



**2373-12**

## **Workshop on Geophysical Data Analysis and Assimilation**

*29 October - 3 November, 2012*

### **Basics of seismic source mechanism Part I**

Torsten Dahm  
*German Research Center for Geosciences (GFZ)  
University of Potsdam  
Germany*

## ICTP 2012: Workshop on Geophysical Data Analysis and Assimilation

### Part I: Basics of seismic source mechanism

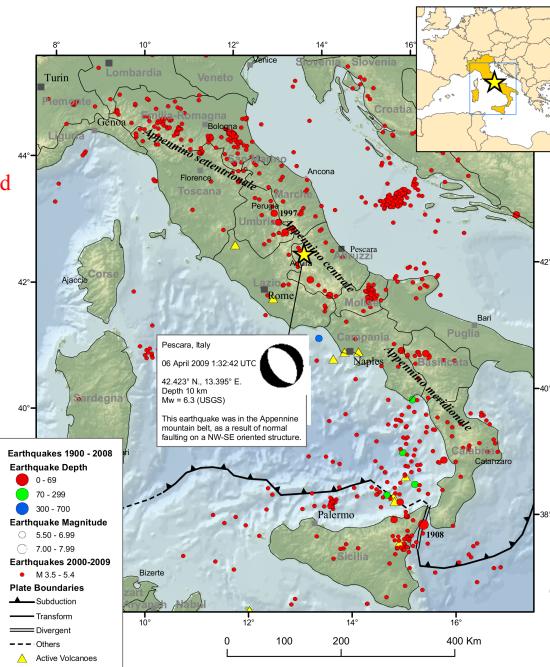
**Prof. Dr. Torsten Dahm**  
*German Research Center for Geosciences (GFZ), Potsdam, Germany  
University of Potsdam, Institute of Earth and Environmental Science, Potsdam*

- Short introduction
- Source parameter: point sources
- Source parameter: finite rupture and directivity



### 6 April 2009 M<sub>W</sub> 6.3 Aquila (Italy) earthquake [Introduction](#)

- ✓ 8.8 km depth
- ✓ 287 killed
- ✓ 1000 injured
- ✓ 40.000 displaced
- ✓ 10.000 houses destroyed



Th. Braun, pers. com.





**seismologists found guilty**  
October 23, 2012

**SCIENTISTS FIND IT 'UNSETTLING' THAT ITALIAN SEISMOLOGISTS WERE FOUND GUILTY OF MANSLAUGHTER FOR INEXACT EARTHQUAKE PREDICTIONS**

CRIME  
Posted on October 22, 2012 at 7:45pm by Liz Klimas

Hon. Posted on October 22, 2012 at 7:45pm by

**US scientists outraged by Italy seismologist ruling**

Sapa-AFP | 23 Oktober, 2012 10:17

**NATURE | BREAKING NEWS**

**Italian court finds seismologists guilty of manslaughter**

Six scientists and one official sentenced to six years in prison over L'Aquila earthquake.

**Nicola Nosengo**  
22 October 2012

Email » Print »

Six Italian scientists and a government official have been found guilty of multiple manslaughter for underestimating the risks of a killer earthquake in L'Aquila in 2009. They were sentenced to six years in jail on Monday in a watershed ruling in a case that has provoked outrage in the international science community. The experts were also ordered to pay more than nine million euros (\$11.5 million) in damages to ~~the~~ and inhabitants. Under the Italian justice system, the seven will remain free men until they ~~have~~ chances to appeal the verdict.

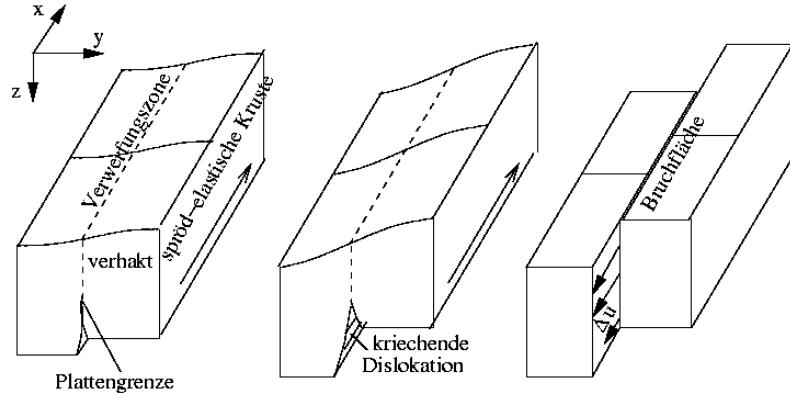
Image by: Gallo Images/Thinkstock

HELMHOLTZ GEMEINSCHAFT

## Introduction

### Earthquake recurrence model by Reid (1910)

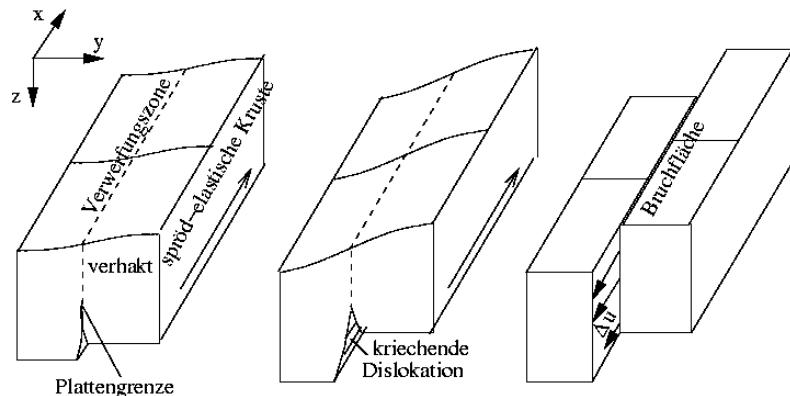
a) Beginn Deformationsaufbau    b) kurz vor dem Beben    c) kurz nach dem Beben



