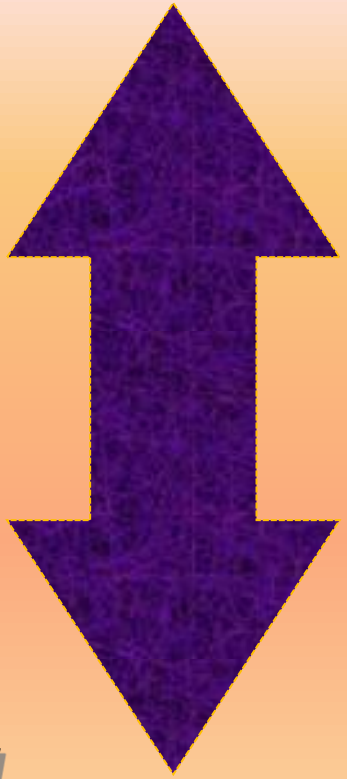
Texture Zero Mass Matrices

Flavor Mixing

H. Fritzsch

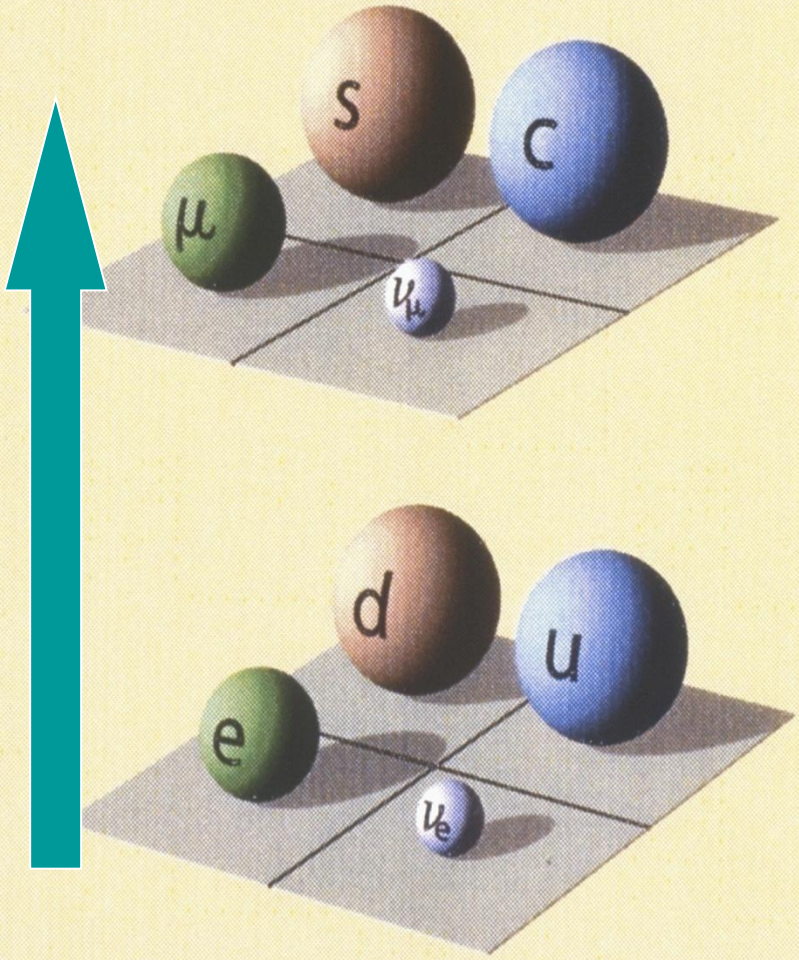
flavor mixing angles



fermion masses

flavor mixing

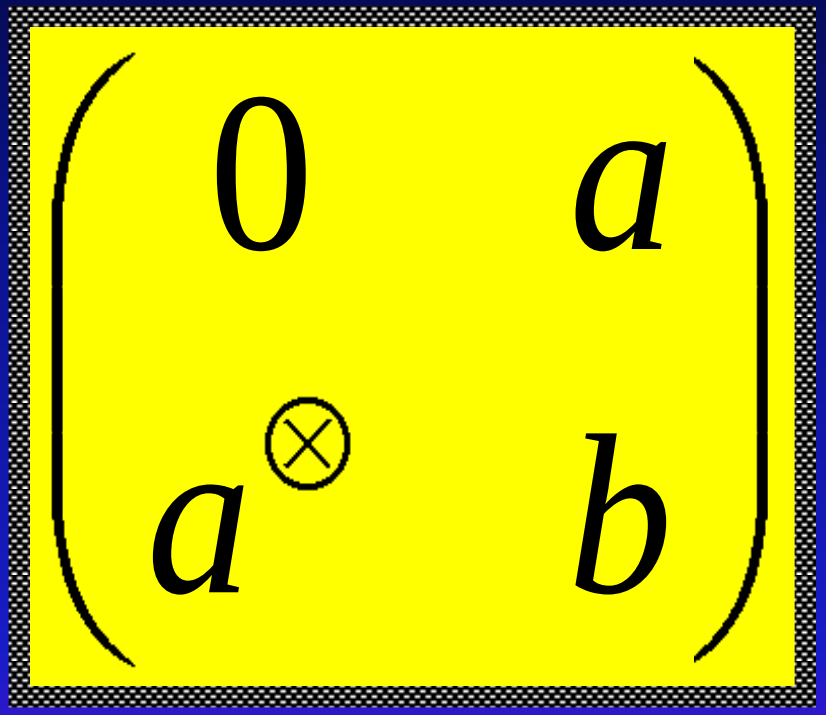
2 families



mass matrices:

texture 0

u,c - d,s



H. Fritzsch
S. Weinberg
1978

mixing angles <=> masses

$$\begin{pmatrix} 0 & a \\ a^{\otimes} & b \end{pmatrix} \Rightarrow \begin{pmatrix} -m_u & 0 \\ 0 & m_c \end{pmatrix}$$

$$\tan 2\theta_u = \frac{2\sqrt{m_u m_c}}{m_c - m_u} \qquad \theta_u \approx \sqrt{\frac{m_u}{m_c}}$$

