Maximal CP Violation in Lepton Mixing

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Maximal CP violation in lepton mixing from a model with $\Delta(27)$ flavour symmetry

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Outline of the talk:

1. Mechanism for maximal CP violation in lepton mixing
2. Model building: charged-lepton sector
3. Model building: neutrino sector—model I
4. Model building: neutrino sector—model II
5. Predictions for neutrino masses and lepton mixing
6. Conclusions

Mechanism for maximal CP violation

Notation:

$$\mathcal{L}_{\rm mass} = -\bar{\ell}_L M_\ell \ell_R + \frac{1}{2} \nu_L^T C^{-1} \mathcal{M}_\nu \nu_L + \text{H.c.}$$

$$\begin{aligned} U_\ell^\dagger M_\ell M_\ell^\dagger U_\ell &= \text{diag}\left(m_e^2, m_\mu^2, m_\tau^2\right) \\ U_\nu^T \mathcal{M}_\nu U_\nu &= \text{diag}\left(m_1, m_2, m_3\right) \end{aligned}$$

Mixing matrix $\boxed{U = U_\ell^\dagger U_\nu}$

Mechanism for maximal CP violation

Observation 1: Harrison, Scott (2002)

$$(*)\ |U_{\mu j}| = |U_{\tau j}| \quad \forall j = 1,2,3 \ \Rightarrow \begin{cases} \cos\theta_{23} = \sin\theta_{23} = \frac{1}{\sqrt{2}} \\ \sin\theta_{13}\cos\delta = 0 \end{cases}$$

$$\sin\theta_{13} \neq 0 \quad \Rightarrow \quad \boxed{\theta_{23} = 45^\circ, \quad \delta = \pm 90^\circ}$$

Compatible with global fits at 2σ (3σ)

Observation 2:

If $U_\ell = U_\omega$ and U_ν real then $U = U_\ell^\dagger U_\nu$ has property $(*)$

$$U_\omega \equiv \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \qquad \omega = \frac{-1 + i\sqrt{3}}{2}$$

Mechanism for maximal CP violation

Proof: $U_\nu = (R_{jk})$ with R_{jk} real $\Rightarrow$

$$U_{\mu j} = (U_\omega^\dagger R)_{\mu j} = \frac{1}{\sqrt{3}}\left(R_{1j} + \omega^2 R_{2j} + \omega R_{2j}\right)$$
$$U_{\tau j} = (U_\omega^\dagger R)_{\tau j} = \frac{1}{\sqrt{3}}\left(R_{1j} + \omega R_{2j} + \omega^2 R_{2j}\right)$$

Therefore, it follows that $U_{\tau j} = U_{\mu j}^*$.

Charged-lepton sector

$$E = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

$$\begin{aligned} [\phi \ell_R]_0 &= \phi_1 \ell_{1R} + \phi_2 \ell_{2R} + \phi_3 \ell_{3R}, \\ [\phi \ell_R]_1 &= \phi_1 \ell_{1R} + \omega \phi_2 \ell_{2R} + \omega^2 \phi_3 \ell_{3R}, \\ [\phi \ell_R]_2 &= \phi_1 \ell_{1R} + \omega^2 \phi_2 \ell_{2R} + \omega \phi_3 \ell_{3R}, \end{aligned}$$

$$\begin{aligned} \mathcal{S}: &\quad \ell_R \to A \ell_R, \ \phi \to A \phi \\ \mathcal{T}: &\quad \ell_R \to E \ell_R, \ \phi \to E \phi \end{aligned}$$

$\mathcal{S}$, $\mathcal{T}$ generate 3-dimensional irrep of A_4

Charged-lepton sector

Transformation property of bilinears: A_4

$$[\phi\ell_R]_j \xrightarrow{\mathcal{S}} [\phi\ell_R]_j$$
$$[\phi\ell_R]_j \xrightarrow{\mathcal{T}} \omega^{2j}\,[\phi\ell_R]_j$$

Yukawa Lagrangian of charged leptons:

$$\mathcal{L}_Y^{(\ell)} = -\sum_{j=1}^{3} h_j \bar{D}_{jL}\,[\phi\ell_R]_{j-1} + \text{H.c.}$$

$$\mathcal{T}: \quad D_L \to C^2 D_L$$

Charged-lepton Yukawa Lagrangian looks very similar to the one of some A_4-based models, however, roles of D_L and ℓ_R reversed!

