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Thermal Conductance of Uniform Quantum Wires

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Thermal conductance of uniform quantum wires

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Outline

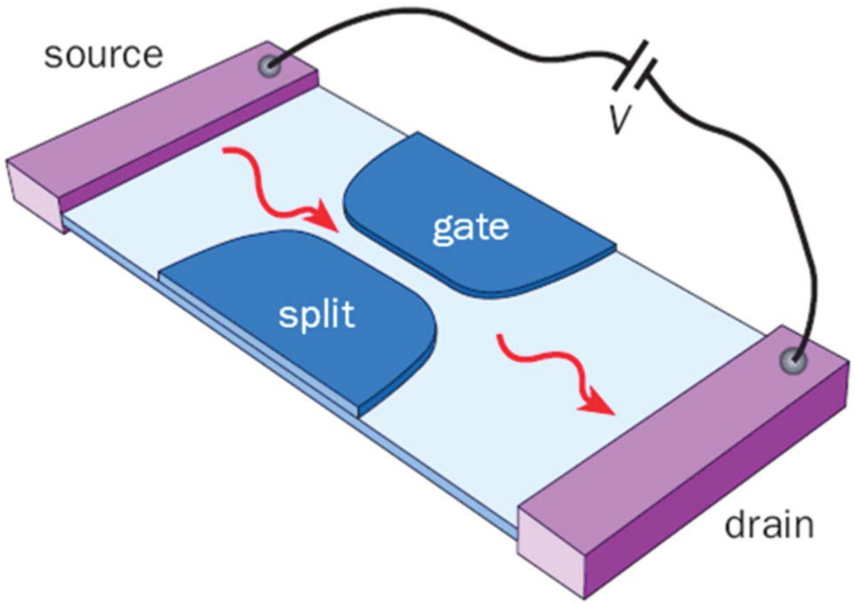
Motivation: Experiments with quantum wires, quantized conductance

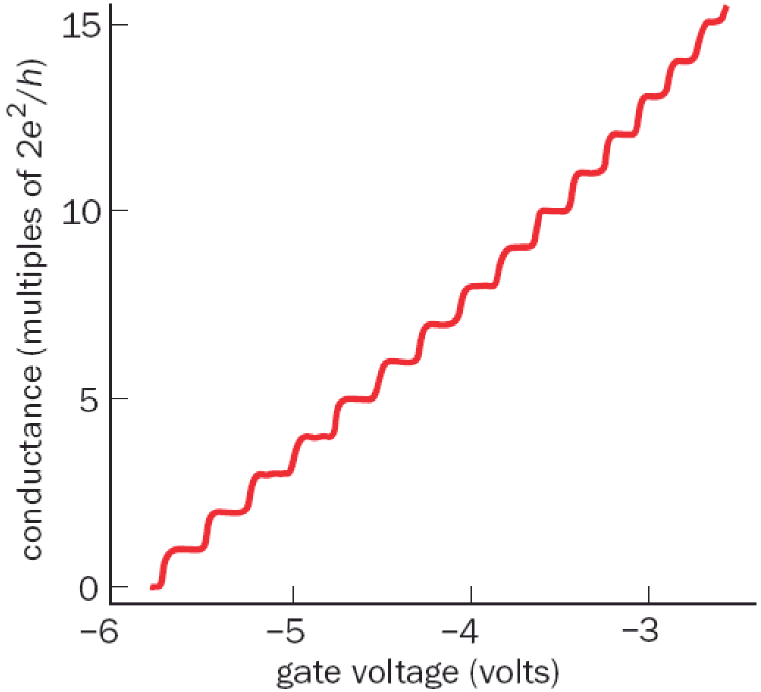
Theory: Thermal conductance of quantum wires

- Non-interacting wires, Wiedemann-Franz law;
- Classification of electron-electron scattering processes;
- Small corrections to thermal conductance in short wires;
- Strong suppression of thermal conductance in long wires.

Quantum wires

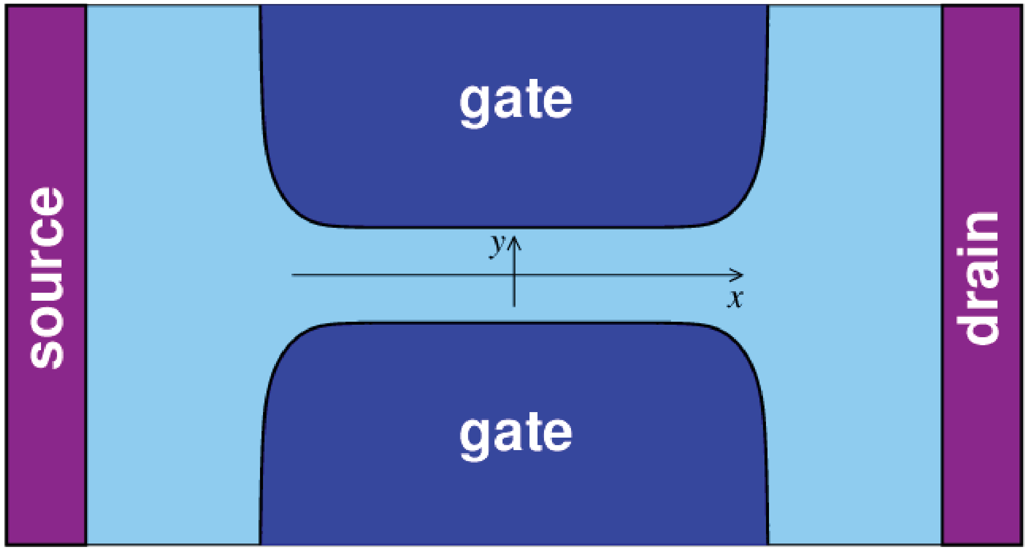
From Berggren & Pepper, 2002



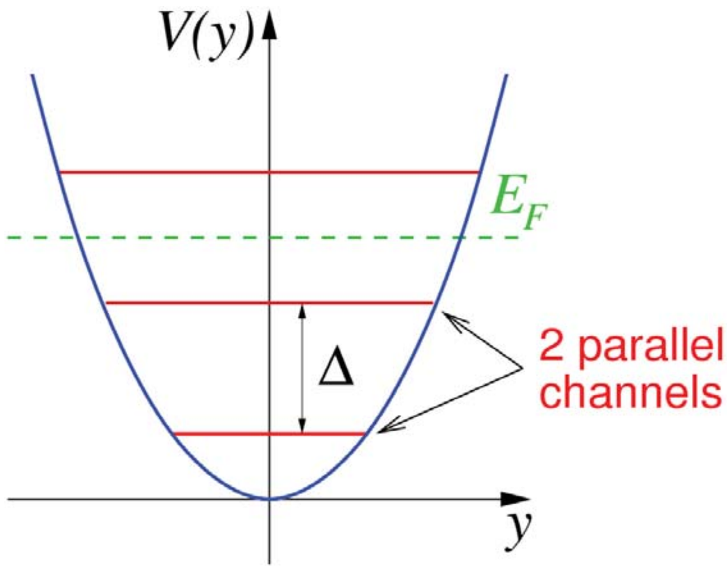


As a function of gate voltage conductance shows multiple steps of height $G_0 = \frac{2e^2}{h}$

Why steps?