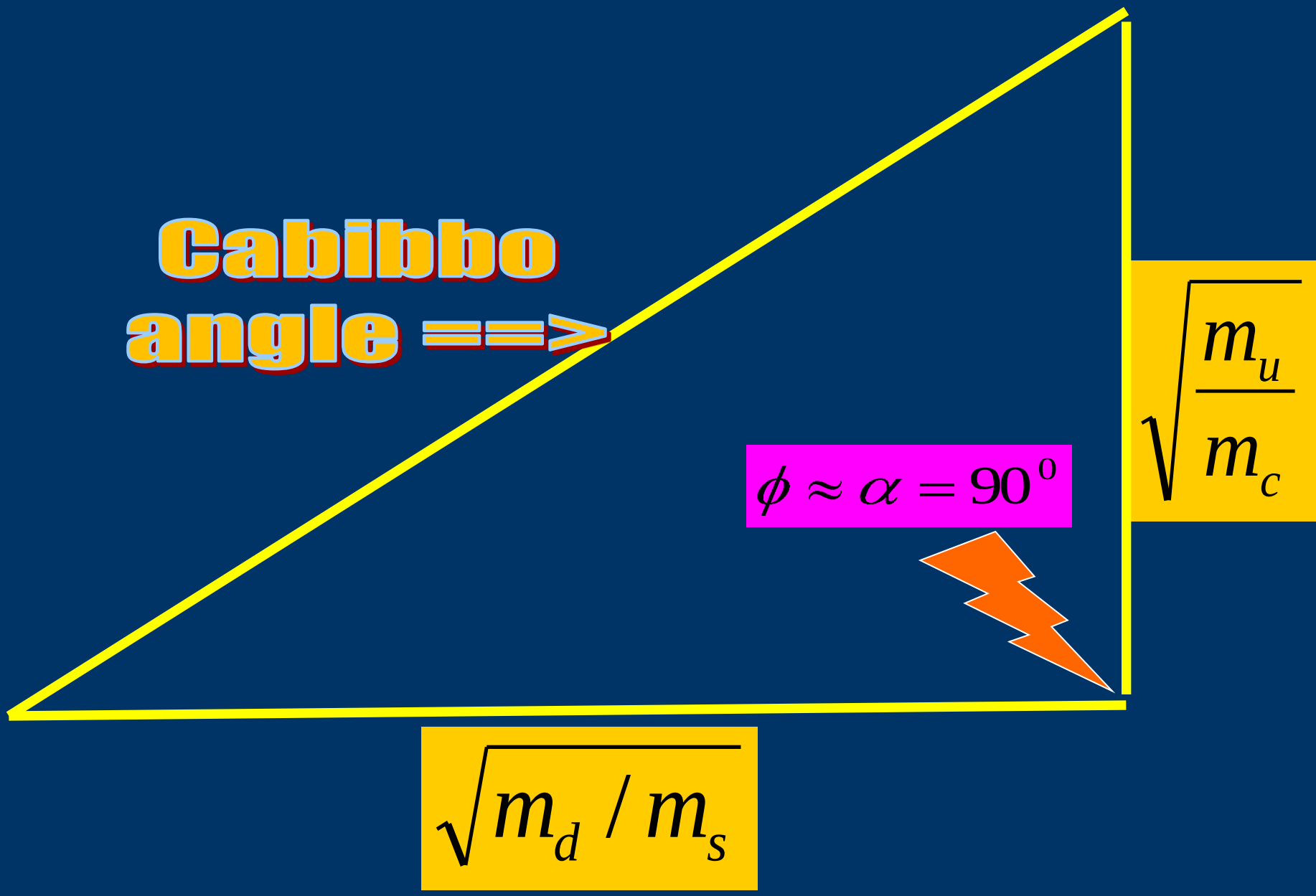
$$\sqrt{\frac{m_d}{m_s}} \approx 0.21$$

$$\sqrt{\frac{m_u}{m_c}} \approx 0.07$$

Cabibbo angle

$$\theta_c \cong \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right|$$

13^o

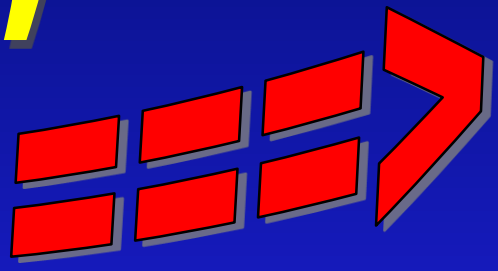

Cabibbo
angle ==>
$\phi \approx \alpha = 90^{\circ}$
$\sqrt{\frac{m_u}{m_c}}$
$\sqrt{m_d / m_s}$

texture zero

$$\begin{pmatrix} 0 & a \\ a^{*} & b \end{pmatrix}$$

texture zero

SU(2) x SU(2)



Grand Unification

Grand Unification

SU(3)xSU(2)xU(1)

==>SO(10)

Fritzsch - Minkowski; Georgi - 1975


SO(10)
SO(6) x SO(4)
SU(4) x SU(2,L) x SU(2,R)
SU(3) x SU(2,L) x U(1)

lefthanded and righthanded neutrinos

electro-weak sector

$$SU(2)_L \times SU(2)_R \times U(1)$$

new energy scale for righthanded SU(2)

U(1) x U(1) symmetry

$$\begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_L \to e^{i\alpha} \begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_L \qquad \begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_R \to e^{-i\alpha} \begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_R$$

$$\begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L \to e^{i\beta} \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L \qquad \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_R \to e^{-i\beta} \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_R$$

