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Determination of seismic source parameters and analysis of uncertainties. Application to studies of strong recent earthquakes.

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We consider the parameterization of seismic source based on the concept of stress glut tensor. We describe techniques for determination of seismic source integral characteristics from analysis of surface wave records. We treat the problem of the nonuniqueness of the moment tensor inversion for shallow earthquakes from long period surface wave data.

I. Parameterization of seismic source

The description of seismic source we will consider is based on the formalism developed by Backus and Mulcahy, 1976.

Statement of the problem.

Motion equation

$$\rho \ddot{u}_i = \sigma_{ij,j} + f_i \tag{1.1}$$

Hook's law for isotropic medium

$$\sigma_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu \varepsilon_{ij} \tag{1.2}$$

Initial conditions

$$\dot{\mathbf{u}} \equiv \mathbf{u} \equiv 0, t < 0 \tag{1.3}$$

Boundary conditions

$$\sigma_{ij} n_j \big|_{S_0} = 0 \tag{1.4}$$

Here **u** – displacement vector; σ_{ij} – elements of symmetric 3x3 stress tensor; i,j=1,2,3 and the summation convention for repeated subscripts is used; $\sigma_{ij,j} = \sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j}$; ε_{ij} – elements of symmetric 3x3 strain tensor and $\varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i})$; ρ - density; f_i – components of external force; n_j – components of the normal to the free surface S_0.

Solution of the problem (1.1)-(1.4) can be given by formula

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega G_{ij}(\mathbf{x},\mathbf{y},t-\tau) f_j(\mathbf{y},\tau) dV_y \tag{1.5}$$

or

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega H_{ij}(\mathbf{x},\mathbf{y},t-\tau) \dot{f}_j(\mathbf{y},\tau) dV_y \tag{1.6}$$

Here G_{ij} is the Green's function,

$$H_{ij}(\mathbf{x},\mathbf{y},t) = \int_0^t G_{ij}(\mathbf{x},\mathbf{y},\tau) d\tau , \tag{1.7}$$

$\mathbf{x} \in \Omega$ and $0 < t < T$ are the space region and time interval where $\dot{f}$ is not identically zero.

Seismic sources

We will consider internal sources only (earthquakes). In this case any external forces are absent. We must then set $\mathbf{f} \equiv 0$ in equation (1.1), so that the only solution that satisfies the homogeneous initial (1.3) and boundary (1.4) conditions, as well as Hook's law (1.2) will be $\mathbf{u} \equiv 0$. Non-zero displacements cannot arise in the medium, unless at least one of the above conditions is not true.

Following Backus and Mulcahy, 1976, we assume seismic motion to be caused by a departure from ideal elasticity (from Hook's law) within some volume of the medium Ω at some time interval $0 < t < T$.

Let $\mathbf{u}(\mathbf{x},t)$ be the actual displacements, $\boldsymbol{\sigma}(\mathbf{x},t)$ - correspondent stresses, if Hook's law is valid, $\mathbf{s}(\mathbf{x},t)$ - actual stresses.
Let the difference

$$\boldsymbol{\Gamma}(\mathbf{x},t) = \boldsymbol{\sigma}(\mathbf{x},t) - \mathbf{s}(\mathbf{x},t), \tag{1.8}$$

called the *stress glut tensor* or *moment tensor density*, is not identically zero for $0 < t < T$ and $\mathbf{x} \in \Omega$.
T we define as source duration, and Ω - source region. Within this region and time interval (and only there) the tensor $\dot{\boldsymbol{\Gamma}}(\mathbf{x},t)$ is not identically zero as well.

Replacing $\boldsymbol{\sigma}(\mathbf{x},t)$ by $\mathbf{s}(\mathbf{x},t)$ in equation (1.1), using definition (1.8) and the absence of external forces ($\mathbf{f} \equiv 0$) we can rewrite the motion equation (1.1) in form

$$\rho \ddot{u}_i = s_{ij,j}$$

or

$$\rho \ddot{u}_i = \sigma_{ij,j} + g_i \tag{1.9}$$

where

$$g_i = -\Gamma_{ij,j} \ . \tag{1.10}$$

Equation (1.10) defines the equivalent force $\mathbf{g}$. Using formula (1.6) with f_i replaced by g_i, definition (1.10) and Gauss theorem we have for displacements

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Omega H_{ij,k}(\mathbf{x},\mathbf{y},t-\tau)\dot{\Gamma}_{jk}(\mathbf{y},\tau)dV_y \,, \tag{1.11}$$

where H_{ij} is differentiated with respect to y_k .
If the inelastic motions are concentrated at a surface Σ, then

$$u_i(\mathbf{x},t) = \int_0^T d\tau \int_\Sigma H_{ij,k}(\mathbf{x},\mathbf{y},t-\tau)\dot{\Gamma}_{jk}(\mathbf{y},\tau)d\Sigma_y \,. \tag{1.12}$$

Relation of stress glut (moment tensor density) with classic definition of moment tensor M :

$$\mathbf{M} = \int_0^T dt \int_\Omega \dot{\boldsymbol{\Gamma}}(\mathbf{y},t)dV_y \quad . \tag{1.13}$$

Normalizing moment tensor we define seismic moment M_0 :

$\mathbf{M}=M_0\mathbf{m}$, where tensor $\mathbf{m}$ is normalized by condition $\mathrm{tr}(\mathbf{m}^{\mathrm{T}}\mathbf{m}) = \sum_{i,j=1}^{3} m_{ij}^2 = 2$, $\mathbf{m}^{\mathrm{T}}$ is transposed tensor $\mathbf{m}$.

Stress glut moment for special types of seismic sources

1. Discontinuity of displacement $\Delta\mathbf{u}$ at a surface Σ in isotropic medium (stress is continuous):

$$\Gamma_{ij}(\mathbf{x},t) = \lambda \Delta u_k(\mathbf{x},t) n_k(\mathbf{x}) \delta_{ij} + \mu [n_i(\mathbf{x}) \Delta u_j(\mathbf{x},t) + n_j(\mathbf{x}) \Delta u_i(\mathbf{x},t)]. \tag{1.14}$$

Here $\mathbf{n}(\mathbf{x})$ is the normal to the surface Σ, and seismic disturbances are given by formula (1.12).
2. In the case of tangential (shear) dislocation we have
$\Delta u_k n_k \equiv 0$ and formula (1.14) takes form

$$\Gamma_{ij}(\mathbf{x},t) = \mu [n_i(\mathbf{x}) \Delta u_j(\mathbf{x},t) + n_j(\mathbf{x}) \Delta u_i(\mathbf{x},t)]. \tag{1.15}$$

3. Instant point tangential dislocation occurred in the point $\mathbf{x}=\mathbf{0}$ at time $t=0$:

$$\dot{\Gamma}_{ij}(\mathbf{x},t) = M_0 m_{ij} \delta(t) \delta(\mathbf{x}), \tag{1.16}$$

where $m_{ij} = n_i a_j + n_j a_i$, $\mathbf{a} = \Delta\mathbf{u}/|\Delta\mathbf{u}|$ and $M_0 = \mu \, |\Delta \mathbf{u}|$.

Phenomena of matrix **m**

Tr**m** = 0. The eigenvalues of matrix **m** are: 1, -1 and 0. The eigenvector correspondent to 1 defines the direction of maximum extension, and the eigenvector correspondent to -1 defines the direction of maximum compression. Such a source is called double-couple.

As it follows from formula (1.12) an instant point double-couple excites a displacement field of the form

$$u_i(\mathbf{x},t) = M_0 H_{ik,l}(\mathbf{x},\mathbf{0},t) m_{kl} \, . \tag{1.17}$$

We have for Fourier transforms $\mathbf{H}(\mathbf{x},\mathbf{y},\omega)$ and $\mathbf{G}(\mathbf{x},\mathbf{y},\omega)$ from equation (1.7):

$$\mathbf{H}(\mathbf{x},\mathbf{y},\omega) = \frac{1}{\mathrm{i}\omega}\mathbf{G}(\mathbf{x},\mathbf{y},\omega) \, , \tag{1.18}$$

where i is the imaginary unit, and ω is angular frequency.
As result the spectrum of displacements is given by formula

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega} M_0 m_{kl} G_{ik,l}(\mathbf{x},\mathbf{0},\omega) \, . \tag{1.19}$$

Relation between the displacement field and stress glut moments

We assume that the time derivative of stress glut tensor can be presented in form:

$$\dot{\boldsymbol{\Gamma}}(\mathbf{x},t) = f(\mathbf{x},t)\mathbf{m} \, , \tag{1.20}$$

where $f(\mathbf{x},t)$ is non-negative function and **m** is a uniform normalized moment tensor.

