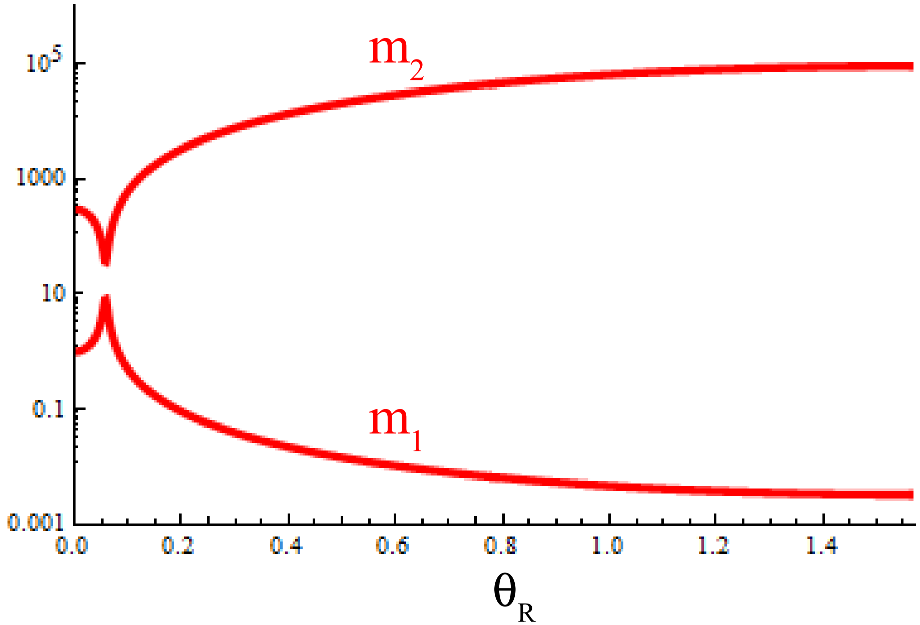

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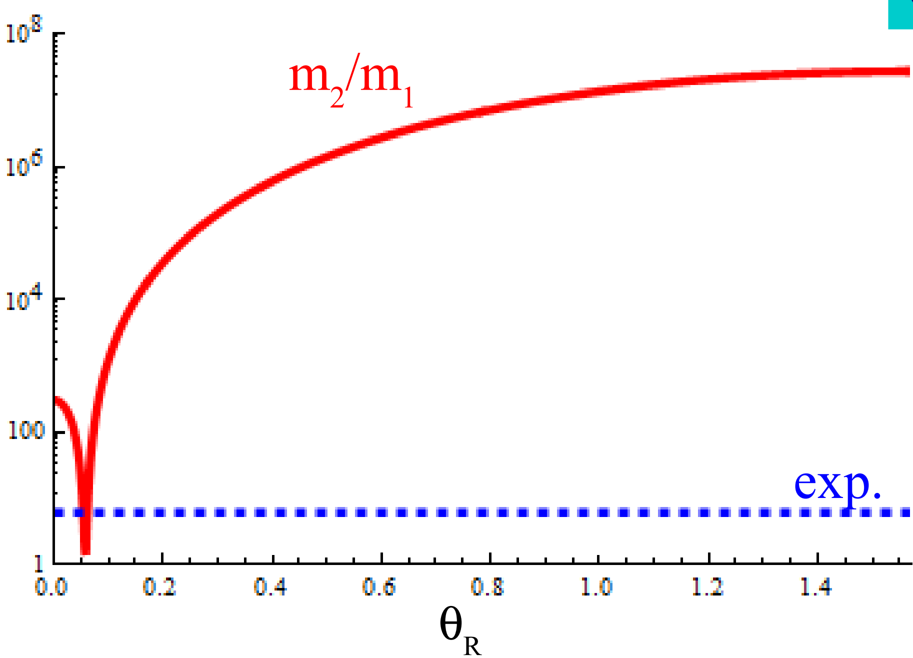
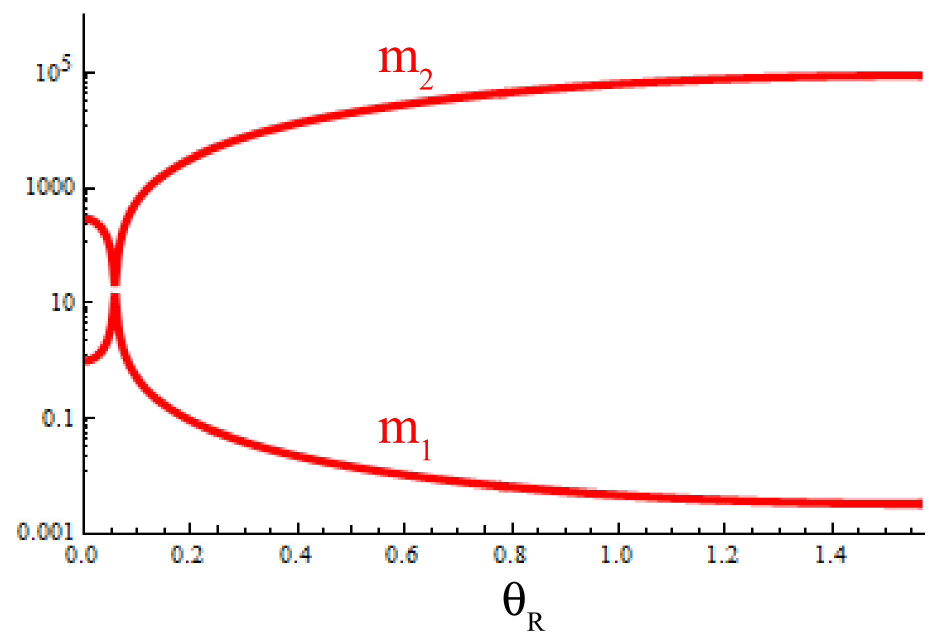




There are two situations where the see-saw mechanism can naturally generate a mild neutrino mass hierarchy (without tunings):

"naive see-saw" $$\frac{m_2}{m_1} \sim \frac{y_2^2}{y_1^2}\frac{M_1}{M_2} \lesssim 6$$

