Neutrino mass & structural logic of NMSO(10)

Charanjit S. Aulakh

(Panjab Univ, Chandigarh)

With Ila Garg, Charanjit Kaur

- Hint Complexes and Scientific desirability.
- NMSO(10) GUT Basics and Virtues
- Phenomenological difficulties and predictive resolution.
- Inflation and the Supersymmetric Type I seesaw.
- Embedding SSI in the NMSO(10)GUT

SM,BSM & Cosmology Hint Complex

- SM paradigm : Gauge Invariance plus Parameters from expt.: $m_{u,d,l}(9), \theta^q_{12,23,13}(3), \delta^q(1), v_{EW}, \alpha_i(M_Z), m_H)$. Over Constrained by experiment! Risky ! : every new measurement threatens adequacy.

- Neutrino oscillations require BSM paradigm to explain: + SM Gauge invariance

$$\Rightarrow m_\nu \neq 0 : \Lambda_{B-L} \sim M_W^2/m_\nu > 10^{13} - 10^{16} GeV$$

OR unnaturally light and weakly coupled ν_Rs New parameters : $m_\nu(2+1?), \theta^L_{12,23,13}(3)$,New Phases $\delta^L(1?), \alpha(?), \beta(?)$

- Unification of couplings in SM $\sim 10^{14}$ GeV $\Rightarrow \not{B} - \not{L}$

- Scalar Higgs $\Rightarrow$ Instability mixes M_W and Λ_{NP} : GHP

- Susy$\Rightarrow$ stabilization & exact g_i^{MSSM} confluence at $M_X^0 \sim 10^{16.25}$ GeV
- Astronomy : Dark, Collision-less Matter $\Omega h^2 \sim 0.1$ necessary; Neutral, colourless to evade limits; Required relic density $\Rightarrow \rho_{CDM} \sim 0.3 GeV/cm^3$, $\tau_\chi >> 10^{18}$ $sec, \sigma_{ann} \sim 10^{-33} cm^2$. About right for a 100 GeV WIMP (χ): MSSM Neutralino $\equiv$ fits the bill !
- CMB Observations: Nearly Scale invariant Power spectrum, flat universe $\Rightarrow$ Inflation
- Ratio of Tensor and scalar power $P_T/P_S < 10^(- 1)$ $\Rightarrow$ scale of inflation $< 2 \times 10^{16}$ GeV
- Reheating : Inflaton-matter coupling ? $\Rightarrow$ MSSM/GUT inflaton ?
- $n_B/n_\gamma, E_{sphaleron} \sim M_W/\alpha \Rightarrow$ Lepto-genesis preferred.

WANTED : FALSIFIABLE THEORIES : RISKY PREDICTIONS

- Ambition increases Risk !
- Limitation and specificity increases Risk of un-reconcilable conflict between parameter regions consistent with disparate BSM phenomenology sectors.
- Piecemeal/global approach : SM as paradigm suggests extraction of predictions and isolation of tensions requires completely specified and calculable theory. 'In principle ' piecemeal approaches to phenomenology inadequate.
- Strategy : Accentuate probability of falsification by over determining theory : demand the same Minimal Higgs and gauge structure and parameter ranges encompass all or as many as possible of the data/hints simultaneously.

- NMSO(10) SSB and hence phenomenological implications are completely calculable and potentially realistic/falsifiable. Forced into distinctive structural choices to remain viable. Predicts scales and parameter magnitudes relevant to BSM phenomenology and cosmology : $y_{d,s}^{MSSM} << y_{d,s}^{SM}$, $M_i \sim TeV$, $m_{\tilde{f}} \sim 5-50\ TeV$ (except $m_{\tilde{\mu}^c} << TeV$!), $A_0, \mu \sim 100TeV$.

- Provides structural explanations for $d = 4, 5, \Delta B \neq 0$ suppression and rates, controlled LFV–$m, \theta_\nu, m_{\tilde{f}}, \Gamma_{\Delta B\neq 0}$ connection. Linked to to $M_{Inflation}$, Lepto-genesis etc. High Potential for falsification. Pursue to the bitter(sweet?) end.

MSSM, R PARITY and B-L

- SM : Gauge Invariance, Renormalizability $\Rightarrow$
  $B, L(perturbative) \Rightarrow B - L$ (Exact, Unique Global U(1)).

- MSSM: Sfermions & Shiggs $\Rightarrow$
  $\mathcal{L}_{\Delta_{B,L}\neq 0} = [W_{\not R_p}]_F = [\mu' LH + \lambda LLe^c + \lambda' LQd^c + \lambda'' u^c d^c d^c]_F \Rightarrow$
  catastrophic B, L violation $\Rightarrow \tau_p^{d=4} \sim (\frac{gM_S}{\lambda_{\not R} M_X})^4 \tau_p^{d=6} \Rightarrow$
  $\lambda\lambda' < 10^{-26}$

- $R_p : Z_2 :$ *Susy Particles odd* : forbids B,L violating terms
  Mohapatra : (1986) : $R_p = (-1)^{3(B-L)+2S} = (-)^{2S} M_p \Rightarrow$
  $M_p \subset U(1)_{B-L} \subset G_{LR} \subset G_{PS} \subset SO(10)$

- Even B-L vevs(M_ν Compatible) $\Rightarrow$ R$\surd\surd \Rightarrow : \Rightarrow$
  LSP Stable : good as Dark Matter
  MSLRMs : CSA,Benakli,Senjanovic, Melfo ....(1995-8)

VIRTUES OF SO(10) UNIFICATION

- $\{Q_L, L_L, u_L^c, d_L^c, l_L^c) \oplus \nu_{\mathbf{L}}^{\mathbf{c}}\} \equiv 16$: Tight and complete
- Simple Tri-band FM Higgs Channel Spectrum

$$16 \otimes 16 = 10 \oplus 120 \oplus 126 \Rightarrow (10 + 120 + \overline{126}_H)$$
$$\overline{126} = (15, 2, 2) + \Delta_R(10, 1, 3) + \Delta_L(\overline{10}, 3, 1) + (6, 1, 1)$$

- $M_p \subset U(1)_{B-L} \subset G_{LR} \subset G_{PS} \subset SO(10) \quad \bigoplus < \Delta_{L,R} > \Rightarrow R_p,$ Stable LSP
- NATURAL HOME TO BOTH SEESAWS :
  $\vec{\Delta}_R(1, 3, -2), \vec{\Delta}_L(3, 1, 2) \subset \overline{126}$ PRESERVE R_p :
  $M_{B-L} \sim < \vec{\Delta}_R >_{SM=0} \Rightarrow M_{\nu^c} \Rightarrow M_\nu^I$
  $\frac{v_W^2}{M_{B-L}} \sim < \vec{\Delta}_L >_{Y=2, T_{3L}=-1} \Rightarrow \Rightarrow M_\nu^{II}$

TWO SCHOOLS OF SO(10)

Renormalizable SO(10)	NON-REN GUTS
Renormalizable couplings	Non Renorm. couplings
No ad-hoc symmetries	Ad-hoc symmetries necessary
Large(126,210,..) few (AS)	Small (10,16,45,54) irreps (AF)
# Parameter minimal	Unlimited # parameters
No Higgs duplication	Duplicates Higgs
$M_p \subset SO(10)$	R_p broken
Only B-L even vevs	"string motivated" Z_2
Higgs-Matter distinct	Higgs-Matter mix
a) $210 \oplus 126 \oplus \overline{126}$	$16_H^n \oplus 10 \oplus 45^m$ plethora
b)$54 \oplus 45 \oplus 126 \oplus \overline{126}$	

