the 3+3 Neutrino Mixing Scheme & Electromagnetic Dipole Moments

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- ♣ Motivation for sterile neutrino physics
- ♣ A description of the 6×6 mixing matrix
- ♣ CP violation from non-unitarity effects
- ♣ Seesaw-induced enhancement of EDM

Xing, 1110.0083 (PRD); Xing, Zhou, 1201.2543 (PLB)

@ Behind the Neutrino Mass, ICTP, 17-21/9/2012

What's Behind ν Mass?

Flavor Symmetry

Texture zeros

Element correlations

GUT relations

They reduce the number of free parameters, and thus lead to predictions for 3 flavor mixing angles in terms of either the mass ratios or constant numbers.

Example (Fritzsch ansatz)

$$M_{l,\nu} = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$$

Dependent on mass ratios

Example (Discrete symmetries)

$$M_{\nu} = \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix}$$

Dependent on simple numbers

PREDICTIONS

Way 1: Constant + Perturbation 2

1st generation:

Cabibbo (78)

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{pmatrix}$$

Wolfenstein (78)

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

2nd generation:

Democratic (96)

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

Bimaximal (97/98)

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{-1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

shift

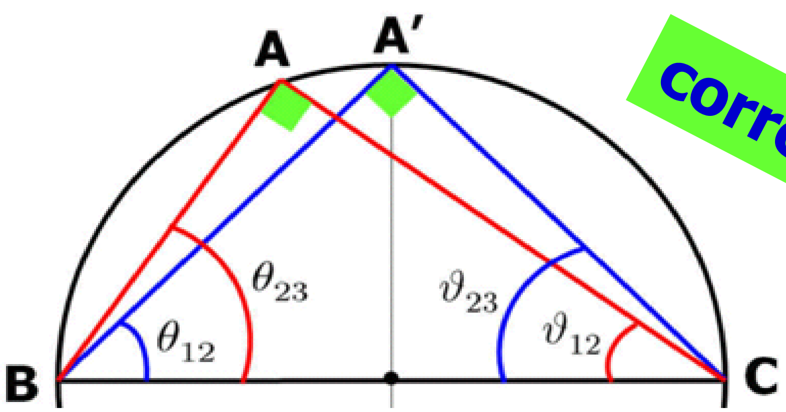
3rd generation:

Tri-bimaximal (02)

$$\begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Golden-ratio (07)

$$\begin{pmatrix} \frac{\sqrt{2}}{\sqrt{5-\sqrt{5}}} & \frac{\sqrt{2}}{\sqrt{5+\sqrt{5}}} & 0 \\ \frac{-1}{\sqrt{5+\sqrt{5}}} & \frac{1}{\sqrt{5-\sqrt{5}}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{5+\sqrt{5}}} & \frac{-1}{\sqrt{5-\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$



correlation

$$\theta_{12} = \pi/4 \qquad \vartheta_{12} = \pi/4 - \theta_*$$
$$\theta_{23} = \pi/4 + \theta_* \qquad \vartheta_{23} = \pi/4$$

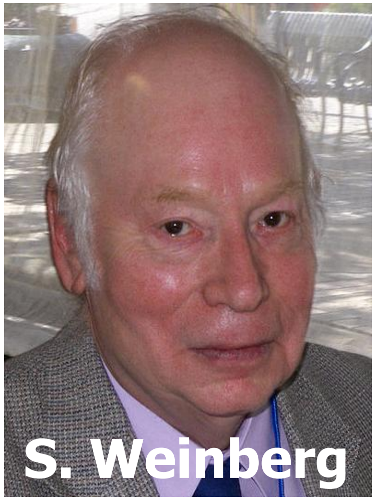
Democratic | **Tri-bimaximal**

Xing, arXiv:1011.2954

$$\boxed{\theta_{13} = \theta_* = \theta_{23} - \theta_{12} \simeq 9.7^\circ}$$

Way 2: Texture Zeros

The flavor mixing angles are simple functions of 4 lepton mass ratios.



1977

2×2 3×3

Texture zeros

1977,78

$$\theta_{ij} = f\left(\frac{m_\alpha}{m_\beta}, \frac{m_k}{m_l}, \cdots\right)$$

$$M_{l,\nu} = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$$

Texture zeros of a fermion mass matrix dynamically mean that some matrix elements are strongly suppressed (in comparison with those weakly suppressed or unsuppressed elements) and may stem from a flavor symmetry (e.g., the Froggatt-Nielsen mechanism 1979)

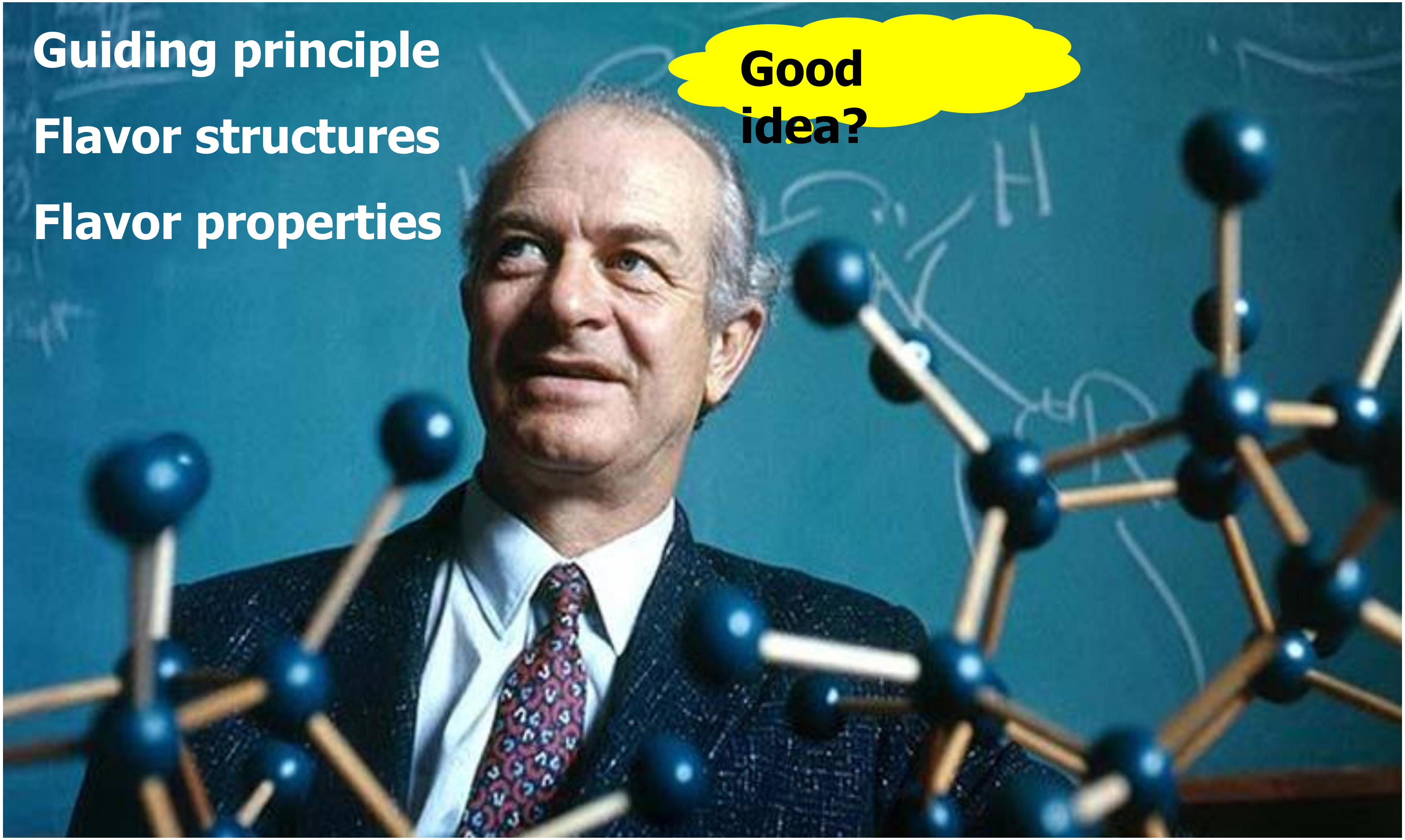
The charged-lepton sector

$$\sqrt{\frac{m_e}{m_\mu}} \simeq 0.069 \Leftrightarrow 4^\circ , \qquad \sqrt{\frac{m_\mu}{m_\tau}} \simeq 0.24 \Leftrightarrow 14^\circ$$

$$\theta_{12} \sim 34^\circ, \ \theta_{23} \sim 40^\circ, \ \theta_{13} \sim 9^\circ$$

So the neutrino sector plays a primary role. e.g. the Fritzsch texture works (Xing 2002)

