

A SUSY $SU(5) \times T'$
Unified Model of Flavour
with large θ_{13}

BENE, ICTP

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In collaboration with
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arXiv:1205.5241



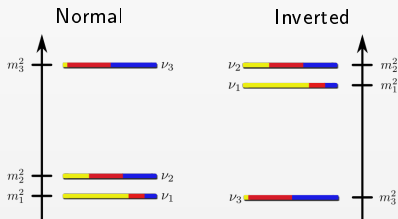
- Motivations for a $SU(5) \otimes T'$ Unified Model of Flavour
- General Setup of the Model
- Neutrino Sector
- Conclusions

Global fit analysis including the Daya Bay and RENO results
 Gonzalez-Garcia et al. [ArXiv: 1209.3023](https://arxiv.org/abs/1209.3023).

Parameter		Best fit	3σ range
Δm_{\odot}^2 [10^{-5} eV 2]	(NH or IH)	7.50	7.00 – 8.09
$ \Delta m_A^2 $ [10^{-3} eV 2]	(NH)	2.47	2.27 – 2.69
	(IH)	2.43	2.24 – 2.65
$\sin^2 \theta_{12}$	(NH or IH)	0.30	0.27-0.34
$\sin^2 \theta_{13}$	(NH or IH)	0.023	0.016-0.030
$\sin^2 \theta_{23}$	(NH or IH)	0.41	0.34-0.67

Open Questions in Neutrino Physics

- Majorana or Dirac?
- Absolute values of neutrino masses
- Hierarchy (normal $m_1 < m_2 < m_3$ or inverted $m_3 < m_1 < m_2$)
- CP-phases: δ , α_{31} and α_{21}



- GUT: $SU(5)$
- * $SU(5)$, Unified picture of quarks and leptons
 - * possibly leads to sizeable θ_{13}

- discrete symmetry: T'
- * corrections to U_{TBM} via lepton sector

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

- * can allow spinorial unitary irreducible reps
- * “geometrical” CP violation

Literature

A model based on $SU(5) \otimes T' \otimes Z_{12} \otimes Z_{12}$ was developed in [M.C. Chen, K.T. Mahanthappa Phys. Lett. B652,34 \(2007\)](#)

- Too small $\theta_{13} \sim \theta^c / (3\sqrt{2}) \Rightarrow \sin^2 \theta_{13} \sim 0.003 \rightarrow$ **RULED OUT!**
See Mahanthappa Talk for modifications of the model

Discrete symmetry: the group T'

- T' is the Double-valued group of $T \sim A_4$ (even permutations of 4 objects)
- inequivalent UIRs : $\underbrace{1, 1', 1'', 3}_{\text{TBM for Neutrinos}} + \underbrace{2, 2', 2''}_{2+1 \text{ assignments for } q \text{ and } \ell}$
- **Complex CG coefficients** when spinorial UIRs are involved !

Clebsch-Gordan coefficients can be **complex** \Rightarrow geometrical origin of **CP** violation!

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{2(2')} \otimes \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix}_{2(2'')} = \begin{pmatrix} \frac{x_1 x'_2 - x_2 x'_1}{\sqrt{2}} \end{pmatrix}_1 \oplus \begin{pmatrix} -\frac{(1+i)}{2}(x_1 x'_2 + x_2 x'_1) \\ x_1 x'_1 \\ -i x_2 x'_2 \end{pmatrix}_3$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_2 \otimes \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}_3 \sim \left[\begin{pmatrix} (1+i)x_2 u_2 + x_1 u_1 \\ (1-i)x_1 u_3 - x_2 u_1 \end{pmatrix}_2 \oplus \begin{pmatrix} (1+i)x_2 u_3 + x_1 u_2 \\ (1-i)x_1 u_1 - x_2 u_2 \end{pmatrix}_{2'} \oplus \begin{pmatrix} (1+i)x_2 u_1 + x_1 u_3 \\ (1-i)x_1 u_2 - x_2 u_3 \end{pmatrix}_{2''} \right]$$

M.-C. Chen, K.T. Mahanthappa Phys.Lett. B652 (2007)

F. Feruglio, C. Hagedorn, Y. Lin, L. Merlo, Nucl.Phys. B775 (2007)

Fundamental Symmetries $SU(5) \otimes T'$ (SUSY!)