NMSO(10) GUT

- **AM Higgs** : $< \mathbf{210}(\Phi_{ijkl}),\ \ \overline{\mathbf{126}}(\overline{\boldsymbol{\Sigma}}_{ijklm})\ \ ,\mathbf{126} > \Rightarrow$
  $Susy\ SO(10) \longrightarrow MSSM$ (CSA, Mohapatra, CKN (1983))
- **Superpotential**

$$
\begin{aligned}
W &= W_{MSGUT} + W_{120} \\
&= m\ 210^2 + \lambda\ 210^3 + M\ 126 \cdot \overline{126} + \eta\ 210 \cdot 126 \cdot \overline{126} \\
&+ 10 \cdot 210(\gamma\ 126 + \bar{\gamma}\ \overline{126}) \\
&+ M_H\ 10^2 + h_{AB}\ 16_A \cdot 16_B + f'_{AB}\ 16_A 16_B + W_{120} \\
W_{120} &= M_O\ 120 \cdot 120 + k\ 10 \cdot 120 \cdot 210 + \rho\ 120 \cdot 120 \cdot 210 \\
&\quad +\zeta\ 120 \cdot 126 \cdot 210 + \bar{\zeta}\ 120 \cdot \overline{126} \cdot 210 + g_{[AB]} 16_A \cdot 16_B \cdot 120
\end{aligned}
$$

MSGUT Parameters : (**25**) **Minimal ! but fails**
ABMSV(2003)

$W_{120} : M_O, k, \rho, \zeta, \bar{\zeta}, g_{AB} : (1+1+1+2+3) \times 2 - 1 = 15 \quad \Rightarrow Total = 39$

(Still Minimal but viable !)

- Calculable SSB at M_X :**GUT scale VEVS** :
  $SO(10) \rightarrow MSSM$

$$\langle(15,1,1)\rangle_{210} \; : \; a \qquad \langle(15,1,3)\rangle_{210} : \omega$$
$$\langle(1,1,1)\rangle_{210} \; : \; p \qquad \langle(10,1,3)\rangle_{126,\overline{126}} : \sigma, \bar{\sigma}$$

- D Terms, preserve SUSY : $|\sigma| = |\overline{\sigma}|$
- F Terms : SSB completely analyzable 4 eqns $\Rightarrow$ **Cubic in** $x = -\lambda\omega/m : \xi = \frac{\lambda M}{\eta m}$. (ABMSV 2003)
  $8x^3 - 15x^2 + 14x - 3 = -\xi(1-x)^2$
- MSGUT : 45+48+ 10+252+210=565 Fields, NMSGUT: 565+120= 685 fields $\Rightarrow$ 26 MSSM-irrep types : Chiral GUT

scale spectra and Threshold effects. :CSA, Girdhar(2003,2004) ; Fukuyama, Ilakovac, Kikuchi, Mejanac, Okada (2004), BMSV (2004).(CSA Garg, 2006)

- Neutrino mass Type I ,Type II GENERIC fits : freedom to choose M_ν scale, Relative strength of Type I / Type II *assumed.*

- NOT JUSTIFIED IN MSGUT where magnitude and relative strength fully specified : is it viable ?? NO! **Demo** CSA(2005-May) **Proof**: CSA,Garg (2005-November), **Checked** Bertolini, Schwetz,Malinski(2006-April)

NMSO(10) FERMION FITS & SUSY THRESHOLDS

- NEW SCENARIO : $h \oplus g >> f \quad \Rightarrow (m_{q,l}, \theta_q^i, \delta_c)$.
  $f << h, g \Rightarrow$ Type I boosted ($\hat{n} \sim \hat{f}^{-1}$).
- $\tan\beta \sim m_t/m_b \sim 45-60$ generic in SO(10) GUTs with single 10-plet mainly responsible for 3-generation charged fermion masses . Single **10**, $t-b-\tau$ unification allows $\tan\beta \sim 45-60 \sim m_t/m_b$ only.
- $10 \oplus 120$ only for Charged fermion fit $\Rightarrow$ TENSIONS :
  (a) $m_{d,s}^{MSSM}(M_Z) \sim m_{d,s}^{SM}(M_Z)/5$
  (b) Tree Level: $m_s - m_\mu = m_b - m_\tau$
  (c) $m_b^{MSSM}(M_Z) \sim 1.1\, m_b^{SM}(M_Z)$ well known tension in b-τ-t unification.
- Well known large $\tan\beta$ driven (H-Hbar mixing) threshold

corrections to down type fermion yukawa masses. (α_s(gluino) and ($A_t y_t^2$ loops for 3d gen)) Also 10-15% gluino corrections for m_{top}.

$$y_i^{GUT}(M_S)\cos\beta = \frac{y_i^{SM}(M_S)}{1+\epsilon_i(m_{\tilde{f}}, M_i, \mu, A_t)\ \tan\beta}$$

- Dominant corrections for quarks:

$$\epsilon_i^G \quad = \quad -\frac{2\alpha_S}{3\pi}\frac{\mu}{M_3}H_2(u_{\tilde{Q}_i}, u_{\tilde{d}_i}) \qquad \epsilon^y = -\frac{y_t^2}{16\pi^2}\frac{A_t^0}{\mu}H_2(v_{\tilde{Q}_3}, v_{\tilde{u}_3})$$

- $H_2 < 0 \ \Rightarrow$ lowering $y_{d,s}^{SGUT} \Rightarrow \ \mu, -A_t >> M_{\tilde{f}}$ with cancellation for y_b. Fitting gives third gen sfermions heavier than first two. Distinct region of Susy parameter space, class of spectra, LHC signatures

M_X THRESHOLD EFFECTS, y_f & $\Gamma^{\Delta B\neq 0}_{d=5}$

- SO(10) $d=5$ B violation : FAMILIAR $\bar{t}[\bar{3},1,-\frac{2}{3}]\oplus t[3,1,\frac{2}{3}]\oplus$ NOVEL $P[3,3,\pm\frac{2}{3}], K[3,1,\pm\frac{8}{3}]$ Multiplet types contribute to baryon violation in SO(10).

- However the effective Superpotential has generic form :

$$W^{\Delta B\neq 0}_{eff} \quad = \quad -\hat{L}_{ABCD}(\frac{1}{2}\epsilon\hat{Q}_A\hat{Q}_B\hat{Q}_C\hat{L}_D) - \hat{R}_{ABCD}(\epsilon\hat{\bar{e}}_A\hat{\bar{u}}_B\hat{\bar{u}}_C\hat{\bar{d}}_D)$$

- SO(10) Yukawas and heavy masses M_i determine both fermion masses and $\Gamma^{\Delta B\neq 0}_{d=5}$

$$\hat{L}_{ABCD}(\{h,g,f\}_{AB},M_i), R_{ABCD}(\{h,g,f\}_{AB},M_i)$$

- $N^{NMSGUT}_{fields} >> N^{MSSM}_{fields} \Rightarrow$ Large wavefunction dressing of

Light fields by heavy fields : Threshold Corrections at M_X to Yukawas can play crucial role :

$$Y_u = (1 + \Delta_{\bar{u}} + \Delta_u)(1 + \Delta_H)Y_u^0$$
$$Y_d = (1 + \Delta_{\bar{d}} + \Delta_d)(1 + \Delta_{\bar{H})} Y_d^0$$

$\Delta_{f,\bar{f},H}$ wavefunction shifts due to loops with 1 or 2 Heavy fields. But $H, \bar{H}$ are mixtures of 6 pairs of doublets from $\mathbf{10}, \mathbf{120}, \mathbf{126}, \overline{\mathbf{126}}, \mathbf{210}$! Thus $\Rightarrow \Delta_f \sim 30\%, \Delta_H \sim 10^2$ easily possible.