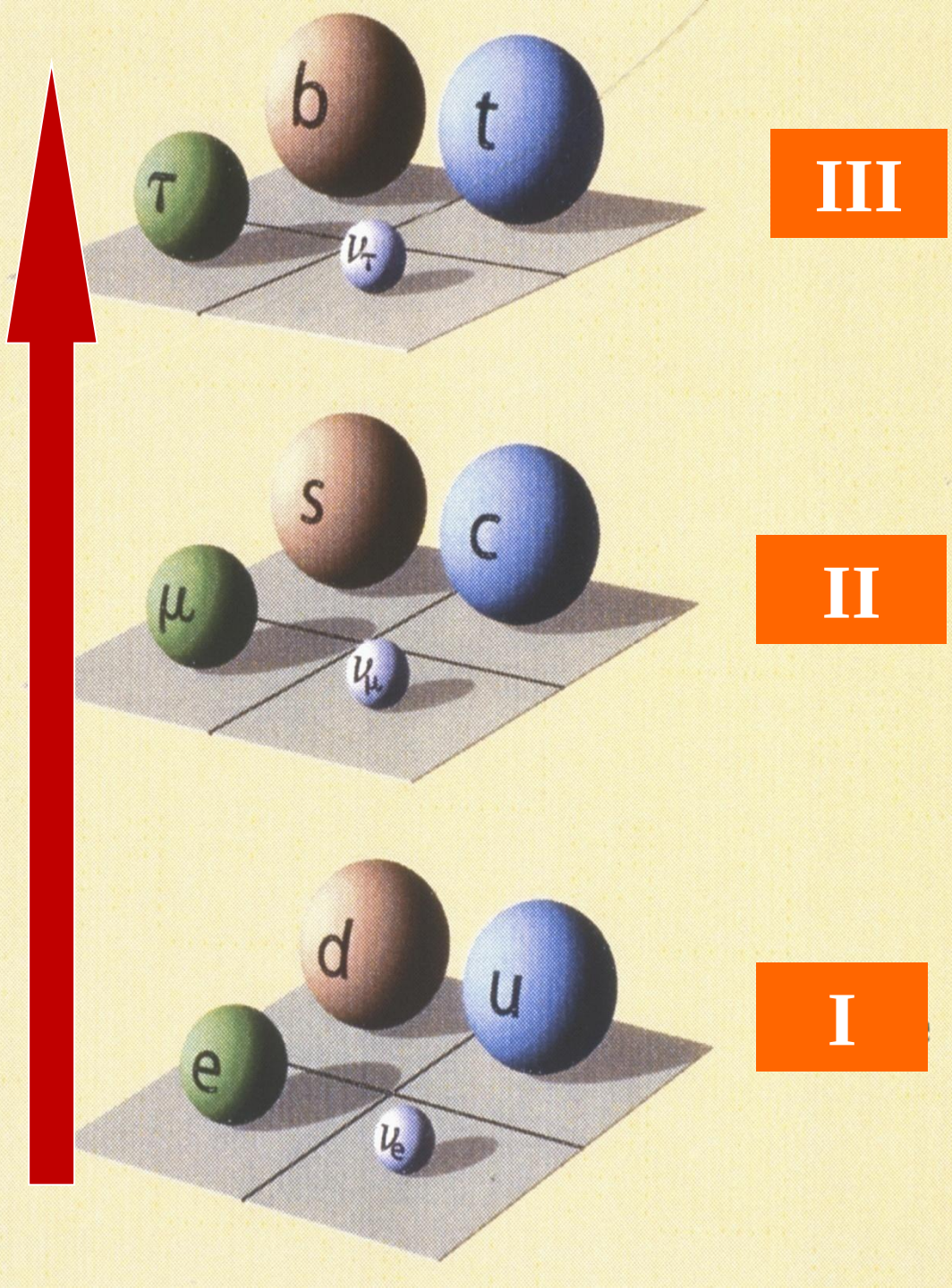
$$U \to e^{2\beta i} U \qquad V \to e^{i(\alpha+\beta)} V$$

$$\rightarrow g\,(\overline{c_0, s_0})_L \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_R + h\,(\overline{c_0, s_0})_L V \begin{pmatrix} u_0 \\ d_0 \end{pmatrix}_R$$

$$+ h'\,(\overline{u_0, d_0})_L V \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_R + \text{h. c.}$$

3 families

flavor mixing



CKM – matrix:

$$\begin{bmatrix} 0{,}97459 & 0{,}2257 & 0{,}00359 \\ 0{,}2256 & 0{,}97334 & 0{,}0415 \\ 0{,}00874 & 0{,}0407 & 0{,}99133 \end{bmatrix}$$

New parametrization :

$$V = \begin{bmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix} \bullet \begin{bmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

H. Fritzsch / Z. Xing

texture zeros

$$\begin{pmatrix} 0 & A & 0 \\ A^{*} & C & B \\ 0 & B^{*} & D \end{pmatrix}$$

$$\begin{pmatrix} 0 & A & 0 \\ A^{*} & C & B \\ 0 & B^{*} & D \end{pmatrix}$$

$$\tan\theta_d = \sqrt{m_d} \,/\, \sqrt{m_s}$$

$$\tan\theta_u = \sqrt{m_u} \,/\, \sqrt{m_c}$$

$$\tan\theta_d = \sqrt{m_d} / \sqrt{m_s}$$

$$\theta_d \approx 13.0 \pm 0.4^o$$

$$Exp: 11.7^o \pm 2.6^o$$

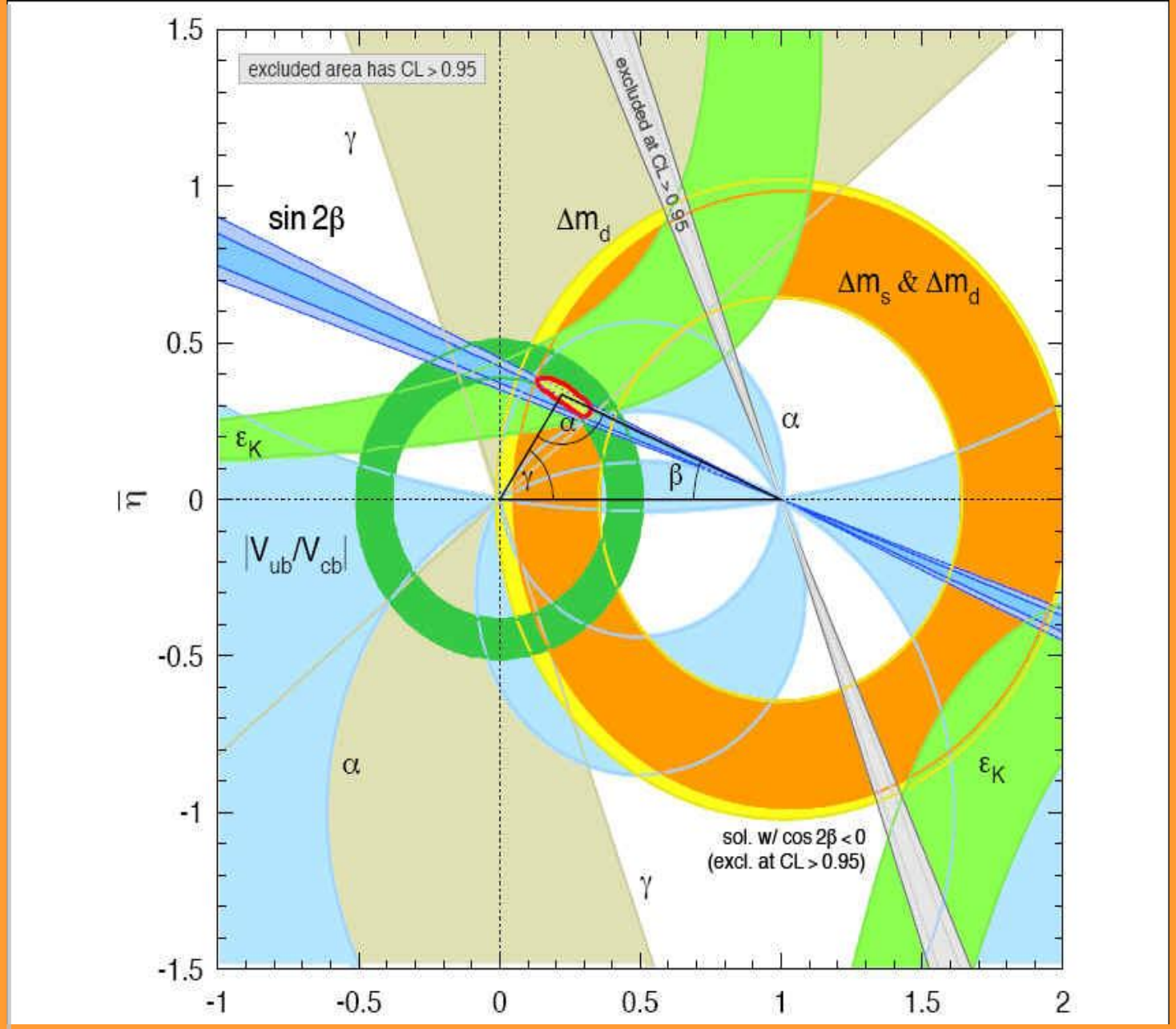
$$\tan\theta_u = \sqrt{m_u} / \sqrt{m_c}$$

