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Advanced Workshop on Energy Transport in Low-Dimensional Systems: Achievements and Mysteries

15 - 24 October 2012

Nonlinear Waves in Low-dimensional Systems - Part III

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Nonlinear Waves in Low-Dimensional Systems: essentials, problems, perspectives

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- **Fermi, Pasta, Ulam and the essentials of statistical physics**
- **discrete breathers – localizing waves on lattices**
- **destruction of Anderson localization**

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THE ENGINE OF THE NEW NEW ZEALAND

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CTCP

MASSEY UNIVERSITY
TE KUNENGA KI PŪREHUROA
UNIVERSITY OF NEW ZEALAND

Glossary

Particles: zero/full transmission below/above barrier, no interference, phase does not matter

Waves: partial transmission below/above barrier, interference, phase matters

Quantum / classical waves:
Identical description for single qm particle / linear case

Quantum many body waves: linear equations in VERY high-dimensional Hilbert (vector) space

Classical nonlinear waves: nonlinear equations, e.g. from mean field approximation for MANY quantum particles

Nonlinearity: wave-wave (mode-mode) interactions

Localization: waves start to travel, but never get away

Anderson localization

Philip W. Anderson
The Nobel Prize in Physics 1977

Nobel Lecture
Nobel Lecture, December 8, 1977

Local Moments and Localized States

I was cited for work both in the field of magnetism and in that of disordered systems, and I would like to describe here one development in each held which was specifically mentioned in that citation. The two theories I will discuss differed sharply in some ways. The theory of local moments in metals was, in a sense, easy: it was the condensation into a simple mathematical model of ideas which were very much in the air at the time, and it had rapid and permanent acceptance because of its timeliness and its relative simplicity. What mathematical difficulty it contained has been almost fully cleared up within the past few years.

Localization was a different matter: very few believed it at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it .

Experimental Evidence for Anderson Localization

waves in disordered media – Anderson localization for:
electrons, phonons, photons, BEC, …

Electrons: in: Akkermans et al 2006

Ultrasound: Weaver 1990

Microwaves: Dalichaoush et al 1991, Chabanov/Pradhan/ et al 2000

Light: Wiersma et al 1997, Scheffold et al 1999, Stoerzer et al 2006, Schwartz et al 2007, Lahini et al 2008

BEC: Billy et al 2008, Roati et al 2008

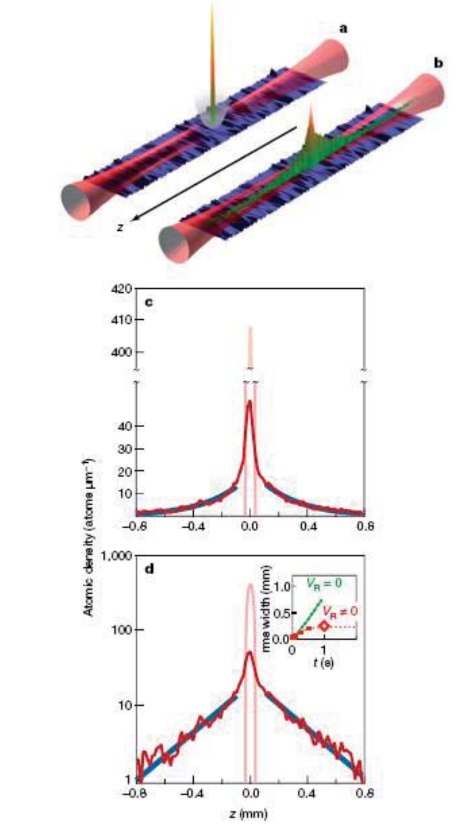
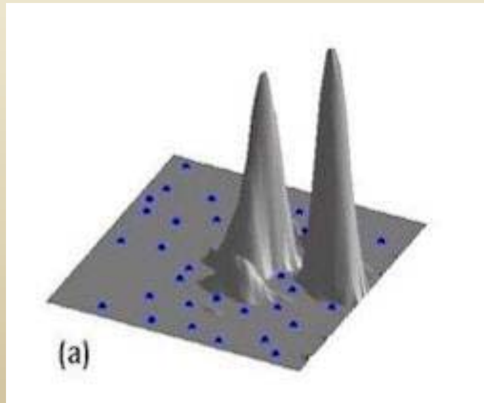


Figure 1 | Observation of exponential localization. a, A small BEC



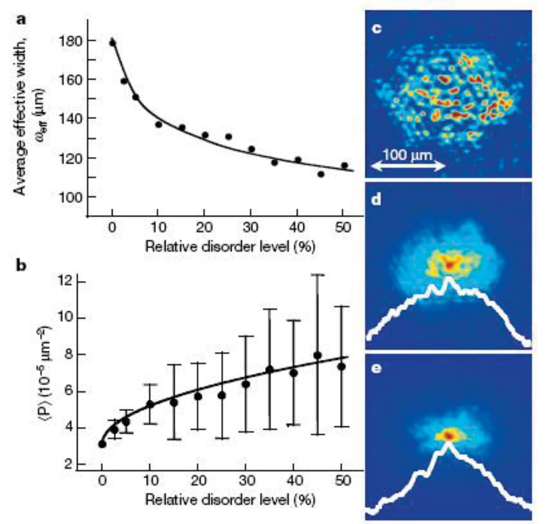


Figure 2 | Experimental results for propagation in disordered lattices.

PRL 98, 210401 (2007) PHYSICAL REVIEW LETTERS week ending 25 MAY 2007

Anderson Localization of Expanding Bose-Einstein Condensates in Random Potentials

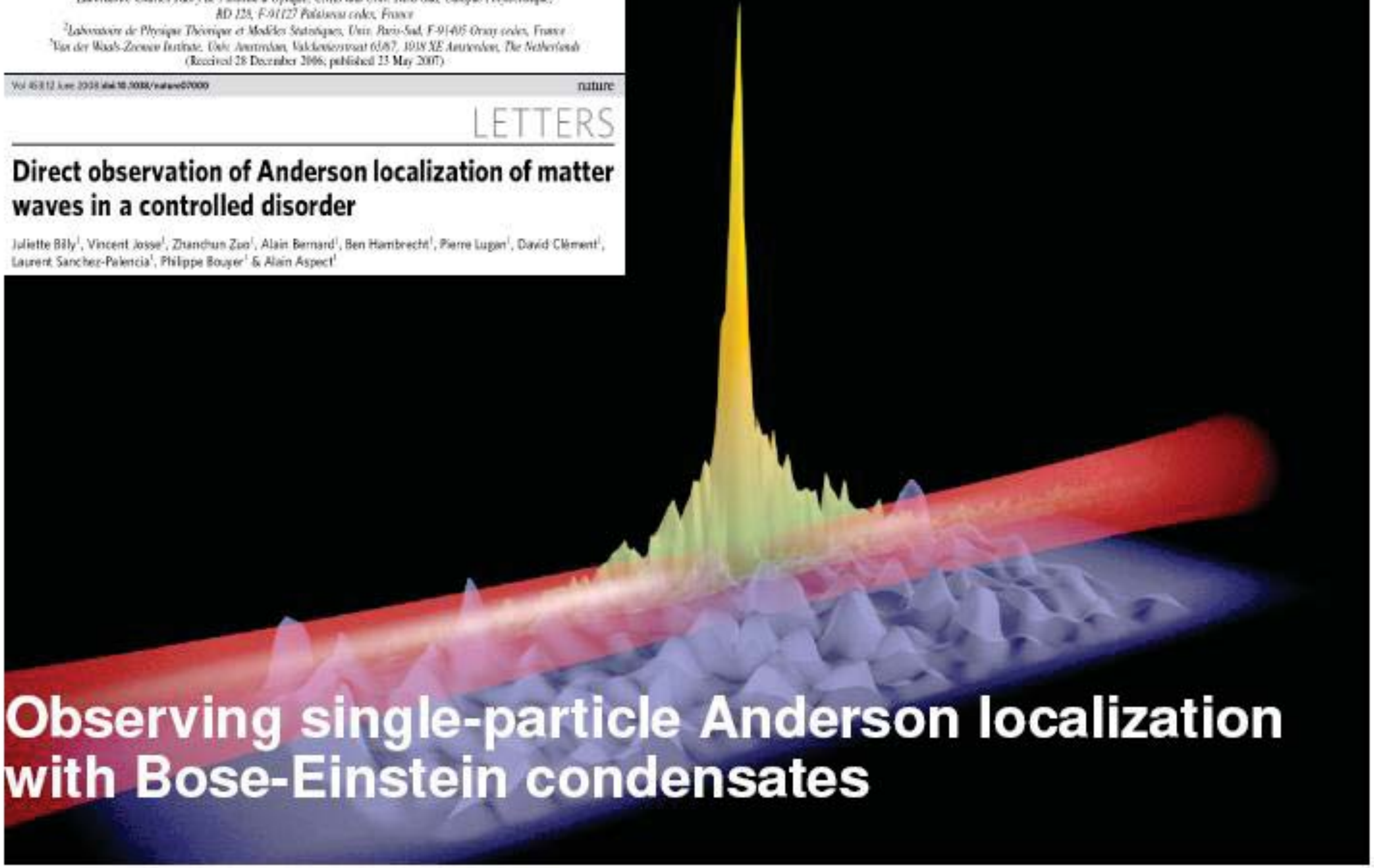
L. Sanchez-Palencia,[1] D. Clément,[1] P. Lugan,[1] P. Bouyer,[1] G. V. Shlyapnikov,[2,3] and A. Aspect[1]
[1]*Laboratoire Charles Fabry de l'Institut d'Optique, CNRS and Univ. Paris-Sud, Campus Polytechnique, RD 128, F-91127 Palaiseau cedex, France*
[2]*Laboratoire de Physique Théorique et Modèles Statistiques, Univ. Paris-Sud, F-91405 Orsay cedex, France*
[3]*Van der Waals-Zeeman Institute, Univ. Amsterdam, Valckenierstraat 65/67, 1018 XE Amsterdam, The Netherlands*
(Received 28 December 2006; published 23 May 2007)

nature

LETTERS

Direct observation of Anderson localization of matter waves in a controlled disorder

Juliette Billy[1], Vincent Josse[1], Zhanchun Zuo[1], Alain Bernard[1], Ben Hambrecht[1], Pierre Lugan[1], David Clément[1], Laurent Sanchez-Palencia[1], Philippe Bouyer[1] & Alain Aspect[1]



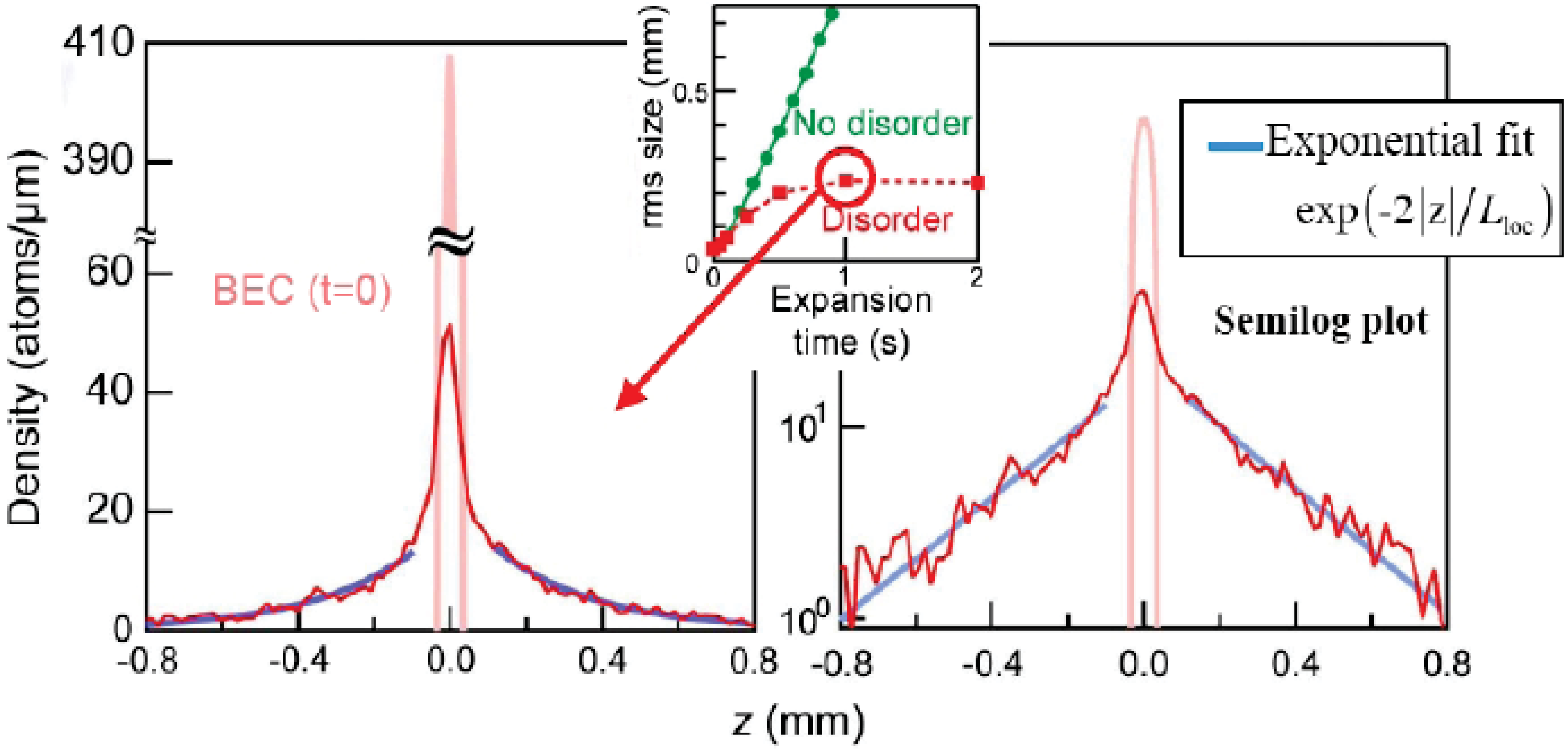
Observing single-particle Anderson localization with Bose-Einstein condensates