- Threshold corrections relax stringent $b - \tau = s - \mu$ and lower required SO(10) yukwawas so much that $\Gamma_{d=5}^{\Delta B \neq 0}$ is suppressed to less than 10^{-36} yrs!

Parameter	$Value$	ParameterandValue
Δ_X	1.21	$\mathrm{A}_0 = -2.3871 \times 10^5$
Δ_G	2.057	$\tan\beta = 50.0000$
$\Delta\alpha_3(M_Z)$	-0.011	$\mathrm{R}_{\frac{b\tau}{s\mu}} = 3.0409$
$\{M^{\nu^c}/10^{12}GeV\}$	$0.000428, 1.69, 71.52$	
$\{M^{\nu}_{II}/10^{-12}eV\}$	$0.1967, 775.53, 32902.84$	
$M_\nu(meV)$	$2.464092, 7.55, 41.04$	
$\{\mathrm{Evals[f]}\}/10^{-6}$	$0.007142, 28.16, 1194.84$	
Soft parameters	$\mathrm{m}_{\frac{1}{2}} = -404.823$	$\mathrm{m}_0 = 6853.139$
at M_X	$\mu = 1.9957 \times 10^5$	$\mathrm{B} = -2.3593 \times 10^{10}$
	$\mathrm{M}^2_{\bar{\mathrm{H}}} = -3.3692 \times 10^{10}$	$\mathrm{M}^2_{\mathrm{H}} = -3.2433 \times 10^{10}$
$Max(\lvert L_{ABCD}\rvert, \lvert R_{ABCD}\rvert)$	$5.1524 \times 10^{-23}\,\mathrm{GeV}^{-1}$	

Unification parameters and Susy breaking parameters at M_X The values of $\mu(M_X), B(M_X)$ are determined by RG evolution from M_Z to M_X of the values determined by the EWRSB conditions.

Parameter	$Value$	Parameter	$Value$
M_1	157.11	$M_{\tilde{\bar{u}}_{1,2}}$	8403.12
M_2	484.02	$M_{\tilde{\bar{u}}_3}$	44405.93
M_3	489.67	$A^{0(l)}_{11,22}$	−152705.06
$M_{\tilde{\bar{l}}_1}$	1597.14	$A^{0(l)}_{33}$	−96337.97
$M_{\tilde{\bar{l}}_2}$	528.63	$A^{0(u)}_{11,22}$	−173939.15
$M_{\tilde{\bar{l}}_3}$	35582.19	$A^{0(u)}_{33}$	−91783.28
$M_{\tilde{L}_{1,2}}$	11785.37	$A^{0(d)}_{11,22}$	−152614.48
$M_{\tilde{L}_3}$	23998.02	$A^{0(d)}_{33}$	−66790.79
$M_{\tilde{\bar{d}}_{1,2}}$	2678.39	$M_{\tilde{\bar{d}}_3}$	59273.46
$M_{\tilde{Q}_{1,2}}$	8647.71	$M_{\tilde{Q}_3}$	52519.99
$\mu(M_Z)$	165349.59	$B(M_Z)$	4.9500×10^9
$M^2_{\bar{H}}$	-2.7594×10^{10}	M^2_H	-2.9665×10^{10}

Values (GeV) in of the soft Susy parameters at M_Z by matching $y^{MSSM}(M_Z)$ to $y^{SM}(M_Z)$. By evolution from soft SUGRY parameters determined at M_X. Note the heavier third sgeneration. $Sign(\mu) = +$.

Field	$Mass(GeV)$
$M_{\tilde{G}}$	489.67
$M_{\chi^{\pm}}$	484.02, 165349.63
M_{χ^0}	157.11, 484.02, 165349.61, 165349.61
$M_{\tilde{\nu}}$	11785.184, 11642.405, 23997.924
$M_{\tilde{e}}$	1597.78, 11785.47, 525.85, 11642.91, 23991.77, 35586.46
$M_{\tilde{u}}$	8403.04, 8647.53, 8401.75, 8646.96, 44403.37, 52522.52
$M_{\tilde{d}}$	2678.51, 8647.93, 2677.55, 8647.23, 52511.76, 59280.79
M_A	497594.34
$M_{H^{\pm}}$	497594.34
M_{H^0}	497594.33
M_{h^0}	123.11

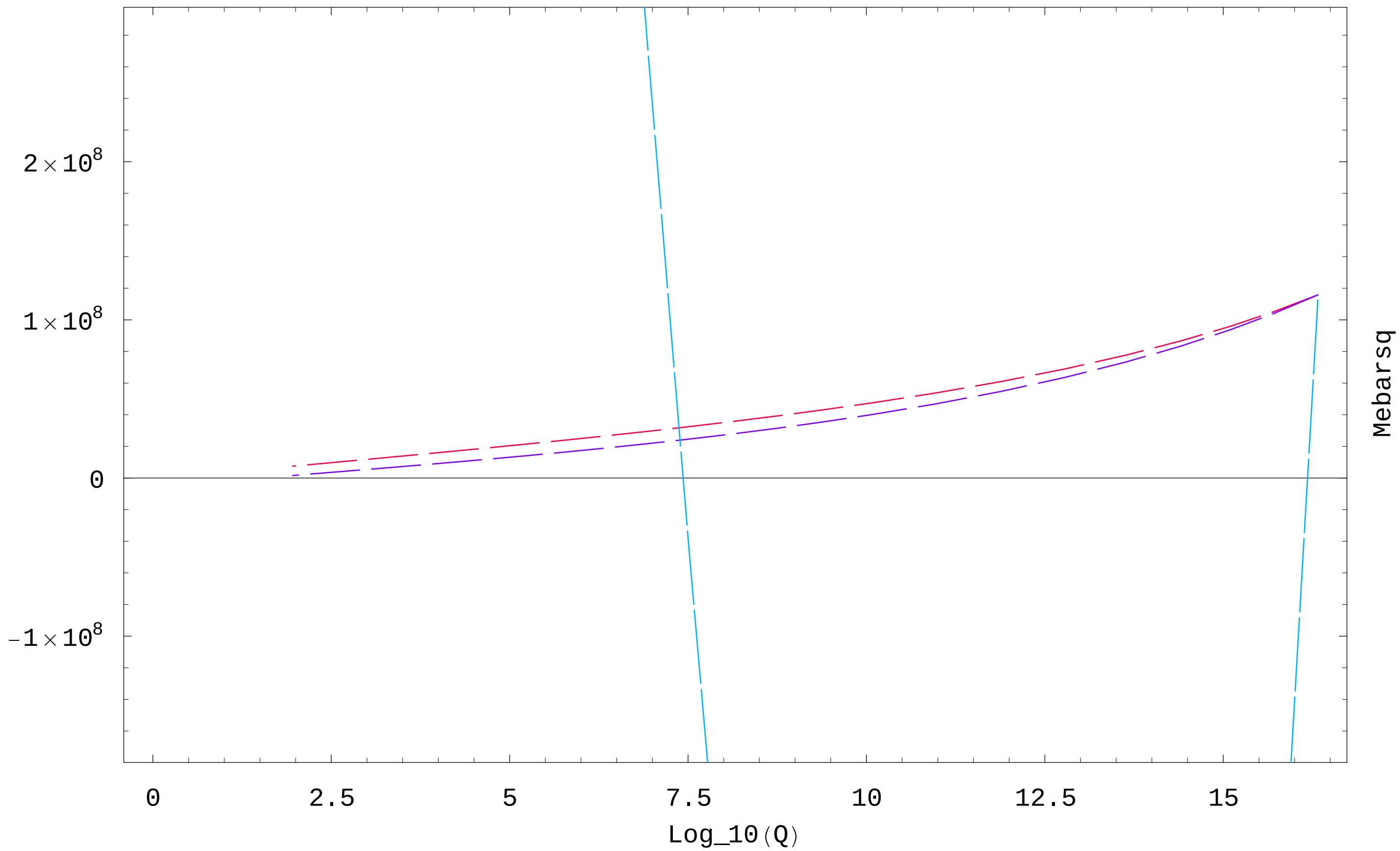
Spectra of supersymmetric partners without generation mixing. Large $\mu, B, A_0 \Rightarrow \chi^{0,+}$ pure Bino and Wino($\tilde{W}_{\pm}$). Right smuon is generically lightest. Other sfermions are multi-TeV . The mini-split supersymmetry spectrum and large μ, A_0 parameters cure FCNC, LFV and CCB/UFB instability. The sfermion masses are ordered by. generation.