Shaping symmetry $Z_{12} \otimes Z_8^3 \otimes Z_6^2 \otimes Z_4$

$U(1)_R$ Continuous version of the usual R -parity

Matter Content $\mathbf{10}(Q, u^c, e^c)_L$: ten-plets into $(T1, T2) \sim \mathbf{2}$, $T3 \sim \mathbf{1}$

$\bar{\mathbf{5}}(d^c, \ell)_L$: five-plets into $(F1, F2, F3) \sim \mathbf{3}$

Three Heavy RH Majorana N_k , $(N_1, N_2, N_3) \sim \mathbf{3}$

light active neutrinos via Type I See-Saw

Higgs sector: copies of $\mathbf{5}$, $\bar{\mathbf{5}}$ and $\mathbf{24}$ assigned to $\mathbf{1}$, $\mathbf{1}''$

up and down quark sector $\langle \tilde{\psi}' \rangle \sim \langle \psi' \rangle \propto \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\langle \tilde{\psi}'' \rangle \sim \langle \psi'' \rangle \propto \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\langle \tilde{\phi} \rangle \sim \langle \phi \rangle \propto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \text{ singlets e.g. } \langle \tilde{\zeta}' \rangle$$

in the ν sector $\langle \xi \rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xi_0$, $\langle \rho \rangle = \rho_0$, $\langle \tilde{\rho} \rangle = \tilde{\rho}_0$

Flavons fields take all **real vevs** when T' is broken!

Flavon Alignment problem !!!

S. Antusch, S. King, C. Luhn, M. Spinrath ArXiv: 1103.5930

Auxiliary Fields: ϵ_i $SU(5) \otimes T' \sim \mathbf{1}$ and $Q_{U(1)_R} = 0$
 transform in a non trivial way under
 the shaping symmetries
 they appear only in the flavon superpotential

$$\mathcal{W}_\epsilon = S_{\epsilon_i} (\epsilon_i^2 - M_{\epsilon_i}^2) + S_{\epsilon_j} \left(\frac{1}{\Lambda} \epsilon_j^3 - M_{\epsilon_j}^2 \right)$$

Driving Fields: $\{S_x \sim \mathbf{1}, D_x \sim \mathbf{3}\}$ $SU(5) \sim \mathbf{1}$ and $Q_{U(1)_R} = 2$
 transform in a non trivial way under
 T' and shaping symmetries

for example in the neutrino sector

$$\mathcal{W}_{\xi, \rho, \tilde{\rho}} = \frac{D_\xi}{\Lambda} (\xi^2 \epsilon_9 + \xi \rho \epsilon_9 + \xi \tilde{\rho} \epsilon_9) + S_\xi (\xi^2 - M_\xi^2) + S_\rho (\rho^2 + \tilde{\rho}^2 - M_\rho^2)$$

$$\xi_i = \xi_0 \neq 0 \quad \text{if} \quad \rho_0 = -\tilde{\rho}_0$$

RL convention, i.e. $-\mathcal{L} = Y_{ij} \bar{f}_R^i f_L^j H + \text{H.c.}$

$$Y_u = \begin{pmatrix} (1-i)a_u & i b_u & 0 \\ i b_u & c_u & (1+i)d_u \\ 0 & (1+i)d_u & e_u \end{pmatrix},$$

$$Y_d = \begin{pmatrix} (1+i)a_d & i b'_d & 0 \\ (1-i)b_d & c_d & 0 \\ 0 & 0 & d_d \end{pmatrix} \quad \text{and} \quad Y_\ell = \begin{pmatrix} -\frac{3}{2}(1+i)a_d & (1-i)b_d & 0 \\ 6 i b'_d & 6 c_d & 0 \\ 0 & 0 & -\frac{3}{2}d_d \end{pmatrix}$$