Very mild hierarchies

$y_1 \simeq y_2$

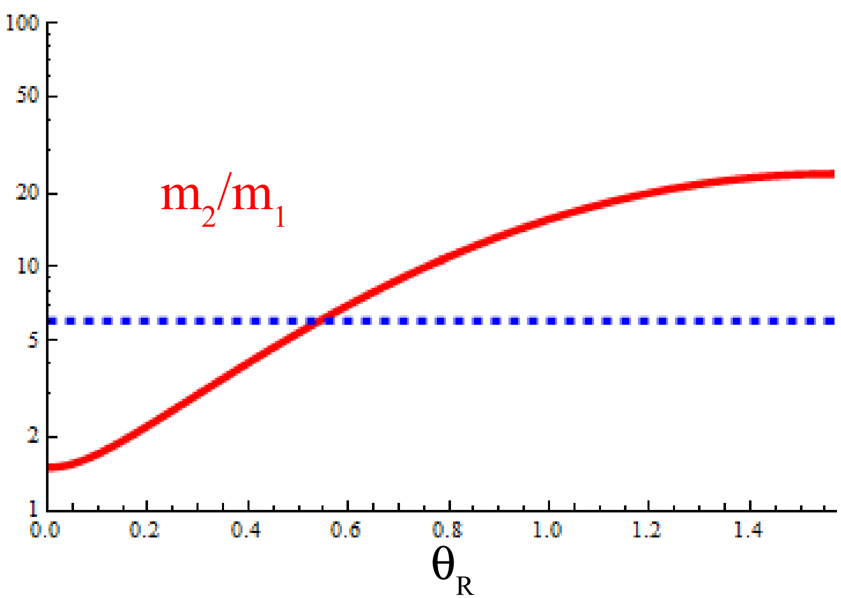
$M_1 \simeq M_2$

$$y_1 : y_2 = \ 1:2$$
$$M_1 : M_2 = \ 1:6$$

Very strong hierarchy in RH masses

$M_1 : M_2 \simeq y_1^2 : y_2^2$

$$y_1 : y_2 = 1:300$$
$$M_1 : M_2 = 1:90000$$



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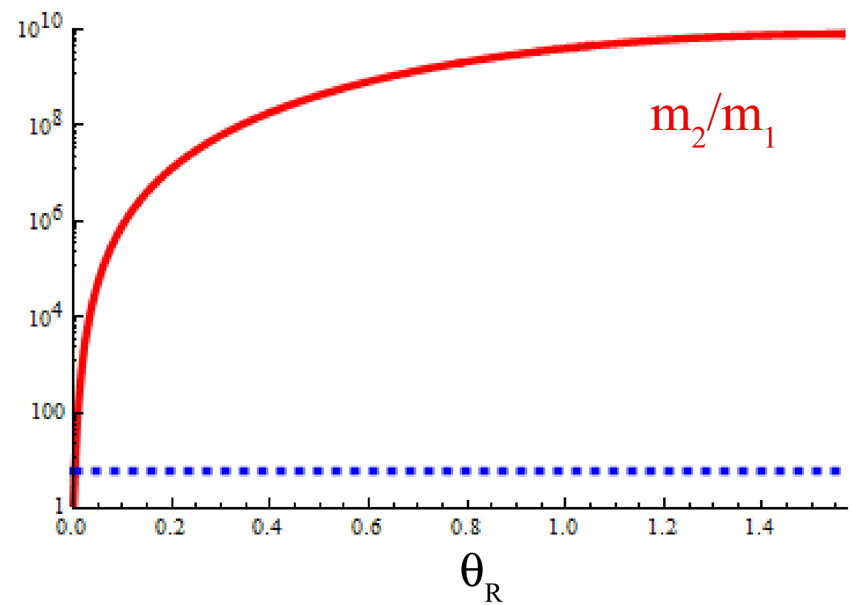

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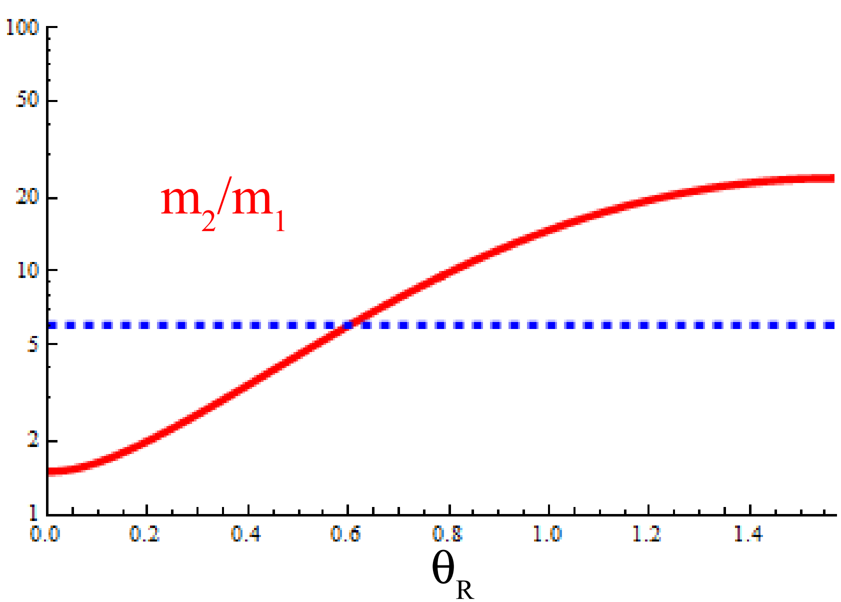

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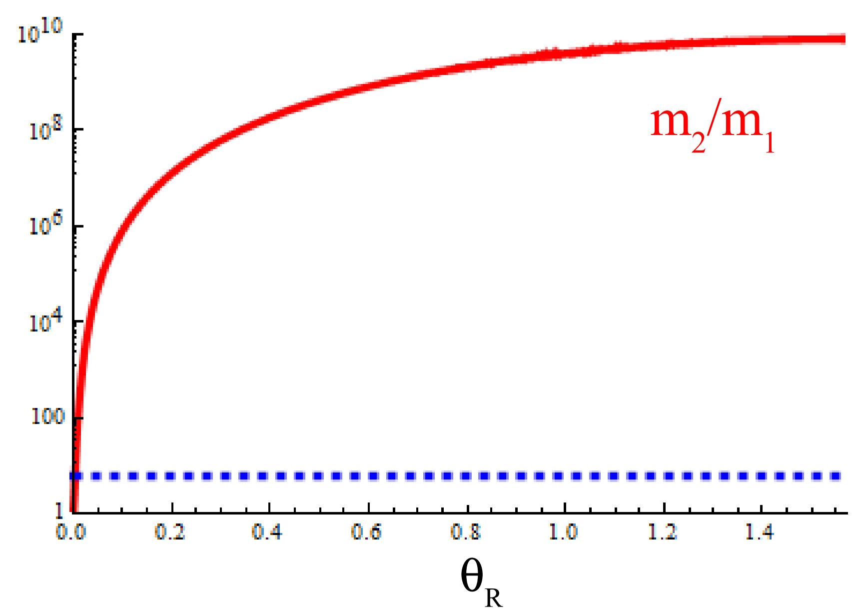

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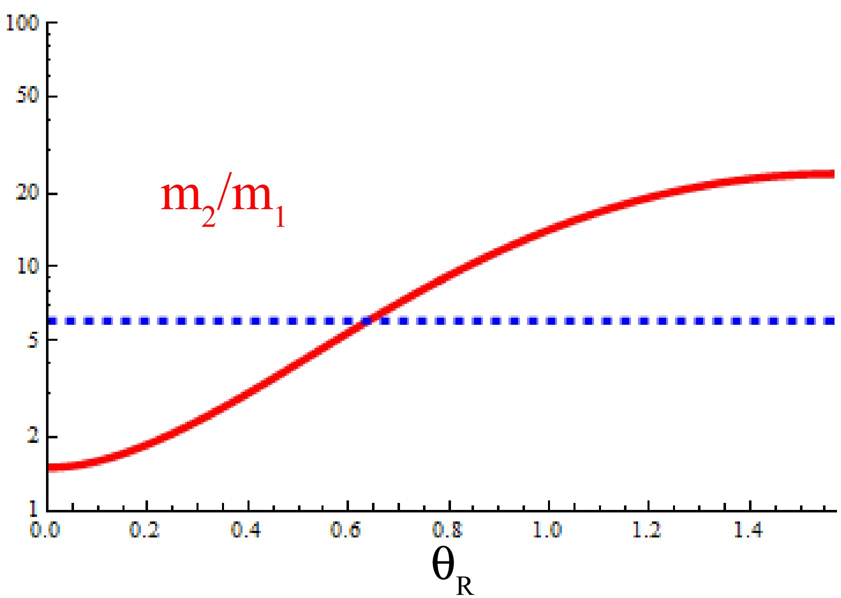

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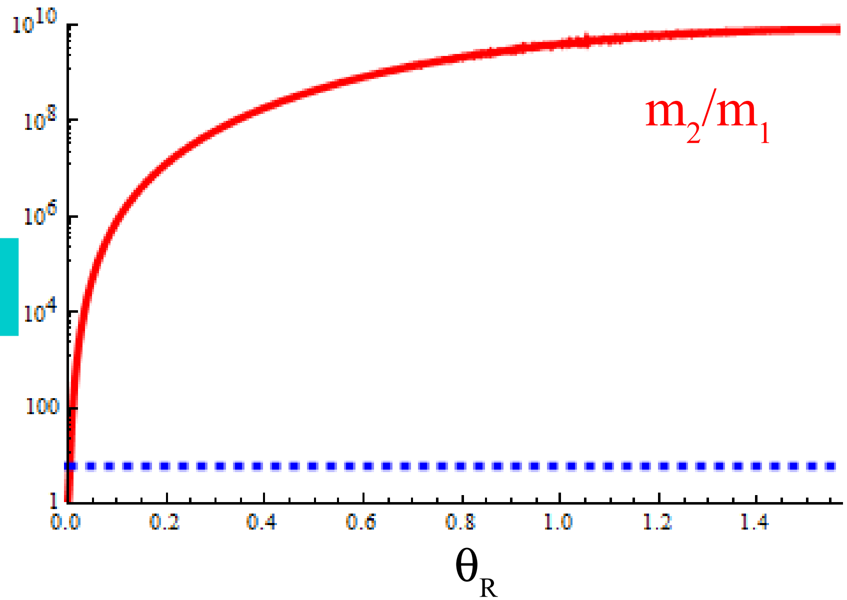