The moment $f^{(l,n)}_{k_1 \ldots k_l}(\mathbf{q},\tau)$ of spatial degree l and temporal degree n with respect to point **q** and instant of time τ is a tensor of order l and is given by formula

$$f^{(l,n)}_{k_1 \ldots k_l}(\mathbf{q},\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_{k_1} - q_{k_1}) \cdots (x_{k_l} - q_{k_l})(t-\tau)^n \, dt \, , \tag{1.21}$$

$k_1,\ldots,k_l$=1,2,3.

Replacing $H_{ij}(\mathbf{x},\mathbf{y},t-\tau)$ in equation (1.11) by its Taylor series in powers of **y** and in powers of τ, we get:

$$u_i(\mathbf{x},t) = \sum_{l=0}^\infty \sum_{n=0}^\infty \frac{(-1)^n}{l!\,n!} m_{jk} f^{(l,n)}_{k_1 \ldots k_l}(\mathbf{0},0) \frac{\partial^n}{\partial t^n} \frac{\partial}{\partial y_{k_1}} \cdots \frac{\partial}{\partial y_{k_l}} \frac{\partial}{\partial y_k} H_{ij}(\mathbf{x},\mathbf{y},t)\Big|_{\mathbf{y}=\mathbf{0}} \, . \tag{1.22}$$

Using formulae (1.18) and (1.22) we have following equation for the spectrum of displacements:

$$u_i(\mathbf{x},\omega) = \sum_{l=0}^\infty \sum_{n=0}^\infty \frac{(-1)^n}{l!\,n!} m_{jk} f^{(l,n)}_{k_1 \ldots k_l}(\mathbf{0},0)(\mathrm{i}\,\omega)^{n-1} \frac{\partial}{\partial y_{k_1}} \cdots \frac{\partial}{\partial y_{k_l}} \frac{\partial}{\partial y_k} G_{ij}(\mathbf{x},\mathbf{y},\omega)\Big|_{\mathbf{y}=\mathbf{0}} \, . \tag{1.23}$$

Here we assume that the point $\mathbf{y}=\mathbf{0}$ and the instant t=0 belong to the source region and the time of the source activity respectively.

When the spectra of displacements $u_i(\mathbf{x},\omega)$ and Green's function $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ have been low pass filtered, the terms in equation (1.23) start to decrease with l and n increasing at least as rapidly as $(\omega \mathrm{T})^{l+n}$ (T is the source duration, and ωT<1), and one might then restrict to considering finite sums only.

We will take into account in the following sections only the first terms in formula (1.23) for $l + n \le 2$.

II. Source inversion in moment tensor approximation

The first term in (1.23) corresponding to l=0, n=0, describes the spectra of displacements $u_i(\mathbf{x},\omega)$ excited by an instant point source (compare with formula (1.19) taking into account that seismic moment is equal to zero moment of function $f(\mathbf{x},t)$: $M_0=f^{(0,0)}$). For a source with nonzero size and duration this term approximates $u_i(\mathbf{x},\omega)$ with high accuracy for periods much longer then source duration. Performing the inversion of long period seismic waves we describe the earthquake by an instant point source. As it was mentioned in previous section, an instant point source can be given by moment tensor - a symmetric 3x3 matrix $\mathbf{M}$. Seismic moment M_0 is defined by equation $M_0 = \sqrt{\tfrac{1}{2}\mathrm{tr}(\mathbf{M}^{\mathrm{T}}\mathbf{M})}$, where $\mathbf{M}^{\mathrm{T}}$ is transposed moment tensor $\mathbf{M}$, and $\mathrm{tr}(\mathbf{M}^{\mathrm{T}}\mathbf{M}) = \sum_{i,j=1}^{3} M_{ij}^{\ 2}$. Moment tensor of any event can be presented in the form $\mathbf{M} = M_0\mathbf{m}$, where matrix $\mathbf{m}$ is normalized by condition $\mathrm{tr}(\mathbf{m}^{\mathrm{T}}\mathbf{m}) = 2$.

We'll consider a double-couple instant point source (a pure tangential dislocation) at a depth h. Such a source can be given by 5 parameters: double-couple depth, its focal mechanism which is characterizing by three angles: strike, dip and slip or by two orthogonal unit vectors (direction of principal tension **T** and direction of principal compression **P**) and seismic moment M_0. Four of these parameters we determine by a systematic exploration of the four dimensional parametric space, and the 5-th parameter M_0 - solving the problem of minimization of the misfit between observed and calculated surface wave amplitude spectra for every current combination of all other parameters.

Under assumptions mentioned above the relation between the spectrum of displacements $u_i(\mathbf{x},\omega)$ and moment tensor $\mathbf{M}$ can be expressed by formula (1.19) rewritten below in slightly different form:

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\,\omega}[M_{jl}\,\frac{\partial}{\partial y_l}G_{ij}(\mathbf{x},\mathbf{y},\omega)] \qquad (2.1)$$

i,j = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (2.1) is the spectrum of Green function for the chosen model of medium and wave type (see Levshin, 1985; Bukchin, 1990), $\mathbf{y}$ - source location. We will discuss the inversion of surface wave spectra, so $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ is the spectrum of surface wave Green function. We assume that the paths from the earthquake source to seismic stations are relatively simple and are well approximated by weak laterally inhomogeneous model (Woodhouse, 1974; Babich *et al.*, 1976). The surface wave Green function in this approximation is determined by the near source and near receiver velocity structure, by the mean phase velocity of wave, and by geometrical spreading. We assume that waves propagate from the source to station along great circles. Under these assumptions the amplitude spectrum $|u_i(\mathbf{x},\omega)|$ defined by formula (2.1) does not depend on the average phase velocity of the wave. In such a model the errors in source location do not affect the amplitude spectrum (Bukchin, 1990). The average phase velocities of surface waves are usually not well known. For this reason as a rule we use only amplitude spectra of surface waves for determining source parameters under consideration. We use observed surface wave phase spectra only for very long periods. Correcting the spectra for attenuation we use laterally homogeneous model for quality factor.

Surface wave amplitude spectra inversion

If all characteristics of the medium are known, the representation (2.1) gives us a system of equations for parameters defined above. Let us consider now a grid in the space of these 4 parameters. Let the models of the media be given. Using formula (2.1) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{x},\omega)$ be any observed value of the spectrum, i = 1,…,N; $\varepsilon^{(i)}_{\text{amp}}$ - corresponding residual of $|u^{(i)}(\mathbf{x},\omega)|$. We define the normalized amplitude residual by formula

$$\varepsilon_{\text{amp}}(h,\mathbf{T},\mathbf{P}) = \left[\left(\sum_{i=1}^{N} \varepsilon^{(i)\,2}_{\text{amp}}\right)\Big/\left(\sum_{i=1}^{N} |u^{(i)}(\mathbf{x},\omega|^2\right)\right]^{1/2}. \tag{2.2}$$

The optimal values of the parameters that minimize ε_{amp} we consider as estimates of these parameters. We search them by a systematic exploration of the four-dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions. Fixing the value of one of varying parameters we put in correspondence to it a minimal value of the residual ε_{amp} on the set of all possible values of the other parameters. In this way we define one residual function on scalar argument and two residual functions on vector argument corresponding to the scalar and two vector varying parameters: $\varepsilon_h(h)$, $\varepsilon_{\mathbf{T}}(\mathbf{T})$ and $\varepsilon_{\mathbf{P}}(\mathbf{P})$. The value of the parameter for which the corresponding function of the residual attains its minimum we define as estimate of this parameter. At the same time these functions characterize the degree of resolution of the corresponding parameters. From geometrical point of view these functions describe the lower boundaries of projections of the 4-D surface of functional ε on the coordinate planes. A sketch illustrating the definition of partial residual functions is given in figure 1.