Lessons from Chemistry

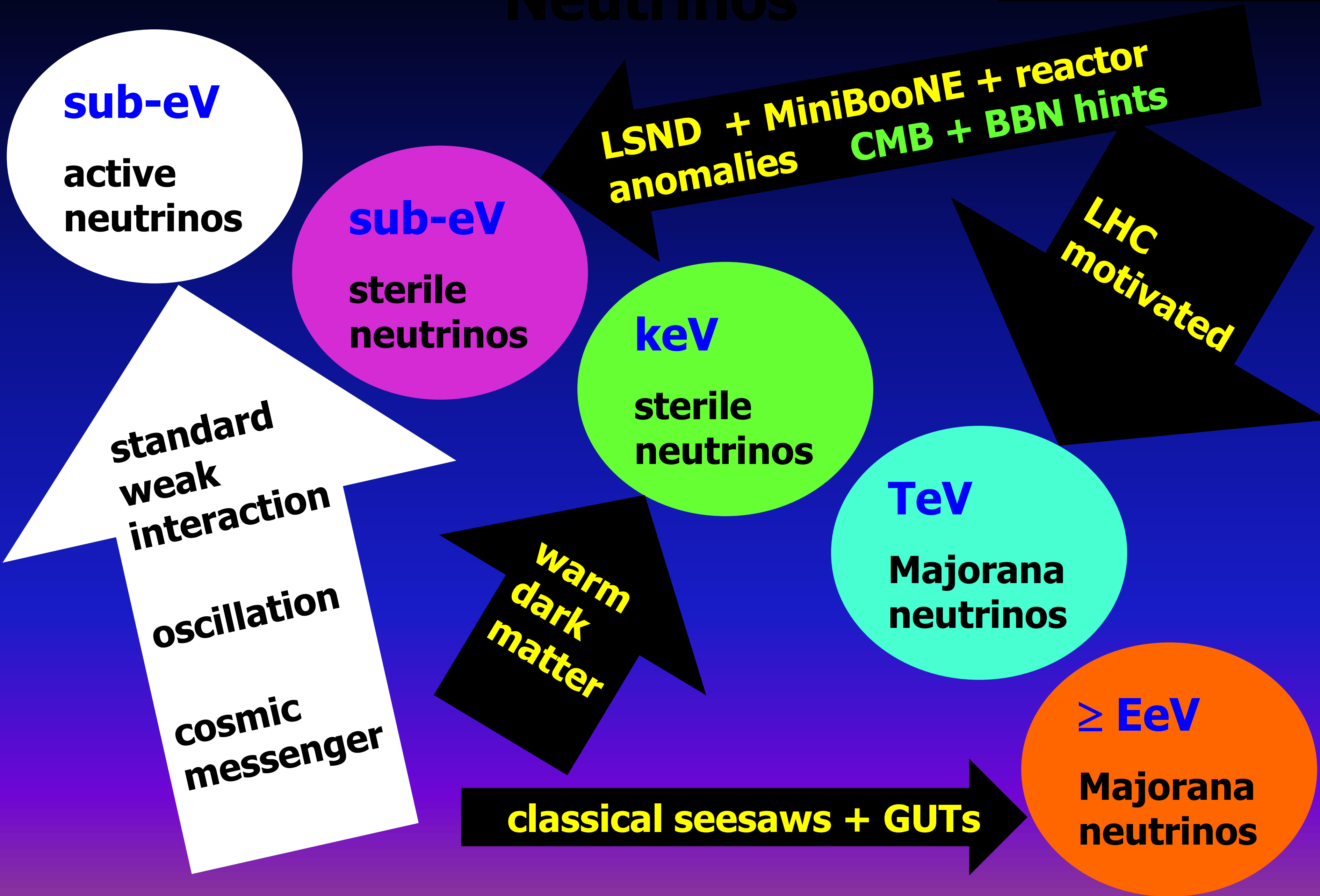


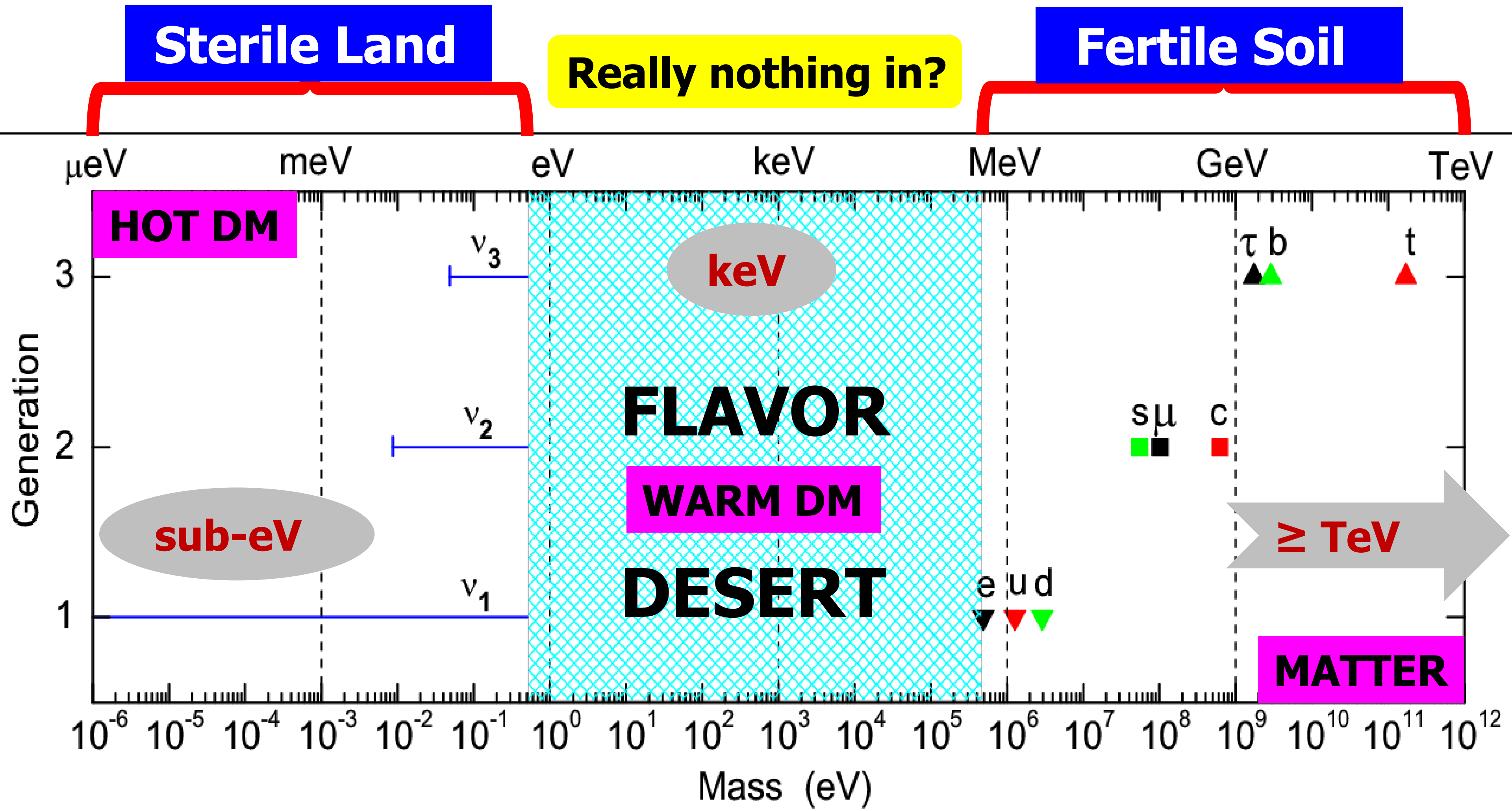
Linus Pauling: The best way …. But so far we have had ….

Sterile Neutrino Physics

Neutrinos

The white paper
arXiv:1204.5379





Weinberg's ***3rd law of progress in theoretical physics*** **(83):**

You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry .... What could be better?

How Many?

Observational hints (really true? harmless in cosmology?)

Theoretical motivation (seesaw + leptogenesis scenarios)

3 + 3

Or just for fun? "3" is a strange and interesting number!

Then Nontrivial...

The generic (3+3) flavor mixing scheme can be simplified to the (3+2) or (3+1) flavor mixing scheme, but

$$\boxed{\begin{array}{l} 3+2 \neq 3+3-1 \\ \\ 3+1 \neq 3+2-1 \neq 3+3-2 \end{array}}$$

by forgetting the heavy neutrino(s) when discussing light neutrino oscillation phenomenology. The reason is

---- the heavy sterile neutrino(s) can result in the indirect

---- the light sterile neutrino(s) will give rise to the direct

violation of unitarity of the standard 3×3 active neutrino mixing matrix! They are distinguishable in ν-oscillations.

Flavor Mixing

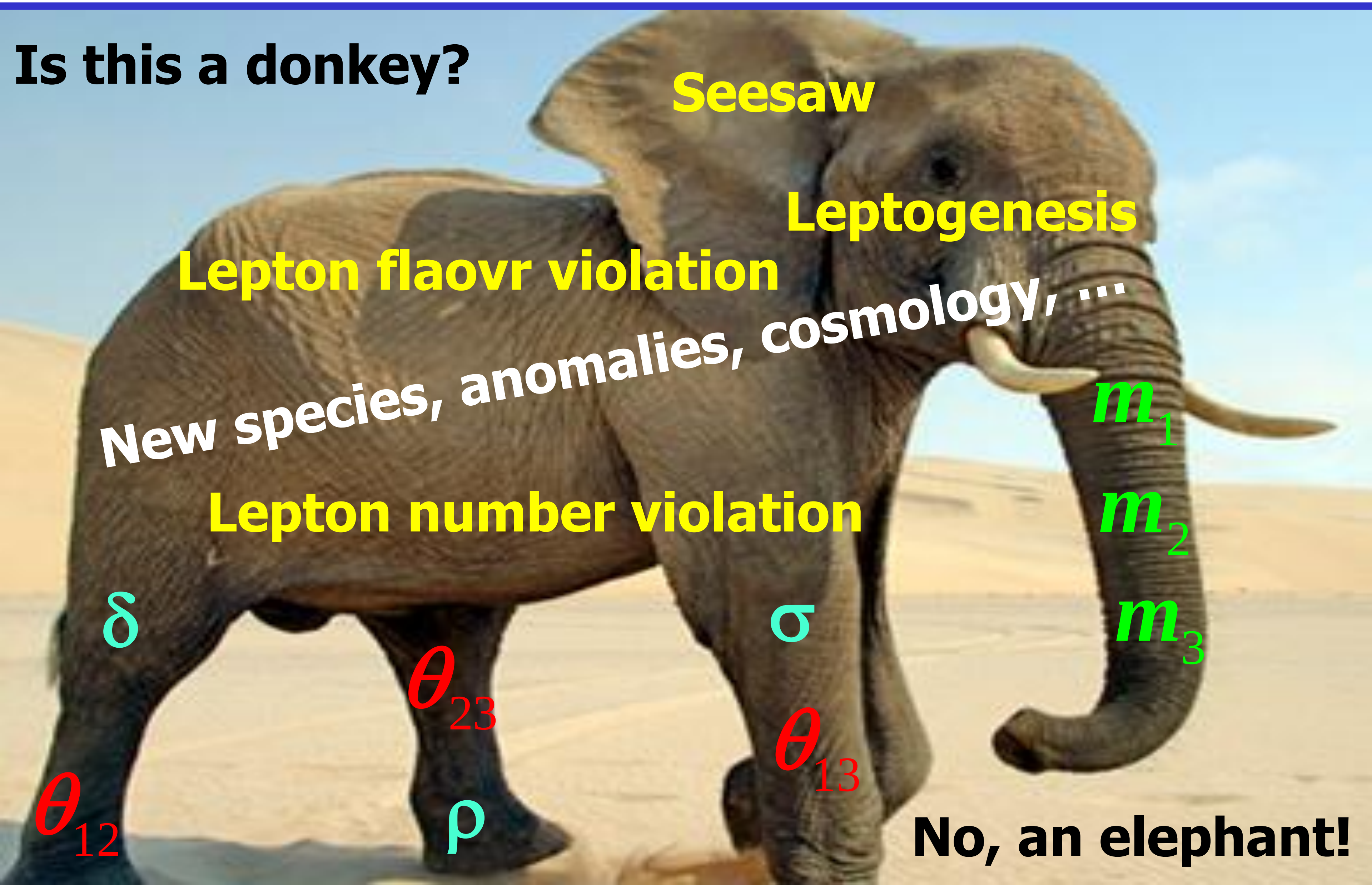
active flavor

sterile flavor

$$\begin{pmatrix} \boxed{\begin{matrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{matrix}} \\ \boxed{\begin{matrix} \nu_x \\ \nu_y \\ \nu_z \end{matrix}} \end{pmatrix} = \mathcal{U} \begin{pmatrix} \boxed{\begin{matrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \end{matrix}} \end{pmatrix}$$

mass state

Why a Full Parametrization?