Observation of the signature of AL

BEC parameters : N=1.7 10^4 atoms, (μ_{in}=220Hz)

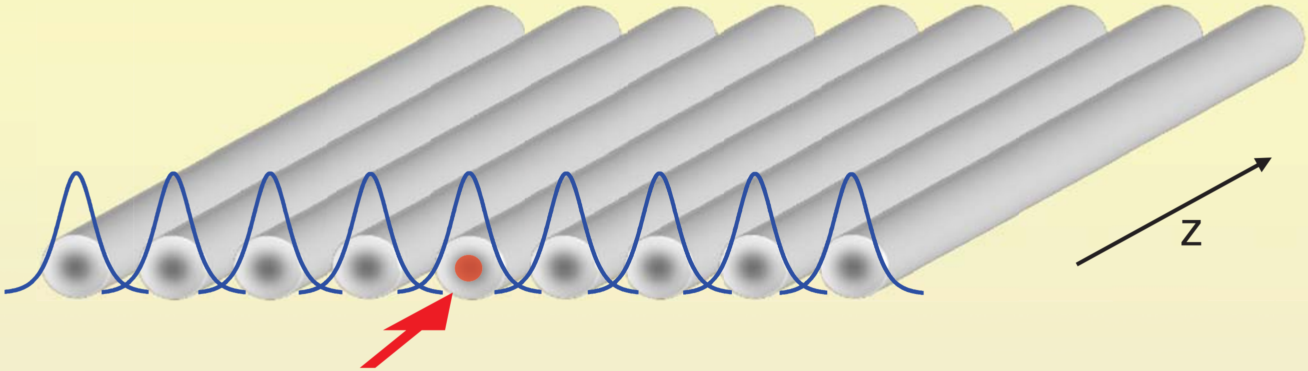
Weak disorder : V_R/μ_{in} =0.12 << 1

k_{max} / k_c = 0.63 +/-0.09



⇒ Exponential decay of the density in the wings : L_{loc}=530 +/- 80 µm

An optical one-dimensional waveguide lattice (Silberberg et al ’08)

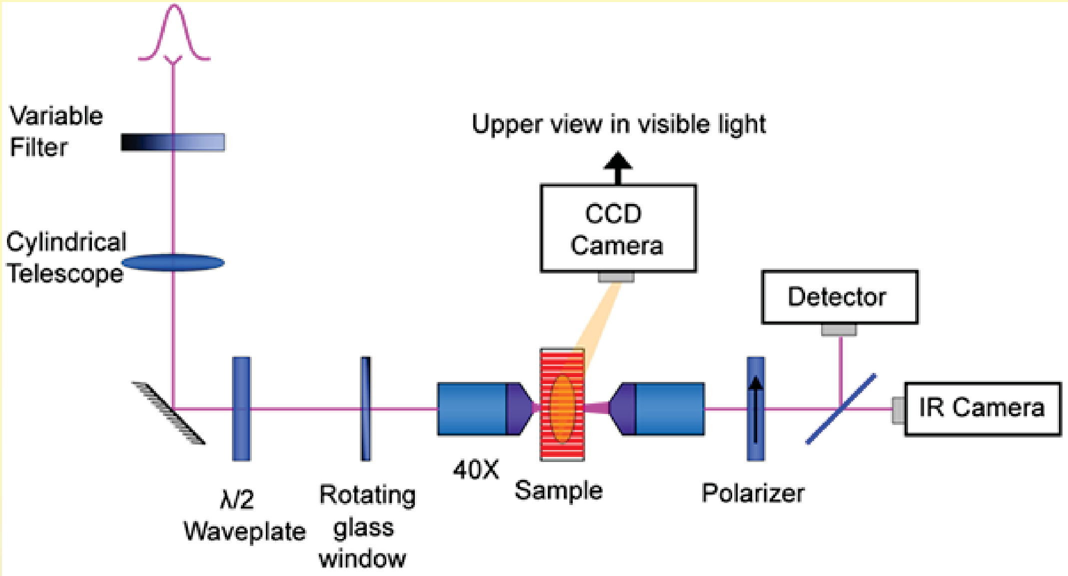


- Evanescent coupling between waveguides
- Light coherently tunnels between neighboring waveguides
- Dynamics is described by the Tight-Binding model

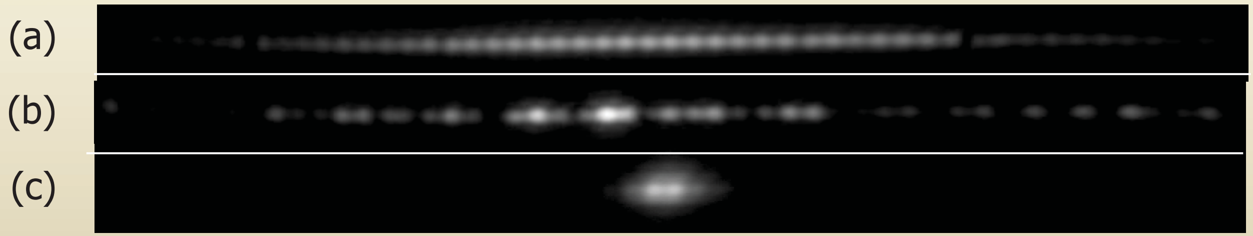
$$i\frac{\partial U_n}{\partial z} = \beta_n U_n + C_{n,n\pm1}\left[U_{n+1} + U_{n-1}\right]$$

β_n – waveguide’s refraction index /width

$C_{n,n\pm1}$ – separation between waveguides

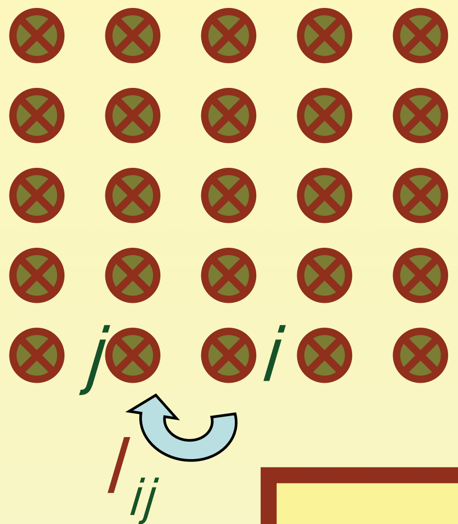


- Injecting a narrow beam (~3 sites) at different locations across the lattice



(a) Periodic array – *expansion*
(b) Disordered array - *expansion*
(c) Disordered array - *localization*

Anderson Model



- *Lattice - tight binding model*
- *Onsite energies* ε_i *- random*
- *Hopping matrix elements* t_{ij}

$$-W/2 < \varepsilon_i < W/2$$

uniformly distributed

$$t_{ij} = \begin{cases} t & i \text{ and } j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$$

Anderson Transition

$t < t_c$

Insulator

All eigenstates are localized

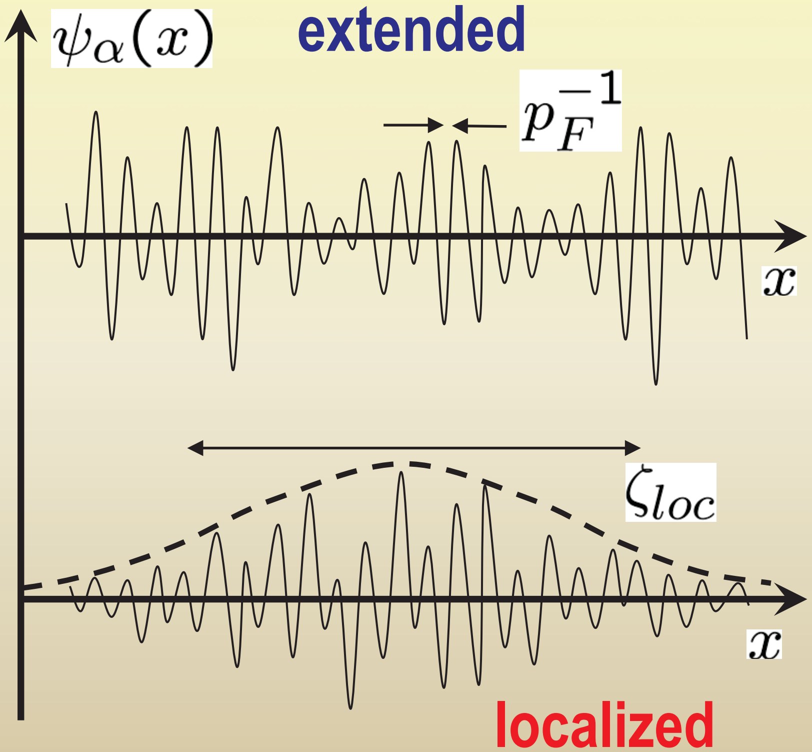
$t > t_c$

Metal

There appear states extended all over the whole system

Localization of single-particle wave-functions. Continuous limit:

$$\left[-\frac{\nabla^2}{2m} + U(\boldsymbol{r}) - \epsilon_F\right]\psi_\alpha(\boldsymbol{r}) = \xi_\alpha\psi_\alpha(\boldsymbol{r})$$



d=1: **All states are localized**

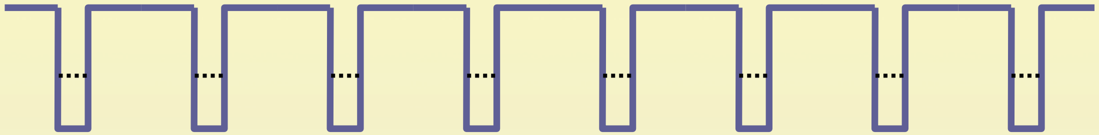
d=2: **All states are localized**

d >2: **Anderson transition**

The one-dimensional tight-binding model

- The periodic Lattice (Bloch, 1928)

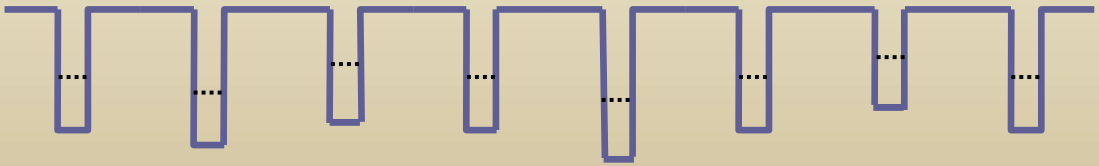
$$-i\frac{\partial \psi_n}{\partial t} = E\psi_n + T\left[\psi_{n+1} + \psi_{n-1}\right]$$

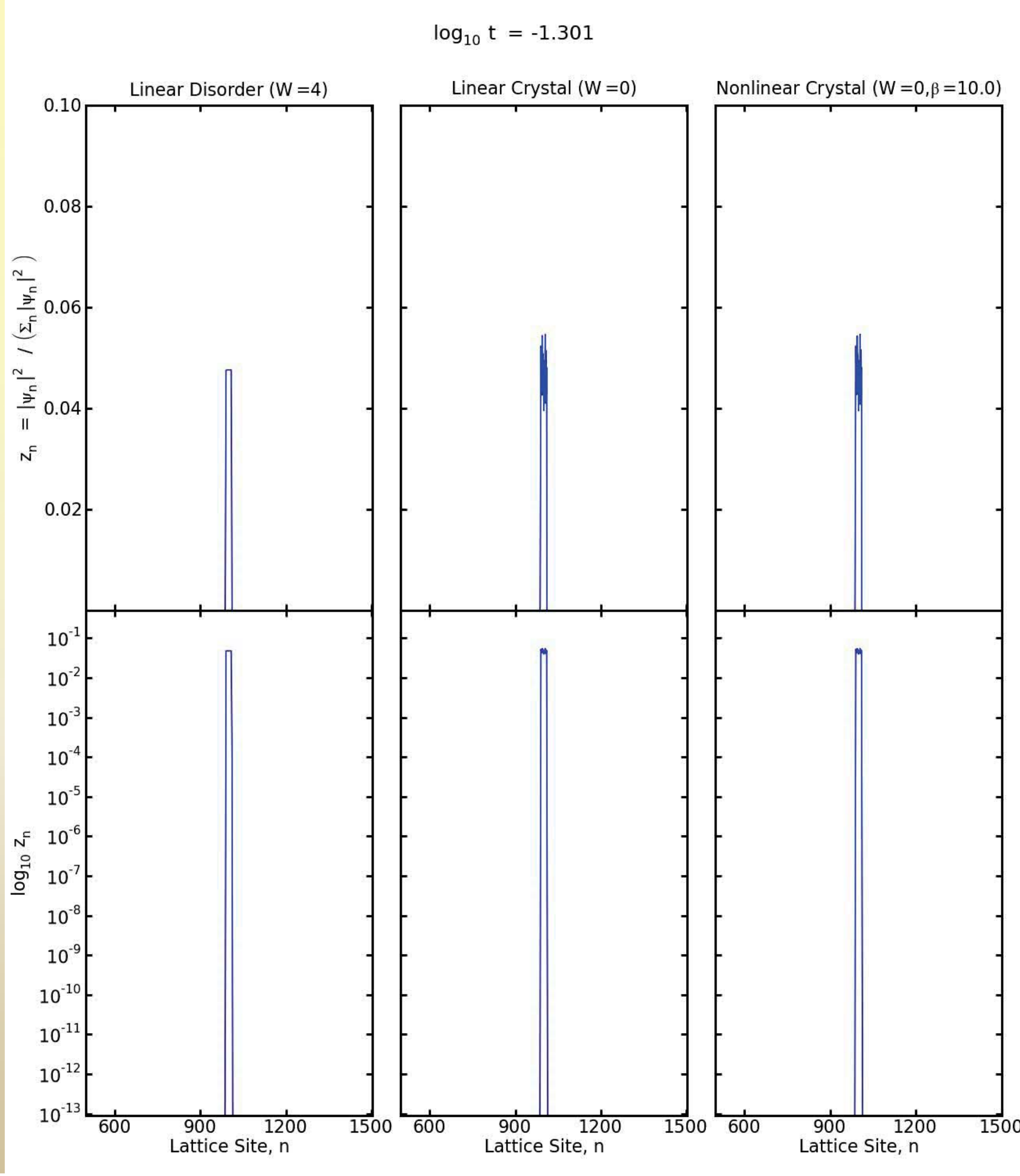


Eigenfunctions extend over entire lattice (Bloch functions)