$$\theta_u \approx 5.0^o \pm 0.7^o$$

$$Exp : 5.4^o \pm 1.1^o$$

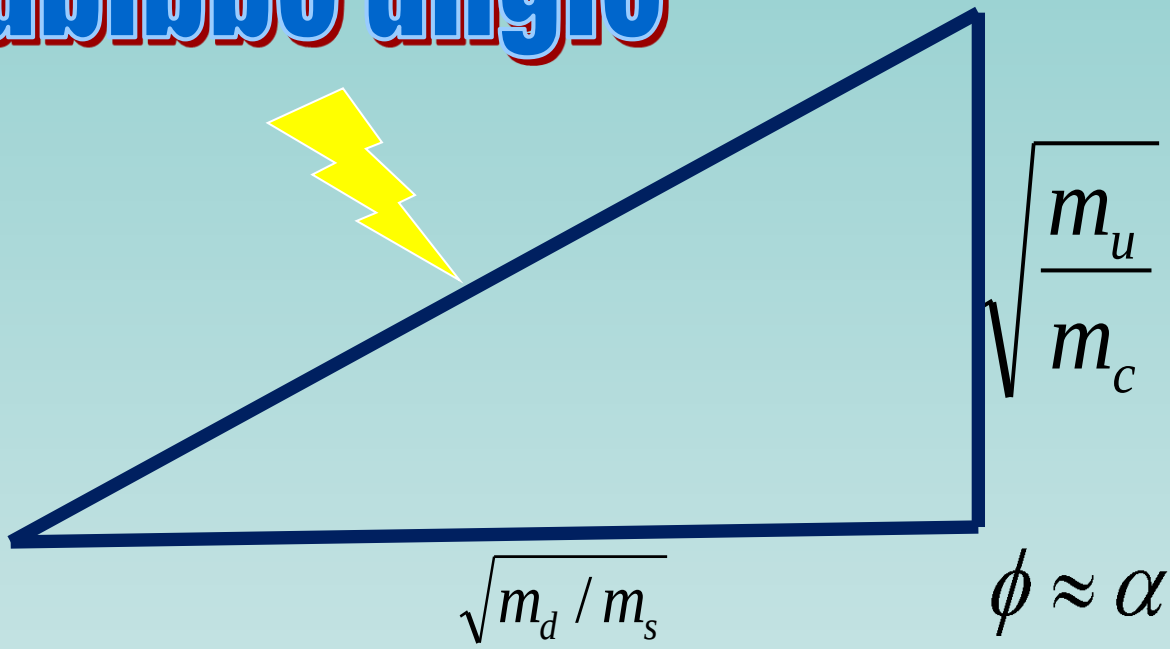
unitarity triangle

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cu} & V_{cs} & V_{cb} \\ V_{tu} & V_{ts} & V_{tb} \end{pmatrix}$$



alpha: 86 ... 95 degrees

Cabibbo angle



Unitarity triangle

(rectangular)

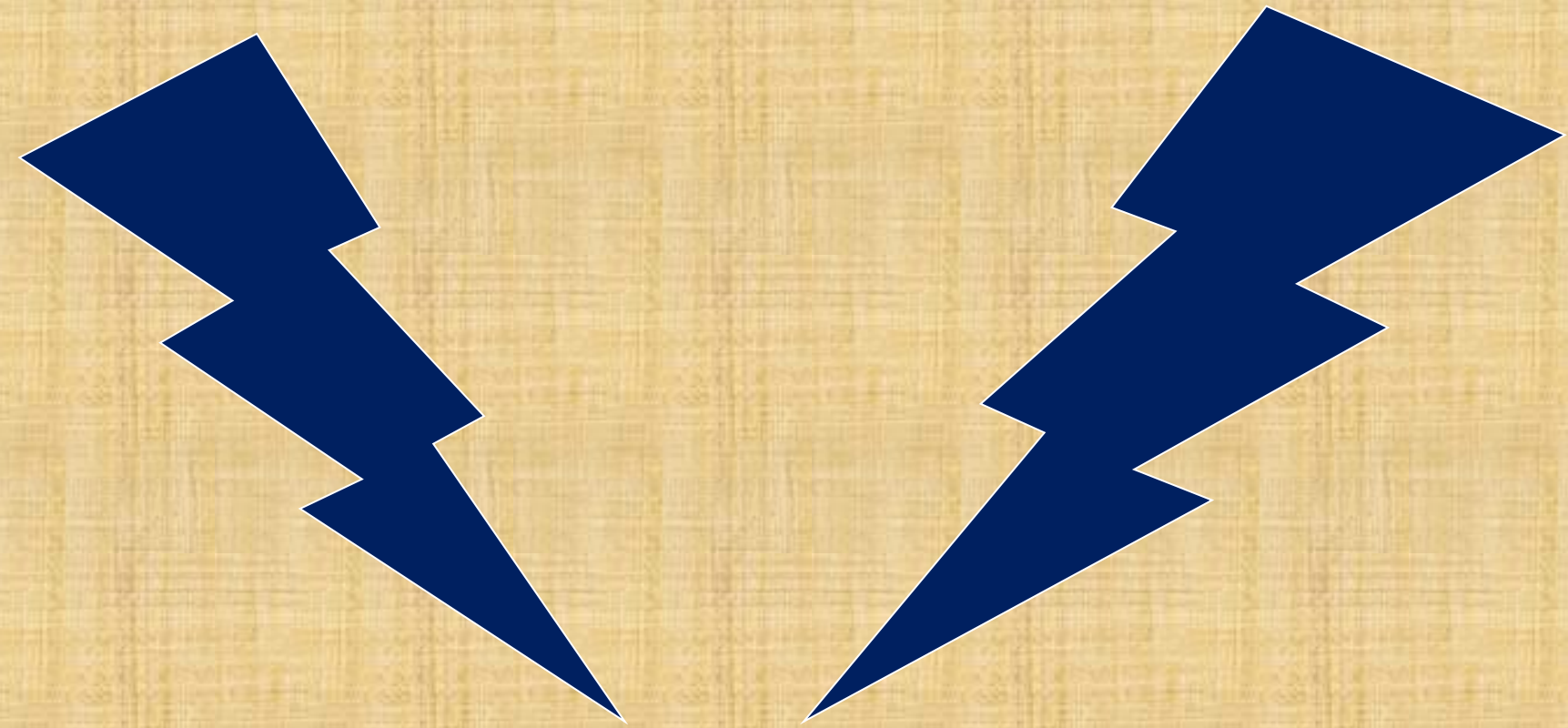
Unitarity triangle:

$$\tan\beta = \frac{\sin\theta_u \cos\theta_d}{\cos\theta_u \sin\theta_d}$$

$$==>$$

$$\sin 2\beta \cong 0.663$$

$$Exp: \sin 2\beta = 0.681 \pm 0.025$$



neutrinos

neutrino mixing matrix

(==> CKM Matrix)

$$V = \begin{pmatrix} V_{1e} & V_{2e} & V_{3e} \\ V_{1\mu} & V_{2\mu} & V_{3\mu} \\ V_{1\tau} & V_{2\tau} & V_{3\tau} \end{pmatrix}$$

V = U x P

$$P = \begin{bmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} \cos\theta_l & \sin\theta_l & 0 \\ -\sin\theta_l & \cos\theta_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \bullet$$

$$\begin{bmatrix} \cos\theta_\nu & -\sin\theta_\nu & 0 \\ \sin\theta_\nu & \cos\theta_\nu & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta \approx \theta_{at}$$

$$\theta_\nu \approx \theta_{sun}$$

$$\theta_l \approx reactor - angle$$

Fritzsch - Xing

Kamiokande, SNO

$$31.7^o \le \theta_{sun} \le 36.3^o$$

$$38^o \le \theta_{at} \le 52^o$$

$$\Delta m_{21}^{\ 2} \approx 7.6 \cdot 10^{-5} eV^2$$

$$\Delta m^2_{32} \approx 2.4 \cdot 10^{-3} eV^2$$

3 texture zeros

$$\begin{pmatrix} 0 & A & 0 \\ A^{*} & C & B \\ 0 & B^{*} & D \end{pmatrix}$$

$$U = \begin{bmatrix} \cos\theta_l & \sin\theta_l & 0 \\ -\sin\theta_l & \cos\theta_l & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} e^{-i\varphi} & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \bullet \begin{bmatrix} \cos\theta_\nu & -\sin\theta_\nu & 0 \\ \sin\theta_\nu & \cos\theta_\nu & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\tan 2\theta_l = \frac{2\sqrt{m_e m_\mu}}{m_\mu - m_e}$$