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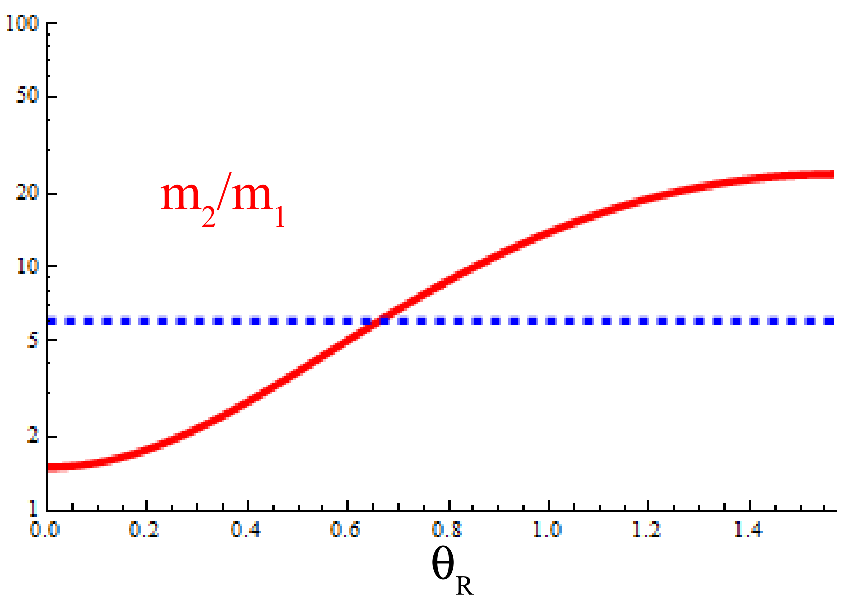

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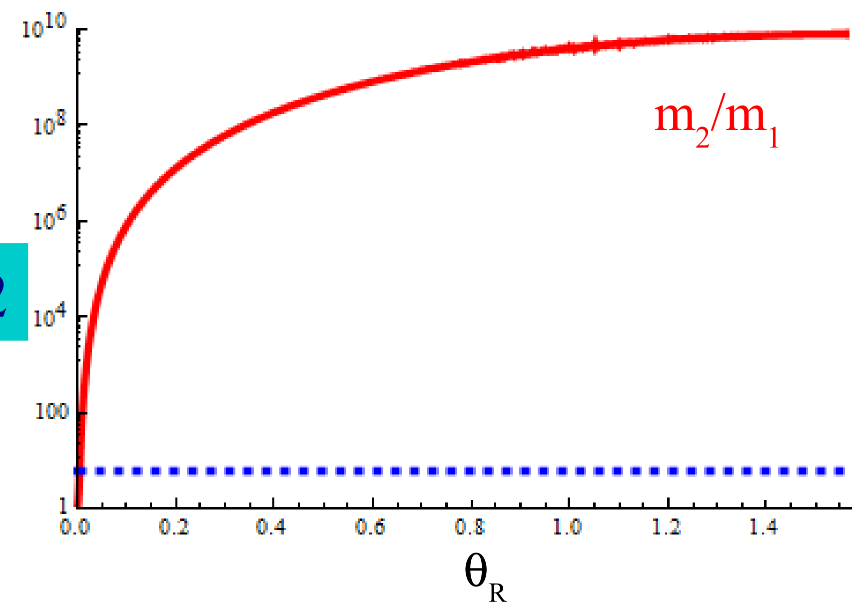

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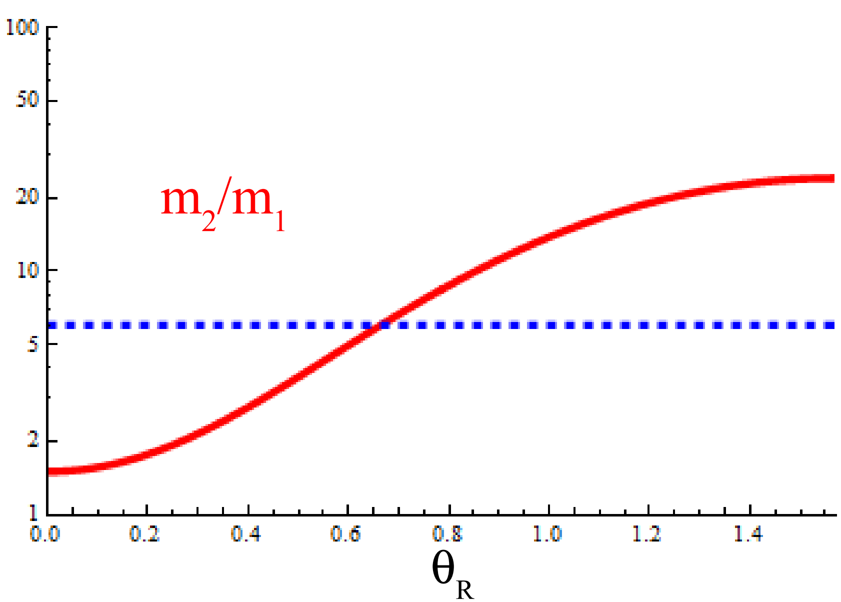

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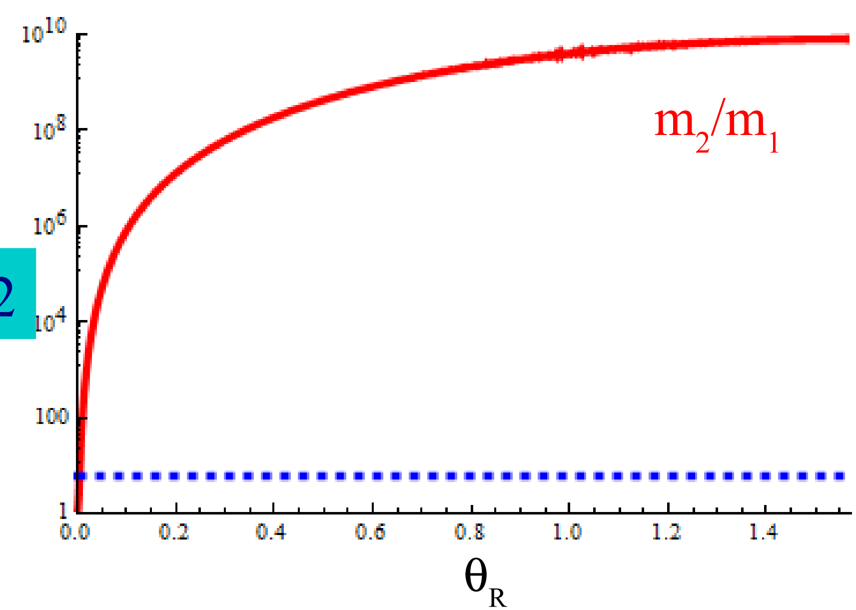

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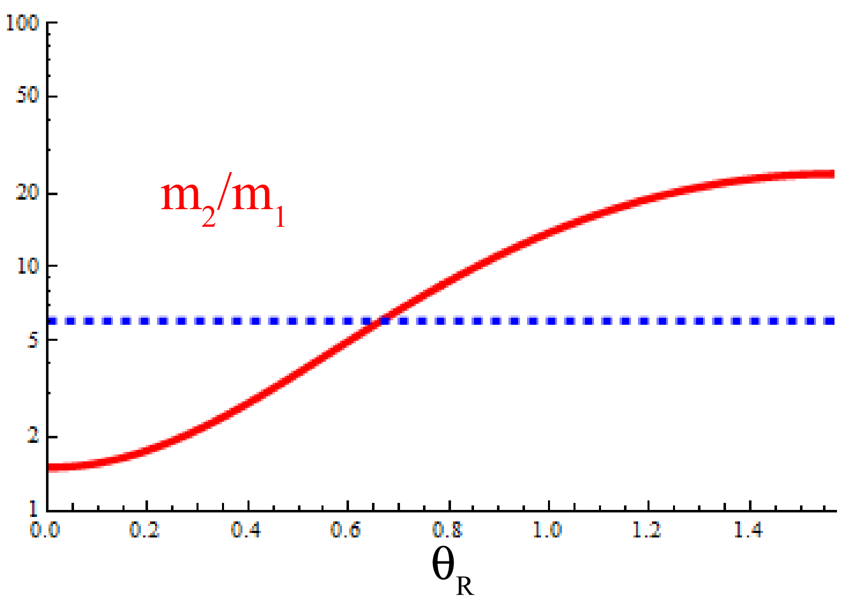

