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Modelling Mantle Convection and Plate Tectonics

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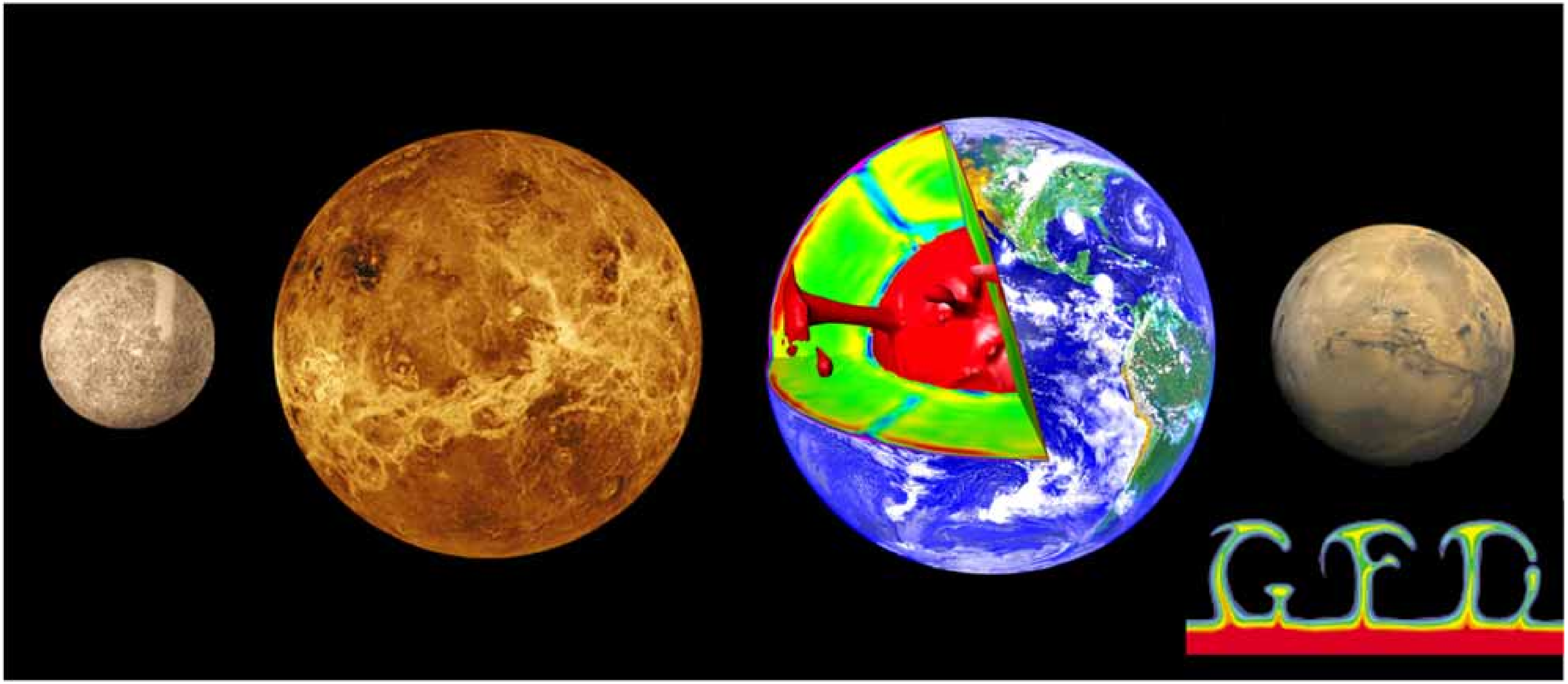
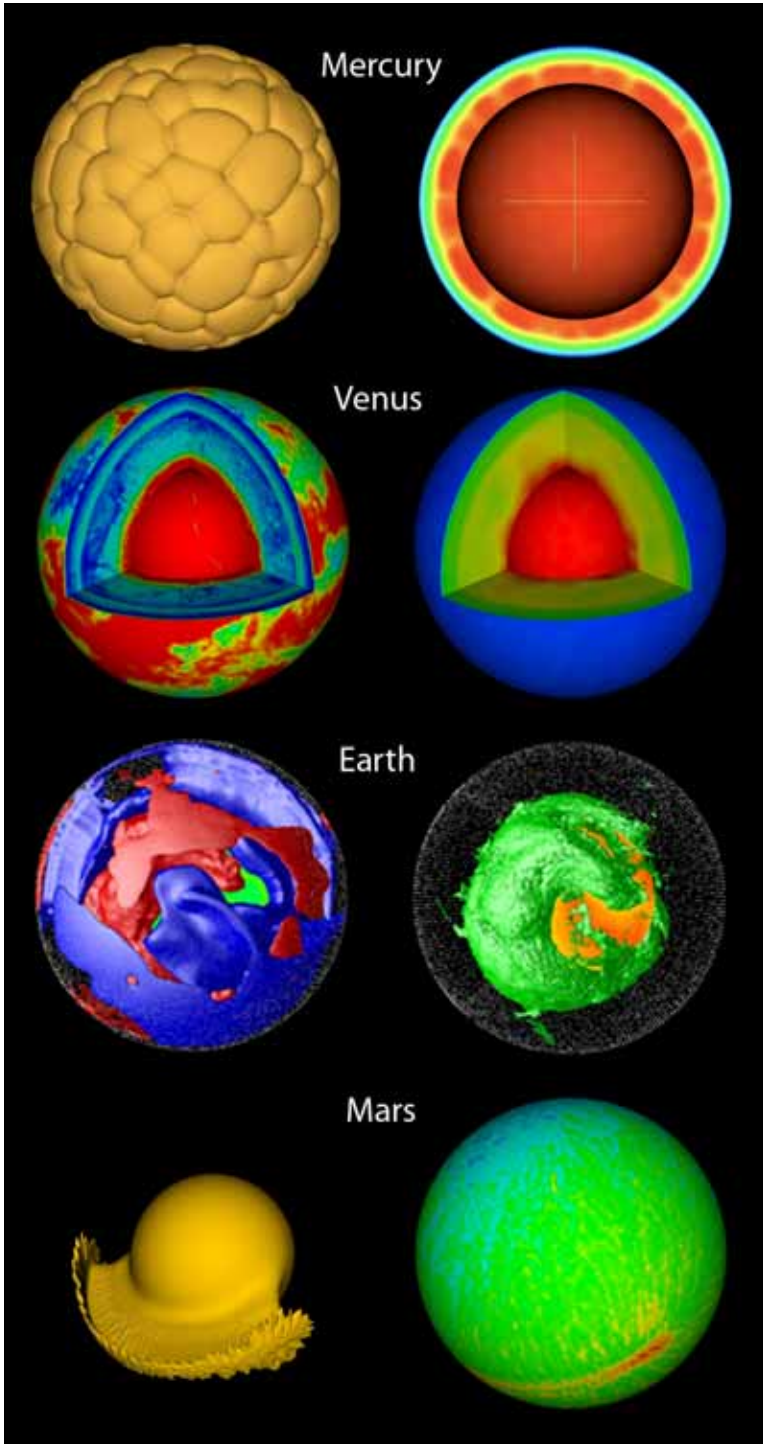


Talk Plan

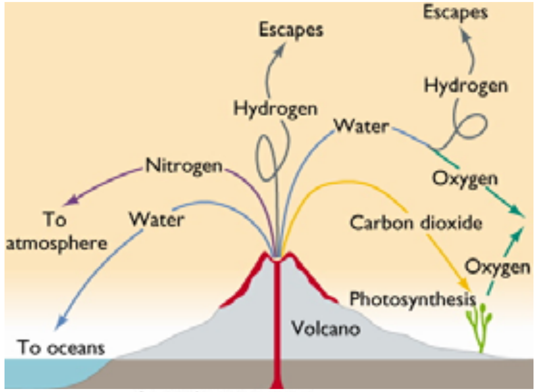
- How to write a simple mantle convection program
- How to deal with more complex physics
- Recent models, including
  - Self-consistent plate tectonics
  - Detailed models of Earth, Mars, Venus, Mercury, super-Earths

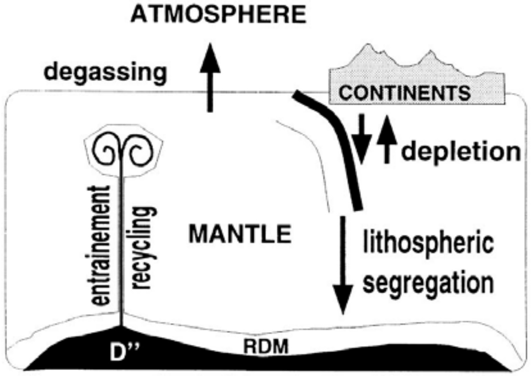


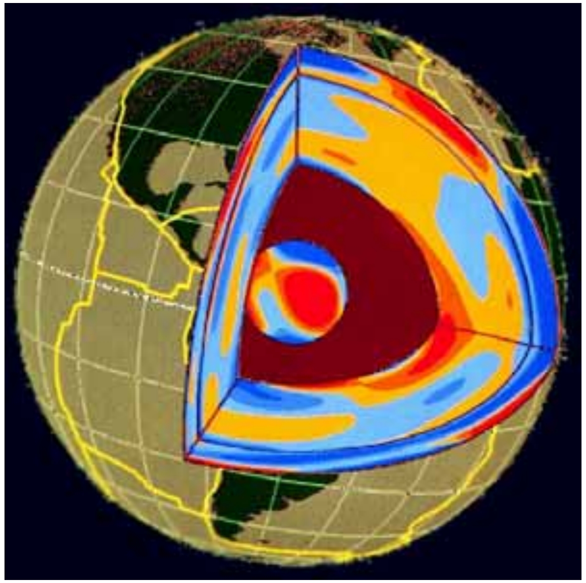
Technical challenges

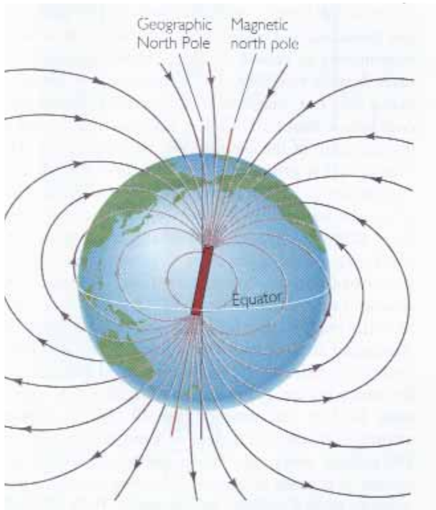
- Rheology
  - Large temperature-dependence (~40+ orders of magnitude)
  - Nonlinear
  - Brittle failure & plasticity
  - Elasticity
- Multi-scale problem
  - Length: mm to 1000s km
  - Time: seconds to billions of years
- Resolution: no limit to what is needed!

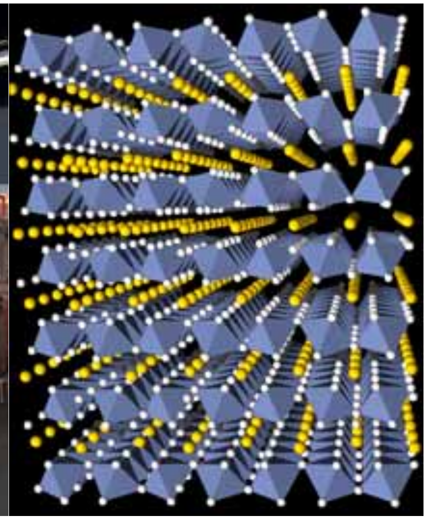
Must consider many things!

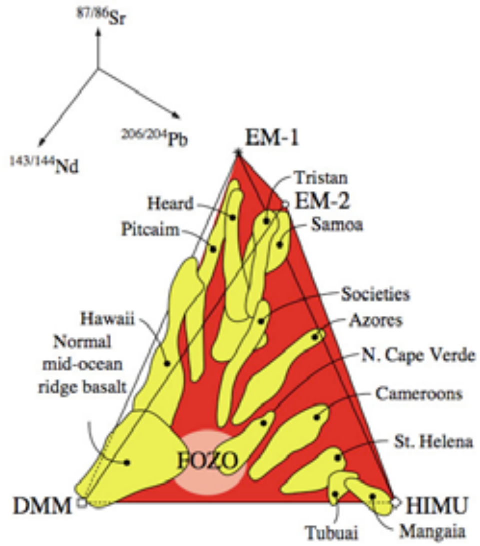






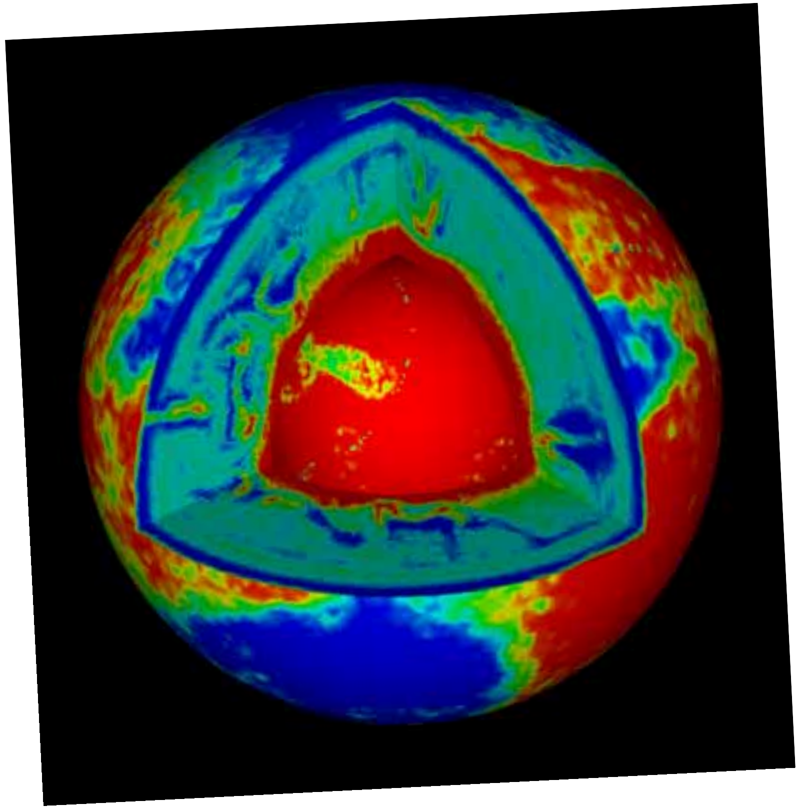






Need huge number of grid points / cells /elements!

- e.g., to fill mantle volume:
- $(8\ \text{km})^3$ cells (oceanic crust) -> 1.9 billion cells
- $(2\ \text{km})^3$ cells -> 123 billion cells



Simplest equations

(Boussinesq, nondimensional, constant properties except viscosity)

Conservation of momentum

$$-\nabla P + \nabla \cdot \underline{\underline{\sigma}} = -Ra.T\hat{z}$$

Conservation of mass

$$\nabla \cdot \vec{v} = 0$$

Conservation of energy

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + H$$

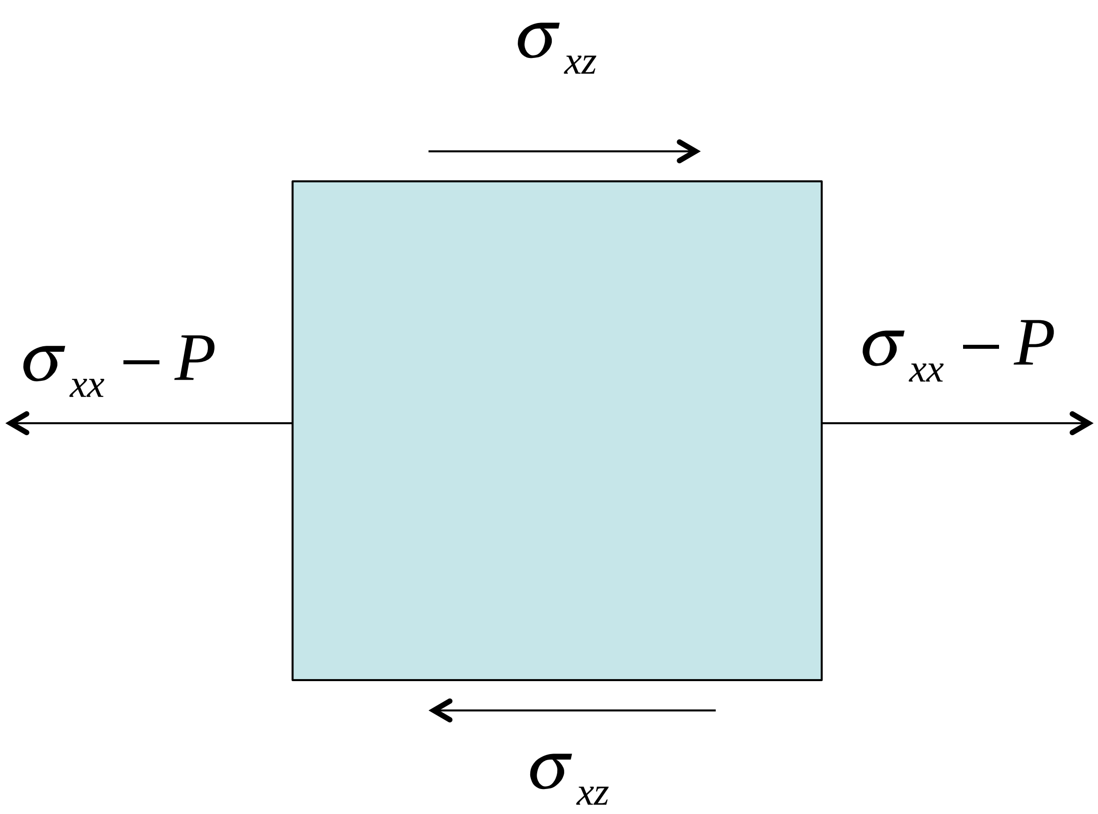
Stresses (2D)

$$\sigma_{xx} = 2\eta \frac{\partial v_x}{\partial x}$$

$$\sigma_{zz} = 2\eta \frac{\partial v_z}{\partial z}$$

$$\sigma_{xz} = \sigma_{zx} = \eta \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)$$

Example: Force balance in x direction (2D)



$$-\frac{\partial P}{\partial x} + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

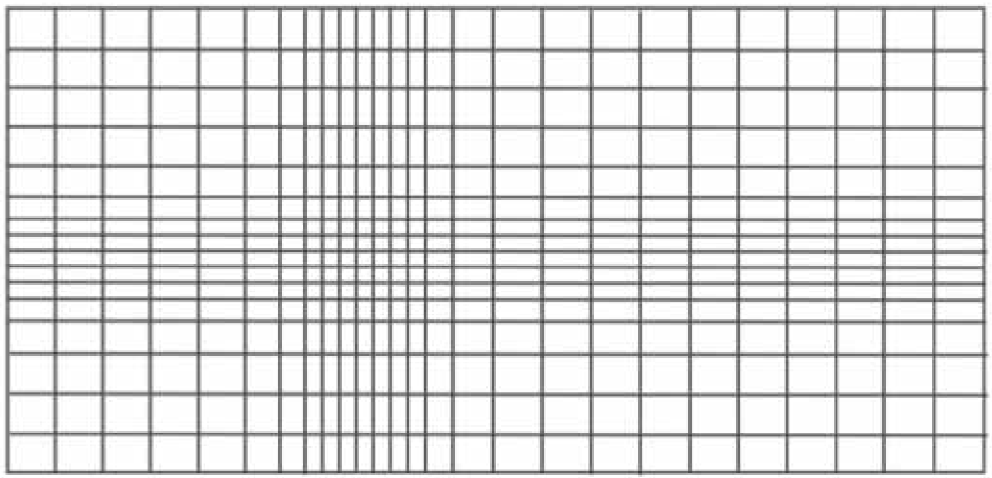
Concept of Discretization

- True solution to equations is continuous in space and time
- In computer, space and time must be discretized into distinct units/steps/points
- Equations are satisfied for each unit/step/point but not necessarily inbetween
- Numerical solution approaches true solution as number of grid or time points becomes larger

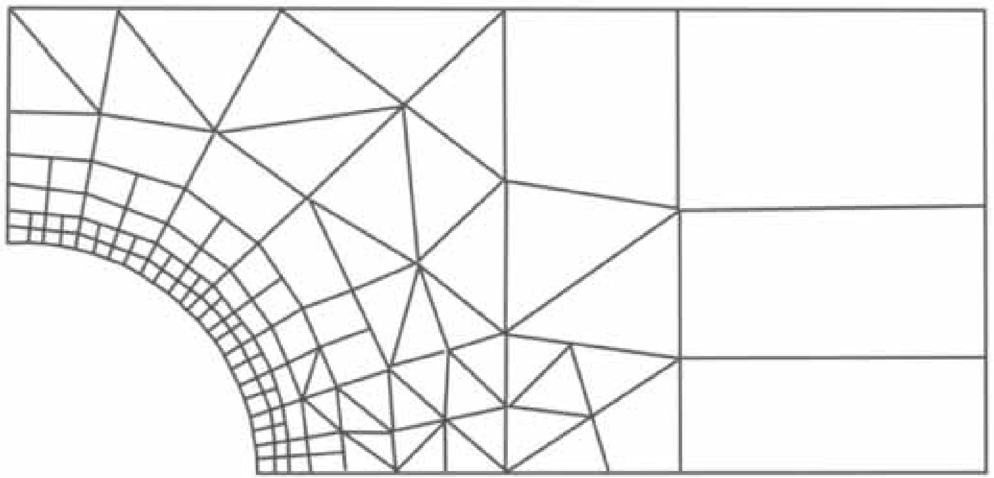
Numerical methods: comparison

Name	How fields represented	Form of equations	Pros	Cons
Finite difference	Grid points	Simple transformation of differential equations	Simple	Structured grids only. Interpolation undefined.
Finite volume	Nodes	Integrated over volumes -> finite difference	Conservative. Unstructured grids. Simple	
Finite element	Nodes + shape functions for interpolation	Integral ('weak') form	Unstructured grids	More complex; equations don't resemble original ones
Spectral transform	Global functions	Decouple for each harmonic (IF constant coefficients)	Accurate, fast	Poor performance for lateral viscosity variations

Structured vs. Unstructured grid



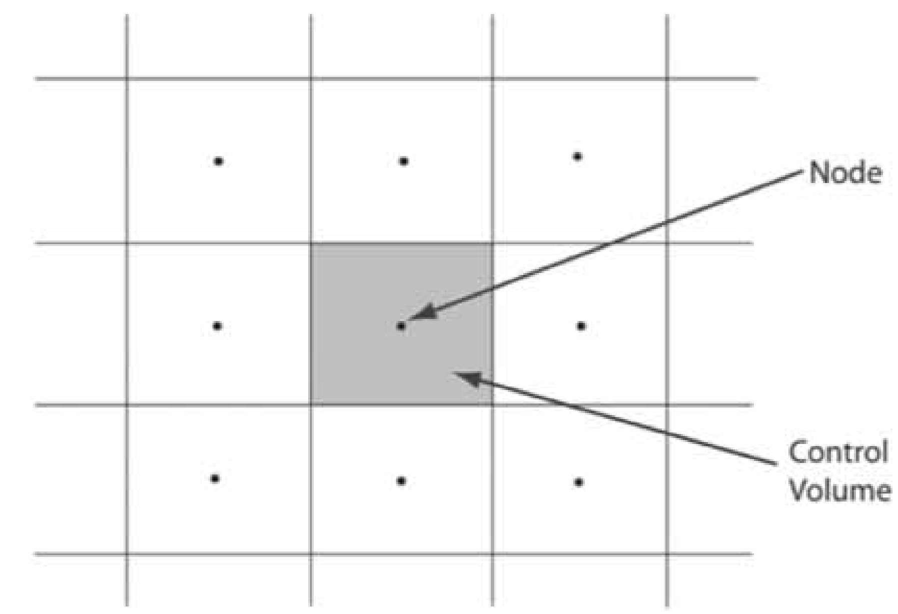
(a)



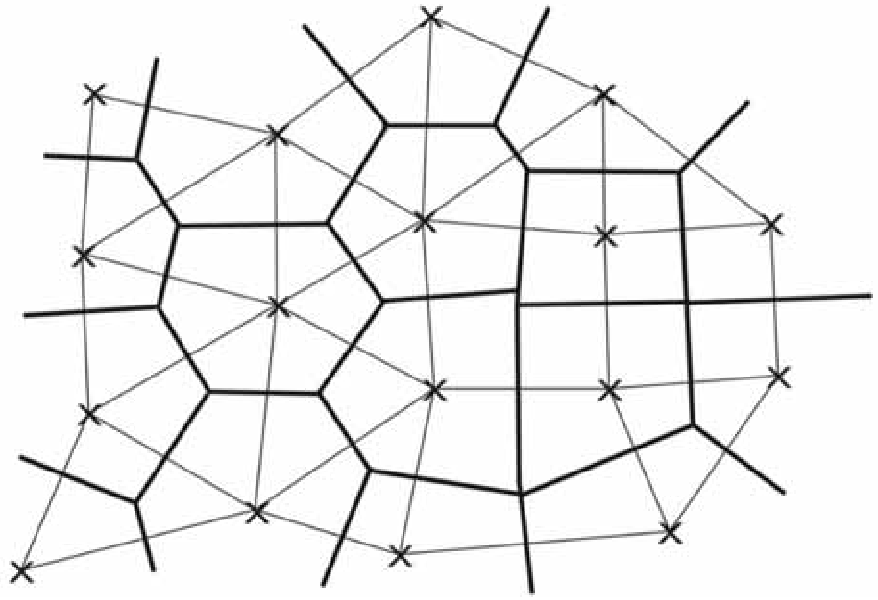
(b)

Examples of two-dimensional structured (a) and unstructured (b) grids.

Finite volume grids



Nodes (dots) and control volumes for a rectangular structured grid.



Derivatives using finite-differences

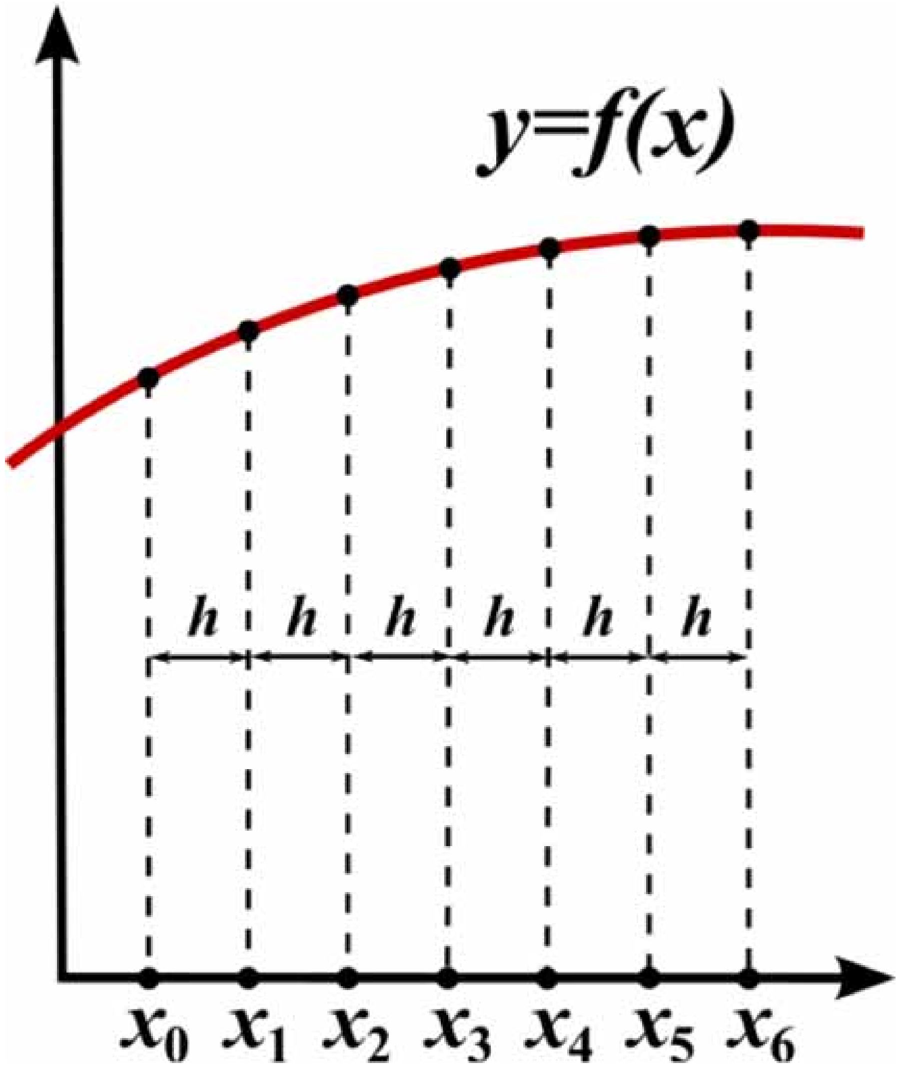
- Graphical interpretation: df/dx(x) is slope of (tangent to) graph of f(x) vs. x
- Calculus definition:

$$\frac{df}{dx} \equiv f'(x) \equiv \lim_{dx \to 0} \frac{f(x+dx) - f(x)}{dx}$$

- Computer version (finite differences):

$$\frac{df}{dx} \approx \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Finite Difference grid in 1-D



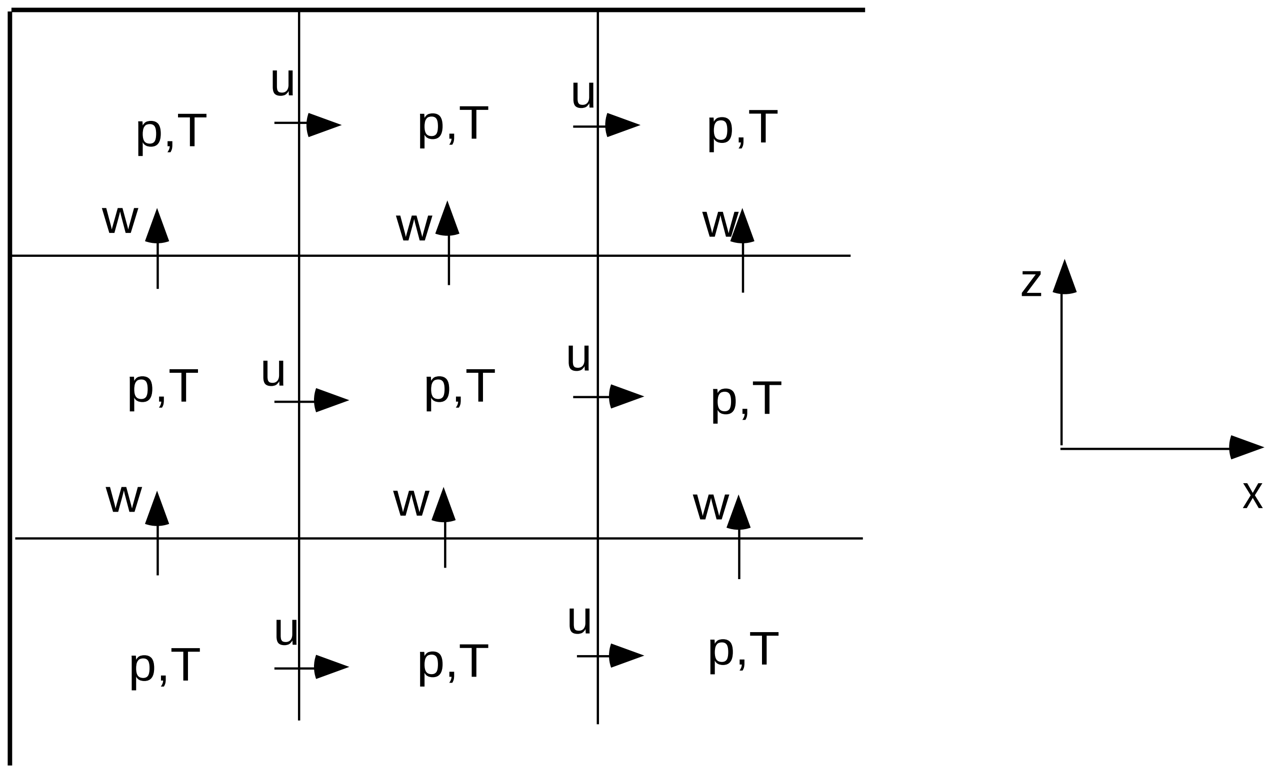
- Grid points $x_0, x_1, x_2 \ldots x_N$
  - Here $x_i = x_0 + i*h$
- Function values $y_0, y_1, y_2 \ldots y_N$
  - Stored in array y(i)

$$\left(\frac{dy}{dx}\right)_i \approx \frac{\Delta y}{\Delta x} = \frac{y(i+1) - y(i)}{h}$$

Second derivative

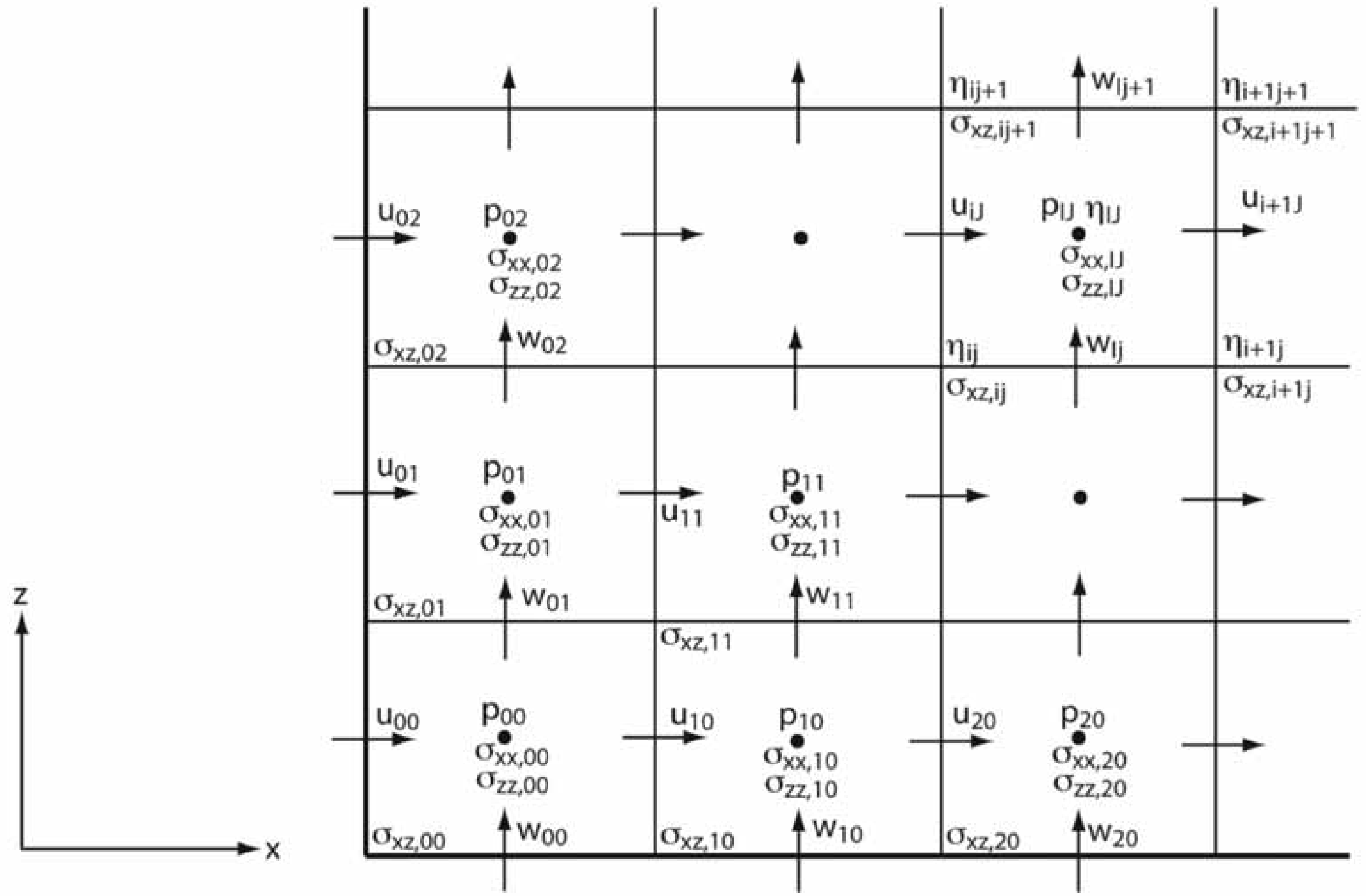
$$\left(\frac{\partial^2 y}{\partial x^2}\right)_i \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

The staggered grid

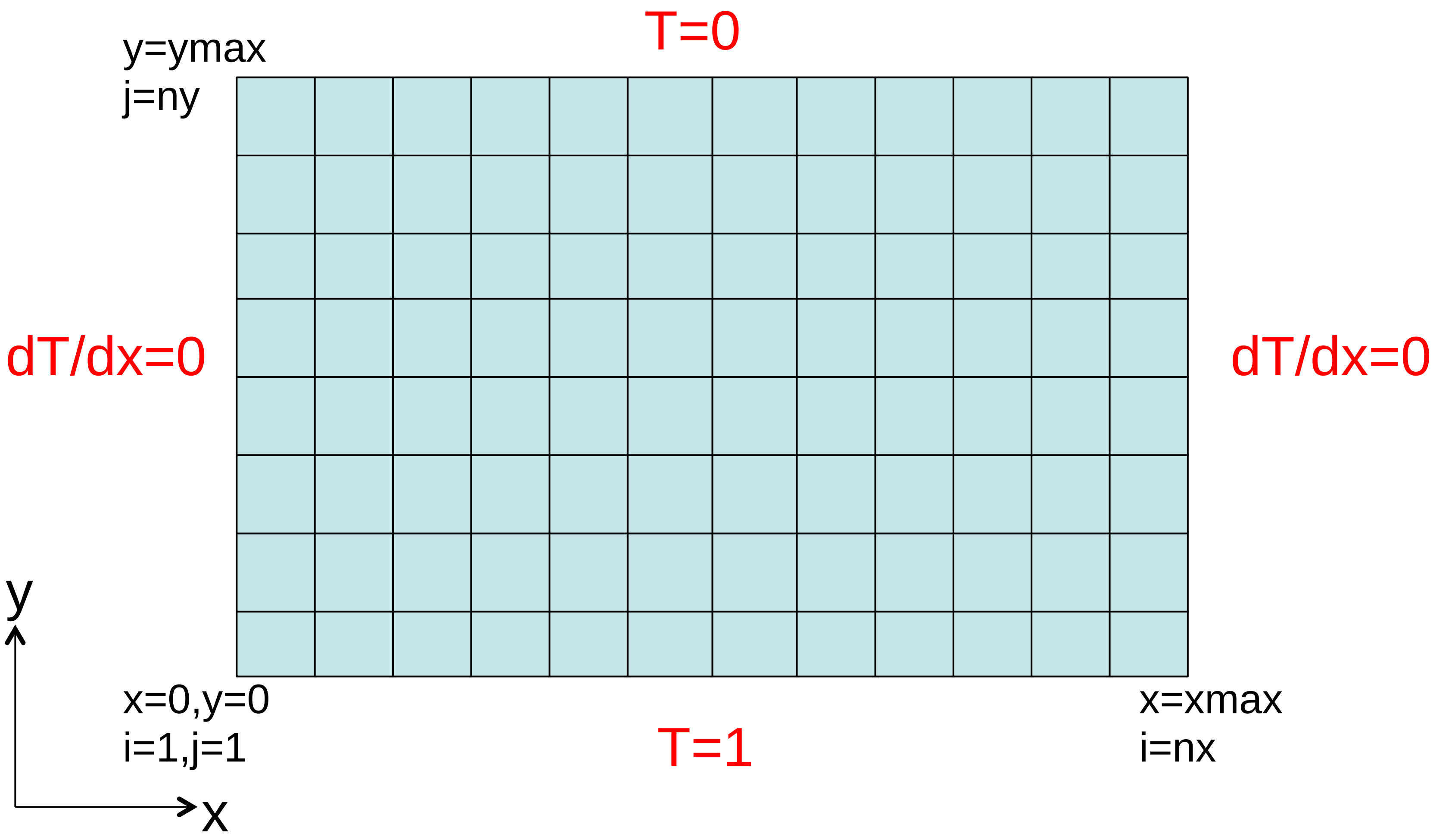


- All derivatives involve adjacent points
- Avoids checkerboard pressure solution
- Extensively used in numerical modelling