Parameter	$Value$
$\epsilon/10^{-7}$	1.94

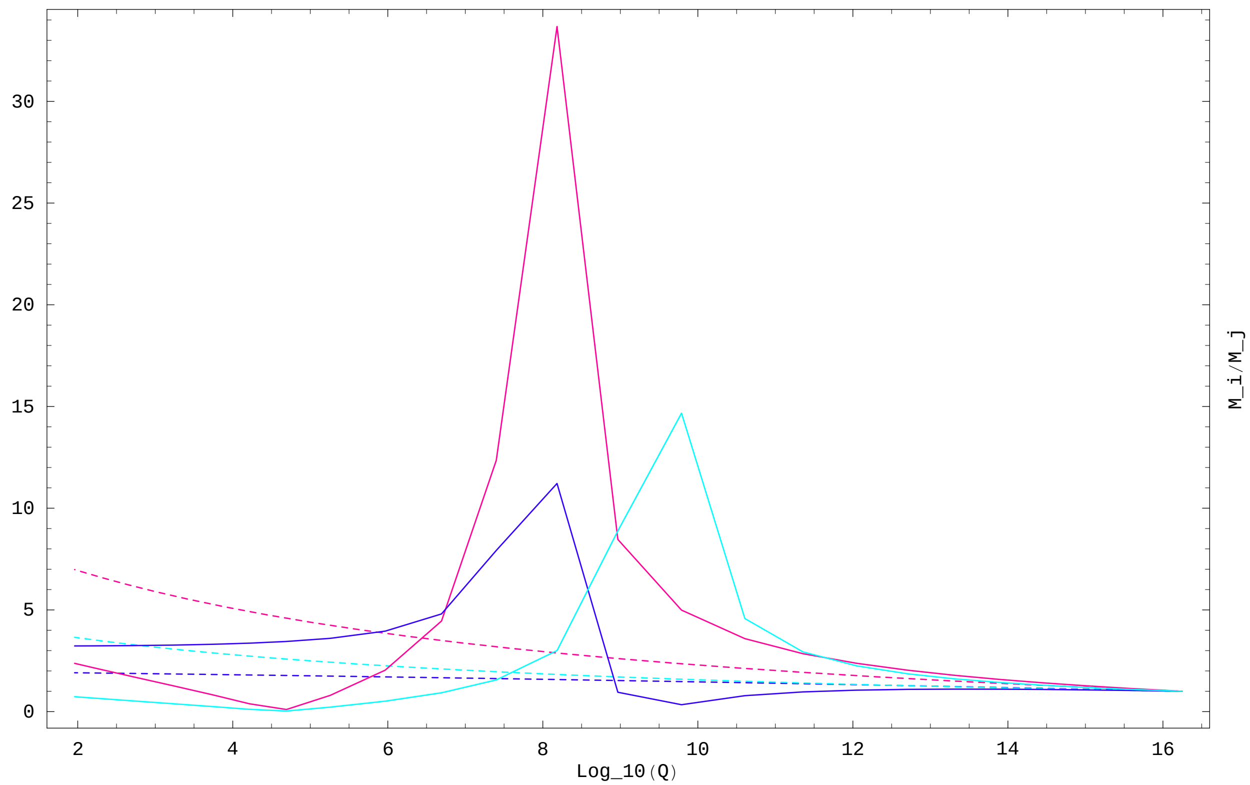
Lepto-genesis relevant CP asymmetry parameter.

Process	Branching Ratio from NMSGUT	Present bound
$\mu \to e\gamma$	3.5317×10^{-13}	2.4000×10^{-12}
$\tau \to \mu\gamma$	2.1051×10^{-12}	4.5000×10^{-8}
$\tau \to e\gamma$	9.9400×10^{-15}	3.0000×10^{-8}

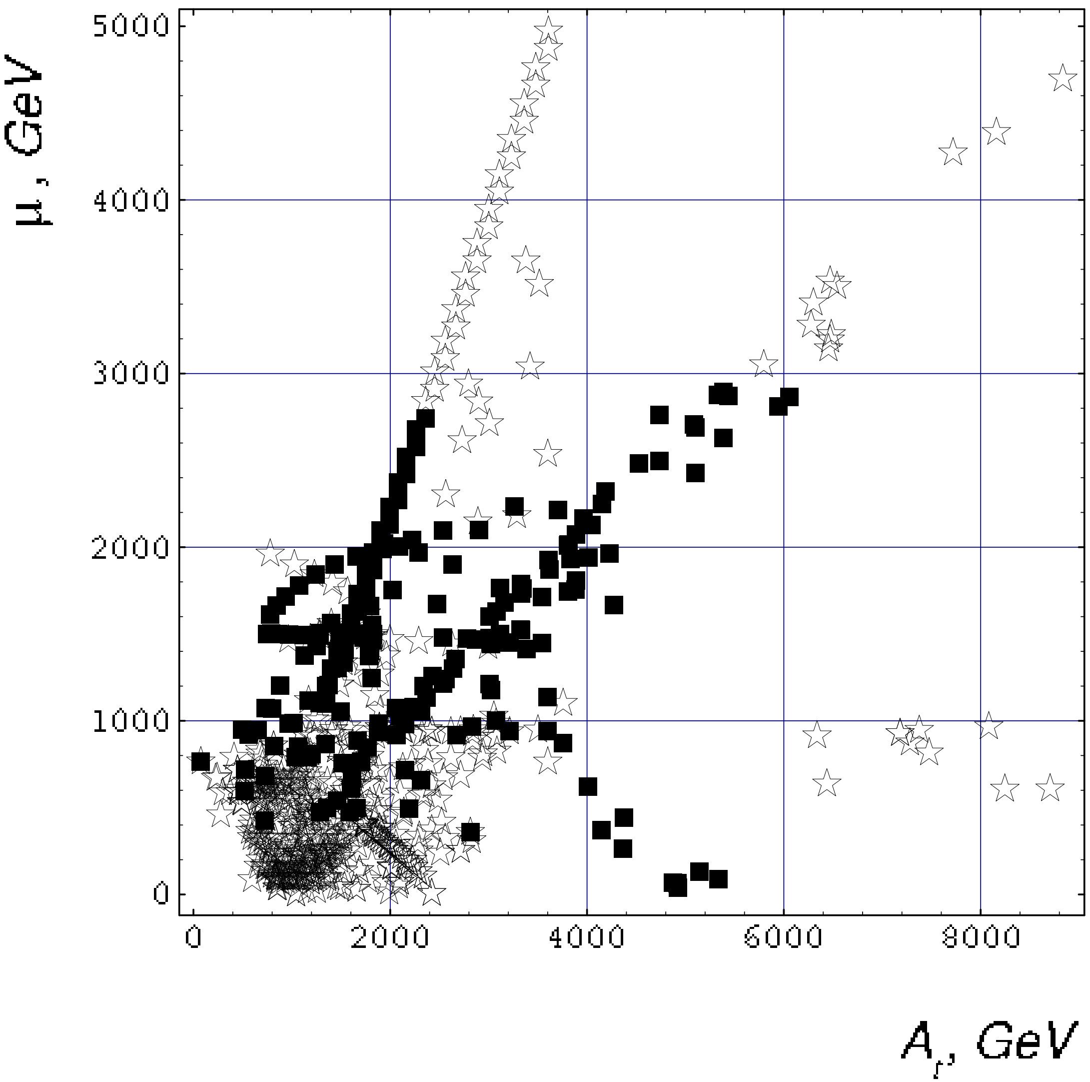
BR of LFV processes.