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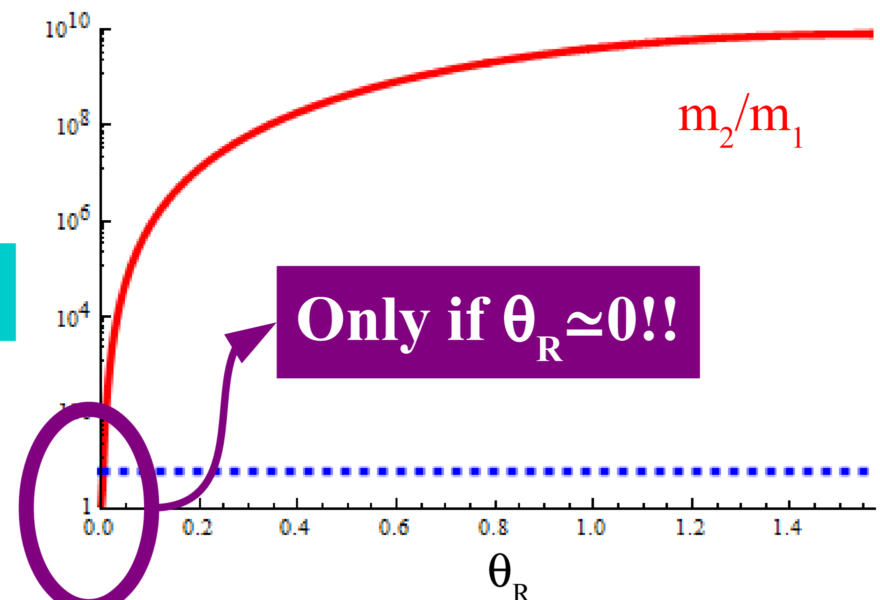


The see-saw mechanism generates a neutrino mass hierarchy much larger than the observed experimentally, except:

- When the Yukawa eigenvalues *and* the right-handed masses present a mild mass hierarchy.

- In the case of hierachical Yukawa eigenvalues, for very special choices of the parameters.

The see-saw mechanism provides a very compelling explanation to the smallness of neutrino masses, while keeping all the successes of the Standard Model. However, it fails to provide a compelling explanation to why the neutrino mass hierarchy is so mild.

A possible solution: introduce a second higgs doublet

With a second higgs doublet, quantum corrections can soften the neutrino mass hierarchy.

Even if at tree level m_3/m_2 is very large, as generically expected in the (standard) see-saw mechanism, the quantum corrections can generate $m_3/m_2 \sim 6$.

Grimus, Neufeld
AI, Simonetto

ν masses in the see-saw model extended with one extra Higgs

Consider the Standard Model extended by right-handed neutrinos and at least one extra Higgs doublet (no ad-hoc discrete symmetries)

$$-\mathcal{L}^{\nu} = (Y_{\nu}^{a})_{ij}\bar{l}_{Li}\nu_{Rj}\tilde{\Phi}_{a} - \frac{1}{2}M_{\mathrm{M}ij}\bar{\nu}_{Ri}^{C}\nu_{Rj} + \mathrm{h.c.}$$

$$M_{\mathrm{Maj}} \gg m_{H}, M_{Z}$$

$$-\mathcal{L}^{\nu,\ \mathrm{eff}} = \frac{1}{2}\kappa_{ij}^{ab}(\bar{l}_{Li}\tilde{\Phi}_{a})(\tilde{\Phi}_{b}^{T}l_{Lj}^{C}) + \mathrm{h.c.}$$

$$\kappa^{ab}(M_1) = (Y_{\nu}^{a}M_{\mathrm{M}}^{-1}Y_{\nu}^{b\,T})(M_1)$$

Work in the basis where only Φ_1 acquires a vev

$$\mathcal{M}_{\nu}(M_1) = \frac{v^2}{2}\kappa^{11}(M_1)$$

The neutrino mass matrix is affected by quantum corrections below M_1

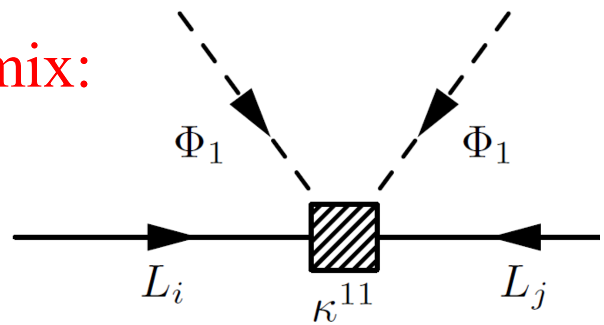
RGE effects

Quantum effects generate a correction to the coefficient of the dimension 5 operator which generates neutrino masses:

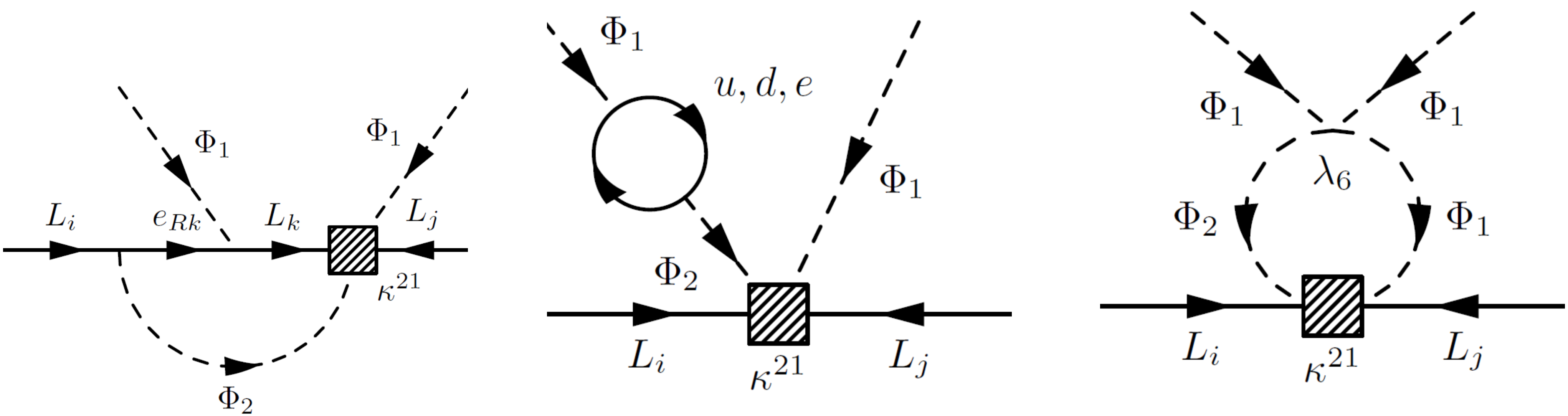
$$\delta\kappa^{11} \simeq B_{1a}\kappa^{a1} + \kappa^{1a}B^T_{1a} + b\kappa^{22}$$

Grimus, Lavoura

Different operators mix:



Is corrected by $B_{12}\,\kappa^{21}$



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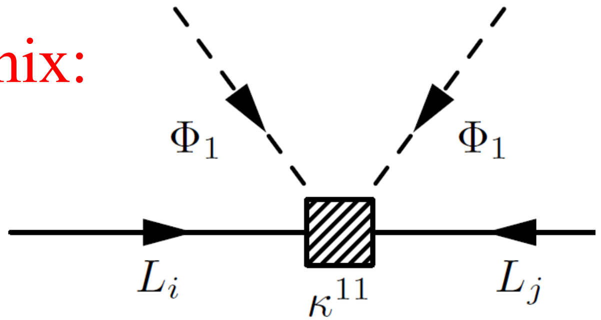
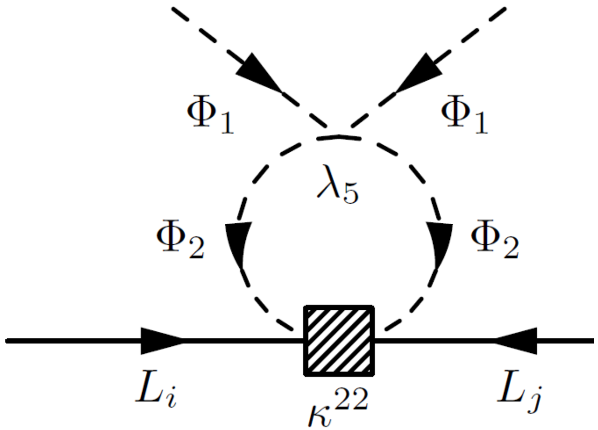


Is corrected by $b\,\kappa^{22}$



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Grimus, Lavoura

Different operators mix.

Compare to the correction in the "one Higgs doublet model":

$$\delta\kappa \simeq B\kappa + \kappa B^T$$

New qualitative features?