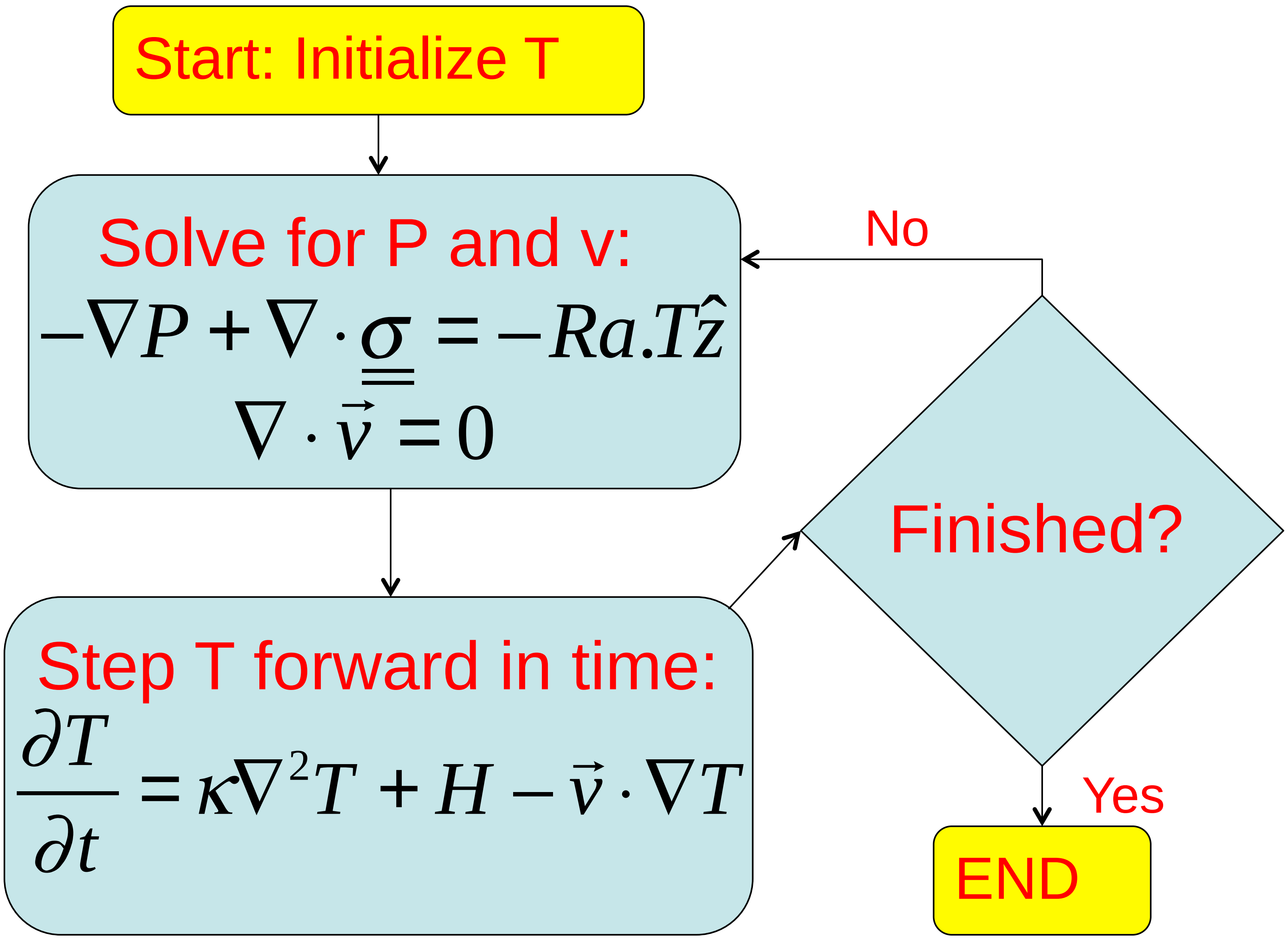
Detail: with stresses



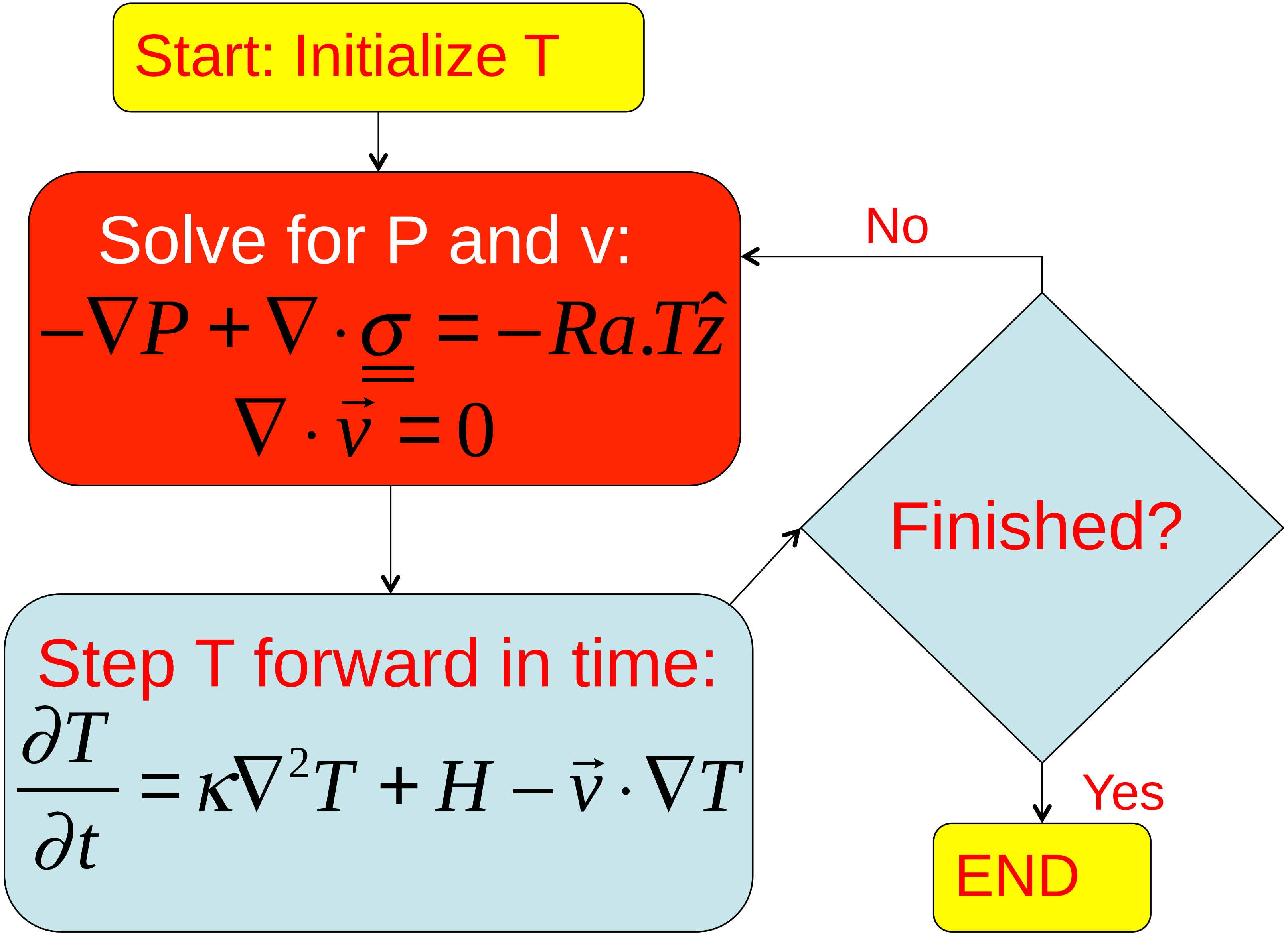
Boundary conditions (typical)



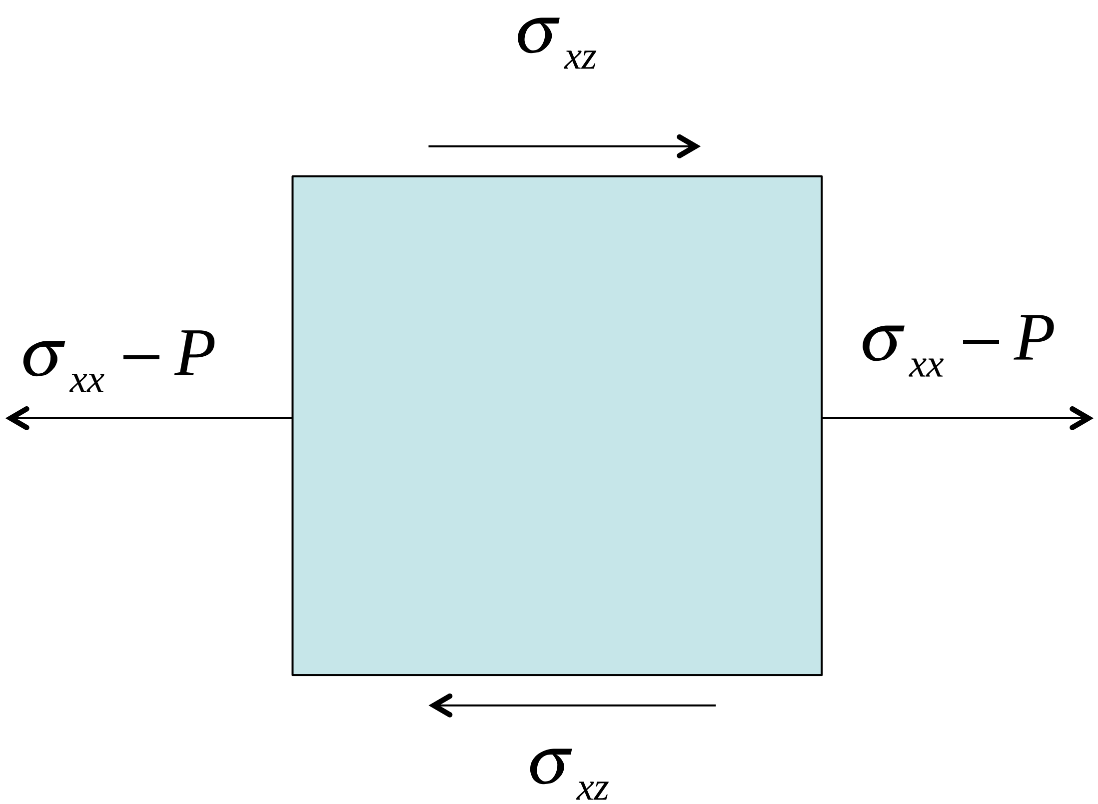
Solution method



1. Solving for P and v

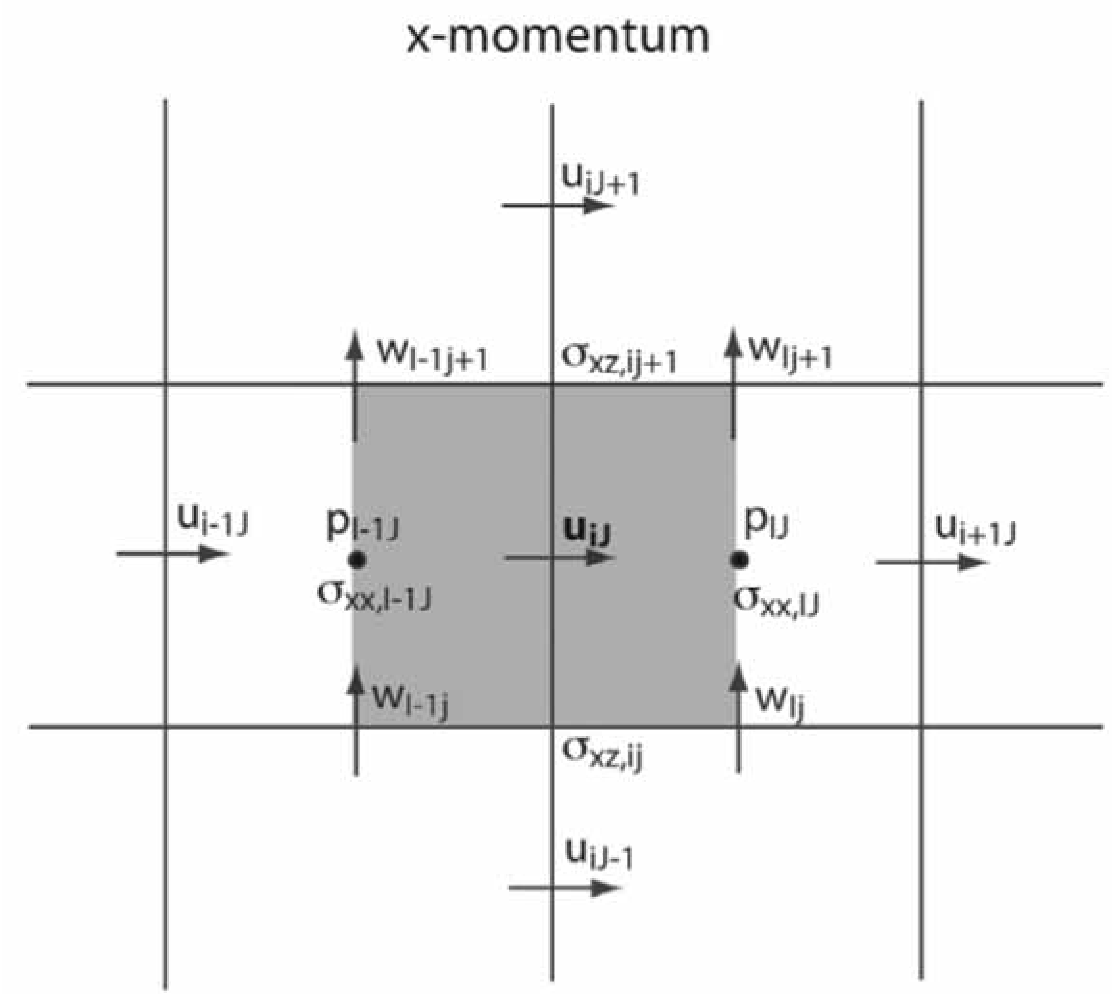


Example: Force balance in x direction (2D)



$$-\frac{\partial P}{\partial x} + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = 0$$

Finite volume version of the x-momentum equation



$$-\frac{p_{IJ} - p_{I-1J}}{\Delta x} + \frac{\sigma_{xx.IJ} - \sigma_{xx.I-1J}}{\Delta x} + \frac{\sigma_{xz.ij+1} - \sigma_{xz.ij}}{\Delta z} = 0$$

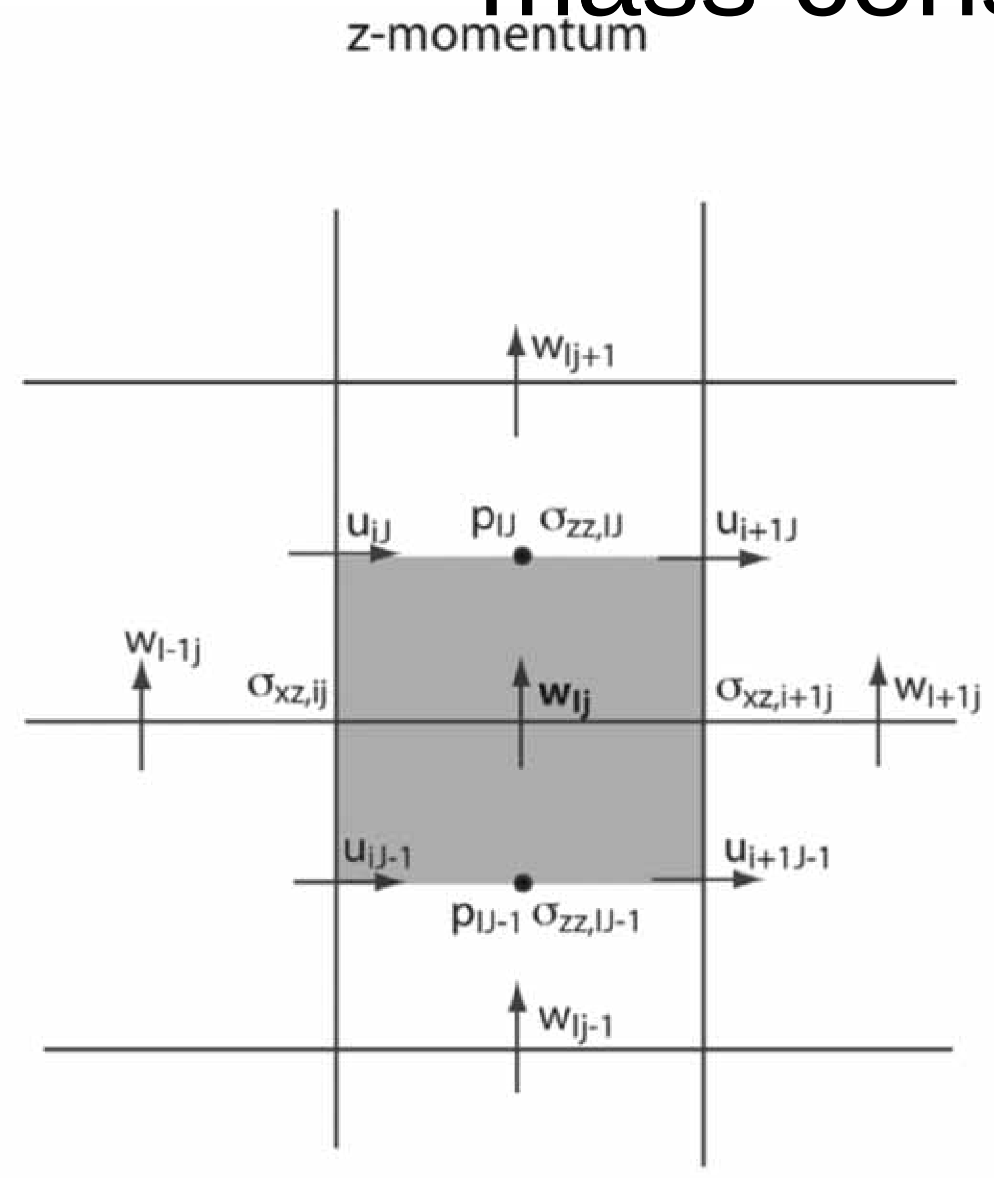
...continued

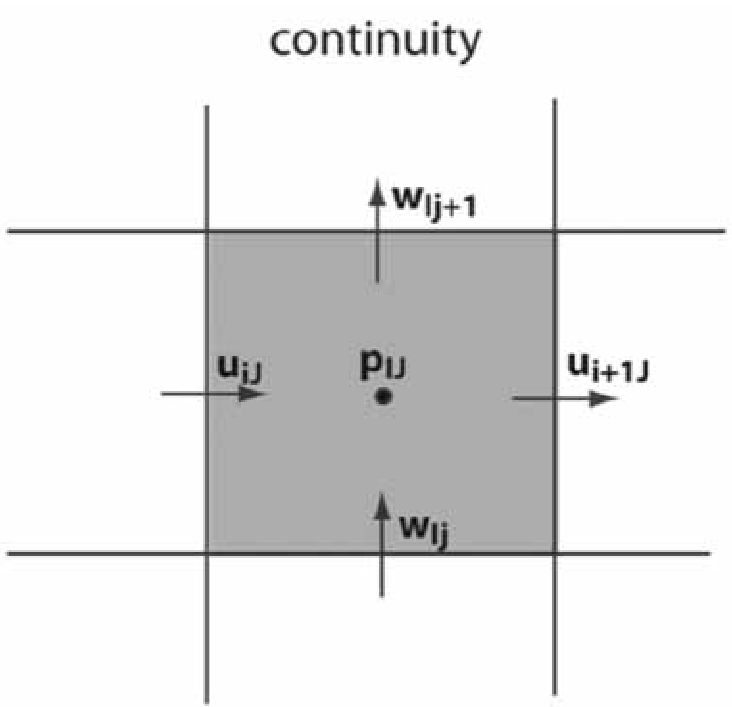
$$-\frac{p_{IJ}-p_{I-1J}}{\Delta x}+\frac{\sigma_{xx.IJ}-\sigma_{xx.I-1J}}{\Delta x}+\frac{\sigma_{xz.ij+1}-\sigma_{xz.ij}}{\Delta z}=0$$

$$-\frac{p_{IJ}-p_{I-1J}}{\Delta x}+\frac{1}{\Delta x}\left(\frac{\eta_{xx,IJ}\left(u_{i+1J}-u_{iJ}\right)-\eta_{xx,I-1J}\left(u_{iJ}-u_{i-1J}\right)}{\Delta x}\right)$$

$$+\frac{1}{\Delta z}\left[\eta_{xz,ij+1}\left(\frac{u_{iJ+1}-u_{iJ}}{\Delta z}+\frac{w_{Ij+1}-w_{I-1J+1}}{\Delta x}\right)-\eta_{xz,ij}\left(\frac{u_{iJ}-u_{iJ-1}}{\Delta z}+\frac{w_{Ij}-w_{I-1J}}{\Delta x}\right)\right]=0$$

Similarly for z-momentum and mass conservation





=> an equation for each velocity point & pressure point. How to solve them all simultaneously?

1. Direct (matrix) solver
2. Iterative multigrid solver

Example: 1D Poisson

Poisson: $\nabla^2 u = f$ In 1-D: $\frac{\partial^2 u}{\partial x^2} = f$

Finite-difference form: $\frac{1}{h^2}\left(u_{i-1} - 2u_i + u_{i+1}\right) = f_i$

Example with 5 grid points:

$$
\begin{aligned}
u_0 &= 0 \\
u_0 - 2u_1 + u_2 &= hf_1 \\
u_1 - 2u_2 + u_3 &= hf_2 \\
u_2 - 2u_3 + u_4 &= hf_3 \\
u_4 &= 0
\end{aligned}
$$

Problem: simultaneous solution needed

Ways to solve Poisson equation

- **Problem**: A large number of finite-difference equations must be solved simultaneously
- **Method 1. Direct**
  - Put finite-difference equations into a matrix and call a subroutine to find the solution
  - Pro: get the answer in one step
  - Cons: for large problems
    - matrix very large (nx*ny)^2
    - solution very slow: time~(nx*ny)^3
- **Method 2. Iterative**
  - Start with initial guess and keep improving it until it is "good enough"
  - Pros: for large problems
    - Minimal memory needed.
    - Fast if use multigrid method: time~(nx*ny)
  - Cons: Slow if don't use multigrid method

Direct (matrix) method

- Arrange unknowns **u** into a single vector
- Put 'unknown' terms **u** on the left-hand side, known terms on the right-hand side vector **f**
- Put coefficients of unknowns into a matrix
- => system of linear equations

$$A_{\ell k} u_k = f_\ell$$

- l^{th} row of A => coefficients of equation at l^{th} point
- Call a standard subroutine to solve this for **u**

Matrix for 2D Poisson

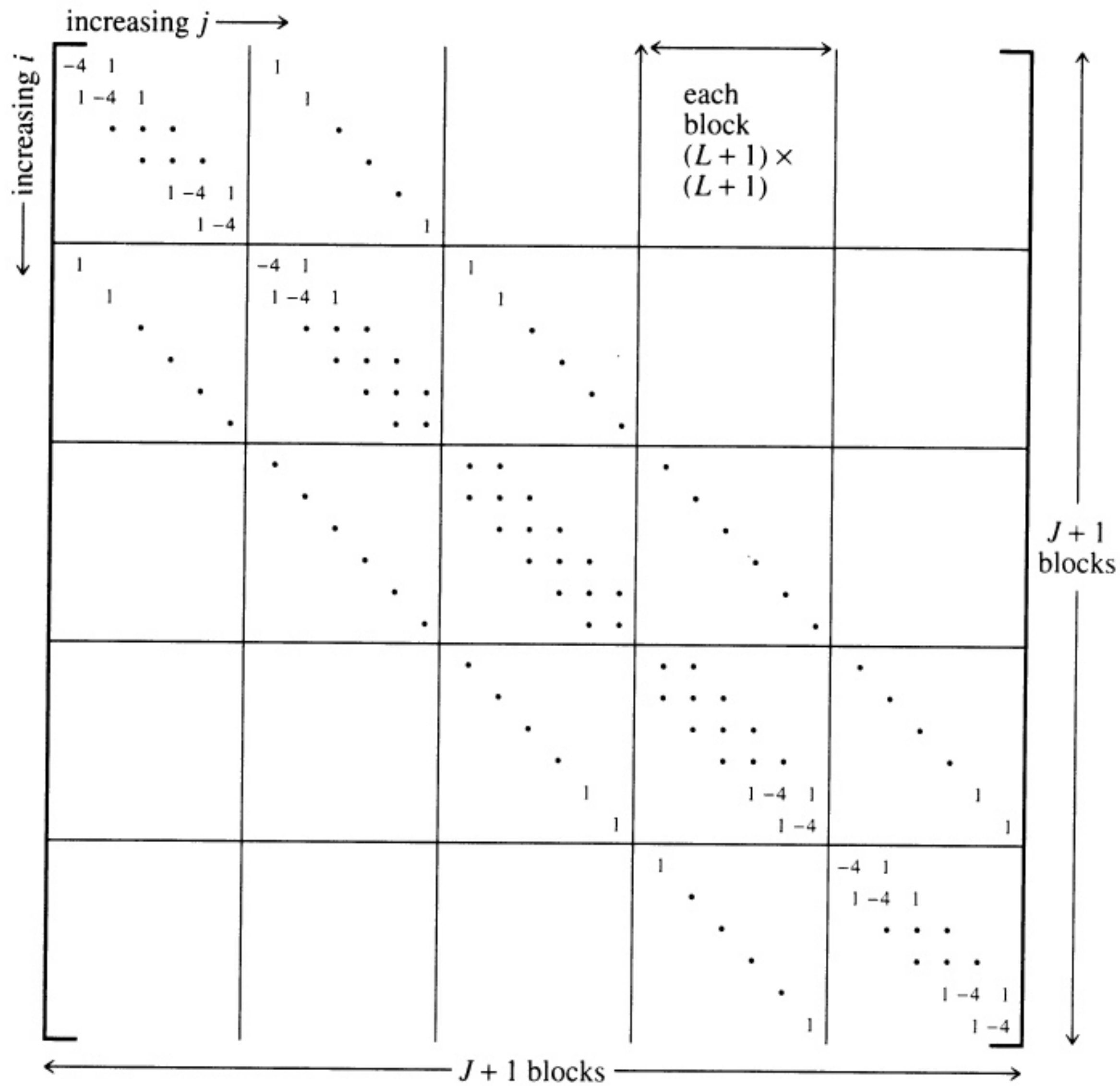


Figure 19.0.3. Matrix structure derived from a second-order elliptic equation (here equation 19.0.6). All elements not shown are zero. The matrix has diagonal blocks that are themselves tridiagonal, and sub- and super-diagonal blocks that are diagonal. This form is called "tridiagonal with fringes." A matrix this sparse would never be stored in its full form as shown here.

From Numerical Recipes

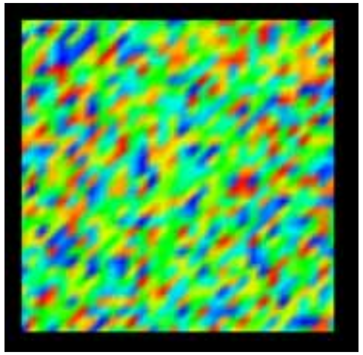
2. Iterative (Relaxation) Methods

- An alternative to using a direct matrix solver for sets of coupled PDEs
- Start with 'guess', then iteratively improve it
- Approximate solution 'relaxes' to the correct numerical solution
- Stop iterating when the error ('residue') is small enough

Why?

- Storage:
  - Matrix method has large storage requirements: (#points)^2. For large problems, e.g., 1e6 grid points, this is impossible!
  - Iterative method just uses #points
- Time:
  - Matrix method takes a long time for large #points: scaling as N^3 operations
  - The iterative **multigrid** method has #operations scaling as N

Now 2D Poisson eqn.



$$\nabla^2 u = f$$

Finite-difference approximation:

$$\frac{1}{h^2}\left(u_{i,j+1} + u_{i,j-1} + u_{i+1,j} + u_{i-1,j} - 4u_{i,j}\right) = f_{ij}$$

Assume we have an approximate solution $\tilde{u}_i$

The error or residue: $R = \nabla^2 \tilde{u} - f$

Now calculate correction to $\tilde{u}_i$ to reduce residue

Correcting $\tilde{u}_i$

From the residue equation note that: $\dfrac{\partial R_{ij}}{\partial \tilde{u}_{ij}} = -\dfrac{4}{h^2}$

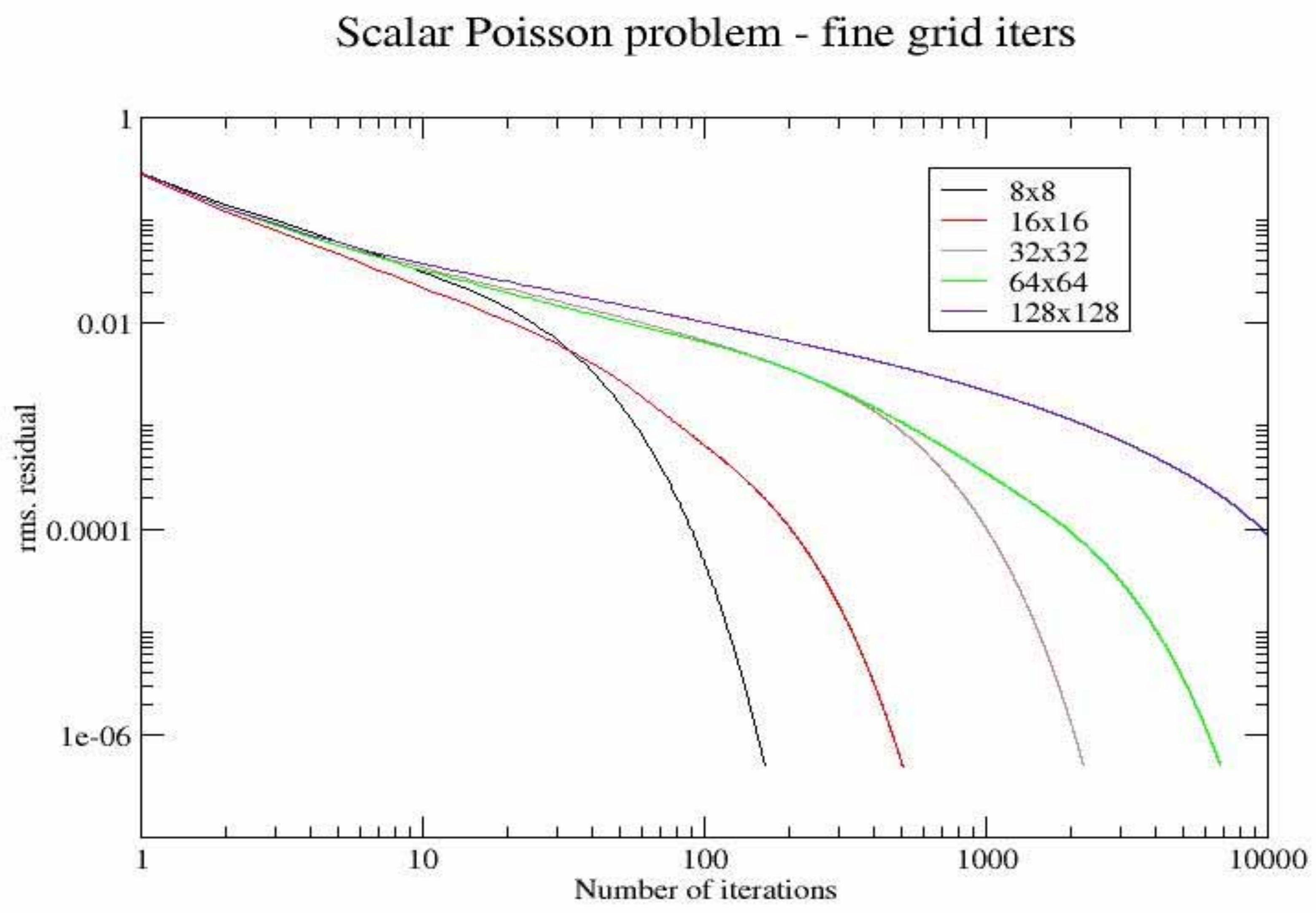
So adding a correction $+\frac{1}{4}h^2 R_{ij}$ to $\tilde{u}_{ij}$ should zero R

i.e., $$\tilde{u}_{ij}^{n+1} = \tilde{u}_{ij}^{n} + \alpha R_{ij}\frac{h^2}{4}$$

Unfortunately it doesn't zero R because the surrounding points also change, but it does reduce R

α is a 'relaxation parameter' of around 1:
α > 1 => 'overrelaxation'
α < 1 => 'underrelaxation'

Convergence of iterations



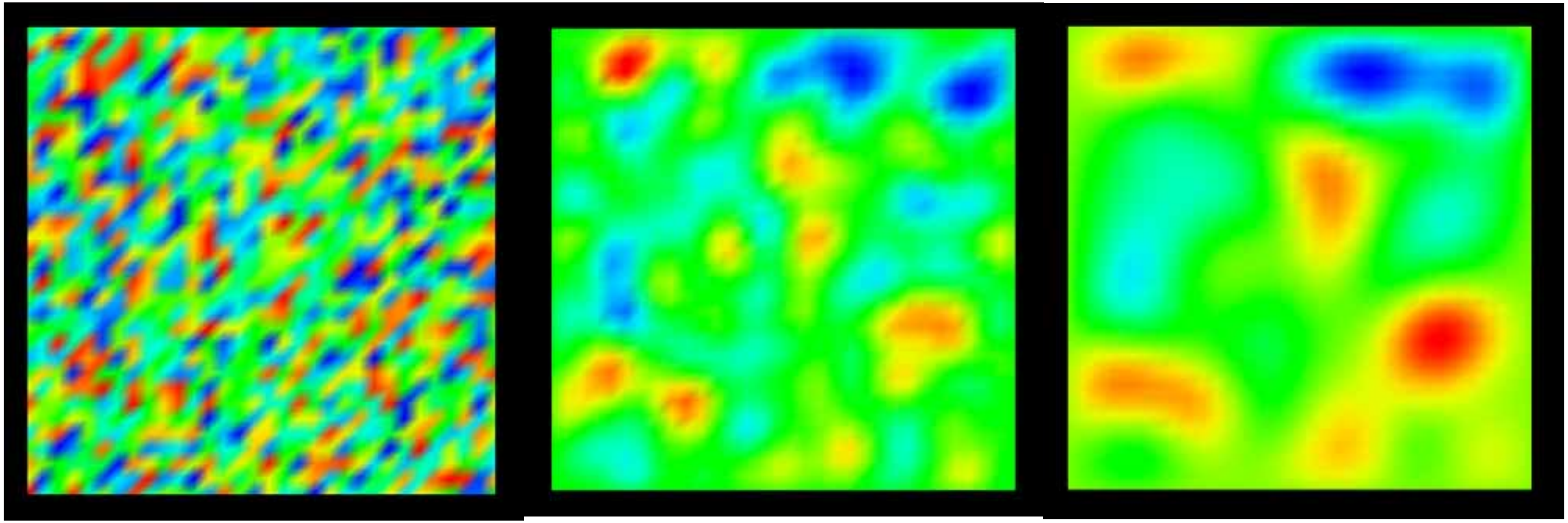
- Higher N => slower convergence

Iterations smooth the residue
=>**solve R on a coarser grid**
=>faster convergence

Start
rms residue=0.5

5 iterations
Rms residue=0.06

20 iterations
Rms residue=0.025



2-grid Cycle

- Several iterations on the fine grid
- Approximate (“**restrict**”) R on coarse grid
- Find coarse-grid solution to R (=correction to u)
- Interpolate (“**prolongate**”) correction=>fine grid and add to u
- Repeat until low enough R is obtained

Multigrid cycle

- Start as 2-grid cycle, but keep going to coarser and coarser grids, on each one calculating the correction to the residue on the previous level
- Exact solution on coarsest grid (~ few points in each direction)
- Go from coarsest to finest level, at each step interpolating the correction from the next coarsest level and taking a few iterations to smooth this correction
- All lengthscales are relaxed @ the same rate!

V-cycles and W-cycles

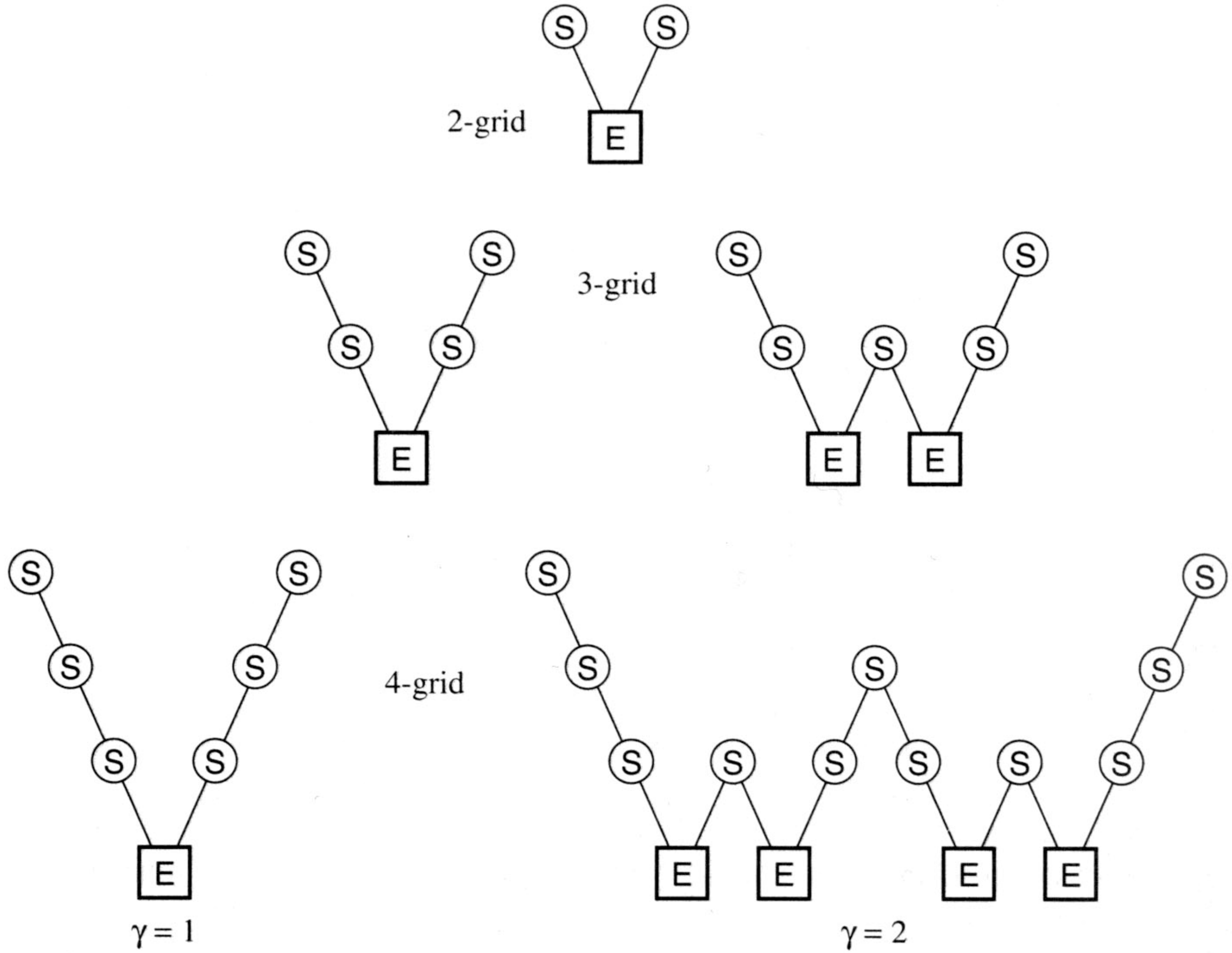
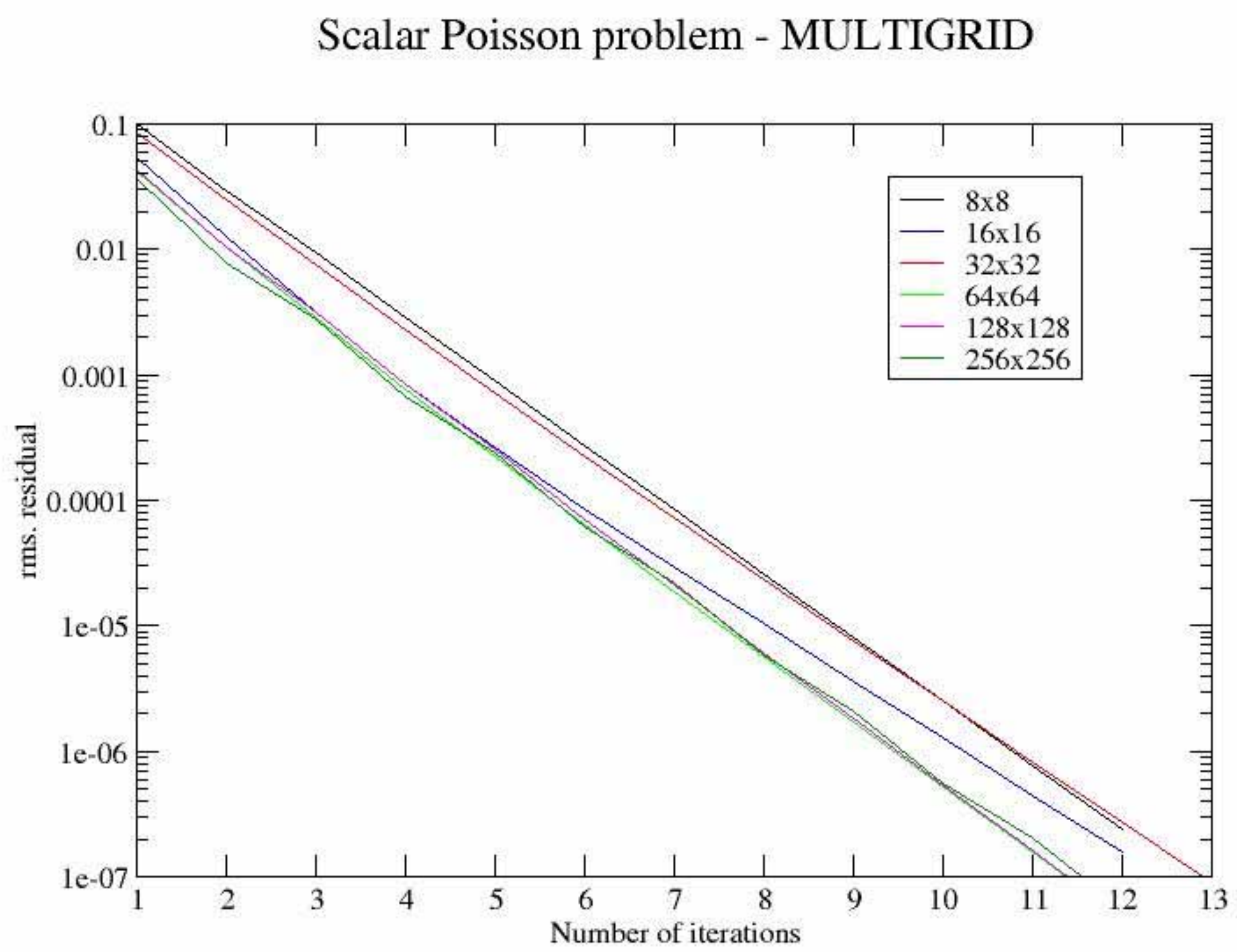


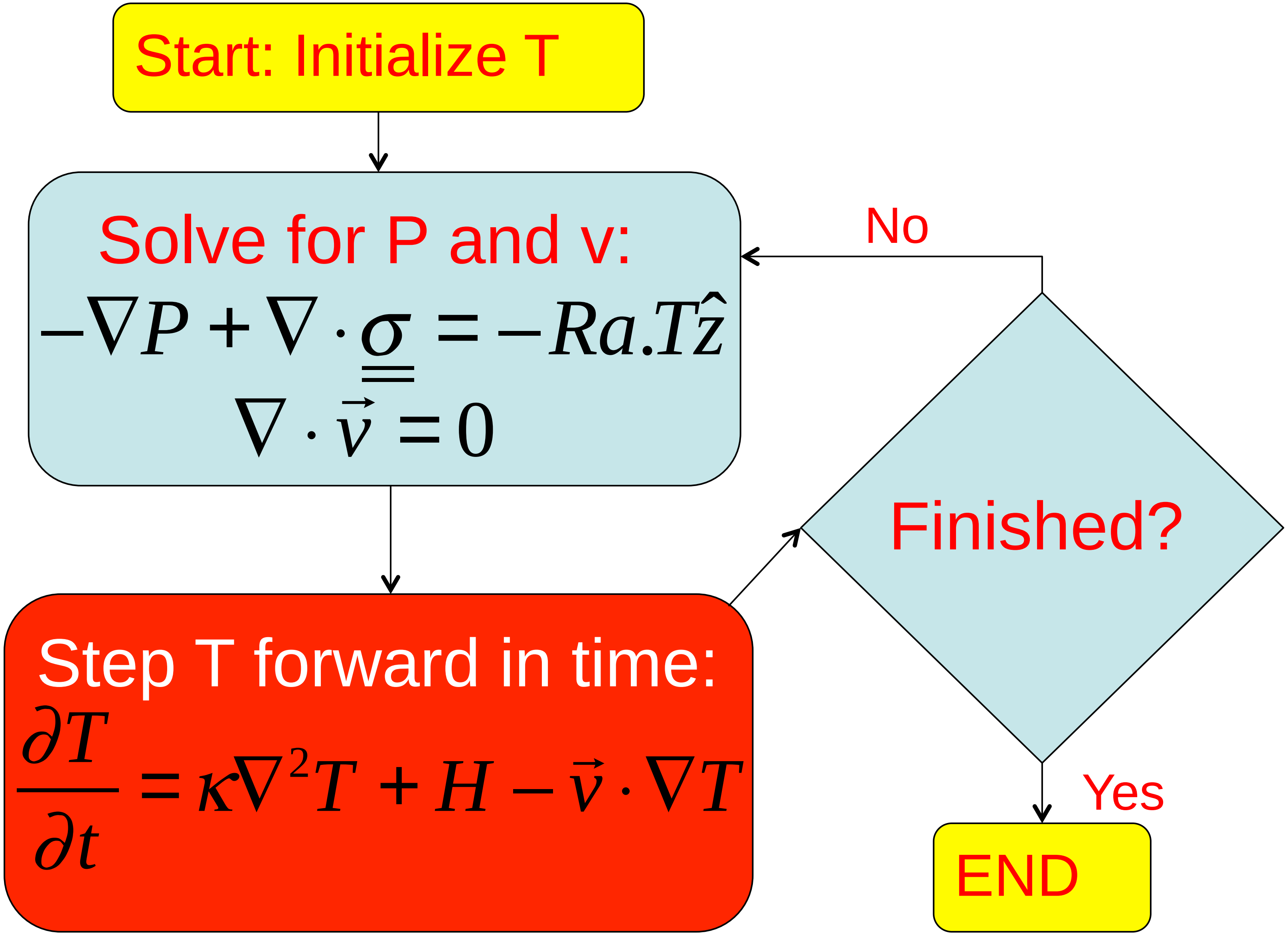
Figure 19.6.1. Structure of multigrid cycles. S denotes smoothing, while E denotes exact solution on the coarsest grid. Each descending line \ denotes restriction ($\mathcal{R}$) and each ascending line / denotes prolongation ($\mathcal{P}$). The finest grid is at the top level of each diagram. For the V-cycles ($\gamma = 1$) the E step is replaced by one 2-grid iteration each time the number of grid levels is increased by one. For the W-cycles ($\gamma = 2$), each E step gets replaced by two 2-grid iterations.