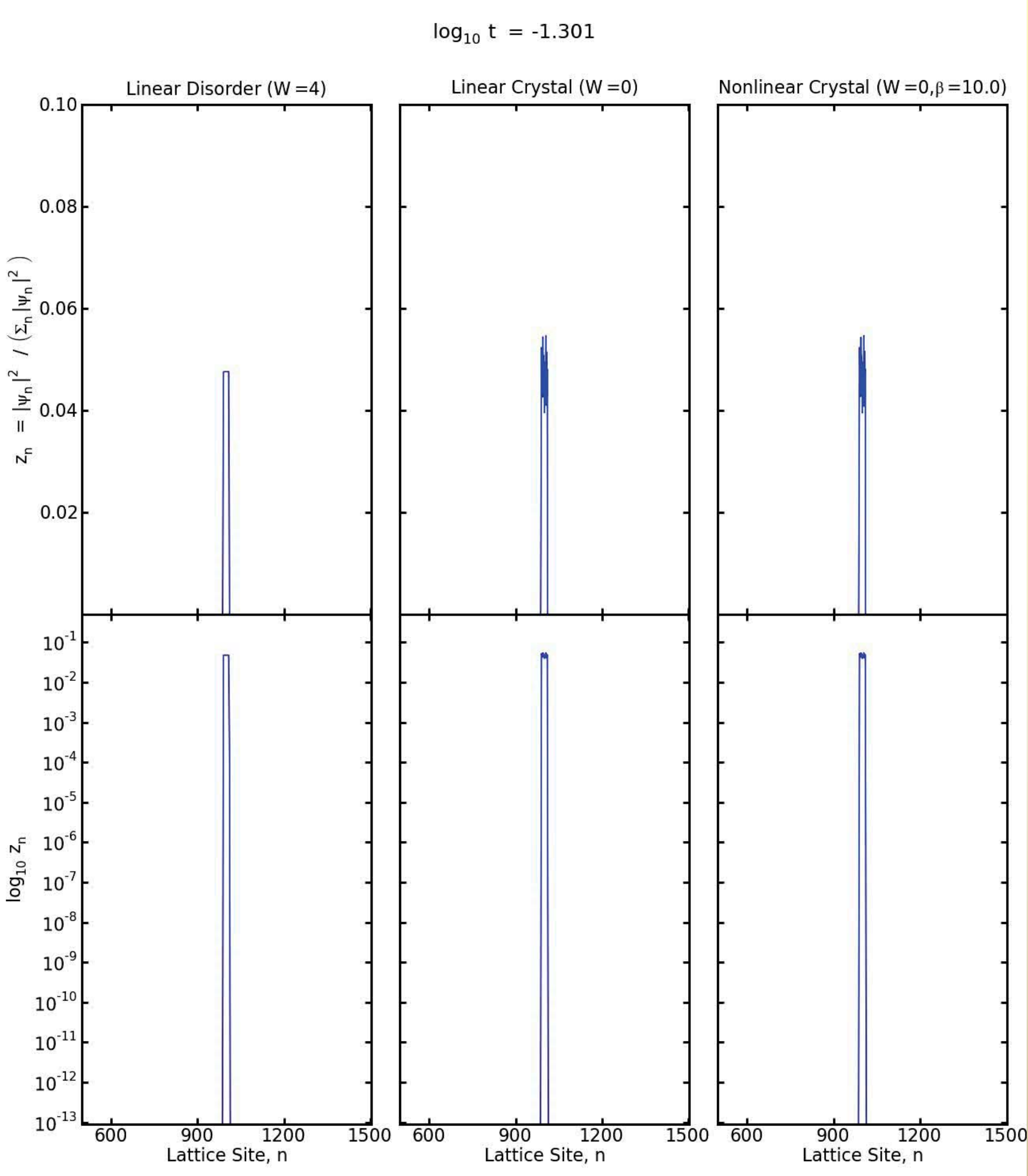
- The disordered lattice (Anderson, 1958)

$$-i\frac{\partial \psi_n}{\partial t} = E_n\psi_n + T_{n,n\pm1}\left[\psi_{n+1} + \psi_{n-1}\right]$$





log10 t = -1.301
Linear Disorder (W =4)
Linear Crystal (W =0)
Nonlinear Crystal (W =0,β =10.0)
zn = |ψn|2 / (Σn |ψn|2)
log10 zn
Lattice Site, n
600
900
1200
1500
0.02
0.04
0.06
0.08
0.10
10-1
10-13


$\log_{10} t = -1.301$
Linear Disorder (W =4)
Linear Crystal (W =0)
Nonlinear Crystal (W =0,β =10.0)
$z_n = |\psi_n|^2 / (\Sigma_n |\psi_n|^2)$
$\log_{10} z_n$
Lattice Site, n
0.10
0.08
0.06
0.04
0.02
600
900
1200
1500

Anderson localization

Anderson (1958)

$$i\frac{\partial \psi_l}{\partial t} = \epsilon_l \psi_l - \psi_{l+1} - \psi_{l-1}$$

$\{\epsilon_l\}$ in $[-W/2, W/2]$

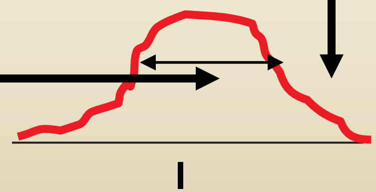
Eigenvalues: $\lambda_\nu \in \left[-2 - \frac{W}{2}, 2 + \frac{W}{2}\right]$

Width of EV spectrum: $\Delta = 4 + W$

Eigenvectors: $A_{\nu,l} \sim e^{-l/\xi(\lambda_\nu)}$

Localization length: $\xi(\lambda_\nu) \leq \xi(0) \approx 100/W^2$

Localization volume of NM: L



Anderson localization

Anderson (1958)

$$\lambda A_l = \epsilon_l A_l - A_{l-1} - A_{l+1}$$

$\{\epsilon_l\}$ **in** $[-W/2, W/2]$

Eigenvalues: $\lambda_\nu \in \left[-2 - \frac{W}{2}, 2 + \frac{W}{2}\right]$

Width of EV spectrum: $\Delta = 4 + W$

Eigenvectors: $A_{\nu,l} \sim \mathrm{e}^{-l/\xi(\lambda_\nu)}$

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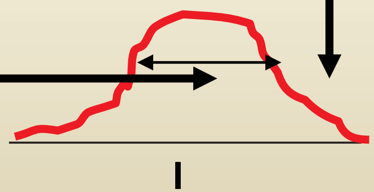


adding nonlinearity

Defining the problem

- a disordered medium
- linear equations of motion: all eigenstates are Anderson localized
- add short range nonlinearity (interactions)
- follow the spreading of an initially localized wave packet

$$i\dot{\psi}_l = \epsilon_l \psi_l \qquad - \psi_{l+1} - \psi_{l-1}$$

Defining the problem

- a disordered medium
- linear equations of motion: all eigenstates are Anderson localized
- add short range nonlinearity (interactions)
- follow the spreading of an initially localized wave packet

$$i\dot{\psi}_l = \epsilon_l \psi_l + \beta |\psi_l|^2 \psi_l - \psi_{l+1} - \psi_{l-1}$$

Defining the problem

- a disordered medium
- linear equations of motion: all eigenstates are Anderson localized
- add short range nonlinearity (interactions)
- follow the spreading of an initially localized wave packet

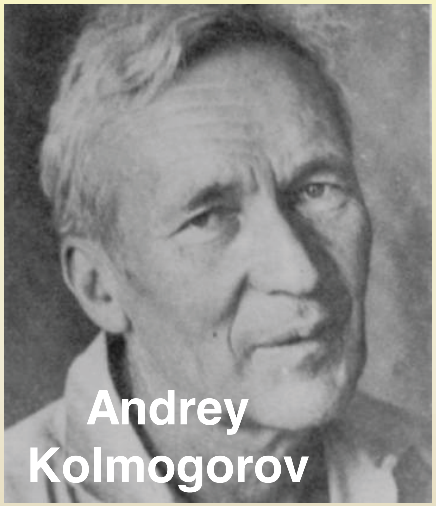
Will it delocalize?

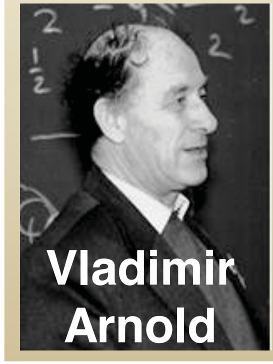
Yes because of nonintegrability and ergodicity

No because of energy conservation –
spreading leads to small energy density,
nonlinearity can be neglected,
dynamics becomes integrable, and
Anderson localization is restored

Kolmogorov – Arnold – Moser (KAM) theory

A.N. Kolmogorov,
Dokl. Akad. Nauk SSSR, 1954.
Proc. 1954 Int. Congress of Mathematics, North-Holland, 1957



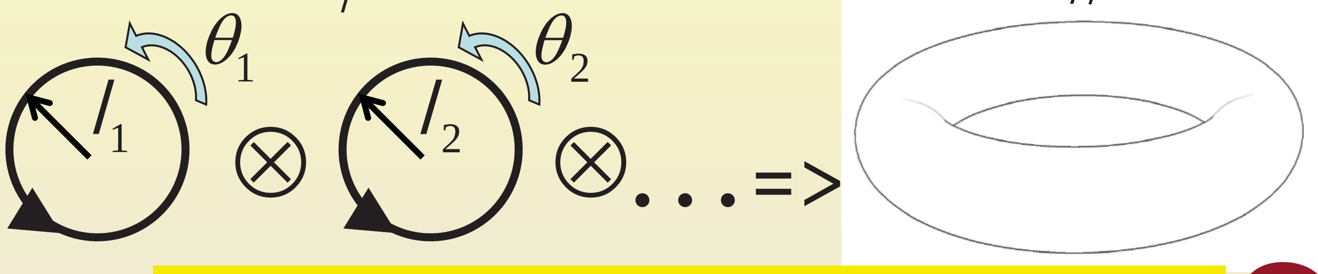


Integrable classical Hamiltonian $\hat{H}_0$, $d>1$:

Separation of variables: d sets of action-angle variables $I_1, \theta_1 = 2\pi\omega_1 t;.., I_2, \theta_2 = 2\pi\omega_2 t;..$

Quasiperiodic motion: set of the frequencies, $\omega_1, \omega_2, .., \omega_d$ which are in general incommensurate

Actions I_i are integrals of motion $\partial I_i / \partial t = 0$



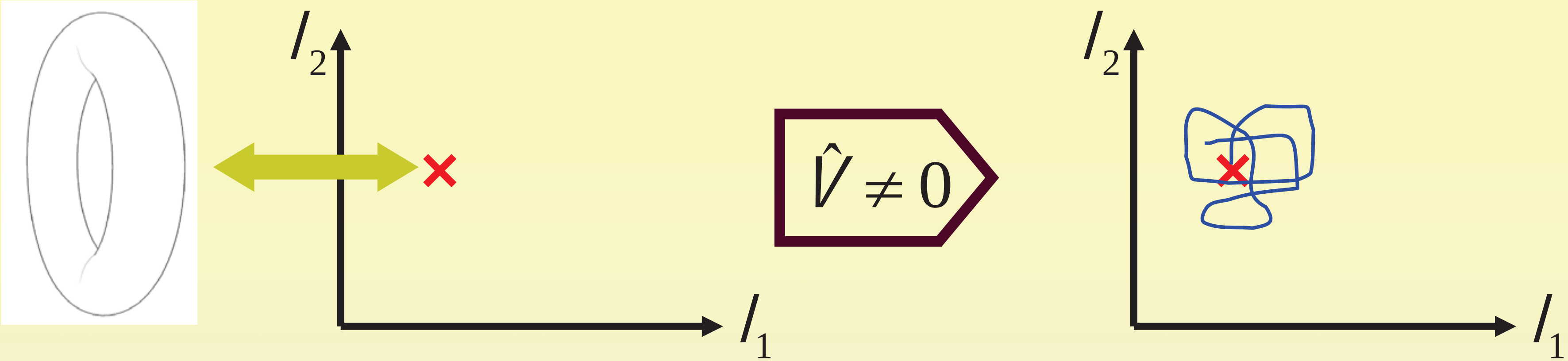
Q: Will an arbitrary weak perturbation $\hat{V}$ of the integrable Hamiltonian $\hat{H}_0$ destroy the tori and make the motion ergodic (when each point at the energy shell will be reached sooner or later) **?**

A: **Most of the tori survive weak and smooth enough perturbations**

KAM theorem

KAM theorem:

Most of the tori survive weak and smooth enough perturbations



Each point in the space of the **integrals of motion** corresponds to a torus and vice versa

Finite motion.
Localization in the **space of the integrals of motion** ?