To highlight the new features, consider a model with one right-handed neutrino and two Higgs doublets (no ad-hoc discrete symmetries imposed):

$$-\mathcal{L}^{\nu} = (Y_{\nu}^{1})_i \bar{l}_{Li} \nu_R \tilde{\Phi}_1 + (Y_{\nu}^{2})_i \bar{l}_{Li} \nu_R \tilde{\Phi}_2 - \frac{1}{2} M_{\rm Maj} \bar{\nu}_R^C \nu_R + \text{h.c.}$$

$$M_{\rm Maj} \gg m_H , M_Z$$

$$-\mathcal{L}^{\nu,\ \rm eff} = \frac{1}{2} \kappa_{ij}^{ab} (\bar{l}_{Li} \tilde{\Phi}_a)(\tilde{\Phi}_b^T l_{Lj}^C) + \text{h.c.}$$

Work in the basis where only Φ_1 acquires a vev

$$\mathcal{M}_{\nu}(M_{\rm Maj}) = \frac{v^2}{2} \kappa^{11}(M_{\rm Maj})$$

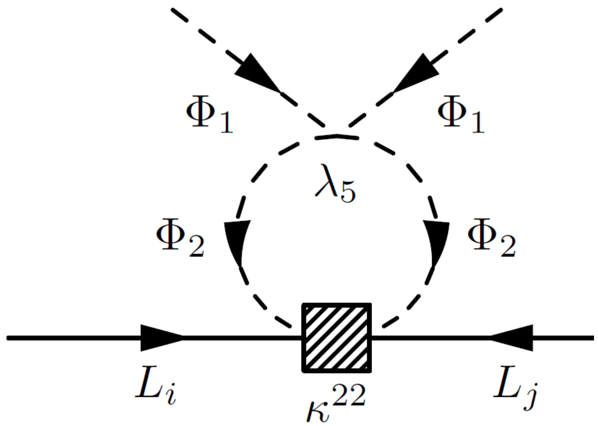
Rank 1. At tree level

$$m_3 = \frac{|Y_{\nu}^{1}|^2 v^2}{2 M_{\rm Maj}}$$

$$m_2,\ m_1 = 0$$

RGE effects

$$\delta\kappa^{11} \simeq B_{1a}\kappa^{a1} + \kappa^{1a}B^T_{1a} + b\kappa^{22}$$



$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}} + \text{small corrections}$$

$$m_2 = \frac{1}{16\pi^2}\frac{|\lambda_5|v^2}{M_{\rm maj}}\left[|Y_\nu^2|^2 - \frac{|Y_\nu^{2\dagger}Y_\nu^1|^2}{|Y_\nu^1|^2}\right]\log\frac{M_{\rm maj}}{m_H}$$

$$m_1 = 0$$

A second neutrino mass is generated from the same right-handed neutrino mass scale M_{maj} → a mild mass hierarchy might be naturally accommodated.

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}}$$

$$m_2 = \frac{1}{16\pi^2}\frac{|\lambda_5| v^2}{M_{\rm maj}}\left[|Y_\nu^2|^2 - \frac{|Y_\nu^{2\dagger} Y_\nu^1|^2}{|Y_\nu^1|^2}\right]\log\frac{M_{\rm maj}}{m_H}$$

Neutrino mass hierarchy:

Assume:

- M_{maj} large, to implement the see-saw mechanism $m_H << M_{maj}$ (e.g m_H=100 GeV-1 TeV)
- Neutrino Yukawa couplings misaligned (new sources of flavour violation are required)
- $|Y_\nu^1| \sim |Y_\nu^2|$
- $\lambda_5 \sim O(1)$

$$\left|\frac{m_2}{m_3}\right| \simeq \frac{|\lambda_5|}{8\pi^2}\frac{|Y_\nu^2|^2}{|Y_\nu^1|^2}\log\left(\frac{M_{\rm maj}}{m_H}\right) \simeq 0.2$$

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}}$$

$$m_2 = -\frac{1}{16\pi^2}\frac{|\lambda_5|v^2}{M_{\rm maj}}\left[|Y_\nu^2|^2 - \frac{|Y_\nu^{2\dagger}Y_\nu^1|^2}{|Y_\nu^1|^2}\right]\log\left(\frac{M_{\rm maj}}{m_H}\right)$$

Neutrino mass hierarchy:

Assume:

- M_{maj} large, to implement the see-saw mechanism $m_H \ll M_{maj}$ (e.g m_H=100 GeV-1 TeV)
- Neutrino Yukawa couplings misaligned (new sources of flavour violation are required)

$$|Y_\nu^1| \sim |Y_\nu^2|$$

Yukawa couplings to the *same* generation of right-handed neutrinos

- $\lambda_5 \sim \mathcal{O}(1)$

$$\left|\frac{m_2}{m_3}\right| \simeq \frac{|\lambda_5|}{8\pi^2}\frac{|Y_\nu^2|^2}{|Y_\nu^1|^2}\log\left(\frac{M_{\rm maj}}{m_H}\right) \simeq 0.2$$

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2M_{\rm maj}}$$

$m_2 = -\frac{1}{\ldots}\frac{|\lambda_5| v^2}{\ldots}\left[\ldots \frac{|Y_\nu^{2\dagger} Y_\nu^1|^2}{\ldots}\right] \ldots \left(\frac{M_{\rm maj}}{m_H}\right)$

LEPTON FLAVOUR VIOLATION?
ELECTRIC DIPOLE MOMENTS?

Neutrino mass hierarchy:

Assume:

- M_{maj} large, to implement the see-saw mechanism $m_H \ll M_{maj}$ (e.g m_H=100 GeV-1 TeV)
- Neutrino Yukawa couplings misaligned (new sources of flavour violation are required)
- $|Y_\nu^1| \sim |Y_\nu^2|$
- $\lambda_5 \sim \mathcal{O}(1)$

$$\left|\frac{m_2}{m_3}\right| \simeq \frac{|\lambda_5|}{8\pi^2}\frac{|Y_\nu^2|^2}{|Y_\nu^1|^2}\log\left(\frac{M_{\rm maj}}{m_H}\right) \simeq 0.2$$

$$m_3 = \frac{|Y_\nu^1|^2 v^2}{2 M_{\rm maj}}$$

$$m_2 = - \frac{1}{\ldots} \frac{|\lambda_5| v^2}{\ldots} \left[\ldots \frac{|Y_\nu^{2\dagger} Y_\nu^1|^2}{\ldots} \right] \ldots \left(\frac{M_{\rm maj}}{m_H} \right)$$

LEPTON FLAVOUR VIOLATION?
ELECTRIC DIPOLE MOMENTS?

Neutrino mass hierarchy:

Assume:

- M_{maj} large, to implement the see-saw mechanism
  $m_H << M_{maj}$ (e.g m_H=100 GeV-1 TeV)
- Neutrino Yukawa couplings misaligned (new sources of flavour violation are required)
- $|Y_\nu^1| \sim |Y_\nu^2|$
- $\lambda_5 \sim O(1)$

Logarithmic dependence with m_H,
while the rate for $\mu \to e\gamma$ decreases as m_H^4

$$\left| \frac{m_2}{m_3} \right| \simeq \frac{|\lambda_5|}{8\pi^2} \frac{|Y_\nu^2|^2}{|Y_\nu^1|^2} \log \left(\frac{M_{\rm maj}}{m_H} \right) \simeq 0.2$$

Message to take home:

The Standard Model extended with ≥1 right-handed neutrinos and ≥1 Higgs doublets can naturally explain the smallness of neutrino masses and the existence of a mild mass hierarchy, without jeopardizing any of the successes of the Standard Model, since all extra particles decouple at low energies.