## Introduction

### Earthquake recurrence model by Reid (1910)

a) Beginn Deformationsaufbau    b) kurz vor dem Beben    c) kurz nach dem Beben



#### **strong**

#### **weak**

seismic cycles (pre-, co-, postseismic)

only qualitative

role of fault zone, fault friction & creep

role of brittle-ductile and brittle-plastic

## Introduction

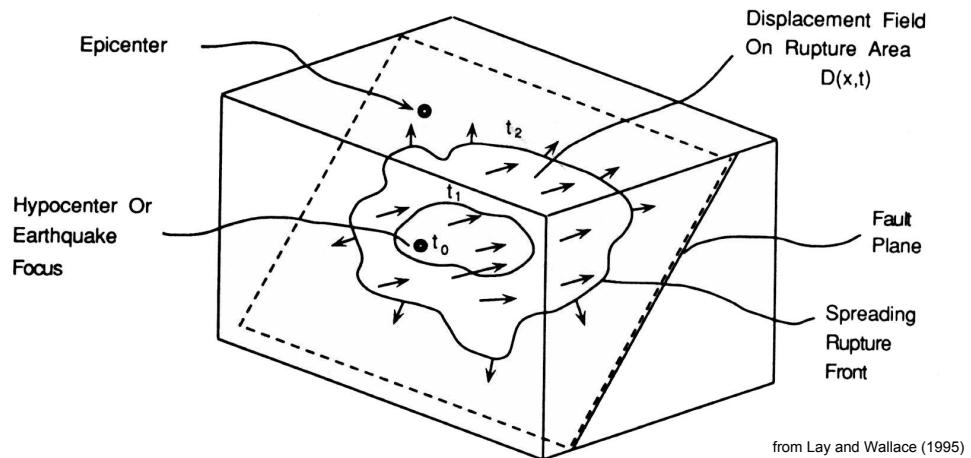
### Other earthquake models

Type / mechanism	Example
Within subducting slab	
• bending related	31 Aug 2012 Philippine $M_W$ 7.5
• deep focus earthquakes	9 Jun 1994 Bolivia $M_w$ 8.2 636 km
Related to fluid movement / intrusions	
• volcanic earthquake swarms	1974-1984 Krafla rift intrusions, Iceland
• non-volcanic earthquake swarms	1985-2011 N. Kostel, Bohemia, swarms
Intra-plate (tectonic) earthquakes	
• at diffuse plate boundaries (oceanic)	4 Apr 2012 Sumatra $M_W$ 8.6 & $M_W$ 8.2
• reactivation of crustal faults	6 Apr 2009 Aquila, Italy, $M_W$ 6.3
• assisted by postglacial rebound	20 Nov 1933 Baffin Bay Canada $M_W$ 7.2



## point source parameter

### Earthquake source parameter



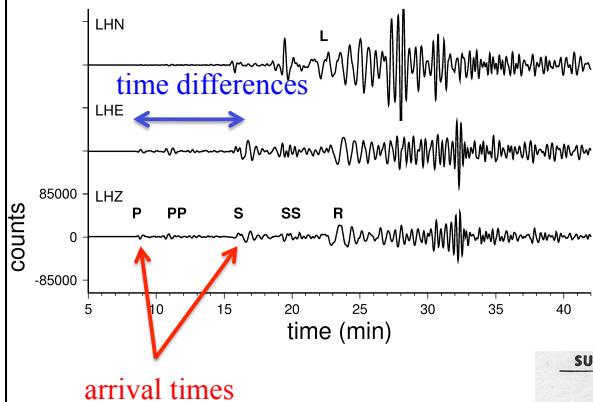
1. point source location (hypocenter or centroid)
2. radiation pattern (mechanism, moment tensor)
3. strength (magnitude and/or moment)



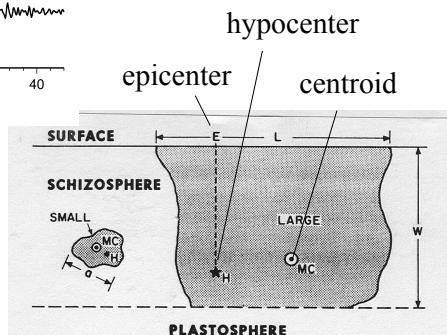
## point source parameter

### 1. Earthquake location (point source, 4 parameter)

Pakistan, 8 Okt. 2005, MS 7.8, LP 30 S



location techniques:  
see textbooks ...



GFZ  
Helmholtz-Zentrum  
POTS DAM

## location methods / codes point source parameter

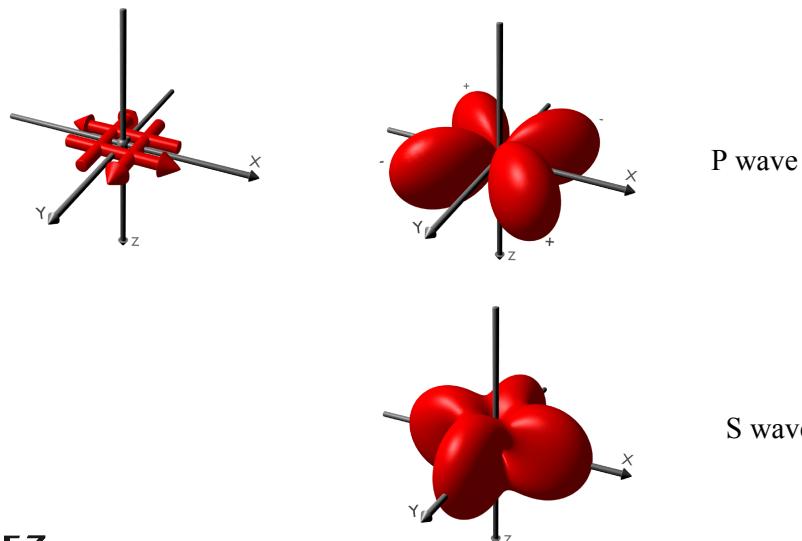
1. Single event absolute location, layered Earth: Geiger method (e.g. **hyposat**: Schweitzer 2003, <ftp://ftp.norsar.no/pub/outgoing/johannes/hyposat>)
2. Single event absolute location, 3D Earth: Directed Grid Search (e.g. **NonLinLoc**: Lomax et al., 2011, <http://alomax.free.fr/nlloc/>)
3. Relative location (master event double differences), layered Earth (e.g. **hypoDD**: Waldhauser 2001, [www.ledo.columbia.edu/~felixw/hypoDD.html](http://www.ledo.columbia.edu/~felixw/hypoDD.html))