- **KAM applies to finite systems**
- **Does it apply to waves in infinite systems?**
- **How are KAM thresholds scaling with number of degrees of freedom?**
- **Will nonlinear waves observe KAM regime?**
- **If they do – then localization remains**
- **If they do not – waves can delocalize**

Equations in normal mode space:

$$i\dot{\phi}_\nu = \lambda_\nu \phi_\nu + \beta \sum_{\nu_1,\nu_2,\nu_3} I_{\nu,\nu_1,\nu_2,\nu_3} \phi^*_{\nu_1} \phi_{\nu_2} \phi_{\nu_3}$$

$$I_{\nu,\nu_1,\nu_2,\nu_3} = \sum_l A_{\nu,l} A_{\nu_1,l} A_{\nu_2,l} A_{\nu_3,l}$$

NM ordering in real space: $X_\nu = \sum_l l A^2_{\nu,l}$

Characterization of wavepackets in normal mode space:

$z_\nu \equiv |\phi_\nu|^2 / \sum_\mu |\phi_\mu|^2$ $\bar{\nu} = \sum_\nu \nu z_\nu$

Second moment: $m_2 = \sum_\nu (\nu - \bar{\nu})^2 z_\nu$ → **location of tails**

Participation number: $P = 1/\sum_\nu z_\nu^2$ → **number of strongly excited modes**

Compactness index: $\zeta = \frac{P^2}{m_2}$

→ **K adjacent sites equally excited:** $\zeta = 12$

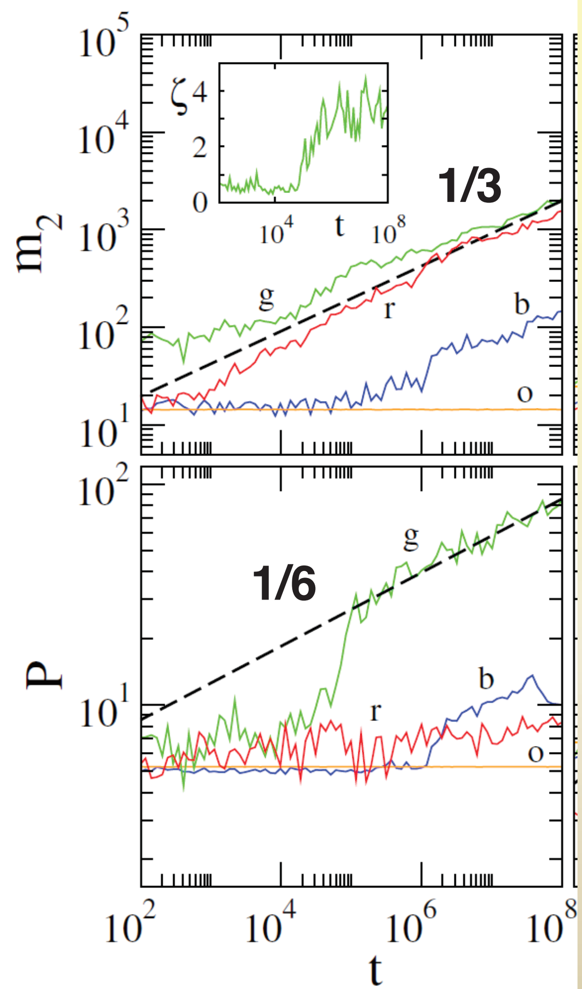
→ **K adjacent sites, every second empty or equipartition:** $\zeta = 3$

Single site excitations: subdiffusion!

$$\psi_l = \delta_{l,l_0} \quad \epsilon_{l_0} = 0$$

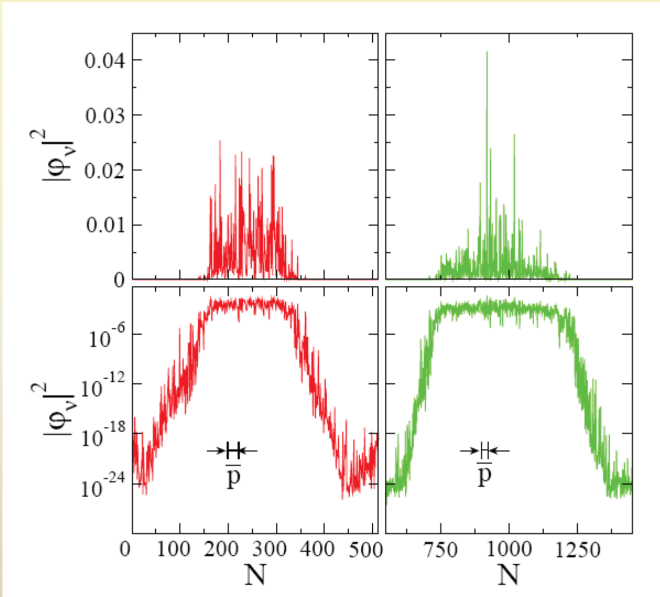
M. Molina PRB 1998, A. Pikovsky et al PRL 2008, G. Kopidakis et al PRL 2008, SF et al PRL 2009

W=4, β= 0, 0.1, 1, 4.5



Wavepacket spreads way beyond localization volume.

$$t = 10^8$$



Integrable approximation keeps Anderson Localization:

$$\mathcal{H}_{int} = \sum_{\nu} \lambda_\nu J_\nu + \beta \sum_{\nu_1,\nu_2,\nu_3,\nu_4} I_{\nu_1,\nu_2,\nu_3,\nu_4} \sqrt{J_{\nu_1} J_{\nu_1} J_{\nu_1} J_{\nu_1}}$$

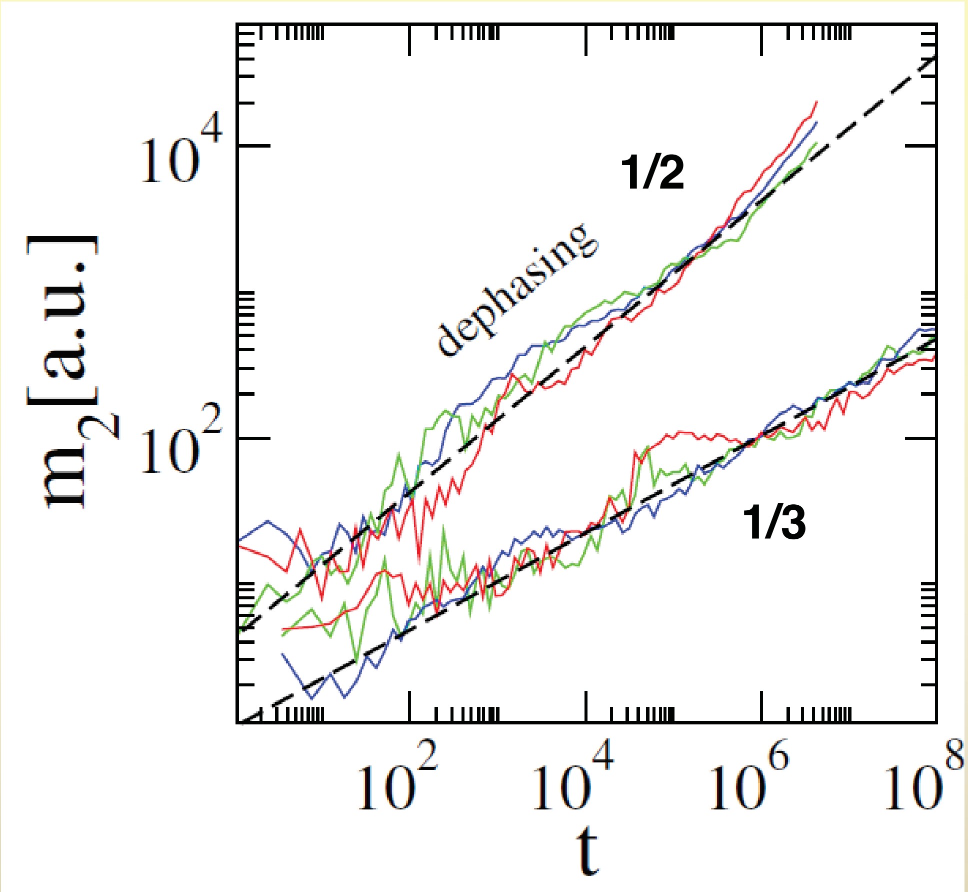
wavepackets stay localized due to overlap integral characteristics

Then observed spreading must be due to

- **deterministic chaos**
- **decoherence of normal mode phases**

Test: additional manual dephasing in normal mode space

W=4,7,10 β= 3,4,6



SF et al PRL 2009

$$m_2 \sim t^\alpha$$

α = ???

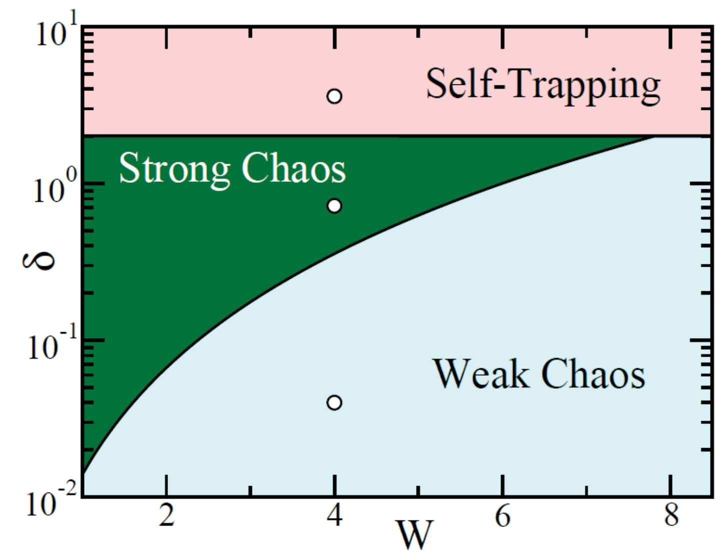
Frequency scales

W=4 :

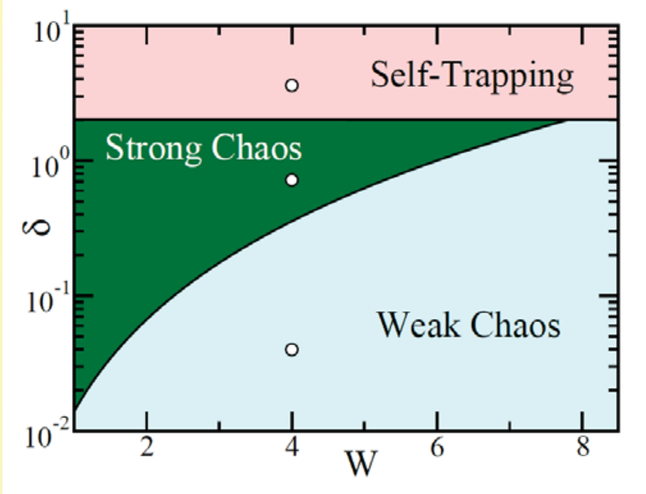
- Eigenvalue (frequency) spectrum width: $\Delta = W + 4$ — **8**
- Localization volume of eigenstate: $V \approx 360/W^2$ — **~18 (sites)**
- Average frequency spacing inside localization volume: $d = \Delta/V$ — **0.43**
- Nonlinearity induced frequency shift: $\delta_l = \beta|\psi_l|^2$

Three expected evolution regimes:
Weak chaos : $\delta < d$
Strong chaos : $d < \delta < 2$
(partial) self trapping : $2 < \delta$

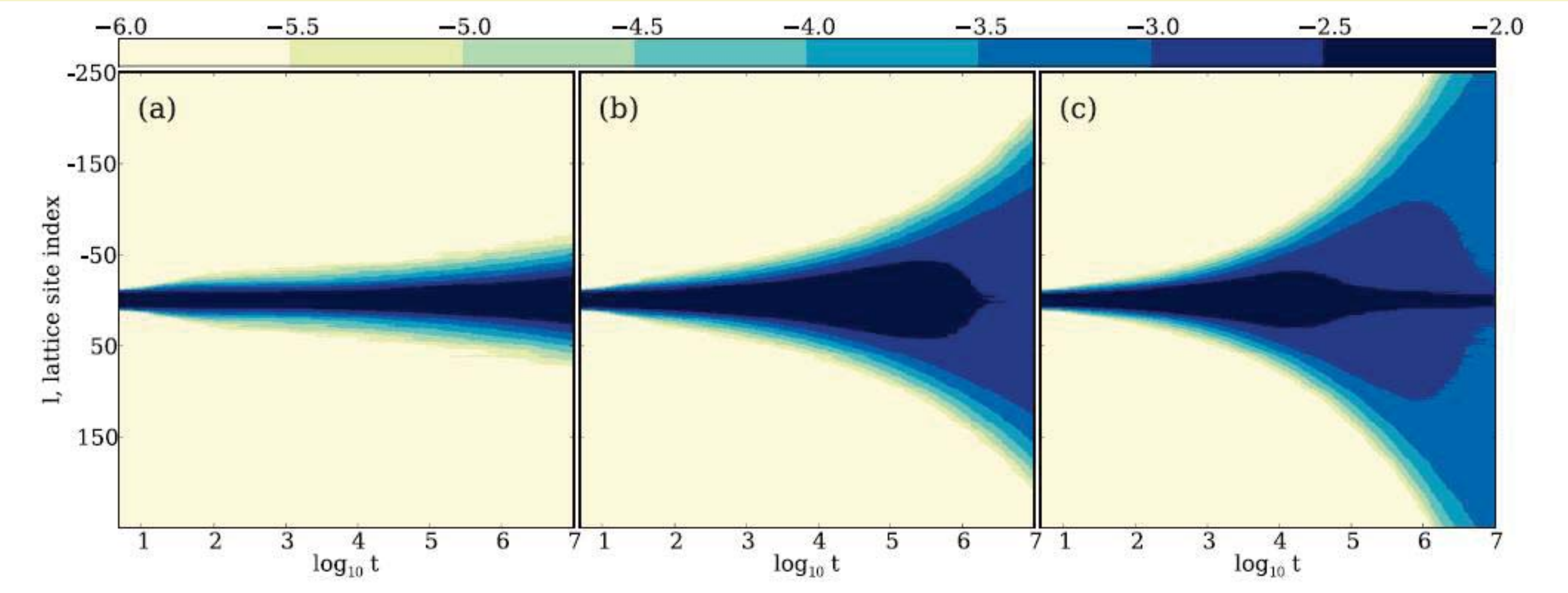
SF Chem Phys 2010, TV Laptyeva et al EPL 2010



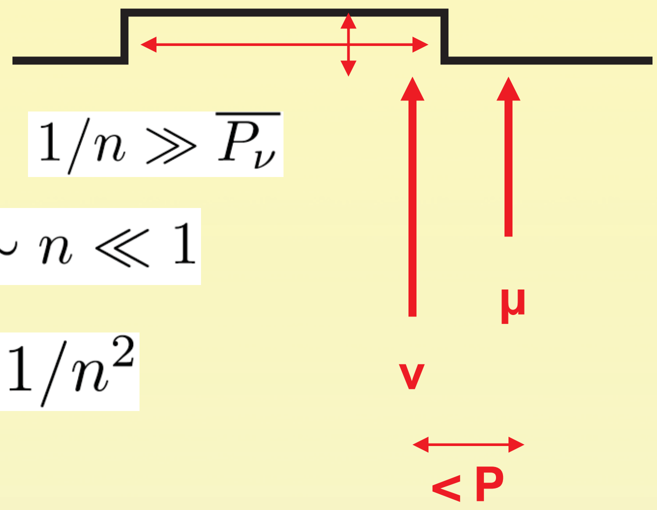
W=4
Wave packet with 20 sites
Norm density = 1
Random initial phases
Averaging over 1000 realizations



J Bodyfelt et al PRE 2011



Explaining subdiffusion?