A remarkable difference with respect to the two right-handed neutrino model:

Possibly, new phenomena at low energies, apart from neutrino masses

LFV processes could be observable, if not too suppressed by m_H.

$$\mathrm{BR}(\mu \to e\,\gamma) = \frac{8\alpha^3}{3\pi^3}\frac{|Y^2_{e12}|^2 + |Y^2_{e21}|^2}{|Y^1_{e22}|^2}\left| f\left(\frac{m_t^2}{m_h^2}\right)\cos\alpha - \frac{Y^2_{u33}}{Y^1_{u33}}\frac{m_t^2}{m_H^2}\log^2\frac{m_t^2}{m_H^2}\right|^2$$

Paradisi
Hisano, Sugiyama, Yamanaka

Could be present at tree level.
If not, generated radiatively by the neutrino Yukawa couplings

Mixing angles: RGE effects on θ_{13} and θ_{23}

New flavour structures in κ^{22} and $Y_e^{\,2}$ can induce radiatively a non-vanishing θ_{13} and a deviation from maximal atmospheric mixing.

$$\delta U_{13} = -\frac{1}{16\pi^2}\frac{Y^{2*}_{\nu 1}}{|Y^1_\nu|}\left[3\mathrm{Tr}(Y_u^{1\dagger}Y_u^2 + Y_d^1 Y_d^{2\dagger}) + 2\lambda_6^* + 2\lambda_5^*\frac{Y_\nu^{2\dagger}Y_\nu^1}{|Y_\nu^1|^2}\right]\log\frac{M_{\mathrm{maj}}}{m_H}$$
$$+\frac{1}{16\pi^2}\frac{(Y_\nu^{1\dagger}(Y_e^1)^{-1}Y_e^{2\dagger})_1}{|Y_\nu^1|}\left[3\mathrm{Tr}(Y_u^{2\dagger}Y_u^1 + Y_d^2 Y_d^{1\dagger})\right]\log\frac{M_{\mathrm{maj}}}{m_H}$$

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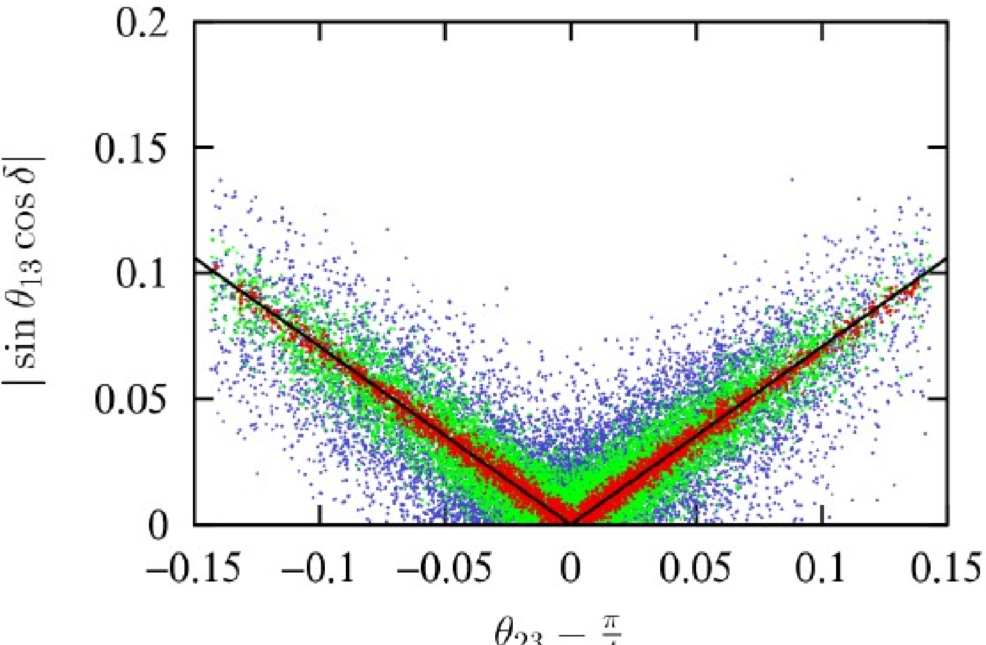
Similar to m_2/m_3 → δU_{13} can easily be ~ 0.1–0.2

Mixing angles: RGE effects on θ_{13} and θ_{23}

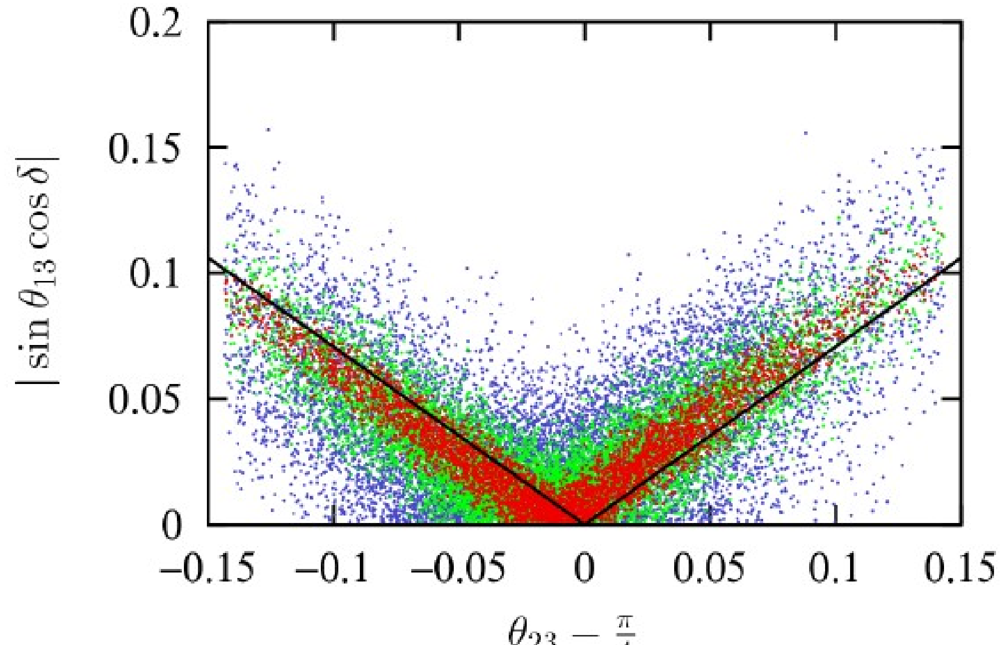
If the neutrino Yukawa couplings are the dominant source of flavour violation in the leptonic sector, there is a correlation between the radiatively generated θ_{13} and the radiatively generated deviation from maximal atmospheric mixing.

$$\left|\theta_{23} - \frac{\pi}{4}\right| \simeq \sqrt{2}\,|\sin\theta_{13}\cos\delta|$$

$\Lambda = M_{maj}$



$\Lambda = 10^{18}$ GeV



Some speculations about the mixing angles

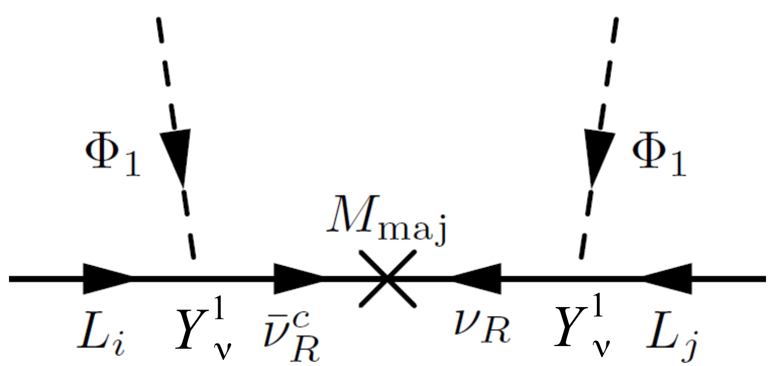
The third column of the leptonic mixing matrix seems to follow a pattern, at least at lowest order: $\theta_{13}=0$, $\theta_{23}=\pi/4$.

$$U_{i3} \approx \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

The second column might not follow any pattern: the solar mixing angle is neither minimal nor maximal.