Typical two loop RG evolution of $M^2_{\tilde{\bar{e}}}$ from M^0_X to M_Z . Red: $M^2_{\tilde{\bar{e}}}$, Blue: $M^2_{\tilde{\bar{\mu}}}$, Green: $M^2_{\tilde{\bar{\tau}}}$. Note the strong growth(off scale) in the the third sgeneration mass at low energies and the anomalous lightness of the smuonbar.



Hypothetical Two loop RG evolution of ratios of gaugino mass ratos with $A_0 \neq 0$ (full lines) and with $A_0 = 0$ (dashed lines) for Case II-1. Red : M_3/M_1, Blue: M_2/M_1 , Green M_3/M_2. In the case $A_0 = 0$, the gaugino masses follow the standard evolution to the $1:2:7$ ratio at low energies



"*Tunneling probability for un-physically large values of A_t and μ The points marked with stars correspond to MSSM "standard/realistic vacua" that are long lived on the scale of the age of the universe. From Kusenko, Langacker and Segre, Phys. Rev. D* **54** *(1996) 5824.*

Case	$\tau_p(M^+\nu)$	$\Gamma(p \to \pi^+\nu)$	$BR(p \to \pi^+\nu_{e,\mu,\tau})$	$\Gamma(p \to K^+\nu)$	$BR(p \to K^+\nu_{e,\mu,\tau})$
$I-1$	2.4×10^{36}	6.2×10^{-38}	$\{3.605 \times 10^{-7}, 0.082, 0.918\}$	3.5×10^{-37}	$\{2.808 \times 10^{-5}, 0.119, 0.881\}$
$I-2$	1.8×10^{34}	7.9×10^{-36}	$\{4.805 \times 10^{-5}, 0.076, 0.924\}$	4.8×10^{-35}	$\{1.124 \times 10^{-4}, 0.114, 0.886\}$
$I-3$	5.7×10^{36}	2.4×10^{-38}	$\{3.226 \times 10^{-7}, 0.100, 0.900\}$	1.5×10^{-37}	$\{2.341 \times 10^{-5}, 0.139, 0.861\}$
$I-4$	5.7×10^{34}	1.7×10^{-36}	$\{7.362 \times 10^{-5}, 0.052, 0.947\}$	1.6×10^{-35}	$\{9.080 \times 10^{-5}, 0.046, 0.953\}$
$II-1$	1.5×10^{36}	9.7×10^{-38}	$\{1.788 \times 10^{-6}, 0.114, 0.886\}$	5.9×10^{-37}	$\{3.608 \times 10^{-5}, 0.170, 0.829\}$
$II-3$	1.7×10^{36}	7.4×10^{-38}	$\{1.903 \times 10^{-6}, 0.153, 0.847\}$	5.1×10^{-37}	$\{2.649 \times 10^{-5}, 0.205, 0.795\}$
$II-4$	6.2×10^{33}	1.6×10^{-35}	$\{5.787 \times 10^{-5}, 0.071, 0.929\}$	1.5×10^{-34}	$\{7.957 \times 10^{-5}, 0.069, 0.931\}$
$III-1$	2.3×10^{36}	6.5×10^{-38}	$\{5.661 \times 10^{-7}, 0.088, 0.912\}$	3.7×10^{-37}	$\{2.536 \times 10^{-5}, 0.128, 0.872\}$
$III-2$	5.0×10^{34}	3.3×10^{-36}	$\{3.325 \times 10^{-5}, 0.050, 0.950\}$	1.7×10^{-35}	$\{9.214 \times 10^{-5}, 0.093, 0.907\}$
$III-3$	2.2×10^{36}	6.2×10^{-38}	$\{5.908 \times 10^{-7}, 0.103, 0.897\}$	4.0×10^{-37}	$\{1.989 \times 10^{-5}, 0.141, 0.859\}$
$III-4$	6.2×10^{33}	1.6×10^{-35}	$\{5.787 \times 10^{-5}, 0.071, 0.929\}$	1.5×10^{-34}	$\{7.957 \times 10^{-5}, 0.069, 0.931\}$

Table of $d = 5$ operator mediated proton lifetimes τ_p (yrs), decay rates $\Gamma(yr^{-1})$ and Branching ratios in the dominant Meson $^+ + \nu$ channels.

Case	$B.R(b \to s\gamma)$	Δa_μ	$\Delta\rho$
$I-1$	3.294×10^{-4}	5.796×10^{-9}	5.985×10^{-6}
$I-2$	3.293×10^{-4}	5.471×10^{-9}	2.397×10^{-5}
$I-3$	3.294×10^{-4}	2.300×10^{-9}	2.825×10^{-6}
$I-4$	3.293×10^{-4}	7.238×10^{-9}	6.064×10^{-7}
$II-1$	3.290×10^{-4}	1.360×10^{-10}	2.503×10^{-6}
$II-3$	3.287×10^{-4}	1.035×10^{-10}	3.385×10^{-6}
$II-4$	3.278×10^{-4}	1.043×10^{-10}	3.612×10^{-6}
$III-1$	3.293×10^{-4}	8.058×10^{-9}	3.718×10^{-6}
$III-2$	3.293×10^{-4}	6.824×10^{-9}	2.105×10^{-5}
$III-3$	3.295×10^{-4}	8.689×10^{-9}	3.743×10^{-6}
$III-4$	3.294×10^{-4}	7.452×10^{-9}	5.989×10^{-7}

Table Low energy constraints from the limits on the branching ratio for $b \to s\gamma, \Delta a_\mu$ and $\Delta\rho$. The $b \to s\gamma$ branching ratio values are in the centre of the region $(3-4\times 10^{-4}) \pm 15\%$ determined by measurements at CLEO,BaBar and Belle. The current difference between experiment and theory for the muon magnetic moment anomaly is $\Delta a_\mu = 255(63)(49) \times 10^{-11}$.