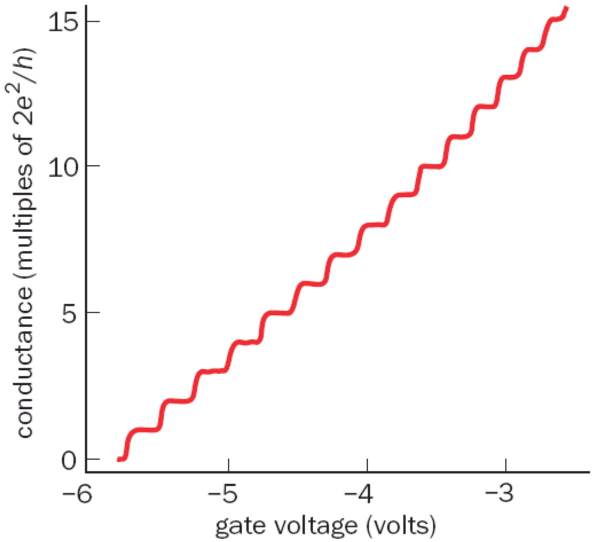


Top view

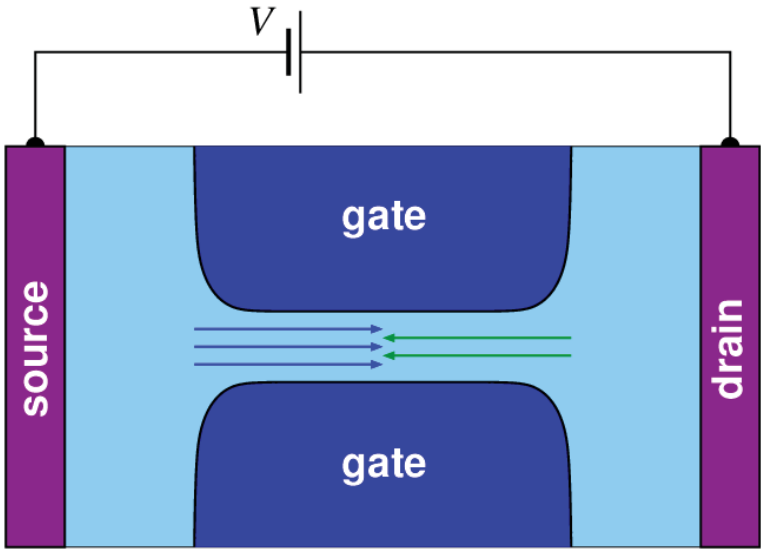


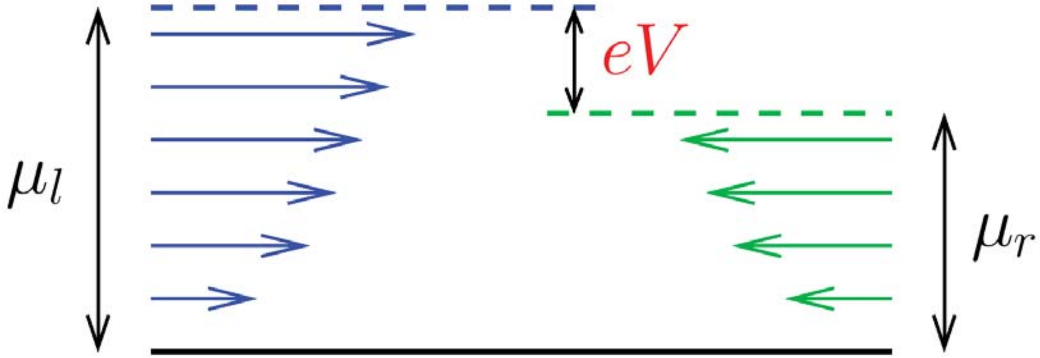
Electron motion across the channel

- Gate voltage changes the number of parallel channels in the wire
- Each channel has conductance $G_0 = 2e^2/h$



Conductance of a single channel





Current:

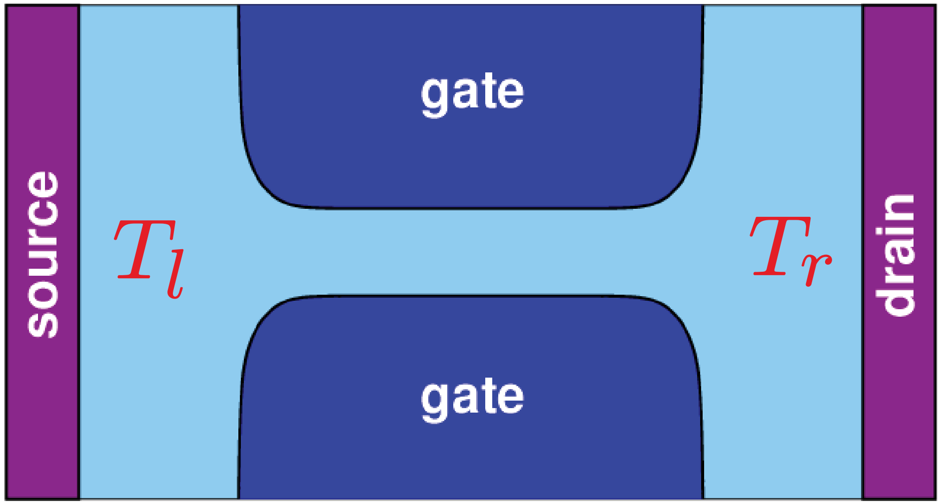
$$
\begin{aligned}
I &= 2e\int_0^\infty \frac{dp}{h} v_p[n_F(\epsilon_p-\mu_l)-n_F(\epsilon_p-\mu_r)] \\
&= \frac{2e}{h}(\mu_l-\mu_r)\int_0^\infty d\epsilon\left(-\frac{\partial n_F}{\partial\epsilon}\right) \\
&= \frac{2e^2}{h} V n_F(0)
\end{aligned}
$$