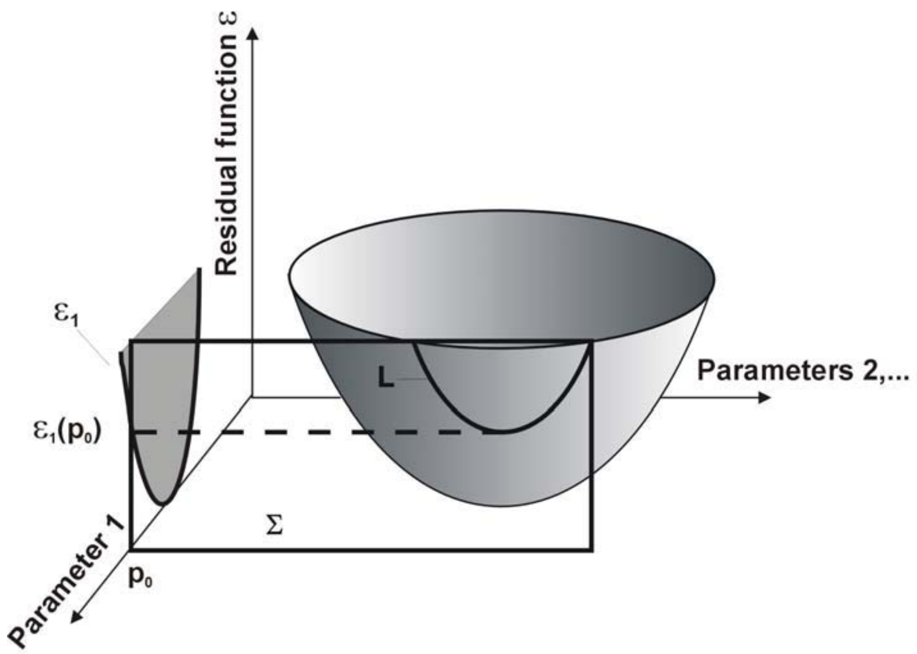

Figure 1. A sketch illustrating the definition of partial residual functions.

Here one of 4 parameters is picked out as ‘parameter 1’, and one of coordinate axis corresponds to this parameter. Another coordinate axis we consider formally as 3-D space of the rest 3 parameters. Plane Σ is orthogonal to the axis ‘parameter 1’ and cross it in a point p_0 . Curve L is the intersection of the plane Σ and the surface of functional ε. As one can see from the figure the point $\varepsilon_1(p_0)$ belong to the boundary of projection of the surface of functional ε, and at the same time it corresponds to a minimal value of the residual ε on the set of all possible values of the other 3 parameters while ‘parameter 1’ is equal to the value p_0.

So, as it is accepted in engineering we characterize our surface by its 4 projections on coordinate planes.

It is well known that the focal mechanism cannot be uniquely determined from surface wave amplitude spectra. There are four different focal mechanisms radiating the same surface wave amplitude spectra. These four equivalent solutions represent two pairs of mechanisms symmetric with respect to the vertical axis, and within the pair differ from each other by the opposite direction of slip.

To get a unique solution for the focal mechanism we have to use in the inversion additional observations. For these purpose we use very long period phase spectra of surface waves or polarities of first arrivals.

Joint inversion of surface wave amplitude and phase spectra

Using formula (2.1) we can calculate for chosen frequency range the phase spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed phase spectra give us a residual $\varepsilon_{ph}^{(i)}$ for every point of observation, every wave and every frequency ω. We define the normalized phase residual by formula

$$\varepsilon_{ph}(h, \varphi, \mathbf{T}, \mathbf{P}) = \frac{1}{\pi}\left[\left(\sum_{i=1}^{N} \varepsilon_{ph}^{(i)2}\right) / N\right]^{1/2} . \tag{2.3}$$

We determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_{ph})(1 - \varepsilon_{amp}) . \tag{2.4}$$

To characterize the resolution of source characteristics we calculate partial residual functions in the same way as was described above.

Joint inversion of surface wave amplitude spectra and P wave polarities

Calculating radiation pattern of P waves for every current combination of parameters we compare it with observed polarities. The misfit obtained from this comparison we use to calculate a joint residual of surface wave amplitude spectra and polarities of P wave first arrivals. Let ε_{amp} be the residual of surface wave amplitude spectra, ε_p - the residual of P wave first arrival polarities (the number of wrong polarities divided by the full number of observed polarities), then we determine the joint residual ε by formula

$$\varepsilon = 1 - (1 - \varepsilon_p)(1 - \varepsilon_{amp}) . \tag{2.5}$$

For this type of inversion we calculate partial residual functions to characterize the resolution of parameters under determination in the same way as it was described for two first types.

Before inversion we apply to observed polarities a smoothing procedure (see Lasserre *et al.*, 2001), which we will describe here briefly.

Let us consider a group of observed polarities (+1 for compression and -1 for dilatation) radiated in directions deviating from any medium one by a small angle. This group is presented in the inversion procedure by one polarity prescribing to this medium direction. If the number of one of two types of polarities from this group is significantly larger then the number of opposite polarities, then we prescribe this polarity to this medium direction. If no one of two polarity types can be considered as preferable, then all these polarities will not be used in the inversion. To make a decision for any group of n observed polarities we calculate the sum $m = n_+ - n_-$, where n_+ is the number of compressions and $n_- = n - n_+$ is the number of dilatations. We consider one of polarity types as preferable if $|m|$ is larger then its standard deviation in the case when +1 and -1 appear randomly with this same probability 0.5. In this case n_+ is a random value distributed following the binomial low. For its average we have $M(n_+) = 0.5n$, and for dispersion $D(n_+) = 0.25n$. Random value m is a linear function of n_+ such that $m = 2n_+ - n$. So following equations are valid for the average, for the dispersion, and for the standard deviation σ of value m

$M(m) = 2M(n_+) - n = n - n = 0$, $\quad D(m) = 4D(n_+) = n$, and $\sigma(m) = \sqrt{n}$.

As a result, if the inequality $|m| \geq \sqrt{n}$ is valid then we prescribe +1 to the medium direction if $m > 0$, and -1 if $m < 0$.

III. Second moments approximation. Characteristics of source shape and evolution in time.

We present here a technique based on the description of seismic source distribution in space and in time by integral moments (see Bukchin *et al.*, 1994; Bukchin, 1995; Gomez, 1997 a, b). We assume that the time derivative of stress glut tensor $\dot{\boldsymbol{\Gamma}}$ can be represented in form (1.20). Following Backus and Mulcahy, 1976 we will define the source region by the condition that function $f(\mathbf{x},t)$ is not identically zero and the source duration is the time interval when nonelastic motion occurs at various points within the source region, i.e., $f(\mathbf{x},t)$ is different from zero.

Spatial and temporal integral characteristics of the source can be expressed by corresponding moments of the function $f(\mathbf{x},t)$ (Backus, 1977a; Bukchin *et al.*, 1994). These moments can be estimated from the seismic records using the relation between them and the displacements in seismic waves, which we will consider later. In general case stress glut rate moments of spatial degree 2 and higher are not uniquely determined by the displacement field (Pavlov, 1994; Das & Kostrov, 1997). But in the case when equation (1.20) is valid such uniqueness takes place (Backus, 1977b; Bukchin, 1995).

Following equations define the spatio-temporal moments of function $f(\mathbf{x},t)$ of total degree (both in space and time) 0, 1, and 2 with respect to point $\mathbf{q}$ and instant of time τ.

$$f^{(0,0)} = \int_V dV \int_0^\infty f(\mathbf{x},t)dt\,, \quad f_i^{(1,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)dt\,,$$

$$f^{(0,1)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)dt\,, \quad f^{(0,2)}(\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(t-\tau)^2 dt\,,$$

$$f_i^{(1,1)}(\mathbf{q},\tau) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(t-\tau)dt\,, \tag{3.1}$$

$$f_{ij}^{(2,0)}(\mathbf{q}) = \int_V dV \int_0^\infty f(\mathbf{x},t)(x_i - q_i)(x_j - q_j)dt$$

Using these moments we will define integral characteristics of the source. Source location is estimated by the spatial centroid $\mathbf{q}_c$ of the field $f(\mathbf{x},t)$ defined as

$$\mathbf{q}_c = \mathbf{f}^{(1,0)}(\mathbf{0}) / M_0\,, \tag{3.2}$$

where $M_0 = f^{(0,0)}$ is the scalar seismic moment.

Similarly, the temporal centroid τ_c is estimated by the formula

$$\tau_c = f^{(0,1)}(0) / M_0\,. \tag{3.3}$$

The source duration is Δt estimated by $2\Delta\tau$, where

$(\Delta\tau)^2 = f^{(0,2)}(\tau_c)\,/\,M_0$. (3.4)

The spatial extent of the source is described by matrix **W**,

$\mathbf{W} = \mathbf{f}^{(2,0)}(\mathbf{q}_c)\,/\,M_0$. (3.5)

The mean source size in the direction of unit vector **r** is estimated by value $2l_r$, defined by formula

$l_r^2 = \mathbf{r}^{\mathrm{T}}\mathbf{W}\mathbf{r}$, (3.6)

where $\mathbf{r}^{\mathrm{T}}$ is the transposed vector **r**. From (3.5) and (3.6) we can estimate the principal axes of the source. There directions are given by the eigenvectors of the matrix **W**. The square of the length of the minor semi-axis is equal to the least eigenvalue, and the square of the length of the major semi-axis is equal to the greatest eigenvalue.