BSM Physics and Inflation

- Inflation Data: Scale invariant power spectrum of curvature (density, scalar) perturbations : $P_R = (2.43 \pm 0.11) \times 10^{-9}$, spectral index $n_s = .967 \pm 0.014$ and scale invariance $kdn_s/dk \simeq 0$. Tensor perturbation(gravity waves) suppressed $P_T/P_R < .1$. Values at a representative "pivot" scale $k_{pivot} \sim (500Mpc)^{-1}$. Scale invariance and scalar dominance generic in slow roll models.
- Standard thermal history : Inflation, Reheating, radiation domination , matter domination, present $\Lambda_{CDM} \Rightarrow$ $N_{pivot} \sim 50 \pm 5$: Number of efolds remaining when representative scale $k_{pivot} > 1/aH$(comoving Horizon)
- Many inflation models manage to fit this skimpy data. A connection of the inflaton field(s) with known Particle physics

is highly desirable. Original and later GUT Inflaton models generally fail on one count or another. $\rho^{1/4} < 2 \times 10^{16}$ GeV from $P_R/P_T \Rightarrow$.

- Inflexion point MSSM inflaton models have Reheating behaviour analyzable in terms of MSSM dynamics but low inflaton scale($\sim M_S$) implies extreme fine tuning of *soft* parameters.
- Allahverdi,Kusenko and Mazumdar : Dirac sneutrino Inflaton is NLH flat direction in the νMSSM$\times U(1)_{B-L}$ connects tiny y_ν required for Dirac m_ν to inflation. Neutrino-Inflation connection. $M \sim M_S \Rightarrow$ But extreme fine tuning !
- CSA, Ila Garg Supersymmetric seesaw neutrino Inflation ? $M_{inflaton} \sim M_{\nu^c} >> M_S$: less fine tuned inflationary scenario fits in well with the(realistic) NMSO(10) GUT. Tuning on Superpotential not Soft parameters. P_T small not tiny.

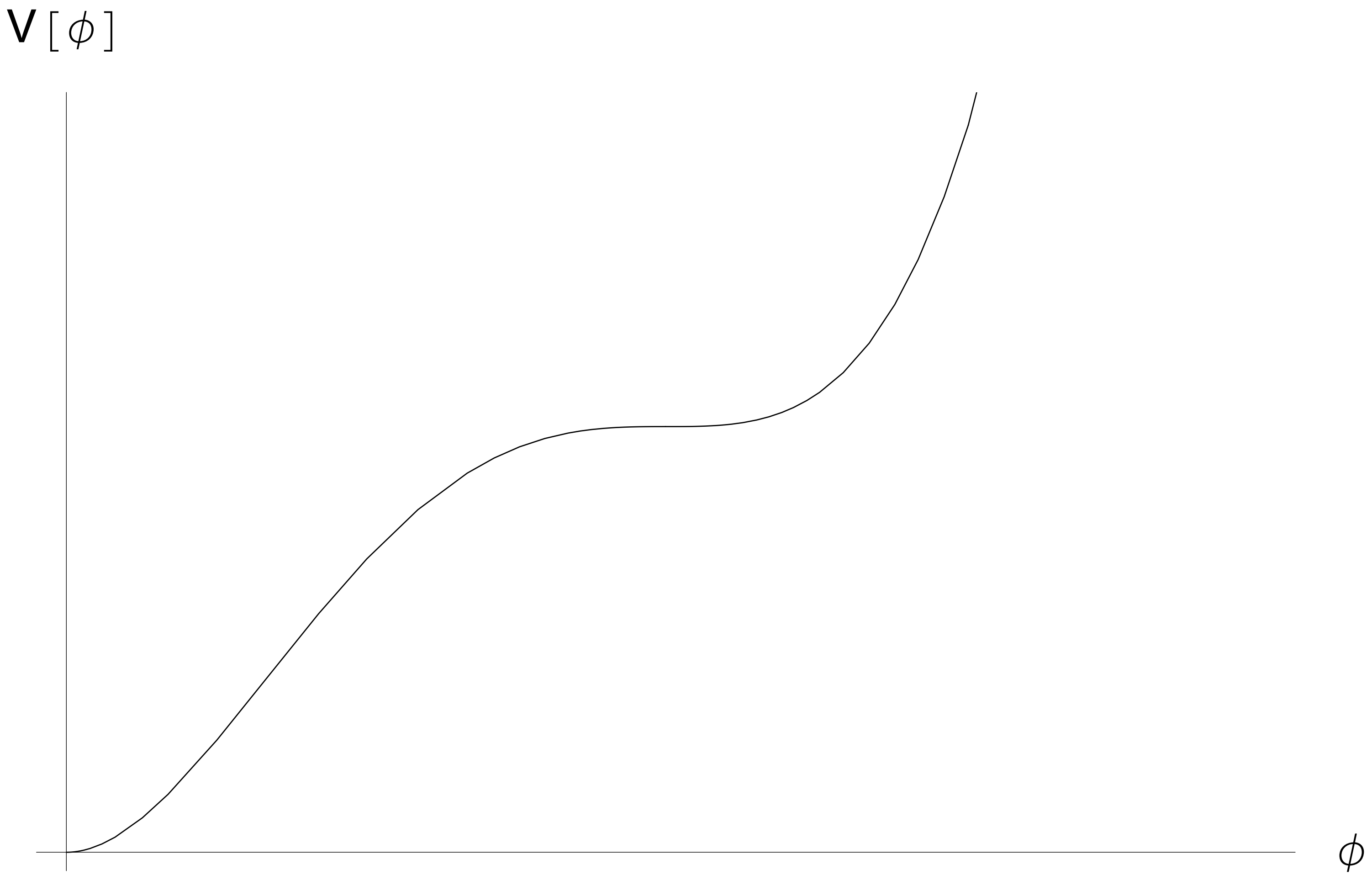


Figure 1: Generic Renormalizable Inflection Point Inflation

Potential : $V = \frac{h^2}{12}\phi^4 - \frac{Ah}{6\sqrt{3}}\phi^3 + \frac{M^2}{2}\phi^2$

Tuning : $A = 4M\sqrt{1-\Delta}$

Numerical Solutions

Accurate analytic solution around $n_s^0 = 0.967, N_C^0 = 50.006$ gives

$$h^2 \quad \sim \quad 10^{-24.95\pm0.17}(\frac{M}{GeV}) \quad ; \quad \Delta \sim 10^{-28.17\pm.13}(\frac{M}{GeV})^2$$