Conductance:

$$G = \frac{2e^2}{h}\frac{1}{1+e^{-\mu/T}}$$

$$G = \frac{2e^2}{h} \text{ at } \mu \gg T$$

Thermal conductance



$$T_l - T_r = \Delta T$$

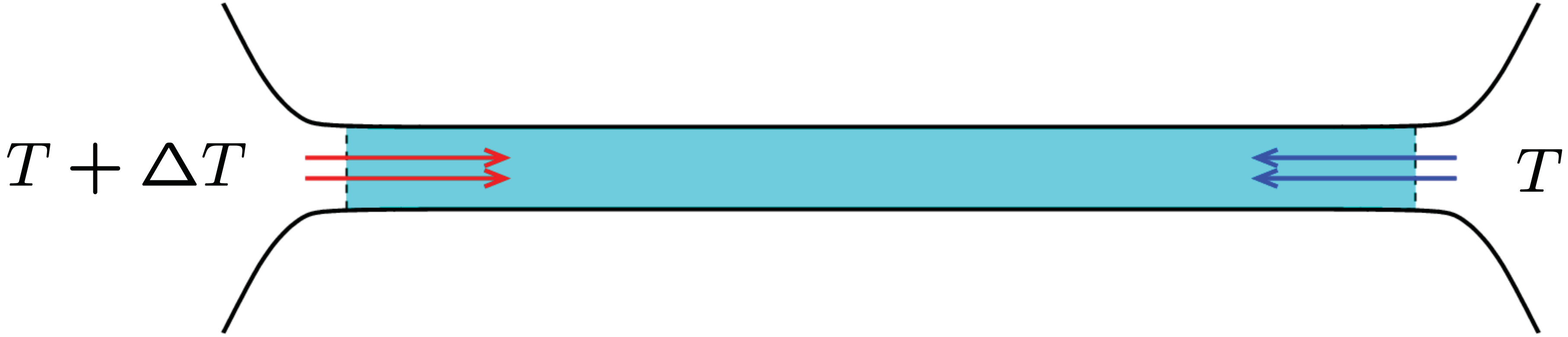
Heat current: $$j = 2\int_0^\infty \frac{dp}{h} v_p(\epsilon_p - \mu)\left[n_F\left(\frac{\epsilon_p-\mu}{T_l}\right) - n_F\left(\frac{\epsilon_p-\mu}{T_r}\right)\right]$$

$$= \frac{2\Delta T}{h}\int_{-\infty}^{\infty} d\xi\,\xi\left(-\frac{\partial n_F}{\partial T}\right) = \frac{2\pi^2 T}{3h}\Delta T$$

Thermal conductance: $K = \frac{2\pi^2 T}{3h}$ cf. $G = \frac{2e^2}{h}$

Wiedemann-Franz law: $K = \frac{\pi^2 T}{3e^2} G$

The problem

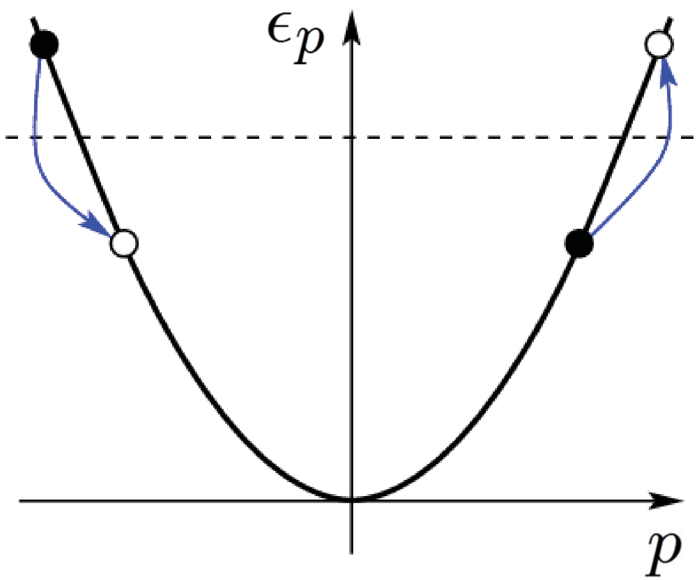


Uniform quantum wire:

- No disorder
- Interactions inside the wire
- No interactions in the leads

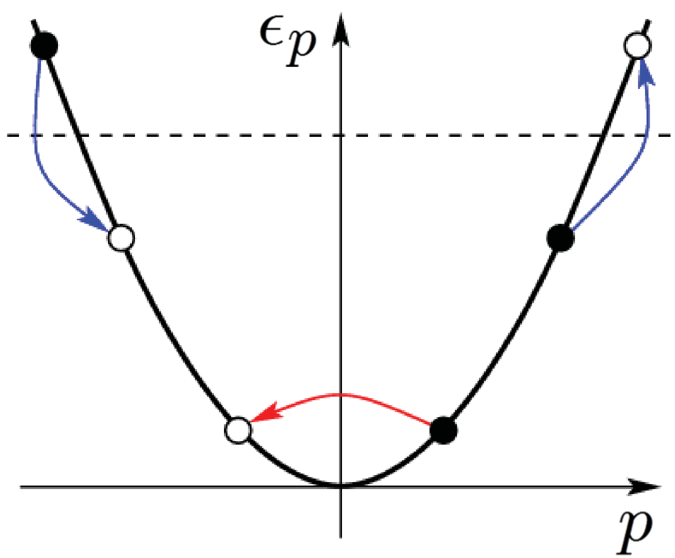
What is the thermal conductance?