- Convergence rate **independent of grid size**
- =>#operations scales as #grid points
- Only a few iterations needed

So, **v** and P are now found using a direct or multigrid solver, next comes....

2. Time-stepping



Advection-diffusion equation
for a known velocity field **v**

$$\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa \nabla^2 T + H$$

advection diffusion

$$\frac{\partial T}{\partial t} = -v_x \frac{\partial T}{\partial x} - v_y \frac{\partial T}{\partial y} + \nabla^2 T + H$$

internal heating

Diffusion first

$$\frac{\partial T}{\partial t} = \nabla^2 T = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

- Now discretise this using finite differences
- Explicit method (use derivatives at current t)

$$\frac{T_{i,j}^{(t_1+\Delta t)} - T_{i,j}^{(t_1)}}{\Delta t} = \left(\frac{T_{i-1,j}^{(t_1)} - 2T_{i,j}^{(t_1)} + T_{i+1,j}^{(t_1)}}{(\Delta x)^2} + \frac{T_{i,j-1}^{(t_1)} - 2T_{i,j}^{(t_1)} + T_{i,j+1}^{(t_1)}}{(\Delta y)^2} \right)$$

$$T_{i,j}^{(t_1+\Delta t)} = T_{i,j}^{(t_1)} + \Delta t \left(\frac{T_{i-1,j}^{(t_1)} - 2T_{i,j}^{(t_1)} + T_{i+1,j}^{(t_1)}}{(\Delta x)^2} + \frac{T_{i,j-1}^{(t_1)} - 2T_{i,j}^{(t_1)} + T_{i,j+1}^{(t_1)}}{(\Delta y)^2} \right)$$

- *$T(t_2)$* only on left, so simple to program!

Finite-volume advection

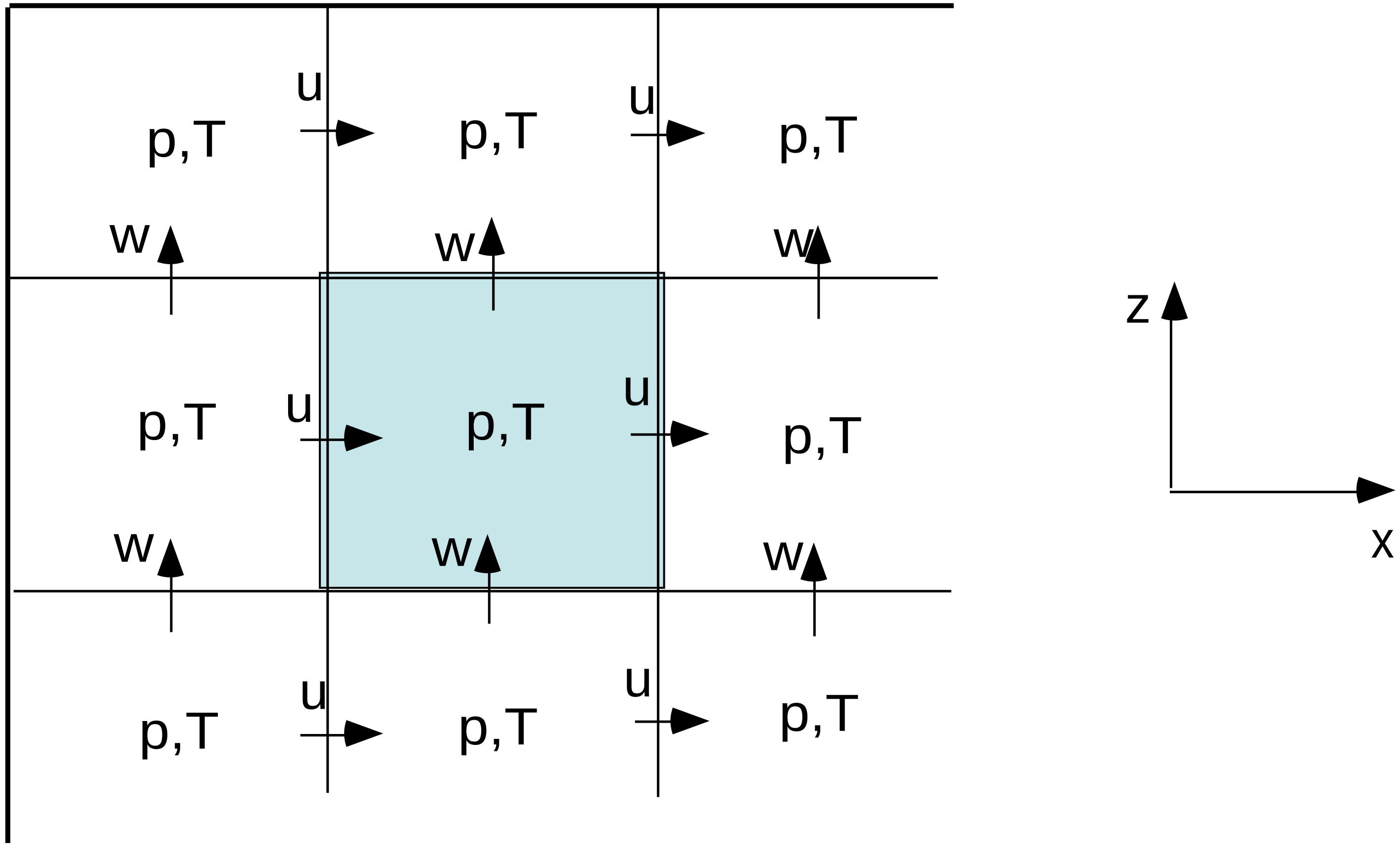
consider $$\nabla \cdot \left(\vec{v}T\right) = \vec{v} \cdot \nabla T + T\nabla \cdot \vec{v}$$

but for incompressible flows, $$\nabla \cdot \vec{v} = 0$$

so, $$\vec{v} \cdot \nabla T = \nabla \cdot \left(\vec{v}T\right)$$

expanding: $$\nabla \cdot \left(\vec{v}T\right) = \frac{\partial}{\partial x}\left(v_x T\right) + \frac{\partial}{\partial y}\left(v_y T\right)$$

(vT) is the flux in/out of the cell



Advantage: CONSERVATIVE (conserves energy) because flux out of one cell=flux into another

but which T to use at each cell side?

Which T to use to calculate fluxes?

For stability reasons:

- **Use the T from the cell that material is coming from** to calculate vT at each cell boundary (‘donor cell’, ‘upwind’)
- Then the advective term is:

$$\left[\nabla \cdot \left(\vec{v}T\right)\right]_{ij} = \frac{(v_x T)_{i+0.5,j} - (v_x T)_{i-0.5,j}}{\Delta x} + \frac{(v_y T)_{i,j+0.5} - (v_y T)_{i,j-0.5}}{\Delta y}$$

A note on numerical advection

Advection is very difficult to treat accurately.

• Simple schemes either go unstable or smear out temperature anomalies (numerical diffusion; the donor cell scheme has plenty of this).

• More sophisticated schemes can cause artificial ripples (numerical dispersion) and other types of distortion.

• Many papers have been written on numerical advection!

• TVD schemes or MPDATA are good ones to choose.

Stability of time stepping

- The explicit method is **unstable** if the time step is too large. This means, you get oscillations whose amplitude grows exponentially with time.
- This happens if material moves/diffuses more than ~1 grid spacing in a time-step
- Diffusion: $$\Delta t_{critical} = 0.25(\Delta x)^2 / \kappa$$
- Advection: $$\Delta t_{critical} \simeq 0.7 \Delta x / \max(|v|)$$

Putting the time-step together

$$T_{i,j}^{(t_1+\Delta t)} = T_{i,j}^{(t_1)} + \Delta t\left[\kappa\left(\nabla^2 T\right)_{i,j}^{(t_1)} - \nabla\cdot\left(\vec{v}T\right)_{i,j}^{(t_1)} + H\right]$$

where

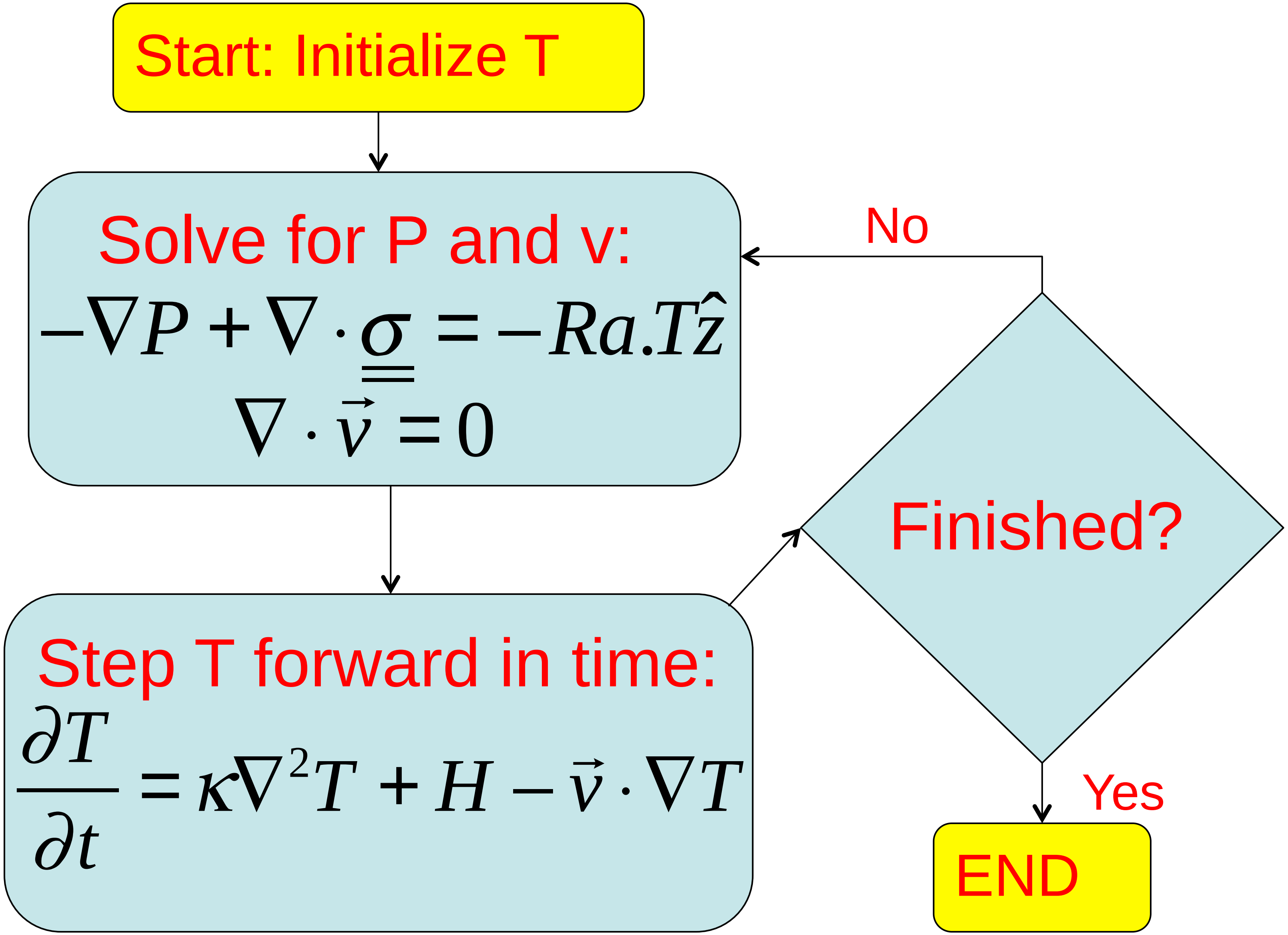
$$\left(\nabla^2 T\right)_{i,j}^{(t_1)} = \left(\frac{T_{i,j-1}^{(t_1)} - 2T_{i,j}^{(t_1)} + T_{i,j+1}^{(t_1)}}{(\Delta x)^2} + \frac{T_{i-1,j}^{(t_1)} - 2T_{i,j}^{(t_1)} + T_{i+1,j}^{(t_1)}}{(\Delta y)^2}\right)$$

Upwind (donor cell) temperatures

$$\left[\nabla\cdot\left(\vec{v}T\right)\right]_{ij} = \frac{(v_x T)_{i+0.5,j} - (v_x T)_{i-0.5,j}}{\Delta x} + \frac{(v_y T)_{i,j+0.5} - (v_y T)_{i,j-0.5}}{\Delta y}$$

$$\Delta t = \min\left[a_{diff}\frac{\left(\min(\Delta x,\Delta z)\right)^2}{\kappa}, a_{adv}\min\left(\frac{\Delta x}{v_{x,\max}}, \frac{\Delta z}{v_{z,\max}}\right)\right]$$

Putting everything together



Convection2D.m

Matlab program for 2D convection

- Solves either
  - Variable-viscosity convection using a direct solver
  - Constant-viscosity convection using streamfunction-vorticity formulation and a multigrid solver (faster)
- Inputs
  - Physical: *Ra, H, initial_temperature, variable_viscosity, viscosity_contrast_temperature, viscosity_contrast_depth*
  - Numerical: *nx, nz, nsteps*

Exercises (later)

1. Determine critical Rayleigh number for onset of convection (try values 1e1 to 1e5)
2. Observe effect of internal heating on convection (with Ra=1e6, try several H values from 0 to 30).
3. Observe effect of box width (try nx from 1* to 4* nz)
4. What temperature-dependent viscosity contrast is needed to obtain stagnant lid?

Explanation: streamfunction-vorticity

For highly viscous flow (e.g., Earth's mantle) with constant viscosity (P=pressure, Ra=Rayleigh number):

$$-\nabla P + \nabla^2 \vec{v} = -Ra.T\hat{z}$$

Substituting the streamfunction for velocity, we get:

$$(v_x, v_z) = \left(\frac{\partial \psi}{\partial z}, -\frac{\partial \psi}{\partial x} \right) \qquad \nabla^4 \psi = -Ra \frac{\partial T}{\partial x}$$

writing as 2 Poisson equations:

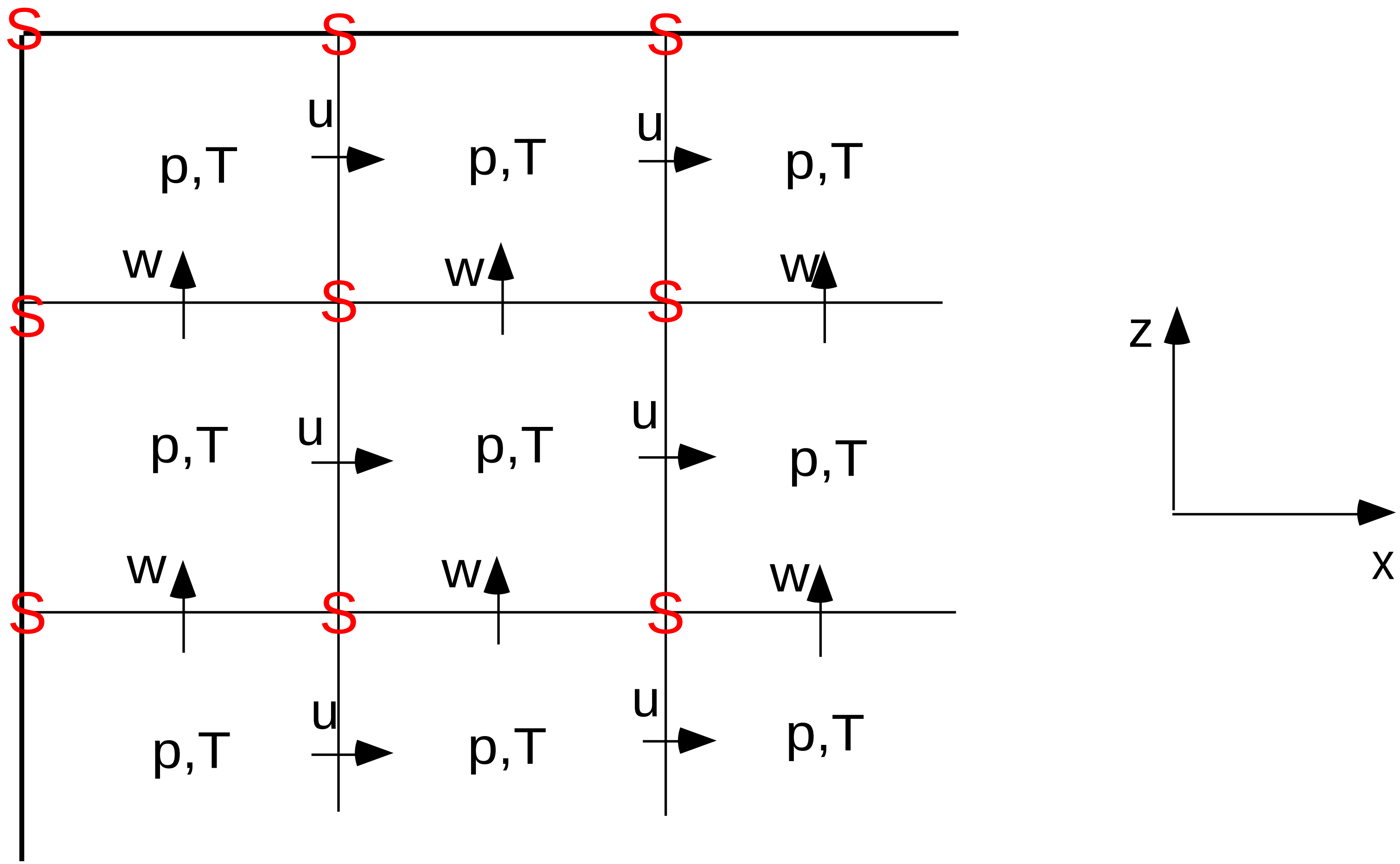
$$\nabla^2 \psi = -\omega \qquad \nabla^2 \omega = Ra \frac{\partial T}{\partial x}$$

the **streamfunction-vorticity** formulation

Advantages of using the streamfunction

- Two vector velocity components are reduced to one scalar
- Continuity is automatically satisfied
- If also solving the Navier-Stokes equation, pressure can be algebraically eliminated from the momentum equation, reducing the number of variables further

Staggered grid
+streamfunction



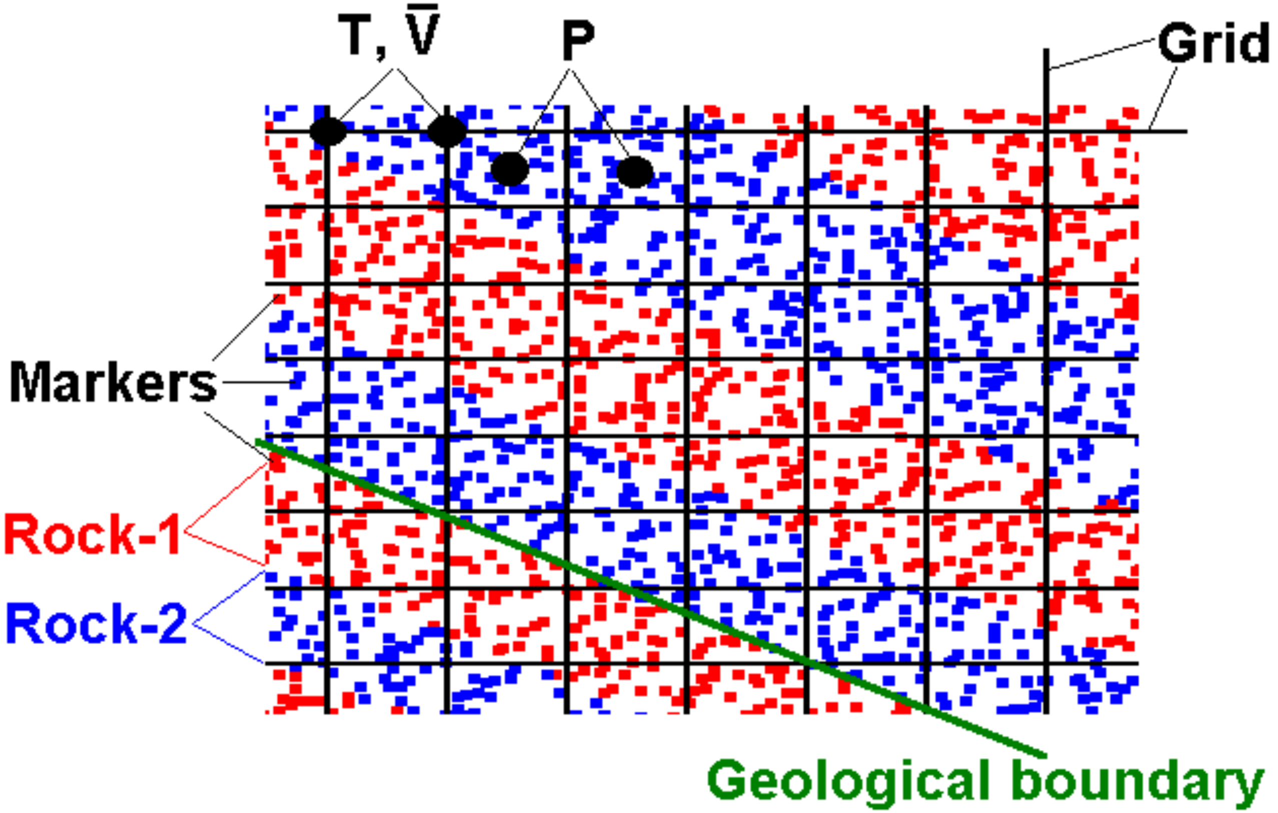
Streamfunction derivatives also involve advacent points

Complexities

"Mantle convection is much more complicated than thermal convection" (G. Schubert)

1. Compositional variations

Lagrangian tracers (markers) for composition



“Marker-In-Cell” or “Particle-In-Cell” method

Figure by T. Gerya

2. Spherical geometry

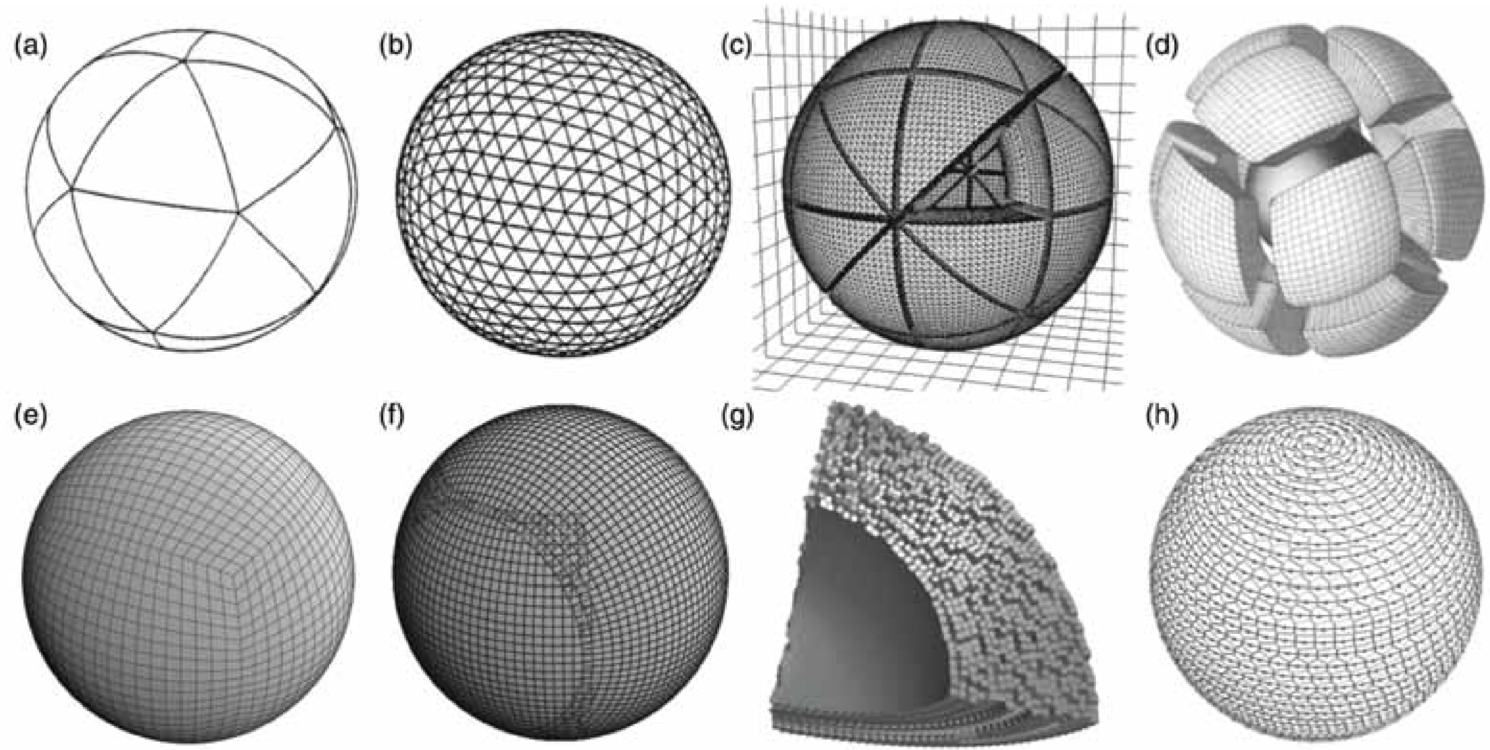
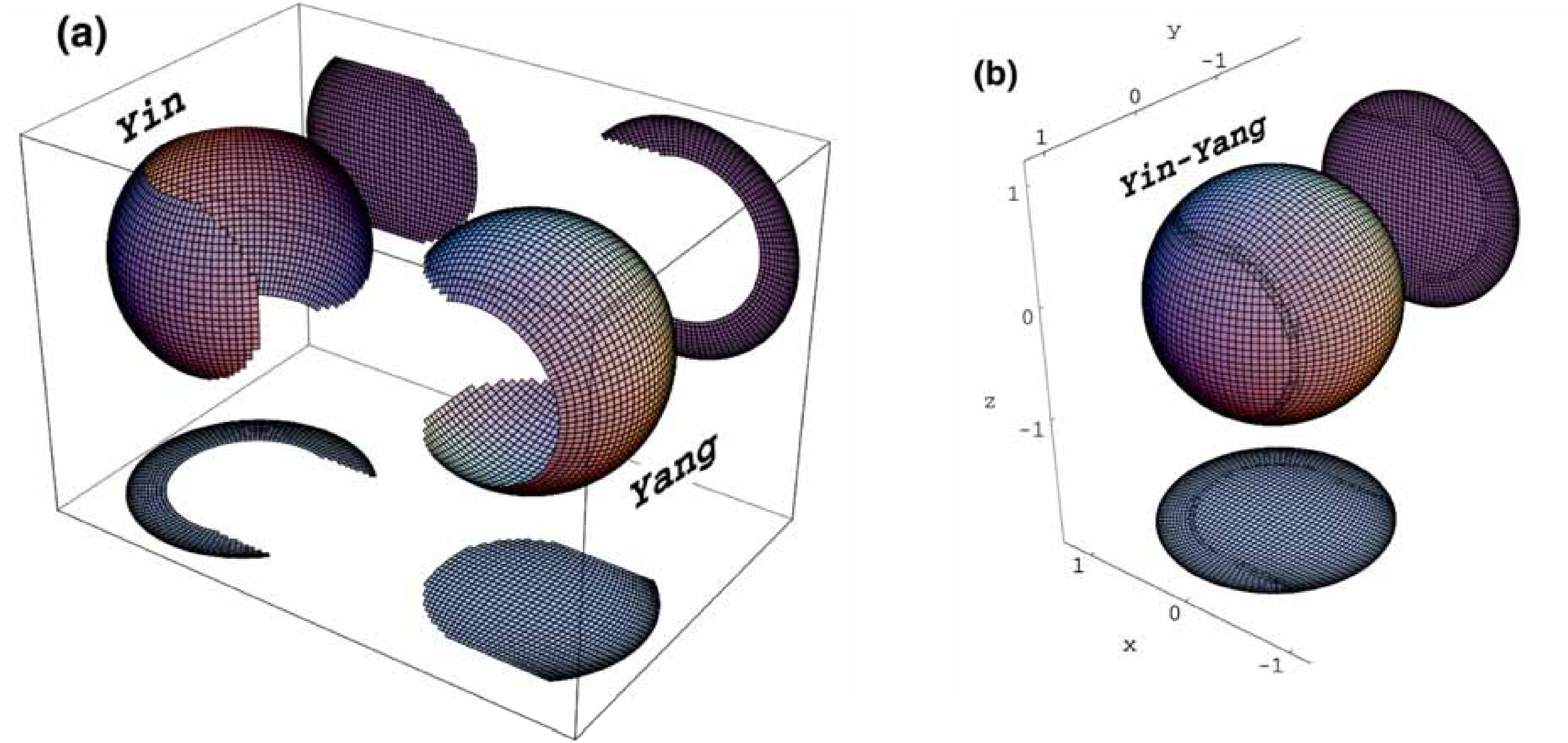


Fig. B1. Spherical meshes. (a) An isocahedron projected onto a sphere, and (b) subdivided eight ways in each direction (reprinted from Baumgardner, 1985). (c) The mesh of Tabata and Suzuki (2000), which can be regarded as subdividing a projected octahedron, first into six triangular blocks for each face then further using tetrahedral finite elements (the figure taken from Tabata, 2006). (d) The mesh used by CitComS, which can be regarded as subdividing each face of a projected tetrahedron into three rhombohedral blocks that are then further subdivided (the figure reprinted from Zhong *et al.*, 2000). (e) The cubed sphere grid (figure reprinted from Hernlund and Tackley, 2003). (f) The Yin-Yang grid (Kageyama and Sato, 2004). (g)(h) The spiral grid (from Huettig and Stemmer, 2008b).

‘Yin-Yang’ spherical grid (Kageyama & Sato 2004 G^3)

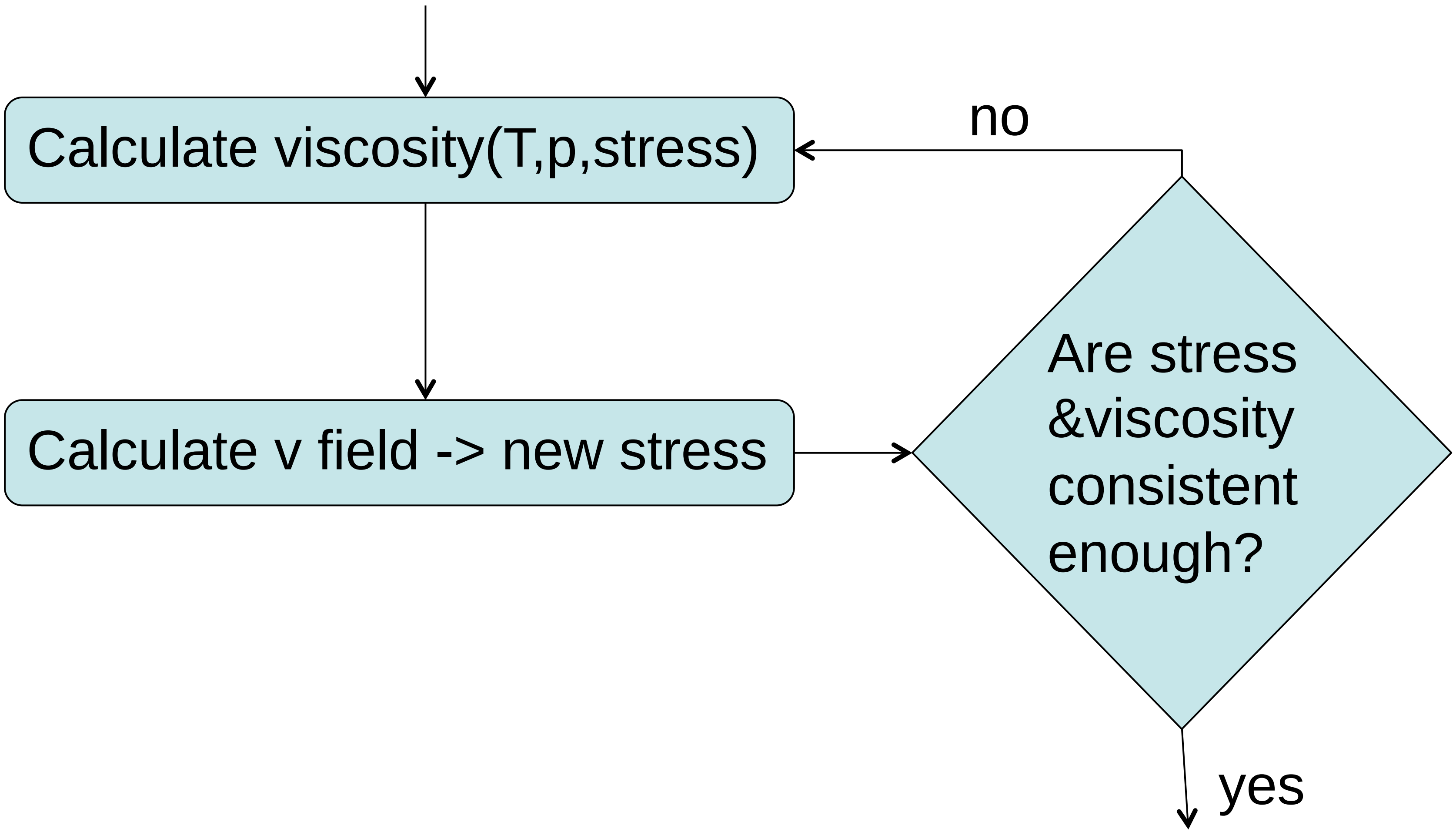


- Orthogonal => finite-differences possible
- Small overlap (minimum overlap version)

3. Nonlinear rheology

(plasticity and/or dislocation creep)

Requires iterations at each moment in time



4. Compressibility

- Modification to continuity & normal stress terms
- Viscous heating & adiabatic heating
- Variable expansivity, conductivity

Conservation of mass:

$$\nabla \cdot (\rho \underline{v}) = 0 \quad , \tag{1}$$

momentum

$$\underline{\nabla} \cdot \underline{\underline{\sigma}} - \underline{\nabla} p = Ra \, . \, \underline{\hat{r}} \, . \rho(C,r,T) / \Delta\rho_{thermal} \tag{2}$$

and energy

$$\rho C_p \frac{DT}{Dt} = -Di_s \alpha \rho T v_r + \underline{\nabla} \bullet (k \nabla T) + \rho H + \frac{Di_s}{Ra} \underline{\underline{\sigma}} : \underline{\underline{\dot{\varepsilon}}} \tag{3}$$

5. Visco-elasticity

Can be treated with a viscous flow solver using

(i) an effective viscosity

(ii) Adding elastic stress to the total stress tensor

(see Moresi, 2002)

$$\eta_{eff} = \eta \frac{\Delta t^e}{\Delta t^e + \alpha}$$

Alpha=relaxation time

$$\tau^{t+\Delta t^e} = \eta_{eff}\left(2\hat{\mathbf{D}}^{t+\Delta t^e} + \frac{\tau^t}{\mu \Delta t^e} + \frac{\mathbf{W}^t \tau^t}{\mu} - \frac{\tau^t \mathbf{W}^t}{\mu} \right)$$

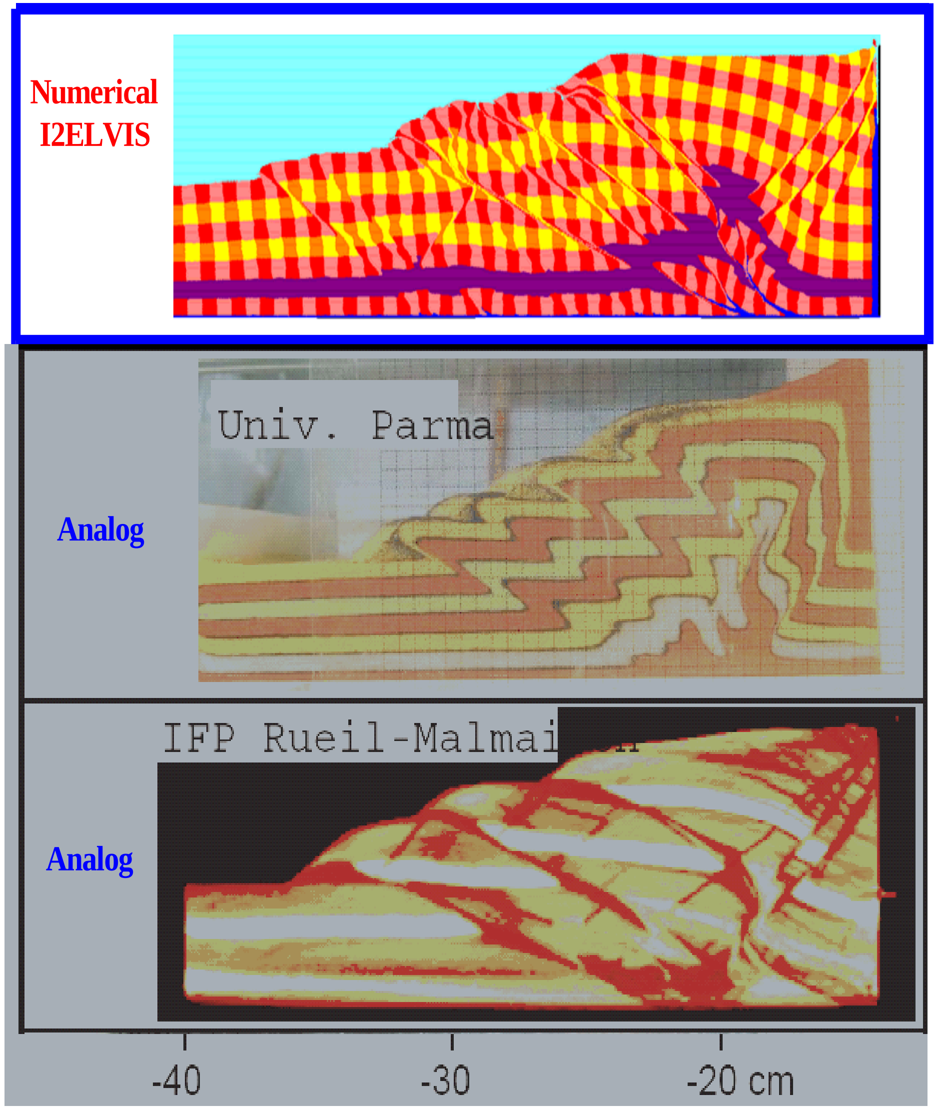
6. Other complexities

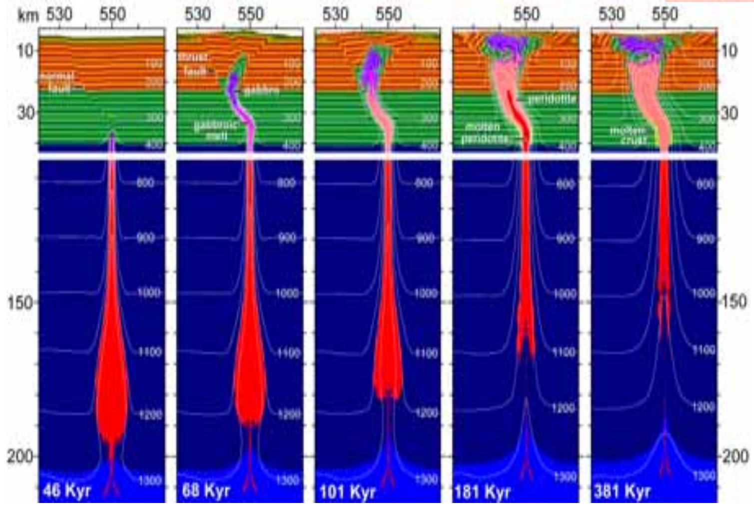
- Phase transitions
  - Modify density/buoyancy, also latent heat term in energy equation
- Melting
  - Calculate melting & form crust instantaneously, OR
  - Melt migration (Darcy's law)
- Tracking of trace element geochemistry
  - On tracers/markers
  - Radiogenic decay, outgassing