In the same way, from the coupled space time moment of order (1,1) the mean velocity **v** of the instant spatial centroid (Bukchin, 1989) is estimated as

$\mathbf{v} = \mathbf{w}\,/\,(\Delta\tau)^2$, (3.7)

where $\mathbf{w} = \mathbf{f}^{(1,1)}(\mathbf{q}_c, \tau_c)\,/\,M_0$.

The relation between integral estimates and real characteristics of source duration and spatial extent depends on the distribution of moment rate density in time and over the fault. Figure 2 illustrates this relation in the case of Gaussian distributions. In this case 99% confidence duration is 2.5 times larger then the integral estimate, and 99% confidence axis length is 3 times larger then correspondent integral estimate.

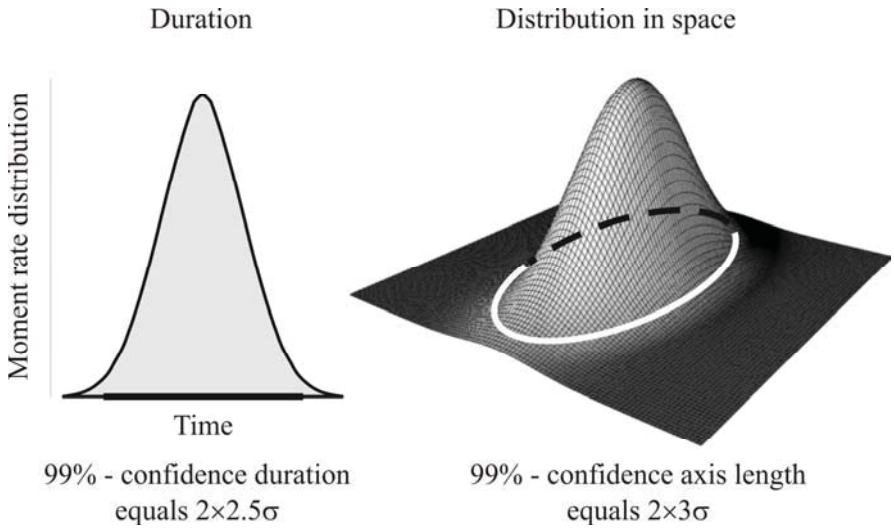

Fig. 2. Relation between integral estimates and real characteristics of source duration and spatial extent.

Now we will consider the low frequency part of the spectra of the i^{th} component of displacements in Love or Rayleigh wave $u_i(\mathbf{x},\omega)$. It is assumed that the frequency ω is small, so that the duration of the source is small in comparison with the period of the wave, and the source size is small as compared with the wavelength. It is assumed that the origin of coordinate system is located in the point of spatial centroid $\mathbf{q}_c$ (i.e. $\mathbf{q}_c = \mathbf{0}$) and that time is measured from the instant of temporal centroid, so that $\tau_c = 0$. With this choice the first degree moments with respect to the spatial origin **x=0** and to the temporal origin t=0 are zero, i.e. $\mathbf{f}^{(1,0)}(\mathbf{0}) = \mathbf{0}$ and $f^{(0,1)}(0) = 0$.

Under this assumptions, taking into account in formula (1.23) only the first terms for $l+n \le 2$ we can express the relation between the spectrum of displacements $u_i(\mathbf{x},\omega)$ and the spatio-temporal moments of the function $f(\mathbf{x},t)$ by following formula (Bukchin,1995)

$$u_i(\mathbf{x},\omega) = \frac{1}{\mathrm{i}\omega} M_0 M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{1}{2\mathrm{i}\omega} f_{mn}^{(2,0)}(\mathbf{0}) M_{jl} \frac{\partial}{\partial y_m}\frac{\partial}{\partial y_n}\frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega)$$

$$- f_m^{(1,1)}(\mathbf{0},0) M_{jl} \frac{\partial}{\partial y_m}\frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega) + \frac{\mathrm{i}\omega}{2} f^{(0,2)}(0) M_{jl} \frac{\partial}{\partial y_l} G_{ij}(\mathbf{x},\mathbf{0},\omega)\,, \qquad (3.8)$$

i,j,l,m,n = 1,2,3 and the summation convention for repeated subscripts is used. $G_{ij}(\mathbf{x},\mathbf{y},\omega)$ in equation (3.8) is the spectrum of Green function for the chosen model of medium and wave type. We assume that the paths from the earthquake source to seismic stations are well approximated by weak laterally inhomogeneous model. Under this assumption, as it was mentioned above, the amplitude spectrum $|u_i(\mathbf{x},\omega)|$ defined by formula (3.8) does not depend on the average phase velocity of the wave, and the errors in source location do not affect the amplitude spectrum.

If all characteristics of the medium, depth of the best point source and seismic moment tensor are known (determined, for example, using the spectral domain of longer periods) the representation (3.8) gives us a system of linear equations for moments of the function $f(\mathbf{x},t)$ of total degree 2. But as we mentioned considering moment tensor approximation the average phase velocities of surface waves are usually not well known. For this reason, we use only amplitude spectrum of surface waves for determining these moments, in spite of non-linear relation between them.

Let us consider a plane source. All moments of the function $f(\mathbf{x},t)$ of total degree 2 can be expressed in this case by formulas (3.2)-(3.7) in terms of 6 parameters: Δt - estimate of source duration, $l_{\max}$ - estimate of maximal mean size of the source (the length of the major axis), φ_l - estimate of the angle between the major axis and strike axis, $l_{\min}$ - estimate of minimal mean size of the source (the length of the minor axis), v - estimate of the absolute value of instant centroid mean velocity $\mathbf{v}$ and φ_v - the angle between $\mathbf{v}$ and strike axis.

Using the Bessel inequality for the moments under discussion we can obtain the following constrain for the parameters considered above (Bukchin, 1995):

$$v^2 \Delta t^2 \left(\frac{\cos^2\varphi}{l_{\max}^2} + \frac{\sin^2\varphi}{l_{\min}^2} \right) \le 1\,, \qquad (3.9)$$

where φ is the angle between major axis of the source and direction of $\mathbf{v}$.
Assuming that the source is a plane fault and representation (1.20) is valid let us consider a grid in the space of 6 parameters defined above. These parameters have to follow inequality (3.9). Let models of the media be given and the moment tensor be fixed as well as the depth of the best point source. Let the fault plane (one of two nodal planes) be identified. Using formula (3.8) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of the varying parameters. Comparison of calculated and observed amplitude spectra give us a residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(\mathbf{r},\omega)$ be any observed value of the spectrum, i = 1,…,N; $\varepsilon^{(i)}$ - corresponding residual of $|u^{(i)}(\mathbf{r},\omega)|$. We define the normalized amplitude residual by formula

$$\varepsilon(\Delta t, l_{\max}, l_{\min}, \varphi_l, v, \varphi_v) = \left[\left(\sum_{i=1}^{\mathrm{N}} \varepsilon^{(i)2} \right) \Big/ \left(\sum_{i=1}^{\mathrm{N}} |u^{(i)}(\mathbf{r},\omega|^2 \right) \right]^{1/2}. \qquad (3.10)$$

The optimal values of the parameters that minimize ε we consider as estimates of these parameters. We search them by a systematic exploration of the six dimensional parameter space. To characterize the degree of resolution of every of these source characteristics we calculate partial residual functions in the same way as was described in previous section. We define 6 functions of the residual corresponding to the 6 varying parameters: $\varepsilon_{\Delta t}(\Delta t)$, $\varepsilon_{l_{max}}(l_{max})$, $\varepsilon_{l_{min}}(l_{min})$, $\varepsilon_{\varphi_l}(\varphi_l)$, $\varepsilon_v(v)$ and $\varepsilon_{\varphi_v}(\varphi_v)$. The optimal values of the parameters that minimize the residual we consider as estimates of these parameters. At the same time these functions characterize the degree of resolution of the corresponding parameters.

IV. Example of application

We illustrate the technique by its application to a study of the recent strong earthquake (M_w = 8.6) occurred to the West of the coast of Northern Sumatra on 11 April 2012.

At the first step inverting long period (from 200 s to 300 s) records of fundamental Love and Rayleigh modes we obtained parameters characterizing the event in point instant double-couple approximation: seismic moment, focal mechanism, and source depth. The records were processed by the frequency-time and polarization analysis package FTAN (Lander, 1989). We selected Love and Rayleigh fundamental mode records of a good quality from 11 stations of GEOSCOPE seismic network. Their azimuthal distribution is given in figure 3.