GFZ  
Helmholtz-Zentrum  
POTS DAM

HELMHOLTZ  
GESELLSCHAFT

point source parameter

## 2. Radiation of waves: e.g. shear crack & double couple

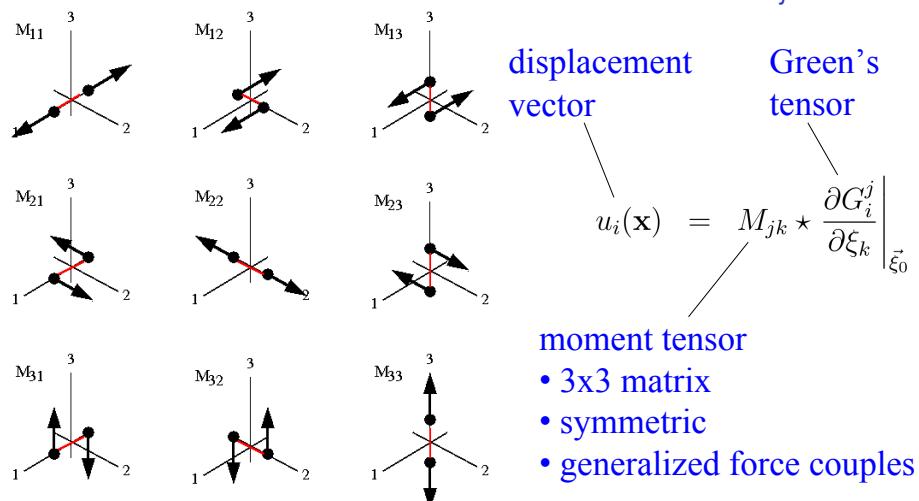


**GFZ**  
Helmholtz-Zentrum  
POTS DAM

HELMHOLTZ  
GEMEINSCHAFT

point source parameter

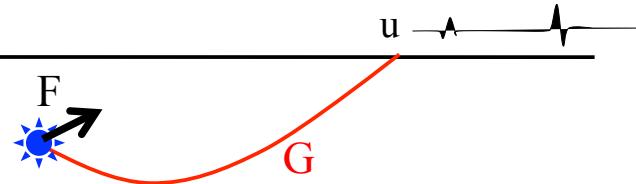
### force dipole representation: moment tensor $M_{jk}$



**GFZ**  
Helmholtz-Zentrum  
POTS DAM

HELMHOLTZ  
GEMEINSCHAFT

Short derivation: (static) point force representation



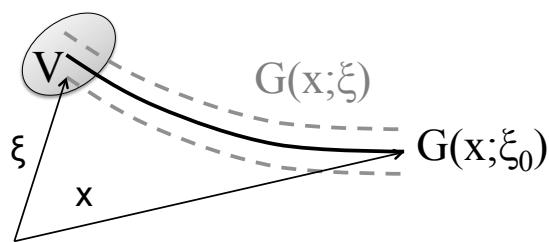
$$u_i(\mathbf{x}) = F_j(\xi_0) G_i^j(\mathbf{x}, \xi_0)$$

↑                                   ↑                                   ↑

displacement  $\mathbf{u}$                       force  $\mathbf{F}$                       Greens „function“ (GF)  $\mathbf{G}$

But: internal sources (earthquakes) cannot be represented by single forces!

Assumption: force distribution within volume  $V$



$$u_i(\mathbf{x}, t) = \int_V f_j(\xi) * G_i^j(\mathbf{x}; \xi) dV$$

Concept: Taylor's series expansion of  $G$  around centroid point

1<sup>st</sup> order tensor

2<sup>nd</sup> order tensor

$$M_j(t) = \int_V f_j(\vec{\xi}) dV$$

$$M_{jk}(t) = \int_V (\vec{\xi} - \vec{\xi}_0)_k f_j(\vec{\xi}) dV$$

## Internal point sources: GF expansion and moment tensor



Taylor's series expansion around centroid  $\xi_0$ :

$$G_i^j(\mathbf{x}; \boldsymbol{\xi}) \approx G_i^j(\mathbf{x}; \boldsymbol{\xi}_0) + \frac{\partial}{\partial \xi_k} G_i^j(\mathbf{x}; \boldsymbol{\xi})|_{\xi_0} \delta \xi_k - \text{where } d\xi_k = (\boldsymbol{\xi} - \boldsymbol{\xi}_0)_k.$$

gives

„internal source“ at centroid!

$$\begin{aligned} u_i(\mathbf{x}) &= G_i^j * \cancel{\int_V f_j dV} + G_{i,k}^j * \int_V f_j \delta \xi_k dV + G_{i,kl}^j * \cancel{\int_V f_j \delta \xi_k \delta \xi_l dV} + \dots \\ &= G_i^j * M_j + G_{i,k}^j * M_{jk} + G_{i,kl}^j * M_{kl} + \dots, \end{aligned}$$

with generalized moment  $M_{j k_1 \dots k_l} = \int_V (\xi_{k_1} - \xi_{0 k_1})(\xi_{k_2} - \xi_{0 k_2}) \dots (\xi_{k_l} - \xi_{0 k_l}) f_j(\xi) dV$

## Seismic moment tensor representation (t-dependent)

$$u_i(\mathbf{x}) = M_{jk} \star \left. \frac{\partial G_i^j}{\partial \xi_k} \right|_{\vec{\xi}_0}$$

↑  
time convolution

pay attention: summation convention applied!