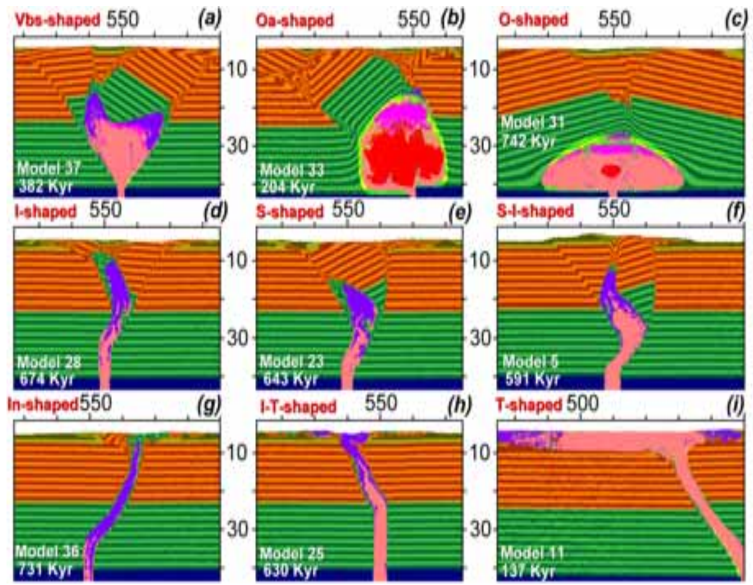
Application to Earth & planets

Recent results from my research group, all calculated using the finite volume + marker-in-cell method

Examples: 2D Crustal shortening and magma pipe intrusion (Taras Gerya, ETH, and coworkers)







Mid-ocean ridges: Gerya (Science 2010)

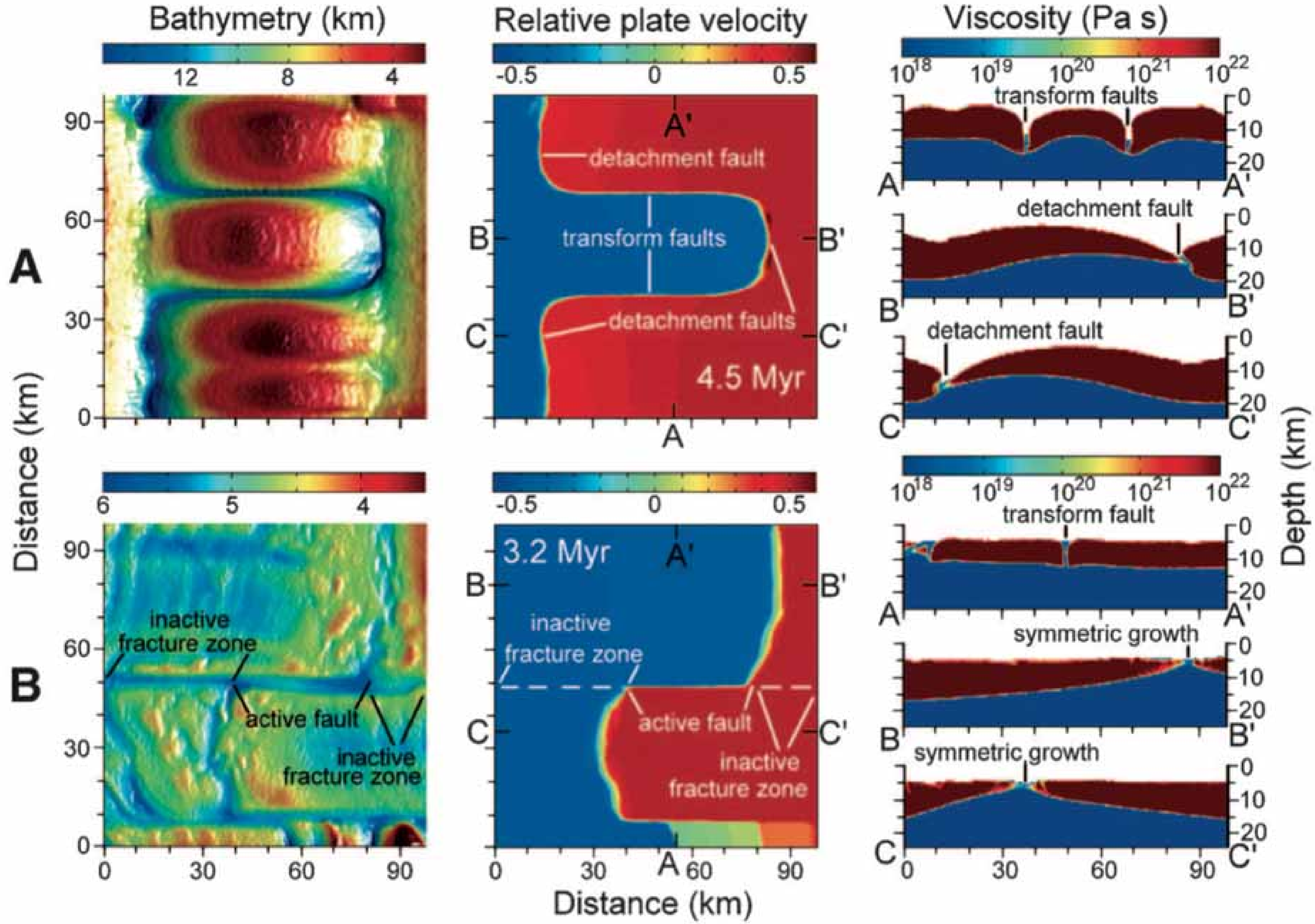


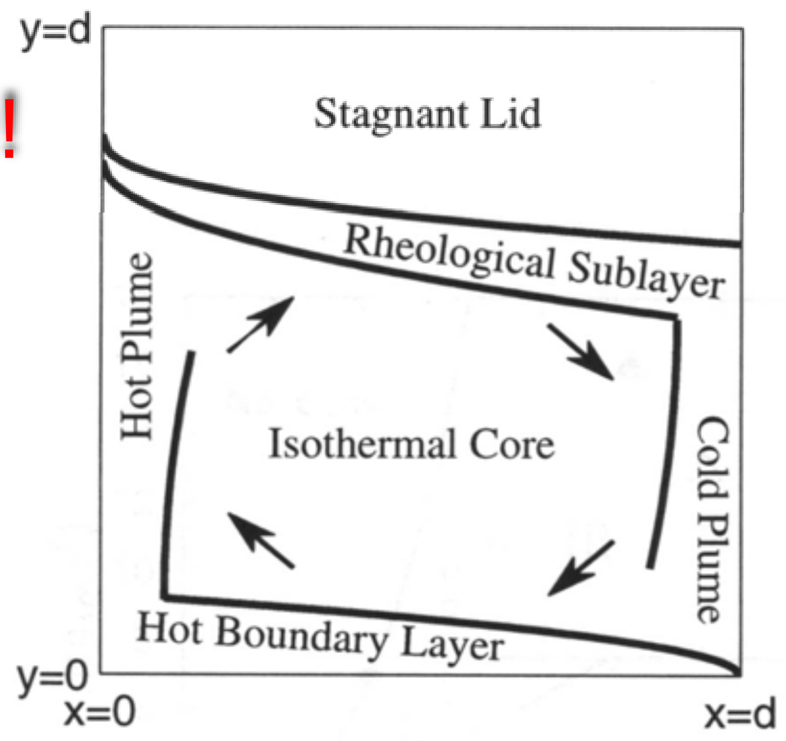
Plate tectonics: Earth unusual ?

- Mars: rigid lid
  - Had plate tectonics early?
- Venus: rigid lid
  - Plate tectonics->rigid lid?
  - Episodic overturn?
- Earth: Different early on?

The plate problem

- Viscous, T-dependent rheology appropriate for the mantle leads to a stagnant lid
- exp(E/kT) where E~340 kJ/mol
- T from 1600 -> 300 K
- =>**1.3x10^{48}** variation
- => RIGID/STAGNANT LID!

Only small ΔT participates in convection: enough to give Δη factor ~10



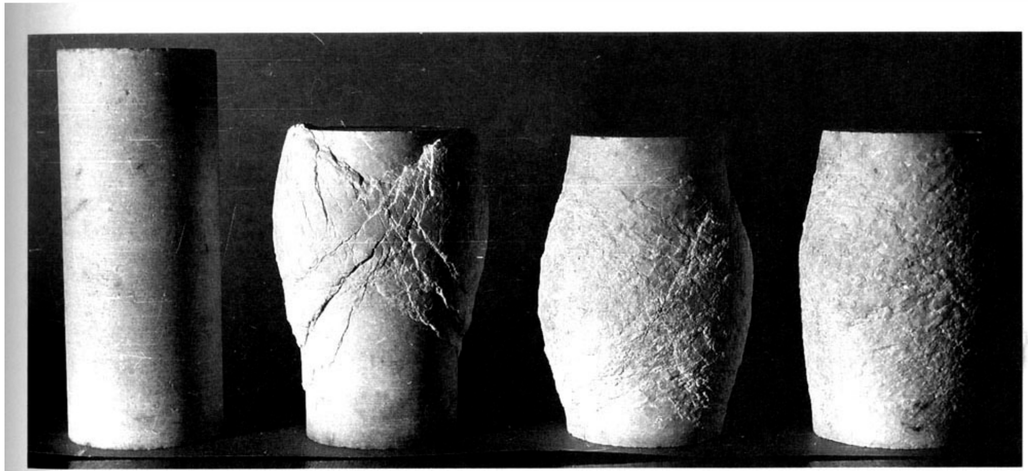
We don't understand plate tectonics at a fundamental level

- Rock deformation is complex
  - Viscous, brittle, plastic, elastic, nonlinear
  - Dependent on grain size, composition (major and trace element, eg water)
- Multi-scale
  - Lengthscales from mm to 1000s km
  - Timescales from seconds - Gyr

Strength of rocks

- Increases with confining pressure (depth) then saturates

Low-T deformation: Effect of P



Undeformed — Low confining pressure — Intermediate confining pressure — High confining pressure

Low T: Effect of P

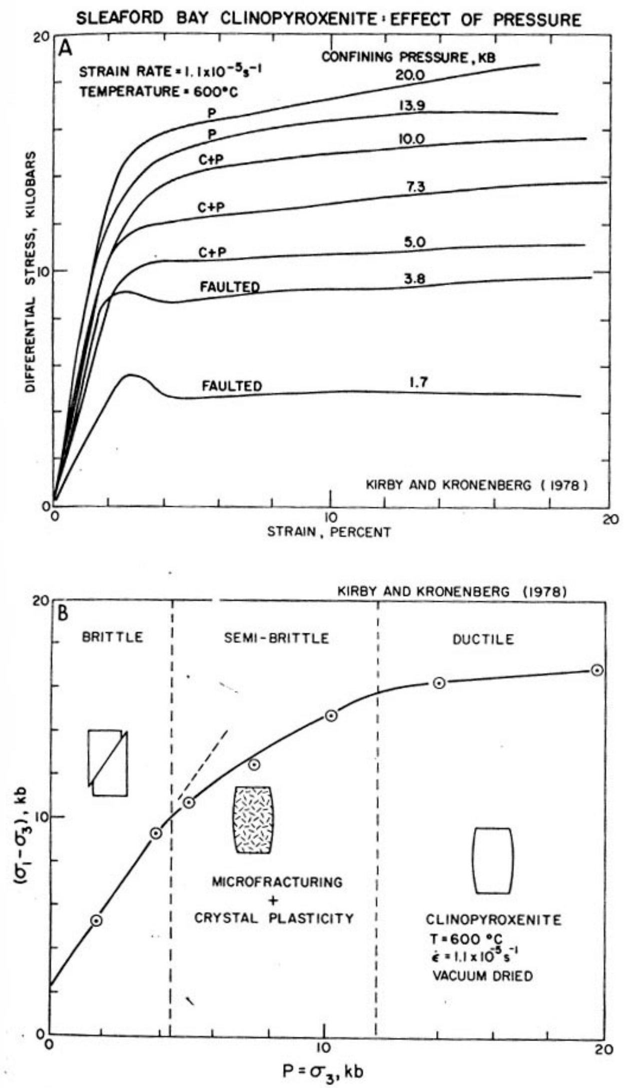
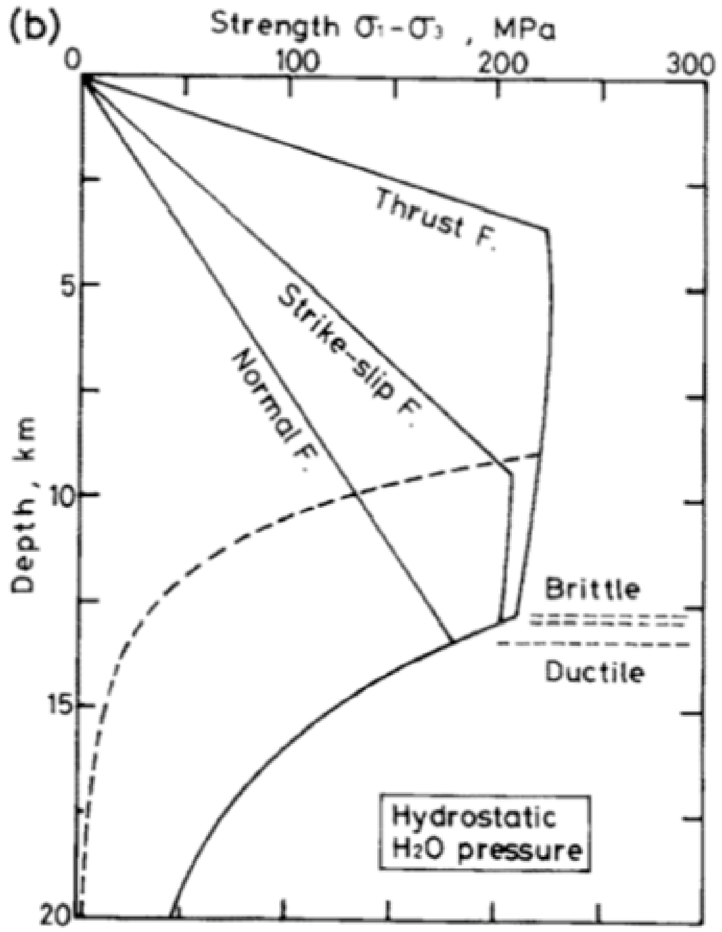


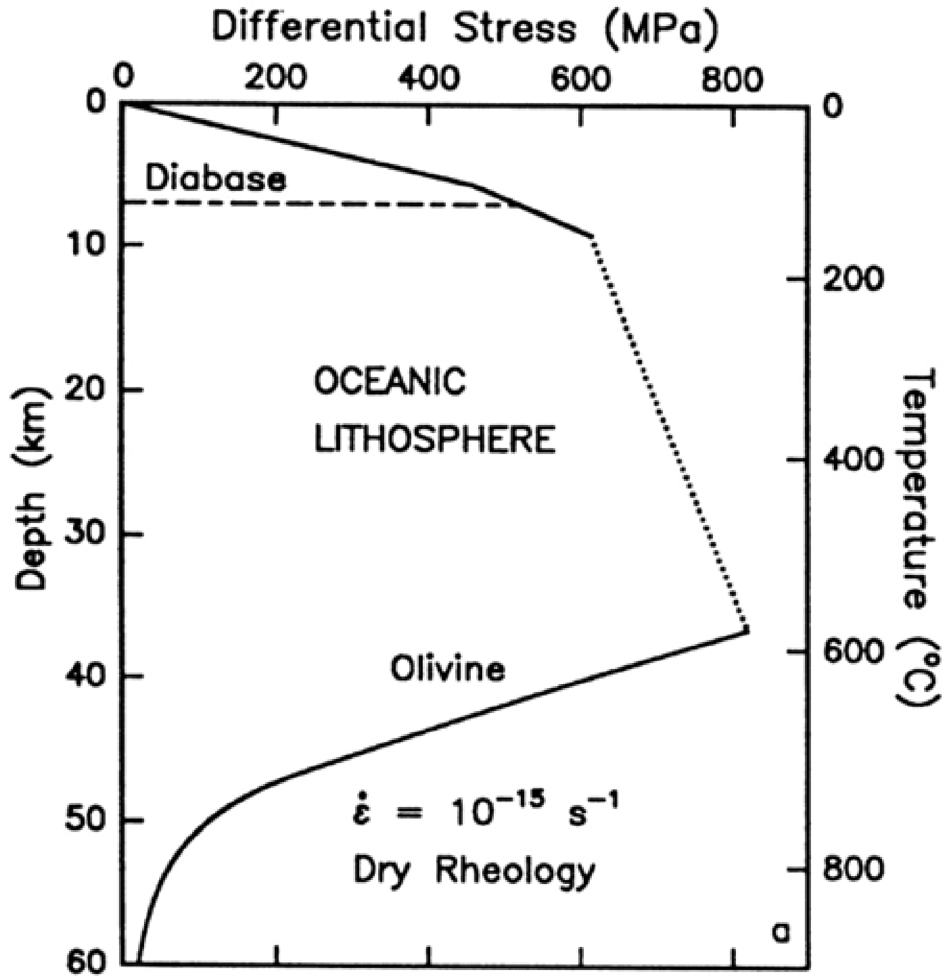
Fig. 6. Effect of confining pressure on the strength of Sleaford Bay clinopyroxenite tested in triaxial compression (S. H. Kirby and A. K. Kronenberg, unpublished data, 1978): (*a*) stress-strain curves, (*b*) ultimate strength or stress at 10% strain as a function of confining pressure.

Strength profile of lithosphere

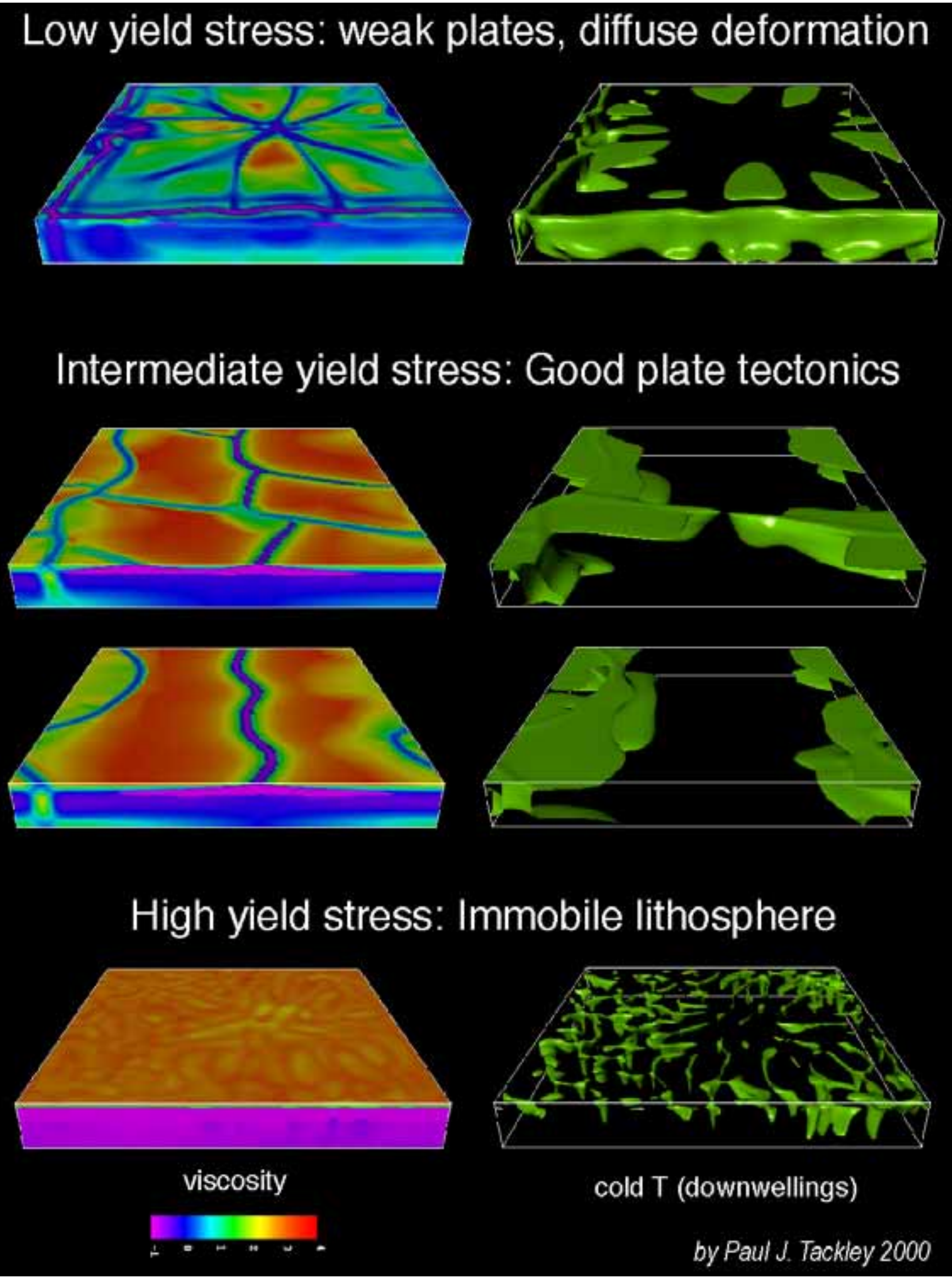
Continental (granite): Shimada 1993



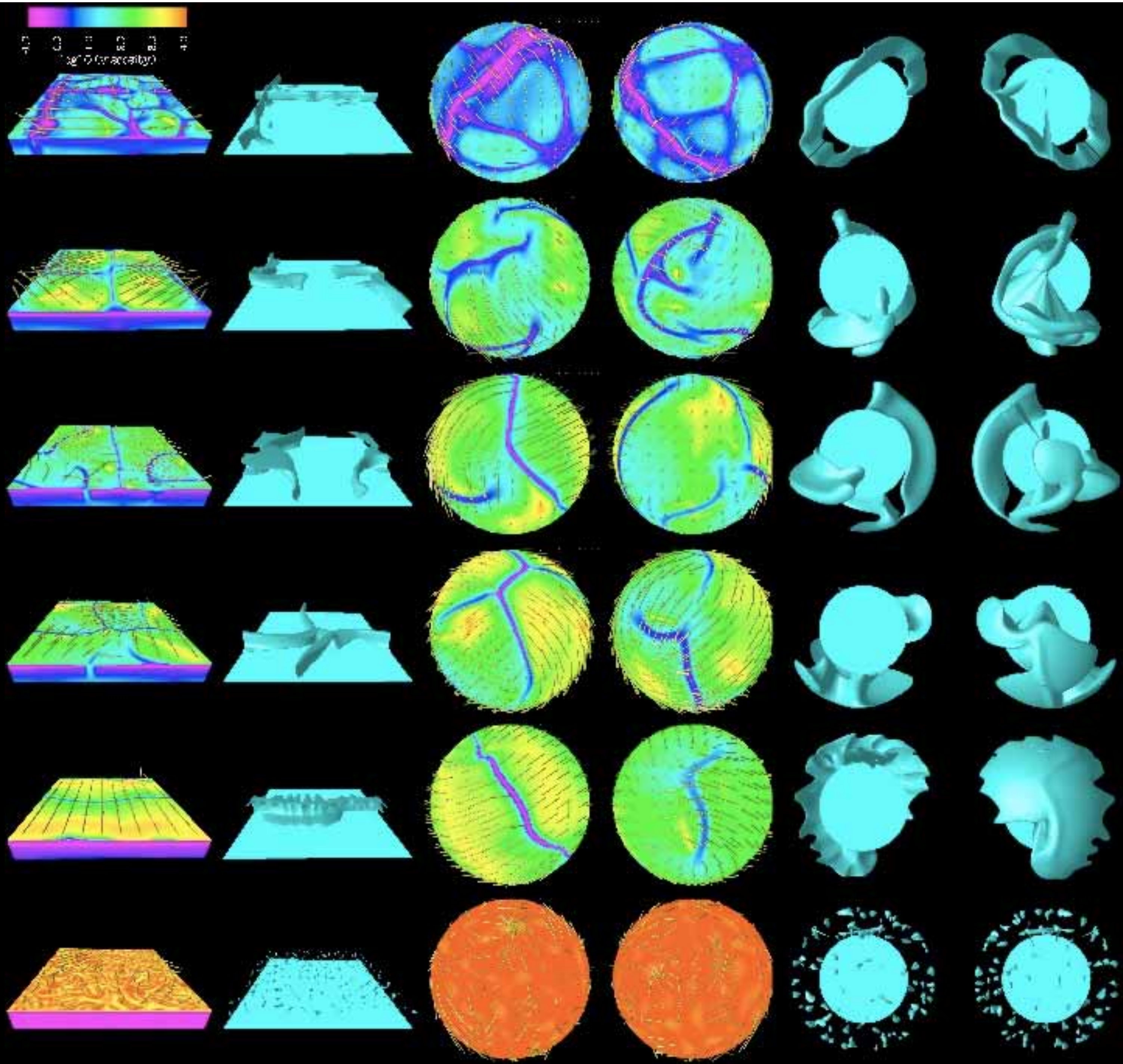
Oceanic: Kohlstedt 1995



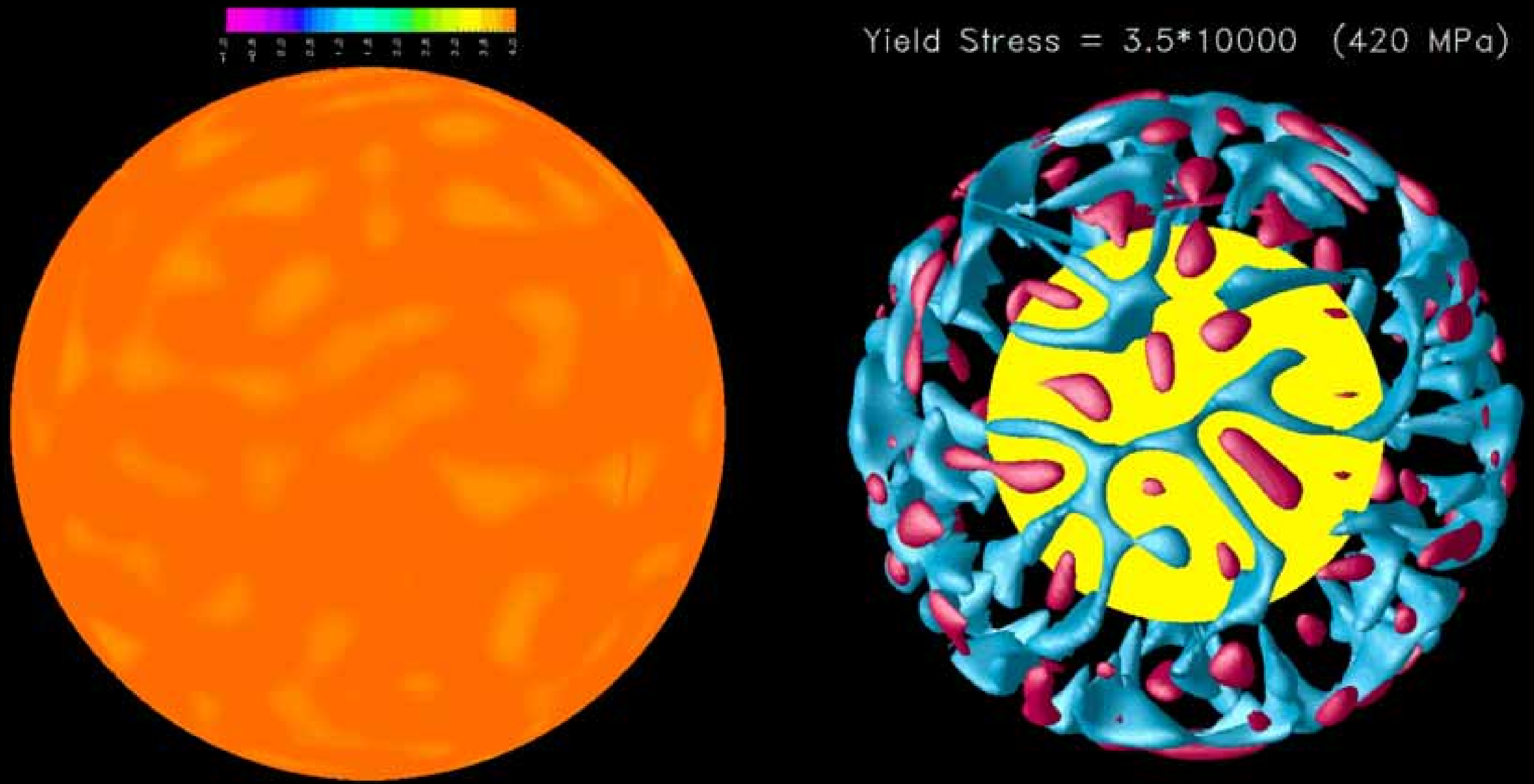
- Varying yield strength, including asthenosph.



Spherical:
van Heck
& me,
GRL 2008

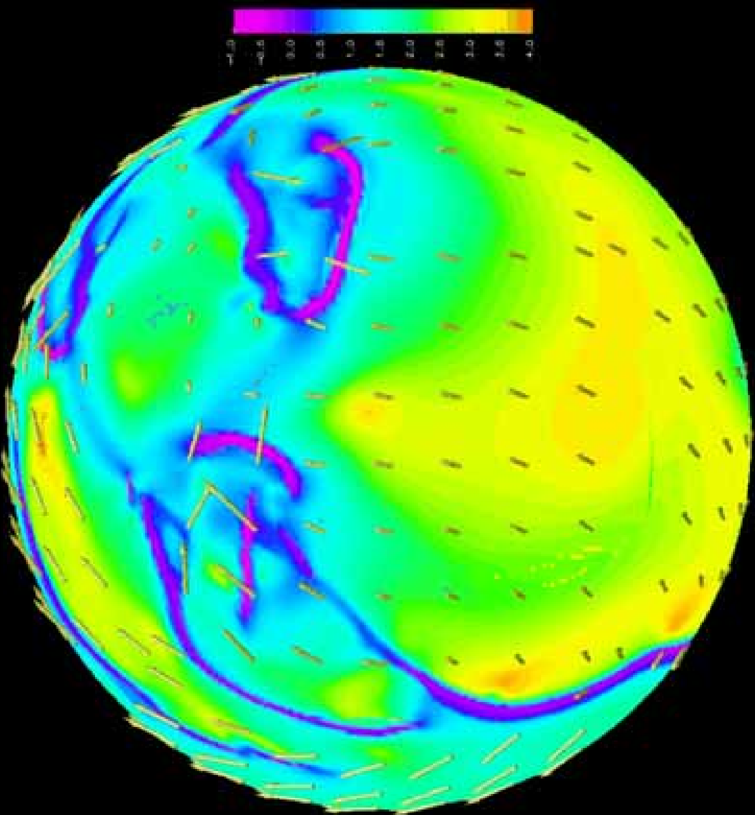


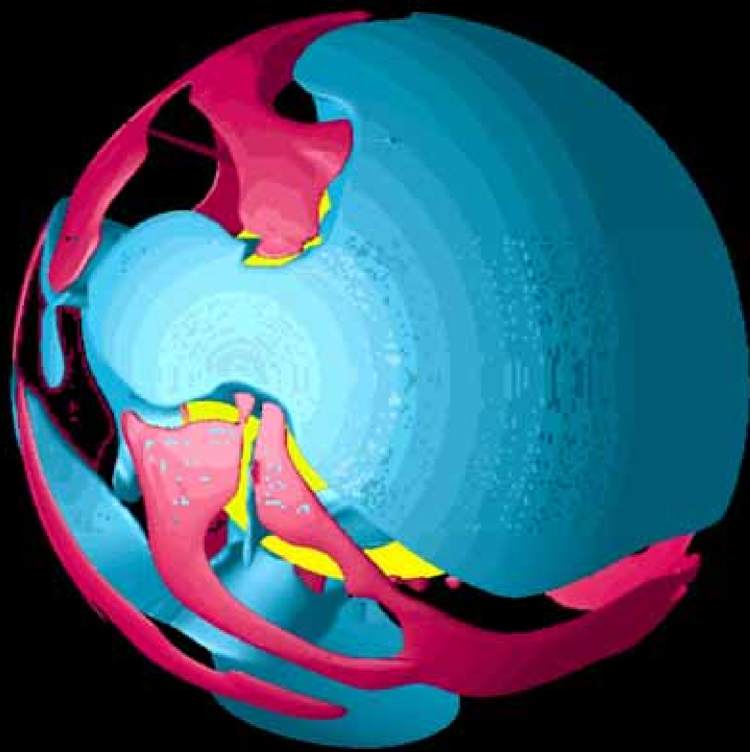
Stagnant lid mode



H. Van Heck

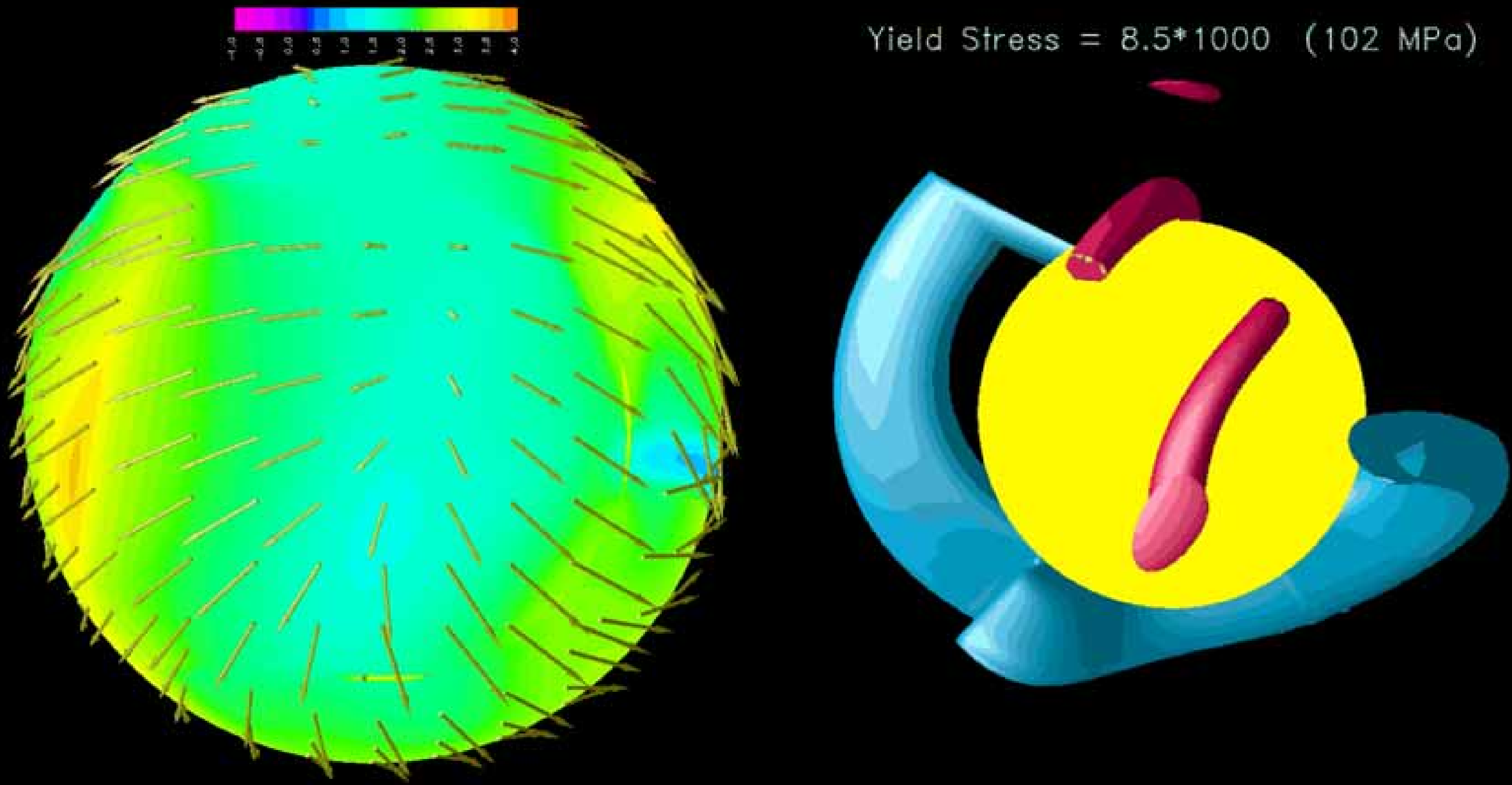
Mobile lid mode





H. Van Heck

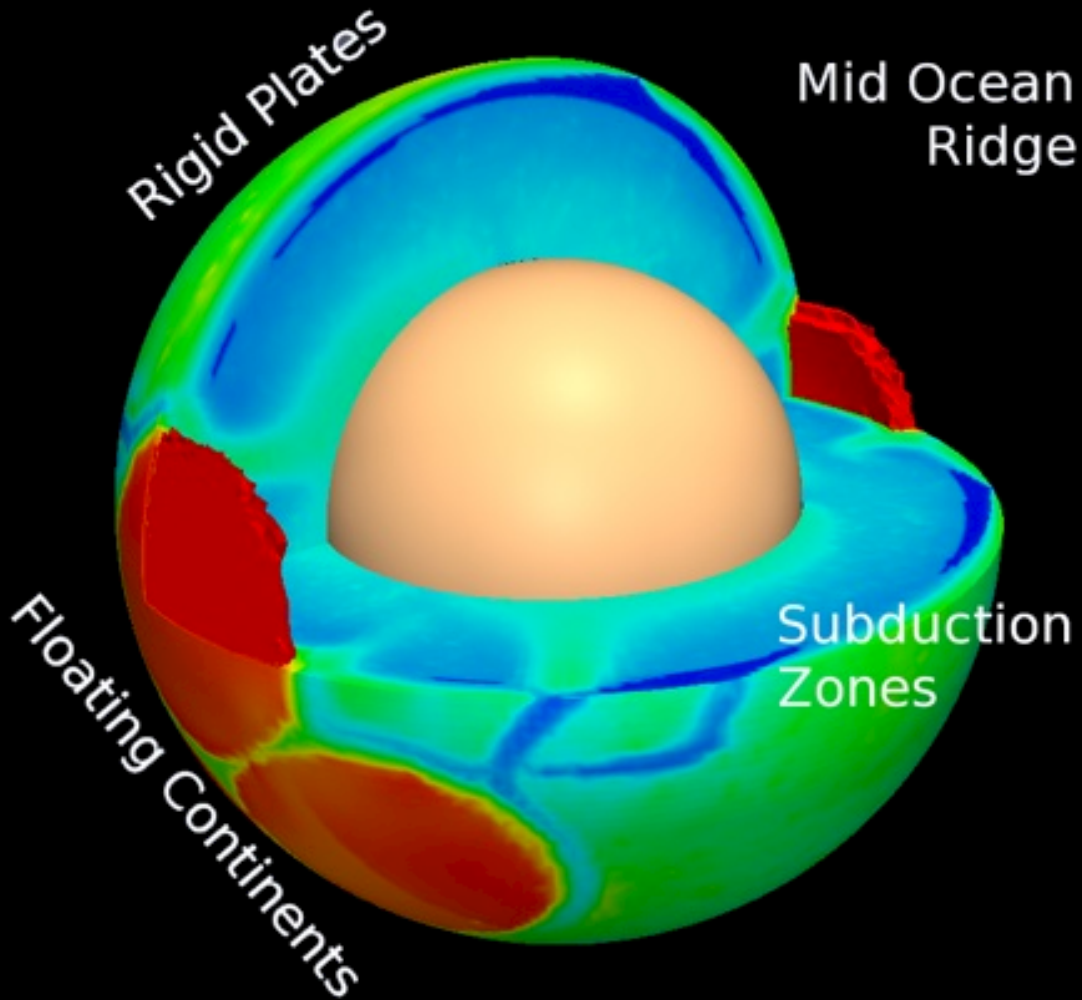
Mobile lid mode



H. Van Heck

Influence of continents on self-consistent plate tectonics?