Scattering of electrons in one dimension



Non-trivial two particle collisions are forbidden by the conservation laws

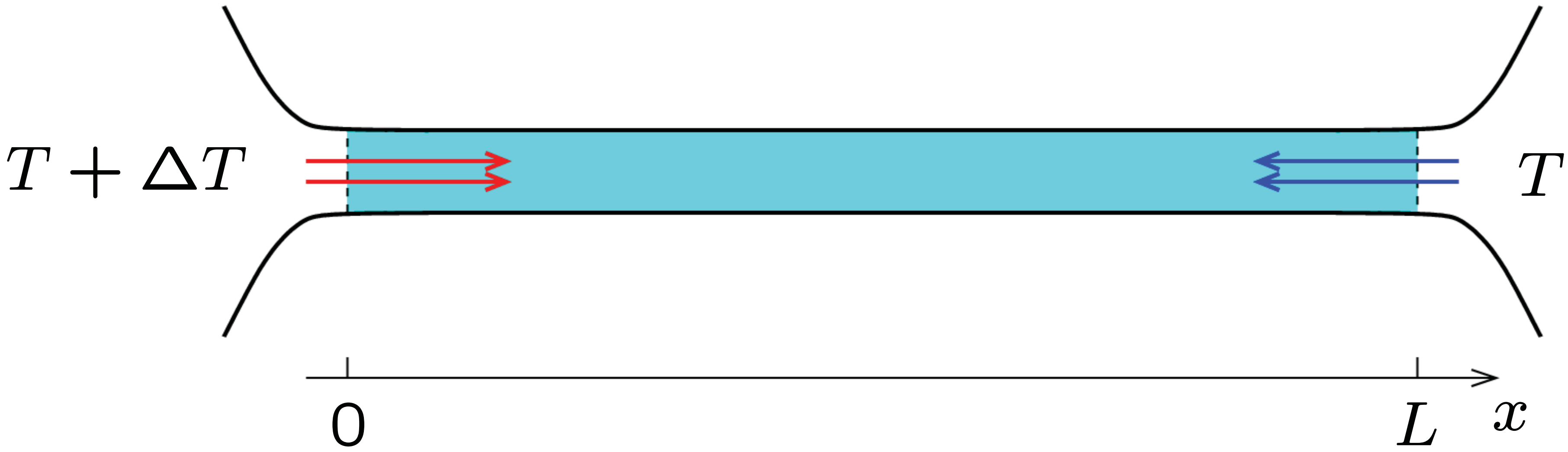
$$\Delta E = 0, \quad \Delta P > 0$$



Three particle collisions conserve both energy and momentum

$$\Delta E = 0, \quad \Delta P = 0$$

The Boltzmann equation



$$\frac{p}{m}\partial_x f(x,p) = I[f(p,x)]$$

$$f(0,p>0) = \frac{1}{e^{(\epsilon_p-\mu)/(T+\Delta T)}+1}$$

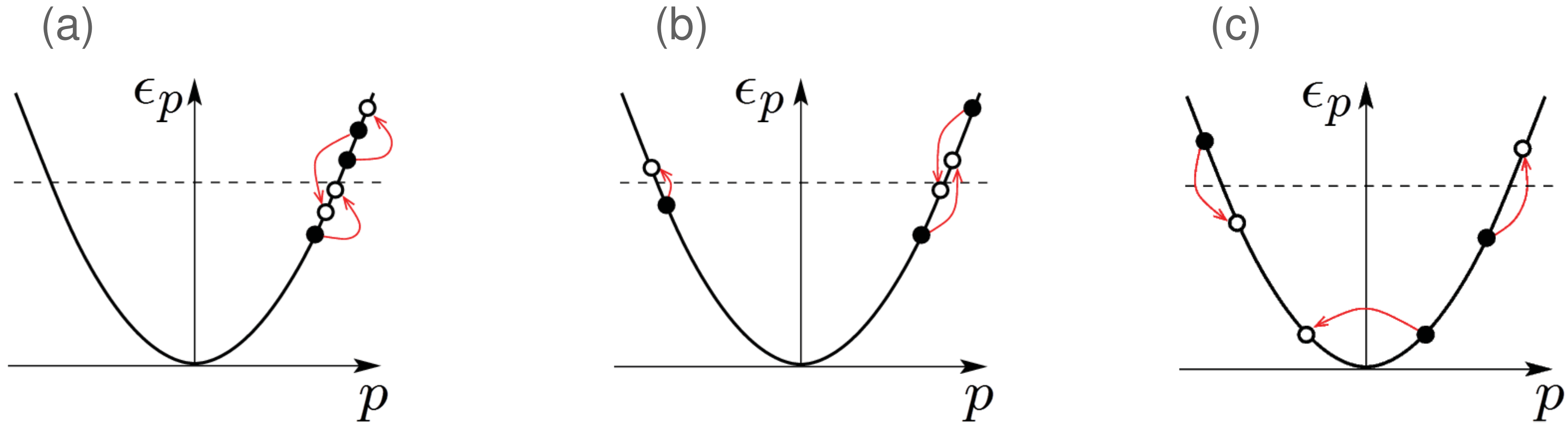
$$f(L,p<0) = \frac{1}{e^{(\epsilon_p-\mu)/T}+1}$$

Collision integral

$$I[f_1] = -\sum_{\substack{p_2,p_3\\ p_{1'},p_{2'},p_{3'}}} W_{123}^{1'2'3'}[f_1 f_2 f_3(1-f_{1'})(1-f_{2'})(1-f_{3'}) - f_{1'}f_{2'}f_{3'}(1-f_1)(1-f_2)(1-f_3)]$$

can be linearized in $\delta f(x,p) = f(x,p) - f^{(0)}(p)$ at $\Delta T \ll T$

Three types of scattering processes



fast — less fast — very slow

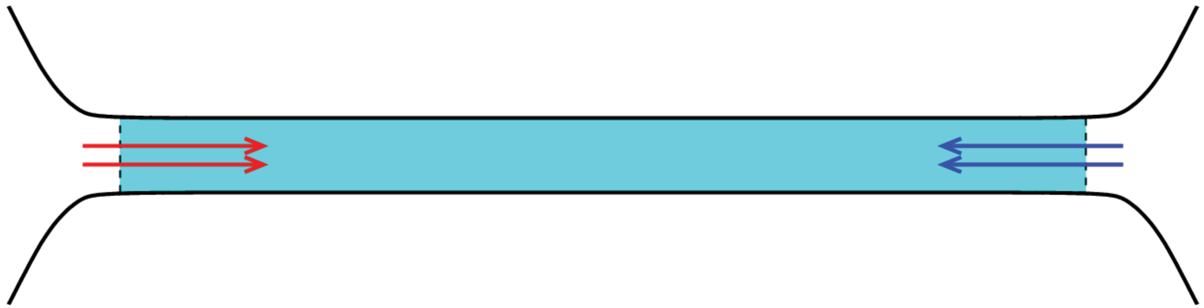
Scattering rates:

$$\tau_a^{-1} \propto T^{\alpha} \qquad \tau_b^{-1} \propto T^{\beta} \qquad \tau_c^{-1} \propto e^{-\mu/T}$$

These scattering processes relax the electron gas toward different equilibrium states

Equilibrium distribution of electrons

Gibbs distribution: $w_i \propto \exp\left(-\frac{1}{T}E_i + \frac{\mu}{T}N_i\right) \Rightarrow f_p = \frac{1}{\exp\left(\frac{\epsilon_p - \mu}{T}\right) + 1}$

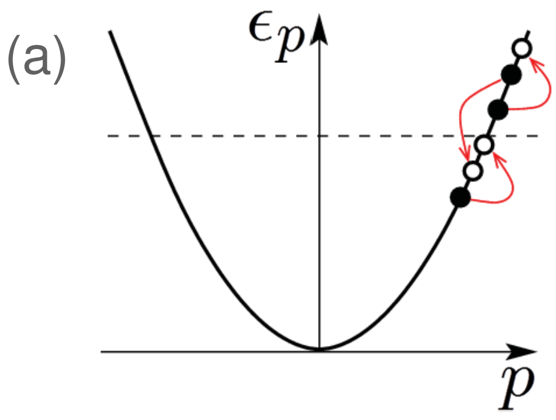


In a uniform wire momentum is conserved, and

$$w_i \propto \exp\left(-\frac{1}{T}E_i + \frac{u}{T}P_i + \frac{\mu}{T}N_i\right) \Rightarrow f_p = \frac{1}{\exp\left(\frac{\epsilon_p - up - \mu}{T}\right) + 1}$$