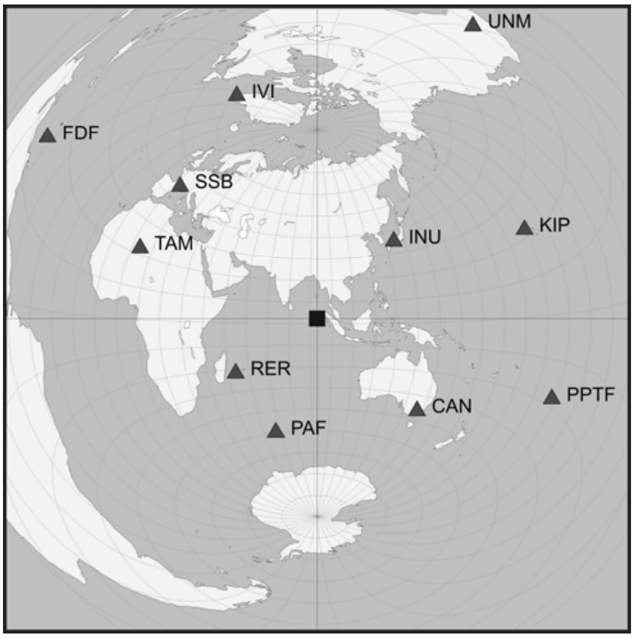

Fig.3. Distribution of stations (triangles) used for moment tensor inversion. The square marks the epicenter.

To improve the resolution we used polarities of direct P-waves as additional information. In the source region and under the receivers, we used the 3SMAC model (Ricard et al. 1996) for the crust and the PREM model below. We used the quality factor given by the PREM model for attenuation correction. The moment tensor describing the source in instant point source approximation is obtained by joint inversion of surface wave amplitude spectra and first arrival polarities at worldwide stations (Lasserre *et al*. 2001). The solution gives a focal mechanism shown in figure 4. The source depth resolution curve is shown in the same figure. The estimate of source depth takes values from 35 to 40 km. The estimated value of seismic moment is equal to $0.88 \cdot 10^{22}$ N·m. It corresponds to the magnitude value $M_w = 8.6$. The resolution maps for the principal stress axes are shown in figure 5.

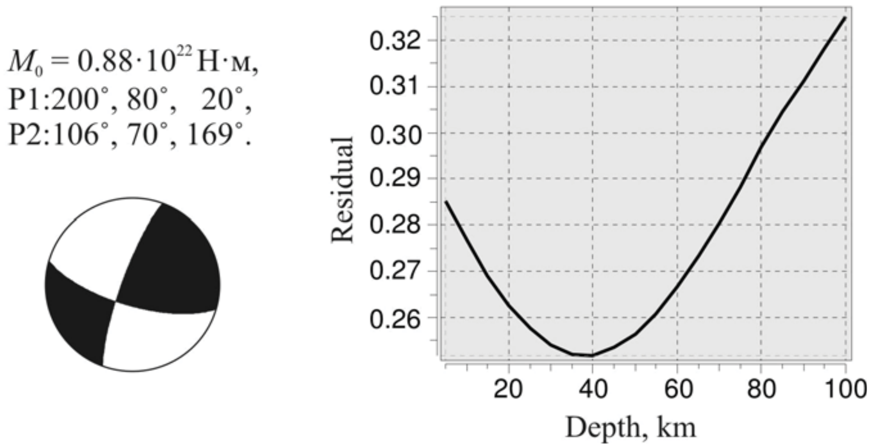

Fig. 4. Double-couple solution and source depth resolution curve.

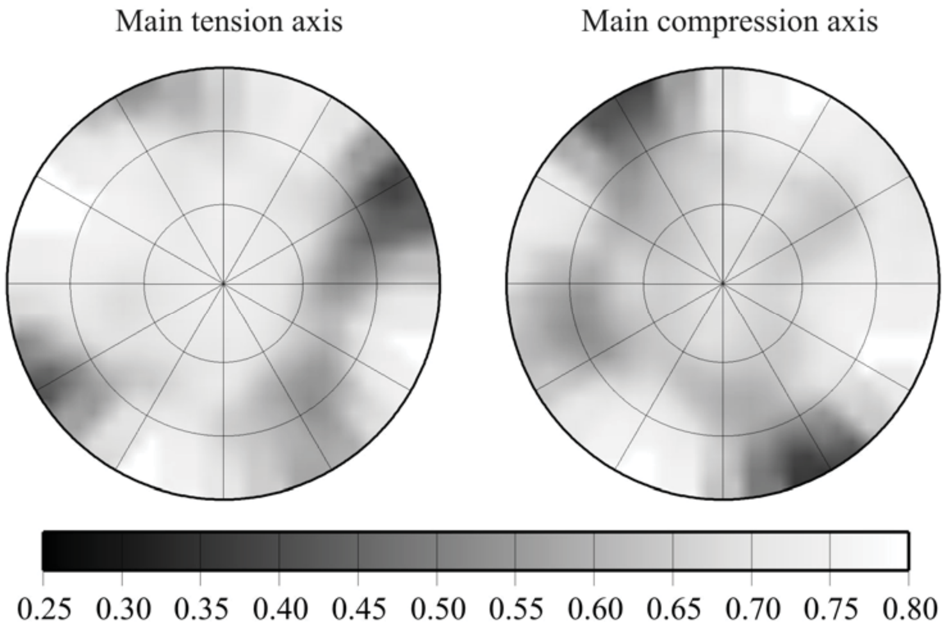

Fig. 5. Resolution maps for the principal stress axes

Determining 2-nd moments of moment tensor density we started from the assumption that the nodal plane with strike angle equal to 106° is the fault plane. We fixed focal mechanism and seismic moment obtained in instant point source approximation. The source depth is recomputed when determining the source 2nd-order moments. Its final estimate takes value 40 km.

The duration and the geometry of the source is estimated from the amplitude spectra of fundamental Love and Rayleigh modes recorded at stations shown in figure 3 in the period band from 200 to 300 seconds.

The residual functions for the integral estimates of the source are given in figure 6.

The inversion yields the integral estimate of duration being about 40 s, a characteristic source length (major axis length) of 125 km. The minor axis length is poorly resolved, lying between 0 and 50 km. The average instant centroid velocity estimate is about 3 km/s. The angles giving the major axis and velocity vector orientations are measured clockwise on the footwall starting from the strike axis. They are consistent with each other and correspondent residual functions attain their minimum values at 0° and 180° correspondingly.

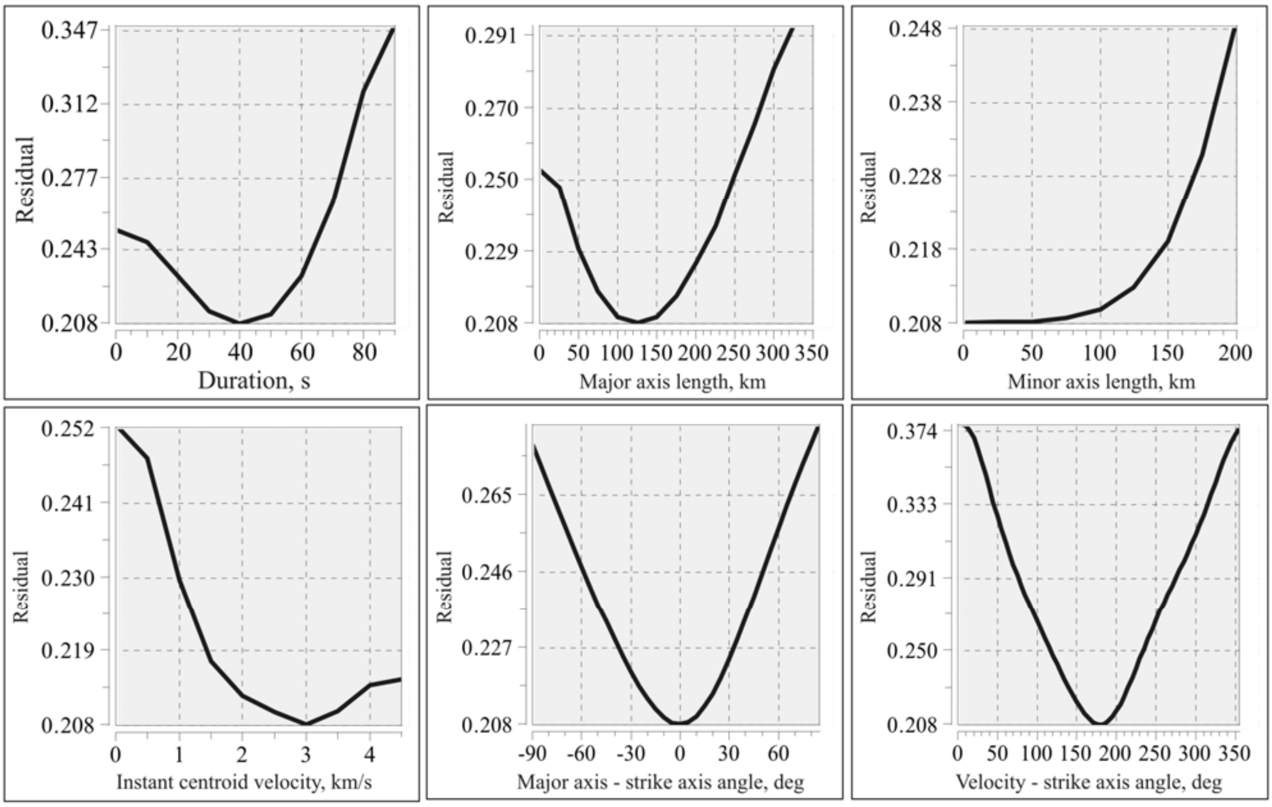

Fig. 6. Residual functions for source integral characteristics.