Tobias Rolf



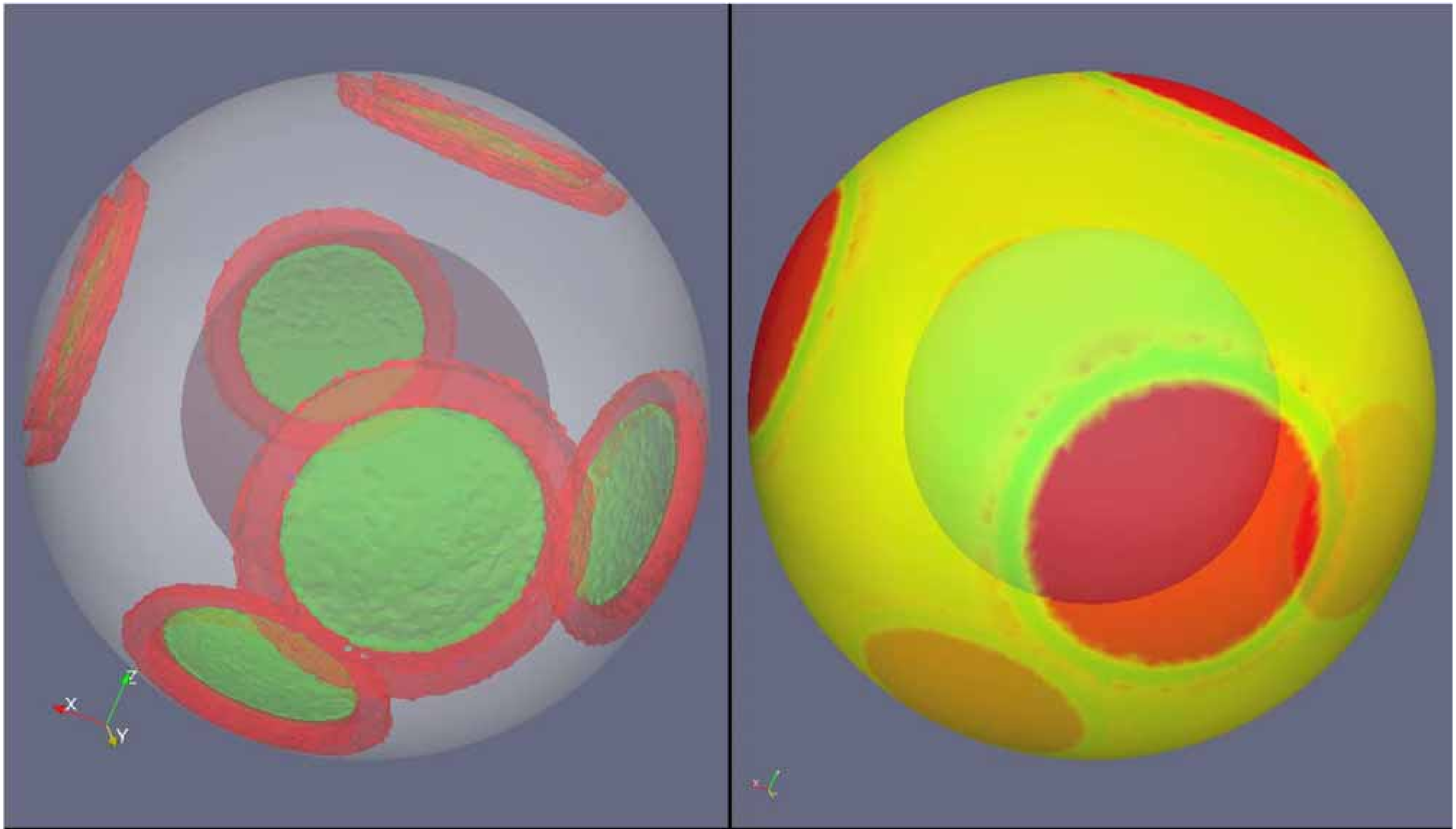
CRYSTAL2PLATE

CRYSTAL2PLATE

How does plate tectonics work: From crystal-scale processes to mantle convection with self-consistent plates

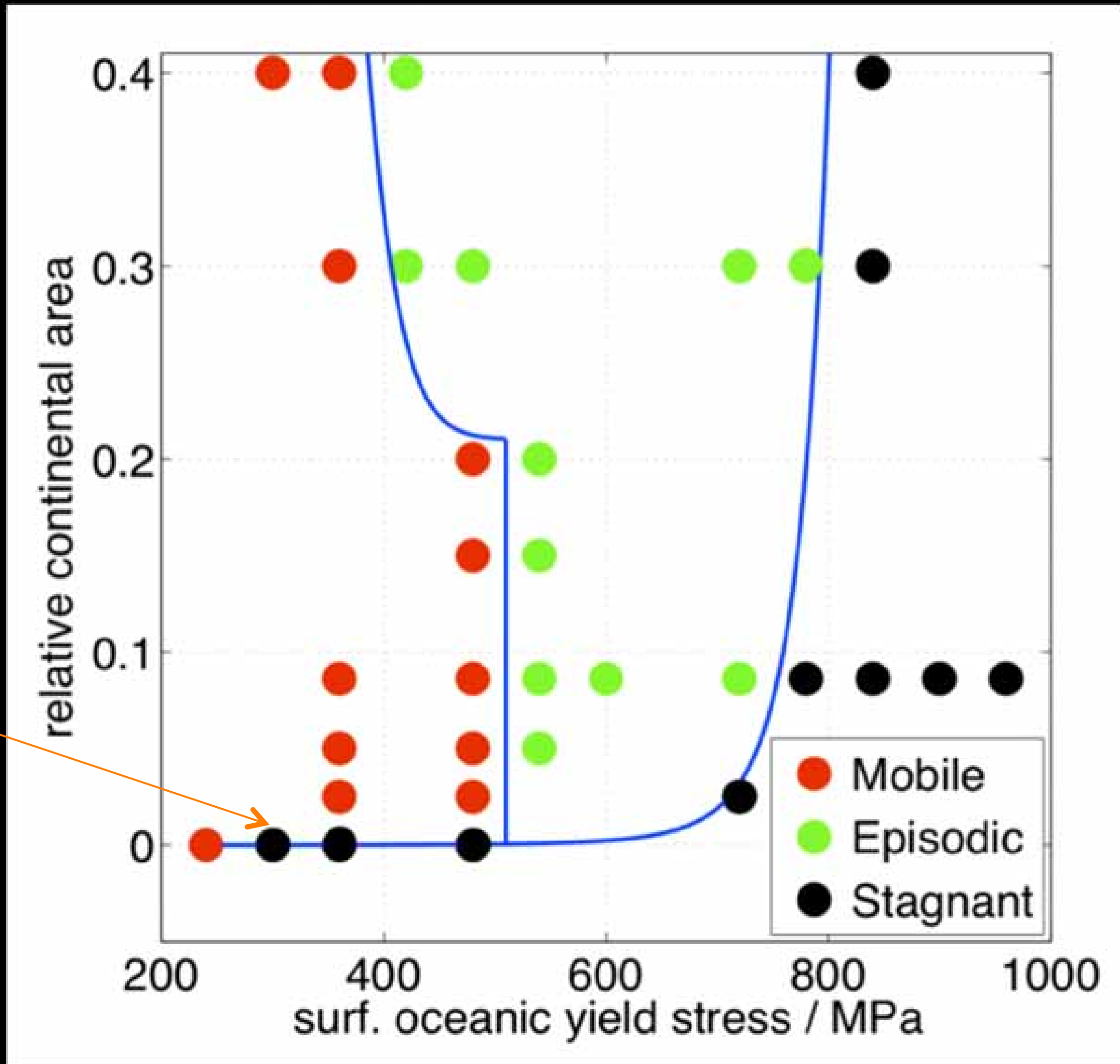
SEVENTH FRAMEWORK PROGRAMME

MARIE CURIE ACTIONS

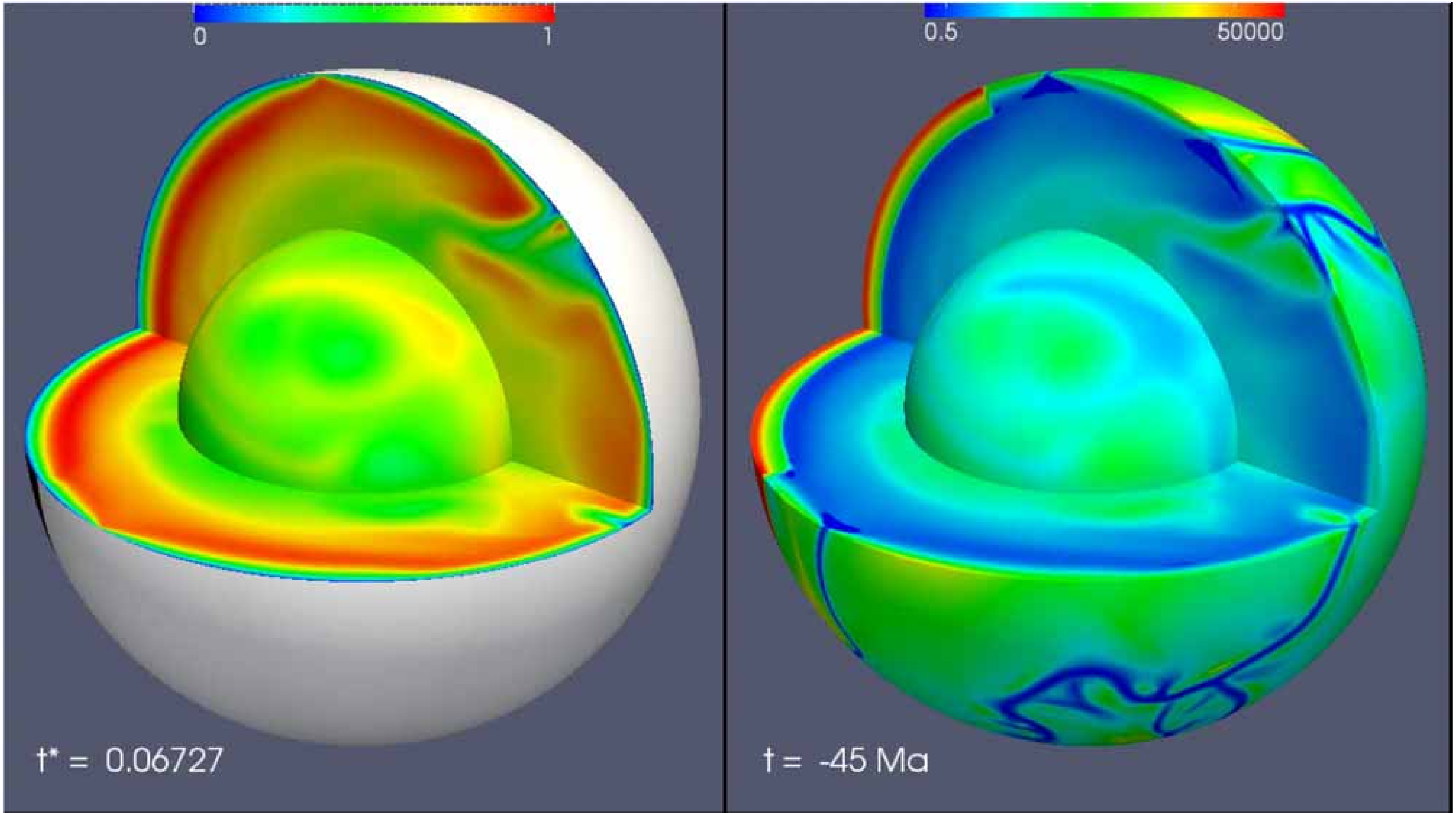

X
Z
Y

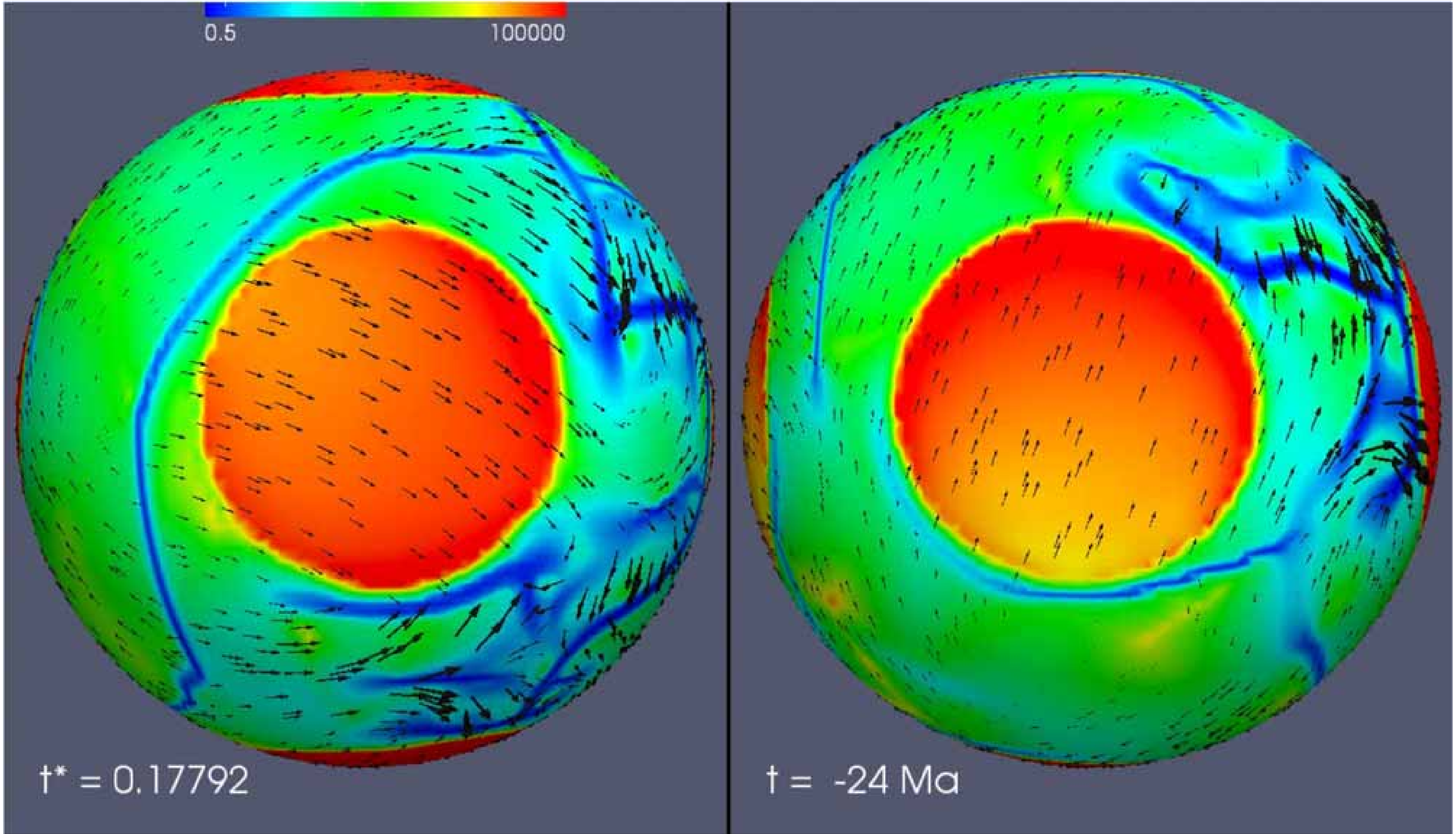
Continents help plate tectonics!

Presence of continent allows plate tectonics at higher yield stress



Rolf and Tackley, GRL 2011


0
1
t* = 0.06727
0.5
50000
t = -45 Ma

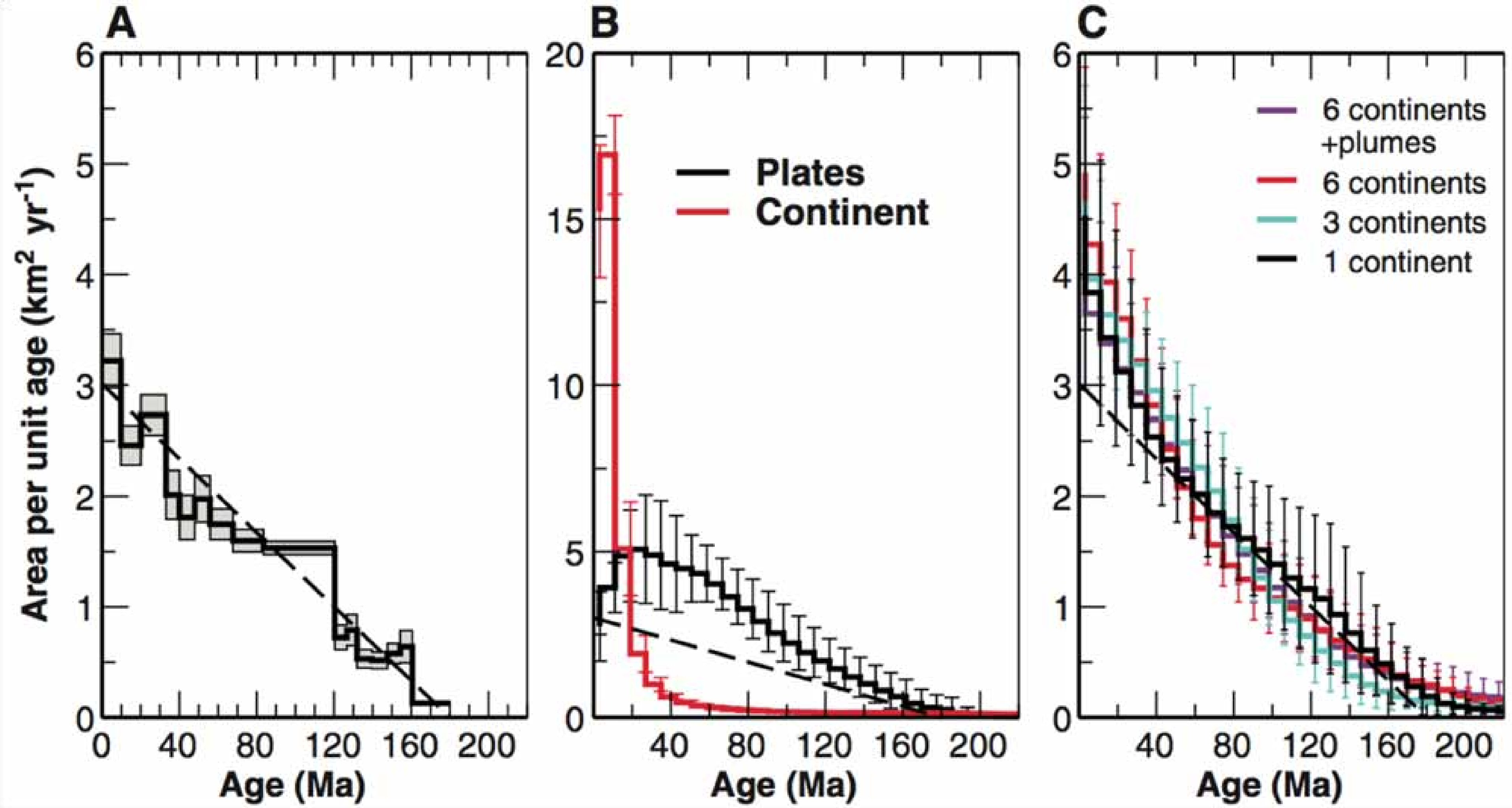

0.5
100000
t* = 0.17792
t = -24 Ma

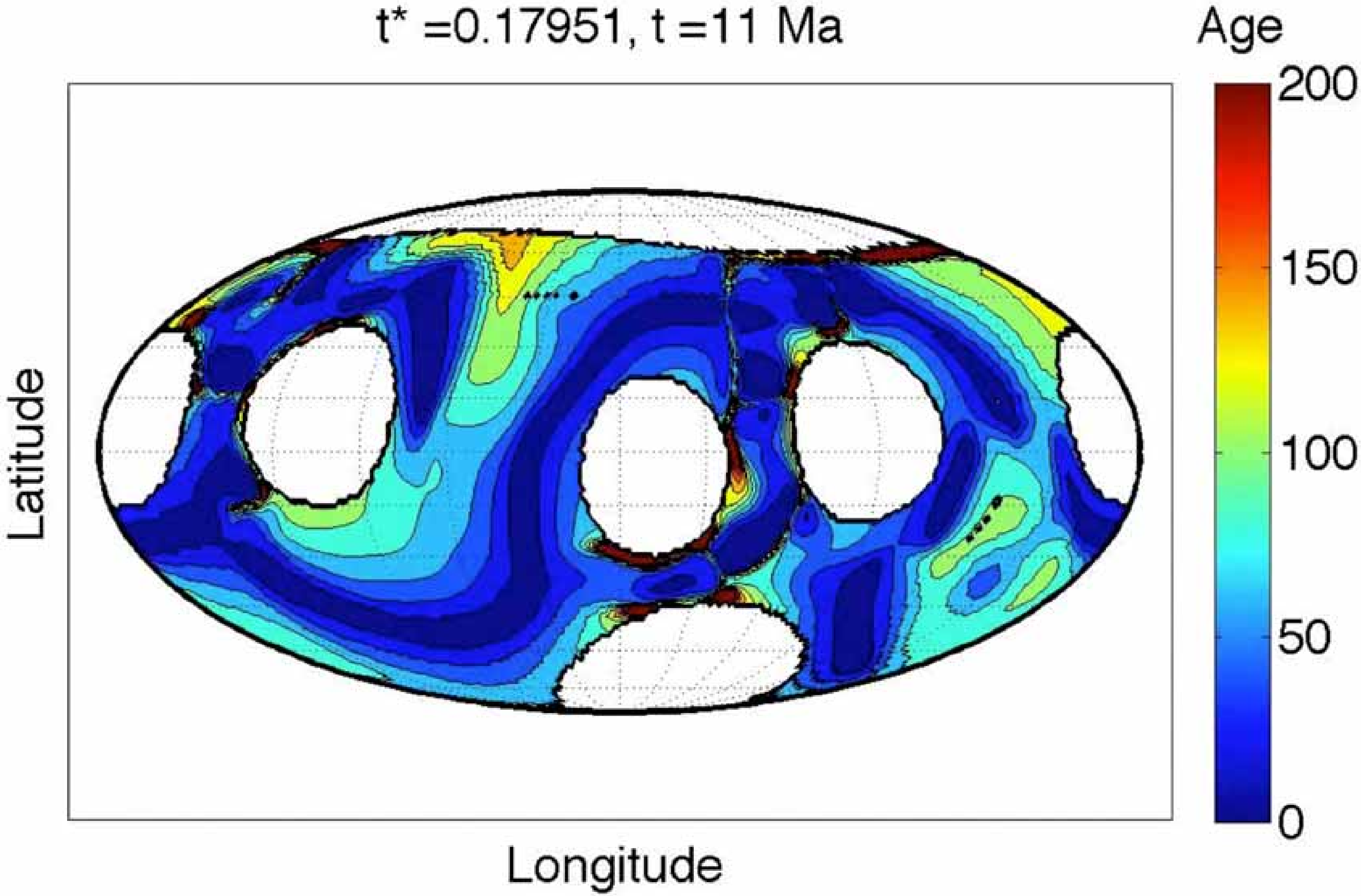
Dynamic Causes of the Relation Between Area and Age of the Ocean Floor

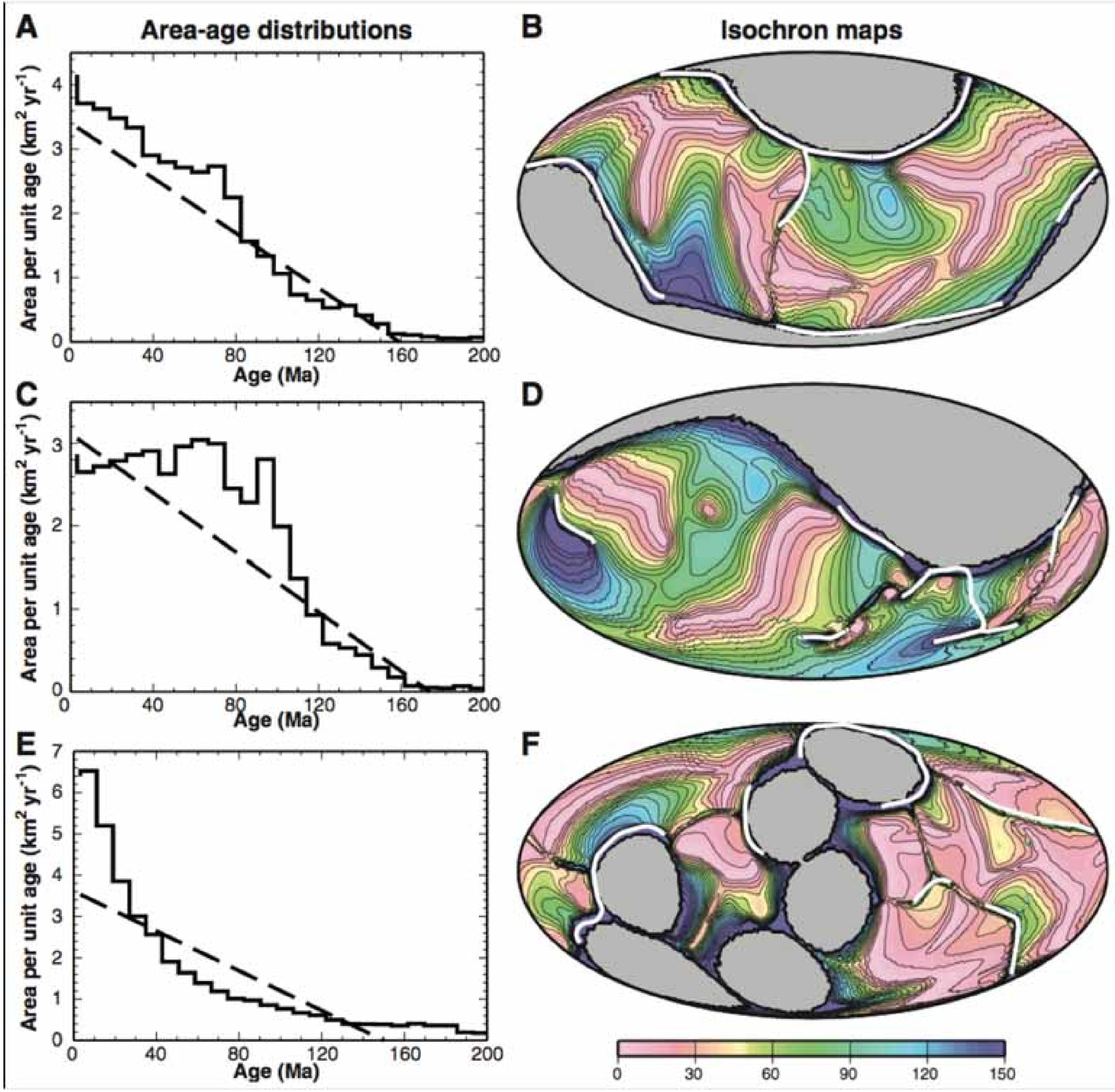
N. Coltice,[1,2]* T. Rolf,[3] P. J. Tackley,[3] S. Labrosse[1,2]

SCIENCE VOL 336 20 APRIL 2012

[redacted]

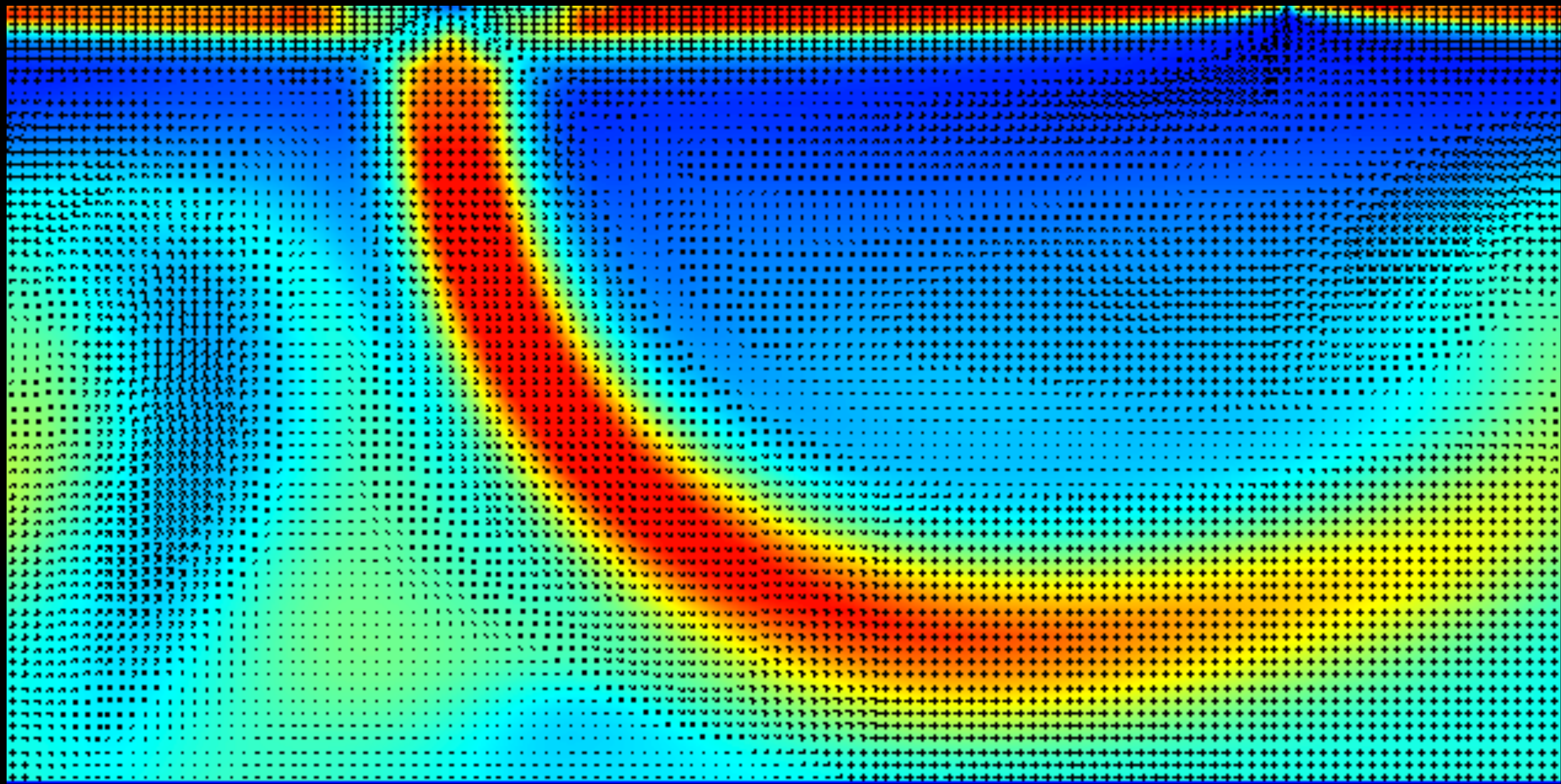



t* =0.17951, t =11 Ma
Age
200
150
100
50
0
Latitude
Longitude



Distribution shape varies with time

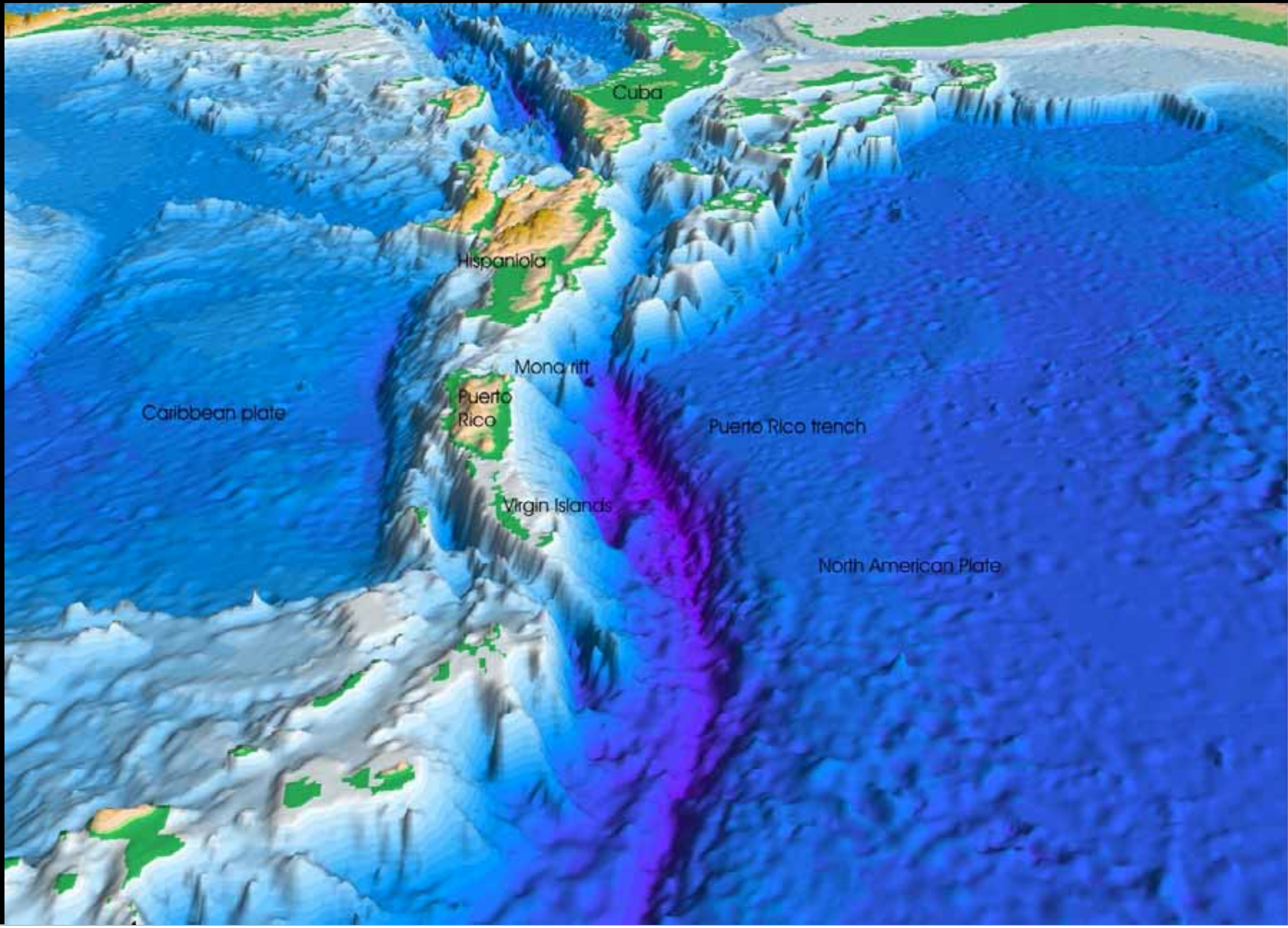
A problem: 2-sided subduction!



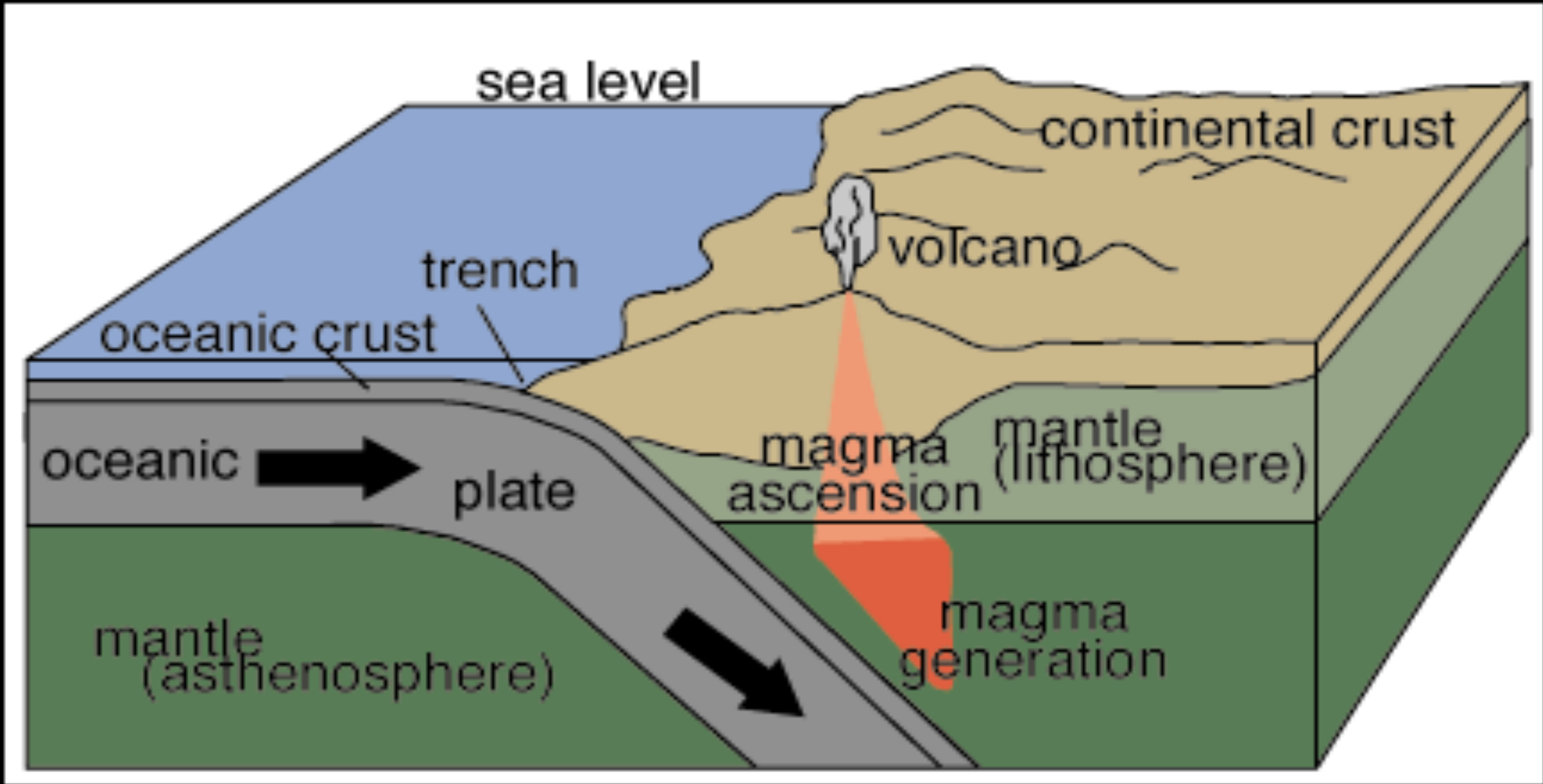
Mantle convection codes assume a free-slip upper boundary: surface is FLAT

- Zero shear stress but finite normal stress, proportional to what the topography would be if allowed.
- But this may create unnatural geometries at subduction zones….