## Far field representation

$$u_i(\mathbf{x}, t) \approx M_{jk} S(t) * \dot{G}_i^j(\mathbf{x}, t) \frac{\partial t'}{\partial \xi_k} = M_{jk} \dot{S}(t) * G_i^j(\mathbf{x}, t) \frac{\partial t'}{\partial \xi_k}$$

*retardation time*

**near field:**  $S(t)$



**far field:**  $dS/dt$  (moment rate function)



## example: full space

*direction cosines*

$$M_{jk} * G_{i,k}^j = \frac{\gamma_i \gamma_j \gamma_k}{4\pi \rho v_p^3 r} \frac{1}{r} \dot{M}_{jk} \left( t - \frac{r}{v_p} \right) - \frac{\gamma_i \gamma_j - \delta_{ij}}{4\pi \rho v_s^3} \frac{1}{\gamma_k r} \dot{M}_{jk} \left( t - \frac{r}{v_s} \right)$$

*geometrical attenuation*

*far field moment rate function*

*retarded time of P and S waves*

*radiation pattern term*

point source parameter

## Double Couple moment tensor (shear crack)

e.g.: slip in 1 direction, fault normal in 3 direction

$$\mathbf{M} = \mathcal{N} \Delta u A \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

[ ]

strength of the shear crack

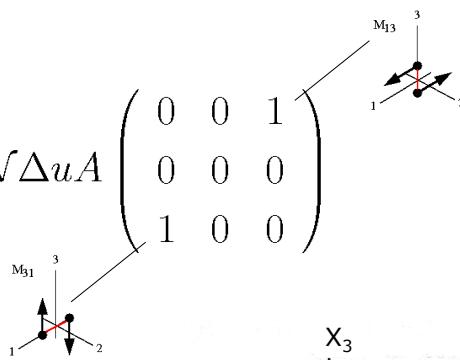
$\mathcal{N}$ : shear module (rigidity) [Nm<sup>-2</sup>]  
 $A$ : rupture plane area [m<sup>2</sup>]  
 $\Delta u$ : average slip [m]



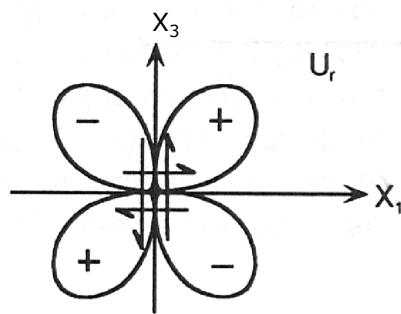
point source parameter

## Double Couple moment tensor (shear crack)

$$\mathbf{M} = \mathcal{N} \Delta u A \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

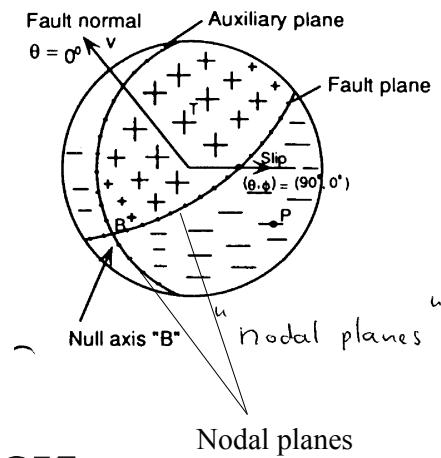


P wave radiation pattern



point source parameter

## Lower hemispherical projection of radiation pattern



**GFZ**  
Helmholtz-Zentrum  
POTS DAM

HEMHOLTZ  
GEMEINSCHAFT

point source parameter

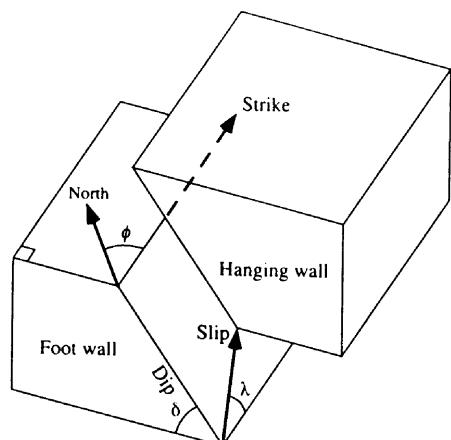
## Fault plane description: strike – dip – rake - slip

- strike :  $0 - 360^\circ$
- dip :  $0 - 90^\circ$
- rake :  $-180^\circ - +180^\circ$
- slip : in meter

*fault normal*      *slip*

$$M_{jk} = N ( n_j \Delta u_k + n_k \Delta u_j )$$

= function of strike, dip, rake and slip



**GFZ**  
Helmholtz-Zentrum  
POTS DAM

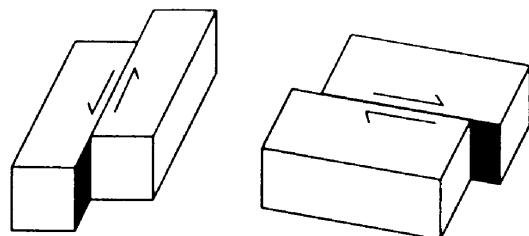
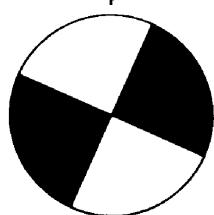
see e.g. Jost and Herrmann (1989)