Maximum variations in the exponents corresponding to the quoted errors in the WMAP 7- year data from the graphs using analytic solution. However for $N_{CMB} \sim 50$, $Z_0 \approx \frac{1.2}{N_{CMB}}$ solves to a good approximation. Then

$$\frac{h^2}{M} \approx \frac{3\pi}{M_P}\frac{\sqrt{P_R}}{N_{CMB}^2} \approx \frac{2.75\times10^{-22}}{N_{CMB}^2} \approx 10^{-25}\ GeV^{-1}$$

$$\frac{\Delta}{M^2} \approx \frac{4.14\times10^{-34}}{N_{CMB}^2 P_R} \approx 10^{-28.2} GeV^{-2}$$

which is effectively the same

Improvement in Naturalness

$$\frac{h^2}{M} \approx \frac{3\pi}{M_P} \frac{\sqrt{P_R}}{N_{CMB}^2} \approx \frac{2.75 \times 10^{-22}}{N_{CMB}^2} \approx 10^{-25}$$

$$\frac{\Delta}{M^2} \approx \frac{4.14 \times 10^{-34}}{N_{CMB}^2 P_R} \approx 10^{-28.2} GeV^{-2}$$

- Dirac mass case: $M \sim M_S$ while here $M \sim M_{\nu^c} >> M_S$.
- Required yukawa coupling $h >>$ Dirac case reaches $10^{-6.5}$ for $M_{\nu^c} \sim 10^{12}$ GeV compare $h \sim 10^{-10.5}$ for $M_S \sim 10^4$ GeV.
- Fine-tuning measure Δ grows with M so that $\beta = \sqrt{\Delta}$ can be as large as 10^{-2} for $M \sim 10^{12}$ GeV compare $\beta \sim 10^{-10}$!!
- No additional dynamics need be invoked to make it plausible .
- Any renormalizable inflection point model must respect these generic constraints.

Inflation scales

Thus we have viable inflation with

$$V_0 \sim \frac{M^4}{h^2} \sim (M)^3 \times 10^{25}\,GeV \sim 10^{43} - 10^{61}\,GeV^4$$

$$H_0 \sim \sqrt{\frac{V_0}{M_P^2}} \sim 10^3 - 10^{12}\,GeV$$

$$T_{max} \sim V_0^{\frac{1}{4}} \sim 10^{11} - 10^{15}\,GeV$$

respects limits from tensor perturbations ($V_0 < 10^{64}$ GeV)

Supersymmetric Seesaw Inflaton Toy model

- $SIMSSM \times U(1)_{B-L} \oplus S[1,1,1,-2] \oplus \{\Omega_i\}$
- $< S >= \bar{\sigma}/\sqrt{2} \Rightarrow M_{\nu^c}$ $(10^6 - 10^{12}$ GeV) via
  $W = 3\sqrt{2} f_{AB} S \nu^c_A \nu^c_B + ...$
- Additional fields Θ_i fix the vev of S in MSLRMs and MSGUTs
- Neutrino Dirac Coupling: $L_A[1,2,0,-1] = (\nu, e)^T_A$ and Higgs
  $H[1,2,1/2,0]$: $W = y^\nu_{AB} N_A L_B H + ...$
- NLH Flat direction Inflaton ϕ rolls out of minimum
  corresponding to

$$SU(3) \times SU(2) \times U(1)_R \times U(1)_{B-L} \rightarrow SIMSSM$$

$$\tilde{N} = \tilde{\nu} = h_0 = \frac{\varphi}{\sqrt{3}} = \phi e^{i\theta}; \quad \phi \geq 0, \quad \theta \in [0, 2\pi)$$

- $Y = 2T_{3R} + (B-L)$. $B-L$ broken at high scale. Γ_N via y_ν.

SSI Potential

- From Seesaw model GRIPI potential follows :

$$V_{tot} = f^2 \left((2 + 9\tilde{y}^2)\phi^4 - (\tilde{A}_0 + 12)\tilde{y}\bar{\sigma}\phi^3 + (\tilde{A}_0 + \tilde{m}_0^2 + 4)\bar{\sigma}^2\phi^2 \right). \tag{1}$$

-

$$\begin{aligned} h &= f\sqrt{12(2 + 9\tilde{y}^2)} \\ A &= \frac{3f(\tilde{A}_0 + 12)\tilde{y}\bar{\sigma}}{\sqrt{(2 + 9\tilde{y}^2)}} \\ M^2 &= 2f^2\bar{\sigma}^2(4 + \tilde{A}_0 + \tilde{m}_0^2) \\ \Delta &= (1 - \frac{A^2}{16M^2}) \\ &= \left(1 - \frac{9\tilde{y}^2(\tilde{A}_0 + 12)^2}{32(2 + 9\tilde{y}^2)(\tilde{A}_0 + \tilde{m}_0^2 + 4)} \right) \end{aligned}$$

- Seesaw models $m_\nu^D > 1MeV$, $M_{\nu^c} > 10^6$ GeV $>> M_S \sim 10$ TeV). $Max(|\tilde{A}_0|, |\tilde{m}_0|) \sim 0.1$ $\tilde{A}_0, \tilde{m}_0 << 1$ for $M_{\nu^c} \sim 10^8$ to 10^{12} GeV *and are thus inessential (compare Dirac).*

- $\Rightarrow \Delta \sim 10^{-12} - 10^{-4} \Rightarrow \tilde{y} = y/f \simeq \tilde{y} = 4/3$ as M increases : $M \sim 10^6$ GeV differs from 1.333 only at the second decimal place: $\tilde{y}^2$ close to

$$\tilde{y}_0^2 = \frac{64}{9} \frac{4 + \tilde{A}_0 + \tilde{m}_0^2}{16 - 8\tilde{A}_0 - 32\tilde{m}_0^2 + \tilde{A}_0{}^2}$$

  $\tilde{A}_0, \tilde{m}_0 \sim O(M_S/M_{\nu^c}) << 1 \Rightarrow, \tilde{y}_0 \rightarrow 4/3$ for larger $M \sim f\bar{\sigma}$

- Fine tuning measure $\beta = \sqrt{\Delta} \sim 10^{-2} - 10^{-6}$ favourable relative MSSM/Dirac neutrino inflaton ($\beta \sim 10^{-12}$ to 10^{-10}) due to $M_\phi \sim 10^3 - 10^4$ GeV.

- Essential fine-tuning is of superpotential parameters, which is radiatively stable due to non renormalization theorems.

$$f \simeq 10^{-26.83\pm0.17}\left(\frac{\overline{\sigma}}{GeV}\right) \quad ; \quad M \simeq 10^{-25.38\pm0.17}\left(\frac{\overline{\sigma}}{GeV}\right)^2$$

$$\Delta \simeq 10^{-78.93\pm0.47}\left(\frac{\overline{\sigma}}{GeV}\right)^4$$

$M \sim 10^{6.6}$ to $10^{10.6}$ GeV $\Leftrightarrow 10^{16}\ GeV < \bar{\sigma} < 10^{18}\ GeV$: Susy SO(10) MSGUTs !

Reheating via Instant Preheating

- Preheating : Inflaton couples strongly to some MSSM modes χ s.t $m_\chi \sim g|\phi|$ or $\sim y_3|\phi| \Rightarrow$ non-perturbative χ mode creation near $\phi = 0$ decay rapidly into ϕ-uncoupled light MSSM (ψ type) (usually coloured) modes as $|\phi| \to |\phi|_{max} :\Rightarrow$ inflaton energy dump into MSSM bath and rapid ϕ decay.