A Possible Way

$$\mathcal{U} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

sterile part **interplay** **active part**

$$\begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} = O_{23} O_{13} O_{12} \; ,$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} = O_{56} O_{46} O_{45} \; ,$$

$$\begin{pmatrix} A & R \\ S & B \end{pmatrix} = O_{36} O_{26} O_{16} O_{35} O_{25} O_{15} O_{34} O_{24} O_{14}$$

Full parametrization:

15 rotation angles
15 phase phases
(Xing, 1110.0083)

$$O_{12} = \begin{pmatrix} c_{12} & \hat{s}_{12}^* & 0 & 0 & 0 & 0 \\ -\hat{s}_{12} & c_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{34} & \hat{s}_{34}^* & 0 & 0 \\ 0 & 0 & -\hat{s}_{34} & c_{34} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{16} = \begin{pmatrix} c_{16} & 0 & 0 & 0 & 0 & \hat{s}_{16}^* \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\hat{s}_{16} & 0 & 0 & 0 & 0 & c_{16} \end{pmatrix}$$

$$O_{13} = \begin{pmatrix} c_{13} & 0 & \hat{s}_{13}^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\hat{s}_{13} & 0 & c_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{15} = \begin{pmatrix} c_{15} & 0 & 0 & 0 & \hat{s}_{15}^* & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\hat{s}_{15} & 0 & 0 & 0 & c_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{26} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{26} & 0 & 0 & 0 & \hat{s}_{26}^* \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\hat{s}_{26} & 0 & 0 & 0 & c_{26} \end{pmatrix}$$

$$O_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{23} & \hat{s}_{23}^* & 0 & 0 & 0 \\ 0 & -\hat{s}_{23} & c_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{25} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{25} & 0 & 0 & \hat{s}_{25}^* & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -\hat{s}_{25} & 0 & 0 & c_{25} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{36} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{36} & 0 & 0 & \hat{s}_{36}^* \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\hat{s}_{36} & 0 & 0 & c_{36} \end{pmatrix}$$

$$O_{14} = \begin{pmatrix} c_{14} & 0 & 0 & \hat{s}_{14}^* & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\hat{s}_{14} & 0 & 0 & c_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{35} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{35} & 0 & \hat{s}_{35}^* & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\hat{s}_{35} & 0 & c_{35} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{46} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{46} & 0 & \hat{s}_{46}^* \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\hat{s}_{46} & 0 & c_{46} \end{pmatrix}$$

$$O_{24} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & \hat{s}_{24}^* & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\hat{s}_{24} & 0 & c_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{45} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{45} & \hat{s}_{45}^* & 0 \\ 0 & 0 & 0 & -\hat{s}_{45} & c_{45} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad O_{56} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{56} & \hat{s}_{56}^* \\ 0 & 0 & 0 & 0 & -\hat{s}_{56} & c_{56} \end{pmatrix}$$

Unitarity

active part

$$V_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}_{12}^* c_{13} & \hat{s}_{13}^* \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}_{23}^* & c_{12}c_{23} - \hat{s}_{12}^*\hat{s}_{13}\hat{s}_{23}^* & c_{13}\hat{s}_{23}^* \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}_{12}^*\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

sterile part

$$U_0 = \begin{pmatrix} c_{45}c_{46} & \hat{s}_{45}^* c_{46} & \hat{s}_{46}^* \\ -\hat{s}_{45}c_{56} - c_{45}\hat{s}_{46}\hat{s}_{56}^* & c_{45}c_{56} - \hat{s}_{45}^*\hat{s}_{46}\hat{s}_{56}^* & c_{46}\hat{s}_{56}^* \\ \hat{s}_{45}\hat{s}_{56} - c_{45}\hat{s}_{46}c_{56} & -c_{45}\hat{s}_{56} - \hat{s}_{45}^*\hat{s}_{46}c_{56} & c_{46}c_{56} \end{pmatrix}$$

unitarity:

$$\begin{aligned} AA^\dagger + RR^\dagger &= BB^\dagger + SS^\dagger = \mathbf{1} \\ AS^\dagger + RB^\dagger &= A^\dagger R + S^\dagger B = \mathbf{0} \\ A^\dagger A + S^\dagger S &= B^\dagger B + R^\dagger R = \mathbf{1} \end{aligned}$$

$$\begin{aligned} VV^\dagger &= AA^\dagger = \mathbf{1} - RR^\dagger \\ U^\dagger U &= B^\dagger B = \mathbf{1} - R^\dagger R \end{aligned}$$

Relations between flavor and mass eigenstates:

$$V \equiv AV_0 \qquad U \equiv U_0 B \qquad \widehat{S} \equiv U_0 S V_0$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} + R \begin{pmatrix} \nu_4 \\ \nu_5 \\ \nu_6 \end{pmatrix} \Longleftrightarrow \begin{pmatrix} \nu_x \\ \nu_y \\ \nu_z \end{pmatrix} = U \begin{pmatrix} \nu_4 \\ \nu_5 \\ \nu_6 \end{pmatrix} + \widehat{S} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

A Blue-Collar Job (1)

Exact results in a triangular pattern (9 angles + 9 phases)
They describe small departures of V_0 and U_0 from unitarity.

$$A = \begin{pmatrix} c_{14}c_{15}c_{16} & 0 & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}\hat{s}^*_{26} - c_{14}\hat{s}_{15}\hat{s}^*_{25}c_{26} \\ -\hat{s}_{14}\hat{s}^*_{24}c_{25}c_{26} \end{array} & c_{24}c_{25}c_{26} & 0 \\ \begin{array}{l} -c_{14}c_{15}\hat{s}_{16}c_{26}\hat{s}^*_{36} + c_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}\hat{s}^*_{36} \\ -c_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}c_{36} + \hat{s}_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} \\ +\hat{s}_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} - \hat{s}_{14}c_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & \begin{array}{l} -c_{24}c_{25}\hat{s}_{26}\hat{s}^*_{36} - c_{24}\hat{s}_{25}\hat{s}^*_{35}c_{36} \\ -\hat{s}_{24}\hat{s}^*_{34}c_{35}c_{36} \end{array} & c_{34}c_{35}c_{36} \end{pmatrix}$$

$$B = \begin{pmatrix} c_{14}c_{24}c_{34} & 0 & 0 \\ \begin{array}{l} -c_{14}c_{24}\hat{s}^*_{34}\hat{s}_{35} - c_{14}\hat{s}^*_{24}\hat{s}_{25}c_{35} \\ -\hat{s}^*_{14}\hat{s}_{15}c_{25}c_{35} \end{array} & c_{15}c_{25}c_{35} & 0 \\ \begin{array}{l} -c_{14}c_{24}\hat{s}^*_{34}c_{35}\hat{s}_{36} + c_{14}\hat{s}^*_{24}\hat{s}_{25}\hat{s}^*_{35}\hat{s}_{36} \\ -c_{14}\hat{s}^*_{24}c_{25}\hat{s}_{26}c_{36} + \hat{s}^*_{14}\hat{s}_{15}c_{25}\hat{s}^*_{35}\hat{s}_{36} \\ +\hat{s}^*_{14}\hat{s}_{15}\hat{s}^*_{25}\hat{s}_{26}c_{36} - \hat{s}^*_{14}c_{15}\hat{s}_{16}c_{26}c_{36} \end{array} & \begin{array}{l} -c_{15}c_{25}\hat{s}^*_{35}\hat{s}_{36} - c_{15}\hat{s}^*_{25}\hat{s}_{26}c_{36} \\ -\hat{s}^*_{15}\hat{s}_{16}c_{26}c_{36} \end{array} & c_{16}c_{26}c_{36} \end{pmatrix}$$

A Blue-Collar Job (2)

Exact results which also depend on 9 angles and 9 phases.
They describe the interplay between active and sterile ν's.