$$U_{i2} \approx \begin{pmatrix} O(1) \\ O(1) \\ O(1) \end{pmatrix}$$

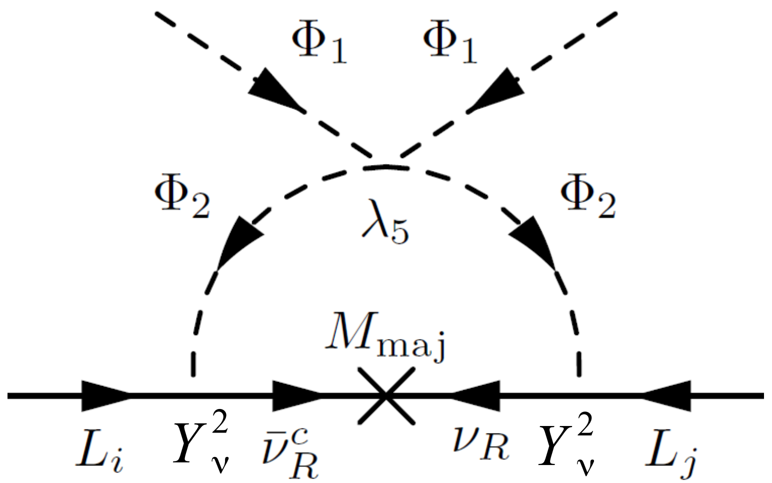
In the 1RHN-2HDM



Third column of U_{lep}

Possibly with a pattern

(if Y_ν^1 has a pattern)



second column of U_{lep}

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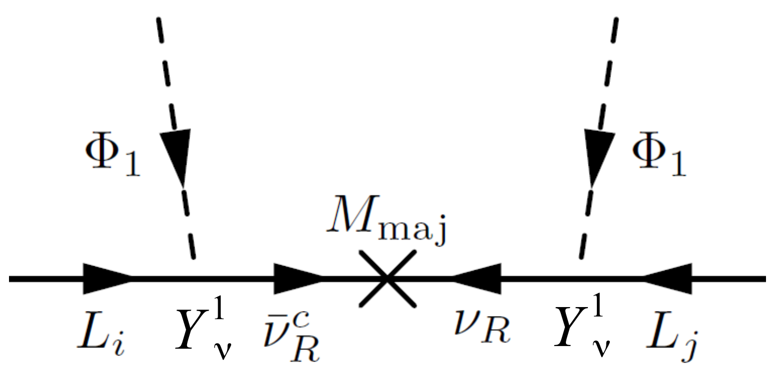
In the 1RHN-3HDM,
(more higgs doublets!)



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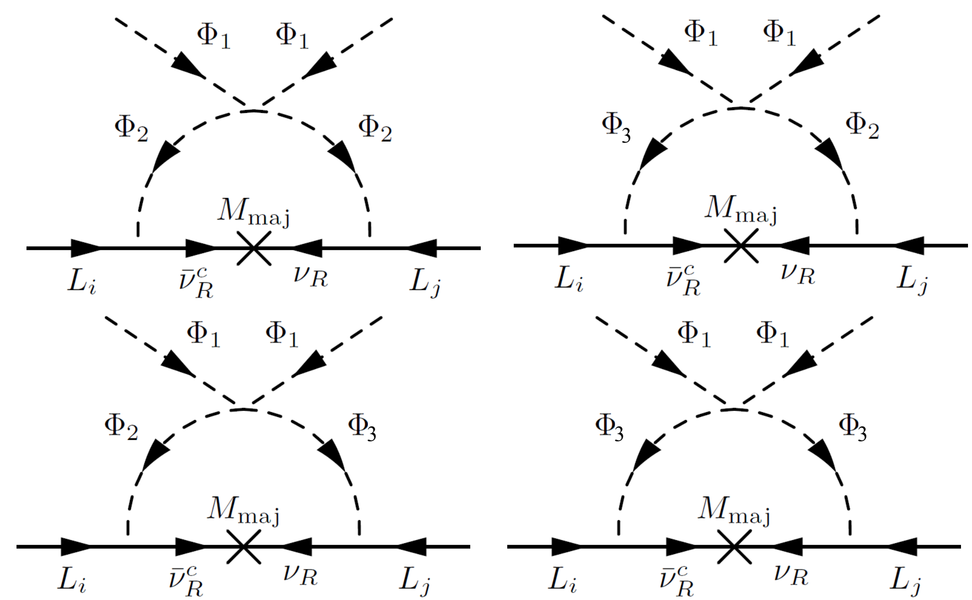

second column of U_{lep}
Even if each Yukawa coupling had an structure, the combination of them gives a “structureless” U_{i2}.

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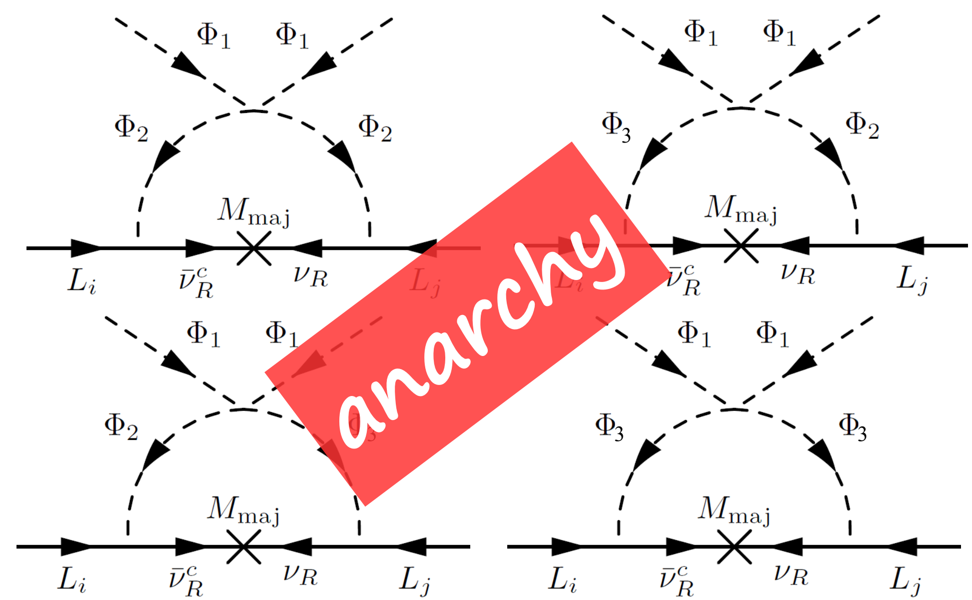

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Neutrino mass models with discrete symmetries

… and the connection to dark matter

Ma's model

Phys.Rev. D73 (2006) 077301

	L_1, L_2, L_3	e_{R1}, e_{R2}, e_{R3}	Φ	χ_1, χ_2, χ_3	η
spin	1/2	1/2	0	1/2	0
$SU(2)_L \times U(1)_Y$	(2,–1/2)	(1,1)	(2,1/2)	(1,0)	(2,1/2)
Z_2	+	+	+	–	–

If the Z_2 symmetry is exact (or very weakly broken) the model contains a dark matter candidate (η or χ_1).

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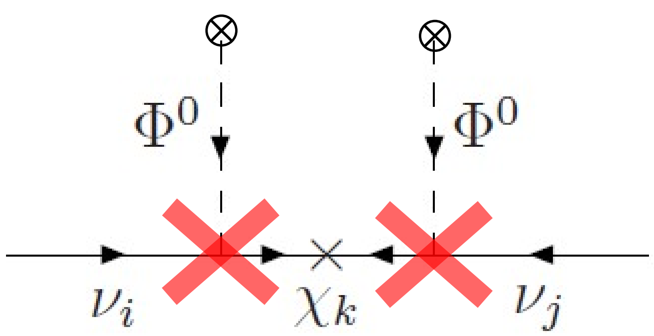
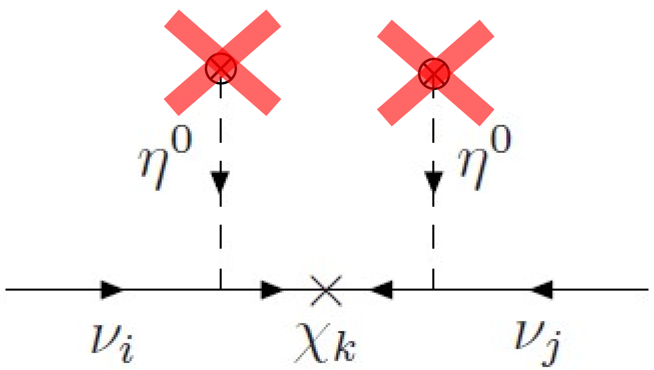
Due to the Z_2 symmetry, η does not acquire a vev → no neutrino mass at tree level.