- at some time t packet contains 1/n modes: $1/n \gg \overline{P_\nu}$
- each mode on average has norm $|\phi_\nu|^2 \sim n \ll 1$
- the second moment amounts to $m_2 \sim 1/n^2$

Simplest assumption:

- some modes in packet interact resonantly and therefore evolve chaotic
- Probability of resonance: P(βn)
- all phases decohere on some time scale

exterior mode:

$$i\dot{\phi}_\mu \approx \lambda_\mu \phi_\mu + \beta n^{3/2} \mathcal{P}(\beta n) f(t)$$

$$\langle f(t) f(t') \rangle = \delta(t - t')$$

E Michaely et al
PRE 2012

momentary diffusion rate:

$$D = 1/T \sim \beta^2 n^2 (\mathcal{P}(\beta n))^2$$

D Krimer et al PRE 2010

$$\mathcal{P} = 1 - \mathrm{e}^{-C\beta n} \qquad \frac{1}{C} \approx d \qquad 1/n^2 \sim \beta(1 - e^{-\beta n/d})t^{1/2}$$

SF ChemPhys 2010

$$m_2 \sim \begin{cases} \beta t^{1/2}, & \beta n/d > 1 \text{ (strong chaos)} \\ d^{-2/3}\beta^{4/3}t^{1/3}, & \beta n/d < 1 \text{ (weak chaos)} \end{cases}$$

Generalizations: higher dimensions, nonlinearity exponent σ:

$$i\dot{\psi}_{\boldsymbol{l}} = \epsilon_{\boldsymbol{l}}\psi_{\boldsymbol{l}} - \beta|\psi_{\boldsymbol{l}}|^{\sigma}\psi_{\boldsymbol{l}} - \sum_{\boldsymbol{m}\in D(\boldsymbol{l})} \psi_{\boldsymbol{m}}$$

$$D \sim \beta^2 n^{\sigma}(\mathscr{P}(\beta n^{\sigma/2}))^2$$

$$m_2 \sim (\beta^2 t)^{\frac{2}{2+\sigma D}}, \text{ strong chaos,}$$
$$m_2 \sim (\beta^4 t)^{\frac{1}{1+\sigma D}}, \text{ weak chaos.}$$

Asymptotic regime of weak chaos

SF et al PRL 2009, Ch. Skokos et al PRE 2009

We averaged the measured exponent over 20 realizations:

$\alpha = 0.33 \pm 0.02$ (DNLS)
$\alpha = 0.33 \pm 0.05$ (KG)

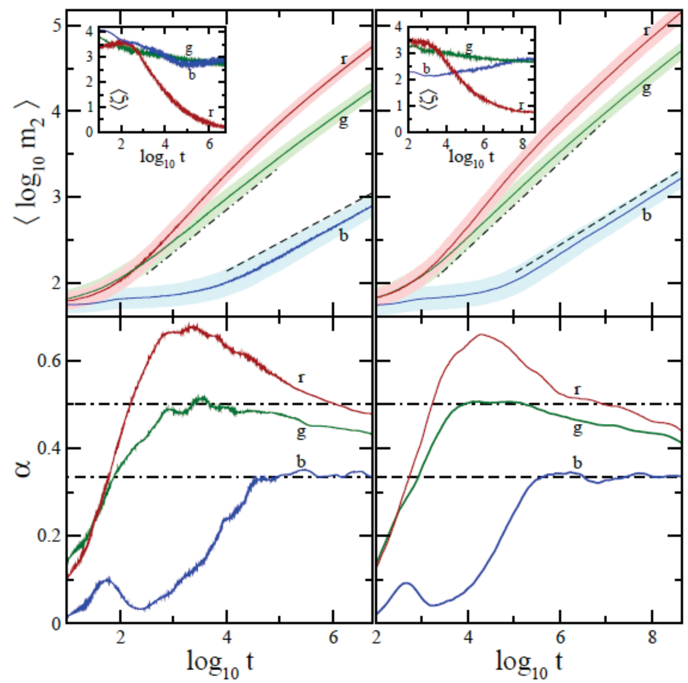
Strong chaos and crossover to weak chaos

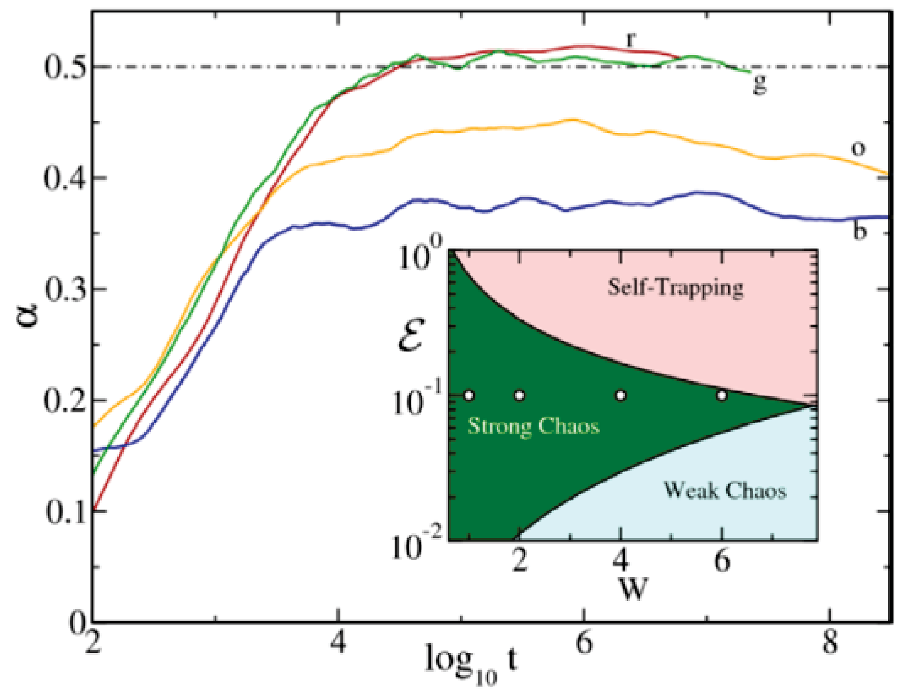
TV Laptyeva et al EPL 2010

Averaging over 1000 realizations, measuring

KG

DNLS, W=4 KG, W=4

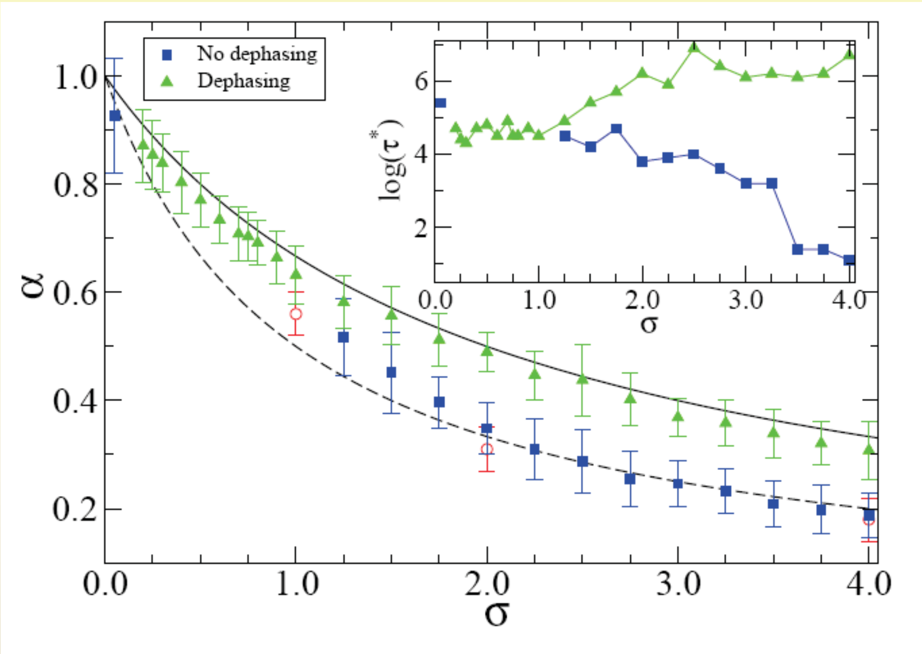




Generalizations: higher dimensions, nonlinearity exponent σ:

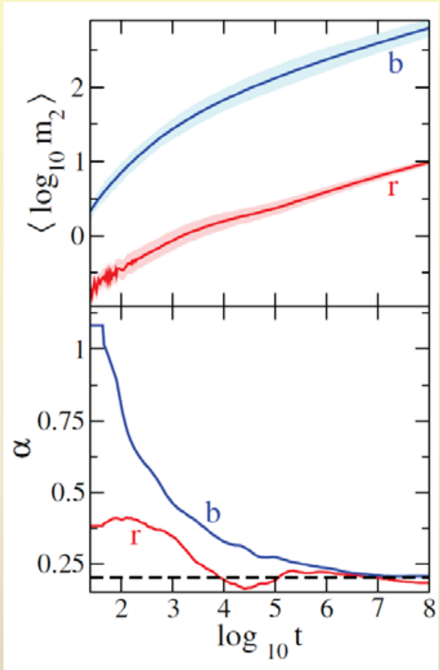
$$i\dot{\psi}_{\boldsymbol{l}} = \epsilon_{\boldsymbol{l}}\psi_{\boldsymbol{l}} - \beta|\psi_{\boldsymbol{l}}|^{\sigma}\psi_{\boldsymbol{l}} - \sum_{\boldsymbol{m}\in D(\boldsymbol{l})}\psi_{\boldsymbol{m}}$$

D=1, 0 < σ < 4 :



Ch Skokos et al PRE 2010

D=2, σ = 2 :

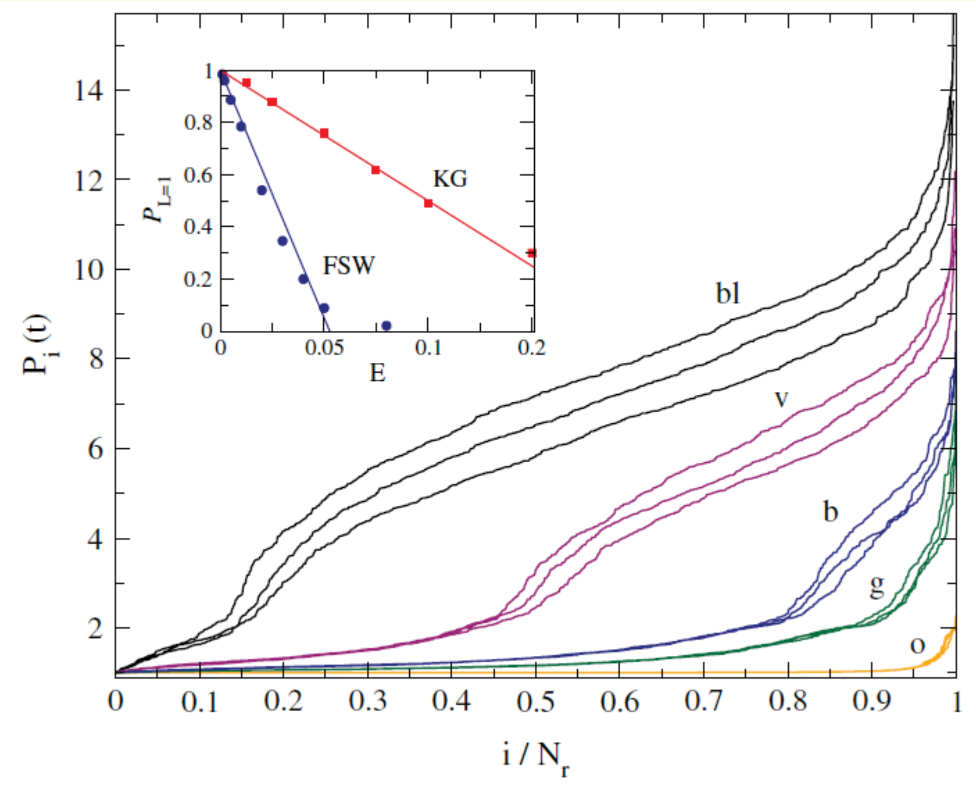


TV Laptyeva et al, EPL 2012

Related results by M Mulansky

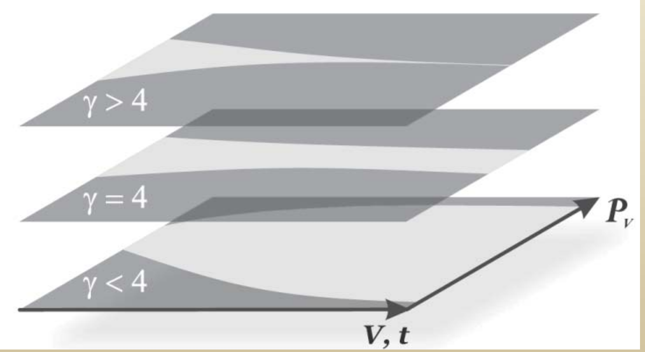
Restoring Anderson localization?   A matter of probability and KAM!