Charged-lepton sector

Mass matrix of the charged leptons: VEVs $v_j = \langle \phi_j^0 \rangle_0$

$$M_\ell = \mathrm{diag}\,(h_1, h_2, h_3) \left(\sqrt{3} U_\omega \right) \mathrm{diag}\,(v_1, v_2, v_3)$$

$$\boxed{U_\omega^\dagger M_\ell \quad \text{diagonal if} \quad h_1 = h_2 = h_3}$$

Our model: equality of Yukawa couplings
Usual A_4 models: equality of VEVs

The equality of the h_j is achieved by assuming invariance of $\mathcal{L}_Y^{(\ell)}$ under

$$\mathcal{T}' : \quad D_L \to E D_L, \ \ell_R \to C \ell_R$$

Therefore, in the following we shall use

$$\boxed{h_1 = h_2 = h_3 \equiv h, \quad U_\omega^\dagger M_\ell = \sqrt{3}\, h\, \mathrm{diag}\,(v_1, v_2, v_3)}$$

Charged-lepton sector

No VEV alignment of Higgs doublets!

$$\begin{aligned} m_e &= \sqrt{3}\,|h v_1| \\ m_\mu &= \sqrt{3}\,|h v_2| \\ m_\tau &= \sqrt{3}\,|h v_3| \end{aligned}$$

Fine-tuning of VEVs:

$$|v_1| : |v_2| : |v_3| = m_e : m_\mu : m_\tau$$

Charged-lepton sector

Side remark:

Basis transformation $D_L = U_\omega D'_L \Rightarrow$

$$\mathcal{L}_Y^{(\ell)} = -\sum_{k=e,\mu,\tau} h\, \bar{D}'_{kL} \phi_k\, \ell_{kR} + \text{H.c.}$$

Yukawa couplings flavour-diagonal!

Neutrino sector—model I

symmetry	D_L	ℓ_R	ϕ	ν_R	η	ϕ_ν
$\mathcal{S}$	$\mathbb{1}$	A	A	$\mathbb{1}$	$\mathbb{1}$	1
$\mathcal{T}$	C^2	E	E	C^2	C^2	1
$\mathcal{T}'$	E	C	$\mathbb{1}$	E	E	1

Multiplets of model I and their transformation properties

Same field content as in A_4 model of He, Keum, Volkas (2006)

Yukawa Lagrangian of the neutrinos: Type I seesaw model

$$\mathcal{L}_Y^{(\nu)} =$$

$$-y_\nu \bar{D}_L \tilde{\phi}_\nu \nu_R + \frac{1}{2} y \sum_{j=1}^{3} \eta_j \nu_{jR}^T C^{-1} \nu_{jR}$$

$$+y' \left(\nu_{2R}^T C^{-1} \nu_{3R} \eta_1 + \nu_{3R}^T C^{-1} \nu_{1R} \eta_2 + \nu_{1R}^T C^{-1} \nu_{2R} \eta_3 \right) + \text{H.c.}$$

Neutrino sector—model I

Neutrino mass matrix: $\langle \eta_j \rangle_0 = s_j \Rightarrow M_R$ determined by VEVs s_j

$$\boxed{M_D \propto \mathbb{1}}$$

$\mathcal{M}_\nu^{-1}$ has typical form of $\Delta(27)$ models (Ma (2006)):

$$\boxed{\mathcal{M}_\nu^{-1} = \begin{pmatrix} \zeta a & c & b \\ c & \zeta b & a \\ b & a & \zeta c \end{pmatrix} \quad \text{with} \quad \zeta^* = y/y'}$$

Neutrino sector—model I

Symmetry group of the model:

Neutrino sector: $\Delta(27)$ generated by E, C

Charged-leptons: $A_4 + \mathcal{T}'$

Column with ℓ_R in table:

suggests AC to be a generator of the symmetry group, has sixth root of unity $-\omega^2$ in its diagonal

Might naively guess $\Delta(108) \equiv \Delta(3 \times 6^2)$ to be the symmetry group. However, beyond the symmetries listed in the table, the model possesses an accidental $2 \leftrightarrow 3$ interchange symmetry:

$$D_{2L} \leftrightarrow D_{3L}, \quad \ell_{2R} \leftrightarrow \ell_{3R}, \quad \phi_2 \leftrightarrow \phi_3, \quad \nu_{2R} \leftrightarrow \nu_{3R}, \quad \eta_2 \leftrightarrow \eta_3$$

Therefore, the full symmetry group is $\Delta(216) = \Delta(6 \times 6^2)$.