$$R = \begin{pmatrix} \hat{s}^*_{14} c_{15} c_{16} & \hat{s}^*_{15} c_{16} & \hat{s}^*_{16} \\ \begin{array}{l} -\hat{s}^*_{14} c_{15} \hat{s}_{16} \hat{s}^*_{26} - \hat{s}^*_{14} \hat{s}_{15} \hat{s}^*_{25} c_{26} \\ +c_{14} \hat{s}^*_{24} c_{25} c_{26} \end{array} & -\hat{s}^*_{15} \hat{s}_{16} \hat{s}^*_{26} + c_{15} \hat{s}^*_{25} c_{26} & c_{16} \hat{s}^*_{26} \\ \begin{array}{l} -\hat{s}^*_{14} c_{15} \hat{s}_{16} c_{26} \hat{s}^*_{36} + \hat{s}^*_{14} \hat{s}_{15} \hat{s}^*_{25} \hat{s}_{26} \hat{s}^*_{36} \\ -\hat{s}^*_{14} \hat{s}_{15} c_{25} \hat{s}^*_{35} c_{36} - c_{14} \hat{s}^*_{24} c_{25} \hat{s}_{26} \hat{s}^*_{36} \\ -c_{14} \hat{s}^*_{24} \hat{s}_{25} \hat{s}^*_{35} c_{36} + c_{14} c_{24} \hat{s}^*_{34} c_{35} c_{36} \end{array} & \begin{array}{l} -\hat{s}^*_{15} \hat{s}_{16} c_{26} \hat{s}^*_{36} - c_{15} \hat{s}^*_{25} \hat{s}_{26} \hat{s}^*_{36} \\ +c_{15} c_{25} \hat{s}^*_{35} c_{36} \end{array} & c_{16} c_{26} \hat{s}^*_{36} \end{pmatrix}$$

$$S = \begin{pmatrix} -\hat{s}_{14} c_{24} c_{34} & -\hat{s}_{24} c_{34} & -\hat{s}_{34} \\ \begin{array}{l} \hat{s}_{14} c_{24} \hat{s}^*_{34} \hat{s}_{35} + \hat{s}_{14} \hat{s}^*_{24} \hat{s}_{25} c_{35} \\ -c_{14} \hat{s}_{15} c_{25} c_{35} \end{array} & \hat{s}_{24} \hat{s}^*_{34} \hat{s}_{35} - c_{24} \hat{s}_{25} c_{35} & -c_{34} \hat{s}_{35} \\ \begin{array}{l} \hat{s}_{14} c_{24} \hat{s}^*_{34} c_{35} \hat{s}_{36} - \hat{s}_{14} \hat{s}^*_{24} \hat{s}_{25} \hat{s}^*_{35} \hat{s}_{36} \\ +\hat{s}_{14} \hat{s}^*_{24} c_{25} \hat{s}_{26} c_{36} + c_{14} \hat{s}_{15} c_{25} \hat{s}^*_{35} \hat{s}_{36} \\ +c_{14} \hat{s}_{15} \hat{s}^*_{25} \hat{s}_{26} c_{36} - c_{14} c_{15} \hat{s}_{16} c_{26} c_{36} \end{array} & \begin{array}{l} \hat{s}_{24} \hat{s}^*_{34} c_{35} \hat{s}_{36} + c_{24} \hat{s}_{25} \hat{s}^*_{35} \hat{s}_{36} \\ -c_{24} c_{25} \hat{s}_{26} c_{36} \end{array} & -c_{34} c_{35} \hat{s}_{36} \end{pmatrix}$$

Non-unitarity ν Mixing?

Example A: light sterile neutrinos ---- some preliminary hints with ??

Example B: heavy Majorana neutrinos ---- popular **seesaw** scenarios.

Example C: whole tower of KK states ---- in extra dimension models.

The scheme of **Minimal Unitarity Violation** (Antusch *et al* 06):
---- Only 3 light neutrino species are considered;
---- Sources of non-unitarity are allowed only in those terms of the SM Lagrangian which involve neutrinos.

Unitarity of the neutrino mixing matrix: good or bad at the 1% level.

Constraint on the 3×3 ν-mixing matrix V ---- data on ν-oscillations, W and Z decays, the LFV modes and leptonic universality tests (Antusch *et al* 06).

$$|VV^\dagger| \approx \begin{pmatrix} 0.994 \pm 0.005 & < 7.0 \cdot 10^{-5} & < 1.6 \cdot 10^{-2} \\ < 7.0 \cdot 10^{-5} & 0.995 \pm 0.005 & < 1.0 \cdot 10^{-2} \\ < 1.6 \cdot 10^{-2} & < 1.0 \cdot 10^{-2} & 0.995 \pm 0.005 \end{pmatrix}$$

$$|V^\dagger V| \approx \begin{pmatrix} 1.00 \pm 0.032 & < 0.032 & < 0.032 \\ < 0.032 & 1.00 \pm 0.032 & < 0.032 \\ < 0.032 & < 0.032 & 1.00 \pm 0.032 \end{pmatrix}$$

Approximation

9 active-sterile mixing angles are constrained to be at most of O(0.1).

$$A \simeq \mathbf{1} - \begin{pmatrix} \frac{1}{2}\left(s_{14}^2 + s_{15}^2 + s_{16}^2\right) & 0 & 0 \\ \hat{s}_{14}\hat{s}_{24}^* + \hat{s}_{15}\hat{s}_{25}^* + \hat{s}_{16}\hat{s}_{26}^* & \frac{1}{2}\left(s_{24}^2 + s_{25}^2 + s_{26}^2\right) & 0 \\ \hat{s}_{14}\hat{s}_{34}^* + \hat{s}_{15}\hat{s}_{35}^* + \hat{s}_{16}\hat{s}_{36}^* & \hat{s}_{24}\hat{s}_{34}^* + \hat{s}_{25}\hat{s}_{35}^* + \hat{s}_{26}\hat{s}_{36}^* & \frac{1}{2}\left(s_{34}^2 + s_{35}^2 + s_{36}^2\right) \end{pmatrix}$$

$$B \simeq \mathbf{1} - \begin{pmatrix} \frac{1}{2}\left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) & 0 & 0 \\ \hat{s}_{14}^*\hat{s}_{15} + \hat{s}_{24}^*\hat{s}_{25} + \hat{s}_{34}^*\hat{s}_{35} & \frac{1}{2}\left(s_{15}^2 + s_{25}^2 + s_{35}^2\right) & 0 \\ \hat{s}_{14}^*\hat{s}_{16} + \hat{s}_{24}^*\hat{s}_{26} + \hat{s}_{34}^*\hat{s}_{36} & \hat{s}_{15}^*\hat{s}_{16} + \hat{s}_{25}^*\hat{s}_{26} + \hat{s}_{35}^*\hat{s}_{36} & \frac{1}{2}\left(s_{16}^2 + s_{26}^2 + s_{36}^2\right) \end{pmatrix}$$

$$R \simeq \mathbf{0} + \begin{pmatrix} \hat{s}_{14}^* & \hat{s}_{15}^* & \hat{s}_{16}^* \\ \hat{s}_{24}^* & \hat{s}_{25}^* & \hat{s}_{26}^* \\ \hat{s}_{34}^* & \hat{s}_{35}^* & \hat{s}_{36}^* \end{pmatrix} \qquad S \simeq \mathbf{0} - \begin{pmatrix} \hat{s}_{14} & \hat{s}_{24} & \hat{s}_{34} \\ \hat{s}_{15} & \hat{s}_{25} & \hat{s}_{35} \\ \hat{s}_{16} & \hat{s}_{26} & \hat{s}_{36} \end{pmatrix} \qquad \boxed{R \simeq -S^\dagger}$$

Standard weak charged-current interactions:

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\, \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}}\, \gamma^\mu \left[V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} \nu_4 \\ \nu_5 \\ \nu_6 \end{pmatrix}_{\rm L} \right] W_\mu^- + {\rm h.c.}$$

Application (1)

In the assumption of 3 light sterile neutrinos, the effective masses of the tritium beta decay / neutrinoless double-beta decay get modified.

The tritium β decay:

$$\boxed{{}^3_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + e^- + \overline{\nu}_e}$$

$$\langle m\rangle'_e \equiv \left[\sum_{i=1}^{6} m_i^2 |V_{ei}|^2\right]^{1/2} = \sqrt{\langle m\rangle_e^2 c_{14}^2 c_{15}^2 c_{16}^2 + m_4^2 s_{14}^2 c_{15}^2 c_{16}^2 + m_5^2 s_{15}^2 c_{16}^2 + m_6^2 s_{16}^2}$$

$$\boxed{\langle m\rangle'_e \geq \langle m\rangle_e}$$

$$\langle m\rangle_e = \sqrt{m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2}$$

The 0ν2β decay:

$$\boxed{A(Z, A) \rightarrow A(Z+2, N-2) + 2e^-}$$

$$\langle m\rangle'_{ee} \equiv \sum_{i=1}^{6} m_i V_{ei}^2 = \langle m\rangle_{ee} \left(c_{14} c_{15} c_{16}\right)^2 + m_4 \left(\hat{s}_{14}^* c_{15} c_{16}\right)^2 + m_5 \left(\hat{s}_{15}^* c_{16}\right)^2 + m_6 \left(\hat{s}_{16}^*\right)^2$$

$$\langle m\rangle_{ee} = m_1 (c_{12} c_{13})^2 + m_2 (\hat{s}_{12}^* c_{13})^2 + m_3 (\hat{s}_{13}^*)^2$$

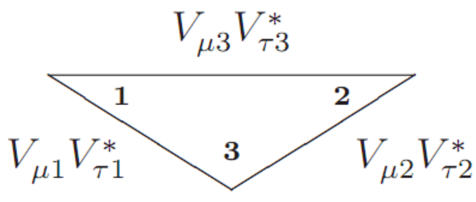
Both $\langle m\rangle_{ee} = 0$ and $\langle m\rangle'_{ee} = 0$ possible due to CP-violating phases.

Application (2)

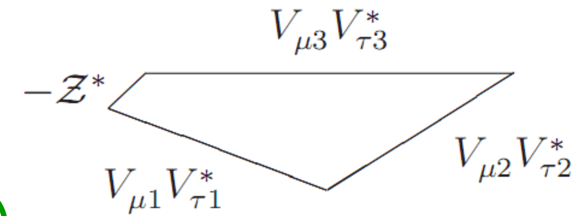
$$\triangle_e: \quad V_{\mu 1}V_{\tau 1}^* + V_{\mu 2}V_{\tau 2}^* + V_{\mu 3}V_{\tau 3}^* = 0$$
$$\triangle_\mu: \quad V_{\tau 1}V_{e1}^* + V_{\tau 2}V_{e2}^* + V_{\tau 3}V_{e3}^* = 0$$
$$\triangle_\tau: \quad V_{e1}V_{\mu 1}^* + V_{e2}V_{\mu 2}^* + V_{e3}V_{\mu 3}^* = 0$$

Deformed unitarity triangles

$$\triangle_e': \quad V_{\mu 1}V_{\tau 1}^* + V_{\mu 2}V_{\tau 2}^* + V_{\mu 3}V_{\tau 3}^* \simeq -\mathcal{Z}^*$$
$$\triangle_\mu': \quad V_{\tau 1}V_{e1}^* + V_{\tau 2}V_{e2}^* + V_{\tau 3}V_{e3}^* \simeq -\mathcal{Y}$$
$$\triangle_\tau': \quad V_{e1}V_{\mu 1}^* + V_{e2}V_{\mu 2}^* + V_{e3}V_{\mu 3}^* \simeq -\mathcal{X}^*$$