The propagation of rupture may be characterized by directivity ratio d proposed by McGuire (2002). This parameter is defined as the ratio of the average velocity of the instant centroid over the apparent rupture velocity equal to $l_{max}/\Delta t$. For a unilateral rupture where slip nucleates at one end of a rectangular fault and propagates to the other at a uniform rupture velocity with a uniform slip distribution, $d = 1$. For a symmetric bilateral rupture that initiates in the middle and propagates to both ends of a fault at uniform rupture velocity with uniform slip distribution, $d = 0$. Predominantly bilateral ruptures correspond to $0 \le d < 0.5$ while predominantly unilateral ruptures correspond to $0.5 < d \le 1$. We find $d = 0.96$ for our model. This value shows almost pure unilateral (westward) rupture propagation.

The relation between integral estimates and real characteristics of source duration and spatial extent depends on the distribution of moment rate density in time and over the fault. In the case of Gaussian distributions the 99% confidence duration is 2.5 times larger then the integral estimate, and 99% confidence axis length is 3 times larger then correspondent integral estimate. Multiplying the integral estimate of duration by factor 2.5 we get for source process duration the value being equal to 100s. Multiplying the integral estimates for principal axes length by factors 3 we get for major axis size 375 km, and for minor axis less than 150 km.

Use of the second nodal plane (with strike angle equal to 200°) as the fault plane gives significantly worse result. It is illustrated by figure 7.

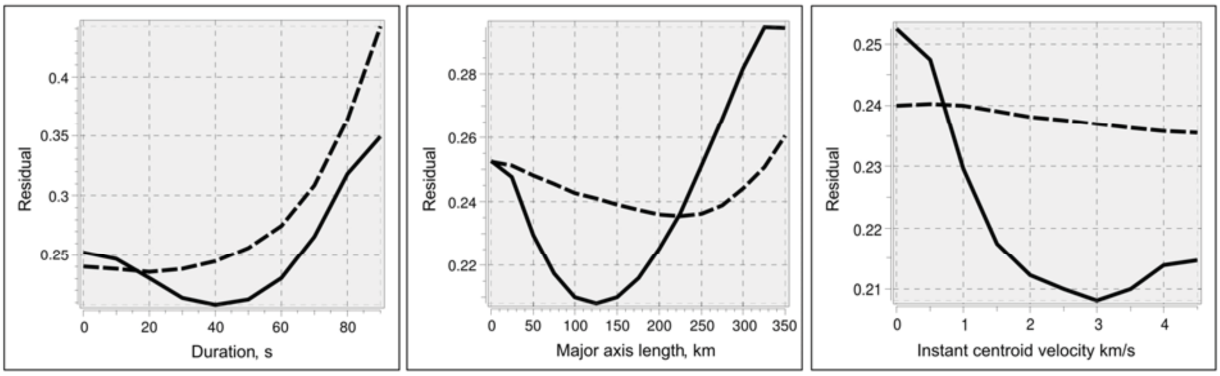

Fig. 7. Comparison of residual functions for two selections of the fault plane. Solid lines correspond to selected plane with strike angle 106°, dashed lines correspond to selected plane with strike angle 200°.

V. Uncertainty of moment tensor determination in case of shallow earthquake.

We consider the uncertainty of moment tensor determination from surface wave records if the wave length is much larger then the source depth (Bukchin *et al.*, 2010).

Such uncertainty for two special cases of double-couple, namely for pure normal (or reverse) fault and for pure strike-slip, was investigated by Kanamori and Given (1981).

It is well known that in case of shallow earthquake moment tensor cannot be uniquely determined from long period surface waves. Only four out of six elements defining a symmetric moment tensor may be reliably determined by such inversion. We consider the consequences of this fact.

We give an existence condition for double-couples radiating the same long period surface waves as the deviatoric moment tensor (symmetric 3x3 matrix with zero trace) obtained by linear inversion.

We describe the family of such double-couples and show that they may provide better estimates of double-couple mechanisms than the traditional ‘‘best double-couple’’ solution.

We describe a family of shallow double-couples which can be uniquely determined from long period surface waves.

Definition of the ‘best double-couple’

In many routine determinations of CMT solutions the best double-couple is calculated from the deviatoric moment tensor. The best double-couple has identical eigenvectors to the deviatoric moment tensor, and seismic moment is given by the average of the absolute values of the most positive and most negative eigenvalues of that moment tensor.

The best double-couple may be shown to be the double-couple moment tensor which deviates least from the original deviatoric moment tensor.

Radiation of surface waves by shallow source

We consider surface waves radiated by an instantaneous point source in a horizontally uniform earth. For the spectrum of displacement in surface wave $\mathbf{u}(\mathbf{r},\omega)$ at a point $\mathbf{r}$ we have

$$\mathbf{u}(\mathbf{r},\omega) = \mathbf{q}(\omega) P(\mathbf{M}, h, \omega, \varphi) \exp[-i\psi(\mathbf{r},\omega)], \qquad (5.1)$$

where $\mathbf{q}(\omega)$ is a complex vector depending on earth structure, $\mathbf{M}$ is the moment tensor, h the source depth, φ the azimuth of surface wave radiation, $\psi(\mathbf{r},\omega)$ the propagation phase, and ω

the circular frequency. The factor P determines the radiation pattern of the source (azimuth dependence of spectral amplitude) and the initial (source) phase.

For a Love wave this factor is given by

$$P(\mathbf{M},h,\omega,\varphi)=\xi V^{(\tau)}(\omega,h)[0.5(M_{33}-M_{22})\sin 2\varphi+M_{23}\cos 2\varphi] \\ +i\frac{\partial V^{(\tau)}(\omega,h)}{\partial z}(M_{12}\sin\varphi-M_{13}\cos\varphi), \tag{5.2}$$

where $V^{(\tau)}$ (z) is the transverse eigenfunction, ξ is the wave number, i is the imaginary unit, and the coordinate system is defined in the following way: 1 – vertical down, 2 – north, and 3 – east.

For a Rayleigh wave the function P is given by

$$P(\mathbf{M},h,\omega,\varphi)=\frac{\partial V^{(z)}(\omega,h)}{\partial z}M_{11} \\ -0.5\xi V^{(r)}(\omega,h)[M_{22}+M_{33}+(M_{22}-M_{33})\cos 2\varphi+2M_{23}\sin 2\varphi] \\ +i[\xi V^{(z)}(\omega,h)+\frac{\partial V^{(r)}(\omega,h)}{\partial z}](M_{12}\cos\varphi+M_{13}\sin\varphi), \tag{5.3}$$

where $V^{(z)}$ (z) , $V^{(r)}$ (z) are vertical and radial components of the eigenfunction, respectively.

The source rotated around the vertical axis by 180° radiates in the direction with azimuth φ the same surface waves as the original one in the direction with azimuth φ -180°. As can be seen from formulas (5.2) and (5.3), the result of this rotation is that the function $P(\mathbf{M},h,\omega,\varphi)$ becomes its complex conjugate.

The coefficients $\frac{\partial V^{(\tau)}(\omega,h)}{\partial z}$ for Love waves and $[\xi V^{(z)}(\omega,h)+\frac{\partial V^{(r)}(\omega,h)}{\partial z}]$ for Rayleigh waves are proportional to the shear traction acting on the horizontal plane. But such a force is vanishing at the free surface (h = 0). As a consequence, if the source depth h is much smaller than the wave length, the moment tensor elements M_{12} and M_{13} almost do not affect the surface wave radiation pattern and the source phase, and they can-not be resolved from the observed spectra. At the same time the imaginary part of $P(\mathbf{M},h,\omega,\varphi)$ is small and the rotation of the source around the vertical axis by 180° doesn't change the radiated surface waves. This property of shallow sources was studied by Henry *et al.* (2002). They explained for the double-couple case the two-fold rotational symmetry of the misfit function around the vertical axis of the moment tensor and demonstrated it for a set of earthquakes.