Neutrino sector—model I

Generalized CP: need real $\mathcal{M}_\nu$ for real U_ν

$$\mathrm{CP}: \quad \left\{ \begin{array}{l} D_L \to iCD_L^*,\ \ell_R \to iSC\ell_R^*,\ \nu_R \to iC\nu_R^*, \\ \phi \to S\phi^*,\ \phi_\nu \to \phi_\nu^*,\ \eta \to \eta^*, \end{array} \right.$$

$$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Renders Yukawa couplings, in particular $\zeta^* = y/y'$, real!

Neutrino sector—model I

VEVs of the scalar gauge singlets: $\langle \eta_j \rangle_0 = s_j$

$$V_\eta = \sum_{j=1}^{3} \left(\mu \left| \eta_j \right|^2 + \lambda_1 \left| \eta_j \right|^4 \right) + \lambda_2 \left(\left| \eta_1 \eta_2 \right|^2 + \left| \eta_1 \eta_3 \right|^2 + \left| \eta_2 \eta_3 \right|^2 \right)$$
$$+ M_1 \left(\eta_1 \eta_2 \eta_3 + \text{H.c.} \right) + M_2 \left(\eta_1^3 + \eta_2^3 + \eta_3^3 + \text{H.c.} \right)$$
$$+ \lambda_3 \left({\eta_1^*}^2 \eta_2 \eta_3 + {\eta_2^*}^2 \eta_1 \eta_3 + {\eta_3^*}^2 \eta_1 \eta_2 + \text{H.c.} \right)$$

Need reals VEVs s_j!

At minimum of V: $\arg s_j = \omega^{p_j}$ with p_j integer,

$p_1 + p_2 + p_3 = 0 \bmod 3$

Model II: type II seesaw

Scalar gauge triplets: $\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$

$$\mathcal{L}_Y^{(\Delta)} = \frac{1}{2}\,\tilde{y}\sum_{j=1}^{3} D_{jL}^T C^{-1}\varepsilon\Delta_j D_{jL}$$
$$+\tilde{y}'\left(D_{2L}^T C^{-1}\varepsilon\Delta_1 D_{3L} + D_{3L}^T C^{-1}\varepsilon\Delta_2 D_{1L} + D_{1L}^T C^{-1}\varepsilon\Delta_3 D_{2L}\right) + \text{H.c.}$$

Neutrino mass matrix: $\langle\Delta_j^0\rangle_0 = \delta_j$

$$\mathcal{M}_\nu = \begin{pmatrix} \tilde{y}\delta_1 & \tilde{y}'\delta_3 & \tilde{y}'\delta_2 \\ \tilde{y}'\delta_3 & \tilde{y}\delta_2 & \tilde{y}'\delta_1 \\ \tilde{y}'\delta_2 & \tilde{y}'\delta_1 & \tilde{y}\delta_3 \end{pmatrix} = \begin{pmatrix} \zeta a & c & b \\ c & \zeta b & a \\ b & a & \zeta c \end{pmatrix}$$ Ma (2006)

Predictions for ν masses and lepton mixing

For definiteness, discussion of model I:

$$\text{Weak basis:} \quad \mathcal{M}_\nu^{(w)} = U_\ell^T \mathcal{M}_\nu U_\ell \quad \text{with} \quad U_\ell = U_\omega$$

$$\mathcal{M}_\nu^{(w)\,-1} = U_\omega^\dagger \mathcal{M}_\nu^{-1} U_\omega^* = \begin{pmatrix} \bar\zeta \bar a & \bar c & \bar b \\ \bar c & \bar\zeta \bar b & \bar a \\ \bar b & \bar a & \bar\zeta \bar c \end{pmatrix}$$

$$\begin{aligned} 3\bar a &= (\zeta - 1)(a + b + c) \\ 3\bar b &= (\zeta - 1)\left(a + \omega^2 b + \omega c\right) \\ 3\bar c &= (\zeta - 1)\left(a + \omega b + \omega^2 c\right) \\ \bar\zeta &= \frac{\zeta + 2}{\zeta - 1} \end{aligned}$$

$$a,\ b,\ c,\ \zeta \text{ real} \Rightarrow \bar a,\ \bar\zeta \text{ real},\ \bar c = \bar b^*$$