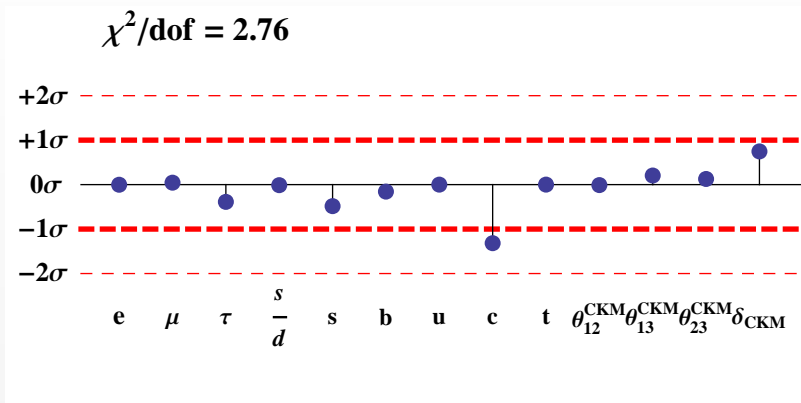
S. Antusch, M. Spinrath arXiv:0902.4644

$$|V_{us}| \sim \left| \frac{b_d}{c_d} \right|$$

$$\theta_{12}^e \sim \left| \frac{6i b'_d}{6c_d} \right| \sim \left| \frac{b'_d}{b_d} \right| \theta^c, \quad (b'_d = 0.9 b_d)$$

$$\theta_{13} = \frac{1}{\sqrt{2}} \theta_{12}^e = \frac{0.9}{\sqrt{2}} \theta^c$$

D. Marzocca, S. T. Petcov, A. Romanino, M. Spinrath arXiv:1108.0614
Antusch, Maurer arXiv:1107.3728



A. M., S. T. Petcov, M. Spinrath arXiv:1205.5241

$$\mathcal{W}_\nu = \underbrace{\lambda_1 NN\xi + NN(\lambda_2\rho + \lambda_3\tilde{\rho})}_{\text{RH Majorana Mass } M_R} + \underbrace{\frac{y_\nu}{\Lambda}(N\bar{F})_1(H_5^{(2)}\rho)_1 + \frac{\tilde{y}_\nu}{\Lambda}(N\bar{F})_1(H_5^{(2)}\tilde{\rho})_1}_{\text{Dirac Yukawa coupling } M_D}$$

$$\langle \xi \rangle = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xi_0, \quad \langle \rho \rangle = \rho_0, \quad \langle \tilde{\rho} \rangle = \tilde{\rho}_0$$

$$M_R = \begin{pmatrix} 2Z + X & -Z & -Z \\ -Z & 2Z & -Z + X \\ -Z & -Z + X & 2Z \end{pmatrix}, \quad M_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{\rho'}{\Lambda}$$

X, Z and ρ' are real parameters

M_R is form diagonalizable

$$U_{\text{TBM}}^T M_R U_{\text{TBM}} = \text{Diag}(3Z + X, X, 3Z - X)$$

Diagonal Majorana Mass Matrix

$$\begin{pmatrix} 3Z + X & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & 3Z - X \end{pmatrix} \rightarrow |X| \begin{pmatrix} |1 + \alpha e^{i\phi}| e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & |1 - \alpha e^{i\phi}| e^{i\phi_3} \end{pmatrix}$$

$$\alpha \equiv |3Z/X| > 0, \quad \phi \equiv \arg(Z) - \arg(X)$$

$$\phi_i = 0, \pi$$

Light neutrino Majorana mass term via type I see-saw mechanism:

$$M_\nu = -M_D^T M_R^{-1} M_D = U_\nu^* \text{Diag}(m_1, m_2, m_3) U_\nu^\dagger$$

$$U_\nu = i U_{\text{TBM}} \text{Diag}(e^{i\phi_1/2}, e^{i\phi_2/2}, e^{i\phi_3/2})$$

$$m_i = \left(\frac{\rho'}{\Lambda}\right)^2 \frac{1}{M_i}, \quad i = 1, 2, 3 \quad m_i > 0$$