Real subduction zone: NOT FLAT



Trench due to bending

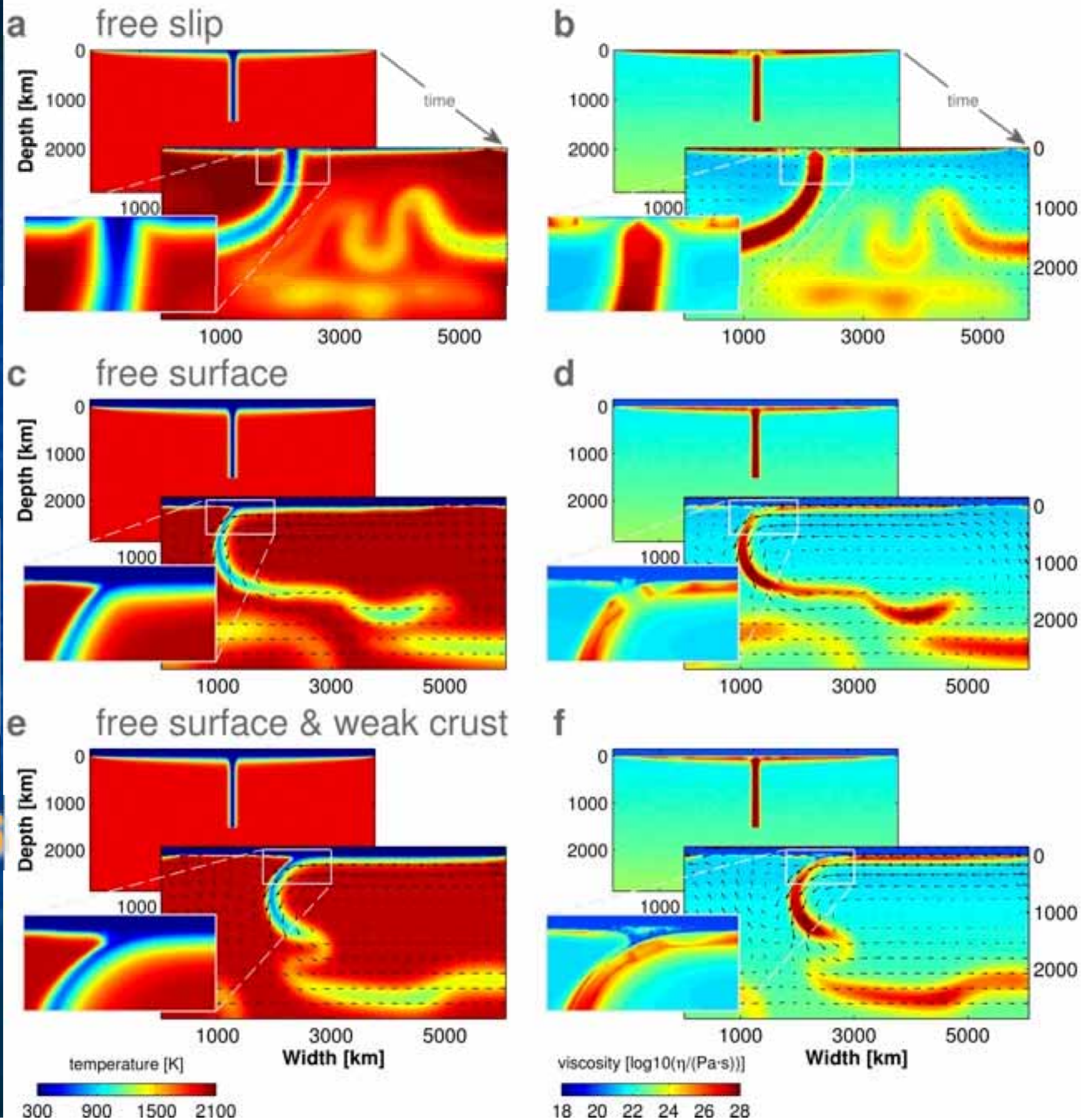


Fabio Crameri

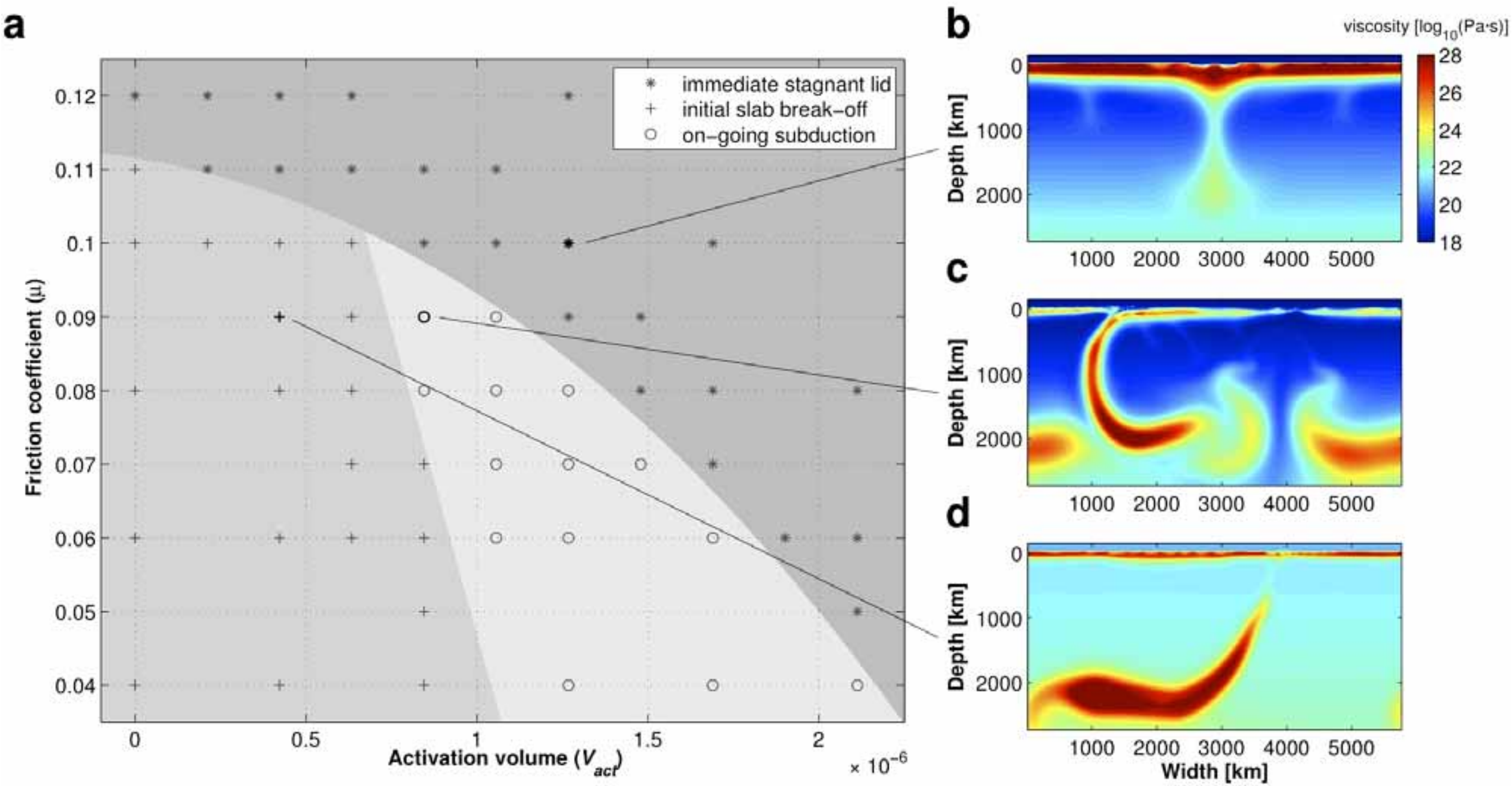
Taras Gerya

Boris Kaus

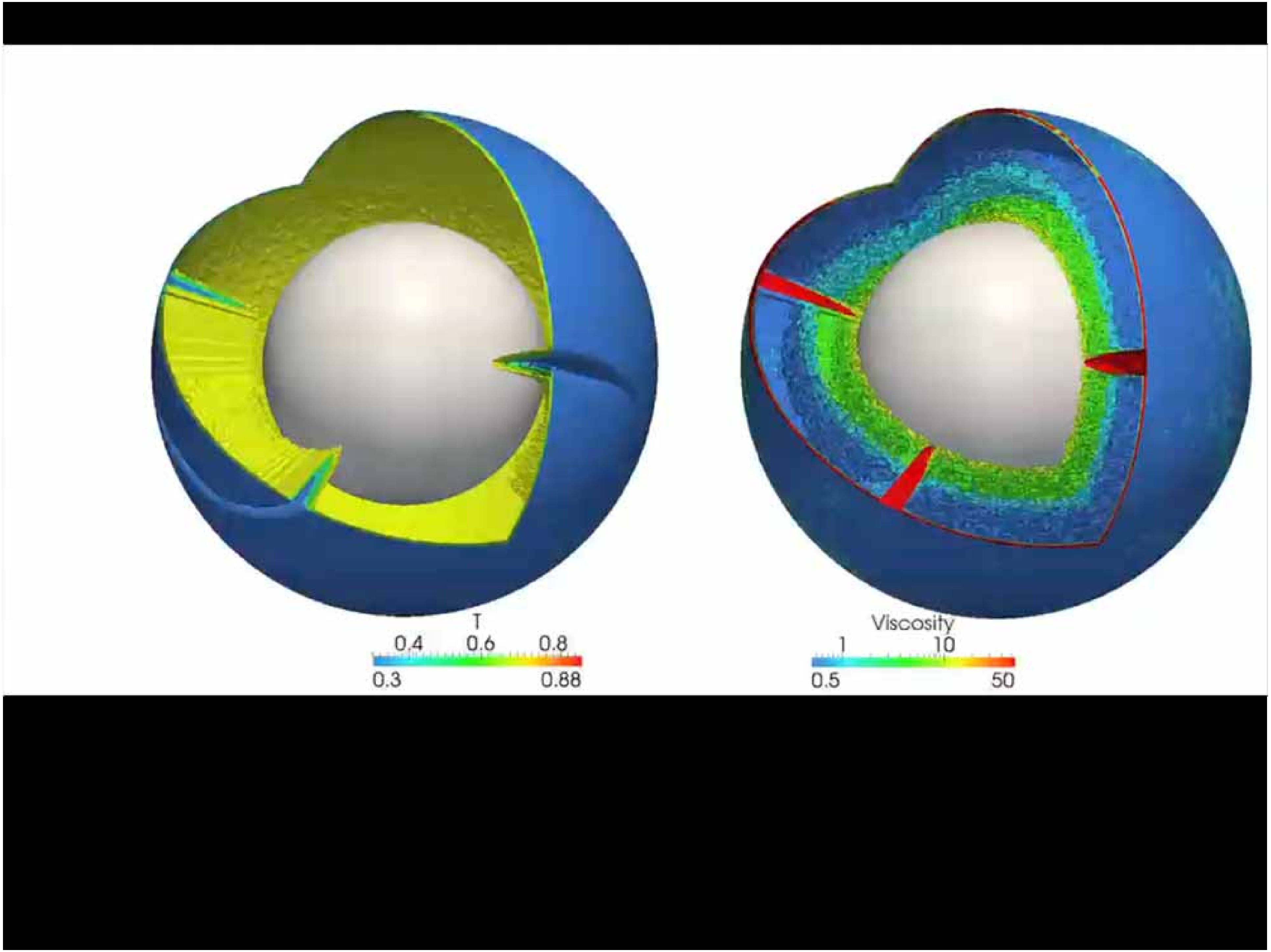
Geophys. Res. Lett. 2012



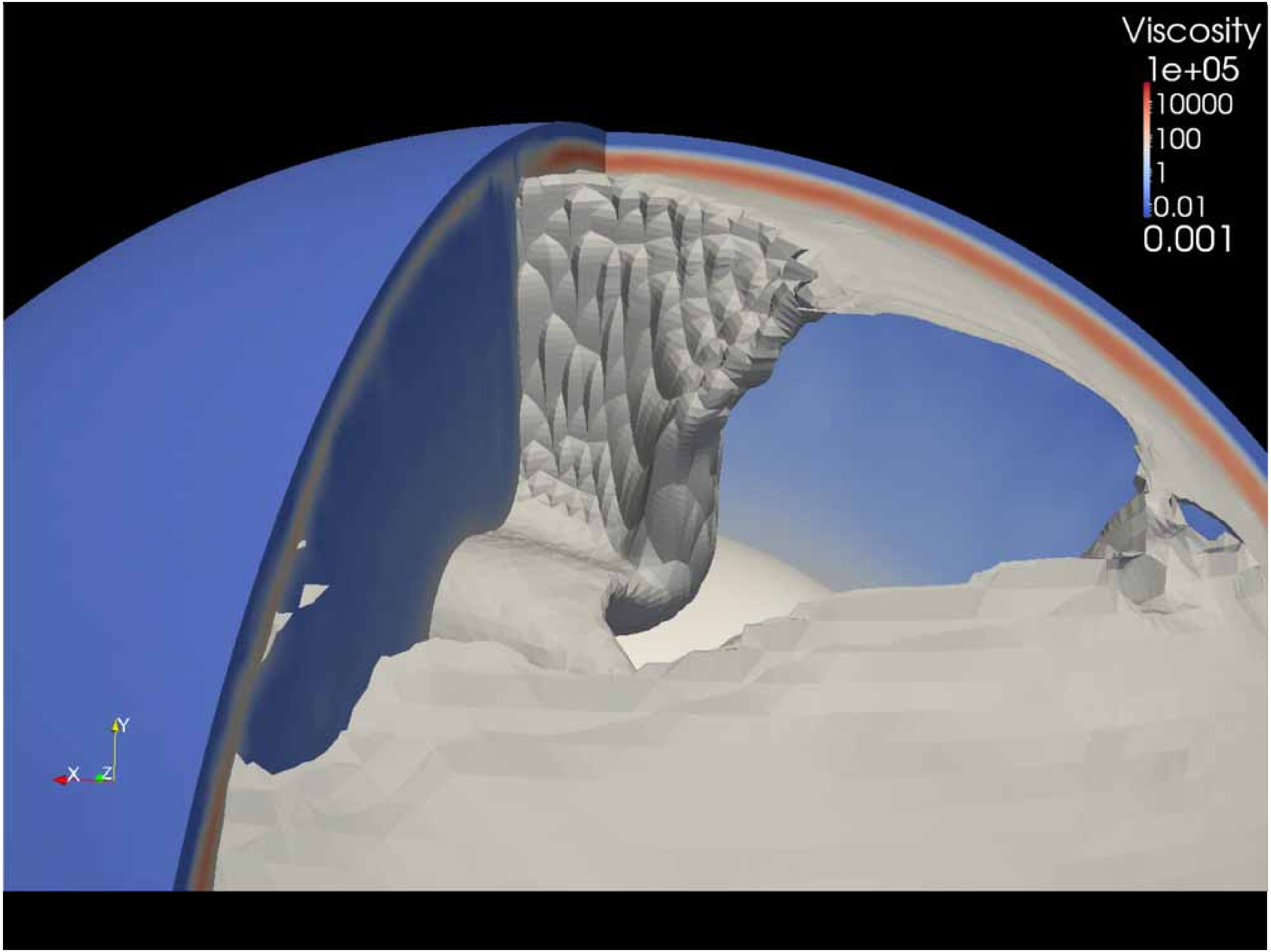
3 regimes



Depends on friction coefficient AND increase of viscosity with depth



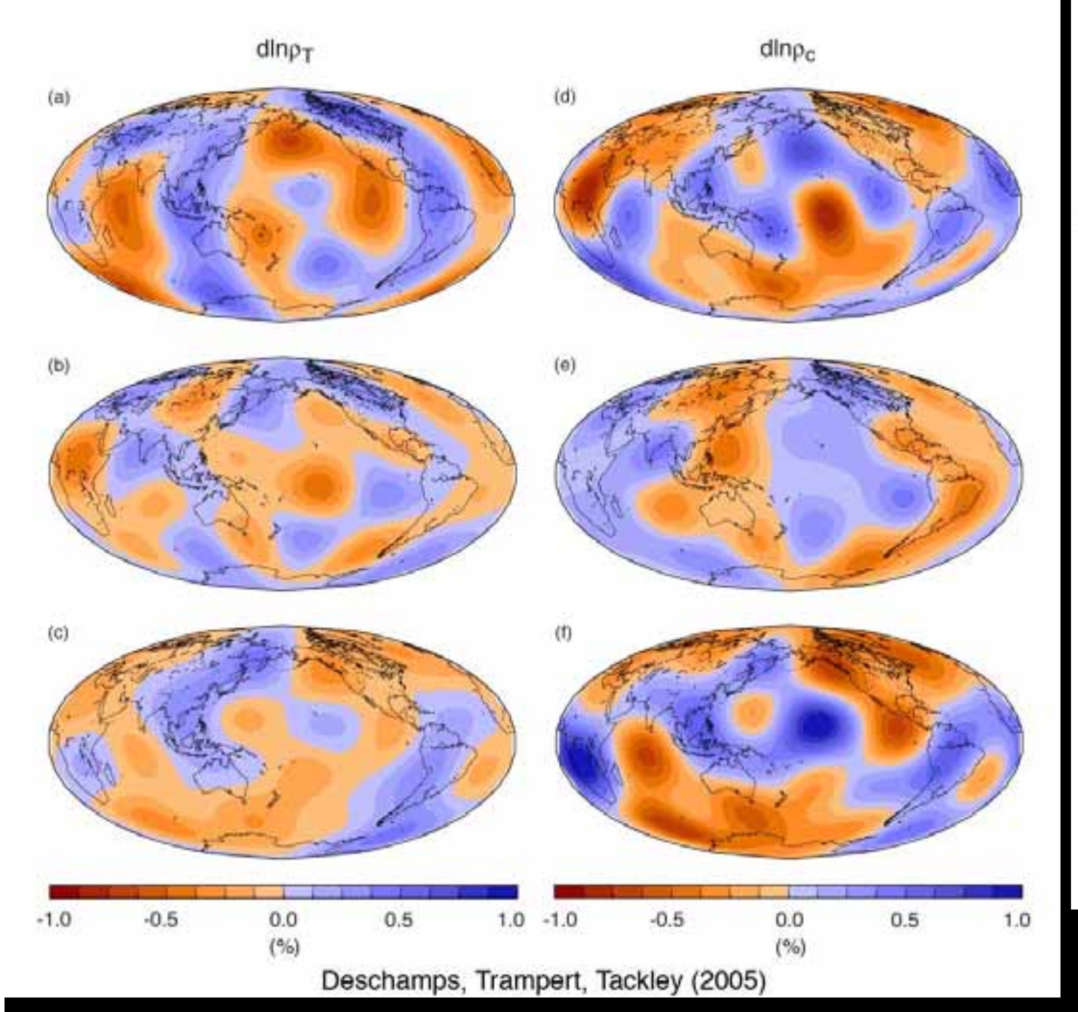
T
0.4
0.6
0.8
0.3
0.88
Viscosity
1
10
0.5
50



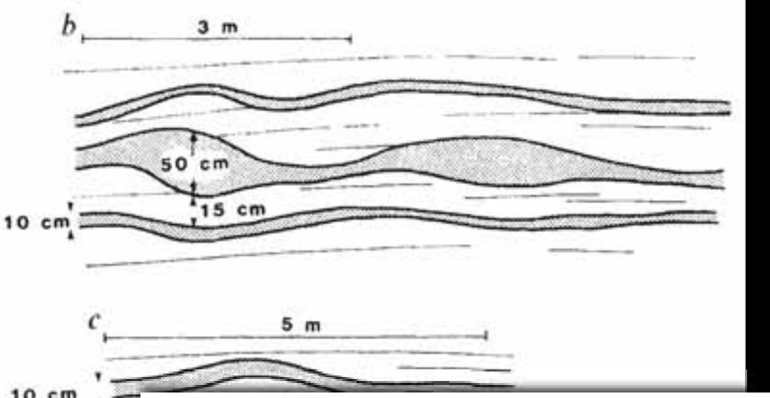
Viscosity
1e+05
10000
100
1
0.01
0.001
Y
X
Z

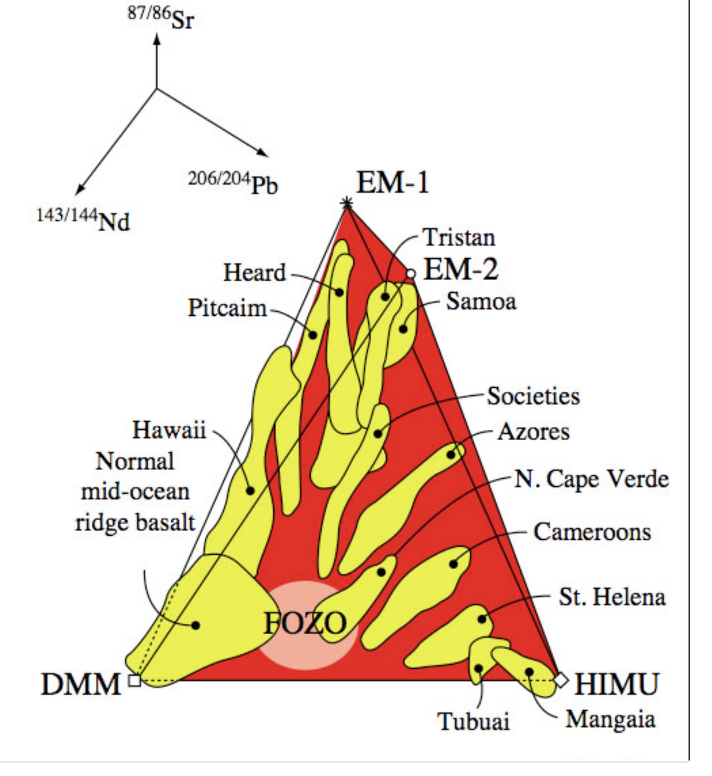
Compositional variations exist at all scales!

Large scale

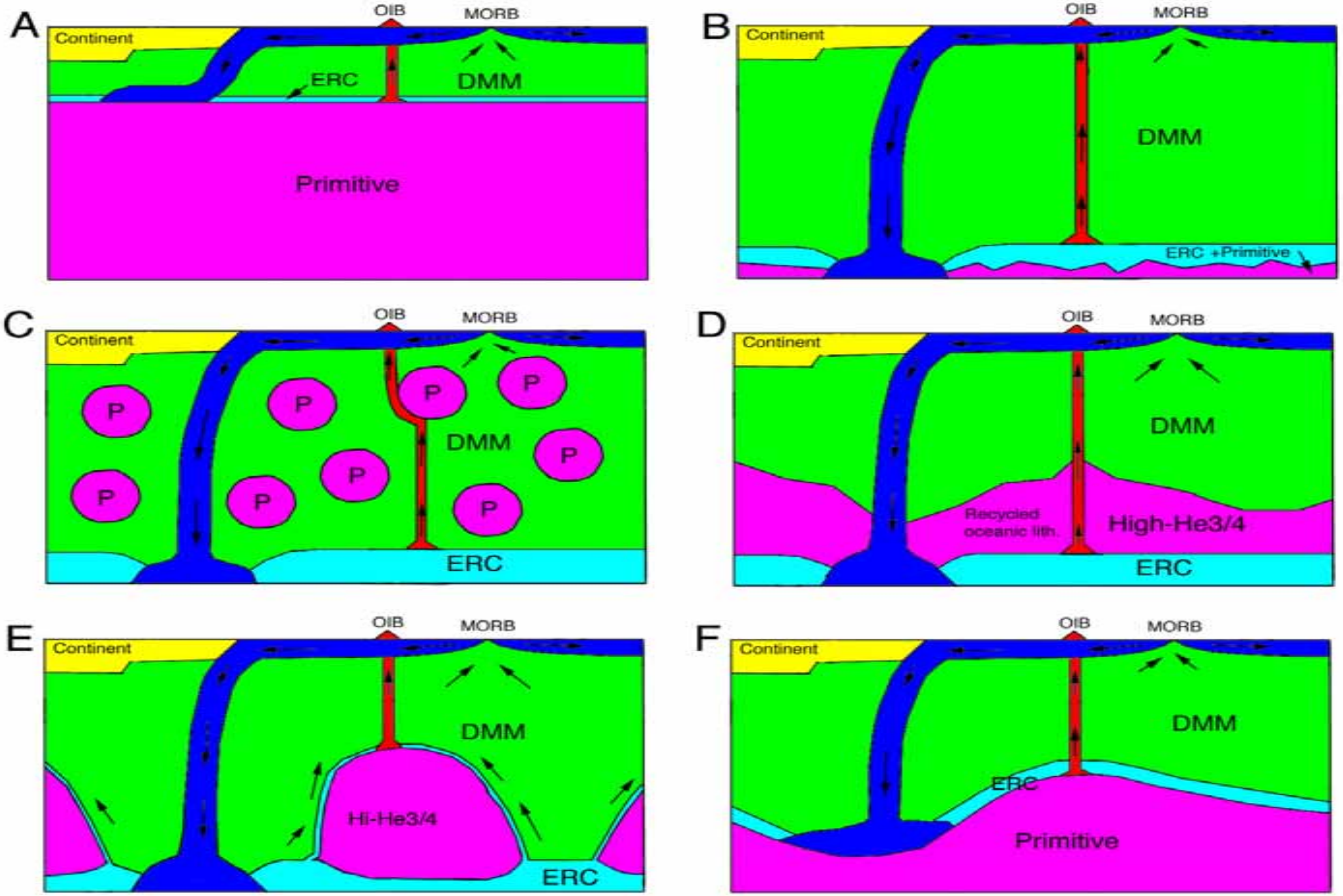


Small scale





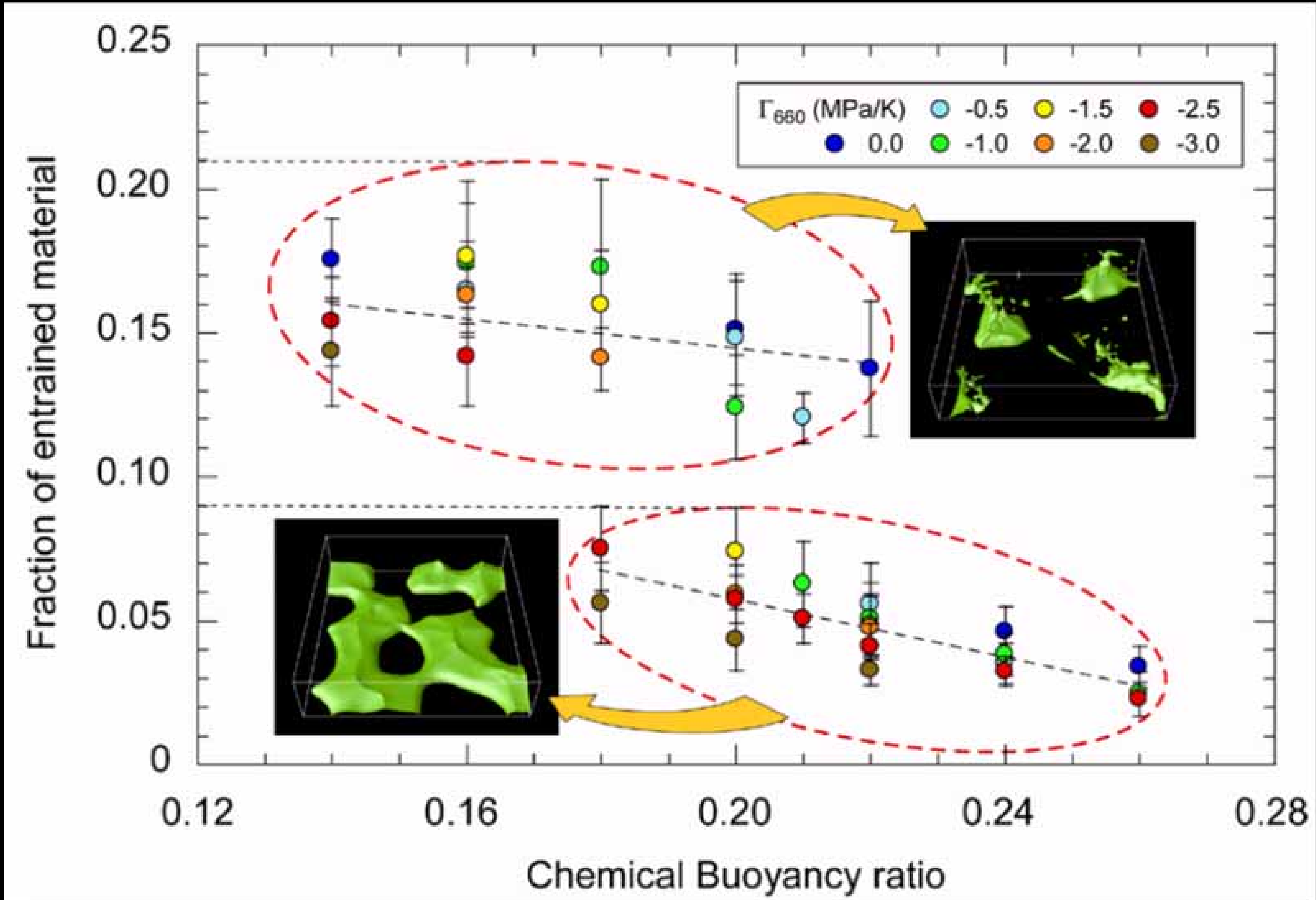
Geochemical mantle: Old cartoons (2000)



from Tackley, Science, 2000: Figure 2

Entrainment of primordial dense piles: can explain high 3He/4He of ocean island basalts

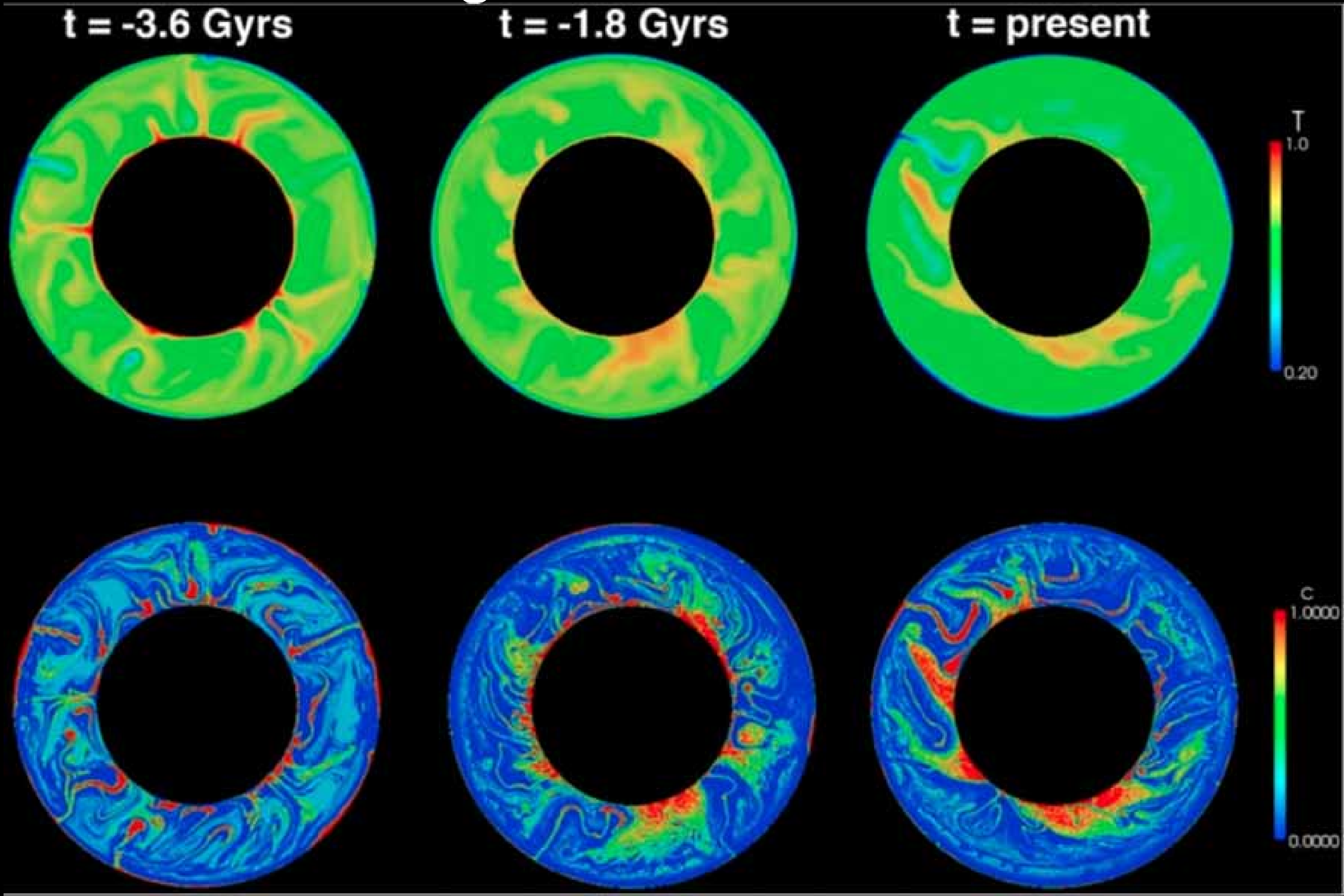
Deschamps, Kaminski & Tackley, Nature Geoscience 2011



Calculations of Earth's mantle thermo-chemical evolution over 4.5 Gyr

- Include melting->crustal production,
  - viscosity dependent on T, d, and stress,
  - self-consistent plate tectonics,
  - decaying radiogenic elements and cooling core,
  - compressible anelastic approximation
- Many papers by Takashi Nakagawa & me

Long-term evolution



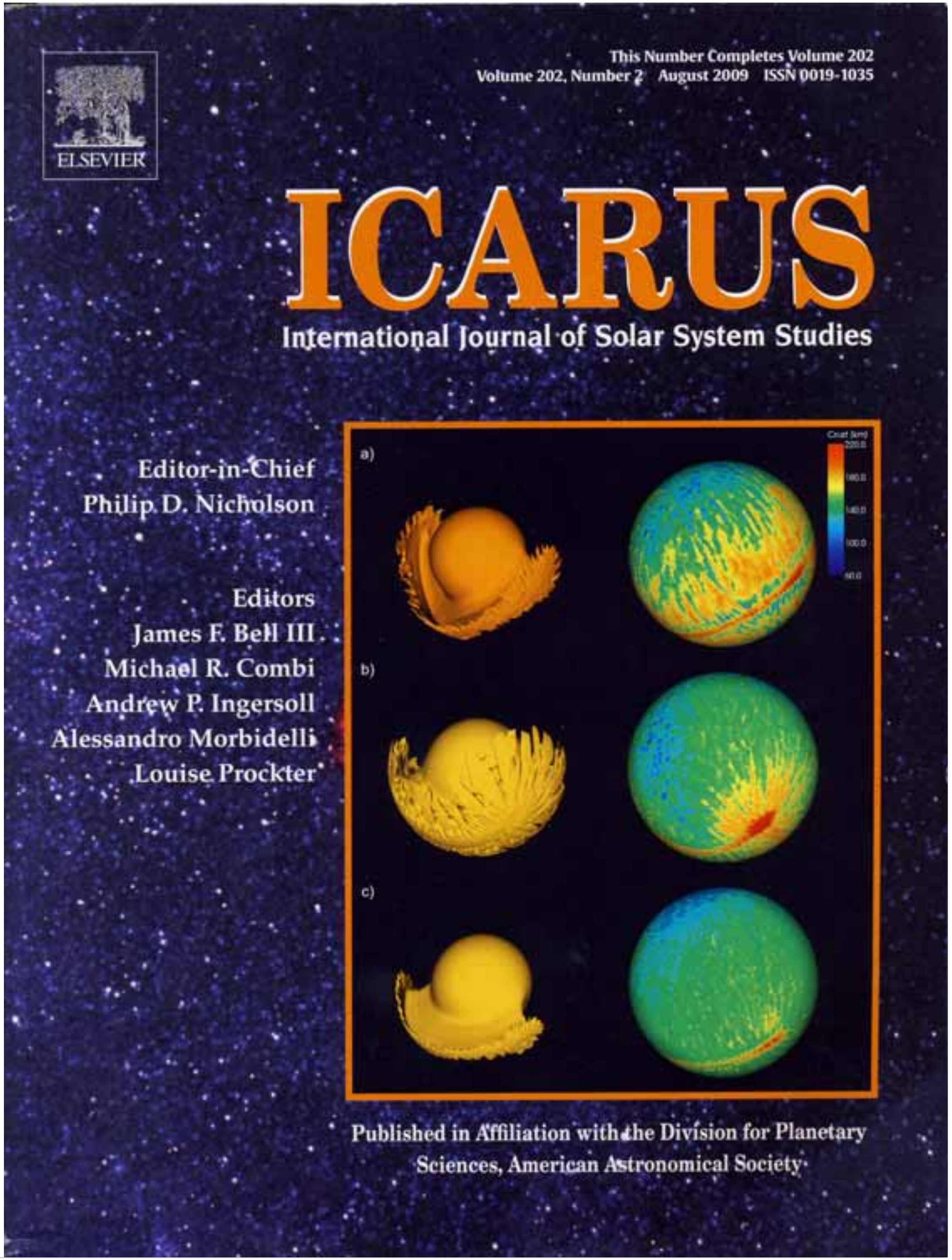
Nakagawa & Tackley 2010 Gcubed

MARS: Modelling mantle dynamics and crustal formation

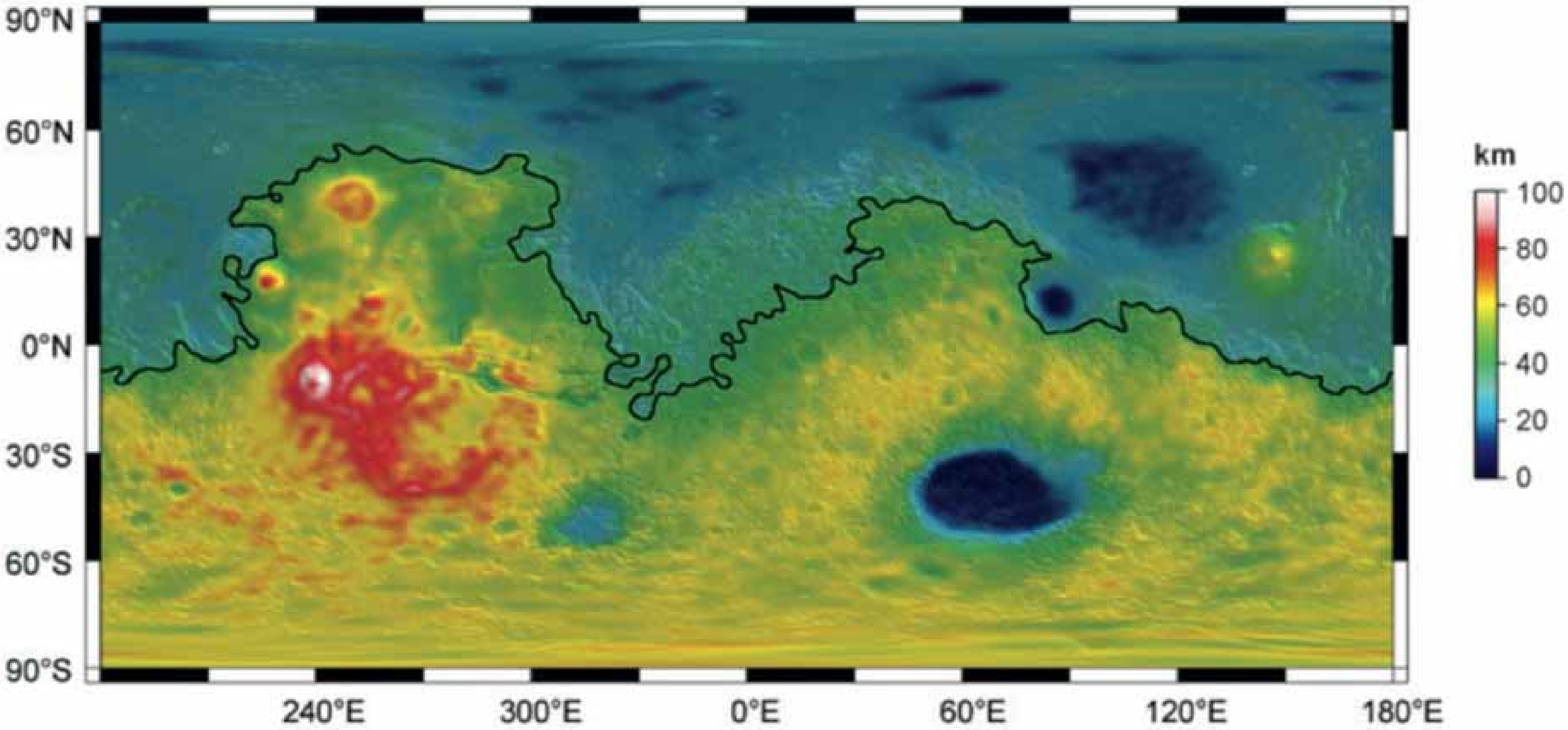
Tobias Keller & Paul J. Tackley

ETH Zürich, Geophysical Fluid Dynamics

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich



The crustal dichotomy



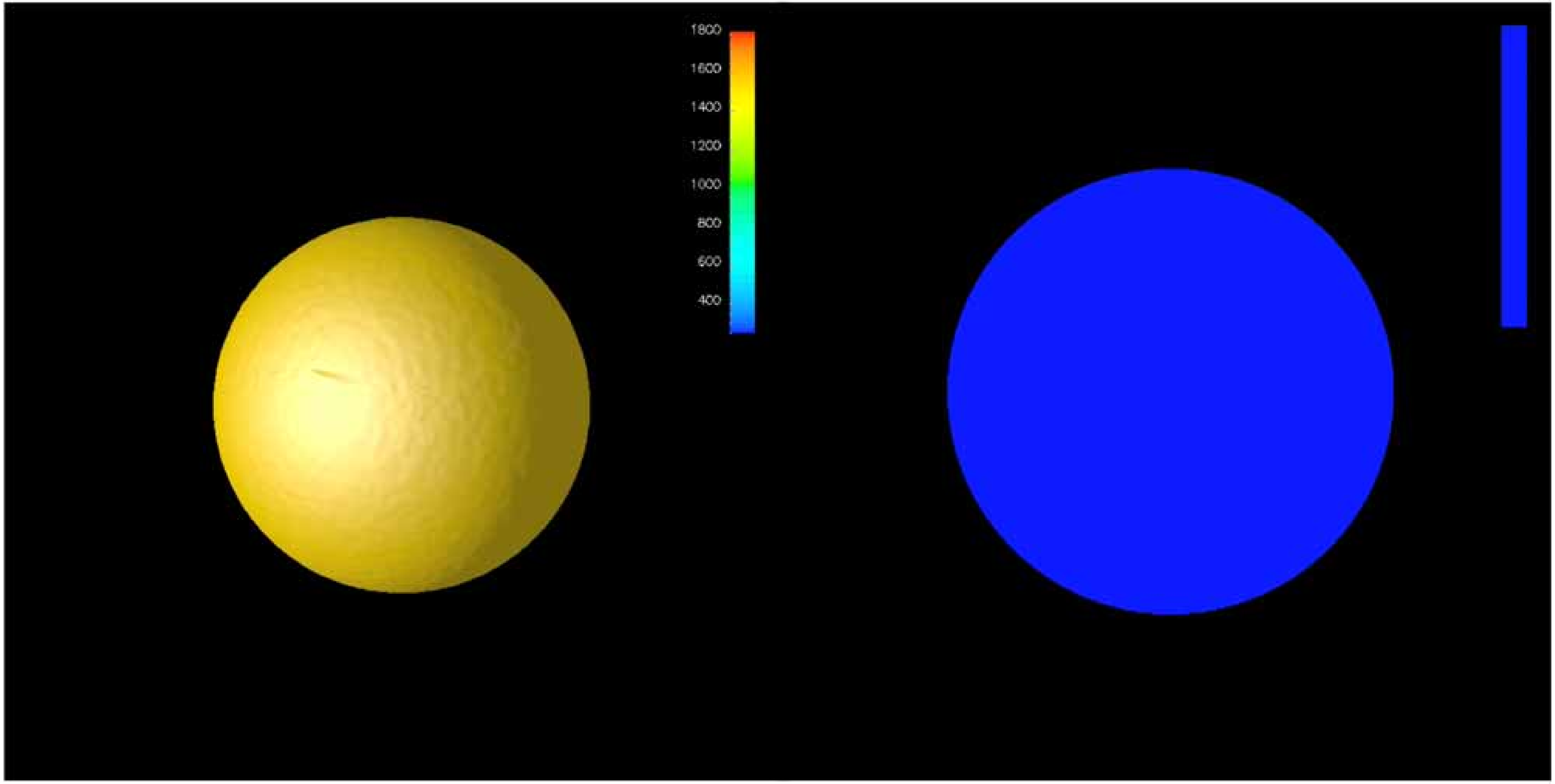
Causes: **Extrinsic** (impacts) or **intrinsic** (degree 1 mantle convection)?