$$\tan 2\theta_\nu = \frac{2\sqrt{m_1 m_2}}{m_2 - m_1}$$

observation

$$\theta_\nu \approx 33^o \quad \theta \approx 45^o$$

$$\Longrightarrow m_1 / m_2 \approx 0.42^{+0.12}_{-0.04}$$

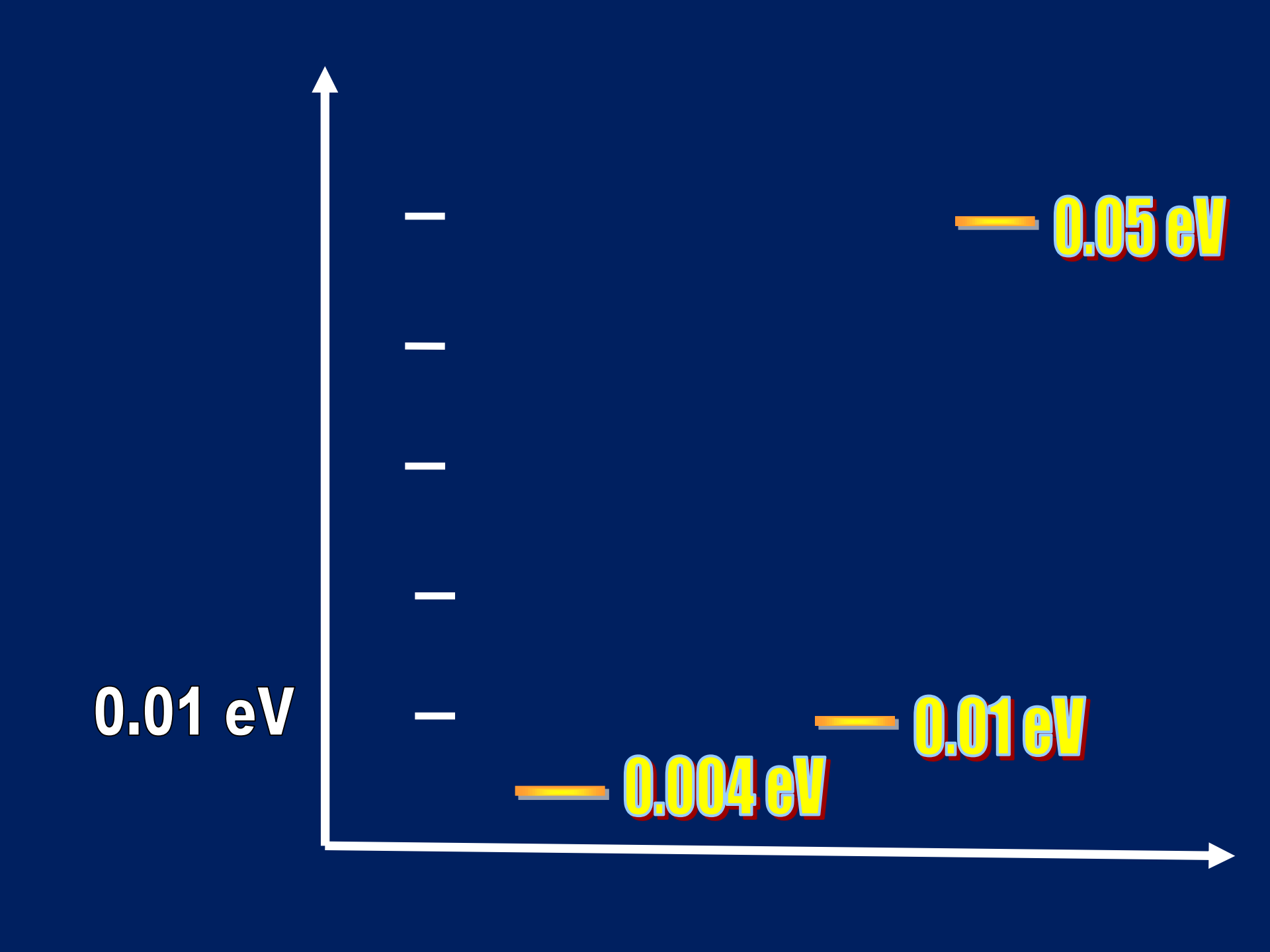
weak mass hierarchy

$$\Delta m_{21}^{\ 2} \approx 7.6 \cdot 10^{-5} eV^2$$

$$\Delta m^2_{32} \approx 2.4 \cdot 10^{-3} eV^2$$

$$m_1 / m_2 \approx 0.42$$

==> neutrino masses


0.05 eV
0.01 eV
0.01 eV
0.004 eV

m(1) = (0.0040 +/- 0.001) eV

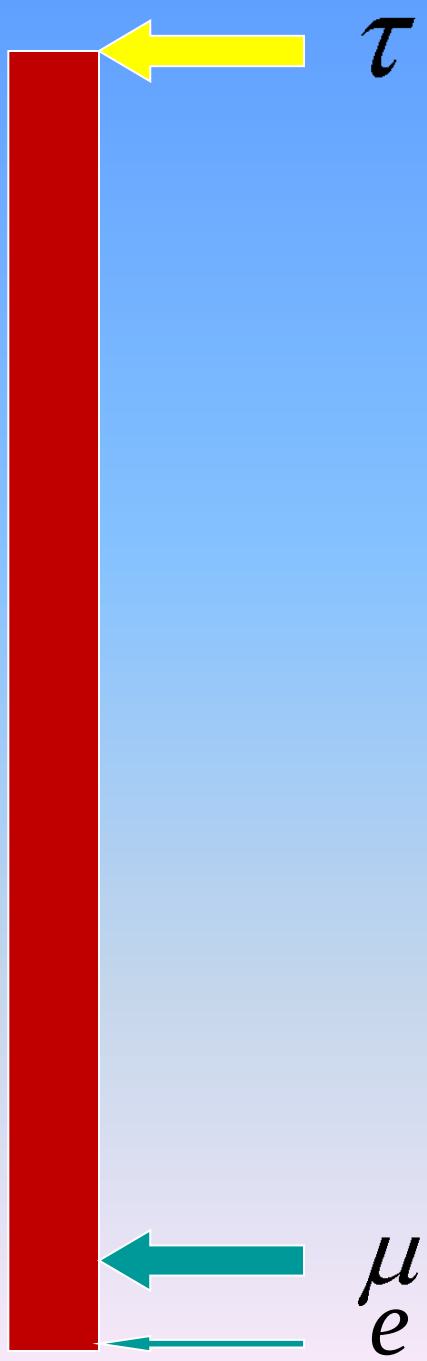
m(2) = (0.0096 +/- 0.002)eV

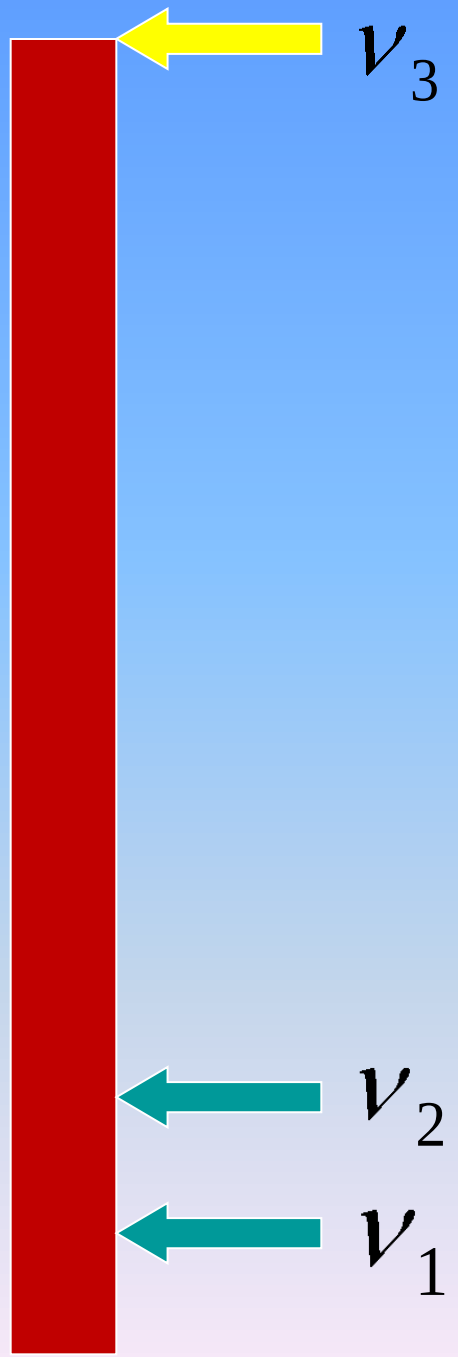
m(3) = (0.049 eV +/- 0-007) eV

normal mass hierarchy
(no inversion)

masses

(relative)





weak mass hierarchy for neutrinos

large mixing angles

Neutrino Mixing Matrix

$$V = \begin{pmatrix} V_{1e} & V_{2e} & V_{3e} \\ V_{1\mu} & V_{2\mu} & V_{3\mu} \\ V_{1\tau} & V_{2\tau} & V_{3\tau} \end{pmatrix}$$

relations between quark masses ?