(remember : $\alpha > 0$ $\cos \phi = \pm 1!$)

$$\Delta m_{31}^2 \equiv \Delta m_A^2 = \frac{1}{|X|^2} \left(\frac{\rho'}{\Lambda} \right)^4 \frac{4\alpha \cos \phi}{|1 + \alpha e^{i\phi}|^2 |1 - \alpha e^{i\phi}|^2}.$$

$\left\{ \begin{array}{l} \text{for } \cos \phi = +1 \quad \text{mass spectrum with NO} \\ \text{for } \cos \phi = -1 \quad \text{mass spectrum with IO} \end{array} \right.$

$$\Delta m_{21}^2 \equiv \Delta m_{\odot}^2 = \frac{1}{|X|^2} \left(\frac{\rho'}{\Lambda} \right)^4 \frac{\alpha (\alpha + 2 \cos \phi)}{|1 + \alpha e^{i\phi}|^2}.$$

$$r = \frac{\Delta m_{\odot}^2}{\Delta m_A^2} = \frac{1}{4} (\alpha + 2 \cos \phi) (1 - 2\alpha \cos \phi + \alpha^2) = \mathbf{0.032 \pm 0.006}.$$

NO Solution A

$$m_1 \cong 4.44 \times 10^{-3} \text{ eV}$$

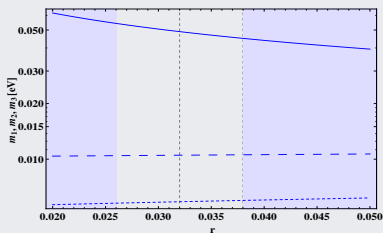
$$m_2 \cong 9.77 \times 10^{-3} \text{ eV}$$

$$m_3 \cong 4.89 \times 10^{-2} \text{ eV}$$

RH Majorana N_k : $M_3 < M_2 < M_1$

$$M_1/M_3 \cong 11.0$$

$$M_2/M_3 \cong 5.0.$$



NO Solution B

$$m_1 \cong 5.89 \times 10^{-3} \text{ eV}$$

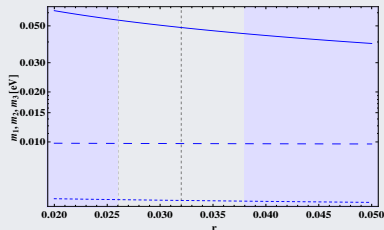
$$m_2 \cong 1.05 \times 10^{-2} \text{ eV}$$

$$m_3 \cong 4.90 \times 10^{-2} \text{ eV}$$

RH Majorana N_k : $M_3 < M_2 < M_1$

$$M_1/M_3 \cong 8.33$$

$$M_2/M_3 \cong 4.67.$$



IO

$$m_1 \cong 5.17 \times 10^{-2} \text{ eV}$$

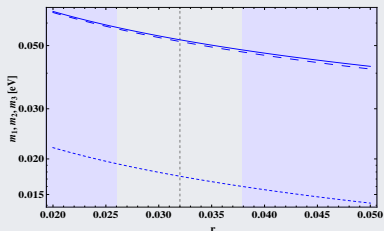
$$m_2 \cong 5.24 \times 10^{-2} \text{ eV}$$