HEMHOLTZ  
GEMEINSCHAFT

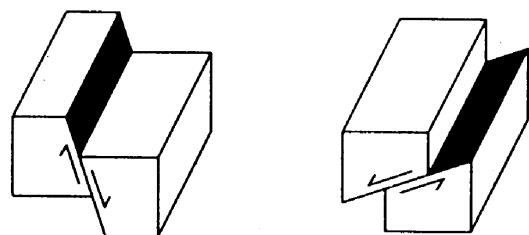
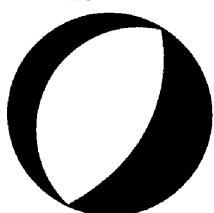
point source parameter

### Basic fault types: focal solutions $\leftrightarrow$ fault geometry

Strike Slip



Normal

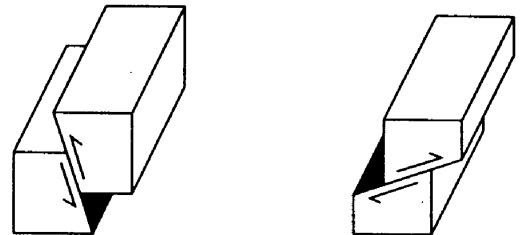
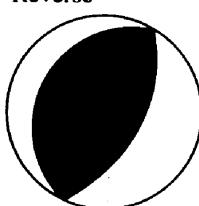


GFZ  
Helmholtz-Zentrum  
POTS DAM

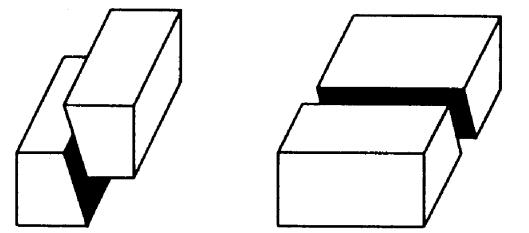
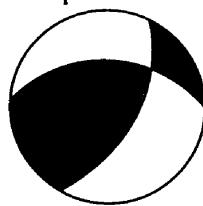
point source parameter

### Basic fault types: focal solutions $\leftrightarrow$ fault geometry

Reverse



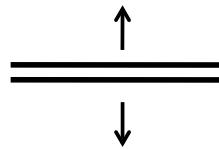
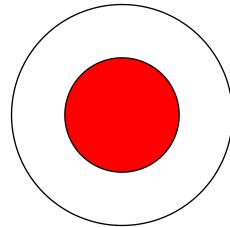
Oblique



GFZ  
Helmholtz-Zentrum  
POTS DAM

point source parameter

### Tensile crack moment tensor tensor



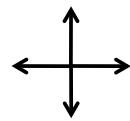
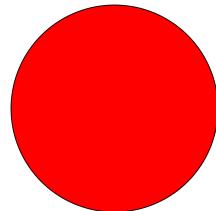
$$\mathbf{M} = \bar{u}A \begin{pmatrix} \mathcal{L} & 0 & 0 \\ 0 & \mathcal{L} & 0 \\ 0 & 0 & \mathcal{L} + 2\mathcal{N} \end{pmatrix}$$

$L$ : 1<sup>st</sup> Lame's parameter [Nm<sup>-2</sup>]



point source parameter

### explosion source moment tensor tensor



$$\mathbf{M} = 4\pi r_e^2 \bar{u}_e (\mathcal{L} + 2\mathcal{N}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$r_e$ : elastic radius

$u_e$ : displacement at  $r_e$

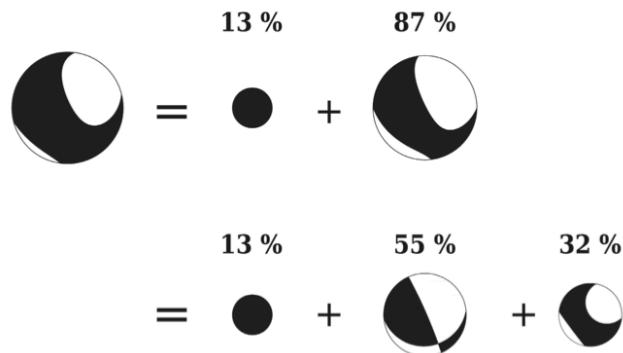


point source parameter

## how to plot “beach balls” with *mopad* ?

see Krieger and Heimann, SRL 2012, doi: 10.1785/gssrl.83.3.589

... python based toolbox for moment tensor plotting and decomposition



point source parameter

### 3. seismic moment and moment magnitude

seismic moment:

$$M_0 = N A \langle u \rangle$$

shear modulus

average slip

rupture plane area

from energy scaling of surface wave magnitude and crack models

$$\text{moment magnitude: } M_W = \log M_0 / 1.5 - 10.73$$



## Memo Plate Point Source

- ✓ hypocenter and centroid location may differ several km
- ✓ focal solution indicates strike, dip and rake of both nodal planes
- ✓ moment tensor defines general internal source
- ✓ seismic moment  $M_0 = N A \langle u \rangle$  defines strength of earthquake (point source)
- ✓ far-field pulse of body waves scales with moment rate function