MV Ivanchenko et al PRL 2011



E : total energy
L : size of initial wave packet

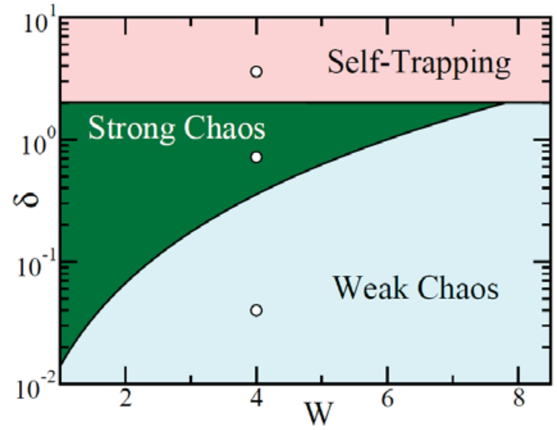
$$\mathcal{P}_L = \left(1 - \frac{3\kappa E}{L}\right)^{2L}$$



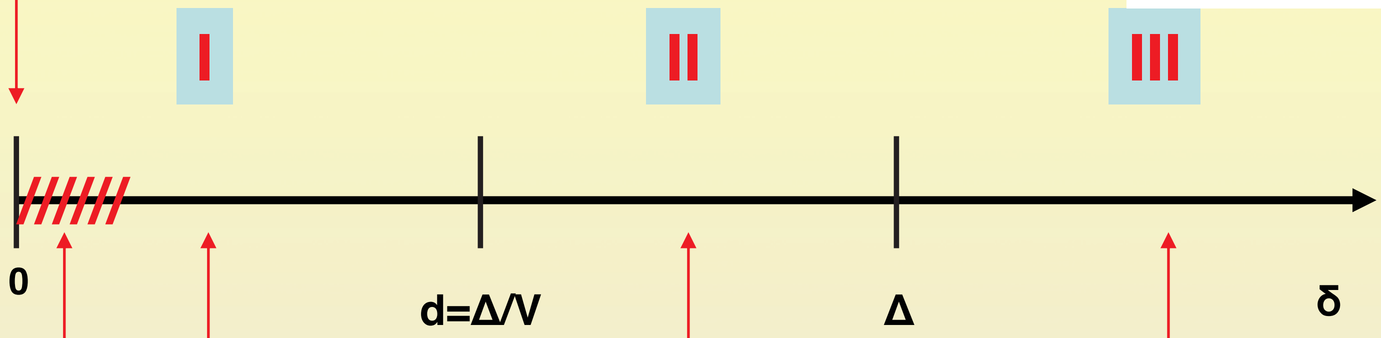
Generalizing:
d: dimension
V: volume of wave packet
γ : = 2σ

Related results by Aubry,Johansson

The emerging picture



Anderson Localization



KAM

Weak Chaos

Strong Chaos

Selftrapping, part of packet IS spreading

SF,Krimer,Skokos (2009)
Shepelyansky and Pikovsky (2008)
Molina (1998)

SF (2010)
Bodyfelt,Lapteva,Krimer,
Skokos,SF (2010)

Kopidakis, Komineas,
SF, Aubry (2008)

In all cases subdiffusive spreading

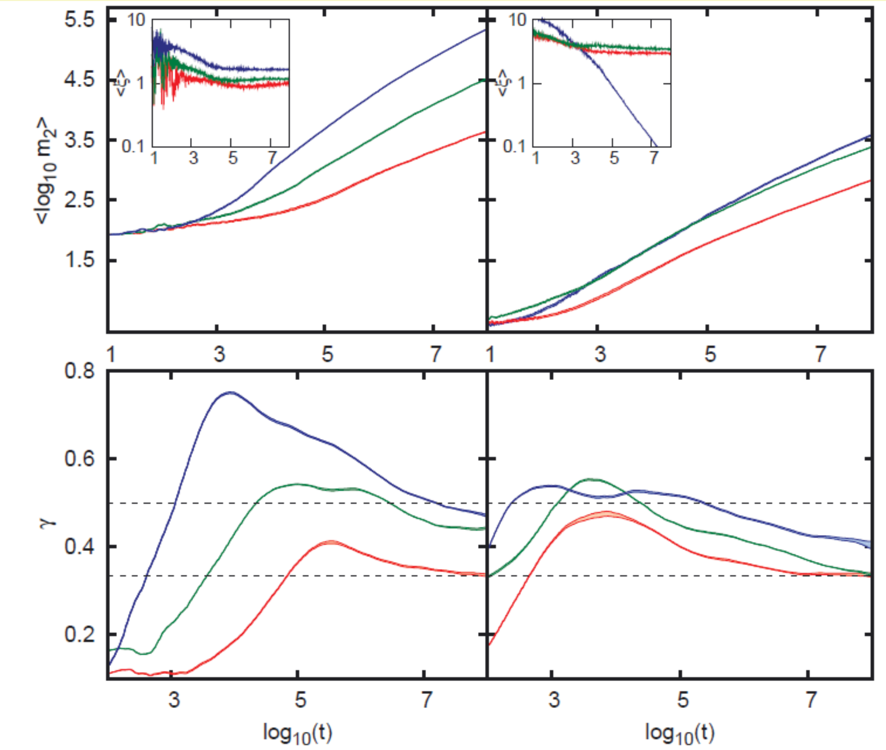
other localizing media

Quasiperiodic potentials (Aubry-Andre):

M Larcher et al, arXiv1206.0833
` New J Phys, in print

$$i\frac{\partial \psi_j}{\partial t} = -(\psi_{j+1} + \psi_{j-1}) + V_j\psi_j + \beta|\psi_j|^2\psi_j$$

$$V_j = \lambda \cos(2\pi\alpha j + \varphi)$$



Pecularities:

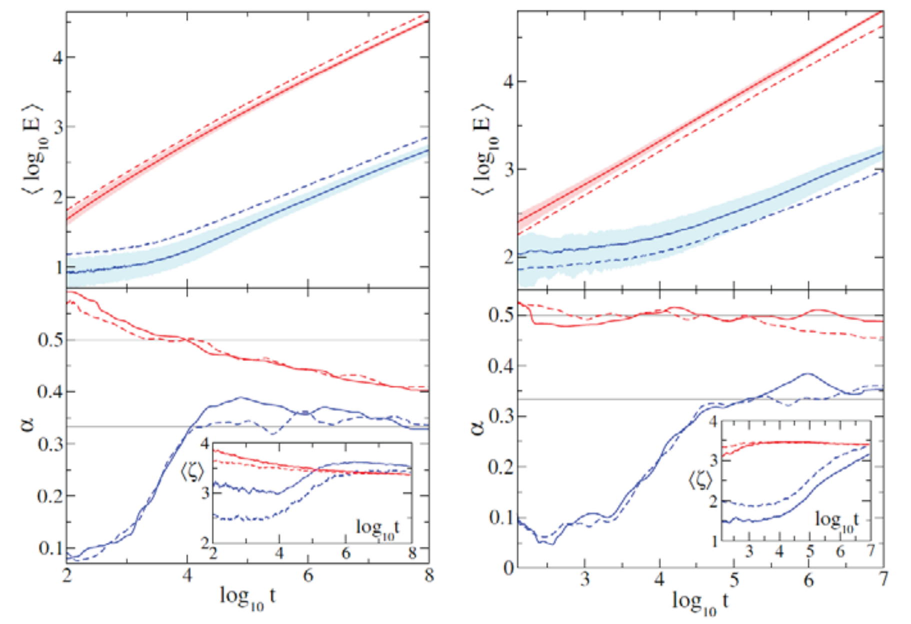
- spectrum with gaps
- subgaps etc
- fractal properties
- gap selftrapping
- hierarchy of level spacings
- α = 1/3

Nonlinear Quantum Kicked Rotor

G Gligoric et al EPL 2011

$$A_n(t+1) = \sum_m (-i)^{n-m} J_{n-m}(k) A_m(t) e^{-i\frac{\tau}{2}m^2 + i\beta|A_m(t)|^2}$$

Spreading first observed by D Shepelyansky PRL 1993



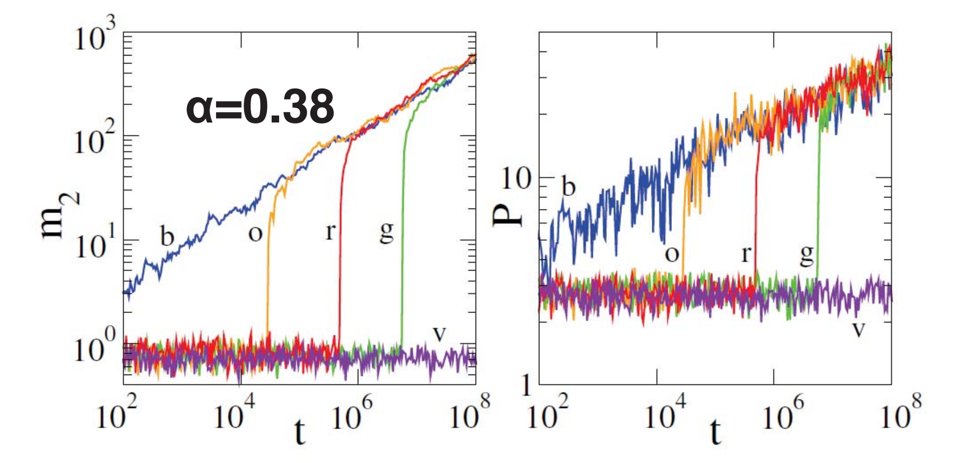
Pecularities:

- **spectrum in compact space**
- **no selftrapping**
- **α = 1/3**

Nonlinear Wannier-Stark ladder

D Krimer et al PRE 2009

$$i\dot{\Psi}_n = -(\Psi_{n+1} + \Psi_{n-1}) + nE\Psi_n + \beta|\Psi_n|^2\Psi_n$$



E=2, β=8, ...,9

Pecularities:

- spectrum is equidistant
- exact resonances
- absence of universality
- exponents depend on E

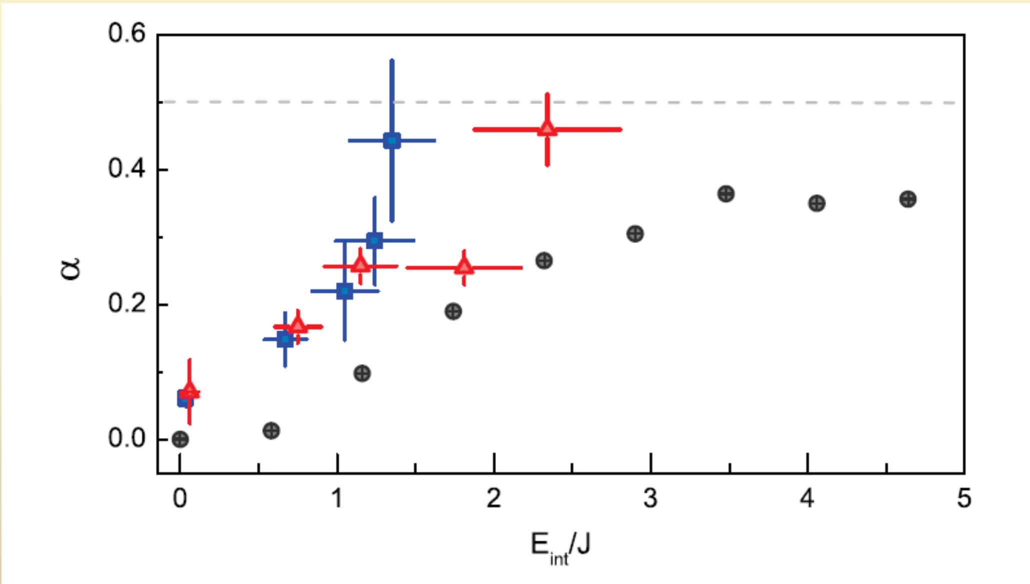
1st experimental confirmation from Firenze

PRL **106,** 230403 (2011) PHYSICAL REVIEW LETTERS wee
10 J

Observation of Subdiffusion in a Disordered Interacting System

E. Lucioni,[1,*] B. Deissler,[1] L. Tanzi,[1] G. Roati,[1] M. Zaccanti,[1,†] M. Modugno,[2,3] M. Larcher,[4]
F. Dalfovo,[4] M. Inguscio,[1] and G. Modugno[1,‡]

Bose-Einstein condensate of ^{39}K atoms



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Pramana J Phys 77 (2011) 1007

heat conductivity

Consequencies for thermal conductivity

Norm density n ~ energy density ε ~ temperature T

Diffusion rate inside wave packet:

$$D \sim \varepsilon^2 \mathcal{P}^2(\varepsilon)$$

$$\mathcal{P}(\varepsilon) \approx 1 - \mathrm{e}^{-a\varepsilon/d}$$

Thermal conductivity:

$$\kappa \sim T^2 \left(1 - \mathrm{e}^{-bT/d}\right)^2$$

$$\kappa \sim T^4 \,, \; T \ll d \text{ (weak chaos)}$$
$$\kappa \sim T^2 \,, \; T \gg d \text{ (strong chaos)}$$