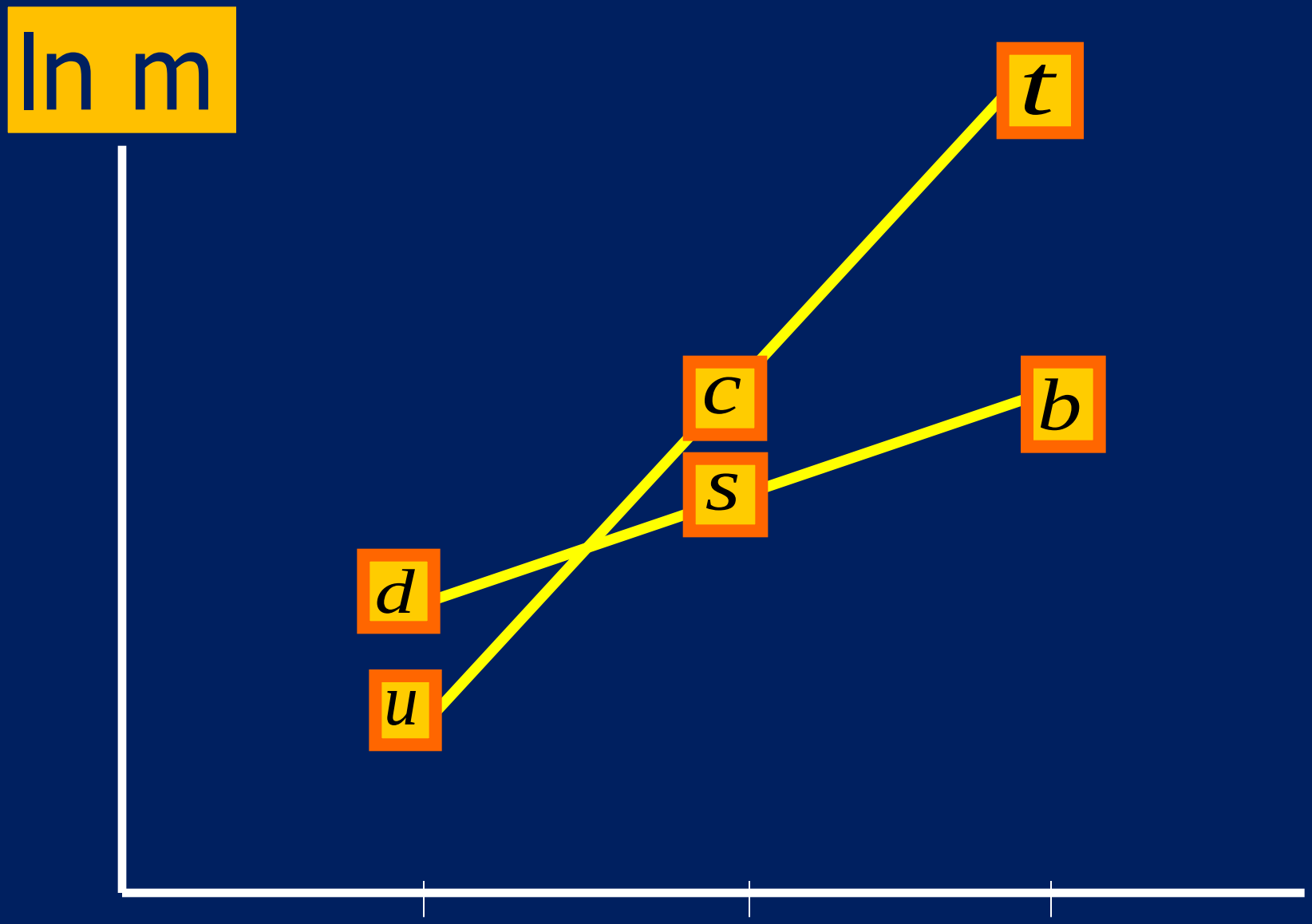
- Observed:

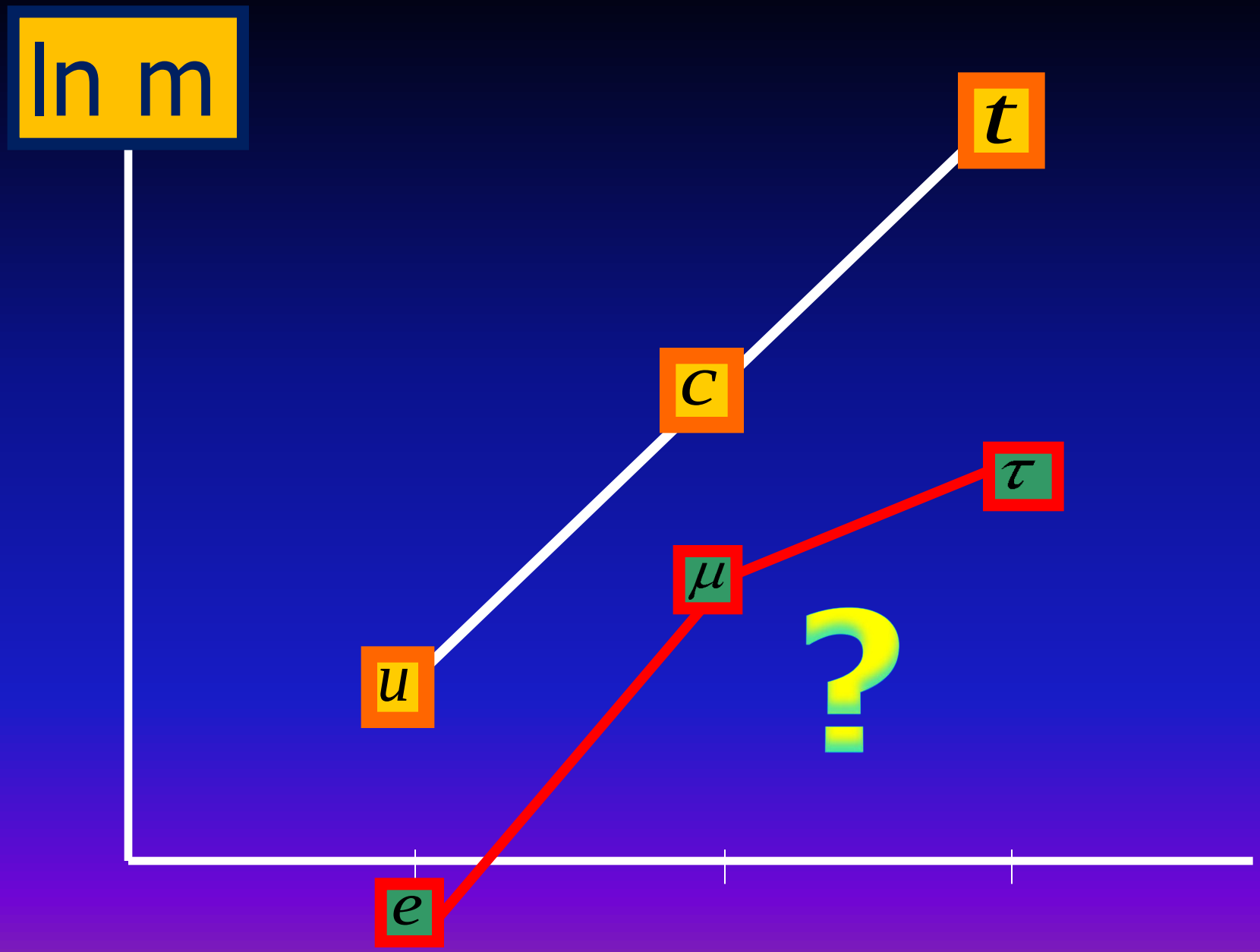
$$m(c) : m(t) = m(u):m(c)$$

1/207 1/207

$$m(s):m(b) = m(d):m(s)$$

1/23 1/23


ln m
t
c
b
s
d
u


ln m
t
c
τ
μ
u
?
e

radiative corrections

$$m(e) = m(e^0) + const.\left(\frac{\alpha}{\pi}\right) m(\tau) + \ldots$$

$$\approx 6.3 \quad MeV - 5.8 \quad MeV \approx 0.511 \quad MeV$$

$$m\left(e^0\right) \approx 6.3 \quad MeV$$

-muon and tauon mass-

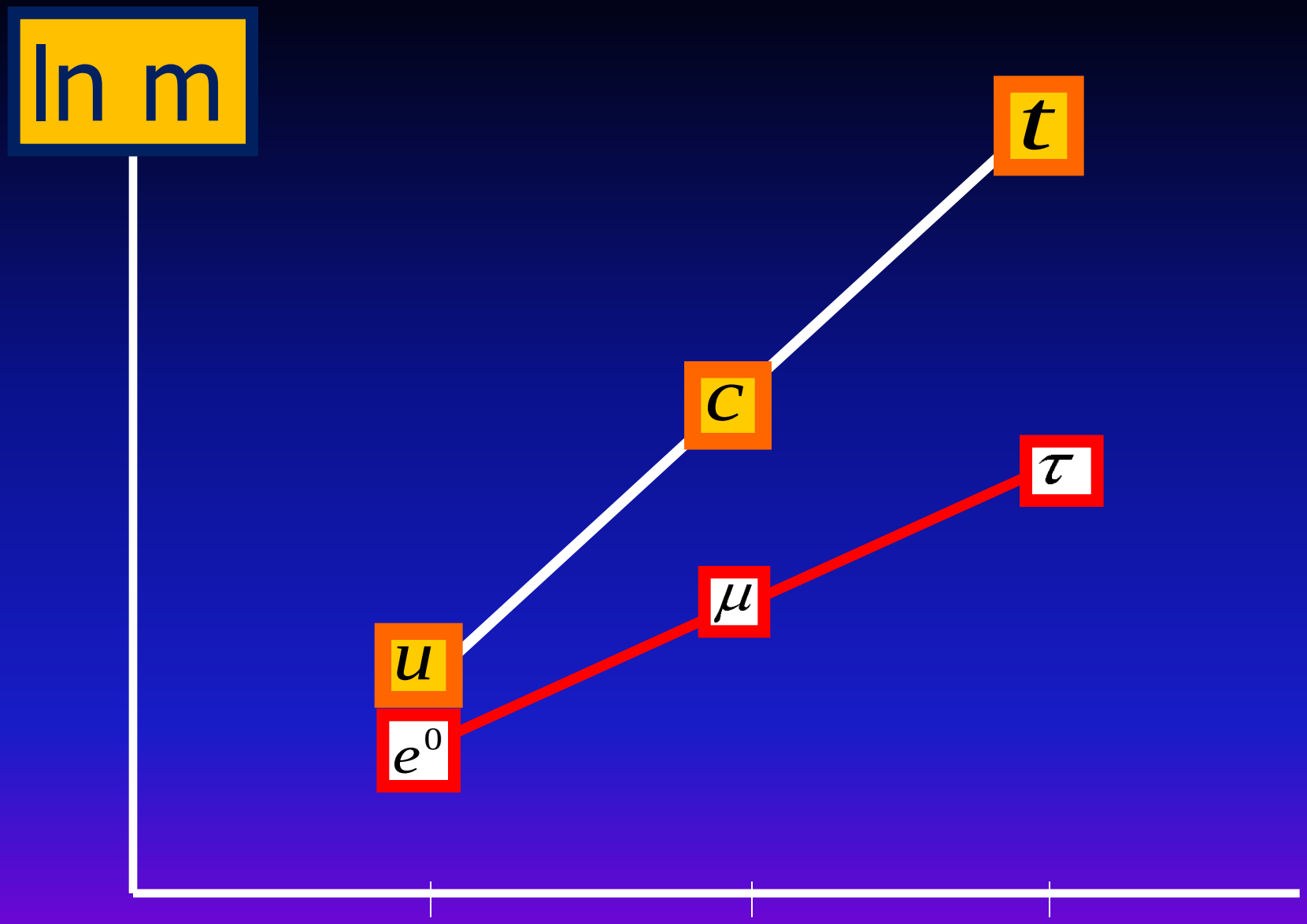
only small changes by radiative corrections

$$m(\mu) = m(\mu^o) + const.\left(\frac{\alpha}{\pi}\right)m(\tau) + \ldots$$

$$\approx 111.5 \quad MeV - 5.8 \quad MeV \approx 105.7 \quad MeV$$

$$\frac{m_{\mu}^{0}}{m_{\tau}^{0}} \cong 0.06$$

$$\frac{m_{e}^{o}}{m_{\mu}^{0}} \cong 0.06$$


ln m
t
c
τ
μ
u
e0

$$V_{e3} = \sin\theta_l \sin\theta_{at}$$

$$\tan\theta_l = \sqrt{\frac{m_e^0}{m_\mu^0}} \cong 0.25$$

$$38^\circ \leq \theta_{at} \leq 52^\circ$$

$$|V_{e3}| = \sin\theta_{13} \Rightarrow 0.148...0.190$$

$$\sin^2 2\theta_{13} \cong 0.1124 \pm 0.027$$

$$= 0.085 \Leftrightarrow 0.139$$

$$\sin^2 2\theta_{13} = 0.1124 \pm 0.027$$

$$\theta_{13} = 9.8^0 \pm 1.3^0$$

Reno

$$\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.005$$

$$\sin^2 2\theta_{13} = 0.1124 \pm 0.027$$

$$\theta_{13} = 9.8^0 \pm 1.3^0$$

Daya Bay

$$\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005$$

Conclude:

neutrino masses

m(1): 0.0041 eV
m(2): 0.0097 eV
m(3): 0.051 eV

$$|V_{e3}| \approx 0.17$$

$$\sin^2 2\theta_{13} \approx 0.11$$

texture zero mass matrices

masses of quarks - leptons

flavor mixing angles