Note that the elements M_{12} and M_{13} do not affect the surface wave radiation so long as they do not exceed significantly (in absolute value) the other elements of the moment tensor. But if these two elements are dominant, then they do contribute into the surface wave radiation and can be resolved for any nonzero h. This takes place for the double-couple in case one of its nodal planes is subhorizontal (Bukchin, 2006). It is important to note that such a source radiates relatively weak surface waves. So, all moment tensor elements for such a shallow double-couple can be resolved from long period surface waves, provided the magnitude of the event is high enough, and as a result, the surface wave records are characterized by high signal-to-noise ratios.

Existence condition and description of equivalent double-couples

Only four out of six elements defining a symmetric moment tensor may be reliably determined from surface wave records if the wave length is much larger then the source depth.

It has in general case nonzero non-double-couple component. It is shown (Bukchin *et al.*, 2010) that given four reliably determined elements is enough to answer the question of existence of pure double-couples radiating long period surface waves similar to that radiating by the original deviatoric moment tensor.

Let elements M_{22}, M_{33} and M_{23} are given. The element M_{11} is defined by zero-trace condition $M_{11} + M_{22} + M_{33} = 0$.

Expressing these moment tensor elements through seismic moment and focal mechanism angles we obtained existence condition for double-couples with given values of these three moment tensor elements, and formulas describing the set of such double-couples.

The existence condition for double-couples with given values of moment tensor elements M_{22}, M_{33} and M_{23} has form of inequality

$$M_{22}M_{33} \le M_{23}{}^2. \tag{5.4}$$

If this condition is satisfied, then such double-couples exist and have the same strike angle given by formulas

$$\psi = 0.5(\pm \arccos \frac{A_1}{\sqrt{A_3{}^2 + A_2{}^2}} - \varphi), \tag{5.5}$$

where A_1, A_2, A_3 and φ are given by

$$A_1 = M_{22} + M_{33},$$

$$A_2 = M_{33} - M_{22},$$

$$A_3 = 2M_{23},$$

$$\sin\varphi = \frac{A_3}{\sqrt{A_3{}^2 + A_2{}^2}},$$

$$\cos\varphi = \frac{A_2}{\sqrt{A_3{}^2 + A_2{}^2}}.$$

+ and – in formula (5.5) corresponds to two nodal planes.

The dip, rake angles and seismic moments M_0 for the set of double-couples characterized by similar long period surface wave radiation patterns and source phases are defined by identities

$$\tan\lambda\cos\delta \equiv C_1, \tag{5.6}$$

$$M_0 \sin\delta\cos\lambda \equiv C_2, \tag{5.7}$$

where constants C_1 and C_2 are given by formulas

$$\begin{cases} C_1 = -\dfrac{A_1}{A_2 \sin 2\psi + A_3 \cos 2\psi} \\ C_2 = 0.5(A_2 \sin 2\psi + A_3 \cos 2\psi) \end{cases}. \tag{5.8}$$

Adding to triples of seismic moment, dip and rake angle values the value of strike angle we describe the first branch of equivalent double-couples. Substitution of the strike angle value ψ by value ψ+180° gives us the second branch of equivalent double-couples.

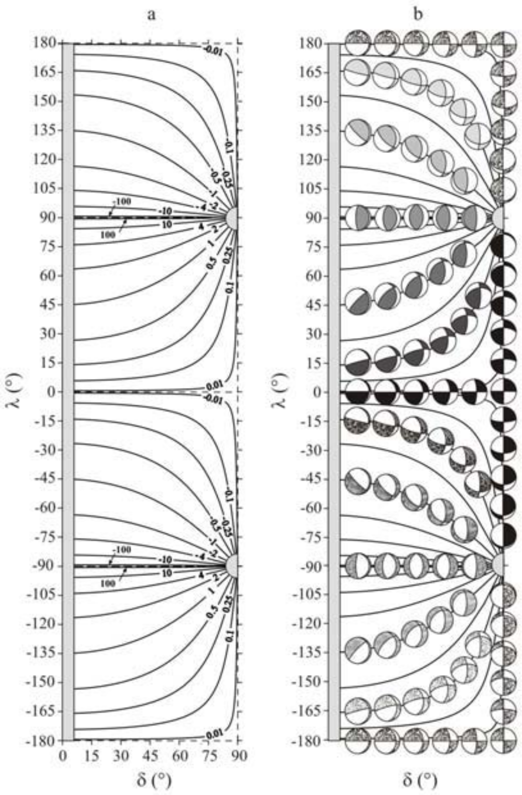

Fig. 8. Examples of families of equivalent double-couples. (a) Contour plot of $f = \tan \lambda \cos \delta$. Contours are marked by the corresponding value of C_1 in (6). Every isoline of this function defines dip and rake angles of a family of equivalent double-couples. (b) The same contour plot with superimposed lower hemispherical projections of focal solutions for some of the isolines. The value of strike angle is fixed equal to 0. Equivalent double-couples are given by focal solutions filled by the same color. Gray strip along the axis $\delta = 0$ and gray sectors centered at points of intersection of the axis $\delta = 90°$ with axes $\lambda = -90°$ and $\lambda = 90°$ show the regions where one of the nodal planes is subhorizontal. Equivalent dip-slips ($\lambda = -90°$ and $\lambda = 90°$) and slips on a vertical fault ($\delta = 90°$) contain the symmetric double-couples rotated around the vertical axis by 180°. All other families of equivalent double-couples must be completed by symmetric solutions with the same corresponding values of dip and rake angles, and with strike angle equal to 180°.

Summing up, if the deviatoric moment tensor **M** in the case of a shallow source is obtained by inversion of long period surface waves, by the CMT method, say, then only the values of four elements M_{11}, M_{22}, M_{33} and M_{23} are reliable. The elements M_{12} and M_{13} are not resolved and incorrect values of these elements can lead to a spurious non-double-couple component even if all other moment tensor elements are correct (Henry *et al.*, 2002).

But it turns out that four reliably determined elements are sufficient to provide the answer to the question of the existence of pure double-couples radiating the same long period surface waves as the original deviator **M** does. If the condition (5.4) is fulfilled, then using formulas (5.5-5.8) one can obtain a complete description of required double-couples. Examples of families of equivalent double-couples are shown in figure 8.

If, on the contrary, the condition (5.4) does not hold, then there is no double-couple radiating the same long period surface waves as the original deviator **M**. In that case we search for equivalent double-couples with values of the elements M_{22}, M_{33} and M_{23} which deviate the least from the given values. Such double-couples are found by minimizing $\sqrt{d_{22}^2 + d_{33}^2 + d_{23}^2}$, where d_{22}, d_{33} and d_{23} are the differences between the corresponding values of the three elements.

Let us consider the equality

$$M_{23}^2 = M_{22}M_{33} \tag{5.9}$$

as the equation of a surface in the 3D Euclidean space M_{22}, M_{33}, M_{23}. This equation describes an elliptic cone. This surface is symmetric with respect to the plane $M_{23} = 0$. The upper part of this surface ($M_{23} \geq 0$) is shown in figure 9. The surface separates the points (M_{22} , M_{33} , M_{23}) corresponding to the moment tensors that satisfy the double-couple existence condition from the points corresponding to the moment tensors that do not satisfy this condition. The existence condition is valid for the exterior of the surface, including the surface itself. To sum up, if for any given values of M_{22} , M_{33} , M_{23} the existence condition (5.4) is not true, then the double-couples with the least-deviation values of these three elements correspond to a point on the surface under consideration. It is shown (Bukchin *et al.*, 2010) that all such double-couples are pure dip-slips ($\delta = \pm 90°$).