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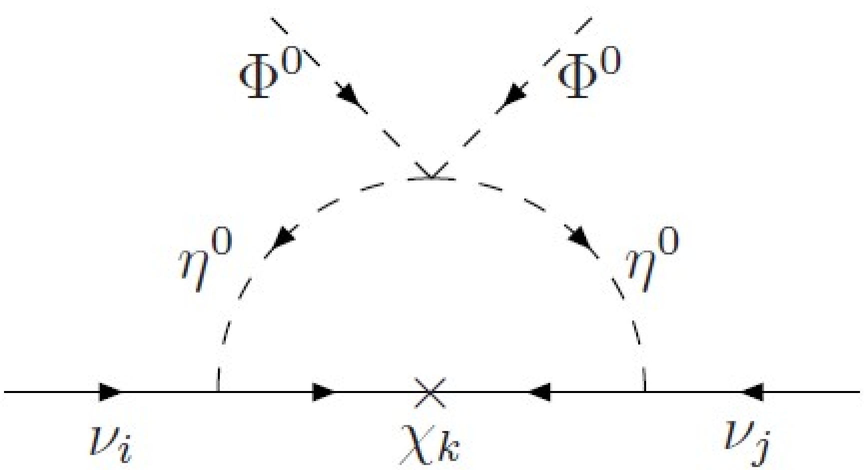
All neutrino masses are generated at the one loop level



For appropriate choices of the parameters, the masses of the new particles could be at the TeV scale
→ Collider signatures

Ma's model

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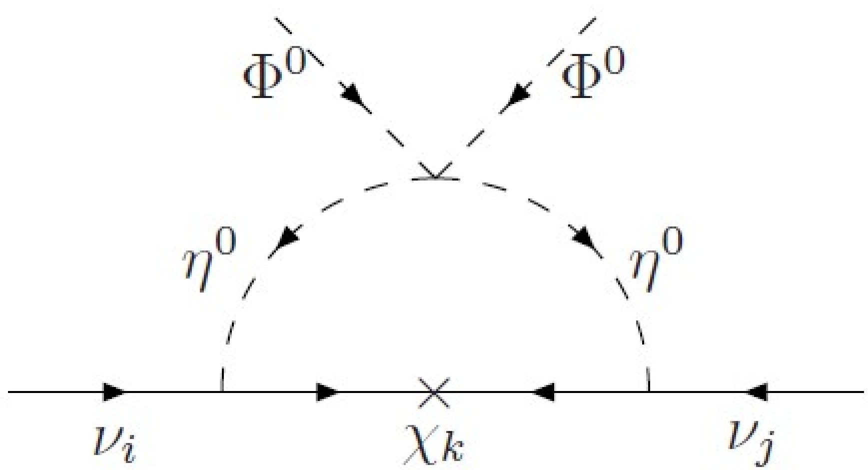


If $M_k^2 \gg m_0^2$, then

$$(\mathcal{M}_\nu)_{ij} = \frac{\lambda_5 v^2}{8\pi^2} \sum_k \frac{h_{ik} h_{jk}}{M_k} \left[\ln \frac{M_k^2}{m_0^2} - 1 \right].$$

However, the model generically predicts large neutrino mass hierarchies

Our modification

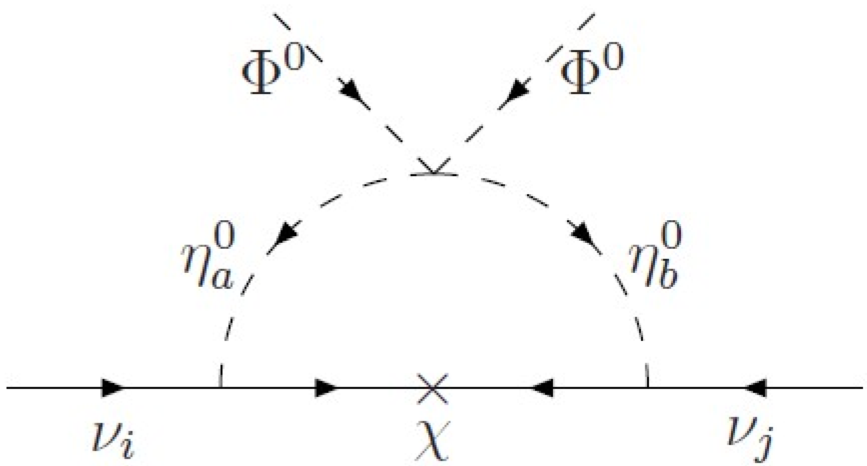
Hehn, AI, arXiv:1208.3162

	L_1, L_2, L_3	e_{R1}, e_{R2}, e_{R3}	Φ	χ	η_1,η_2
spin	1/2	1/2	0	1/2	0
$SU(2)_L \times U(1)_Y$	(2,–1/2)	(1,1)	(2,1/2)	(1,0)	(2,1/2)
Z_2	+	+	+	–	–

If the Z_2 symmetry is exact (or very weakly broken) the model contains a dark matter candidate (η_1 or χ).

Due to the Z_2 symmetry, η_1,η_2 do not acquire a vev → no neutrino mass at tree level.

All neutrino masses are generated at the one loop level

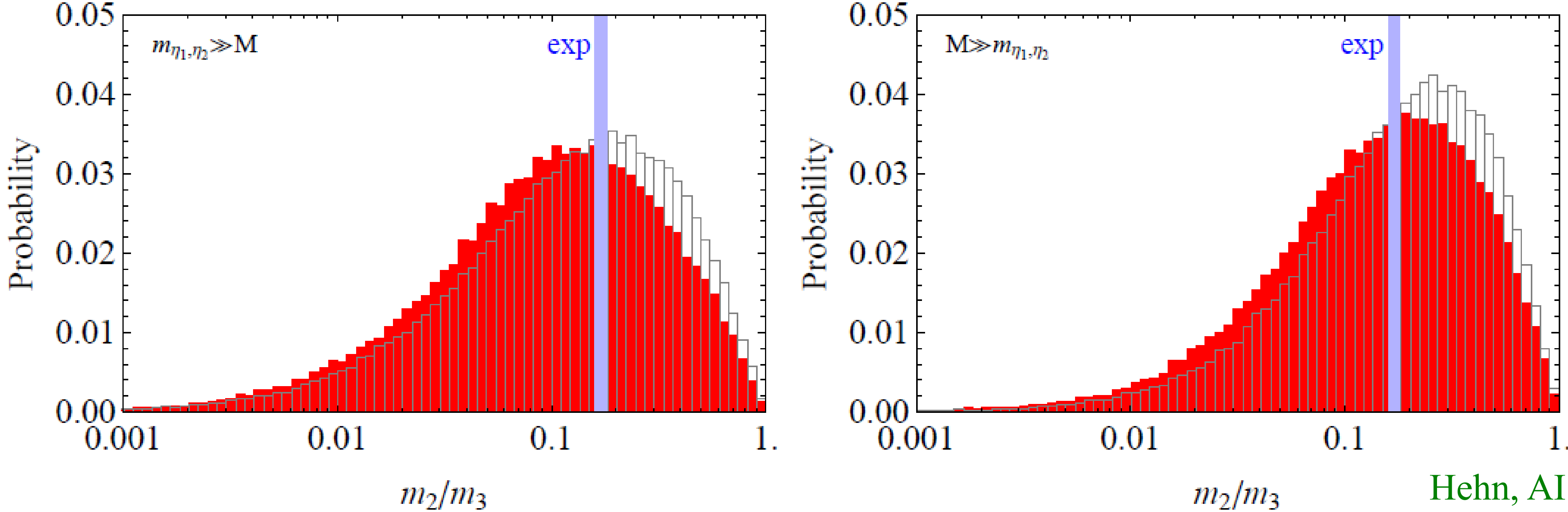


$$(\mathcal{M}_\nu)_{ij} \simeq -\frac{Y_i^{(a)} Y_j^{(b)} \lambda_5^{(ab)} v^2}{8\pi^2} \frac{1}{M} \times \left\{ \frac{m_{\eta_b}^2}{m_{\eta_a}^2 - m_{\eta_b}^2} \log \frac{m_{\eta_a}^2}{m_{\eta_b}^2} + \log \frac{m_{\eta_a}^2}{M^2} \right\}$$

$|Y^{(1)}| \sim |Y^{(2)}|$ (coupling to *the same* χ)

$\lambda_5^{(11)} \sim \lambda_5^{(12)} \sim \lambda_5^{(22)}$

Mild mass hierarchy generically expected



Features:

■ Small neutrino mass due to the see-saw mechanism.

■ Mild neutrino mass hierarchy due to the presence of a second scalar doublet.

■ No need to invoke a flavor symmetry to explain the intergenerational mass hierarchy in the neutrino sector, although it might be necessary to explain the pattern of mixing angles.

■ The model contains a dark matter candidate and can generate the observed matter-antimatter asymmetry in the Universe through leptogenesis.

Conclusions

	SM	SM + Heavy RH νs	SM + Heavy RH νs + scalar doublets	SM + Heavy RH νs + scalar doublets + Z_2
Flavour, CP, EWPD				
Tiny neutrino masses				
Mild ν mass hierarchy				
Neutrino Mixing angles				
Baryogenesis				
Dark matter				
Strong CP problem				
Hierarchy problem				
Cosmological constant problem				

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Thank you for your attention!