MOLA data: Zuber (2001), Watters et. al (2007)

Results

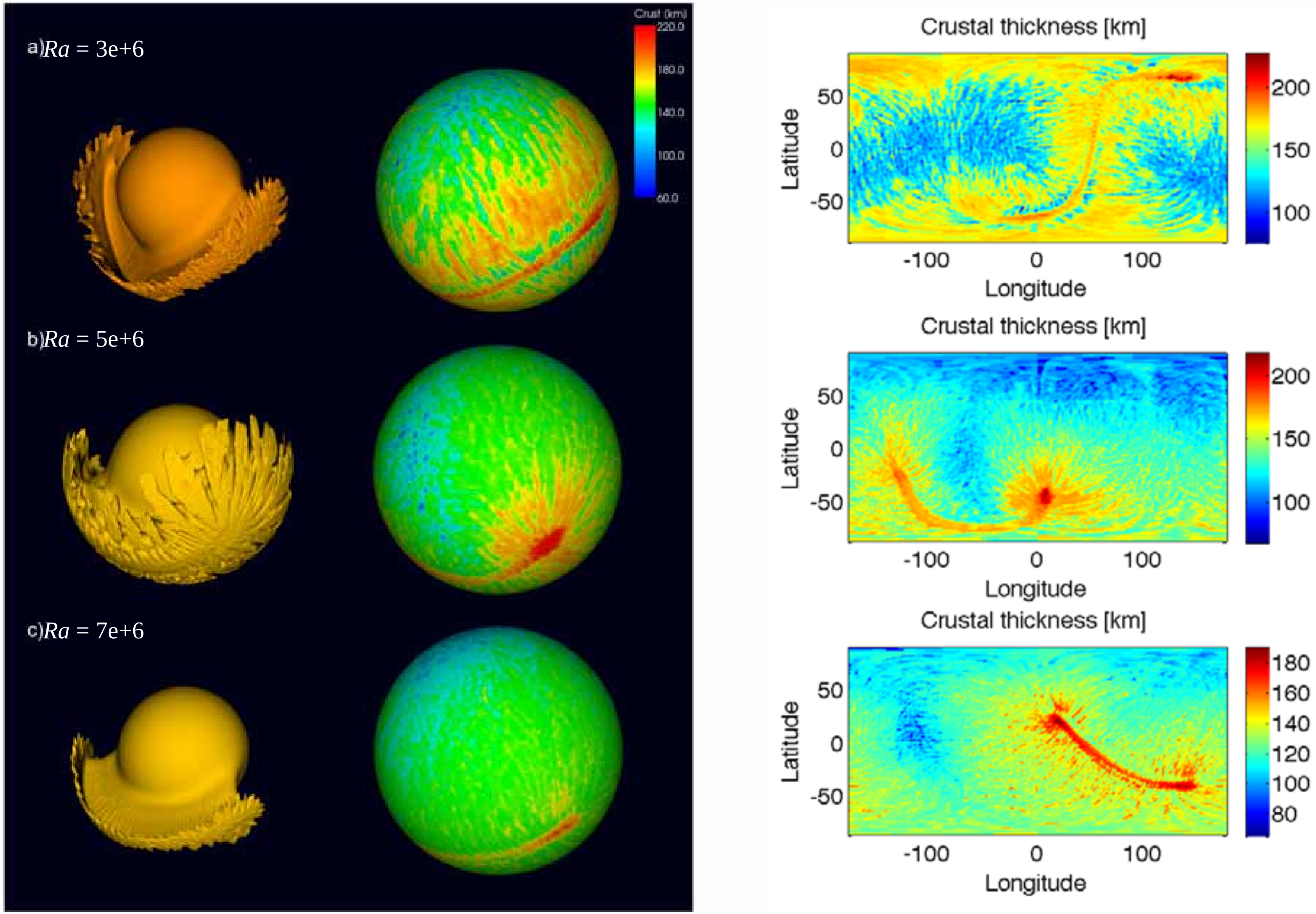
Temperature [K]

Crustal thickness [km]

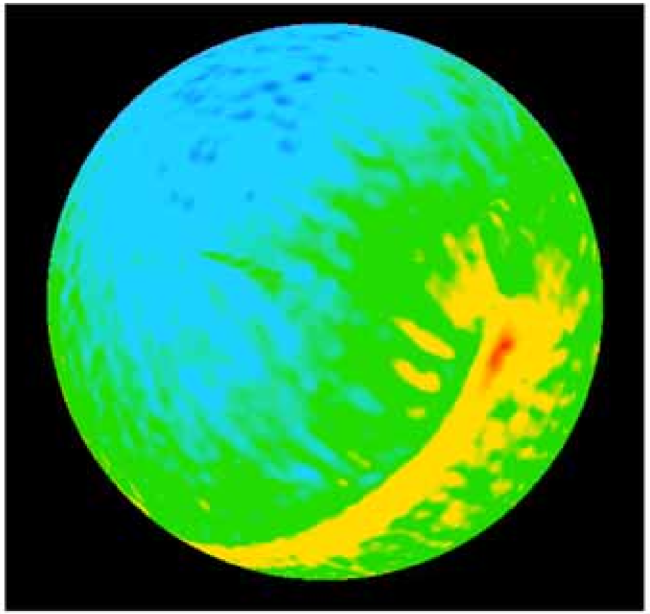


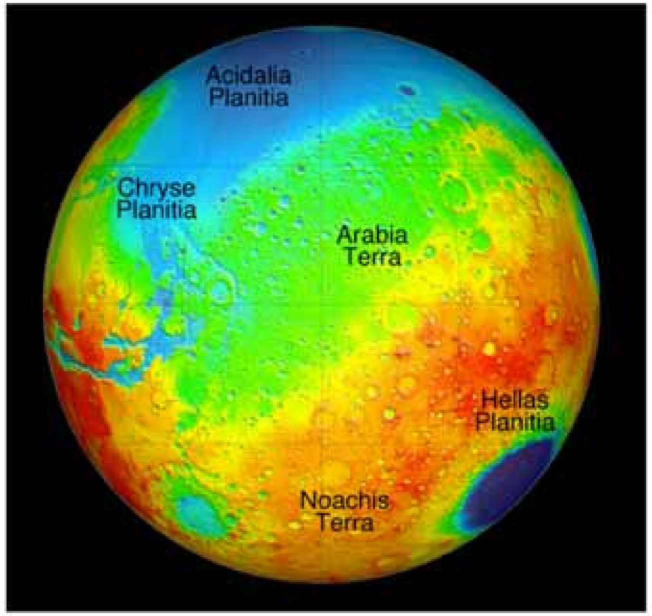
Ra = 7.0 e+6

Results at time = 1.0 Gyr



Interpretation





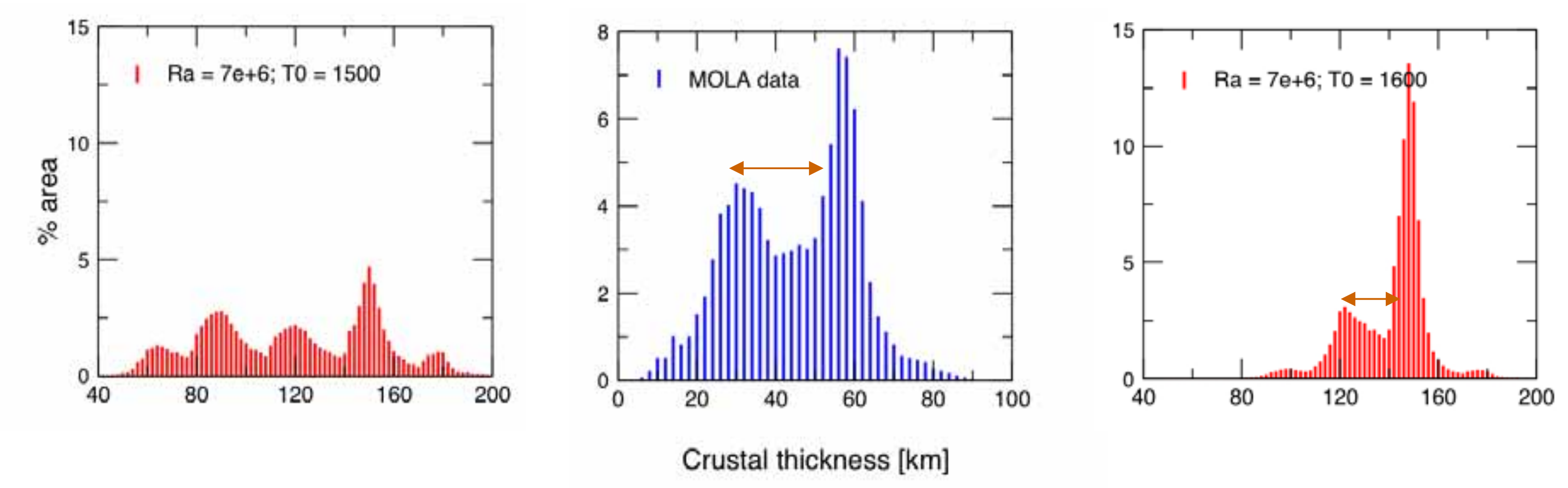
Striking first-order similarity!

Discussion

Crustal thickness distribution histograms

- two peaks for northern plains and southern highlands

bad fit Mars data best fit

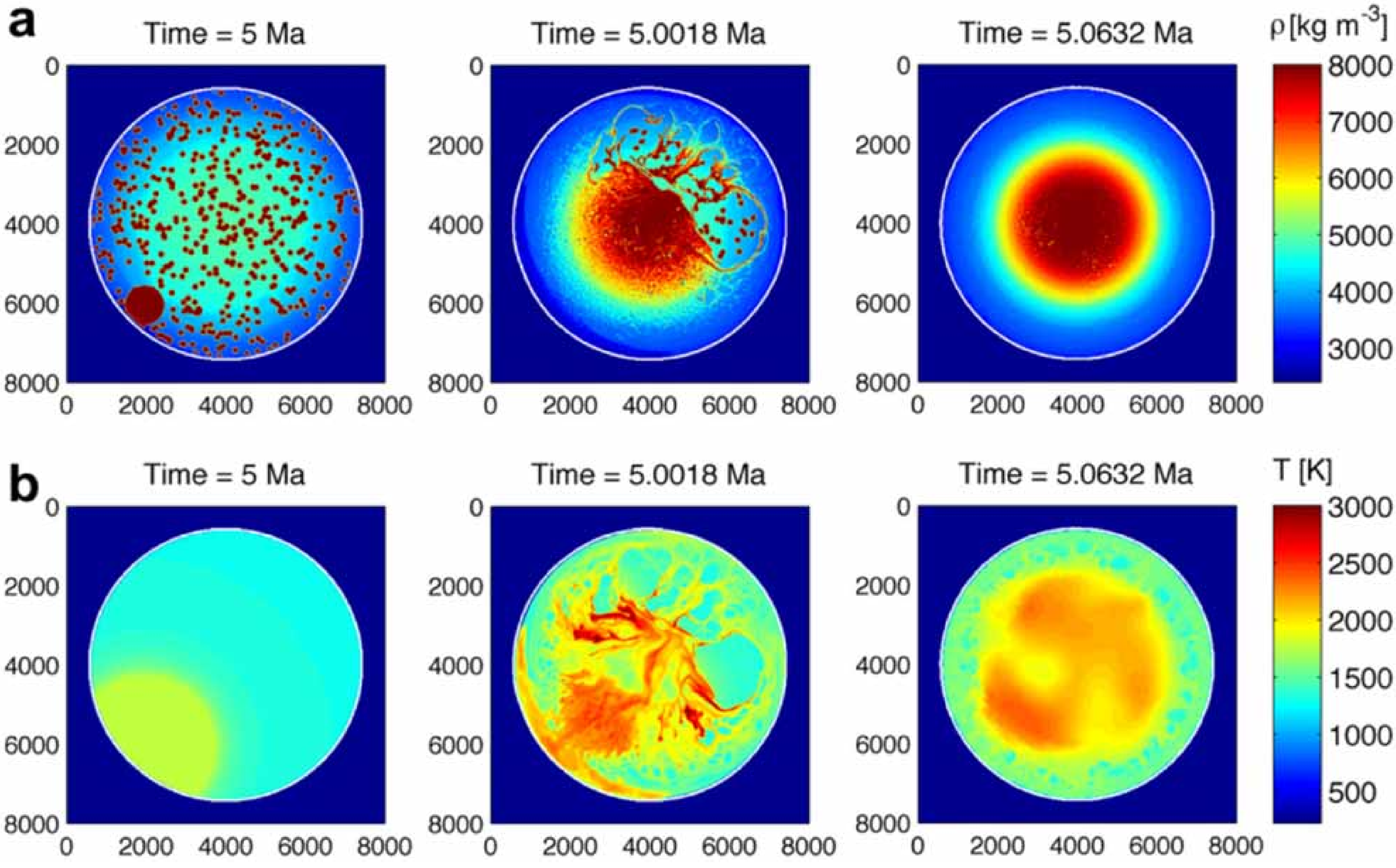


N-S difference = 26 km

PROBLEM: Takes 100s Myr to form – probably too slow

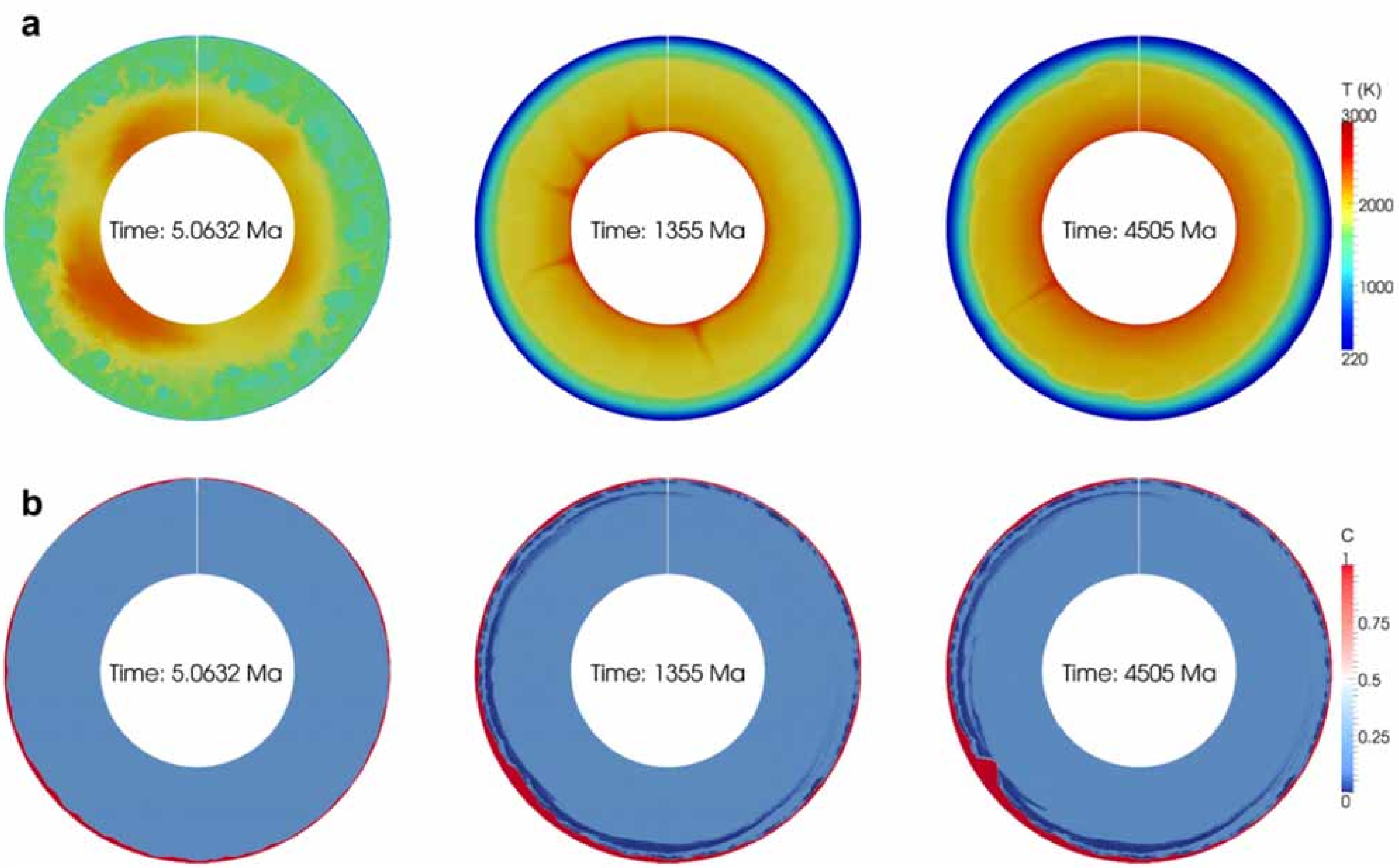
MOLA data from Watters et al. (2007)

Impact -> higher T -> more melting -> thicker crust



Golabek, Keller et al., 2011

Subsequent evolution



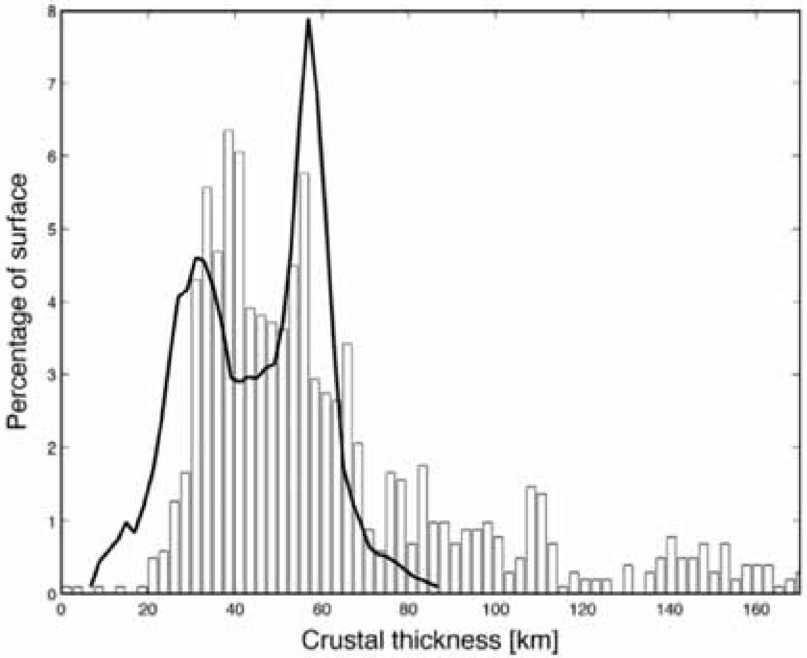
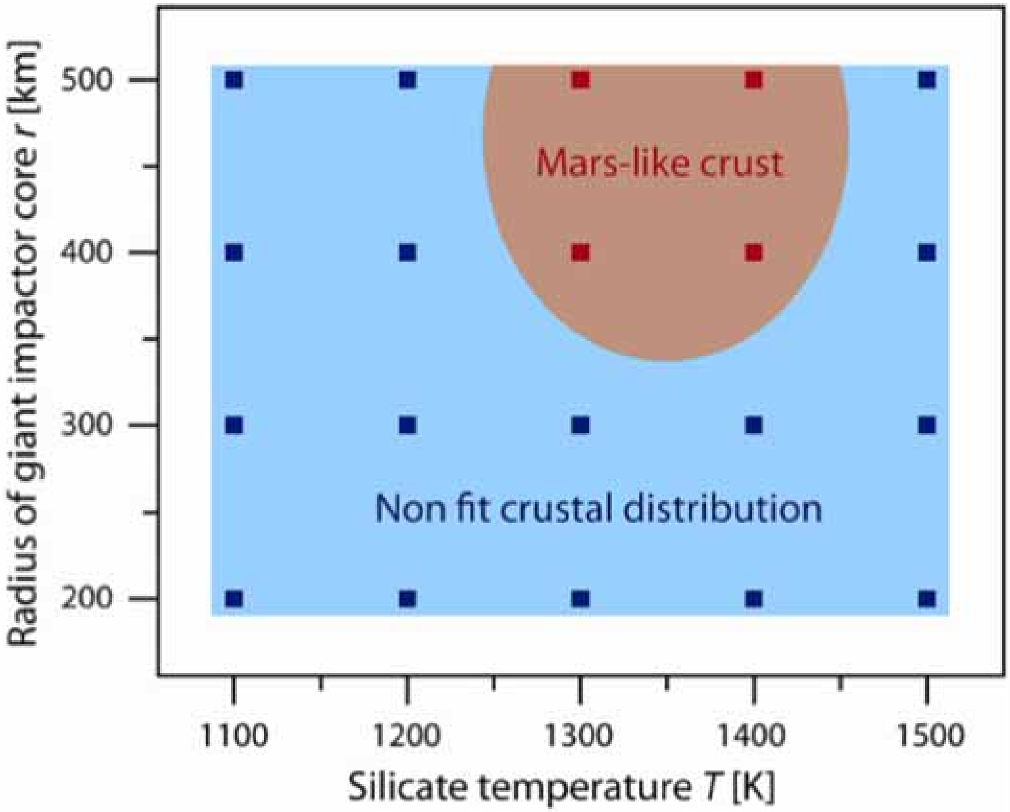
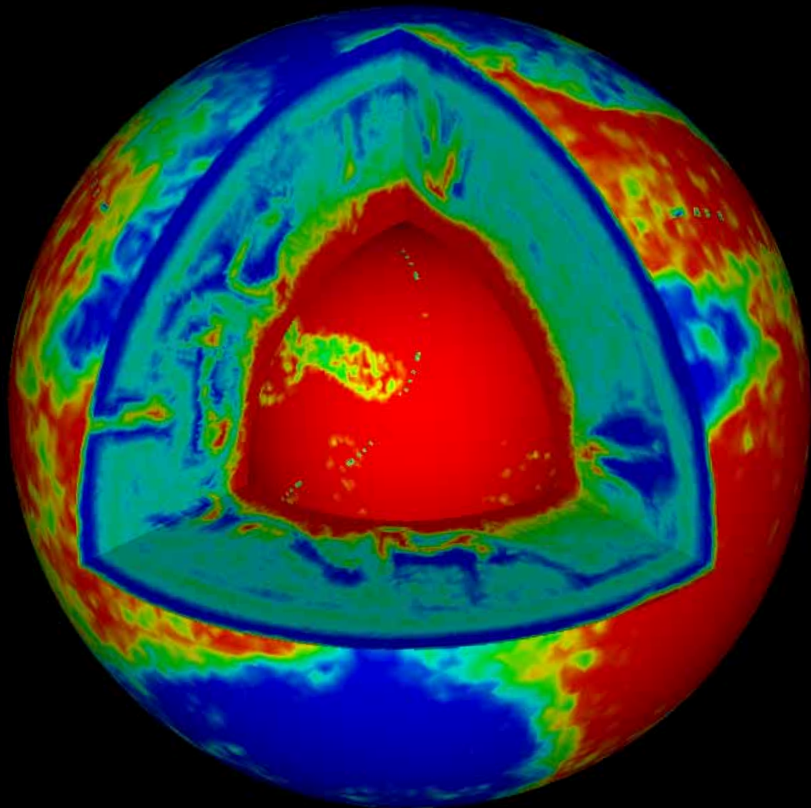


Fig. 4. Histogram of crustal distribution after 4.5 Ga model evolution. The model crust (bars) displays two peaks at around 40 and 55 km thickness comparable to the observed crustal thicknesses (Neumann et al., 2004) of the martian plains and southern highlands (black line).



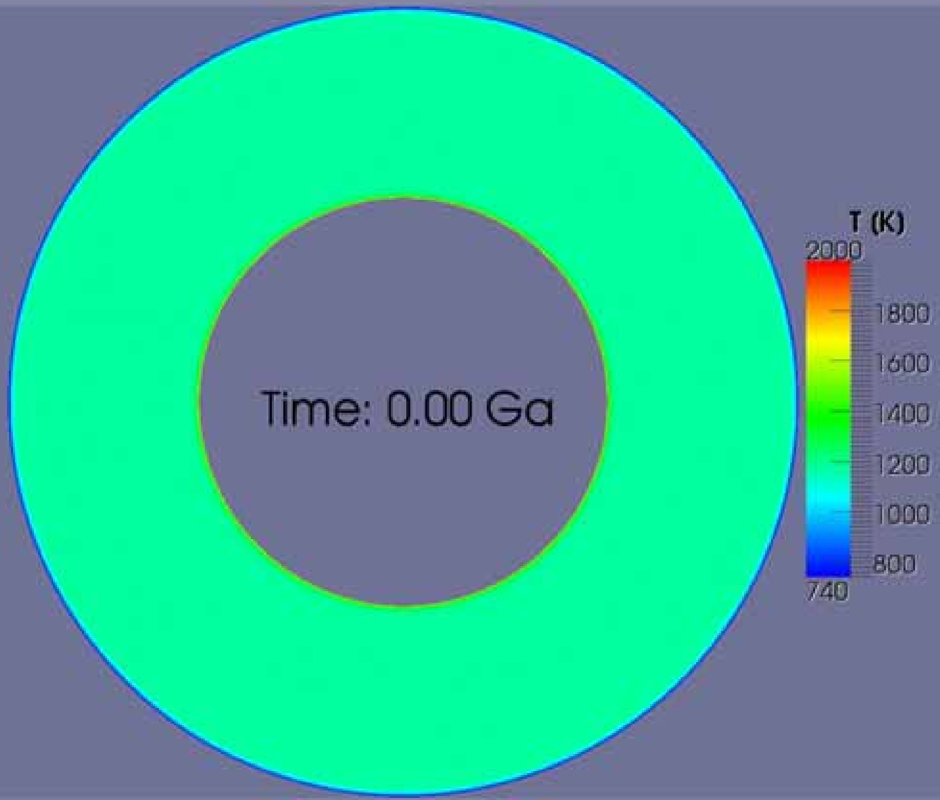
Venus



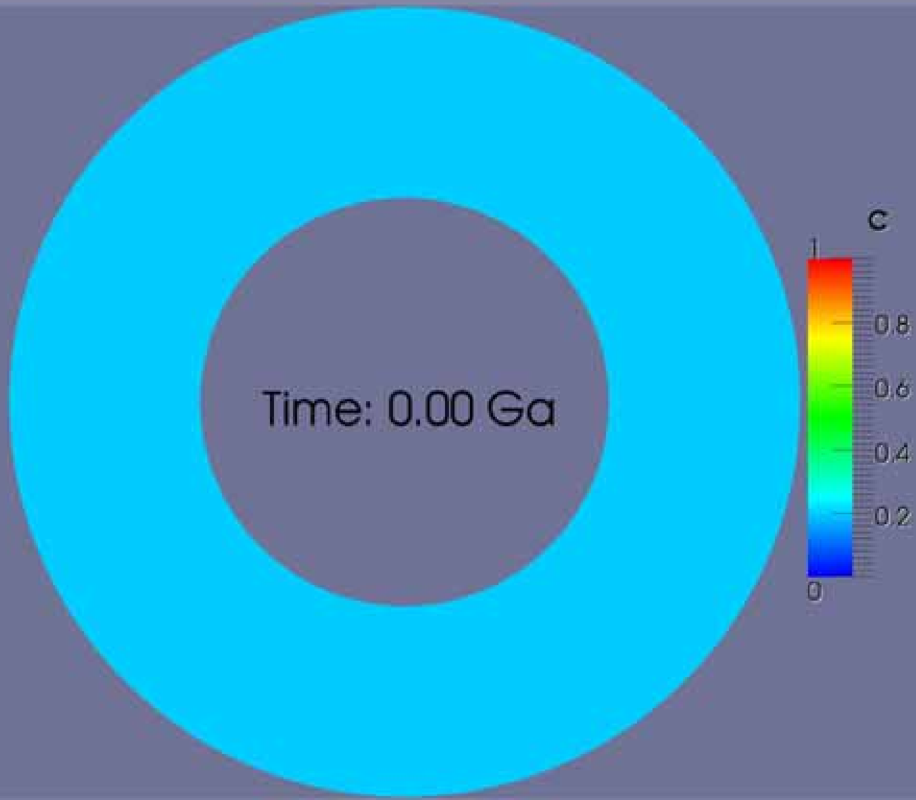
Stagnant Lid Cases

Reference Case

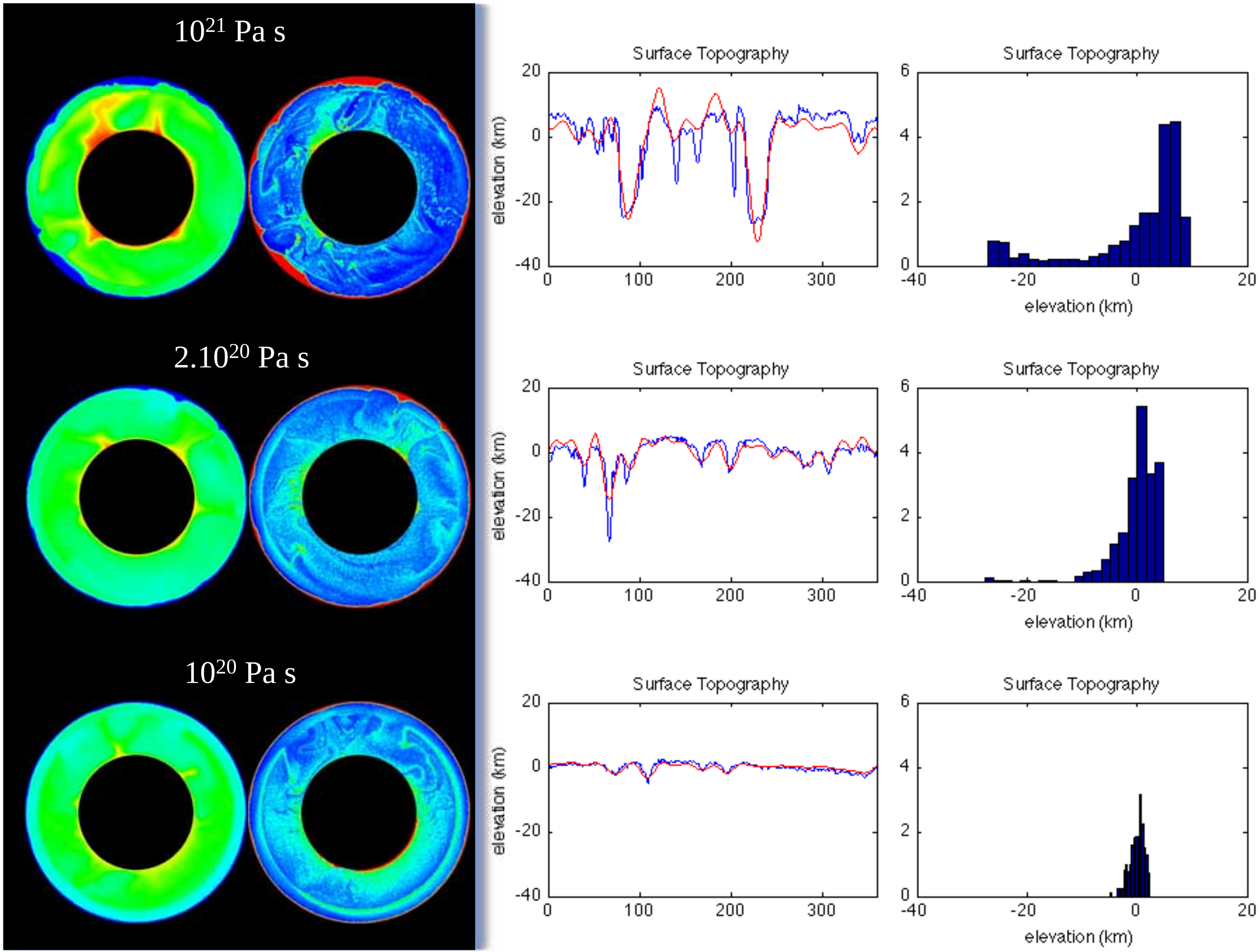
Ra = 6.15E8 -> $\eta = 2 \times 10^{20}$ Pa s



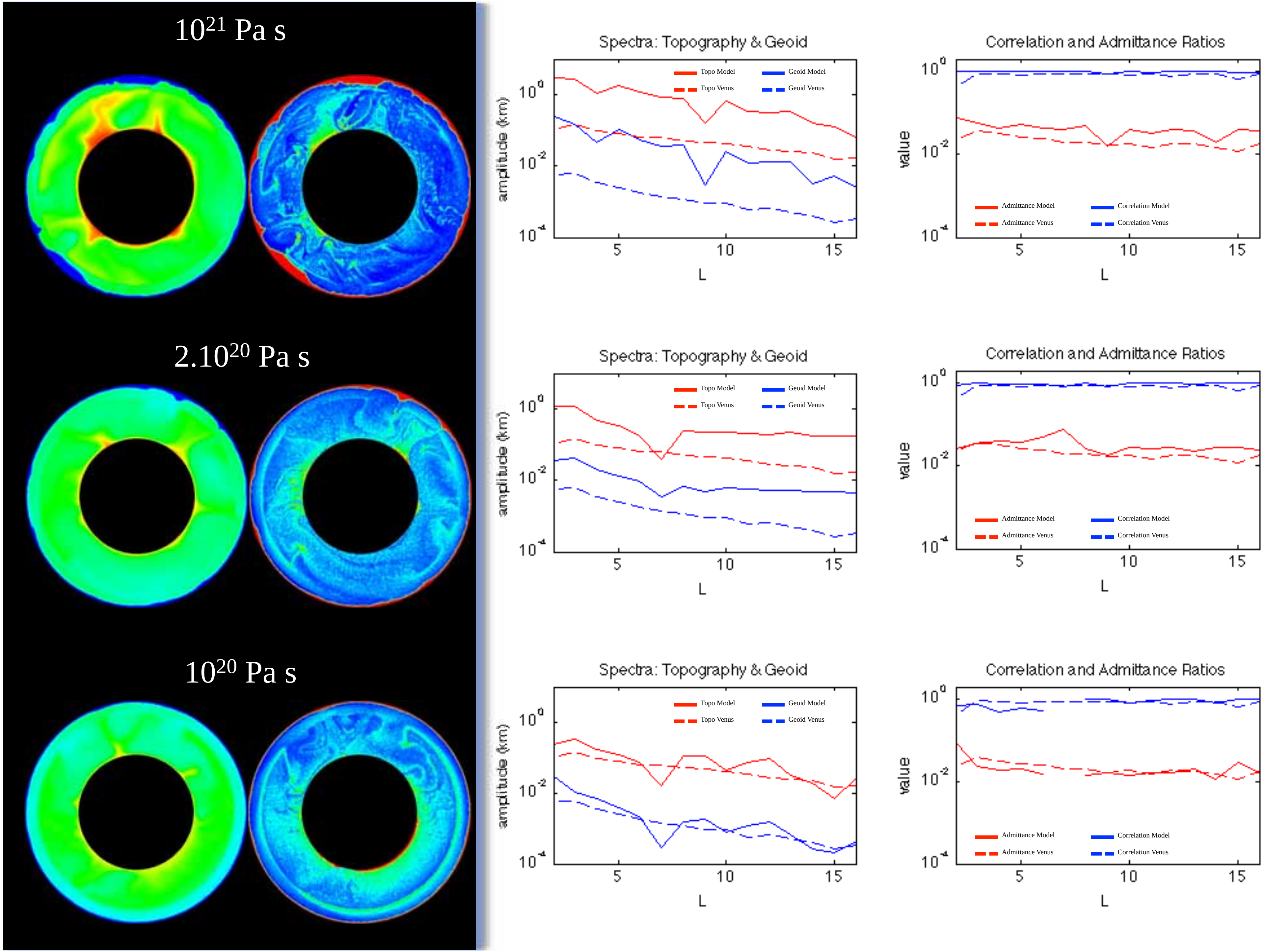
Temperature



Composition



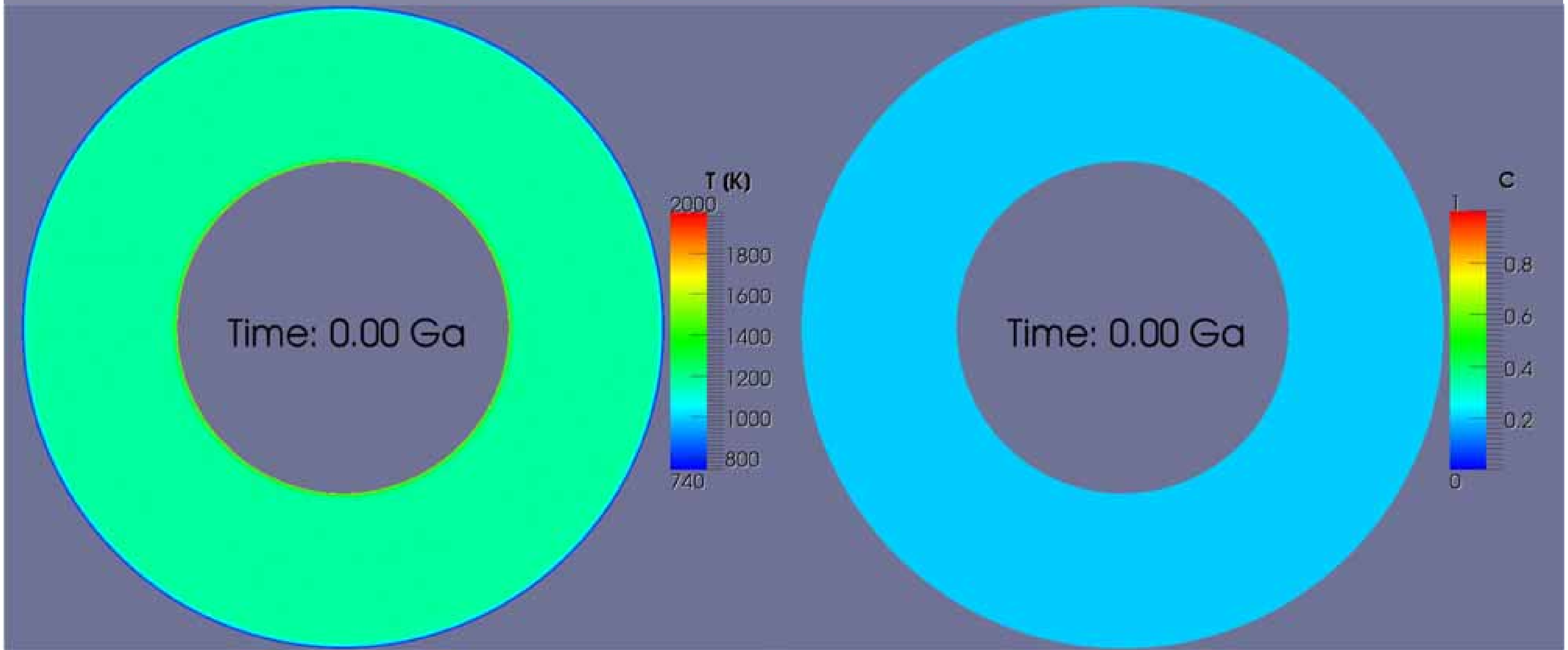
10^21 Pa s
2.10^20 Pa s
10^20 Pa s
Surface Topography
elevation (km)
Surface Topography
elevation (km)



10^21 Pa s
2.10^20 Pa s
10^20 Pa s
Spectra: Topography & Geoid
Topo Model
Topo Venus
Geoid Model
Geoid Venus
amplitude (km)
L
Correlation and Admittance Ratios
Admittance Model
Admittance Venus
Correlation Model
Correlation Venus
value

Episodic Lid Cases

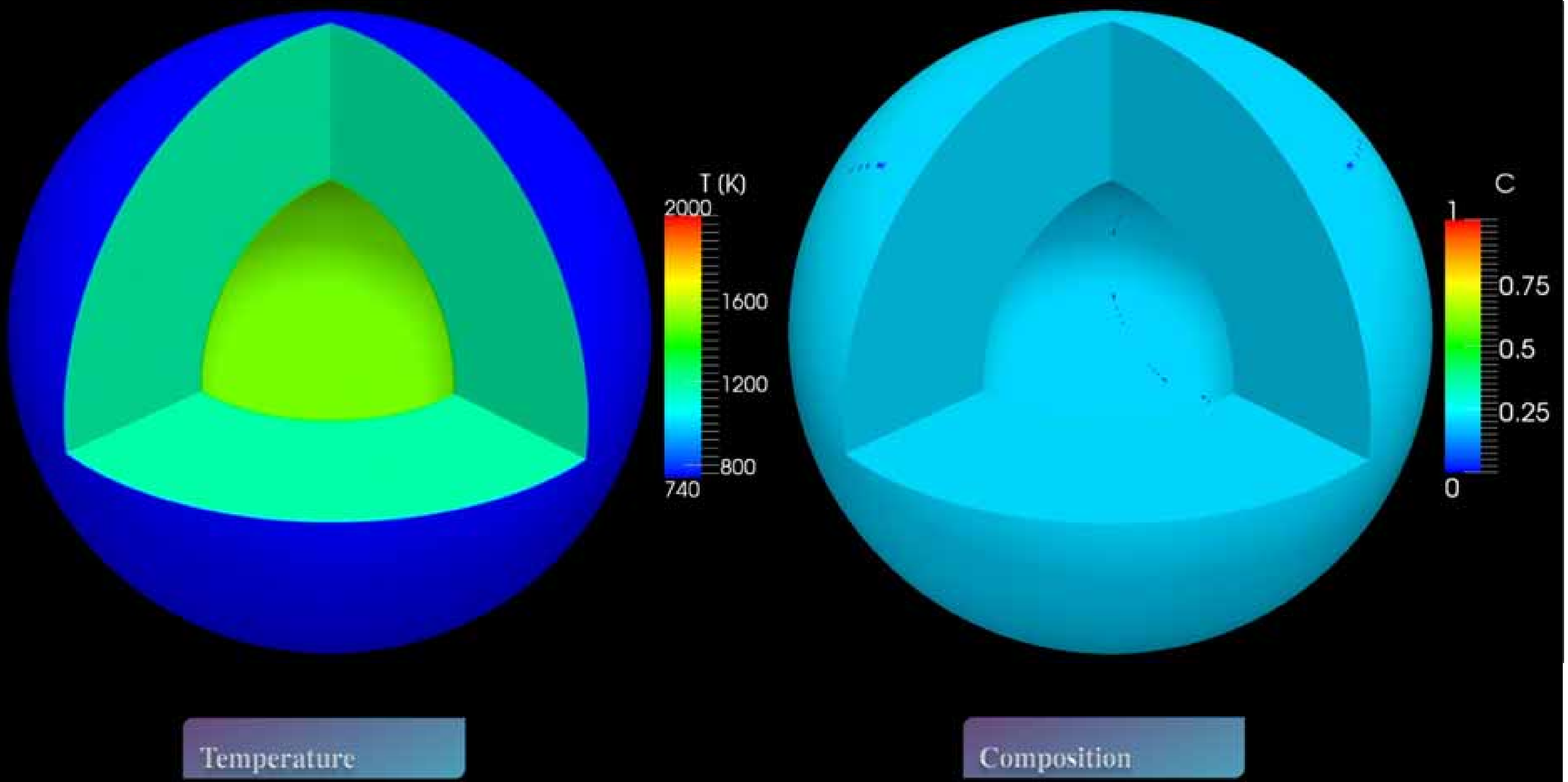
Ra = 1.23E9 -> $\eta = 1 \times 10^{20}$ Pa s
σy = 100 MPa