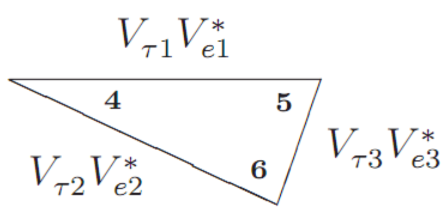


$(\triangle_e)$

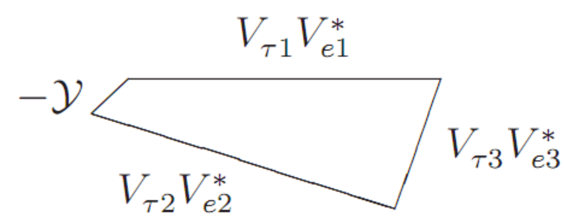


$(\triangle_e')$

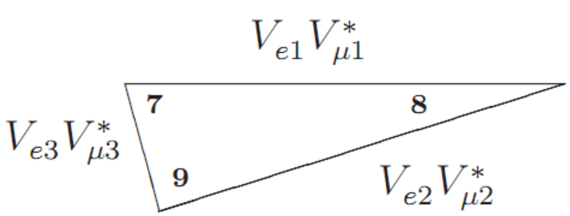
$$\mathcal{X} \equiv \hat{s}_{14}\hat{s}_{24}^* + \hat{s}_{15}\hat{s}_{25}^* + \hat{s}_{16}\hat{s}_{26}^*$$
$$\mathcal{Y} \equiv \hat{s}_{14}\hat{s}_{34}^* + \hat{s}_{15}\hat{s}_{35}^* + \hat{s}_{16}\hat{s}_{36}^*$$
$$\mathcal{Z} \equiv \hat{s}_{24}\hat{s}_{34}^* + \hat{s}_{25}\hat{s}_{35}^* + \hat{s}_{26}\hat{s}_{36}^*$$



$(\triangle_\mu)$



$(\triangle_\mu')$

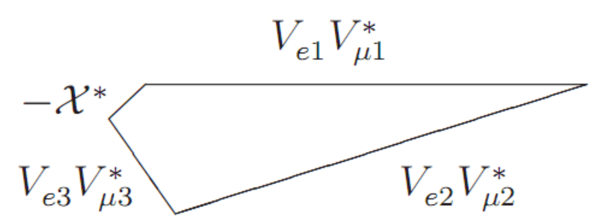


$(\triangle_\tau)$

New effects

of

CP violation

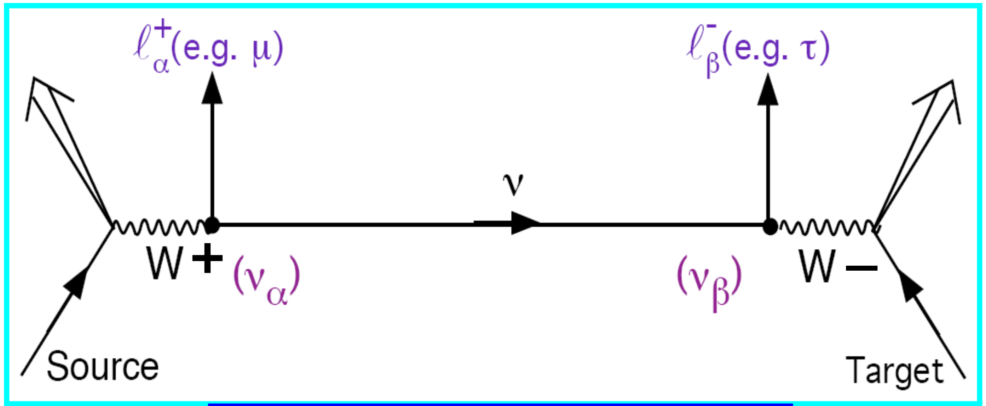


$(\triangle_\tau')$

Neutrino Oscillations

Production + detection of a neutrino beam via CC weak interactions:

$$A(W^+ \to l_\alpha^+ \nu_i) = \frac{V_{\alpha i}^*}{\sqrt{(VV^\dagger)_{\alpha\alpha}}}$$



$$A(\nu_i W^- \to l_\beta^-) = \frac{V_{\beta i}}{\sqrt{(VV^\dagger)_{\beta\beta}}}$$

$$\sum_i |A(W^+ \to l_\alpha^+ \nu_i)|^2 = 1$$

$$\mathrm{Prop}(\nu_i) = \exp\left(-i\frac{m_i^2}{2E}L\right)$$

$$\sum_i |A(\nu_i W^- \to l_\beta^-)|^2 = 1$$

$$\begin{aligned} A(\nu_\alpha \to \nu_\beta) &= \sum_i \left[A(W^+ \to l_\alpha^+ \nu_i) \cdot \mathrm{Prop}(\nu_i) \cdot A(\nu_i W^- \to l_\beta^-)\right] \\ &= \frac{1}{\sqrt{(VV^\dagger)_{\alpha\alpha}(VV^\dagger)_{\beta\beta}}} \sum_i \left[V_{\alpha i}^* \exp\left(-i\frac{m_i^2}{2E}L\right) V_{\beta i}\right] \end{aligned}$$

Like the case of the non-standard interactions in initial + final states.

Direct/Indirect New Effects

Neutrino oscillation probability in vacuum (Antusch et al 06, Xing 08):

$$\boxed{P(\nu_\alpha \to \nu_\beta) = \frac{\sum_i |V_{\alpha i}|^2 |V_{\beta i}|^2 + 2\sum_{i<j} \mathrm{Re}\left(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}\right)\cos\Delta_{ij} - 2\sum_{i<j} J^{ij}_{\alpha\beta}\sin\Delta_{ij}}{\left(VV^\dagger\right)_{\alpha\alpha}\left(VV^\dagger\right)_{\beta\beta}}}$$

$$\boxed{\Delta_{ij} \equiv \Delta m^2_{ij} L/(2E) \text{ with } \Delta m^2_{ij} \equiv m^2_i - m^2_j} \qquad \boxed{|\Delta m^2_{13}| \approx |\Delta m^2_{23}| \gg |\Delta m^2_{12}|}$$

If (3+3) → (3+2) or (3+1), both direct and indirect non-unitary effects occur.

Jarlskog invariants of CP violation: $\boxed{J^{ij}_{\alpha\beta} \equiv \mathrm{Im}(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i})}$

Unitary: universal Jarlskog invariant = 2 area of each unitarity triangle.

Non-unitary: 9 different Jarlskog invariants, and triangles → polygons.

"Zero-distance" (near-detector) effect at *L* = 0 :

$$\boxed{P(\nu_\alpha \to \nu_\beta)\,|_{L=0} = \frac{|(VV^\dagger)_{\alpha\beta}|^2}{(VV^\dagger)_{\alpha\alpha}(VV^\dagger)_{\beta\beta}}}$$

Jarlskog's

In the standard case:

$$J_0 \equiv J_{e\mu}^{12} = J_{e\mu}^{23} = J_{e\mu}^{31} = J_{\mu\tau}^{12} = J_{\mu\tau}^{23} = J_{\mu\tau}^{31} = J_{\tau e}^{12} = J_{\tau e}^{23} = J_{\tau e}^{31} = c_{12}s_{12}c_{13}^2 s_{13}c_{23}s_{23}\sin\delta$$

In the presence of non-unitary effects:

$$\boxed{J_{\alpha\beta}^{ij} \equiv \mathrm{Im}(V_{\alpha i}V_{\beta j}V_{\alpha j}^{*}V_{\beta i}^{*})}$$

$$J_{e\mu}^{23} \simeq J_{e\mu}^{31} \simeq J_{\tau e}^{23} \simeq J_{\tau e}^{31} \simeq J_0$$

$$X \equiv \mathcal{X}e^{-i\delta_{12}}$$
$$Y \equiv \mathcal{Y}e^{-i(\delta_{12}+\delta_{23})}$$
$$Z \equiv \mathcal{Z}e^{-i\delta_{23}}$$

$$J_{e\mu}^{12} \simeq J_0 + c_{12}s_{12}c_{23}\mathrm{Im}X\ ,$$
$$J_{\tau e}^{12} \simeq J_0 + c_{12}s_{12}s_{23}\mathrm{Im}Y\ ,$$
$$J_{\mu\tau}^{12} \simeq J_0 + c_{12}s_{12}c_{23}s_{23}\left(s_{23}\mathrm{Im}X + c_{23}\mathrm{Im}Y\right)\ ,$$
$$J_{\mu\tau}^{23} \simeq J_0 + c_{12}c_{23}s_{23}\left(s_{12}s_{23}\mathrm{Im}X + s_{12}c_{23}\mathrm{Im}Y + c_{12}\mathrm{Im}Z\right)$$
$$J_{\mu\tau}^{31} \simeq J_0 + s_{12}c_{23}s_{23}\left(c_{12}s_{23}\mathrm{Im}X + c_{12}c_{23}\mathrm{Im}Y - s_{12}\mathrm{Im}Z\right)$$

For heavy sterile ν's

CPV: $A_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta) = \dfrac{4}{\left(AA^\dagger\right)_{\alpha\alpha}\left(AA^\dagger\right)_{\beta\beta}}\sum_{i<j} J_{\alpha\beta}^{ij}\sin\Delta_{ij}$

$$\simeq 4\left[J_{\alpha\beta}^{12}\sin\Delta_{21} + \left(J_{\alpha\beta}^{13} + J_{\alpha\beta}^{23}\right)\sin\Delta_{32}\right]$$

$$\boxed{\Delta_{ij} \equiv \Delta m_{ij}^2 L/(2E)}$$

CP Violation

CP-violating asymmetries:

$$\mathcal{A}_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$$

$$\mathcal{A}_{\mu e} \simeq -4\left(J_0 + c_{12}s_{12}c_{23}\mathrm{Im}X\right)\sin\frac{\Delta m^2_{21}L}{2E} \; ,$$

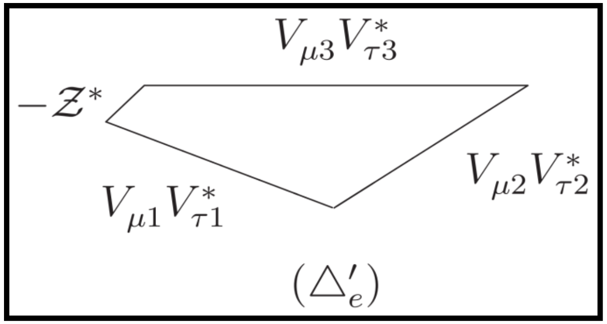
$$\mathcal{A}_{e\tau} \simeq -4\left(J_0 + c_{12}s_{12}s_{23}\mathrm{Im}Y\right)\sin\frac{\Delta m^2_{21}L}{2E} \; ,$$