In general, all conserved quantities enter the exponent of the Gibbs distribution

Conserved quantities and the Fermi distribution (a)



$$N^R,\ N^L,\ P^R,\ P^L,\ E^R,\ E^L$$

Numbers, momenta, and energies of the left- and right-moving particles are conserved separately

Equilibrium distribution:

$$f_p = \frac{\theta(-p)}{e^{(\epsilon_p - u^L p - \mu^L)/T^L} + 1} + \frac{\theta(p)}{e^{(\epsilon_p - u^R p - \mu^R)/T^R} + 1}$$

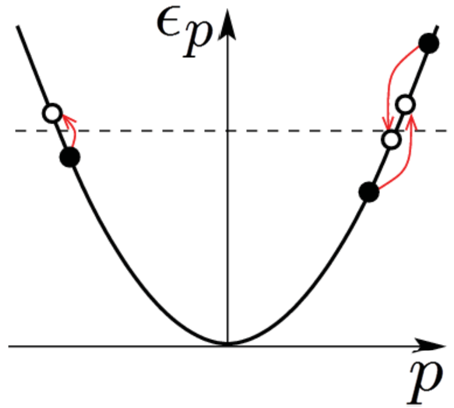
The distribution supplied by the leads belongs to this class:

$$f_p = \frac{\theta(-p)}{e^{(\epsilon_p - \mu)/T} + 1} + \frac{\theta(p)}{e^{(\epsilon_p - \mu)/(T+\Delta T)} + 1}$$

No effect on thermal transport

Conserved quantities and the Fermi distribution (b)

(b)



$$N^R,\ N^L,\ P = P^R + P^L,\ E = E^R + E^L$$

Numbers of the left- and right-moving particles are conserved separately, in addition to the total momentum and energy

Equilibrium distribution:

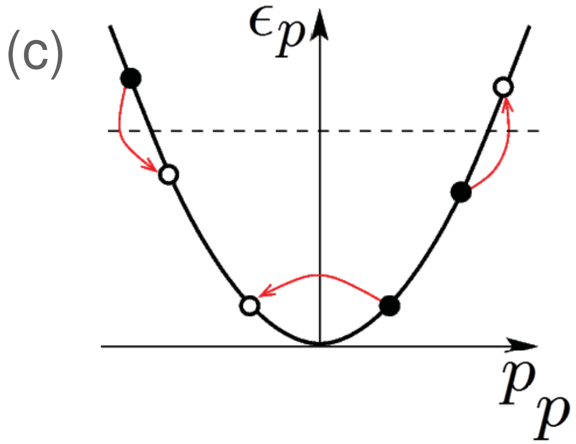
$$f_p = \frac{\theta(-p)}{e^{(\epsilon_p - up - \mu^L)/T} + 1} + \frac{\theta(p)}{e^{(\epsilon_p - up - \mu^R)/T} + 1}$$

The distribution supplied by the leads does not belong to this class:

$$f_p = \frac{\theta(-p)}{e^{(\epsilon_p - \mu)/T} + 1} + \frac{\theta(p)}{e^{(\epsilon_p - \mu)/(T+\Delta T)} + 1}$$

Type (b) processes affect the electron distribution and thermal transport

Conserved quantities and the Fermi distribution (c)

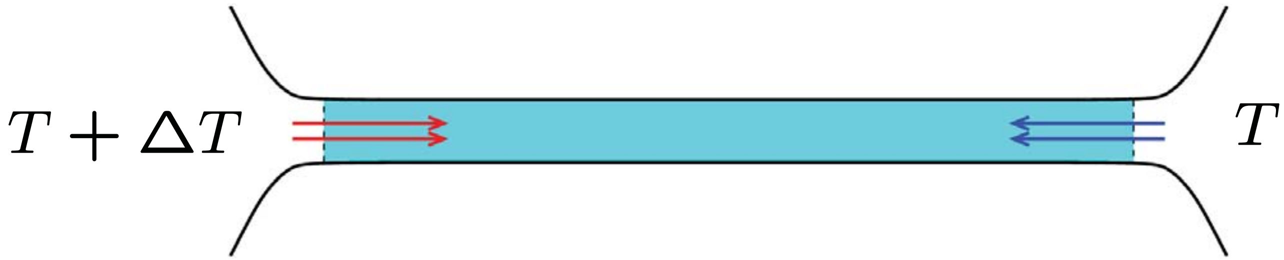


$$N,\ P,\ E$$

Only the total number of particles, momentum, and energy are conserved

Equilibrium distribution:

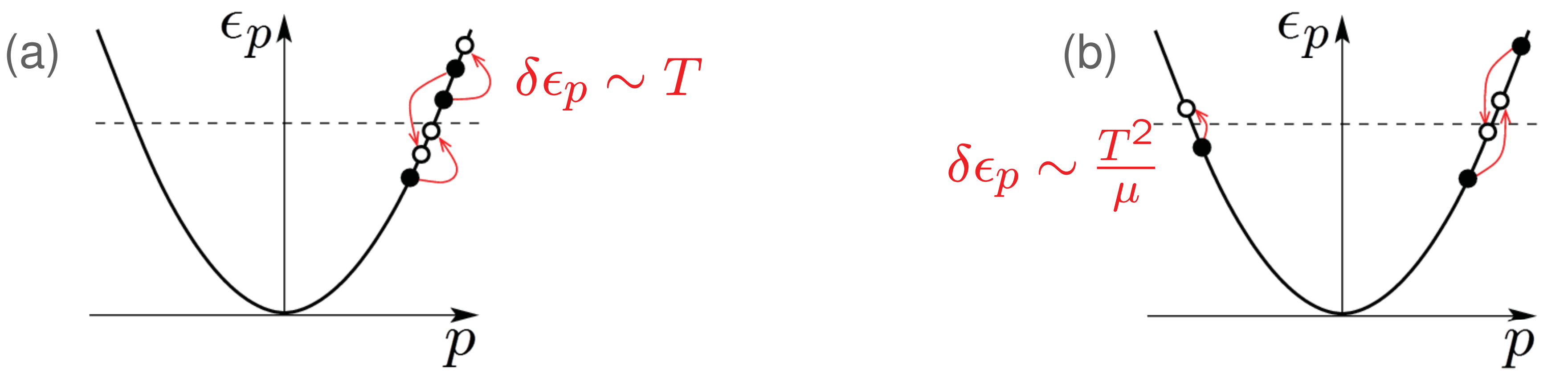
$$f_p = \frac{1}{e^{(\epsilon_p - up - \mu)/T} + 1}$$