Temperature

Composition

Episodic 3-D model

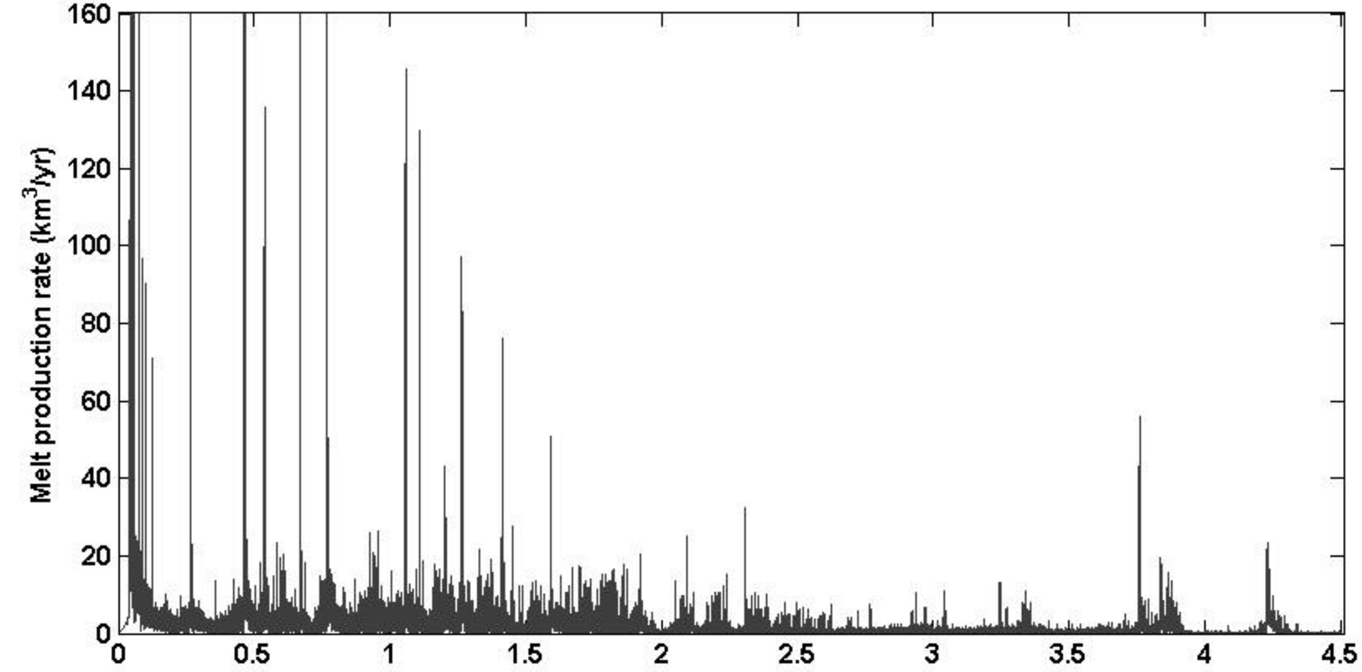


Interior-Atmosphere coupling

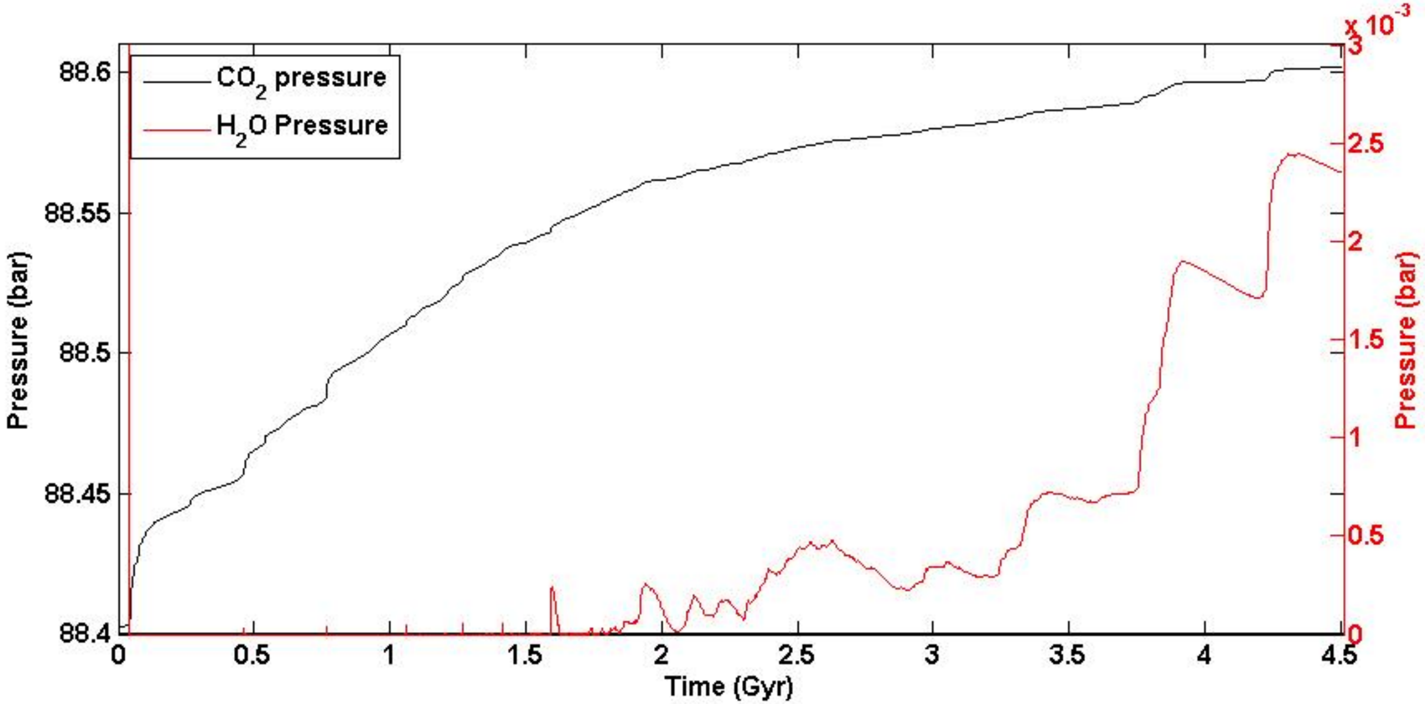
Cédric Gillmann

- Volcanism -> volatiles (CO2, H2O) to atmosphere
- Amospheric escape removes them
- Surface T acts as a boundary condition for mantle convection.
- Atmosphere model:
  - 1D gray radiative convective model
  - Greenhouse gases (CO2, H2O) modify surface T
  - Takes into account the faint young sun hypothesis
- Escape model:
  - Hydrodynamic escape during first 100 Myr
  - Only non-thermal escape mechanisms (sputtering, ionospheric outflow, dissociative recombination, ion pick up) during main evolution
  - Efficiency decreases with time, as depends on Extreme UV flux from the Sun (not total luminosity)

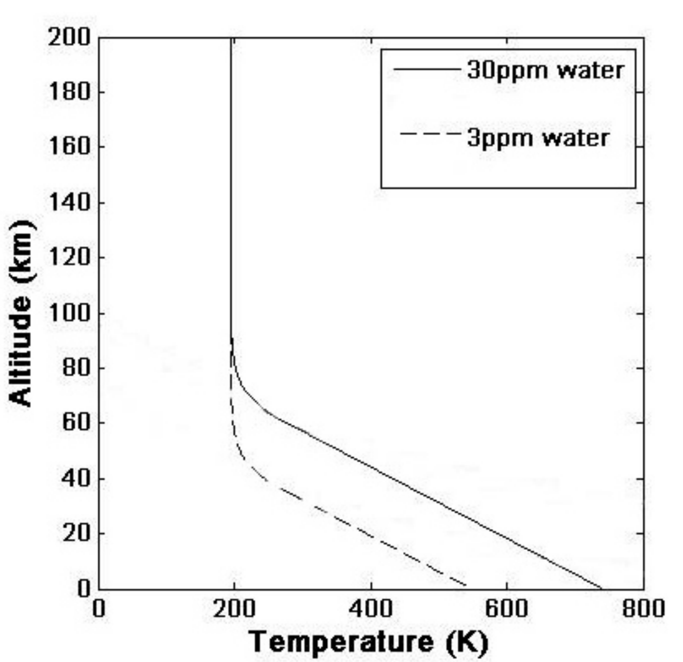
Outgassing

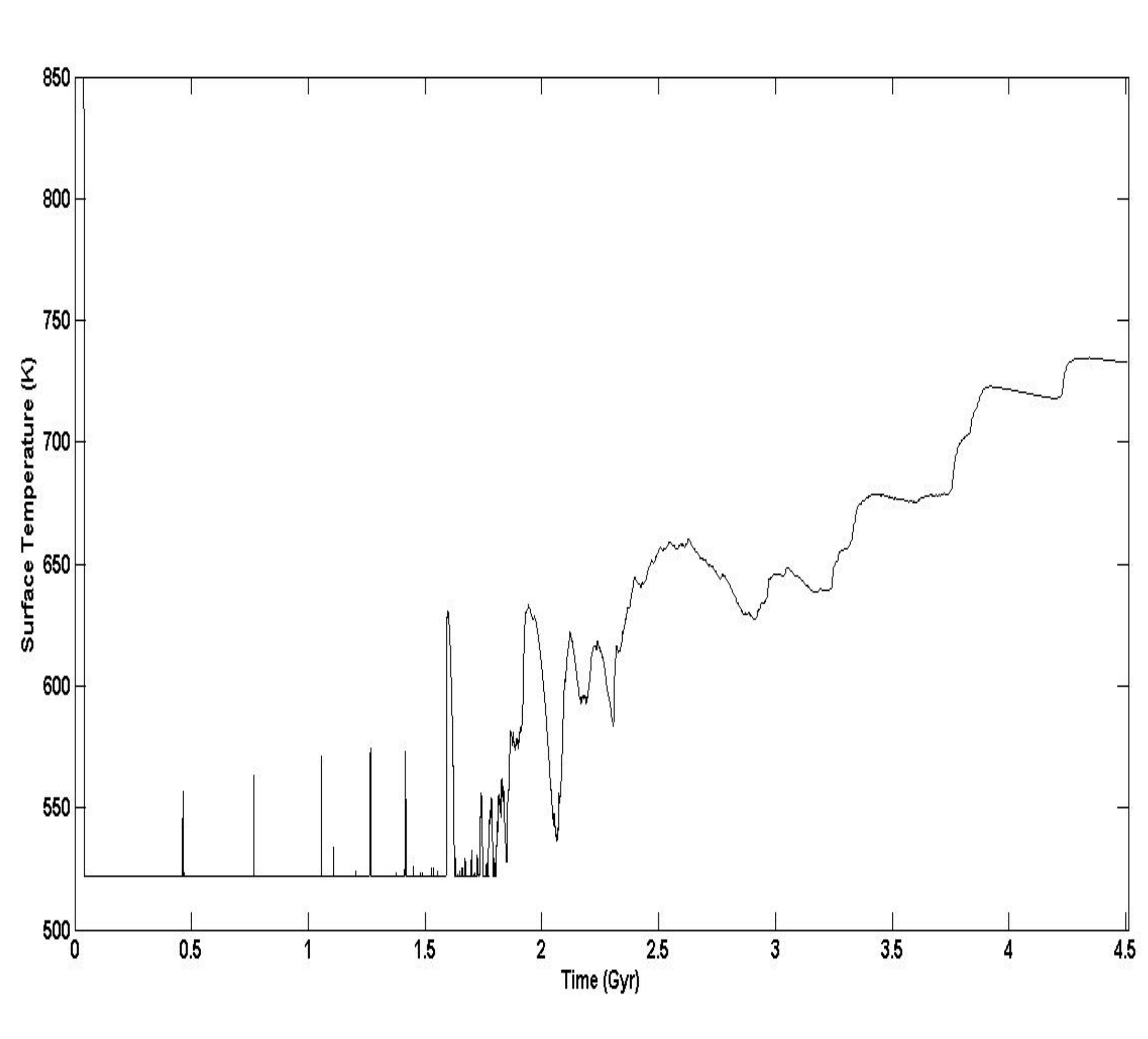


Atmosphere



Surface T evolution

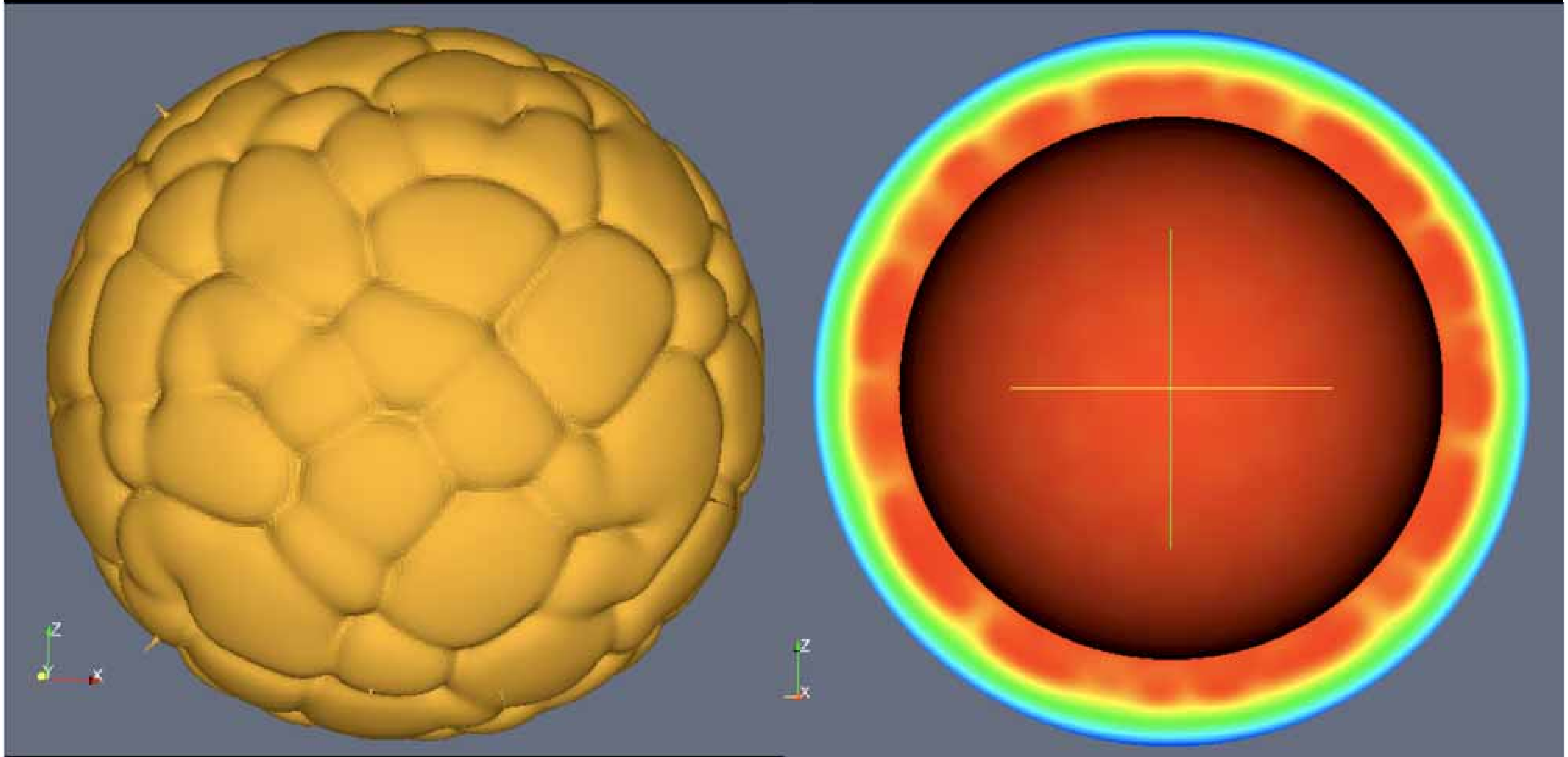




Venus Conclusions

- Rigid lid: Magmatism dominant heat transport mode, crustal delamination. Match geoid & topography for reference viscosity ~10^{20} Pa s
- Episodic overturn: deep crustal recycling, conduction more important, geoid & topo OK
- Geoid, topography, admittance ratios favour viscosity~10^{20}-10^{21} Pa s
- Preferred case: episodic yielding with ys 100 Mpa
- Atmosphere-surface coupling under evaluation

Mercury: 3D spherical model



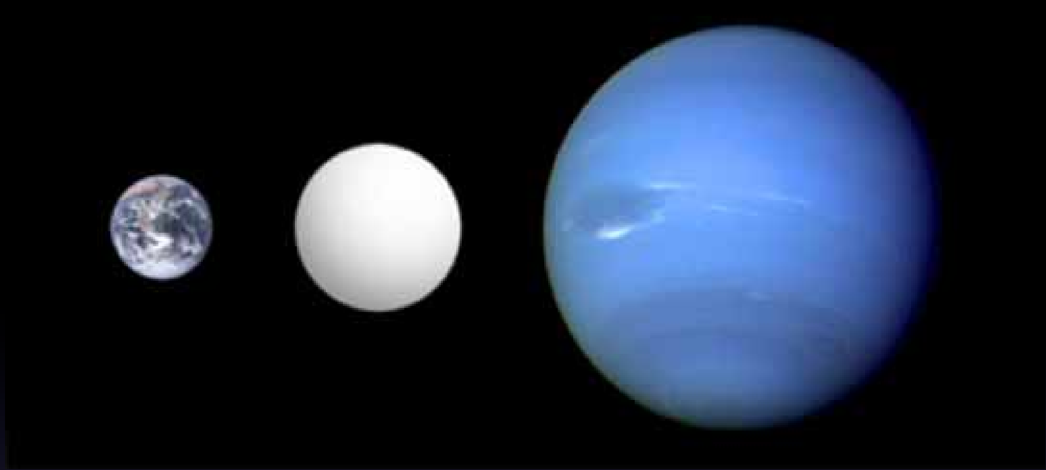
Effect of Rheology on Mantle Dynamics and Plate Tectonics in Super-Earths

P. J. Tackley (ETH Zurich)

M. Ammann, J. P. Brodholt, D. P. Dobson (UCL)

D. Valencia (MIT)

Dynamics of extrasolar super-Earths?

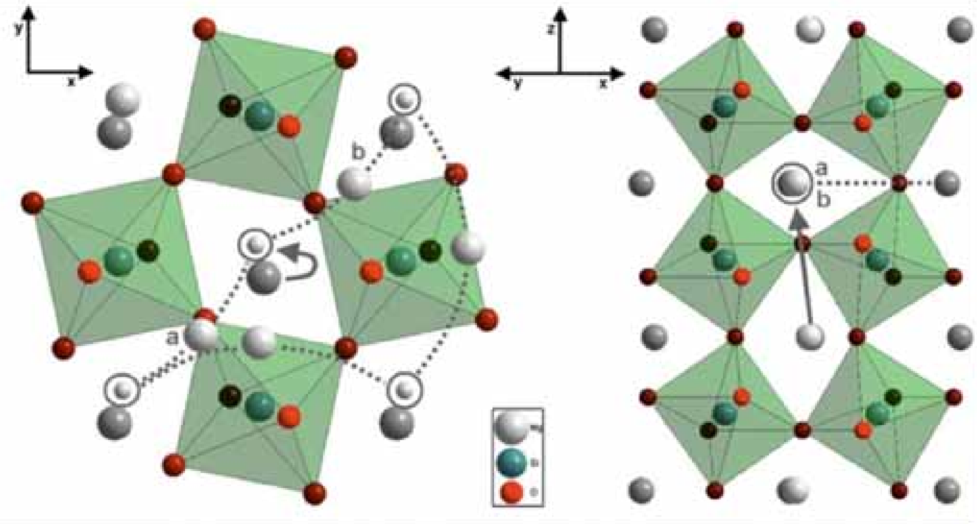


COROT-7b

- Several super-Earths (1-10 * mass of Earth) have been found; many more expected.
- Habitability: Atmosphere & interior strongly linked -> understand interior dynamics & evolution
  - Plate tectonics? (*van Heck & Tackley 2011*)
  - High interior pressure influences rheology: effect of this?

Activation enthalpy(p): Density Function Theory

Fig. 4 Sketch of magnesium migration pathways in orthorhombic $MgSiO_3$ perovskite [*left* view in *z*-direction, *right* projection onto (110)]. *Straight-line* pathways are indicated as *solid arrows*. *Darker* atoms are farther away from the observer. On the curved pathways, the migrating magnesium is positioned at the saddle-point location (only in the left figure). Vacancy locations are indicated with *circles*



Phys Chem Minerals (2009) 36:151–158
DOI 10.1007/s00269-008-0265-z

ORIGINAL PAPER

DFT study of migration enthalpies in $MgSiO_3$ perovskite

M. W. Ammann · J. P. Brodholt · D. P. Dobson

nature Vol 465 27 May 2010 doi:10.1038/nature09052

LETTERS

First-principles constraints on diffusion in lower-mantle minerals and a weak D′′ layer

M. W. Ammann[1], J. P. Brodholt[1], J. Wookey[2] & D. P. Dobson[1]

Reviews in Mineralogy and Geochemistry

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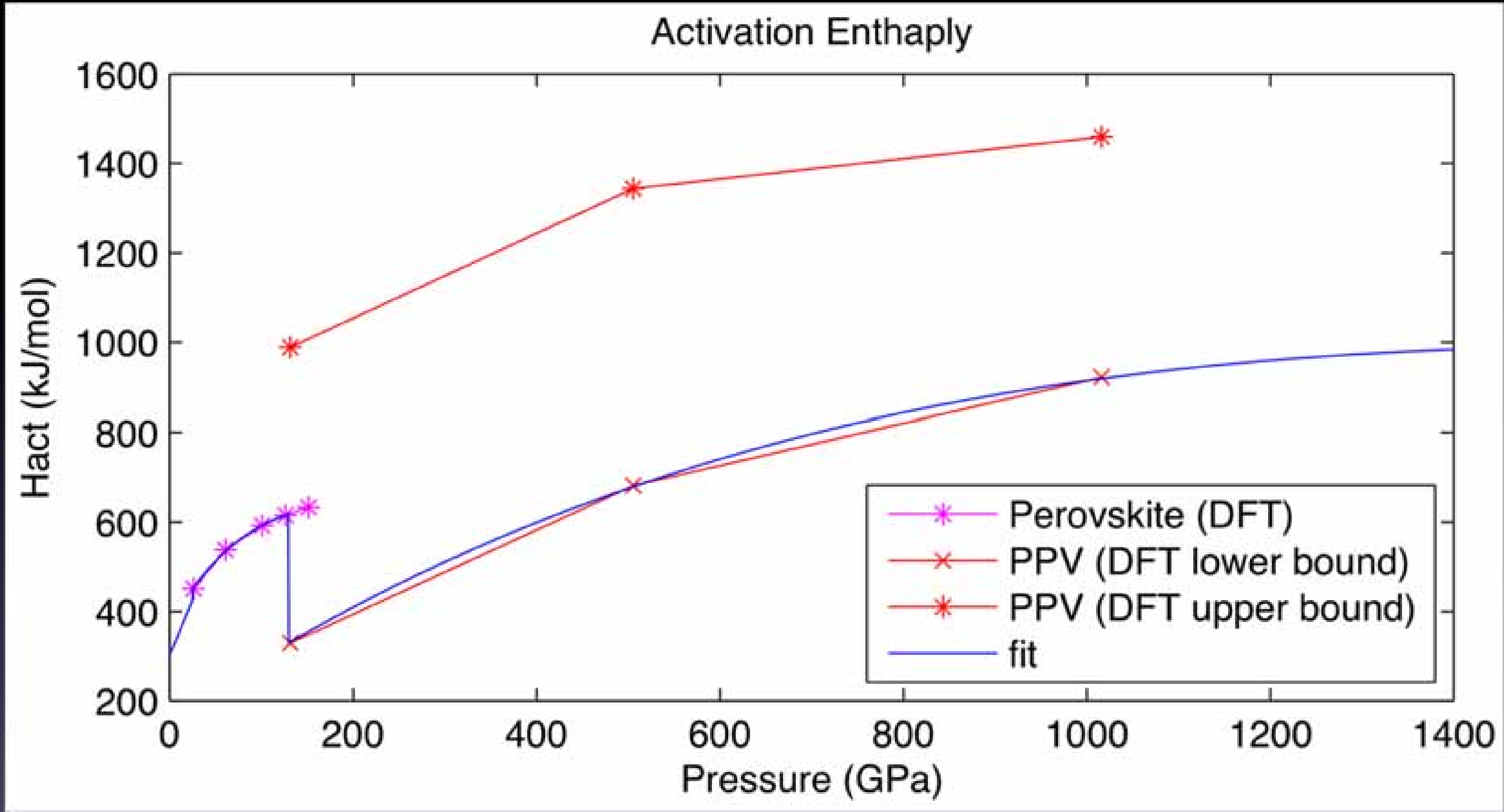
Simulating Diffusion

Michael W. Ammann, John P. Brodholt and David P. Dobson

H(p) results & fit



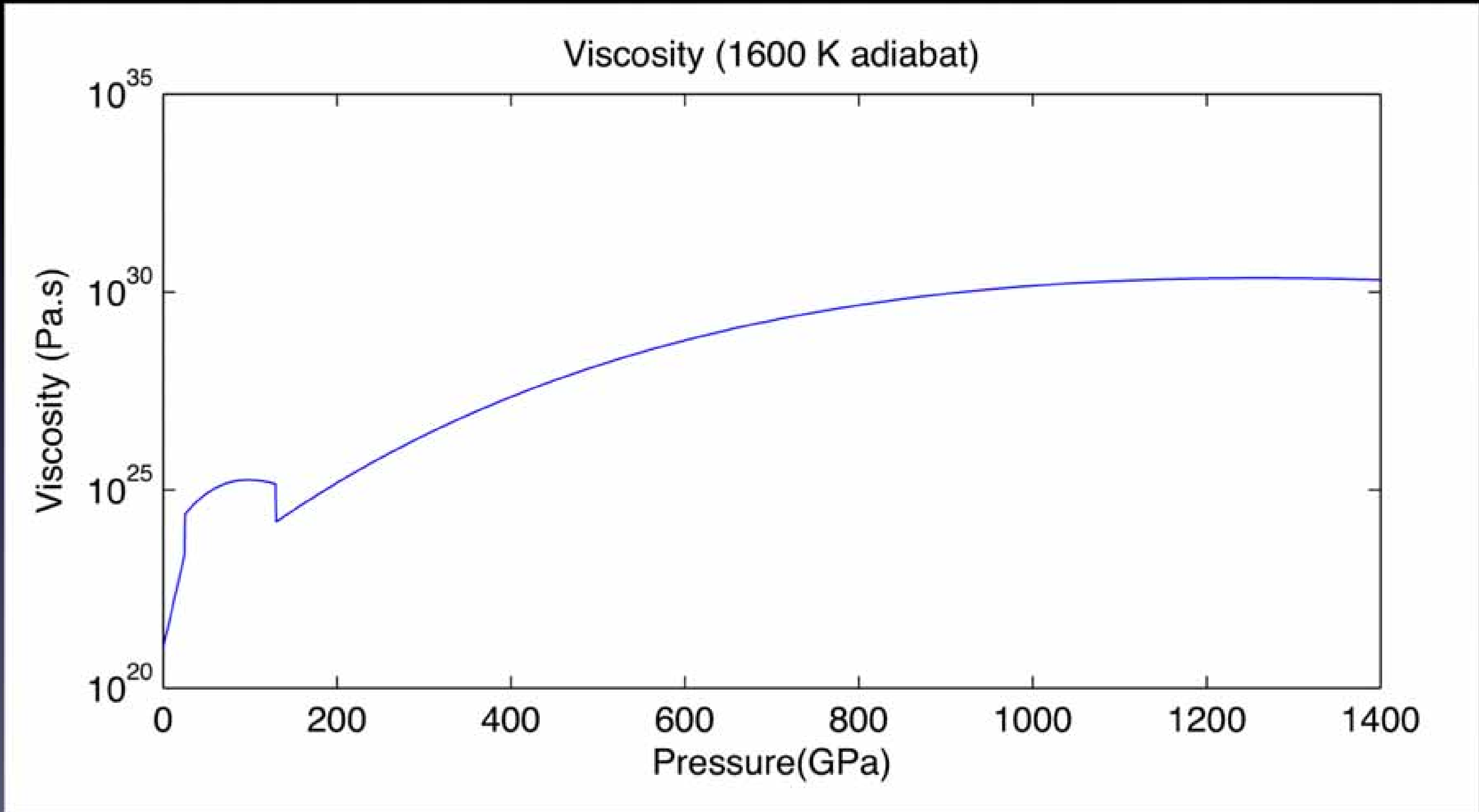
Arrhenius viscosity law

$$\eta(T,p)=\eta_0 \exp\left[\frac{H(p)}{RT}-\frac{H(0)}{RT_0}\right]$$

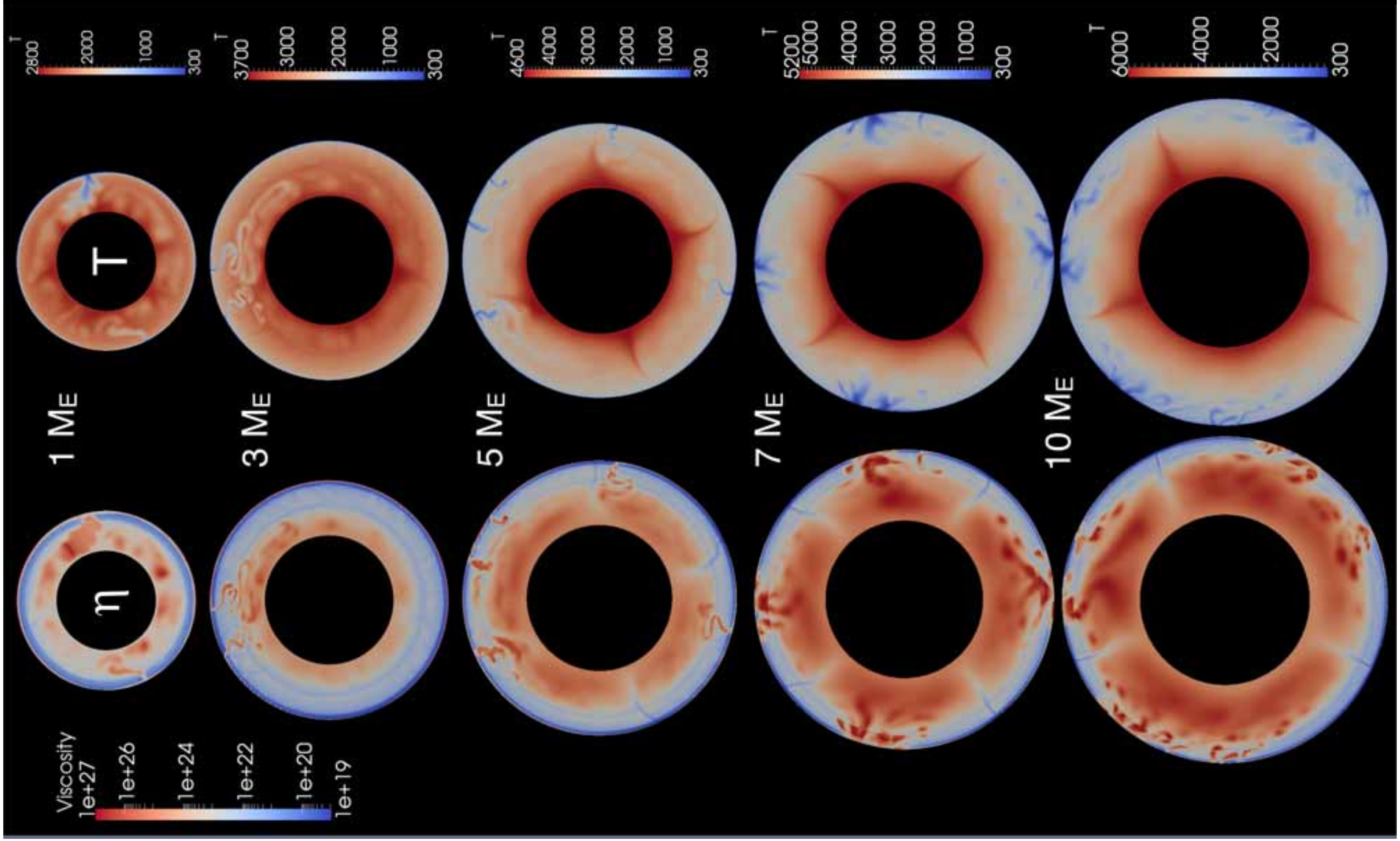
Plastic yielding

$$\eta_{eff}=\left(\eta_{diff}^{-1}+\frac{2\dot{e}}{\sigma_Y}\right)^{-1}$$

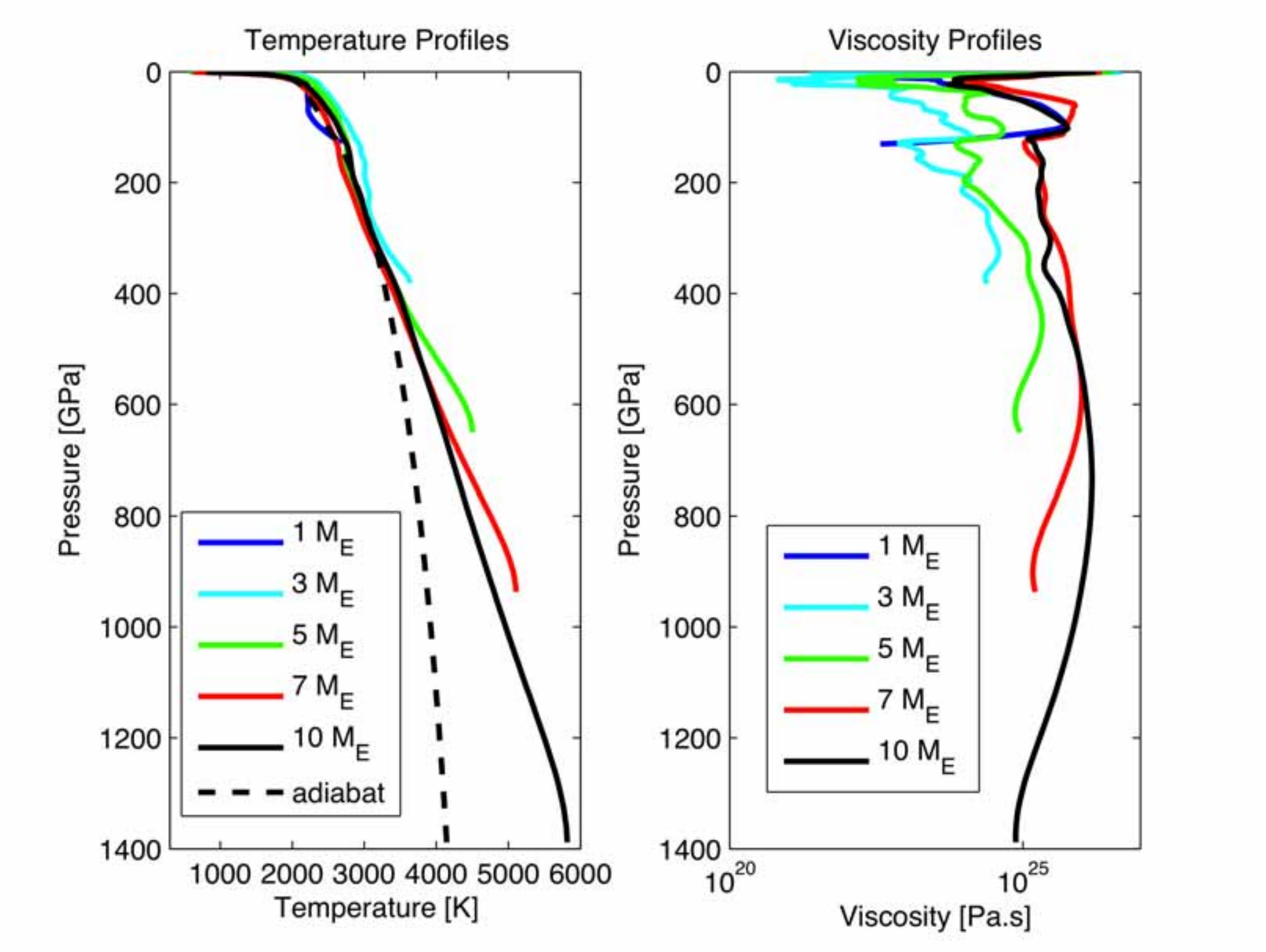
Viscosity profile along adiabat



T & viscosity structures

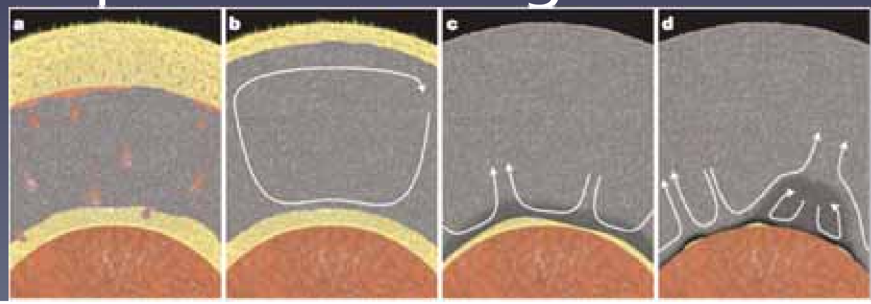


Mean profiles

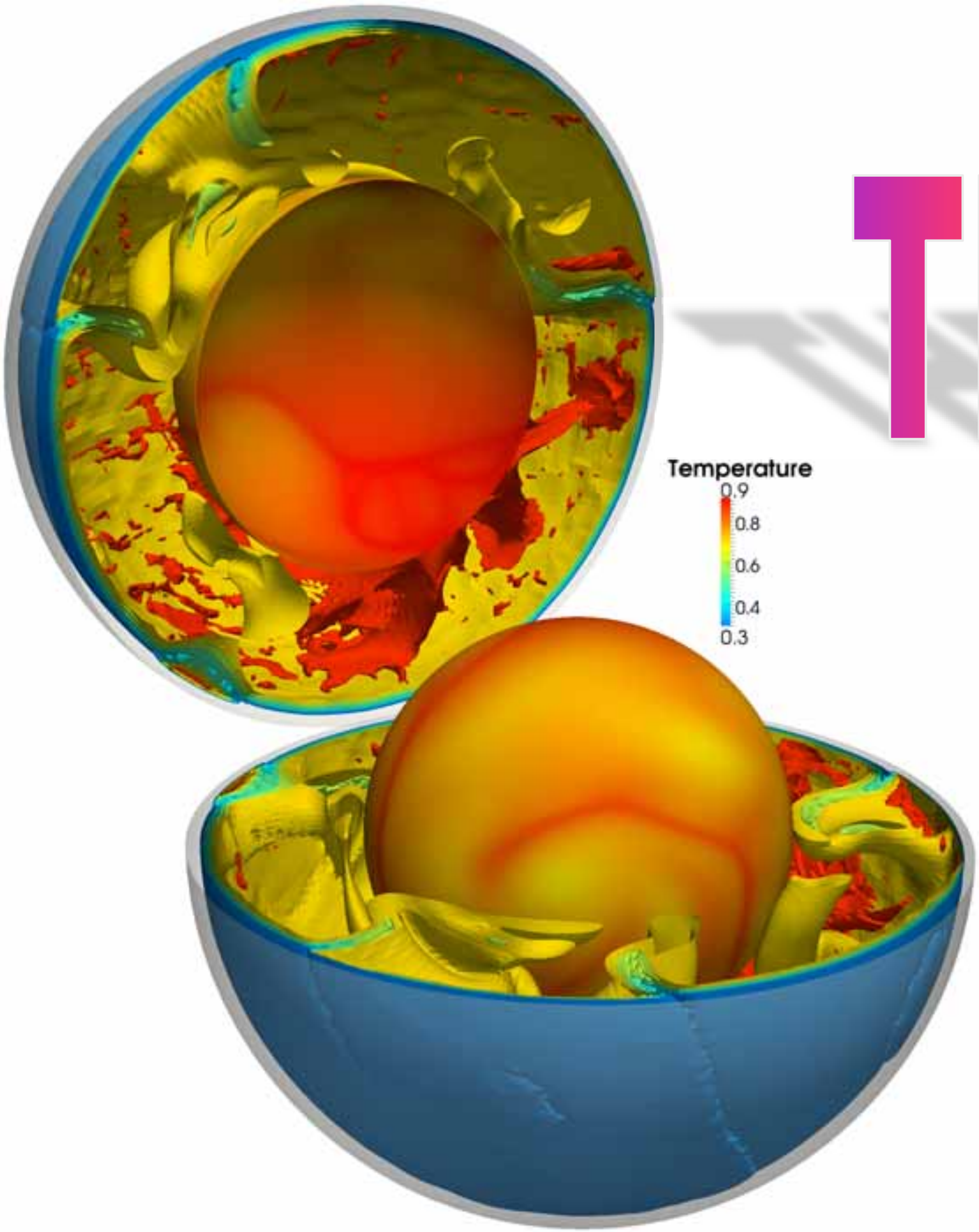


Super-E Discussion/Conclusions

- Plate tectonics easier on larger planets (*Valencia et al. 2007, 2009; van Heck & Tackley 2011; Korenaga 2011*)
- Self-regulation of viscosity: if adiabatic viscosity too high, T increases until mantle can lose radiogenic heat
  - Superadiabatic T profile
  - ~Isoviscous viscosity profile
- Results in hot super-Earth: super-Basal Magma Ocean?



Labrosse et al. 2007



THE END