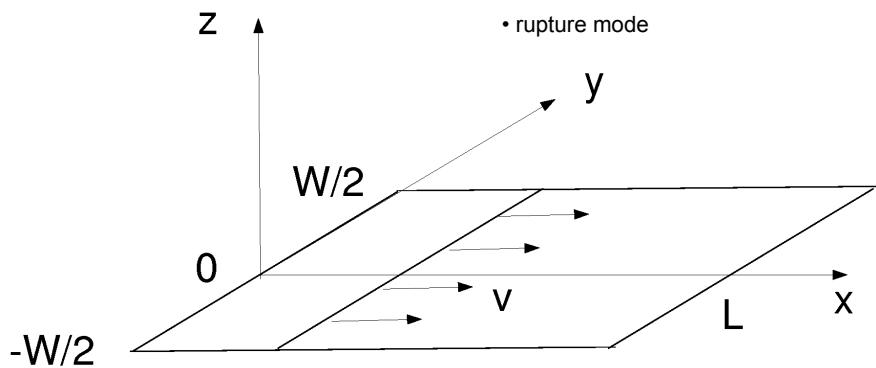
## extended source parameter

*Finite rupture and directivity*

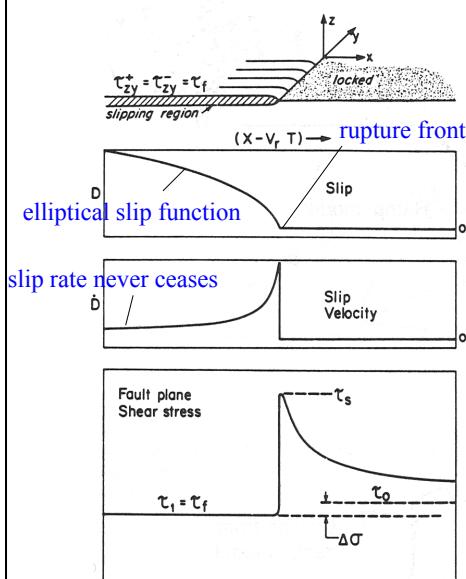


## Haskell model: *a simple physical kinematic model*

- rupture front (velocity and shape)
- rise time and slip function (temporal)
- healing front (velocity and shape)
- slip direction and slip pattern (spatial)
- rupture mode



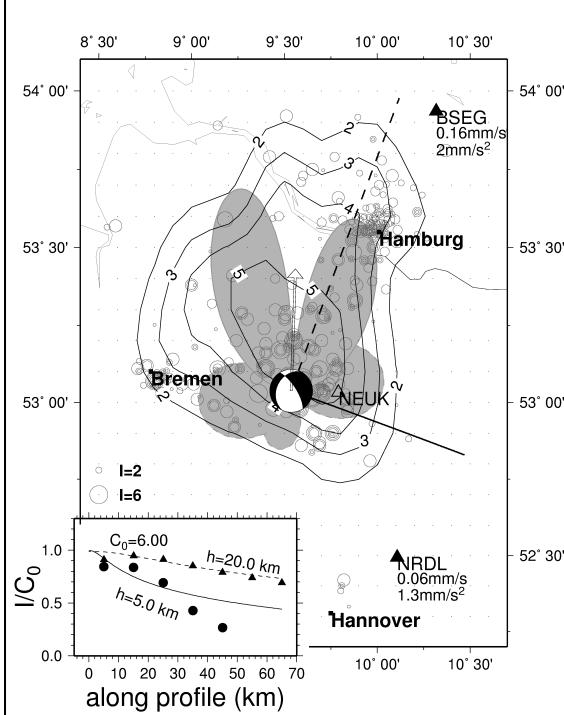
## crack model or slip pulse model ?



Kostrov (1966)

## extended source parameter

- ✓ How does the source time function look like?
- ✓ What are directivity effects?

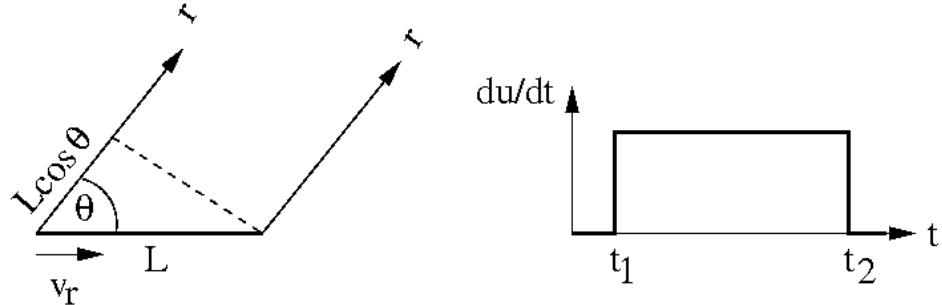


The Rotenburg 2004,  
Germany, M 4.5  
earthquake

The directivity of surface waves (grey) may explain that ground motion peak amplitudes (contourlines = intensities) is direction dependent