$$m_3 \cong 1.74 \times 10^{-2} \text{ eV}$$

 RH Majorana N_k : $M_1 \cong M_2 < M_3$

$$M_1/M_2 \cong 1.014$$

$$M_3/M_2 \cong 3.01.$$



$$U^{\text{PMNS}} = U_{eL}^\dagger U_\nu$$

$$U_{eL} = \text{diag}(1, e^{i\varphi}, 1) R_{12}(\theta_{12}^e), \quad U_\nu \sim U_{\text{TBM}}$$

$$U_{\text{appx}}^{\text{PMNS}} = \begin{pmatrix} \frac{e^{-i\frac{\beta_1}{2}}}{\sqrt{6}} & \frac{e^{-i\frac{\beta_2}{2}}}{\sqrt{3}} & \frac{\sin\theta_{12}^e e^{-i(\pi-\theta_{12}^e)/2}}{\sqrt{2}} \\ * & * & * \\ * & * & * \end{pmatrix}$$

$$\theta_{12} = \arcsin(1/\sqrt{3}) + \sqrt{2}/8(\theta_{12}^e)^2$$

$$\theta_{13} = \theta_{12}^e/\sqrt{2}$$

$$\theta_{23} = \pi/4 - 1/4(\theta_{12}^e)^2$$

$$\delta = \frac{\pi}{2} - \frac{1}{2}\theta_{12}^e$$

$$\beta_1 = 2\pi - 2\theta_{12}^e + \phi_3$$

$$\beta_2 = 2\pi + \theta_{12}^e + \phi_3 - \phi_2$$

Quantity	Experiment (2σ ranges)	Model
$\sin^2 \theta_{12}$	0.275 – 0.342	0.340
$\sin^2 \theta_{23}$	0.36 – 0.60	0.490
$\sin^2 \theta_{13}$	0.015 – 0.032	0.020
δ	-	84.3°
β_1	-	$337.1^\circ + \phi_3$
β_2	-	$11.5^\circ + \phi_3 - \phi_2$
J_{CP}	-	0.0324

We consider as well RG corrections
 The largest correction we find is for $\delta = 81.2^\circ$

NO Solution A

$$\sum_{k=1}^3 m_k = 6.31 \times 10^{-2} \text{ eV}$$

$$|\langle m \rangle| = 4.90 \times 10^{-3} \text{ eV}$$

NO Solution B

$$\sum_{k=1}^3 m_k = 6.54 \times 10^{-2} \text{ eV}$$

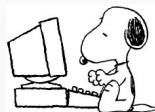
$$|\langle m \rangle| = 7.95 \times 10^{-3} \text{ eV}$$

IO

$$\sum_{k=1}^3 m_k = 12.1 \times 10^{-2} \text{ eV}$$

$$|\langle m \rangle| = 2.17 \times 10^{-2} \text{ eV}$$

- We construct a unified model of flavor with large θ_{13} based on $S(5) \otimes T'$ with type I See-Saw
- T' leads to TBM in the neutrino sector + corrections coming from the charged lepton sector
- Geometrical CP violation via CGs of the T' group
 - Essential ingredient: real flavon vev alignment
- Predictions:
 - $\sin^2 \theta_{13} \sim 0.02$
 - $\delta \cong \pi/2 - 0.45\theta^c \sim 84.3^\circ$
 - Neutrino mass spectra: NO and IO possible
 - unambiguous predictions for $\beta\beta 0\nu$ -Decay $|\langle m \rangle|$



Looking forward to see upcoming exp results!



Thank you



Back up slides

$SU(5) \otimes T' \otimes Z_{12} \otimes Z_{12}$
was developed in

M.C. Chen, K.T. Mahanthappa Phys. Lett. B652,34 (2007);
PoS ICHEP2010:407,2010. Phys. Lett. B681, 444 (2009)

- Too small $\theta_{13} \sim \frac{\theta^c}{3\sqrt{2}} \Rightarrow \sin^2 \theta_{13} \sim 0.003$
- Different flavour structure \rightarrow different values for observables!
- Only **NO** possible
- Flavon vacuum alignment problem
- Messenger sector