In wires longer than $l_c \propto e^{\mu/T}$ type (c) processes ensure complete equilibration of the electron distribution and **dramatically affect thermal transport**

Equilibration lengths

In a long wire electron distribution reaches the equilibrium form corresponding to the dominant scattering process



Equilibration due to type (a) processes is more efficient: $l_b \sim \frac{\mu}{T} l_a \gg l_a$

In wires shorter than l_b, type (b) processes can be accounted for perturbatively

At $L \sim l_b \gg l_a$ the distribution function has the form

$$f_p = \frac{\theta(-p)}{e^{(\epsilon_p - u^L p - \mu^L)/T^L} + 1} + \frac{\theta(p)}{e^{(\epsilon_p - u^R p - \mu^R)/T^R} + 1}$$

The 6 parameters $u^L, u^R, \mu^L, \mu^R, T^L, T^R$ depend on position x

Simplification of the Boltzmann equation

$$\frac{p}{m}\partial_x f(x,p) = I[f(p,x)]$$

Integro-differential equation: $I[f] = \iiint W \ldots$

6 ordinary differential equations:

$$\partial_x \mu^{R(L)} = \mp p_F \left(1 - \frac{\pi^2 T^2}{24\mu^2} - \frac{7\pi^4 T^4}{384\mu^4}\right) \partial_x u^{R(L)}$$

$$\partial_x (T^R \pm T^L) = -\frac{T}{v_F}\left(1 + (25 \pm 4)\frac{\pi^2 T^2}{120\mu^2}\right) \partial_x (u^R \mp u^L)$$

4 conservation laws

$N^R,\ N^L,\ P,\ E$

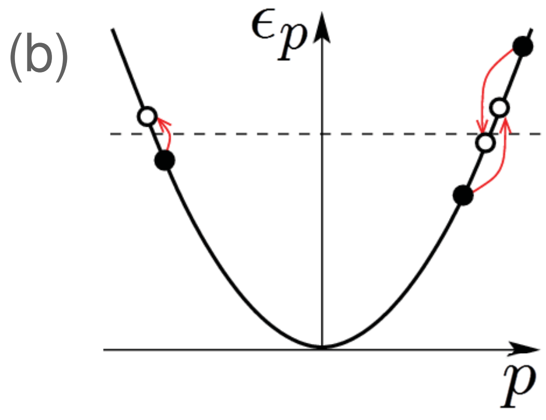
$$\partial_x (T^R - T^L) = -\frac{1}{l_b}\frac{T}{v_F}(u^R - u^L)$$

$$\partial_x (u^R - u^L) = -\frac{1}{l_b}\frac{v_F}{T}(T^R - T^L)$$

Easily solvable!

Correction to the thermal conductance

$$K = K_0 \left[1 - \frac{\pi^2 T^2}{30\mu^2} \left(1 - e^{-L/l_b} \right) \right]$$



The correction is due to the curvature of electron spectrum near the Fermi level. Even in long wires it remains small as $(T/\mu)^2$

The equilibration length l_b is model-specific. For unscreened Coulomb interactions we found

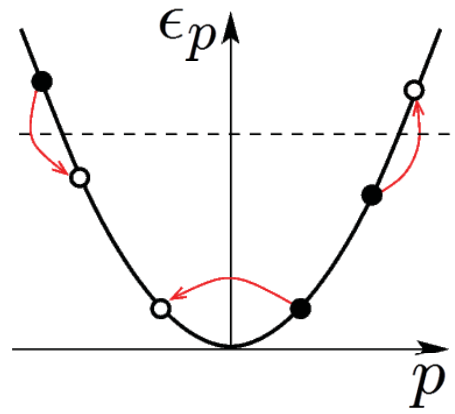
without spins $\qquad l_b^{-1} \propto (T/\mu)^3$

with spins $\qquad l_b^{-1} \propto (T/\mu)\ln^2(\mu/T)$

[Levchenko, Micklitz, Ristivojevic & KM, PRB **84**, 115447 (2011)]

Thermal conductance of a long wire

$$l_{a,b} \ll L \sim l_c \propto e^{\mu/T}$$

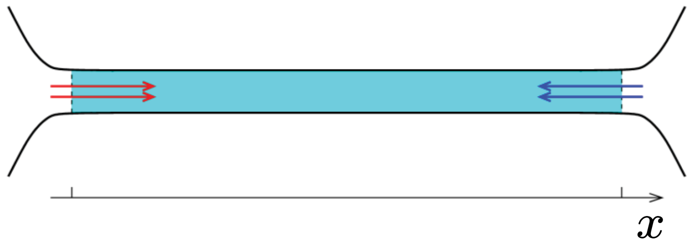


Electron distribution:

$$f_p = \frac{\theta(-p)}{e^{(\epsilon_p - up - \mu^L)/T} + 1} + \frac{\theta(p)}{e^{(\epsilon_p - up - \mu^R)/T} + 1}$$

Heat current

$$\begin{aligned} j &= 2\int_0^\infty \frac{dp}{h} v_p(\epsilon_p - \mu)\left[n_F(\epsilon_p - up - \mu^R) - n_F(\epsilon_p + up - \mu^L)\right] \\ &= \frac{2}{h}\int d\xi\, \xi\, n_F'(\xi)[-2up - \mu^R + \mu^L] \end{aligned}$$



depends only on the velocity u

$$j = \frac{2\pi}{3\hbar}\frac{T^2}{v_F}u \quad \Rightarrow \quad u = \text{const} = ?$$

Energy conservation

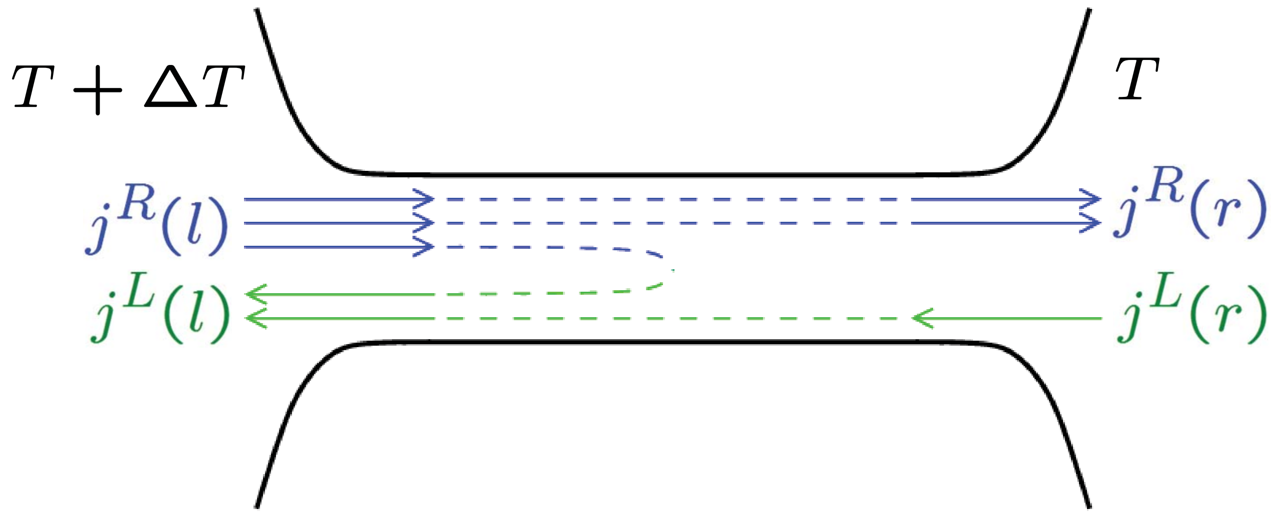
- Consider the energy current of right-moving electrons