Same mechanism in Mohapatra, Nishi (2012)

Predictions for ν masses and lepton mixing

Theorem: Grimus, Lavoura (2003)

$$\mathcal{M}_\nu^{(w)\,-1} = \begin{pmatrix} x & y & y^* \\ y & z & w \\ y^* & w & z^* \end{pmatrix} \quad \text{with } x \text{ and } w \text{ real}$$

Then $\mathcal{M}_\nu^{(w)\,-1}$ is diagonalized by

$$U' = \begin{pmatrix} u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \\ w_1^* & w_2^* & w_3^* \end{pmatrix} \quad \text{with} \quad u_j \in \mathbb{R}$$

and

$$U'^\dagger \mathcal{M}_\nu^{(w)\,-1} U'^* = \mathrm{diag}\left(\frac{1}{\mu_1}, \frac{1}{\mu_2}, \frac{1}{\mu_3}\right) \quad \text{with } \mu_j = \epsilon_j m_j,\ \epsilon_j = \pm 1$$

Consequences of the theorem:

1. $|U_{\mu j}| = |U_{\tau j}| \; \forall j = 1,2,3$
2. The ϵ_j correspond to the Majorana phase factors

Effective mass in $(\beta\beta)_{0\nu}$ decay:

$$m_{\beta\beta} = \left| \sum_{j=1}^{3} u_j^2 \mu_j \right| = \left| \sum_{j=1}^{3} u_j^2 \epsilon_j m_j \right|$$

Predictions for ν masses and lepton mixing

Counting of parameters and predictions:

4 real parameters in ${\mathcal{M}_\nu^{(w)}}^{-1}$

9 physical quantities: three masses, mixing angles, phases $\Rightarrow$

$9 - 4 = 5$ predictions:

$s_{23}^2 = 1/2$, $\delta = \pm\pi/2$, Majorana phases 0 or π

Extra relation from

$$\left({\mathcal{M}_\nu^{(w)}}^{-1}\right)_{11}\left({\mathcal{M}_\nu^{(w)}}^{-1}\right)_{13} = \bar{\zeta}\bar{a}\bar{b} = \left({\mathcal{M}_\nu^{(w)}}^{-1}\right)_{22}\left({\mathcal{M}_\nu^{(w)}}^{-1}\right)_{23}$$

Only equality of moduli of both sides physically meaningful!

Predictions for ν masses and lepton mixing

Abbreviation: $U_j \equiv u_j^2 = |U_{ej}|^2$

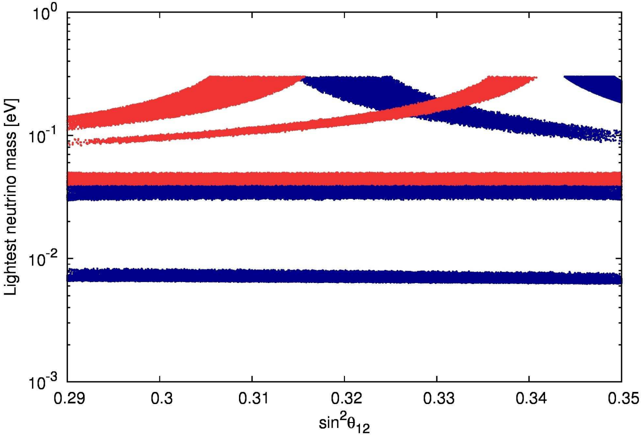
Extra relation

$$\left(\sum_j \frac{U_j^2}{m_j^2} + \sum_{j<j'} \frac{2U_j U_{j'}}{\mu_j \mu_{j'}} \right) \left[\sum_j \frac{U_j (1 - U_j)}{2m_j^2} - \sum_{j<j'} \frac{U_j U_{j'}}{\mu_j \mu_{j'}} \right]$$

$$= \left[\sum_j \frac{(1 - U_j)^2}{4m_j^2} + \sum_{j<j'} \frac{-1 + U_j + U_{j'} + U_j U_{j'}}{2\mu_j \mu_{j'}} \right]$$