- Thermalization via strong and electroweak interactions (once $\phi \simeq 0$) in 1-100 oscillation times

$$\tau_{osc} \sim m_\phi^{-1} << H_{infln}^{-1} \sim hM_p\tau_{osc}/M \sim (1-150)\tau_{osc}$$

So

$$T_{rh} \sim (\frac{30}{\pi^2 g_*})^{1/4} V_0^{1/4} \sim T_{max} \sim V_0^{1/4} \sim M/h^{1/2} \sim 10^{11} - 10^{15}\, GeV$$

- Gravitino problem unless

$$\tau_{grav} \sim 10^5\, sec(\frac{1\,TeV}{m_{3/2}})^3 << \tau_N \sim 1sec \Rightarrow$$
$$m_{3/2} > 50TeV \qquad (2)$$

As in NMSO(10)GUT !!

Leptogenesis

- High T_{reheat} implies ample creation of right handed neutrinos and thus allows thermal leptogenesis.
- Generation of (n_B/n_γ) via non-thermal Leptogenesis may also be natural in this model since the Higgs is itself a χ mode as required in the mechanism of Ahn and Kolb(PRD2006)
- Our model is just what is needed. *Non-thermal* and Thermal leptogenesis can occur. Higgs field H is χ type and coupled to ν^c. Higgs mass $m_h \sim g_2\phi$ fluctuates below and above $M_{\nu^c} \sim f\bar{\sigma}$ even in the presence of the inflaton (i.e $\tilde{\nu}, h, N$) background since $g_2 >> f, y$). CP violating -therefore net lepton number producing - inter-conversion of the Higgs with righthanded Neutrinos. Requires detailed Boltzmann dynamics of coupled N, L, H, χ, ψ system (PhD thesis Charanjit Kaur, Ila Garg).

$\text{SSI} \subset \text{NMSO}(10)$

- NMSGUT allows embedding of SSI. Embedding viability non-trivially depends on derivation of 2 MSSM doublets from 6 pairs of GUT Higgs doublets .

- D-flatness conditions solved by

$$\sum_A |\tilde{\nu}_A|^2 = \sum_i |h_{i0}^2| = \sum_A |\tilde{\bar{\nu}}_A|^2 + 2|h_{40}|^2$$

$h_i \to H\alpha_i$ in effective theory.

- Single generation of each lepton in inflaton for simplicity, small yukawas required so take $\nu_A = \nu_1$. But $\tilde{\bar{\nu}}_A = \tilde{\bar{\nu}}_1$ requires tuning $|y_{11}|^2 \sim 10(|y_{21}|^2 + |y_{31}|^2)$: very hard with normal neutrino mass hierarchy. $\nu_A^c = \nu_3^c$ better :

$$\tilde{\nu}_1 = \frac{\phi}{\sqrt{3}} \qquad h_{i0} = \frac{\alpha_i \phi}{\sqrt{3}} \qquad \tilde{\bar{\nu}}_3 = \frac{\phi}{\sqrt{3}}\sqrt{1-2|\alpha_4|^2} \quad (3)$$

- peculiar α_4 entry as $\Gamma = 1 - 2|\alpha_4|^2$ enables tuning !
- NMSO(10) : $|y_{33}| >> |y_{32}| >> |y_{31}| >> |y_{21}| > |y_{11}| \Rightarrow$

$$\Gamma \simeq 0 \quad \text{i.e} \qquad |\alpha_4| \simeq \frac{1}{\sqrt{2}} \qquad (4)$$

H is 50% from 210 plet !

- Then effectively

$$|y^\nu_{31}|^2 \quad = \quad 8(|y^\nu_{11}|^2 + |y^\nu_{21}|^2)$$

easy to enforce in the NMSO(10)

Remaining Difficulty and Threshold effects

- The condition for adequate e-folds $h^2/M_3 \sim (y^{\nu\dagger} y^{\nu})_{11}/(M_3\sqrt{\Gamma}) \sim 10^{-25} \Rightarrow$ is harder to achieve !
- Threshold effects may again be crucial :

$$Y_f = (1+\Delta_f^T)\cdot(Y_f)_{tree}\cdot(1+\Delta_f)(1+\Delta_{H^\pm})$$

  Large number of fields can give $\Delta_H >> 10$. Lowers GUT yukawas by large factor.
- In arXiv:1107.2963 [hep-ph] Δ calculated in approx that only 10-plet contribution significant. Now complete calculation available ($|\alpha_4| \simeq 0.7 \Rightarrow$ Absolutely necessary!!).
- Large wavefunction correction in comparing NMSO(10) and SIMSSM fields not necessarily fatal but two loop effect to see if calculations convergent.

Parameter	$Value$	$Parameter$	$Value$
χ_X	0.45	M_{h^0}	123.0
χ_Z	0.143	M_X	$7.08E17$
f_3	1.07×10^{-3}	f_1, f_2	$2.59 \times 10^{-8}, 4.41E{-5}$
h	$2.44E{-4}$	Λ_n	0.999999
M	$3.043E11$	Λ_d	0.999999
Γ	$4.34E{-5}$	Δ_{tuning}	0.989
$\lvert\overline{\sigma}\rvert$	$4.69E15$	M_X	$5.25E17$
$A_0(M_X), m_0(M_X)$	$-5.24E5, 1.26E4$	$\mu, B(M_X)$	$4.32E5, -1.13E11$
$M^2_{\bar{H}}$	$-1.5E11$	M^2_H	$-1.45E11$
$\lvert\Delta_{H_0}\rvert, \lvert\Delta_{\bar{H}_0}\rvert$	$50.25, 63.93$	$\lvert\alpha_4\rvert$	0.707
$M_3^{\nu^c}$	$4.86E13$	$M_{1,2}^{\nu^c}$	$1.181E9, 2.01E12$
$\lvert y^{\nu}_{31tree}\rvert$	$2.0E{-4}$	$\lvert y^{\nu}_{21tree}\rvert, \lvert y^{\nu}_{11tree}\rvert$	$4.49E{-5}, 1.64E{-6}$
$Log_{10}(h^2/M)$	-18.71	V_0, ϕ_{end}	$3.58E52, 2.15E15$
N_{pivot}, N_{CMB}	$54.22, 4.78E{-4}$	Δ, β	$8.82E{-12}, 5.92E{-6}$

Masses in GeV. $\chi_{X,Z}$ accuracies of fits to fermion mass data at $M_{X,Z}$.

Conclusions

- Even loaded with complete (MS)SM, neutrino mass data, Flavour violation and B-violation constraints the NMSGUT still stands alive on the edge of falsifiability/viability as a good scientific theory should.

- It made(in 2008) definite and distinctive predictions about the nature of sparticle spectra to lie in an unexpected (Large A_0, μ) region of parameter space which is highly preferred after Higgs discovery.

- It also connects Supersymmetric (type I) seesaw with Inflation and Lepto-genesis connecting.

- The distinctive and viable RG evolution of sfermion masses may lead to novel and falsifiable Cosmological histories.

- Fine tuning is less severe and more stable than in the MSSM/Dirac neutrino case because superpotential couplings are tuned $M_{inflation} \sim 10^6 - 10^{12} >> M_S$GeV rather than M_S as in MSSM inflaton models.

- $T_{rh} \sim 10^{11} - 10^{15} \Rightarrow$ GeV. $m_{3/2} > 50\, TeV$ as also required in NMSO(10) from fits.

- SSI may allow thermal and implementation of Ahn-Kolb non thermal Leptogenesis.

- SSI can be embedded in NMSO(10) with completely realistic parameters extending its claims to viable inflationary and leptogenetic cosmology in addition to DM.

- Complete calculation of the wave function corrections to the tree level relations between SIMSSM and NMSGUT yukawa couplings is complete.