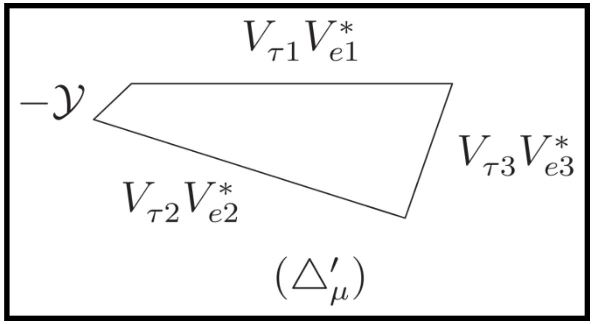
$$\mathcal{A}_{\mu\tau} \simeq +4\left[J_0 + c_{12}s_{12}c_{23}s_{23}\left(s_{23}\mathrm{Im}X + c_{23}\mathrm{Im}Y\right)\right]\sin\frac{\Delta m^2_{21}L}{2E} + 4c_{23}s_{23}\mathrm{Im}Z\sin\frac{\Delta m^2_{32}L}{2E}$$

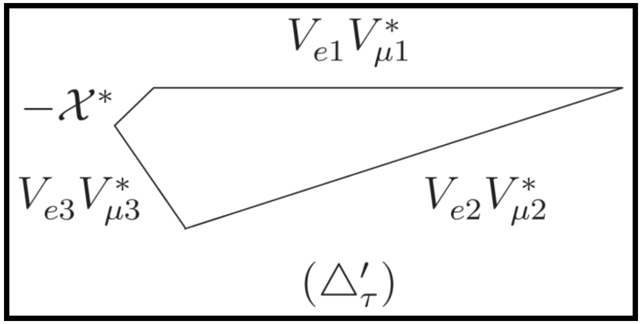
Correlation:

$$\boxed{\mathcal{A}_{\mu\tau} + \left(s^2_{23}\mathcal{A}_{\mu e} + c^2_{23}\mathcal{A}_{e\tau}\right) \simeq 4c_{23}s_{23}\mathrm{Im}Z\sin\frac{\Delta m^2_{32}L}{2E}}$$

Testing the unitarity of the PMNS matrix at low energies: a window to possible new physics at superhigh energies.

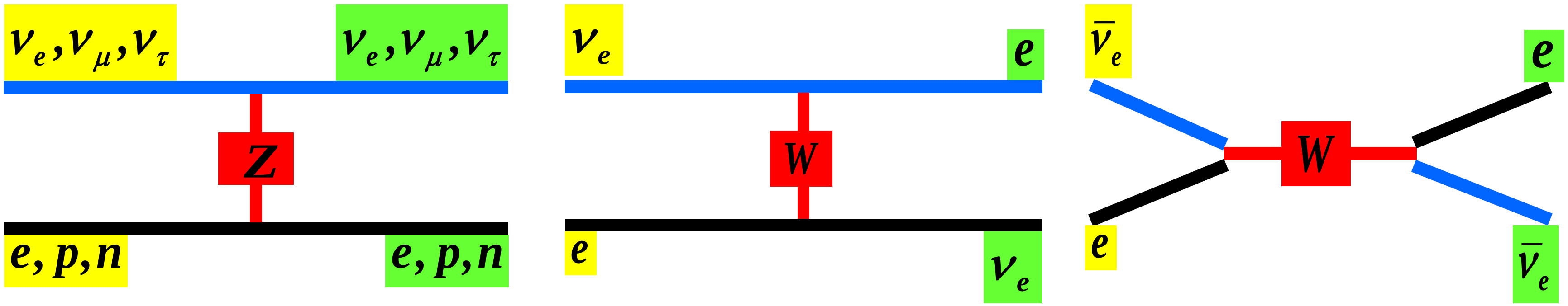






MSW Matter Effects

Illustration: 3 heavy Majorana neutrinos and constant matter density.



$$P(\nu_\mu \to \nu_\tau) \approx \sin^2 \frac{\Delta_{23}}{2} - \sum_{l=4}^{6} s_{2l} s_{3l} \left[\sin(\delta_{2l} - \delta_{3l}) + A_{\rm NC} L \cos(\delta_{2l} - \delta_{3l})\right] \sin \Delta_{23}$$

$$P(\overline{\nu}_\mu \to \overline{\nu}_\tau) \approx \sin^2 \frac{\Delta_{23}}{2} + \sum_{l=4}^{6} s_{2l} s_{3l} \left[\sin(\delta_{2l} - \delta_{3l}) + A_{\rm NC} L \cos(\delta_{2l} - \delta_{3l})\right] \sin \Delta_{23}$$

(Goswami, Ota 08; Luo 08; Xing 09)

Genuine CPV **Matter effect**

$$A_{\rm NC} = G_{\rm F} N_n / \sqrt{2}$$

$$\sim 2.6 \times 10^{-4} \text{ km}$$

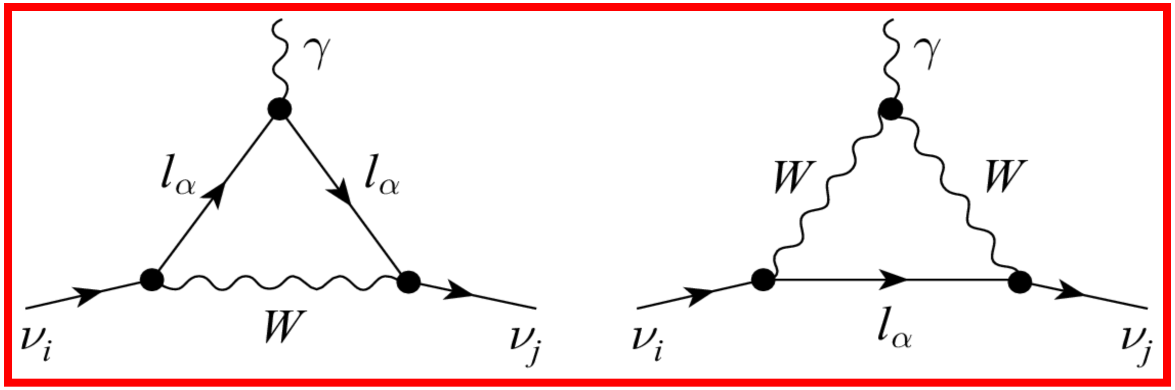
The same matter term appears in disappearance ν_μ → ν_μ oscillation.

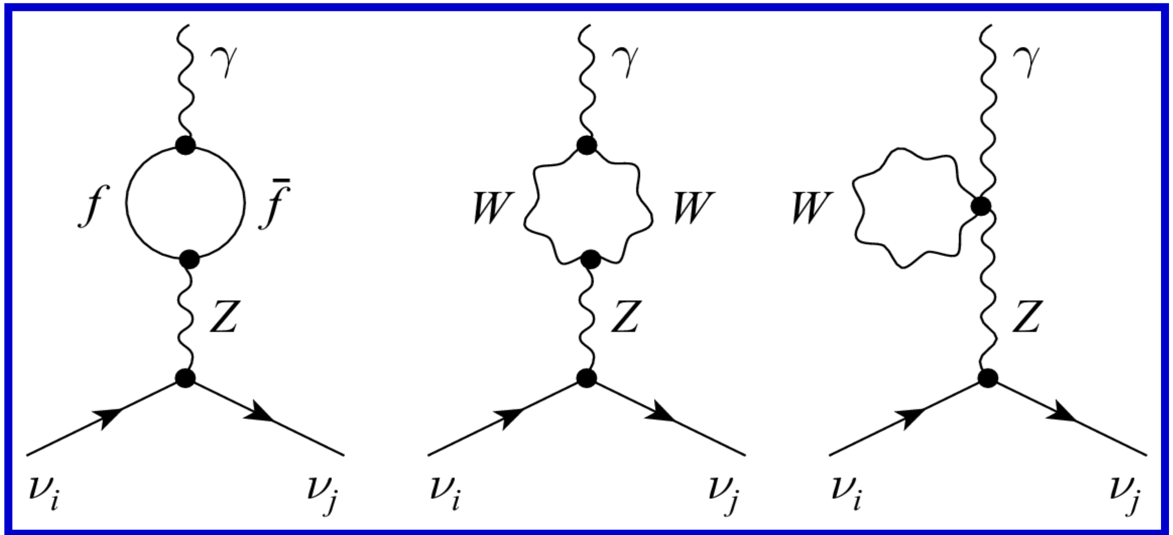
Dipole Moments

Given the SM interaction, a massive Dirac neutrino can have a tiny magnetic dipole moment $\leq 10^{-20}\,\mu_B$.

A Majorana neutrino can not have the magnetic & electric dipole moments.

EX bound:$\leq$ a few$\times 10^{-11}\,\mu_B$





No net contribution to the DMs

Both Dirac and Majorana neutrinos have transition dipole moments:

— **neutrino decays**

— **scattering with electrons**

— **interaction with external magnetic fields**

— **correlation to the origin of neutrino mass**

the canonical seesaw

Xing, Zhou, 1201.2543

Canonical Seesaw

The canonical seesaw mechanism:

$$-\mathcal{L}_\nu = \overline{\ell_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^c_{\rm R}} M_{\rm R} N_{\rm R} + {\rm h.c.}$$

$$\mathcal{U}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \mathcal{U}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix}$$

The basis transformation:

$$\mathcal{U} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

Physical masses: $\widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$ and $\widehat{M}_N \equiv {\rm Diag}\{M_1, M_2, M_3\}$

Charged-current interactions:

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\,\overline{(e \quad \mu \quad \tau)_{\rm L}}\,\gamma^\mu \left[V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}$$

Exact seesaw: $V \widehat{M}_\nu V^T + R \widehat{M}_N R^T = \mathbf{0}$ $\qquad$ $VV^\dagger + RR^\dagger = \mathbf{1}$

Comment (1): neutrino masses and mixing parameters are entangled;

Comment (2): the crucial question is where the seesaw mass scale is;

Comment (3): lower the seesaw scale and magnify non-unitarity of V.