from Dahm et al., 2007, BSSA

### unilateral propagation of Haskell (line) source

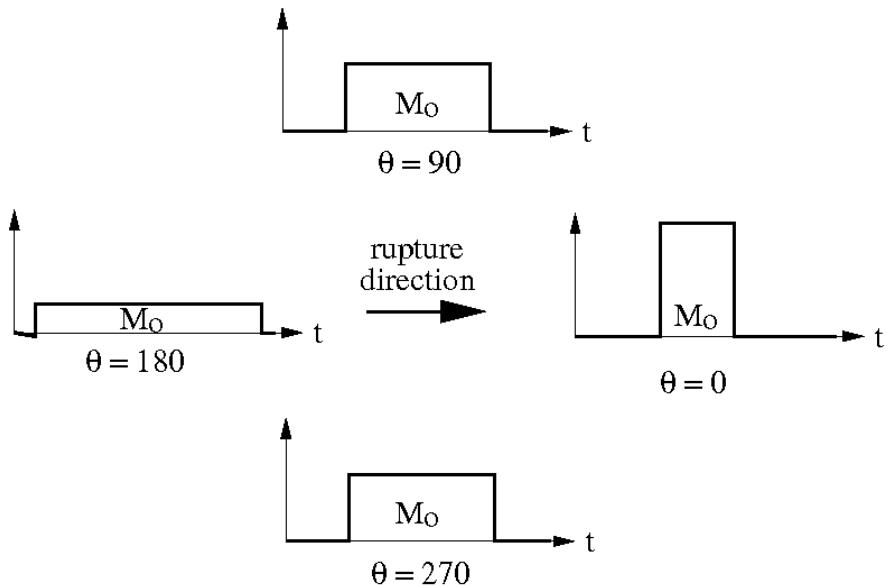


$$T_r = t_2 - t_1$$

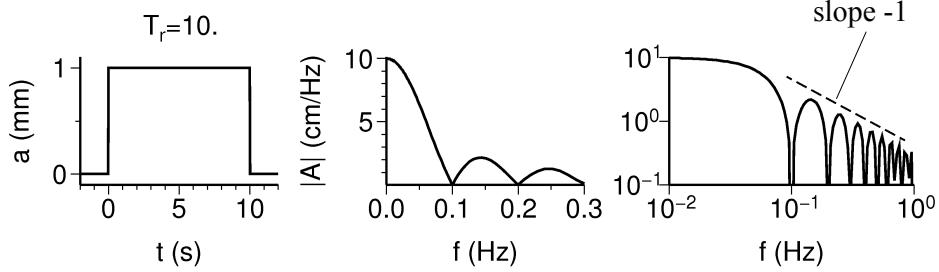
= ...

$$= L \left( \frac{1}{v_r} - \frac{\cos \Theta}{\beta} \right)$$

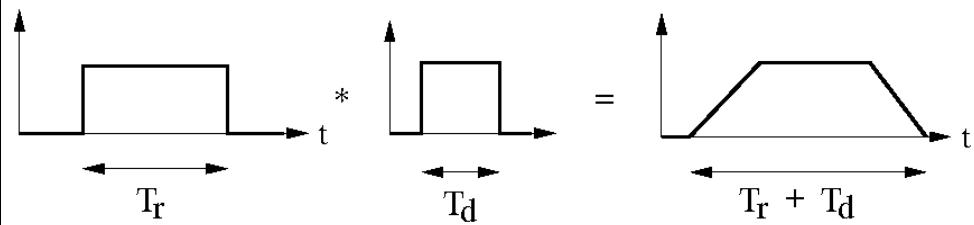
### directivity of far-field pulse



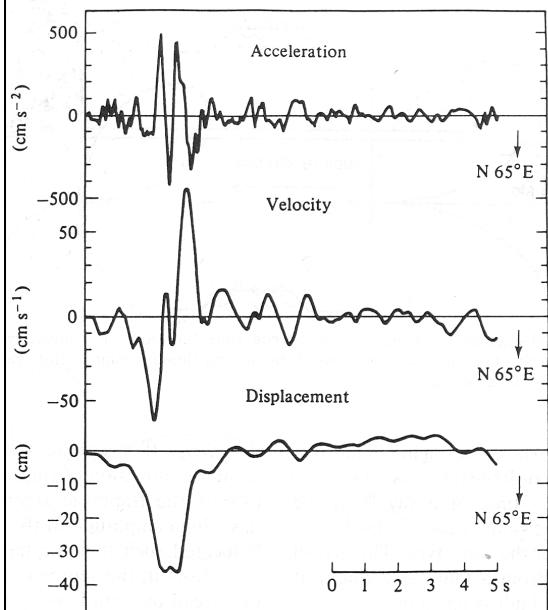
## source spectra



Rupture duration and slip duration  
= trapezoidal displacement pulses

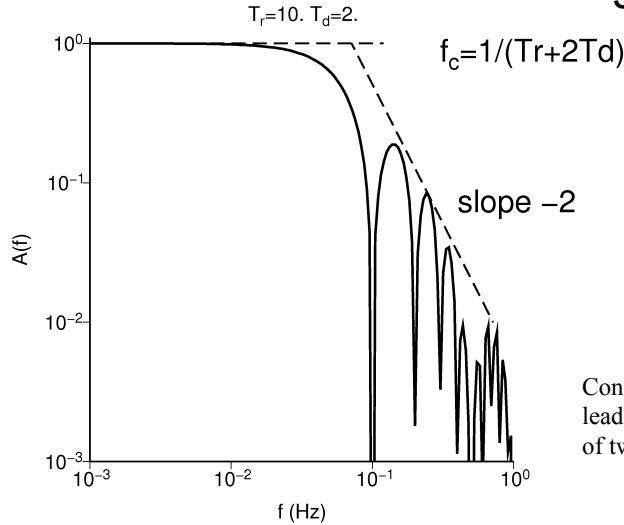


## trapezoidal slip



SH Ground motion near the Epicenter of an earthquake at Parkfield. SH radiation is maximal, P-waves are nodal (Aki, 1968)

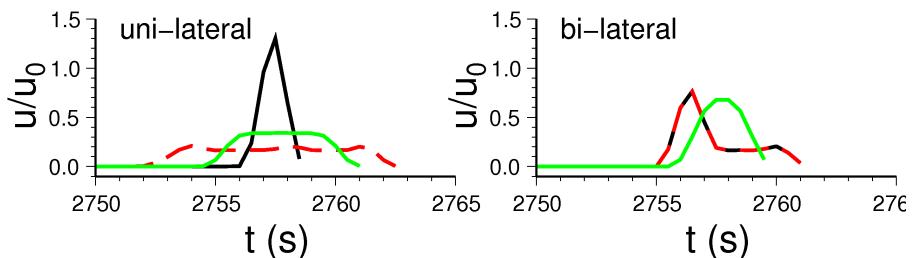
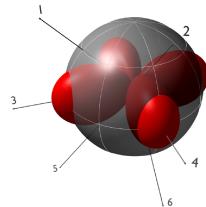
## source spectra