Auxiliary Fields: ϵ_i $SU(5) \otimes T' \sim \mathbf{1}$ and $Q_{U(1)_R} = 0$
 transform in a non trivial way under
 the shaping symmetries
 they appear only in the flavon superpotential

$$\mathcal{W}_\epsilon = S_{\epsilon_i} (\epsilon_i^2 - M_{\epsilon_i}^2) + S_{\epsilon_j} \left(\frac{1}{\Lambda} \epsilon_j^3 - M_{\epsilon_j}^2 \right)$$

Driving Fields: $\{S_x \sim \mathbf{1}, D_x \sim \mathbf{3}\}$ $SU(5) \sim \mathbf{1}$ and $Q_{U(1)_R} = 2$
 transform in a non trivial way under
 T' and shaping symmetries

for example in the neutrino sector

$$\mathcal{W}_{\xi, \rho, \tilde{\rho}} = \frac{D_\xi}{\Lambda} (\xi^2 \epsilon_9 + \xi \rho \epsilon_9 + \xi \tilde{\rho} \epsilon_9) + S_\xi (\xi^2 - M_\xi^2) + S_\rho (\rho^2 + \tilde{\rho}^2 - M_\rho^2)$$

$$\xi_i = \xi_0 \neq 0 \quad \text{if} \quad \rho_0 = -\tilde{\rho}_0$$

Discrete symmetry: UIRs of the group T'

- T' is the Double-valued group of $T \sim A_4$ (even permutations of 4 objects)
- inequivalent UIRs: $\underbrace{1, 1', 1'', 3}_{\text{TBM for Neutrinos}}$ + $\underbrace{2, 2', 2''}_{2+1 \text{ assignments for } q \text{ and } \ell}$
- **Complex CG coefficients** when spinorial UIRs are involved !
- 3 generators: $s^2 = r$, $r^2 = t^3 = (st)^3 = e$, $rt = tr$

$$3: \quad r = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad s = \frac{1}{3} \begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}$$

where $1 + \omega + \omega^2 = 0$ conventionally.

$$\begin{aligned}
 1 \otimes \Gamma^p &= \Gamma^p, & 1 \otimes 1'(1'') &= 1'(1''), & 1' \otimes 1'(1'') &= 1''(1), & 1'(1'') \otimes 1'' &= 1(1') \\
 2 \otimes 1'(1'') &= 2'(2''), & 2' \otimes 1'(1'') &= 2''(2), & 2'' \otimes 1'(1'') &= 2(2') \\
 2(2') \otimes 2(2'') &= 1 \oplus 3, & 2'(2) \otimes 2'(2'') &= 1 \oplus 3, & 2''(2) \otimes 2''(2') &= 1 \oplus 3 \\
 3 \otimes 1'(1'') &= 3, & 3 \otimes 2 &= 2 \oplus 2' \oplus 2'', & 3 \otimes 2'(2'') &= 2'(2'') \oplus 2''(2) \oplus 2(2') \\
 & & 3 \otimes 3 &= 1 \oplus 1' \oplus 1'' \oplus 3 \oplus 3
 \end{aligned}$$

T' group: Clebsch-Gordan series $\Gamma^p \otimes \Gamma^q = \Gamma^q \otimes \Gamma^p$. Here $\Gamma^p = 1, 2, 3$

Table of Charge Assignments: Matter and Higgs fields

	T_3	T_a	\bar{F}	N	$H_5^{(1)}$	$H_5^{(2)}$	$H_5^{(3)}$	$\bar{H}_5^{(1)}$	$\bar{H}_5^{(2)}$	$\bar{H}_5^{(3)}$	\bar{H}_5''	H_{24}''	\tilde{H}_{24}''
$SU(5)$	10	10	$\bar{5}$	1	5	5	5	5	5	5	5	24	24
T'	1	2	3	3	1	1	1	1	1	1	1''	1''	1''
$U(1)_R$	1	1	1	1	0	0	0	0	0	0	0	0	0
Z_{12}^u	2	11	1	9	8	8	2	9	3	6	3	0	3
Z_8^d	4	0	2	6	0	4	0	1	4	7	7	4	2
Z_8^v	7	6	2	0	2	6	4	1	1	5	7	4	0
Z_8	0	5	2	2	0	0	6	0	0	6	6	4	2
Z_6	5	0	1	0	2	5	2	2	0	2	2	0	0
Z_6'	2	3	1	0	2	5	2	5	0	2	2	0	0
Z_4	3	3	0	0	2	0	2	0	1	1	0	0	1