TeV Scale?

Planck scale $\Lambda \sim 10^{19}\,\mathrm{GeV}$

GUT scale? $\Lambda \sim 10^{16}\,\mathrm{GeV}$

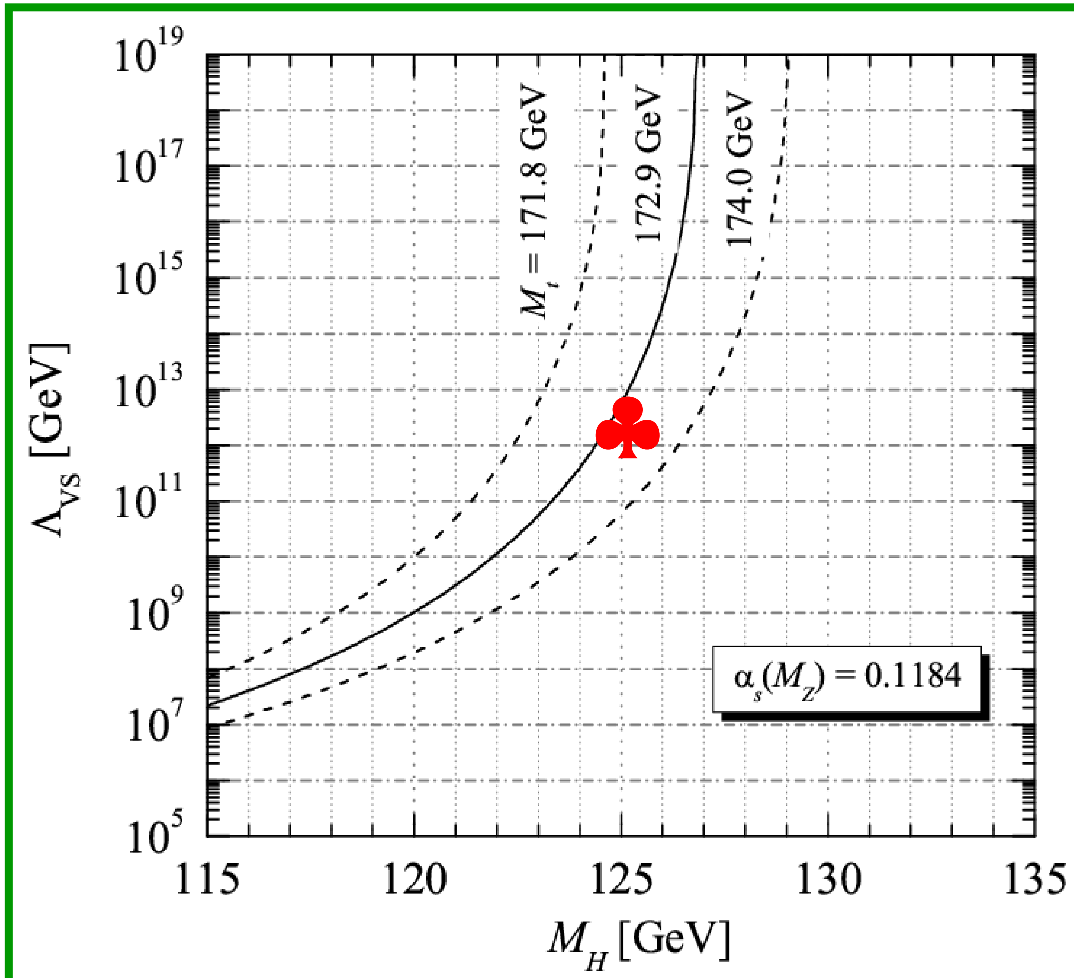
Seesaw scale? $\Lambda \sim 10^{12}\,\mathrm{GeV}$

TeV / SUSY? $\Lambda \sim 10^{3}\,\mathrm{GeV}$

Fermi scale $\Lambda \sim 10^{2}\,\mathrm{GeV}$

QCD scale $\Lambda \sim 10^{2}\,\mathrm{MeV}$

The SM vacuum stability for a light Higgs



e.g., Holthausen et al.; Masina, Notari; Elias-Miro et al.; Xing, Zhang, Zhou 12

Lift the GIM

After a careful treatment of infinities + non-unitary effects, we obtain

$$\Gamma^{\mu}_{ij}(0) = \mu_{ij}\left(i\ \sigma^{\mu\nu} q_\nu\right) + \epsilon_{ij}\left(\sigma^{\mu\nu} q_\nu \gamma_5\right)$$

The same as Shrock's result (82) but contains non-unitary effects.

$$\mu_{ij} = \frac{ieG_{\rm F}}{4\sqrt{2}\pi^2}\left(m_i + m_j\right)\sum_\alpha F_\alpha {\rm Im}\left(V_{\alpha i} V^*_{\alpha j}\right)$$

$$\epsilon_{ij} = \frac{eG_{\rm F}}{4\sqrt{2}\pi^2}\left(m_i - m_j\right)\sum_\alpha F_\alpha {\rm Re}\left(V_{\alpha i} V^*_{\alpha j}\right)$$

$$F_\alpha = \frac{3}{4}\left[\frac{2-\xi_\alpha}{1-\xi_\alpha} - \frac{2\xi_\alpha}{\left(1-\xi_\alpha\right)^2} + \frac{2\xi_\alpha^2 \ln \xi_\alpha}{\left(1-\xi_\alpha\right)^3}\right]$$

$$\xi_\alpha \equiv m_\alpha^2 / M_W^2 \ \ (\text{for } \alpha = e, \mu, \tau)$$

Non-unitary effects:

$$V = AV_0 \simeq V_0 - TV_0$$

Lift the GIM suppression:

$$\sum_\alpha F_\alpha \left(V_{\alpha i} V^*_{\alpha j}\right) \simeq \underbrace{-\frac{3}{2}\sum_\alpha \left[\left(V_0\right)_{\alpha i}\left(TV_0\right)^*_{\alpha j} + \left(TV_0\right)_{\alpha i}\left(V_0\right)^*_{\alpha j}\right]}_{\text{new term / non-unitary effect}} \underbrace{- \frac{3}{4}\sum_\alpha \left[\xi_\alpha \left(V_0\right)_{\alpha i}\left(V_0\right)^*_{\alpha j}\right]}_{\text{conventional term}}$$

The 1st term is the seesaw-induced effect which may be comparable with or even larger than the 2nd term.

$$\xi_\alpha \lesssim 4.9 \times 10^{-4}$$

Parameters

Input parameters:

$$\Delta m_{12}^2 \simeq +7.6\times 10^{-5}\,\mathrm{eV}^2$$
$$\Delta m_{23}^2 \simeq \pm 2.5\times 10^{-3}\,\mathrm{eV}^2$$

$$\theta_{12} \simeq 34^\circ$$
$$\theta_{13} \simeq 9^\circ$$
$$\theta_{23} \simeq 45^\circ$$

$$V_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}_{12}^* c_{13} & \hat{s}_{13}^* \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}_{23}^* & c_{12}c_{23} - \hat{s}_{12}^*\hat{s}_{13}\hat{s}_{23}^* & c_{13}\hat{s}_{23}^* \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}_{12}^*\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

$$T = \begin{pmatrix} \frac{1}{2}\sum_{k=4}^{6} s_{1k}^2 & 0 & 0 \\ \sum_{k=4}^{6} \hat{s}_{1k}\hat{s}_{2k}^* & \frac{1}{2}\sum_{k=4}^{6} s_{2k}^2 & 0 \\ \sum_{k=4}^{6} \hat{s}_{1k}\hat{s}_{3k}^* & \sum_{k=4}^{6} \hat{s}_{2k}\hat{s}_{3k}^* & \frac{1}{2}\sum_{k=4}^{6} s_{3k}^2 \end{pmatrix}$$

In addition,

$$\min\{m_1, m_2, m_3\} = 5\ \mathrm{meV}$$
$$M_1, M_2, M_3 \simeq O(1)\ \mathrm{TeV}$$

Current constraints (Antusch et al. 06)

$$T_{11} < 5.5\times 10^{-3}\,, \quad |T_{21}| < 7.0\times 10^{-5}$$
$$T_{22} < 5.0\times 10^{-3}\,, \quad |T_{31}| < 1.6\times 10^{-2}$$
$$T_{33} < 5.0\times 10^{-3}\,, \quad |T_{32}| < 1.0\times 10^{-2}$$

Arbitrary CP phases to satisfy:

$$V\widehat{M}_\nu V^T + R\widehat{M}_N R^T = 0$$

$$VV^\dagger + RR^\dagger = 1$$

Seesaw + unitarity

Conventional Case

Comment (a): **almost degenerate 3 heavy neutrinos to assure that the radiative correction to the masses of 3 light neutrinos are sufficiently small (Pilaftsis 1992); e.g., smaller than 0.5 meV .**

Comment (b): **to present numerical results in a convenient way, let us define a parameter to measure the strength of unitarity violation of V:**

$$\varepsilon_{\rm uv} \equiv \left[\sum_{k=4}^{6}\left(s_{1k}^2 + s_{2k}^2 + s_{3k}^2\right)\right]^{1/2}$$

$$0 \leq s_{ik} < 0.15 \quad (\text{for } i = 1,2,3 \text{ and } k = 4,5,6)$$

$$\mu_{\rm eff} \equiv \sqrt{|\mu_{ij}|^2 + |\epsilon_{ij}|^2}$$

The conventional case (0 non-unitary effects)

$$\mu_{\rm eff} \simeq \begin{cases} (0.8 \sim 3.0) \times 10^{-25}\ \mu_{\rm B} & (\nu_2 \to \nu_1 + \gamma) \\ (0.8 \sim 1.5) \times 10^{-24}\ \mu_{\rm B} & (\nu_3 \to \nu_1 + \gamma) \\ (1.1 \sim 2.1) \times 10^{-24}\ \mu_{\rm B} & (\nu_3 \to \nu_2 + \gamma) \end{cases}$$