$$j^R(r) = j^R(l) + \dot{E}^R$$

- The total energy current

$$j = j^R(r) + j^L(r)$$

- Exclude the outgoing current $j^R(r)$



$$j = j^R(l) + j^L(r) + \dot{E}^R$$

- Substitute the known expression for the incoming currents

$$j = K_0 \Delta T + \dot{E}^R$$

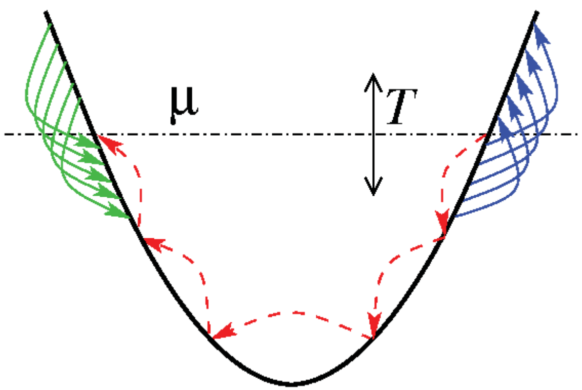
Momentum conservation

Momentum of the electron gas

$$f_p = \frac{\theta(-p)}{e^{(\epsilon_p - up - \mu^L)/T} + 1} + \frac{\theta(p)}{e^{(\epsilon_p - up - \mu^R)/T} + 1}$$

$$P = p_F(N^R - N^L) + \frac{2\pi}{3\hbar}\frac{T^2}{v_F^3} Lu$$

The total momentum is conserved, $\dot{P} = 0$ ⇒ $$\dot{N}^R = -\frac{\pi}{3\hbar}\frac{T^2}{v_F^3 p_F} L\dot{u}$$



Momentum change: $$\Delta p^L + \Delta p^R = 2p_F$$

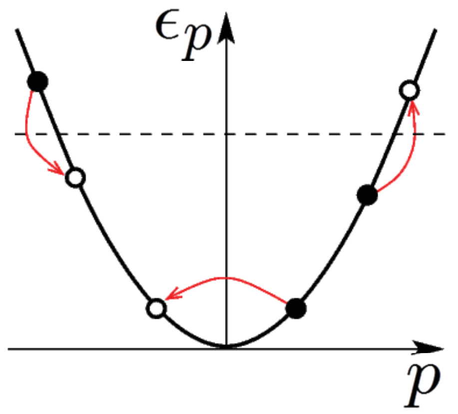
Energy change: $$-v_F \Delta p^L + v_F \Delta p^R = 0$$

$$\Delta p^L = \Delta p^R = p_F$$

$$\Delta E^R = -v_F p_F \Delta N^R$$

Rate of energy change due to backscattering: $$\dot{E}^R = \frac{\pi}{3\hbar}\frac{T^2}{v_F^2} L\dot{u}$$

Relaxation towards complete equilibrium



$$f_p = \frac{\theta(-p)}{e^{(\epsilon_p - up - \mu^L)/T} + 1} + \frac{\theta(p)}{e^{(\epsilon_p - up - \mu^R)/T} + 1}$$

In a system without net electric current the electron distribution relaxes towards the usual Fermi function

$$\mu^L - \mu^R \to 0, \quad u \to 0$$

Standard relaxation law: $\dot{u} = -\frac{u}{\tau_c}, \quad \tau_c^{-1} \propto e^{-E_F/T}$

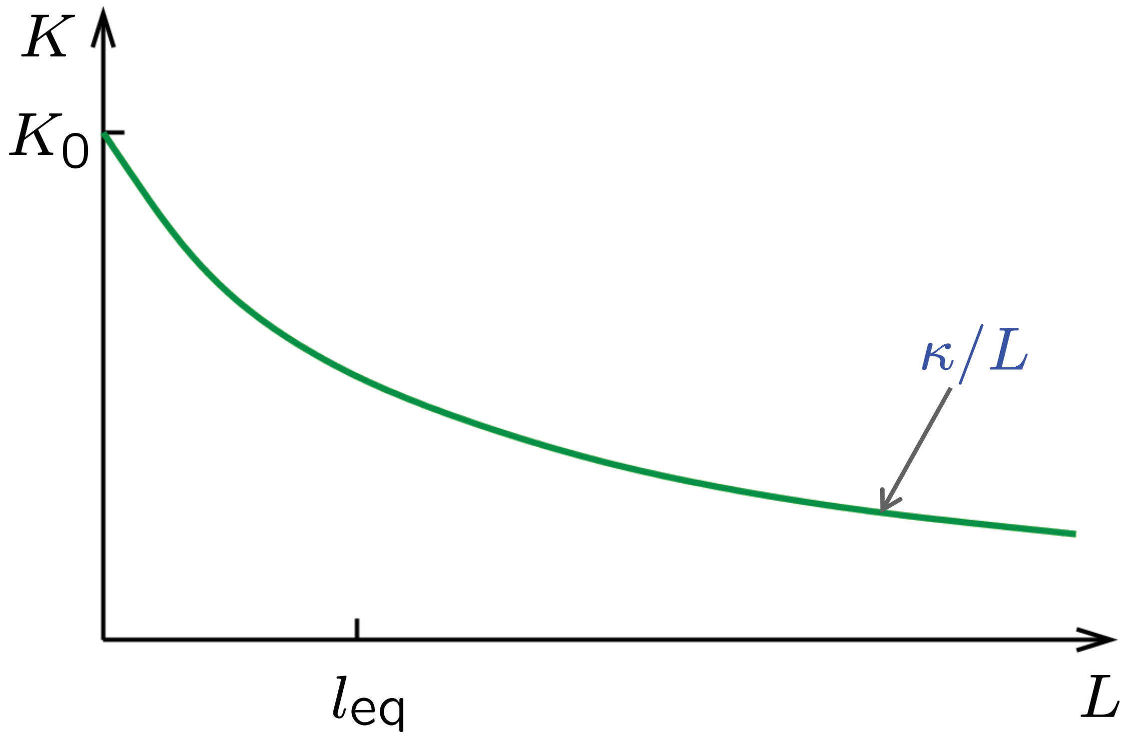
$$\dot{E}^R = -\frac{\pi}{3\hbar} \frac{T^2}{v_F^2} L \frac{u}{\tau_c}$$