Convolution of 2 boxcar function leads in  $f$ -domain to multiplication of two sinc-functions:

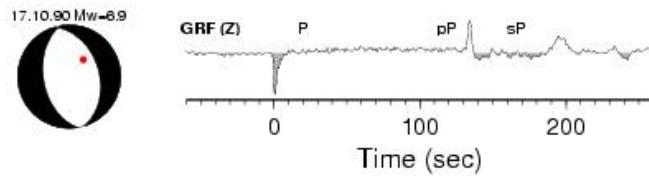
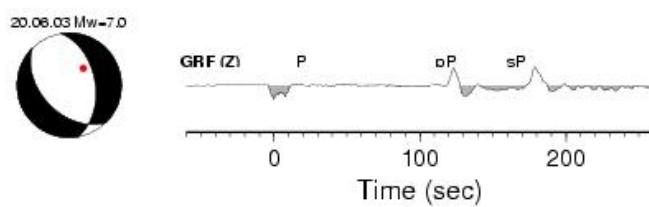
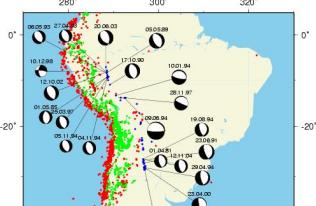
$$A(f) \sim M_0 \left| \frac{\sin \pi f T_r}{\pi f T_r} \right| \left| \frac{\sin \pi f T_d}{\pi f T_d} \right| \sim f^{-2}$$

## Synthetic body waves: uni- and bilateral rupture

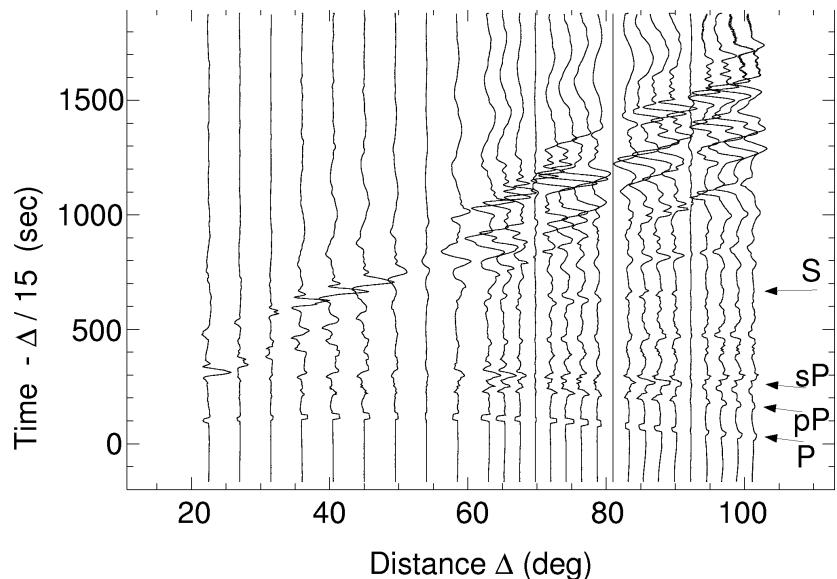


by S. Heimann

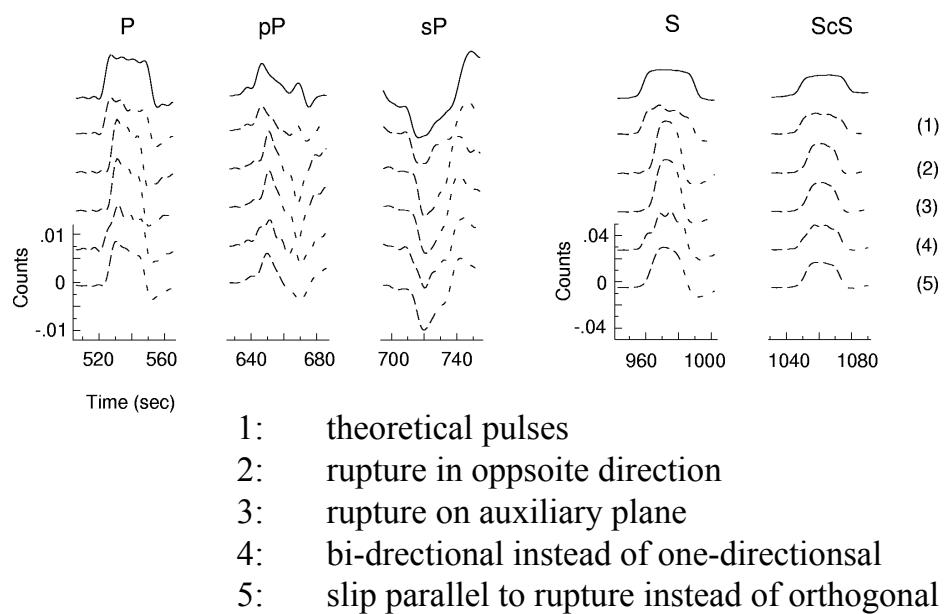
## Brasilian deep focus earthquakes



## synthetic seismograms: deep earthquake



## Waveforms at 58 deg epicentral distance



## Memo Plate Directivity

- ✓ directivity effects can change pulse duration, corner frequency  $f_c$  and pulse amplitude
- ✓ moment of body wave pulses remains constant (e.g. constant level of low frequency plateau of spectrum)
- ✓ Directivity effects are in general small and only visible at frequencies above  $f_c$



## Summary Seismic Sources

- **Seismic point source parameters:** hypocenter, centroid, seismic moment (magnitude), moment tensor (source mechanism)
- **Extended source parameter:** directivity, rupture style, distinguishing rupture and auxiliary plane, length/area of rupture, slip distribution, patches of high slip, rupture velocity, etc.
- **Moment tensor concept** can handle general point and extended sources