$m_1 \simeq 5$ meV

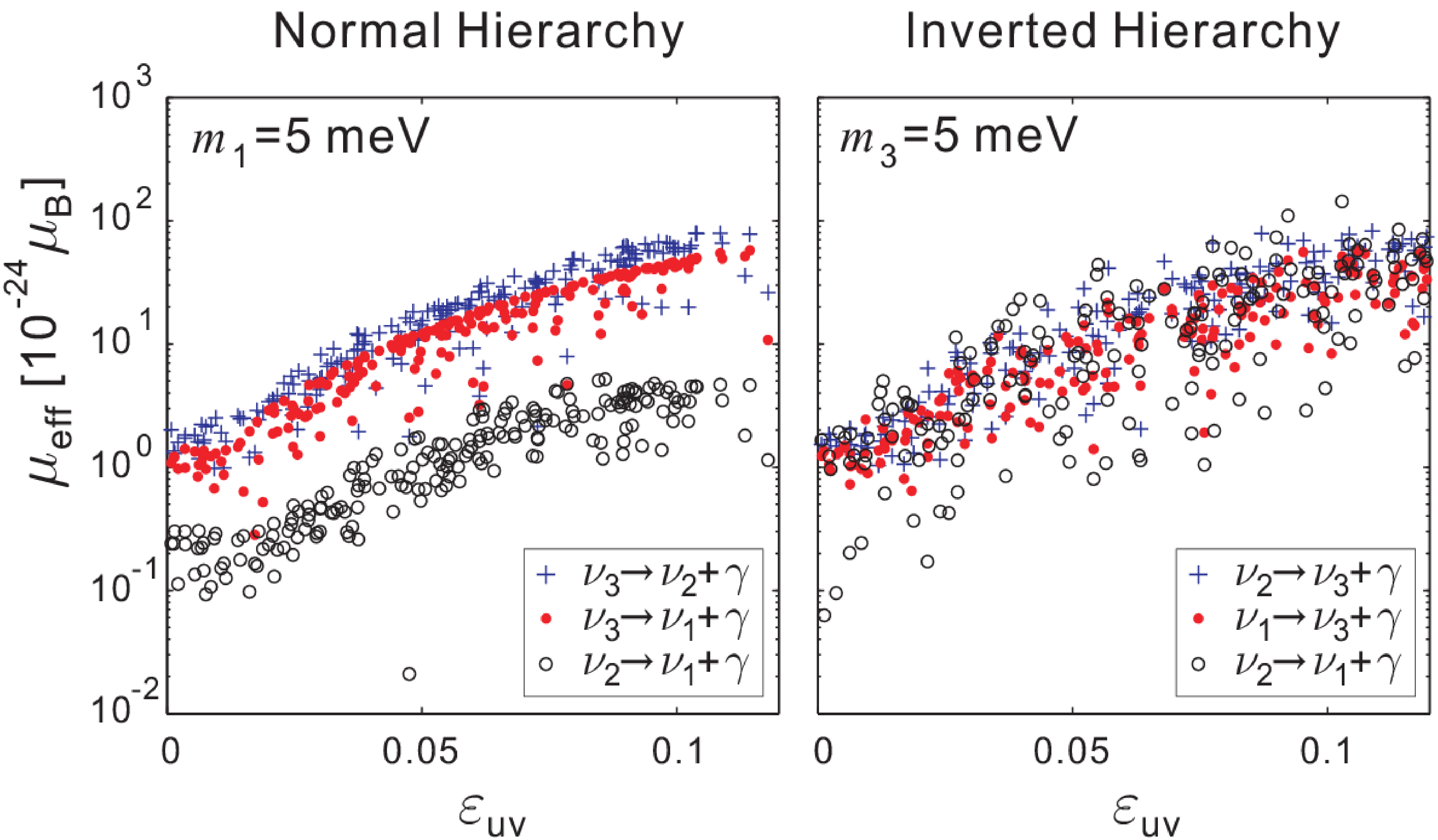
EX bound: $\leq$ a few $\times 10^{-11}\ \mu_{\rm B}$

$$\mu_{\rm eff} \simeq \begin{cases} (0.01 \sim 2.0) \times 10^{-24}\ \mu_{\rm B} & (\nu_2 \to \nu_1 + \gamma) \\ (0.8 \sim 1.5) \times 10^{-24}\ \mu_{\rm B} & (\nu_3 \to \nu_1 + \gamma) \\ (1.3 \sim 2.0) \times 10^{-24}\ \mu_{\rm B} & (\nu_3 \to \nu_2 + \gamma) \end{cases}$$

$m_3 \simeq 5$ meV

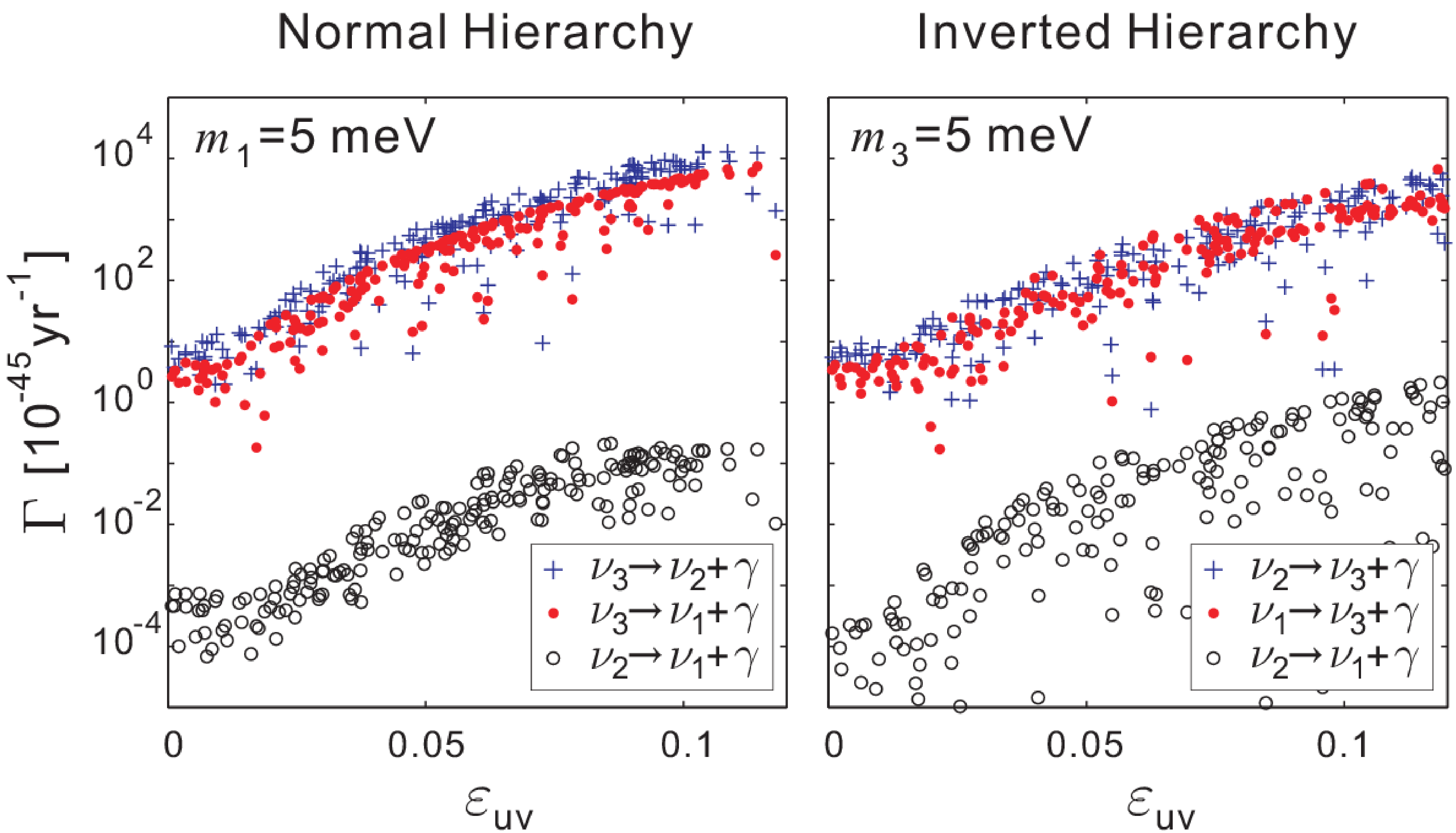
Enhanced electromagnetic DMs:

$$\mu_{ij} = \frac{ieG_{\rm F}}{4\sqrt{2}\pi^2}\left(m_i + m_j\right)\sum_\alpha F_\alpha {\rm Im}\left(V_{\alpha i}V^*_{\alpha j}\right) \ , \qquad \epsilon_{ij} = \frac{eG_{\rm F}}{4\sqrt{2}\pi^2}\left(m_i - m_j\right)\sum_\alpha F_\alpha {\rm Re}\left(V_{\alpha i}V^*_{\alpha j}\right)$$

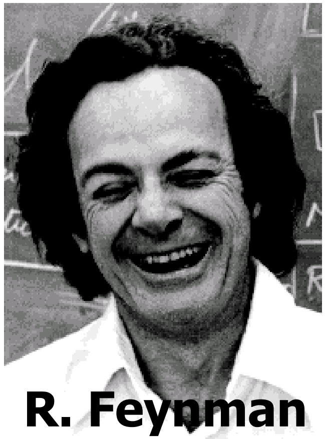


Enhanced decay rates: (too small to have impact on the CIB)

$$\Gamma_{\nu_i \to \nu_j + \gamma} = \frac{\left(m_i^2 - m_j^2\right)^3}{8\pi m_i^3}\left(|\mu_{ij}|^2 + |\epsilon_{ij}|^2\right) \simeq 5.3 \times \left(1 - \frac{m_j^2}{m_i^2}\right)^3 \left(\frac{m_i}{1\ \mathrm{eV}}\right)^3 \left(\frac{\mu_{\rm eff}}{\mu_{\rm B}}\right)^2 \mathrm{s}^{-1}$$



Summary: Open Questions



R. Feynman

1) Do we feel happy/painful/sorry to add sterile neutrinos into the SM (Weinberg's theorem)?

2) How many species of sterile neutrinos should be taken into account for this or that purpose?

3) If all the current experimental and observational hints disappear, will the sterile neutrino physics still survive?

4) How strong is the correlation between electromagnetic properties of neutrinos and the origin of their masses?

5) How about the flavor structures of charged and neutral fermions, including the extra species? (Flavor Theory) ....

THANK YOU FOR YOUR ATTENTION!