Combining with $j = K_0 \Delta T + \dot{E}^R$ and $j = \frac{2\pi}{3\hbar} \frac{T^2}{v_F} u$ we find

$$j = \frac{K_0 \Delta T}{1 + L/l_{\text{eq}}}, \quad l_{\text{eq}} = 2 v_F \tau_c$$

Result for the thermal conductance

$$K = \frac{K_0}{1 + L/l_{\mathrm{eq}}}$$



- Old result $K = K_0$ in a short wire, $L \ll l_{eq}$.
- Small thermal conductance, but finite thermal conductivity in a long wire, $L \gg l_{eq}$.

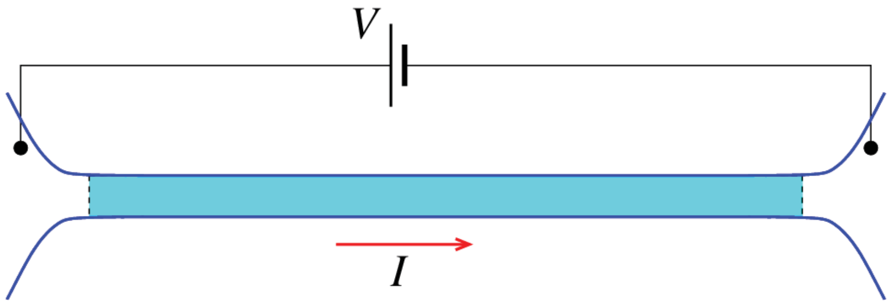
$$K = \frac{\kappa}{L}, \qquad \kappa = K_0\, l_{\mathrm{eq}}$$

[Micklitz, Rech & KM, PRB **81**, 115313 (2010)]

Electrical vs. thermal conductance

In long wires the electrical conductance saturates

$$G \to \frac{2e^2}{h}\left[1 - \frac{\pi^2}{3}\left(\frac{T}{v_F p_F}\right)^2\right] \simeq \frac{2e^2}{h}$$



Momentum conservation results in infinite conductivity

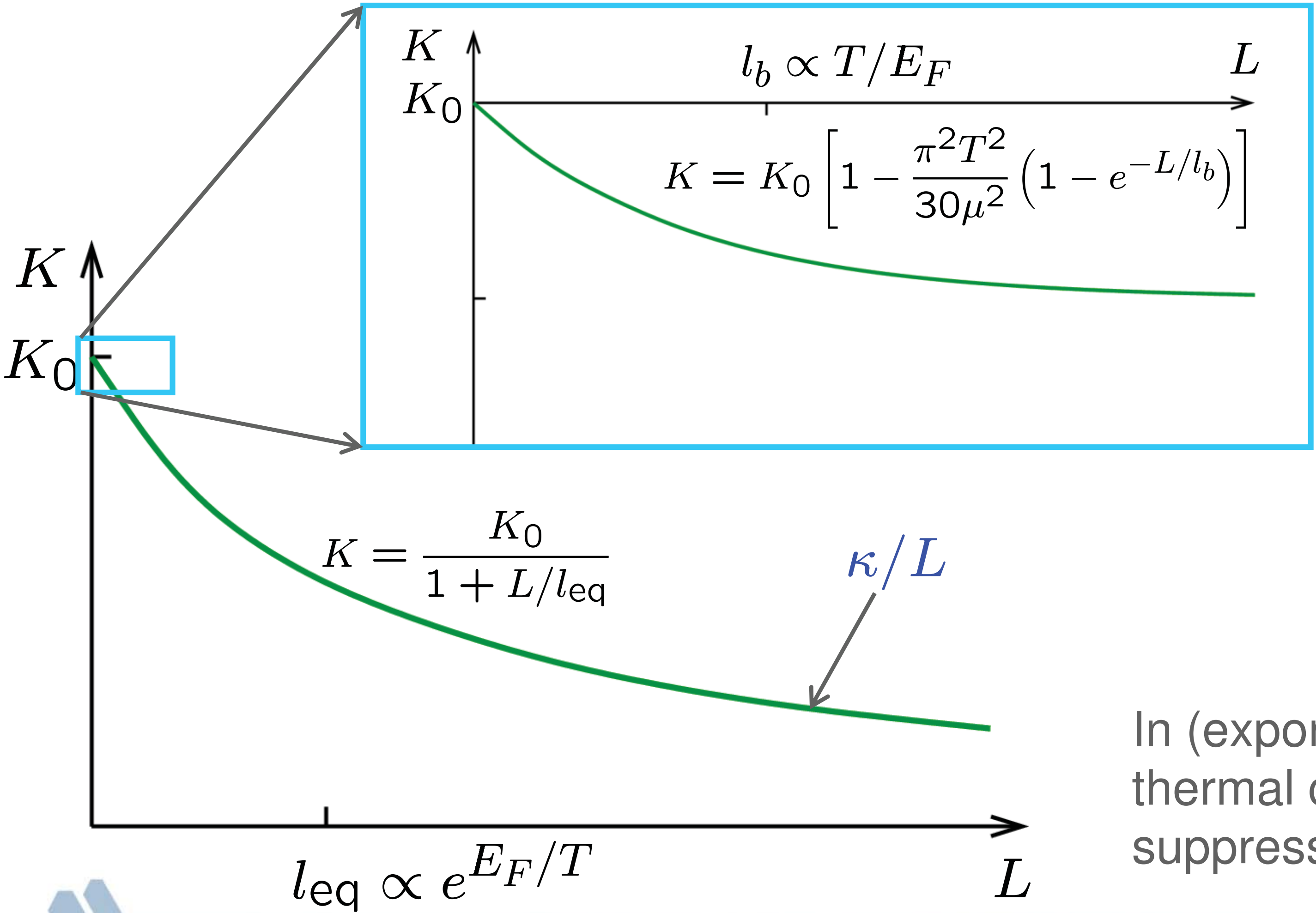
But the thermal conductance is small: $K = \frac{\kappa}{L}$

Strong violation of the Wiedemann-Franz law: $K \ll \frac{\pi^2 T}{3e^2} G$

Thermoelectric figure of merit: $ZT = \frac{G S^2 T}{K} \propto L \to \infty$

Summary

Electron-electron scattering reduces thermal conductance of quantum wires



Small reduction $\sim T^2$ of thermal conductance in short wires

In (exponentially) long wires the thermal conductance is strongly suppressed