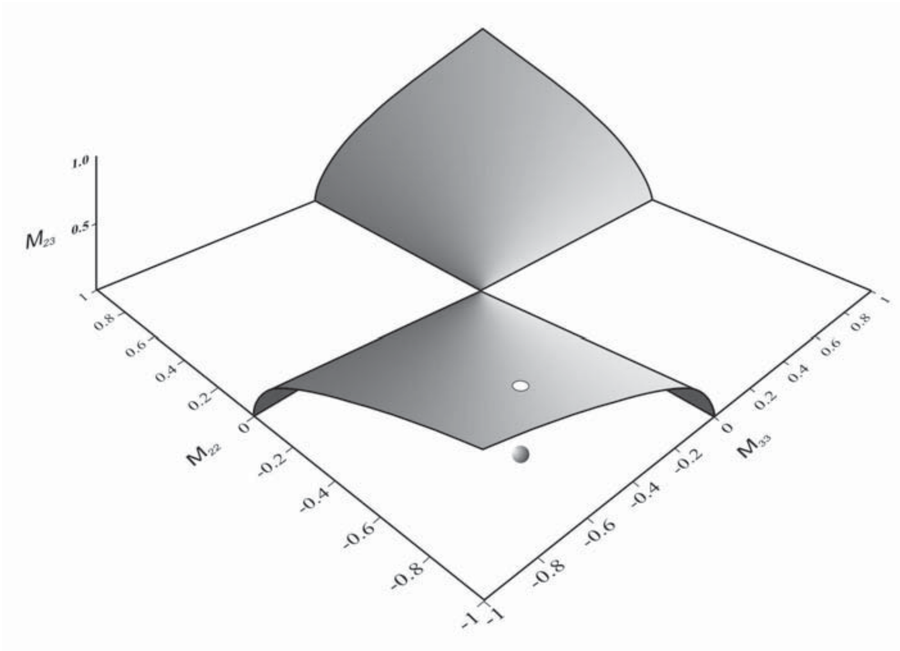

Fig. 9. Upper half of the surface , $M^2_{23} = M_{22}\ M_{33}$. The small ball specifies the values of moment tensor elements M_{22} , M_{33} , M_{23} not satisfying the existence condition. The small ellipse marks the double-couple with the least deviating values of these elements.

Described double-couples and reference deviatoric moment tensor are radiating similar surface wave fields if the depth of the source is much smaller than the wave length. To control the similarity of radiated wave fields for given values of source depth and period we calculate the normalized misfit between synthetic surface wave spectra calculated for any double-couple with current values of dip and rake angles and for the reference deviatoric moment tensor. This misfit is defined as the ratio of the root mean square misfit to the root mean square spectra calculated for the reference moment tensor.

We present the misfit contour plot in plane (δ, λ) in the same way as Henry *et al.* (2002) . The left and the right parts of the picture correspond to two values of strike angle different from each other by 180°. These two parts are rotated by 180° with respect to each other. This allows us to consider these two misfit plots as a single map which is continuous at the line $\delta = 90°$. The continuity follows from the equality

$\mathbf{m}(\psi, 90°, \lambda) = \mathbf{m}(\psi + 180°, 90°, -\lambda,)$,

where $\mathbf{m}(\psi, \delta, \lambda)$ is a double-couple moment tensor with given values of strike angle ψ, dip angle δ and rake angle λ.

Application to the March 25, 1998 Antarctic Plate earthquake

We consider the large shallow earthquake studied in detail by Henry *et al.*, 2002. They compared the best fitting double-couple mechanisms obtained from mantle wave inversion with the traditional 'best double-couple' obtained from the best fitting deviatoric moment tensor and with the results of body wave analysis.

We present here a complete description of double-couples radiating long period surface waves similar to the radiation pattern of the deviatoric moment tensor from the Global CMT catalog, and compare them with the results reported by Henry *et al.* (2002).

We introduce the same notation as in Henry *et al.* (2002). We shall refer to the best fitting deviatoric moment tensor as the optimal deviatoric (ODV) moment tensor. We shall refer to the best fitting double-couple as the optimal pure double-couple (OPDC). And finally we shall refer to the so-called best double-couple which is calculated from the ODV moment tensor in many routine determinations of CMT solutions as the BDC.

The ODV solution for this M_w = 8.1 earthquake from the Global CMT catalog has a large non-double-couple component, characterized by the parameter ε, which is the ratio of minimum (in absolute magnitude) eigenvalue to the maximum (in absolute magnitude) eigenvalue. For this solution ε = 0.41. We use the ODV solution from the Global CMT catalog as the reference deviatoric moment tensor. The values of its elements M_{22}, M_{33} and M_{23} satisfy the equivalent double-couple existence condition (5.4). Using these values we calculate from (5.5) the strike angle 95° for one of the two branches of equivalent double-couples. Then we use (5.6-5.8) to find the set of pairs of dip and rake angle values and the corresponding seismic moments. Replacing the strike angle value 95° by the value 95° +180° = 275° gives us the second branch of equivalent double-couples.

A contour plot of the misfit function for period 200 s and source depth 10 km is shown in figure 10. The same figure shows two branches of equivalent double-couples and the different moment tensor solutions for this earthquake. It can be seen that the misfit is negligible on long segments of the equivalent double-couple curves. Both optimal pure double-couples obtained by Henry *et al.* (2002) from mantle wave inversion (solutions 3 and 4) fit well the curves of equivalent double-couples, and the values of the strike angle for these solutions practically coincide with the predicted values. On the contrary, the BDC from the Global CMT catalog is far from the lines of minimum misfit and its values of strike angle differ by 6° from the predicted values. The large misfit for the BDC solution can be explained by the dependence of all the BDC moment tensor elements on the values of ODV elements M_{12} and M_{13} which are not estimated reliably. Large non-double-couple components for the ODV solution can be explained as well by inaccurate estimates of M_{12} and M_{13}. As a preferable estimate of the double-couple mechanism for this earthquake we suggest the double-couple (marked in figure 10 by the star) from the family of equivalent double-couples nearest to the solution obtained from body wave inversion by Henry *et al.* (2000).

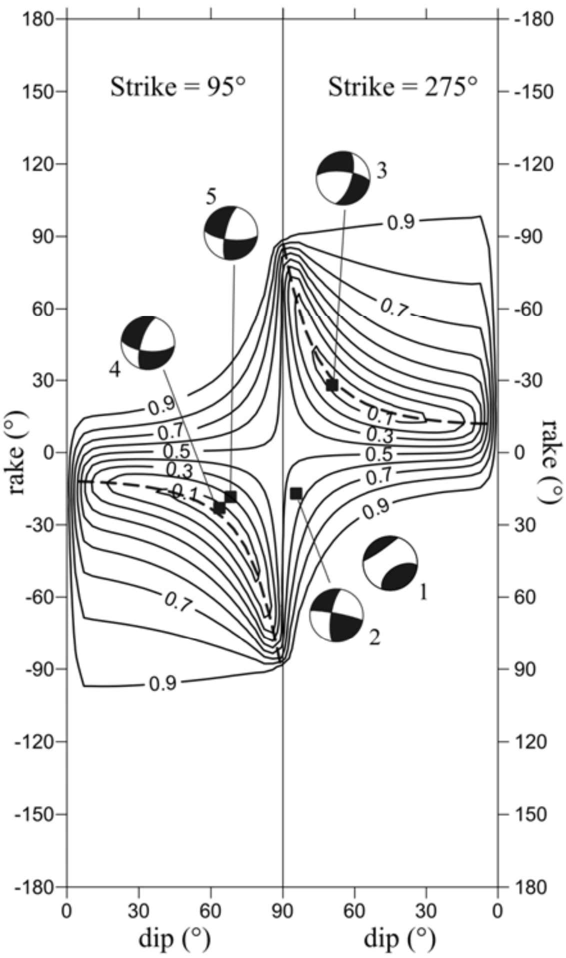

Fig. 10. The 1998 Antarctic earthquake. Misfit function calculated with respect to the radiation of ODV moment tensor from Global CMT catalog for period 200 s and source depth 10 km. Red dashed lines show two branches of equivalent double-couples. Points to the left of the vertical line at the center of the figure have strike 95° and rakes corresponding to the left ordinate. Points to the right of the line have the strike 275° and rakes corresponding to the right ordinate. 1 – CMT solution from the Global CMT catalog; 2 – BDC from the Global CMT catalog (281°, 84°, 17°); 3, 4 – optimal DCs (Henry *et al.* 2002) (276°, 69°, -28°) and (96°, 64°, -23°); 5 – DC obtained from body waves analysis (Henry *et al.* 2000) (95°, 63°, -26°);.

Main results of the uncertainty analysis

– Traditional ‘‘best double-couple’’ may provide inadequate estimate of double-couple mechanism caused by wrong values of elements M_{12} and M_{13} which are not estimated reliably.
– The existence of equivalent double-couples shows that non-double-couple component of moment tensor may be spurious and can be explained by errors in estimates of elements M_{12} and M_{13}.
– Four reliably determined moment tensor elements is enough to answer the question of existence of equivalent double-couples.
– If the existence condition is valid a complete set of equivalent double-couples can be described analytically.
– If the existence condition is not valid, equivalent double-couples can be determined for values of moment tensor elements M_{22}, M_{33} and M_{23} nearest to given values.

- To select the preferable solution from the set of equivalent double-couples, additional data are required. These can be solution obtained from body wave inversion, first motion data or long period P-wave polarities.
- If one of nodal planes of shallow pure double-couple is subhorizontal then all elements of moment tensor are well resolved and the solution is unique.

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