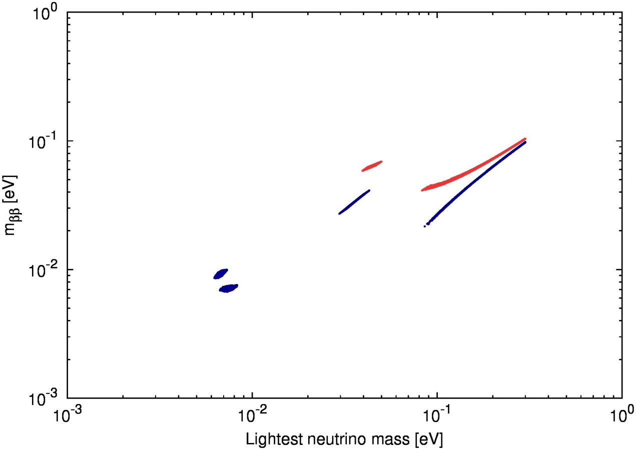
$$\times \left[\sum_j \frac{(1 - U_j)^2}{4m_j^2} + \sum_{j<j'} \frac{1 - U_j - U_{j'} + U_j U_{j'}}{2\mu_j \mu_{j'}} \right]$$

$$U_3 = \sin^2\theta_{13}, \quad U_2 = \cos^2\theta_{13}\sin^2\theta_{12}, \quad U_1 = \cos^2\theta_{13}\cos^2\theta_{12}$$



Model I: Lightest neutrino mass as a function of $\sin^2\theta_{12}$
blue: = normal, red = inverted

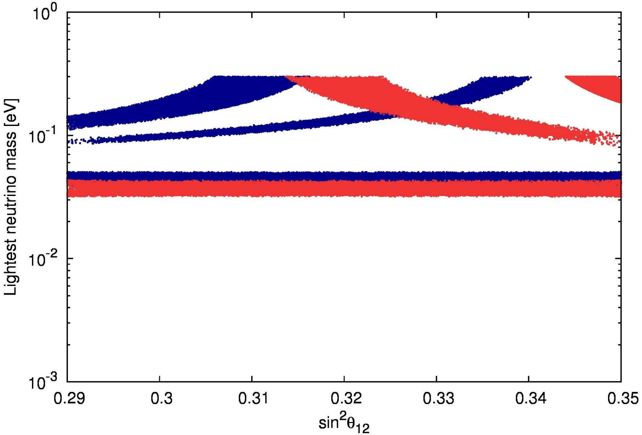
Predictions for ν masses and lepton mixing



Model I: $m_{\beta\beta}$ versus lightest neutrino mass

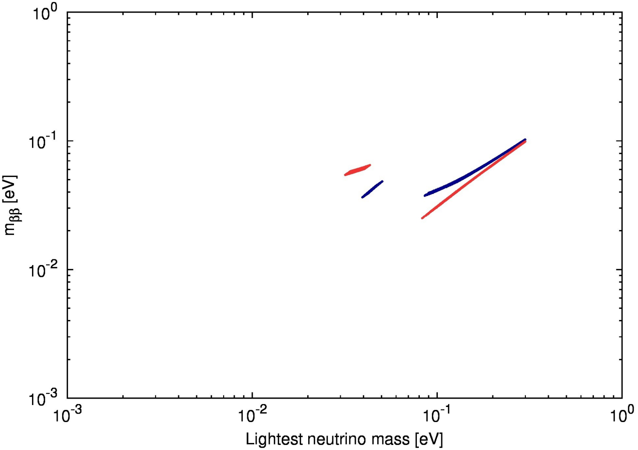
blue = normal, red = inverted

Predictions for ν masses and lepton mixing



Model II: Lightest neutrino mass as a function of $\sin^2\theta_{12}$
blue = normal, red = inverted

Predictions for ν masses and lepton mixing



Model II: $m_{\beta\beta}$ versus lightest neutrino mass

blue = normal, red = inverted

Conclusions

1. Observation: $U_\ell = U_\omega$, U_ν real $\Rightarrow$
   $|U_{\mu j}| = |U_{\tau j}| \quad \forall j = 1, 2, 3$
2. Predictions: $\delta = \pm 90^\circ$, $\theta_{23} = 45^\circ$, Majorana phases 0 or π
3. Models: Seesaw type I or type II
4. Minimal VEV alignment: real VEVs of gauge singlet scalars
5. No FCNI
6. Neutrino sector has $\Delta(27)$ symmetry
7. One additional prediction from that symmetry:
   Lightest ν mass as function of θ_{12}, θ_{13}, Δm^2_{21}, Δm^2_{31}