Table of Charge Assignments: Flavons

	$\bar{\phi}$	$\bar{\psi}''$	$\bar{\psi}'$	$\bar{\zeta}''$	$\bar{\zeta}'$	ϕ	ψ''	ψ'	ζ''	ζ'	ξ	ρ	$\tilde{\rho}$
$SU(5)$	1	1	1	1	1	1	1	1	1	1	1	1	1
T'	3	2''	2'	1''	1'	3	2''	2'	1''	1'	3	1	1
$U(1)_R$	0	0	0	0	0	0	0	0	0	0	0	0	0
Z_{12}^u	0	3	9	0	0	6	3	9	6	0	6	6	6
Z_8^d	0	0	0	0	0	2	1	7	6	4	4	4	4
Z_8^v	4	1	7	0	0	2	7	1	6	4	0	0	0
Z_8	4	7	5	4	0	2	5	3	6	4	4	4	4
Z_6	4	4	2	4	2	0	3	3	0	0	0	0	0
Z_6'	4	4	2	4	2	3	0	0	0	0	0	0	0
Z_4	0	2	2	0	0	0	3	1	2	0	0	0	0

$$\begin{aligned} \mathcal{W}_{Y_u} = & y_{33} H_5^{(1)} T_3 T_3 + \frac{y_{23}}{\Lambda^2} (T_a \tilde{\phi})_{2'} H_5^{(2)} (T_3 \tilde{\psi}'')_{2''} + \frac{y_{22}}{\Lambda^3} (T_a \tilde{\psi}'')_3 (H_5^{(1)} \tilde{\zeta}')_{1'} (T_a \tilde{\psi}'')_3 \\ & + \frac{y_{21}}{\Lambda^4} (T_a \tilde{\phi})_{2'} (H_5^{(1)} \tilde{\zeta}')_{1'} (\tilde{\psi}' (T_a \tilde{\psi}')_3)_{2'} + \frac{y_{11}}{\Lambda^4} ((T_a \tilde{\phi})_2 \tilde{\zeta}'')_{2''} H_5^{(3)} (\tilde{\zeta}'' (T_a \tilde{\phi})_{2''})_{2'} , \end{aligned}$$

$$\begin{aligned} \mathcal{W}_{Y_{d,e}} = & \frac{y_{33}}{\Lambda^2} ((\bar{H}_5^{(2)} \bar{F})_3 \phi)_{1'} (H_{24}'' T_3)_{1''} + \frac{y_{22}}{\Lambda^3} ((\phi T_a)_2 H_{24}'')_2 (\psi' (\bar{H}_5^{(1)} \bar{F})_3)_2 \\ & + \frac{y_{12}}{\Lambda^4} (((T_a \tilde{H}_{24}'')_{2''} (\bar{F} \psi')_{2''})_3 \psi')_{2''} (\bar{H}_5^{(3)} \psi')_{2'} + \frac{y_{21}}{\Lambda^4} ((\bar{F} \psi')_{2''} (\zeta'' \bar{H}_5^{(1)})_{1''} \zeta'')_2 (T_a \phi)_2 \\ & + \frac{y_{11}}{\Lambda^4} ((\bar{F} \psi'')_{2'} (H_{24}'' \psi'')_{2'} \bar{H}_5'')_{1'} (T_a \psi'')_{1''} , \end{aligned}$$