Model 1: The Klein-Gordon chain

$$\mathcal{H}_K = \sum_l \frac{p_l^2}{2} + \frac{\tilde{\epsilon}_l}{2} u_l^2 + \frac{1}{4} u_l^4 + \frac{1}{2W}(u_{l+1} - u_l)^2$$

$$\ddot{u}_l = -\partial \mathcal{H}_K / \partial u_l$$

$\tilde{\epsilon}_l$ **uniformly from** $\left[\frac{1}{2}, \frac{3}{2}\right]$

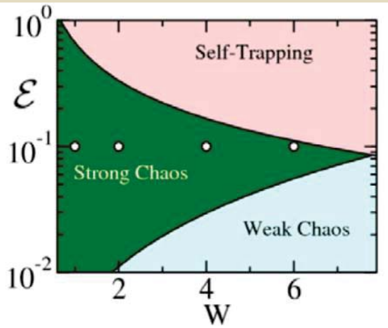
$$\ddot{u}_l = -\tilde{\epsilon}_l u_l - u_l^3 + \frac{1}{W}(u_{l+1} + u_{l-1} - 2u_l)$$

Inducing a heat flow and measuring conductivity:

$$\ddot{u}_{1,N} = -\partial H / \partial u_{1,N} + \xi_{1,N} - \lambda \dot{u}_{1,N}$$

$$\langle \xi_{1,N}(t) \xi_{1,N}(0) \rangle = 2 \lambda T_{1,N} \delta(t)$$

$$j_{KG} = -\frac{1}{2W} \sum_l (\dot{u}_{l+1} + \dot{u}_l)(u_{l+1} - u_l)$$



$$\kappa = jN/(T_N - T_1)$$

$$T = (T_N + T_1)/2$$
$$(T_N - T_1)/T = 0.5$$

Model 2: The Fröhlich-Spencer-Wayne chain

$$\mathcal{H}_{FSW} = \sum_l \frac{p_l^2}{2} + \frac{\tilde{\epsilon}_l}{2} u_l^2 + \frac{1}{4}(u_{l+1} - u_l)^4$$

J. Fröhlich, T. Spencer and C. E. Wayne, J. Stat. Phys. **42**, 247 (1986)

- limit of KG model for strong disorder
- renormalized frequencies
- artificial total mechanical momentum conservation in anharmonic FPU part
- only doublet interactions, but completely uncorrelated frequencies

- only weak chaos and crossover to pseudo-FPU regime since d~1

KG: size effects

SF, MV Ivanchenko, N Li
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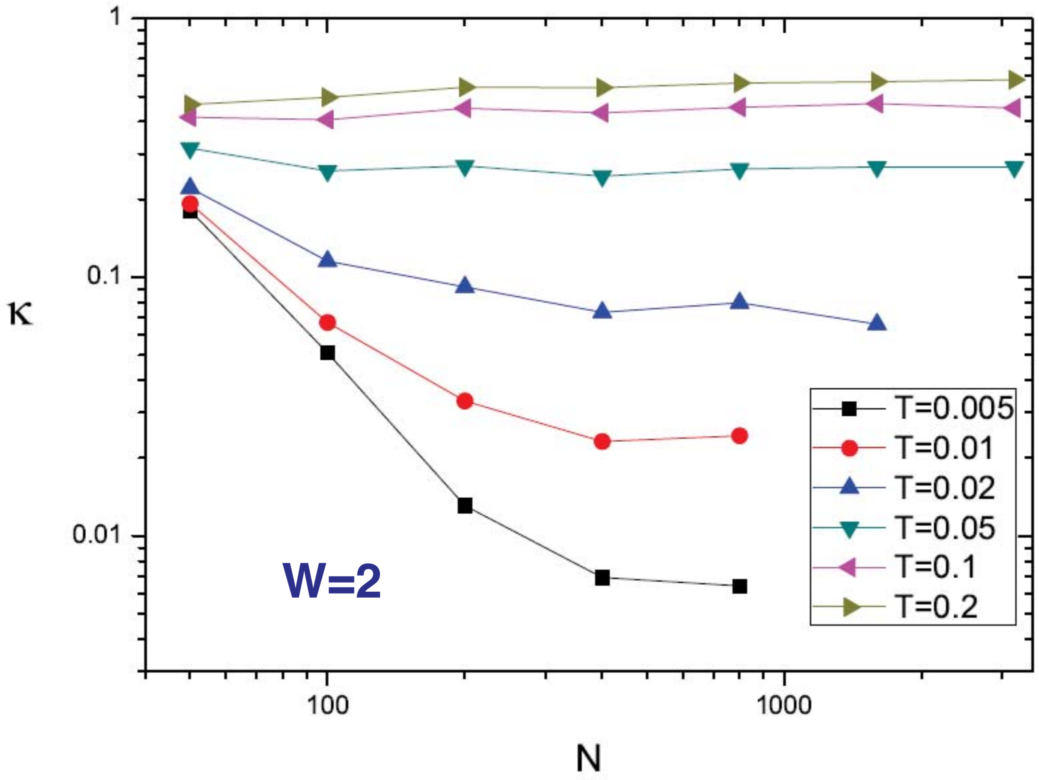
Linear chain: $\kappa_{linear} \sim \mathrm{e}^{-N/\xi}$

Assume for nonlinear chain: $\kappa \sim T^{\alpha}$

Needed system size: $N \sim -\alpha\xi \ln T$



KG: results

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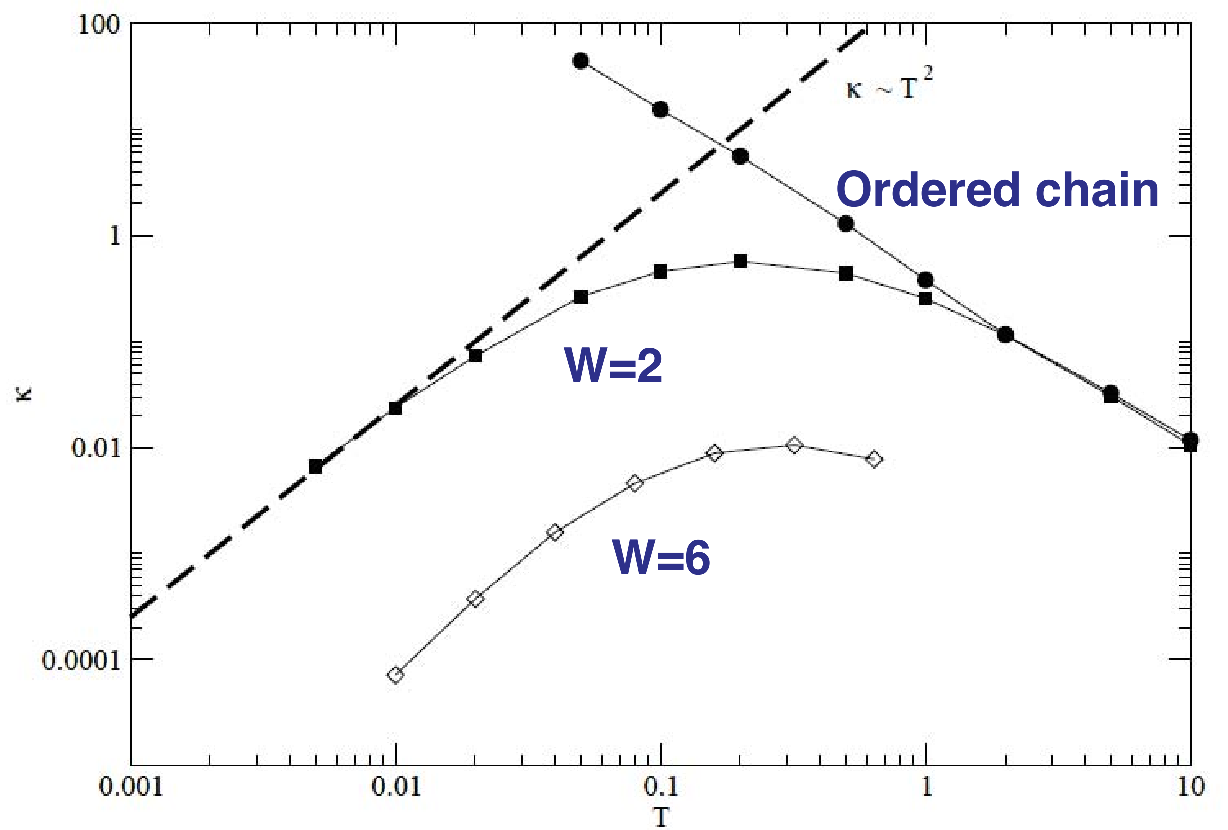

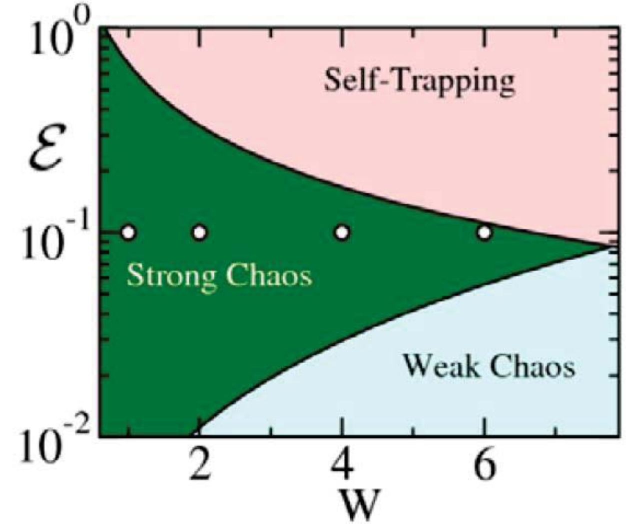

- evidence for dropping of k with dropping T
- no contradiction to predictions
- no real confirmation either
- much better computations needed, other comp. architechture required
- or another splendid idea wanted

FSW: results

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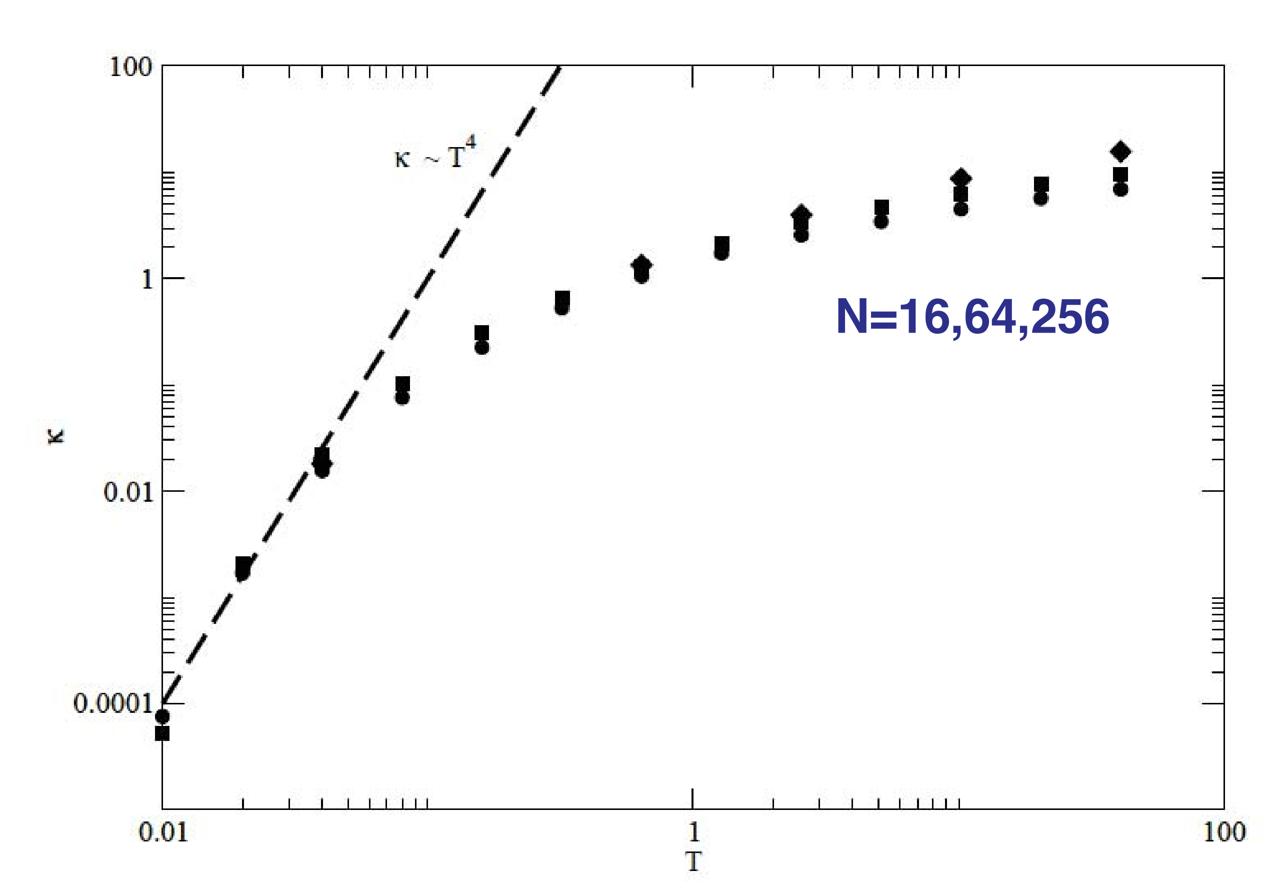

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going quantum

Two interacting particles in a quasiperiodic potential

$$\hat{\mathcal{H}} = \sum_j \left[\hat{b}^+_{j+1}\hat{b}_j + \hat{b}^+_j\hat{b}_{j+1} + \epsilon_j \hat{b}^+_j\hat{b}_j + \frac{U}{2}\hat{b}^+_j\hat{b}^+_j\hat{b}_j\hat{b}_j \right]$$

basis:

$$|q\rangle = \sum_{m,l\leqslant m}^{N} \mathcal{L}^{(q)}_{l,m}|l,m\rangle, \qquad |l,m\rangle \equiv \frac{b^+_l b^+_m|0\rangle}{\sqrt{1+\delta_{lm}}}$$

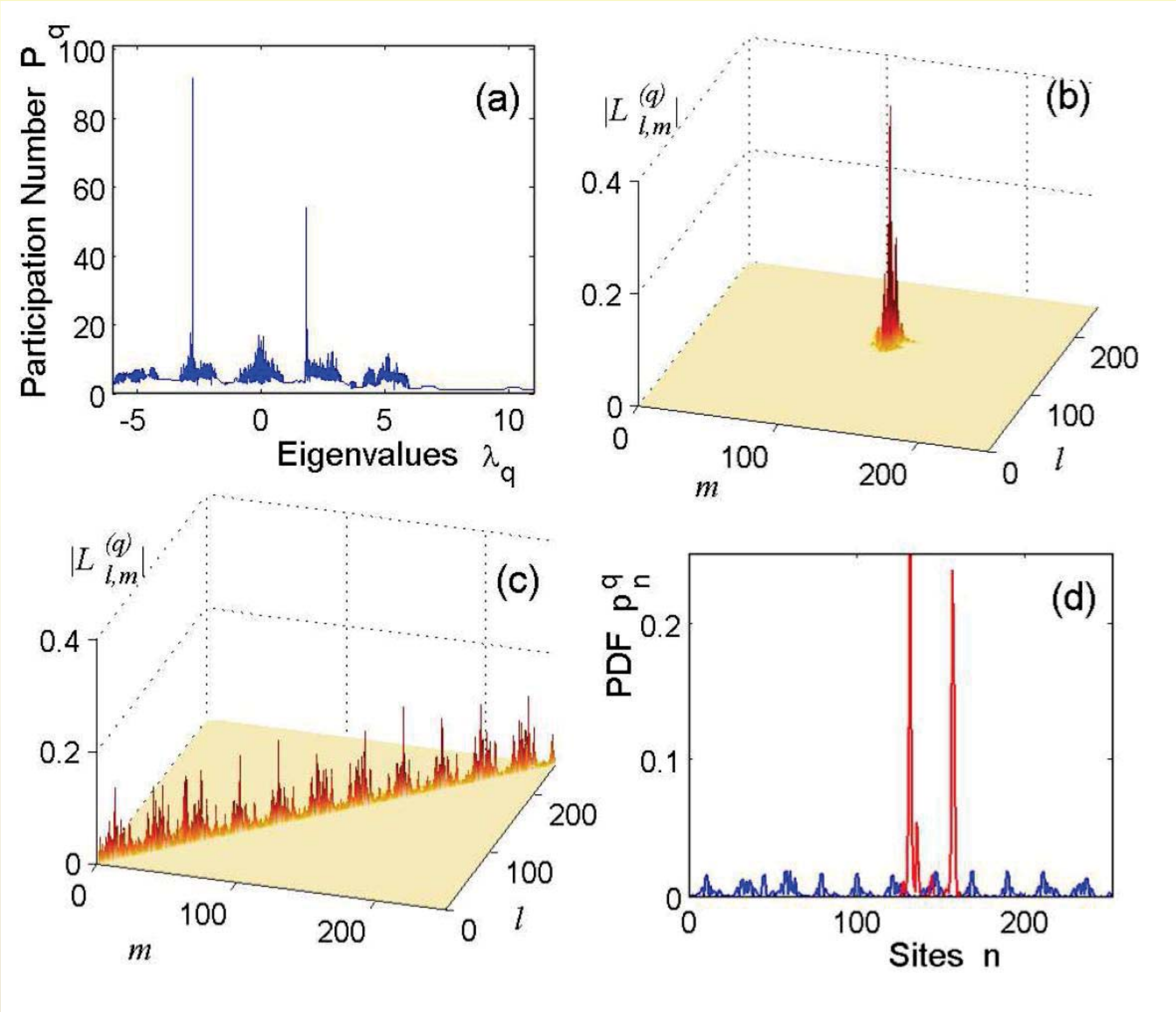
M.V. Ivanchenko
R. Khomeriki
S. Flach
EPL 98 66002 (2012)

pdf of particle density:

$$p^{(q)}_l = \frac{\langle q|\hat{b}^+_l\hat{b}_l|q\rangle}{2} = \frac{1}{2}\left(\sum_{k,l\leqslant k}^{N} \mathcal{L}^{(q)2}_{l,k} + \sum_{m,l\geqslant m}^{N} \mathcal{L}^{(q)2}_{m,l} \right)$$

Participation number of density pdf: $P_q = 1/\sum_l^N (p^{(q)}_l)^2$

Results: eigenfunctions

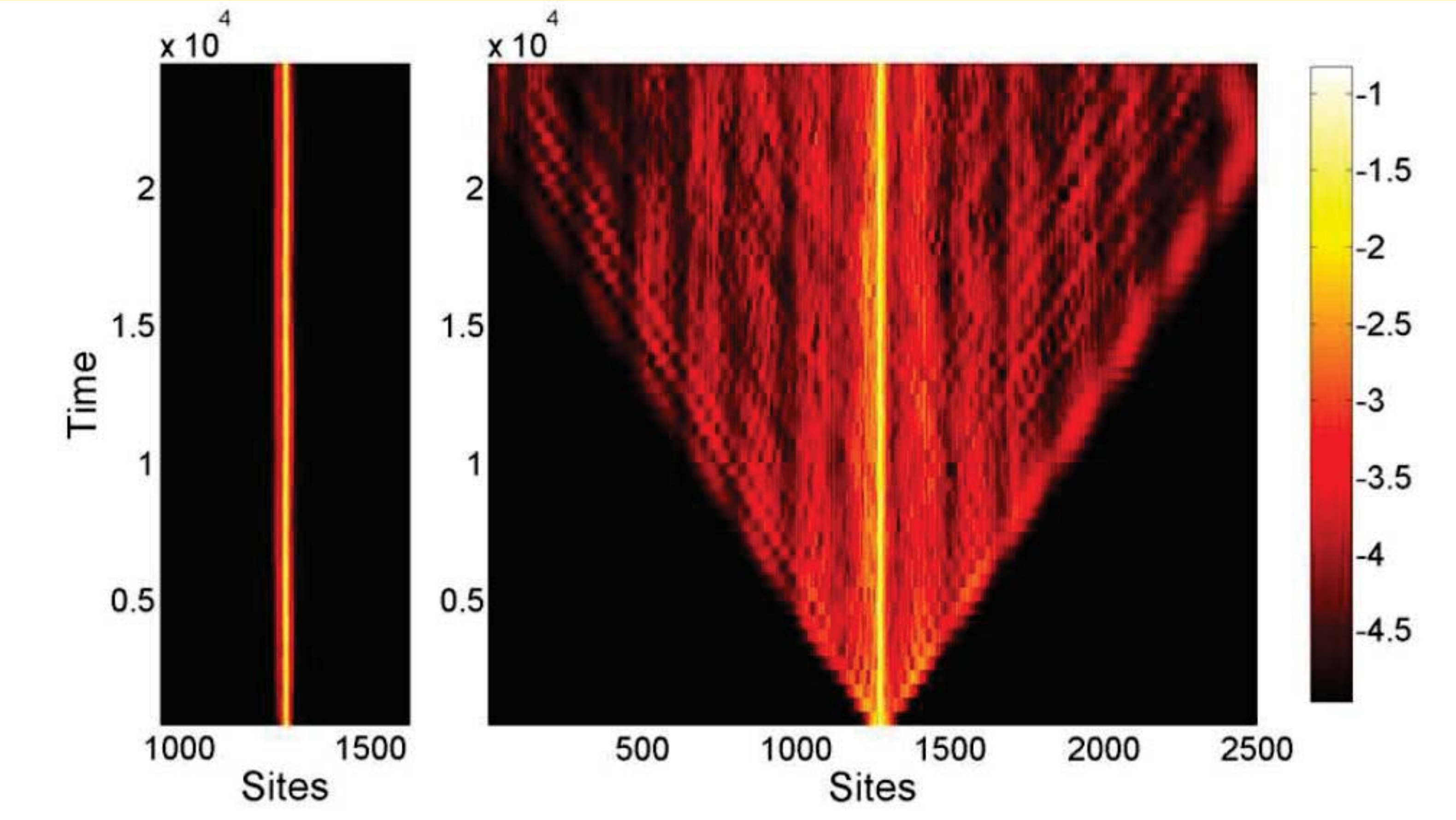


M.V. Ivanchenko
R. Khomeriki
S. Flach
EPL 98 66002 (2012)

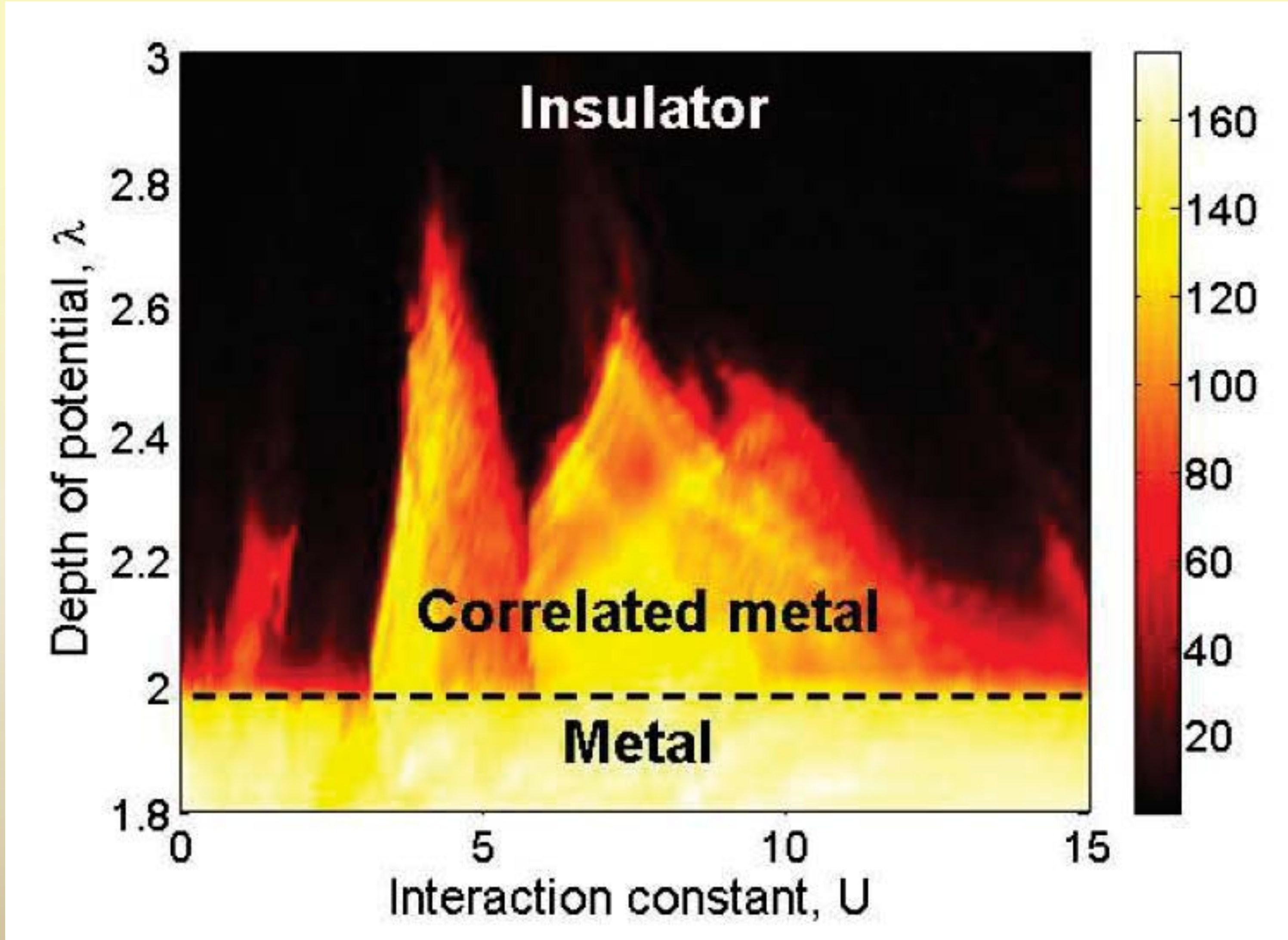
$U = 7.9$ and $\lambda = 2.5$

Results: PDF of spreading of wave packet with λ=2.5 and N=2500 and two particles initially at adjacent sites

U=2 **U=4.5**



Results: the complete picture from spreading wave packets: square rooted 2nd moment for 60 different realizations



Summary of Lecture III

Fishman,Pikovsky,Basko,Aubry: slowing down of spreading?
we searched and did not find ANY signature of slowing down

Shepelyansky: different predictions for exponents?
our numerics exclude them. Some arguments appear to be incorrect

Heat conductivity measurements (SF et al Pramana J Phys 2011):
energy density dependence follows predictions

Two interacting quantum particles:
disorder: weak enhancement of localization length
D. Krimer et al JETP Letters 2011
quasiperiodic potentials: complete delocalization
MV Ivanchenko et al EPL 2012

Nonlinear diffusion equations and scaling (arXiv:1206.6085):
very good agreement with spreading of wave packets

Speeding up subdiffusion for easier experimental studies (arXiv:1210:3148):
is possible and agrees with main underlying assumptions

Take Home Messages

- nonlinear dynamical systems – nonintegrability, chaos
- quasiperiodic motion destroyed, BUT:
- periodic orbits are generic low-d invariant manifolds
- breathers are essential periodic orbits which describe the evolution of relevant mode-mode interactions, correlations in and relaxations of complex systems
- in the long run chaos destroys coherence and therefore wave localization is lost in any of its forms